Beta decay rates of neutron-rich nuclei

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Physics at SPES with non re-accelerated beams
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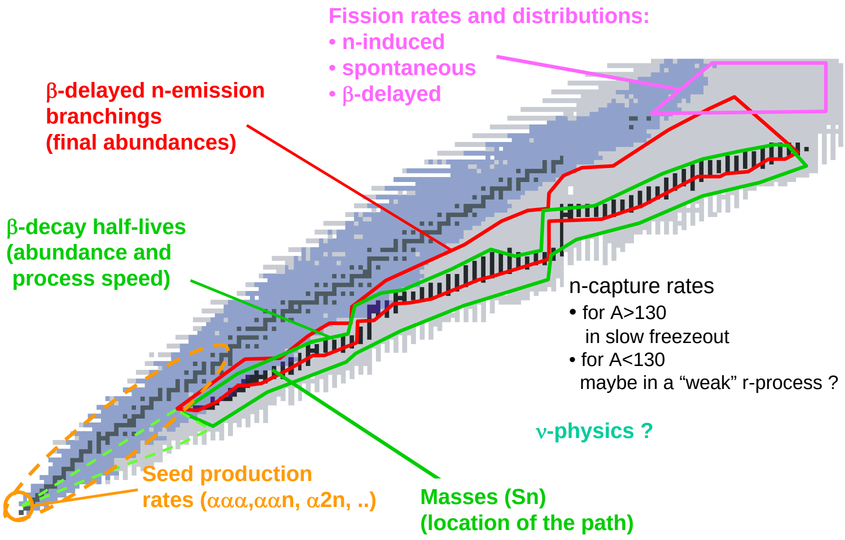
Introduction



Transitions are obtained by solving the pn-RQRPA equations

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^\lambda \\ Y^\lambda \end{pmatrix} = E_\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^\lambda \\ Y^\lambda \end{pmatrix}$$

Residual interaction is derived from the Lagrangian density

$$\mathcal{L}_{\rho+\pi} = -g_\rho \bar{\psi} \gamma_\mu \vec{\rho}^\mu \vec{\tau} \psi - \frac{f_\pi}{m_\pi} \bar{\psi} \gamma_5 \gamma^\mu \partial_\mu \vec{\pi} \vec{\tau} \psi$$

Total strength of a particular transition

$$B_{\lambda,J}(GT) = \left| \sum_{pn} \left\langle p \left\| \hat{O}_J \right\| n \right\rangle \left(X_{pn}^{\lambda,J} u_p v_n - Y_{pn}^{\lambda,J} v_p u_n \right) \right|^2$$

Decay rate:

$$\lambda_i = D \int_1^{W_{0,i}} W\sqrt{W^2-1}\,\left(W_{0,i}-W\right)^2 F(Z,W)C(W)dW$$

$$T_{1/2} = \frac{\ln 2}{\lambda}, \qquad D = \frac{(G_F V_{ud})^2}{2\pi^3}\frac{\left(m_e c^2\right)^5}{\hbar}$$

Allowed decays shape factor:

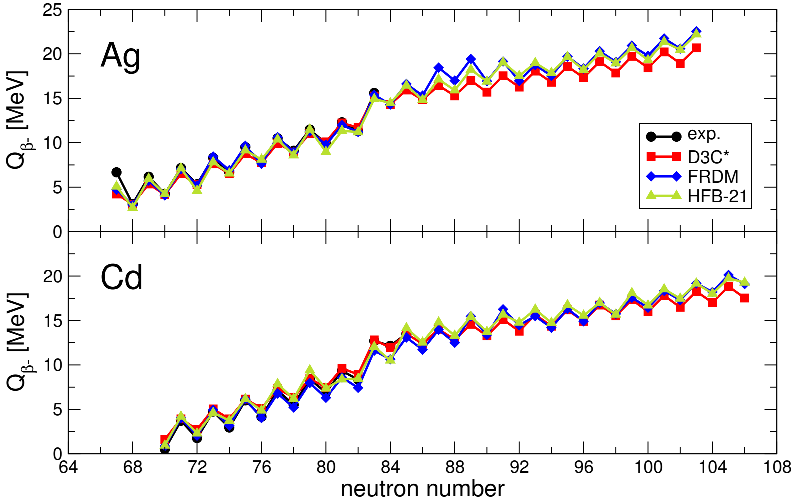
$$C(W) = B(GT)$$

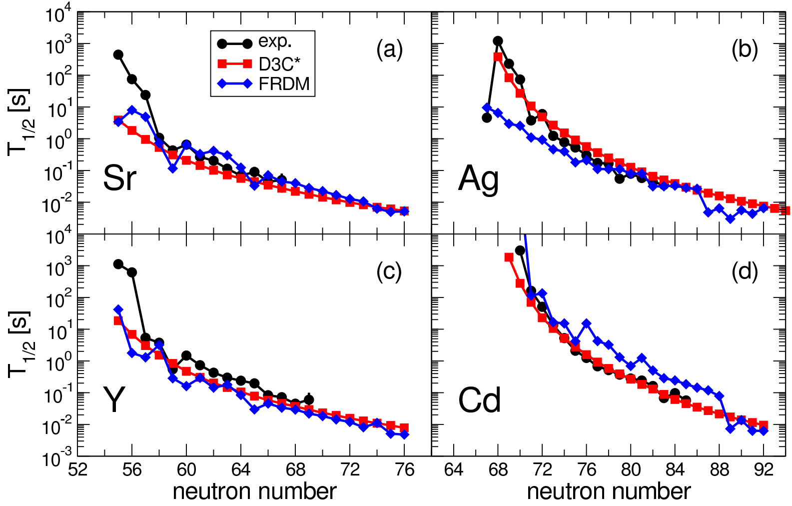
First-forbidden decays shape factor:

