Solving the inhomogeneous Bethe-Salpeter Equation in Minkowski space

M. Viviani

INFN, Sezione di Pisa &
Department of Physics, University of Pisa
Pisa (Italy)

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Outline

1. Motivations and generalities: BS Amplitude and BS Equation for a two-scalar system $\to \mathcal{L} = g\phi^2\chi$, all massive scalars
2. Nakanishi perturbation-theory integral representation (PTIR) and the BS Amplitude
3. Results for bound states in ladder approximations
4. Results for zero-energy scattering in ladder approximation
5. Positive energies
6. Conclusions & Perspectives

In collaboration with

Tobias Frederico (ITA - S. José dos Campos)
Gianni Salmè (INFN - Roma)

FSV [PRD **85**, 036009 (2012)] & [PRD **89**, 016010 (2014)] & [EPJC **75**, 398 (2015)]

Motivations

- To achieve a fully covariant description for a few-body system, in Minkowski space
- To take properly into account the dynamics, within a field-theoretical framework
- To make feasable numerical calculations

Well-known non perturbative approches: lattice calculations in Euclidean space

Direct solution of the BSE in Minkovski or Euclidean space (for bound states)

Notation

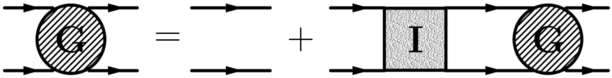
- Two bosons interacting via $\mathcal{L} = g\phi^2\chi$
- $m =$ mass of the particle ϕ
- $\mu =$ mass of the exchanged particle
- $\alpha = g^2/16\pi m^2$ adimensional parameter
- $\kappa^2 = m^2 - M^2/4$, $M =$ "energy" of the system
- Bound states $\kappa^2 > 0$
- Zero energy scattering $\kappa^2 = 0$
- Positive energy scattering $\kappa^2 < 0$

The BSE in a nutshell

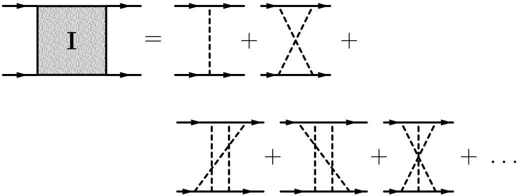
The 4-point Green's Function,

$$G(x_1, x_2; y_1, y_2) =< 0 \,|\, T\{\phi_1(x_1)\phi_2(x_2)\phi_1^+(y_1)\phi_2^+(y_2)\} \,|\, 0 > \quad ,$$

fulfills an integral equation $G = G_0 \; + \; G_0 \; \mathrm{I} \; G$



I $\equiv$ kernel given by the infinite sum of irreducible Feynmann graphs



Iterations produce all the expected contributions

For bound-states

Insert a complete Fock basis in

$$G(x_1, x_2; y_1, y_2) = < 0 \,|\, T\{\phi_1(x_1)\phi_2(x_2)\phi_1^+(y_1)\phi_2^+(y_2)\} \,|\, 0 >$$

then in the Fourier space, the bound state contribution (assuming only one non degenerate bound state for the sake of simplicity) appears as a pole, i.e.

$$G_B(k, q; p_B) \simeq \frac{i}{(2\pi)^4} \, \frac{\phi(k; p_B) \; \bar{\phi}(k; p_B)}{2\omega_B(p_0 - \omega_B + i\epsilon)} \qquad \omega_B = \sqrt{M_B^2 + |\mathbf{p}|^2}$$

$\phi(k; p_B)$ is the Bethe-Salpeter Amplitude

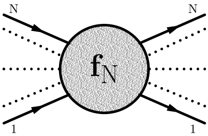
For $p_0 \to \omega_B$ the 4-point Green's function can be approximate by $G \simeq G_B + \; \textit{regular terms}$ and one deduces from $G = G_0 \; + \; G_0 \; \mathrm{I} \; G$, the integral equation determining the BS Amplitude for a bound state, i.e. the homogeneous BS Eq.

$$\phi(k; p_B, \beta) = G_0(k; p_B, \beta) \; \int d^4q' \; \mathrm{I}(k, q'; p_B) \; \phi(q'; p_B, \beta)$$

(with nor self-energy neither vertex corrections, at the present stage)
Notice: $\mathrm{I}(k, q'; p_B)$, the irreducible kernel in BSE, is the same in $G = G_0 + G_0 \mathrm{I} G$.

Nakanishi perturbation-theory integral representation (PTIR) for the N-leg transition amplitude

In the sixties, Nakanishi (PR **130**, 1230 (1963)) proposed an integral representation for N-leg transition amplitudes, based on the parametric formula for the Feynman diagrams.



