Beta decay rates of neutron-rich nuclei

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Introduction

Masses (Sn) (location of the path) /c45-decay half-lives (abundance and process speed)

Fission rates and distributions:
• n-induced
• spontaneous
• β-delayed

β-delayed n-emission branchings (final abundances)

β-decay half-lives (abundance and process speed)

n-capture rates
• for A>130 in slow freezeout
• for A<130 maybe in a “weak” r-process?

Seed production rates (ααα, ααn, α2n, ..)

Masses (Sn) (location of the path)

ν-physics?
Transitions are obtained by solving the pn-RQRPA equations

\[
\begin{pmatrix}
A & B \\
B^* & A^*
\end{pmatrix}
\begin{pmatrix}
X^\lambda \\
Y^\lambda
\end{pmatrix}
= E^\lambda
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
X^\lambda \\
Y^\lambda
\end{pmatrix}
\]

Residual interaction is derived from the Lagrangian density

\[
\mathcal{L}_{\rho+\pi} = -g_\rho \bar{\psi} \gamma_\mu \rho^\mu \tau \psi - \frac{f_\pi}{m_\pi} \bar{\psi} 5 \gamma^\mu \partial_\mu \tau \psi
\]

Total strength of a particular transition

\[
B_{\lambda,J} (GT) = \left| \sum_{pn} \left\langle p \right| \hat{O}_J \left| n \right\rangle \left( X_{pn}^\lambda \, u_p v_n - Y_{pn}^\lambda \, v_p u_n \right) \right|^2
\]
Decay rate:

\[ \lambda_i = D \int_{1}^{W_{0,i}} W \sqrt{W^2 - 1} \left( W_{0,i} - W \right)^2 F(Z, W) C(W) dW \]

\[ T_{1/2} = \frac{\ln 2}{\lambda}, \quad D = \frac{(G_F V_{ud})^2}{2\pi^3} \frac{(m_e c^2)^5}{\hbar} \]

Allowed decays shape factor:

\[ C(W) = B(GT) \]

First-forbidden decays shape factor:

\[ C(W) = k \left( 1 + aW + bW^{-1} + cW^2 \right) \]
\[ k = \left[ \zeta_0^2 + \frac{1}{9} w^2 \right]_{(0)} + \left[ \zeta_1^2 + \frac{1}{9} (x + u)^2 - \frac{4}{9} \mu_1 \gamma_1 u(x + u) \right]_{(1)} + \frac{1}{18} W_0^2 (2x + u)^2 - \frac{1}{18} \lambda_2 (2x - u)^2 \]\\
\[ + \left[ \frac{1}{12} z^2 \left( W_0^2 - \lambda_2 \right) \right]_{(2)} \]

\[ ka = \left[ - \frac{4}{3} u Y - \frac{1}{9} W_0 \left( 4x^2 + 5u^2 \right) \right]_{(1)} - \left[ \frac{1}{6} W_0 z^2 \right]_{(2)} \]

\[ kb = \frac{2}{3} \mu_1 \gamma_1 \left\{ - \left[ \zeta_0 w \right]_{(0)} + \left[ \zeta_1 (x + u) \right]_{(1)} \right\} \]

\[ kc = \frac{1}{18} \left[ 8u^2 + (2x + u)^2 + \lambda_2 (2x - u)^2 \right]_{(1)} + \frac{1}{12} \left[ (1 + \lambda_2) z^2 \right]_{(2)} \]
\[
\begin{align*}
    w &= -g_A \sqrt{3} \left\langle f \left| \sum_k r_k [C_1^k \otimes \sigma^k]^0 t_k^- \right| i \right\rangle \frac{1}{\sqrt{2J_i+1}}, \\
    x &= -\frac{\left\langle f \left| \sum_k r_k C_1^k t_k^- \right| i \right\rangle}{\sqrt{2J_i+1}}, \\
    u &= -g_A \sqrt{2} \left\langle f \left| \sum_k r_k [C_1^k \otimes \sigma^k]^1 t_k^- \right| i \right\rangle \frac{1}{\sqrt{2J_i+1}}, \\
    z &= 2g_A \left\langle f \left| \sum_k r_k [C_1^k \otimes \sigma^k]^2 t_k^- \right| i \right\rangle \frac{1}{\sqrt{2J_i+1}}, \\
    w' &= -g_A \frac{2}{\sqrt{3}} \left\langle f \left| \sum_k r_k l(1,1,1,1,r_k) [C_1^k \otimes \sigma^k]^0 t_k^- \right| i \right\rangle \frac{1}{\sqrt{2J_i+1}}, \\
    x' &= -\frac{2}{3} \frac{\left\langle f \left| \sum_k r_k l(1,1,1,1,r_k) C_1^k t_k^- \right| i \right\rangle}{\sqrt{2J_i+1}}, \\
    u' &= -g_A \frac{2\sqrt{2}}{3} \left\langle f \left| \sum_k r_k l(1,1,1,1,r_k) [C_1^k \otimes \sigma^k]^1 t_k^- \right| i \right\rangle \frac{1}{\sqrt{2J_i+1}}.
\end{align*}
\]
The diagrams show plots of $T_{1/2}$ (half-life in seconds) on a logarithmic scale against neutron number. The data is presented for four elements: Sr, Ag, Y, and Cd.

- In (a), the graph for Sr shows a clear distinction between experimental (exp.) data and calculations using D3C* and FRDM models.
- In (b), Ag's data highlights the same comparison, showing the agreement and deviation between the models.
- The plot for Y (c) and Cd (d) follows a similar pattern, comparing experimental results with theoretical predictions from D3C* and FRDM.