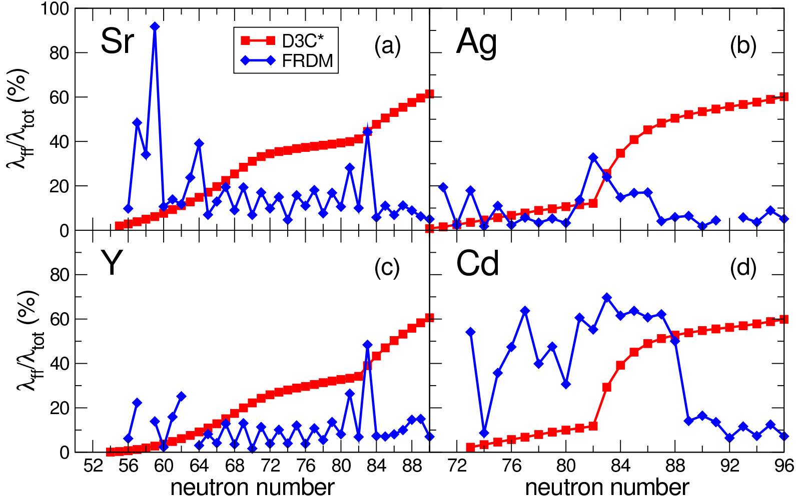
$$C(W) = k\left(1 + aW + bW^{-1} + cW^2\right)$$

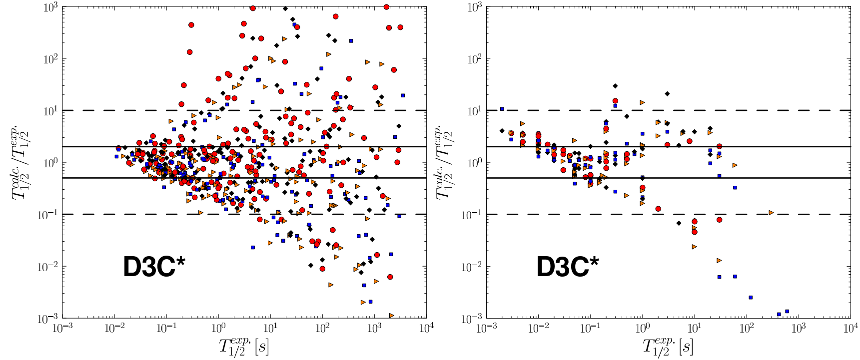
$$
\begin{aligned}
k &= \left[\zeta_0^2 + \frac{1}{9}w^2\right]_{(0)} + \left[\zeta_1^2 + \frac{1}{9}(x+u)^2 - \frac{4}{9}\mu_1\gamma_1 u(x+u)\right. \\
&\quad \left. + \frac{1}{18}W_0^2(2x+u)^2 - \frac{1}{18}\lambda_2(2x-u)^2\right]_{(1)} \\
&\quad + \left[\frac{1}{12}z^2\left(W_0^2 - \lambda_2\right)\right]_{(2)} \\
ka &= \left[-\frac{4}{3}uY - \frac{1}{9}W_0\left(4x^2+5u^2\right)\right]_{(1)} - \left[\frac{1}{6}W_0 z^2\right]_{(2)} \\
kb &= \frac{2}{3}\mu_1\gamma_1\left\{-\left[\zeta_0 w\right]_{(0)} + \left[\zeta_1(x+u)\right]_{(1)}\right\} \\
kc &= \frac{1}{18}\left[8u^2 + (2x+u)^2 + \lambda_2(2x-u)^2\right]_{(1)} \\
&\quad + \frac{1}{12}\left[\left(1+\lambda_2\right)z^2\right]_{(2)}
\end{aligned}
$$

$$
\begin{aligned}
w &= -g_A\sqrt{3}\frac{\left\langle f \middle\| \sum_k r_k \left[\boldsymbol{C}_1^k \otimes \boldsymbol{\sigma}^k\right]^0 \boldsymbol{t}_-^k \middle\| i \right\rangle}{\sqrt{2J_i+1}}, \\
x &= -\frac{\left\langle f \middle\| \sum_k r_k \boldsymbol{C}_1^k \boldsymbol{t}_-^k \middle\| i \right\rangle}{\sqrt{2J_i+1}}, \\
u &= -g_A\sqrt{2}\frac{\left\langle f \middle\| \sum_k r_k \left[\boldsymbol{C}_1^k \otimes \boldsymbol{\sigma}^k\right]^1 \boldsymbol{t}_-^k \middle\| i \right\rangle}{\sqrt{2J_i+1}}, \\
z &= 2g_A\frac{\left\langle f \middle\| \sum_k r_k \left[\boldsymbol{C}_1^k \otimes \boldsymbol{\sigma}^k\right]^2 \boldsymbol{t}_-^k \middle\| i \right\rangle}{\sqrt{2J_i+1}}, \\
w' &= -g_A\frac{2}{\sqrt{3}}\frac{\left\langle f \middle\| \sum_k r_k I(1,1,1,1,r_k) \left[\boldsymbol{C}_1^k \otimes \boldsymbol{\sigma}^k\right]^0 \boldsymbol{t}_-^k \middle\| i \right\rangle}{\sqrt{2J_i+1}}, \\
x' &= -\frac{2}{3}\frac{\left\langle f \middle\| \sum_k r_k I(1,1,1,1,r_k) \boldsymbol{C}_1^k \boldsymbol{t}_-^k \middle\| i \right\rangle}{\sqrt{2J_i+1}}, \\
u' &= -g_A\frac{2\sqrt{2}}{3}\frac{\left\langle f \middle\| \sum_k r_k I(1,1,1,1,r_k) \left[\boldsymbol{C}_1^k \otimes \boldsymbol{\sigma}^k\right]^1 \boldsymbol{t}_-^k \middle\| i \right\rangle}{\sqrt{2J_i+1}}.
\end{aligned}
$$


Ag
Cd
exp.
D3C*
FRDM
HFB-21
$Q_{\beta-}$ [MeV]
neutron number
0
5
10
15
20
25
64
68
72
76
80
84
88
92
96
100
104
108


exp.
D3C*
FRDM
(a)
(b)
(c)
(d)
Sr
Ag
Y
Cd
$T_{1/2}$ [s]
$T_{1/2}$ [s]
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
52
56
60
64
68
72
76
80
84
88
92
neutron number
neutron number


Sr
(a)
D3C*
FRDM
Ag
(b)
Y
(c)
Cd
(d)
λff/λtot (%)
neutron number
100
80
60
40
20
0
52 56 60 64 68 72 76 80 84 88
72 76 80 84 88 92 96

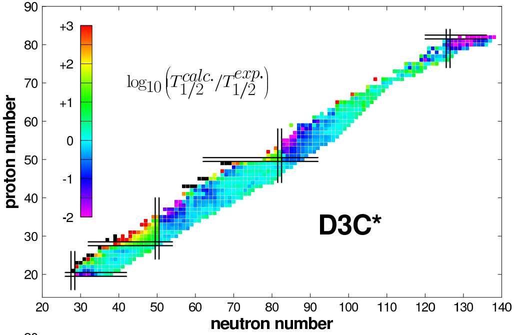


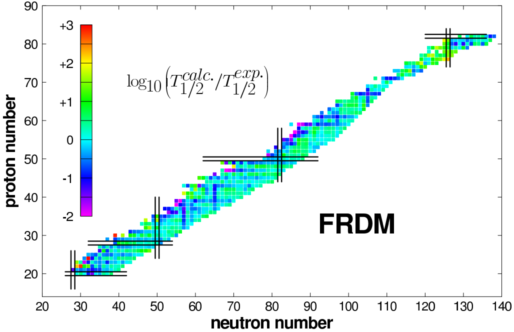
$$\bar{r} = \frac{1}{N}\sum_i \log\frac{T_{\text{th.}}}{T_{\text{exp.}}}$$

$$\sigma = \left[\frac{1}{N}\sum_i (r_i - \bar{r})^2\right]^{1/2}$$

	D3C*		FRDM	
$T_{\text{exp.}}$ [s]	$\bar{r}$	σ	$\bar{r}$	σ
< 1000	0.011	0.889	0.021	0.660
< 100	0.057	0.791	0.040	0.580
< 10	0.061	0.645	0.046	0.515
< 1	0.011	0.436	0.019	0.409
< 0.1	0.041	0.195	0.021	0.354

G. Audi *et al.*, CPC 36, 1157 (2012)



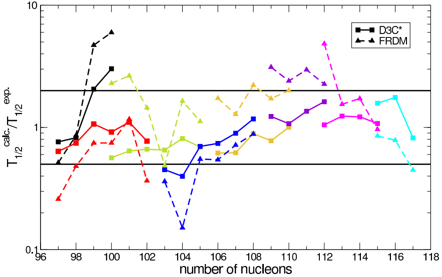


D3C*

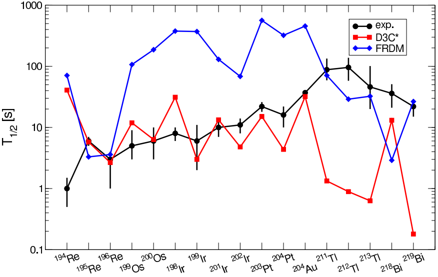
	$\bar{r}$	σ
even-even	-0.037	0.331
odd-Z	0.054	0.328
odd-N	-0.086	0.387
odd-odd	0.089	0.582
total	0.011	0.436

FRDM

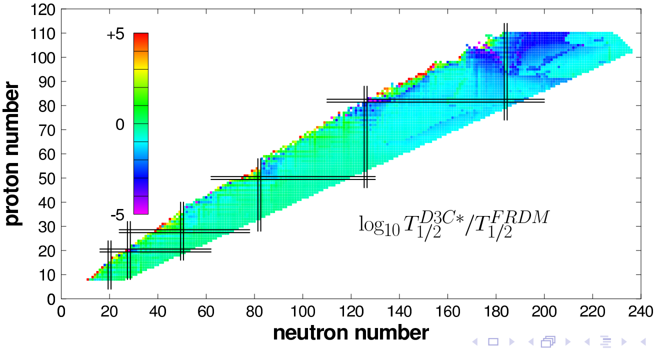
	$\bar{r}$	σ
even-even	0.333	0.226
odd-Z	-0.128	0.288
odd-N	0.124	0.436
odd-odd	-0.179	0.409
total	0.019	0.409