The graphs illustrate the half-lives of these elements as a function of neutron number, with the models providing a comparative analysis to the experimental data.
\[
\bar{r} = \frac{1}{N} \sum_i \log \frac{T_{\text{th}}}{T_{\text{exp.}}}
\]

\[
\sigma = \left[ \frac{1}{N} \sum_i (r_i - \bar{r})^2 \right]^{1/2}
\]

<table>
<thead>
<tr>
<th>( T_{\text{exp.}} ) [s]</th>
<th>( \bar{r} )</th>
<th>( \sigma )</th>
<th>( \bar{r} )</th>
<th>( \sigma )</th>
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<td>0.195</td>
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G. Audi et al., CPC 36, 1157 (2012)
**D3C*:**

<table>
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<th>Type</th>
<th>$\bar{\tau}$</th>
<th>$\sigma$</th>
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<td>0.331</td>
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<td>odd-Z</td>
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<td>odd-N</td>
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<td>odd-odd</td>
<td>0.089</td>
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<tr>
<td>total</td>
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**FRDM:**

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<td>0.226</td>
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<tr>
<td>odd-Z</td>
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<td>0.288</td>
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<tr>
<td>odd-N</td>
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<tr>
<td>odd-odd</td>
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<td>0.409</td>
</tr>
<tr>
<td>total</td>
<td>0.019</td>
<td>0.409</td>
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</table>
\[
P_{xn} = \frac{1}{\lambda_{tot}} \sum_{E_i=S_{xn}}^{S_{(x+1)n}} \lambda_i
\]

\[
\langle n \rangle = \sum_i i P_{in}
\]
In reactors, 99% of the electrons come from decay of fission products of 4 nuclei.

\[ S_{tot}(E) = \sum_{k=235U, 238U, 239Pu, 241Pu} \alpha_k S_k(E), \]

- \( \alpha_k \) - number of fissions at considered time
- \( S_k(E) \) - \( \beta \) spectrum normalized to one fission
- \( E \) - kinetic energy of emitted electrons

Electrons (and antineutrinos) come from the \( \beta \)-decay of resulting fission fragments.

\[ S_k(E) = \sum_{f=1}^{N_f} Y_f S_f(E) \]
\[ S_f(E) = \sum_{i=i}^{N_t} \frac{\lambda_i}{\lambda_{tot}} S_f^i(Z, A, E_{max}, E). \]
For allowed transitions the spectrum reads

\[ S_i^f = F(Z, A, E) \cdot pE(E - E_{\text{max}})^2 \cdot L_0(Z, E) \cdot C'(Z, E) \]

- \( F(Z, A, E) \) - Fermi function, correction for the Coulomb field
- \( L_0(Z, E) \) - correction for the finite size of the charge distribution
- \( C'(Z, E) \) - correction for the nucleon moving within a nuclear potential
- other corrections are neglected

but if we include first-forbidden transitions

\[ S_i^f = F(Z, A, E) \cdot pE(E - E_{\text{max}})^2 \cdot C(E) \cdot L_0(Z, E) \cdot C(Z, E) \]

where \( C(E) \) is the shape factor
sve korekcije
$^{78}$Cu

\[ \lambda [s^{-1}] \]

\[ n(e) [\text{MeV}^{-1}] \]

\[ n(\nu\_\_)[\text{MeV}^{-1}] \]

- GT
- GT + ff

E [MeV]
$238\text{U}$

The graph shows the neutron emission rates (n(e)) and absorption rates (n(ν_)) as a function of energy (E) in MeV. The rates are normalized to the ground state transition (GT) and the ground state transition plus feedforward (GT + ff) rates. The experimental data (exp.) is also plotted.

The y-axis represents the energy in MeV, and the x-axis represents the energy in MeV. The graph includes lines for electron (e^{-}) and antineutrino (\bar{\nu}) absorption rates.
$^{208}\text{Pb}$

E [MeV]

$B(\text{GT})$ [1/MeV]

RRPA, GT
RTBA, GT
RTBA, GT + IVSM

### Table

<table>
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<tr>
<th>Element</th>
<th>Z</th>
<th>Mass Number</th>
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<tbody>
<tr>
<td>$^{137}_{55}$Xe</td>
<td>55</td>
<td>137</td>
</tr>
<tr>
<td>$^{138}_{54}$Xe</td>
<td>54</td>
<td>138</td>
</tr>
<tr>
<td>$^{139}_{53}$Cs</td>
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<td>139</td>
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<tr>
<td>$^{140}_{54}$Zr</td>
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<td>138</td>
</tr>
<tr>
<td>$^{137}_{55}$I</td>
<td>55</td>
<td>137</td>
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<tr>
<td>$^{138}_{54}$I</td>
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<tr>
<td>$^{139}_{53}$I</td>
<td>53</td>
<td>139</td>
</tr>
</tbody>
</table>

### Diagrams

#### $^{241}$Pu

**$E = 2$ MeV**

**$E = 6$ MeV**

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$^{235}\text{U}$

**E = 2 MeV**

- $^{90}\text{Sr}$
- $^{90}\text{Sr}$
- $^{100}\text{Zr}$
- $^{91}\text{Y}$
- $^{91}\text{Rb}$
- $^{90}\text{Rb}$

**E = 6 MeV**

- $^{135}\text{Te}$
- $^{86}\text{Ge}$
- $^{136}\text{Te}$
- $^{134}\text{Sb}$
- $^{137}\text{I}$
- $^{138}\text{I}$

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