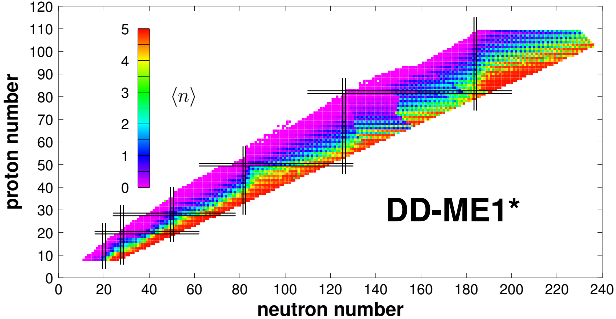
D3C*
FRDM
number of nucleons



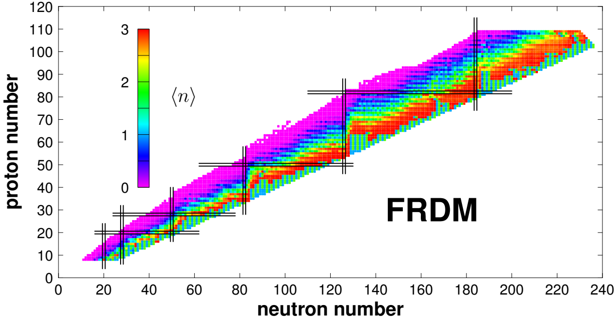
exp.
D3C*
FRDM



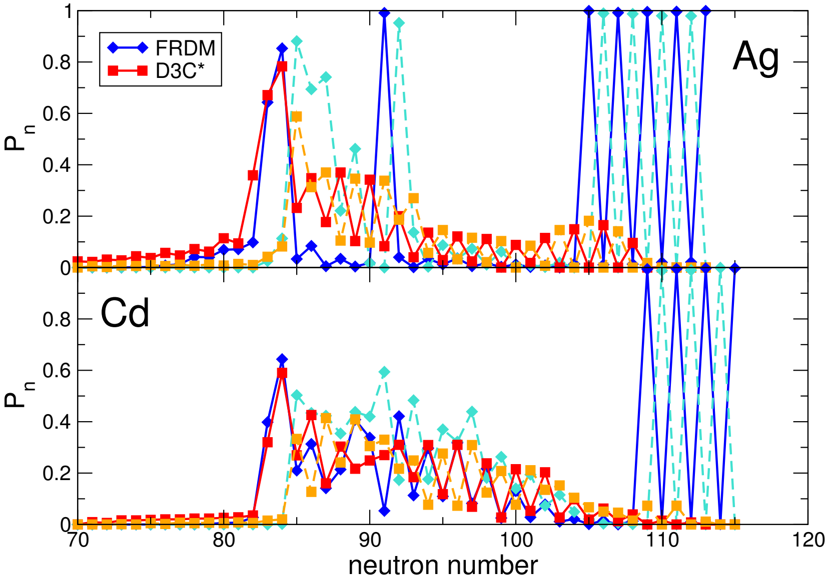
proton number
neutron number



$$P_{xn} = \frac{1}{\lambda_{tot}} \sum_{E_i=S_{xn}}^{S_{(x+1)n}} \lambda_i$$



$$\langle n \rangle = \sum_i i P_{in}$$


FRDM
D3C*
Ag
Cd
P
n
1
0.8
0.6
0.4
0.2
0
70
80
90
100
110
120
neutron number

Evaluation of reactor antineutrino spectra

In reactors, 99% of the electrons come from decay of fission products of 4 nuclei.

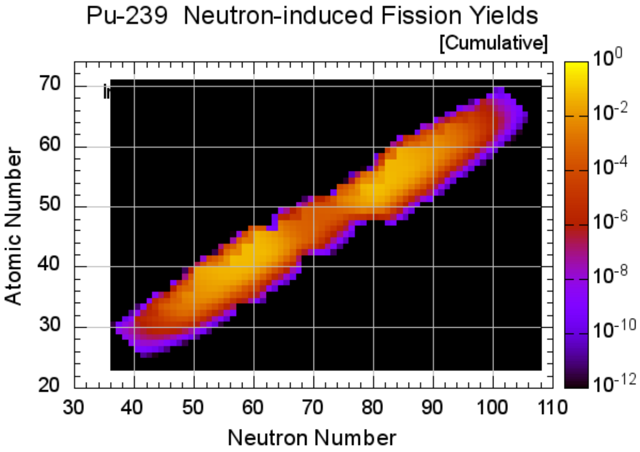
$$S_{tot}(E) = \sum_{k={}^{235}U,{}^{238}U,{}^{239}Pu,{}^{241}Pu} \alpha_k S_k(E),$$

- α_k - number of fissions at considered time
- $S_k(E)$ - β spectrum normalized to one fission
- E - kinetic energy of emitted electrons

Electrons (and antineutrinos) come from the β-decay of resulting fission fragments.

$$S_k(E) = \sum_{f=1}^{N_f} Y_f S_f(E)$$

$$S_f(E) = \sum_{i=i}^{N_t} \frac{\lambda_i}{\lambda_{tot}} S_f^i(Z, A, E_{max}, E).$$



JAEA Nuclear Data Center

For allowed transitions the spectrum reads

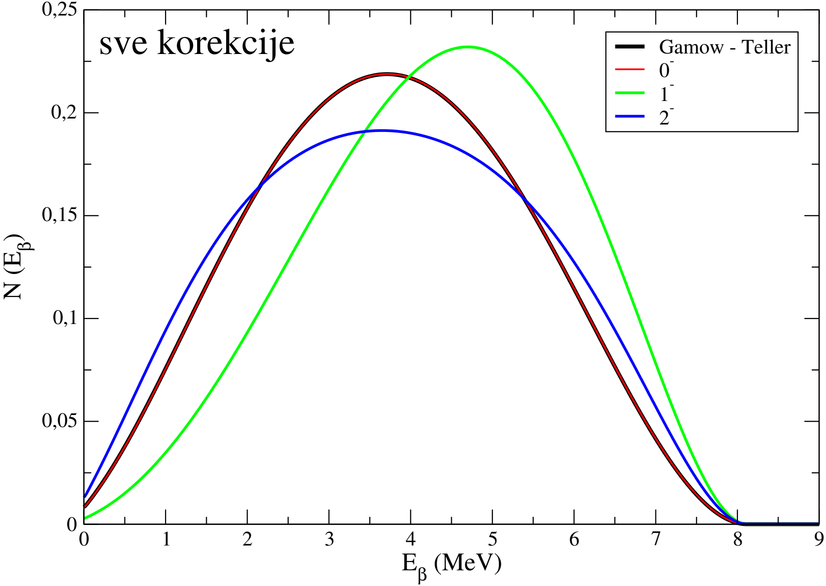
$$S_f^i = F(Z, A, E) \cdot pE(E - E_{max})^2 \cdot L_0(Z, E) \cdot C'(Z, E)$$

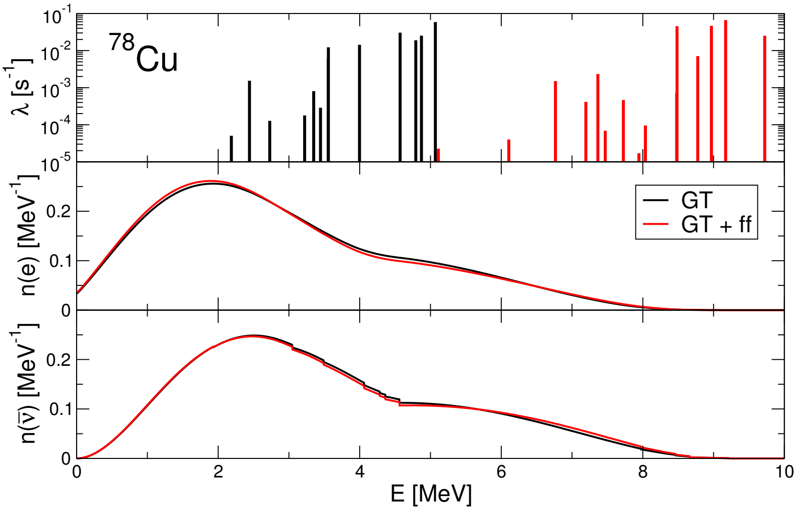
- $F(Z, A, E)$ - Fermi function, correction for the Coulomb field
- $L_0(Z, E)$ - correction for the finite size of the charge distribution
- $C'(Z, E)$ - correction for the nucleon moving within a nuclear potential
- other corrections are neglected

but if we include first-forbidden transitions

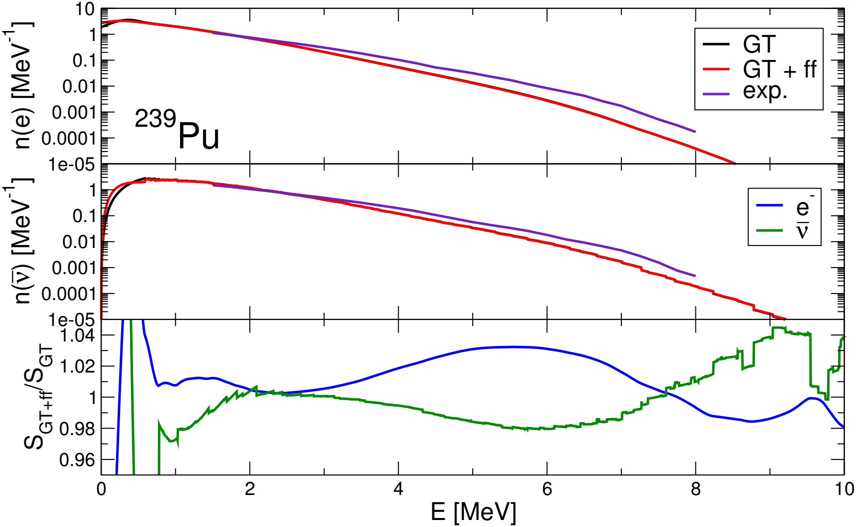
$$S_f^i = F(Z, A, E) \cdot pE(E - E_{max})^2 \cdot C(E) \cdot L_0(Z, E) \cdot C(Z, E)$$

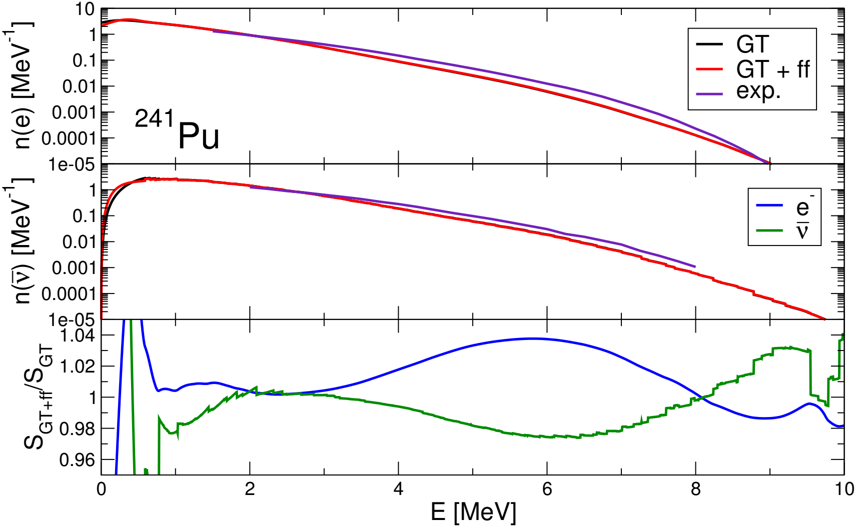
where $C(E)$ is the *shape factor*


sve korekcije
Gamow - Teller
0-
1-
2-
0,25
0,2
0,15
0,1
0,05
0
N (Eβ)
0
1
2
3
4
5
6
7
8
9
Eβ (MeV)

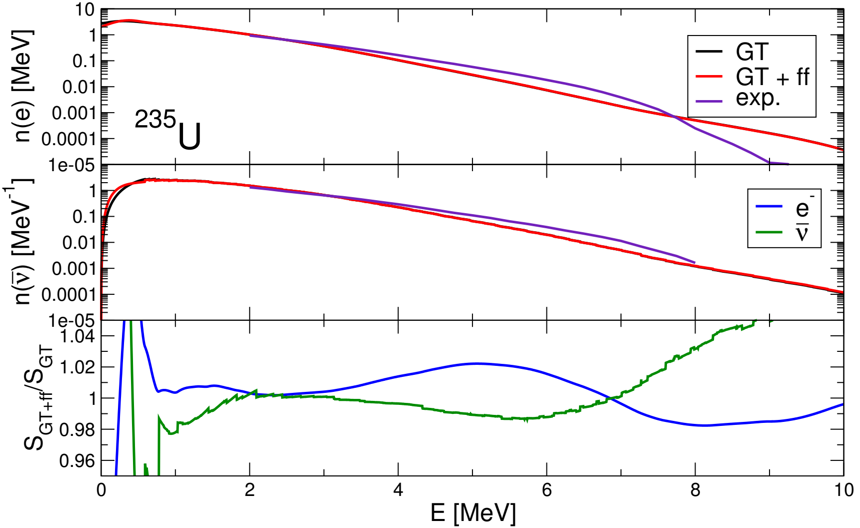


^{78}Cu
λ [s^{-1}]
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
n(e) [MeV^{-1}]
0.2
0.1
0
GT
GT + ff
n($\bar{\nu}$) [MeV^{-1}]
0.2
0.1
0
0
2
4
6
8
10
E [MeV]

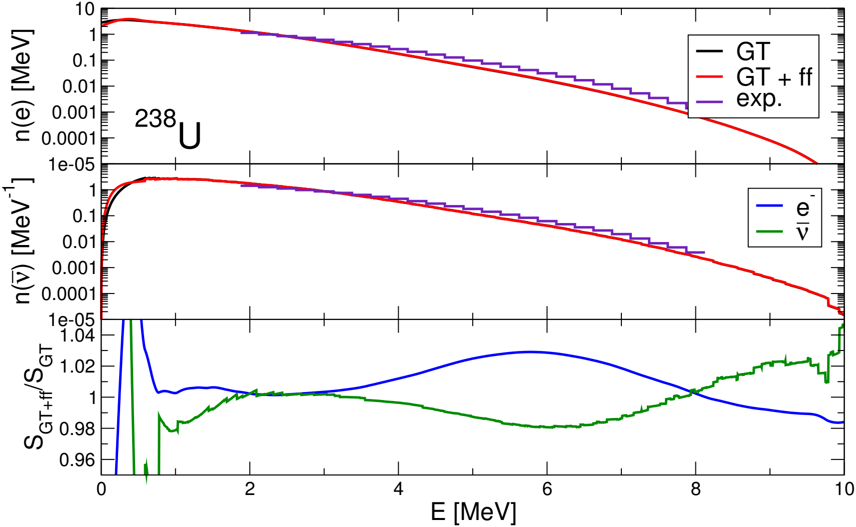

n(e) [MeV^{-1}]
10
1
0.1
0.01
0.001
0.0001
1e-05
^{239}Pu
GT
GT + ff
exp.
n($\bar{\nu}$) [MeV^{-1}]
e^-
$\bar{\nu}$
S_{GT+ff}/S_{GT}
1.04
1.02
1
0.98
0.96
0
2
4
6
8
10
E [MeV]



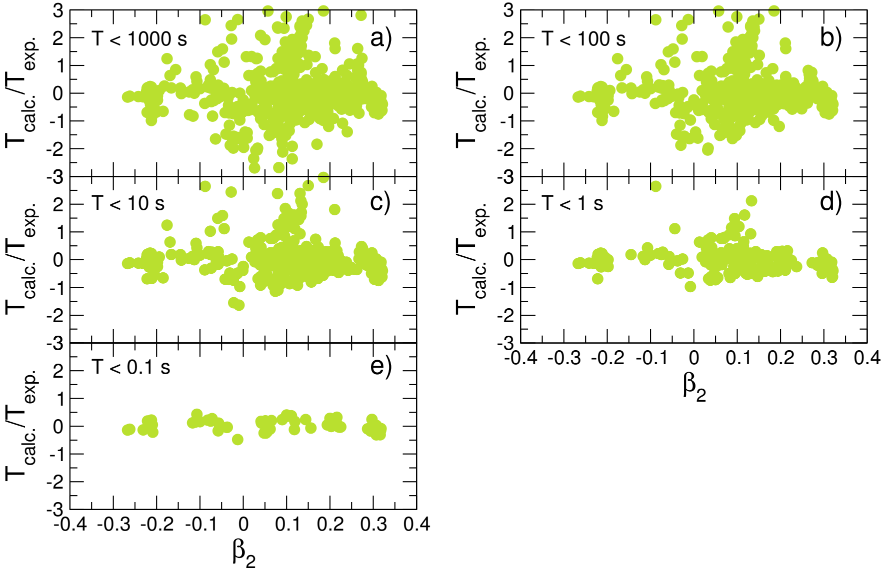
10
1
0.1
0.01
0.001
0.0001
1e-05
n(e) [MeV^{-1}]
GT
GT + ff
exp.
^{241}Pu
$n(\bar{\nu})$ [MeV^{-1}]
e$^-$
$\bar{\nu}$
1.04
1.02
1
0.98
0.96
S_{GT+ff}/S_{GT}
0
2
4
6
8
10
E [MeV]

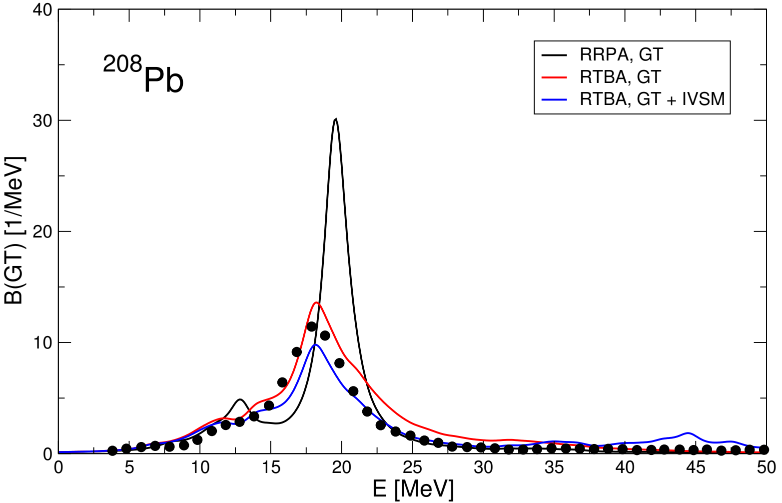


n(e) [MeV]
10
1
0.1
0.01
0.001
0.0001
1e-05
^{235}U
GT
GT + ff
exp.
$n(\bar{\nu})$ [MeV^{-1}]
e$^-$
$\bar{\nu}$
S_{GT+ff}/S_{GT}
1.04
1.02
1
0.98
0.96
0
2
4
6
8
10
E [MeV]

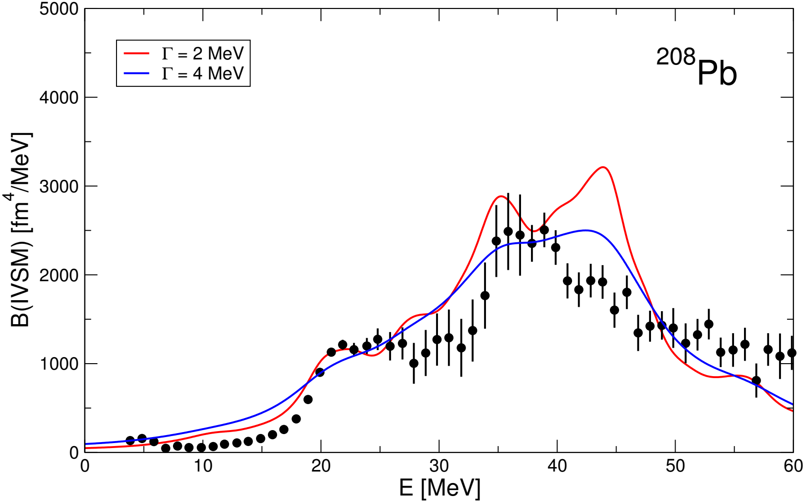


10
1
0.1
0.01
0.001
0.0001
1e-05
n(e) [MeV]
238U
GT
GT + ff
exp.
$n(\bar{\nu})$ [MeV^{-1}]
e^-
$\bar{\nu}$
1.04
1.02
1
0.98
0.96
S_{GT+ff}/S_{GT}
0
2
4
6
8
10
E [MeV]

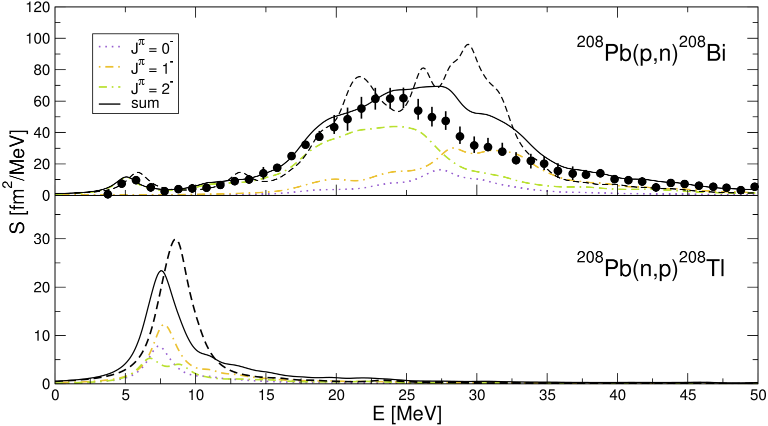

T < 1000 s
a)
T < 100 s
b)
T < 10 s
c)
T < 1 s
d)
T < 0.1 s
e)
$T_{calc.}/T_{exp.}$
β_2
3
2
1
0
-1
-2
-3
-0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4



T. Wakasa *et al.*, Phys. Rev. C 85, 064606 (2012)

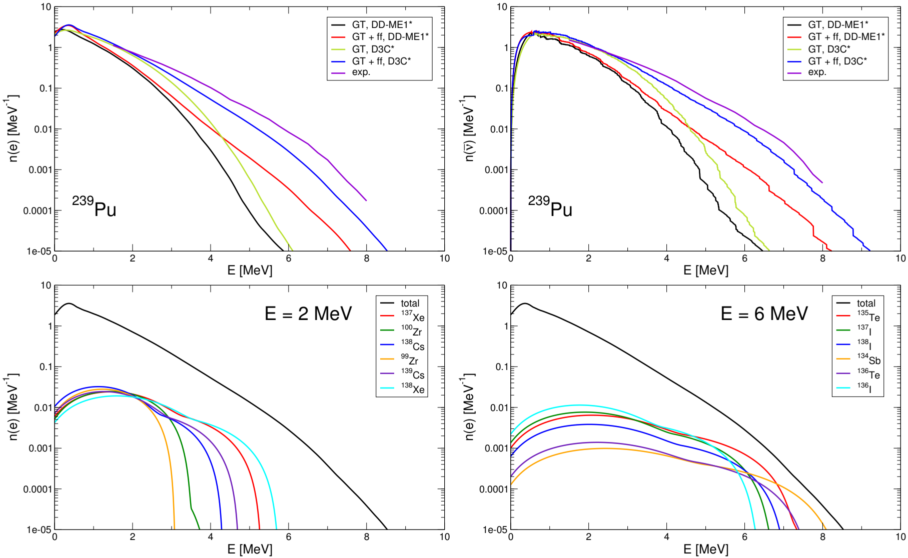


Γ = 2 MeV
Γ = 4 MeV
208Pb
B(IVSM) [fm4/MeV]
E [MeV]
5000
4000
3000
2000
1000
0
0
10
20
30
40
50
60



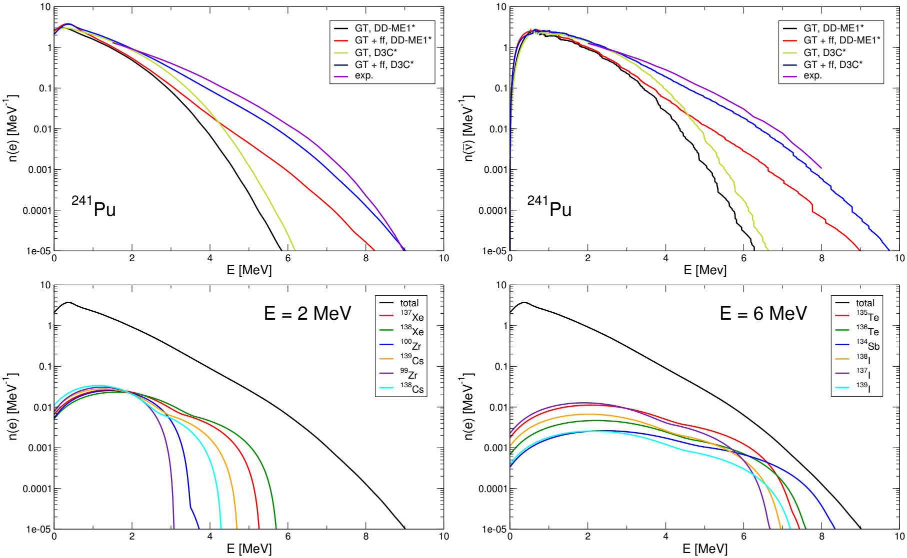
^{208}Pb(p,n)^{208}Bi
^{208}Pb(n,p)^{208}Tl
$J^{\pi} = 0^-$
$J^{\pi} = 1^-$
$J^{\pi} = 2^-$
sum
S [fm^2/MeV]
E [MeV]

^{239}Pu



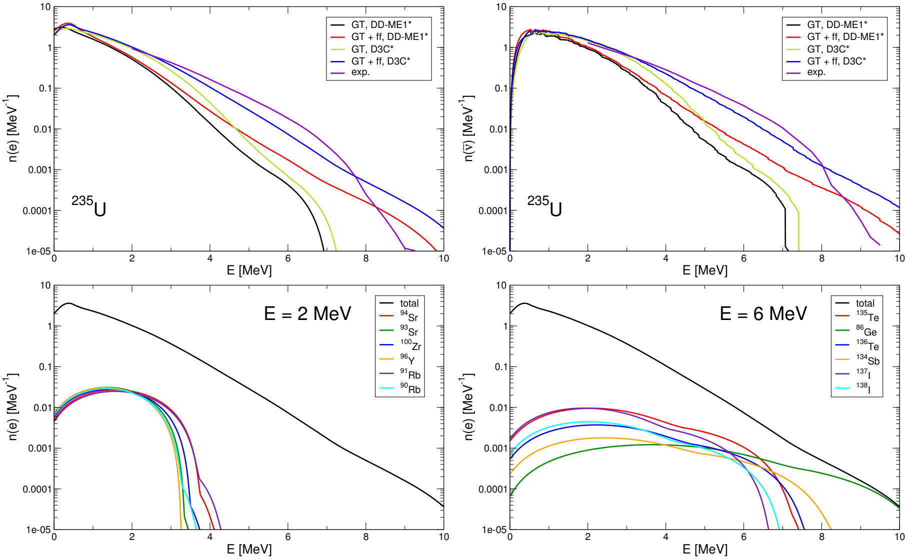
GT, DD-ME1*
GT + ff, DD-ME1*
GT, D3C*
GT + ff, D3C*
exp.
239Pu
n(e) [MeV-1]
n(ν̄) [MeV-1]
E [MeV]
E = 2 MeV
total
137Xe
100Zr
138Cs
99Zr
139Cs
138Xe
E = 6 MeV
135Te
137I
138I
134Sb
136Te
136I

^{241}Pu

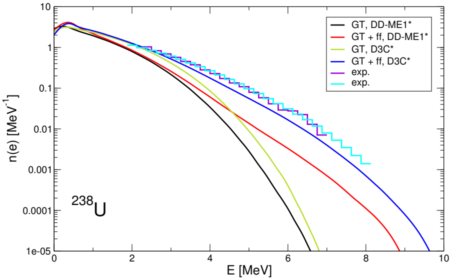


GT, DD-ME1*
GT + ff, DD-ME1*
GT, D3C*
GT + ff, D3C*
exp.
241Pu
n(e) [MeV-1]
E [MeV]
n(ν̄) [MeV-1]
E [Mev]
E = 2 MeV
total
137Xe
138Xe
100Zr
139Cs
99Zr
138Cs
E = 6 MeV
135Te
136Te
134Sb
138I
137I
139I

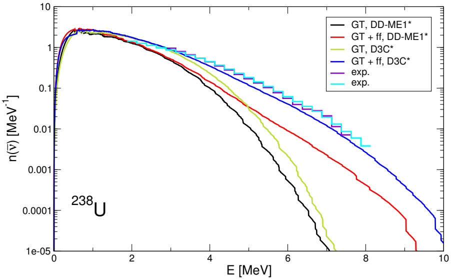
^{235}U


GT, DD-ME1*
GT + ff, DD-ME1*
GT, D3C*
GT + ff, D3C*
exp.
235U
n(e) [MeV-1]
n(ν̄) [MeV-1]
E [MeV]
E = 2 MeV
total
94Sr
93Sr
100Zr
96Y
91Rb
90Rb
E = 6 MeV
135Te
86Ge
136Te
134Sb
137I
138I

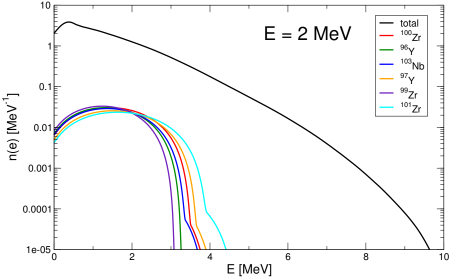
^{238}U



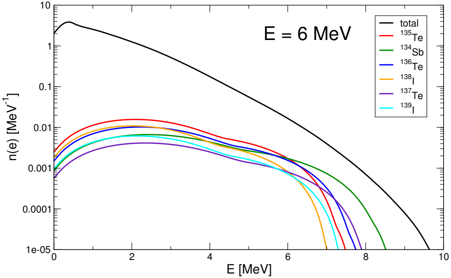
GT, DD-ME1*
GT + ff, DD-ME1*
GT, D3C*
GT + ff, D3C*
exp.
exp.
238U
n(e) [MeV-1]
E [MeV]



GT, DD-ME1*
GT + ff, DD-ME1*
GT, D3C*
GT + ff, D3C*
exp.
exp.
238U
n(ν̄) [MeV-1]
E [MeV]



E = 2 MeV
total
100Zr
96Y
103Nb
97Y
99Zr
101Zr
n(e) [MeV-1]
E [MeV]



E = 6 MeV
total
135Te
134Sb
136Te
138I
137Te
139I
n(e) [MeV-1]
E [MeV]