Generic contribution to the transition amplitude is given by

$$f_{\mathcal{G}}(p_1, p_2, ..., p_N) \propto \prod_{r=1}^{k} \int d^4 q_r \frac{1}{(\ell_1^2 - m_1^2)(\ell_2^2 - m_2^2)\ \dots\ (\ell_n^2 - m_n^2)}$$

where one has n propagators and k loops

The sum over all Feynman diagram $\mathcal{G}$ for a full N-leg transition amplitude can be formally written as

$$f_N(s) = \sum_{\mathcal{G}} f_{\mathcal{G}}(s) \propto \prod_h \int_0^1 dz_h \int_0^\infty d\gamma \frac{\delta(1 - \sum_h z_h)\ \phi_N(z_1, z_2, \dots, \gamma)}{\gamma - \sum_h z_h s_h}$$

the dependence upon the external momenta, $p_1, p_2 \dots p_N$, $p'_1, p'_2 \dots p'_N$, traded off in favour of all the independent scalar products $s \equiv \{s_1, s_2, \dots s_h, \dots\}$, one can construct.

Nakanishi PTIR - II

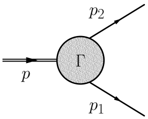
Within the BS framework, such an elegant expression can be exploited for obtaining

- the 3-leg transition amplitude (vertex function → bound-state BS amplitude) (Kusaka et al, PRD **56** (1997), Carbonell-Karmanov EPJA **27** (2006))
- the 4-leg one (off-shell or half-off-shell T-matrix → scattering-state BS amplitude) (FSV, PRD **85** (2012))

The PTIR of the vertex (three-leg) function

$$f_3(s) \equiv \Gamma(s) = \int_0^1 dz \int_0^\infty d\gamma \frac{\phi_3(z,\gamma)}{\gamma - \frac{p^2}{4} - k^2 - zk \cdot p - i\epsilon}$$

with $p = p_1 + p_2$ and $k = (p_1 - p_2)/2$



How can the Nakanishi weight function, ϕ_3, be determined for an actual, dynamical model?

Can the Nakanishi expression, elaborated in perturbation theory, be used in a non perturbative realm, as the BS framework does (one has to face with an integral equation, i.e. one has an infinite set of contributions)?

The exact projection of the BSE onto the null plane

Integrating the BSE on the LF variable $k^- = k^0 + k_z$ Karmanov & Carbonell, 2006

Let us take the Nakanishi vertex function as an Ansatz for the BS amplitude and then, integrate it on the Light-Front variable $k^- = k^0 + k_z$.

One gets the valence component of the state of the interacting system (after expanding on the Fock basis)

$$\psi_{n=2}(\xi, k_\perp) = \frac{p^+}{\sqrt{2}}\, \xi\, (1-\xi) \int \frac{dk^-}{2\pi} \overbrace{\Phi_b(k,p)}^{\text{BS Amplit.}}$$

$\xi = \frac{1-z}{2} = \frac{1}{p^+}\left(\frac{p^+}{2} + k^+\right)$ fraction of longitudinal momentum

The LF projection of the BSE amplitude produces $\psi_{n=2}(\xi, k_\perp)$, a non-singular function!

Therefore, let us consider the LF projection of BSE $\Rightarrow$

$$\int_0^\infty d\gamma' \frac{g_b(\gamma', z; \kappa^2)}{[\gamma' + \gamma + z^2 m^2 + (1-z^2)\kappa^2 - i\epsilon]^2} =$$

$$= \int_0^\infty d\gamma' \int_{-1}^{1} dz'\ V_b^{LF}(\gamma, z; \gamma', z') g_b(\gamma', z'; \kappa^2).$$

with $V_b^{LF}(\gamma, z; \gamma', z')$ determined by the irr. kernel $\mathrm{I}(k, k', p)$!

Applying the uniqueness of the Nakanishi weight function

Nakanishi enriched his theoretical invetigation by demonstrating a theorem on the uniqueness of the weight function for a given N-leg amplitude.

If such a theorem is valid also in the non perturbative context of the BSE a simpler integral equation for the weight function can be written

$$g_b(\gamma, z; \kappa^2) = \int_0^\infty d\gamma' \int_{-1}^{1} dz' \; \mathcal{V}_b(\gamma, z; \gamma', z'; \kappa^2) g_b(\gamma', z'; \kappa^2)$$

where $\mathcal{V}_b(\gamma, z; \gamma', z'; \kappa^2)$ is a new kernel, properly related to $V_b^{LF}(\gamma, z; \gamma', z')$!

$$V_b^{LF}(\gamma, z; \gamma'', z') = \int_0^\infty d\gamma' \frac{\mathcal{V}_b(\gamma', z; \gamma'', z'; \kappa^2)}{[\gamma' + \gamma + z^2 m^2 + (1 - z^2)\kappa^2 - i\epsilon]^2}$$

a Fredholm integral equation of first kind.

First in a canonical approach (Kusaka et al PRD **56**, (1997)), recently in a LF approach (FSV PRD **85**,(2012))

Numerical results for the bound state in ladder approx.

We have carried out a comprehensive investigation, in ladder approximation, of the simple scalar model, $\mathcal{L} = g\phi^2\chi$,

- In ladder approximation V_b and $\mathcal{V}_b$ proportional to $g^2 \sim \alpha$
- Standard procedure: we fix $M = 2m - B$, $B =$ binding energy
- Study for different binding energies $0 < B/m \leq 2$ and mass of the exchanged scalar, μ/m
- using the two eigen-equations: the one involving directly the valence wave function and the one based on the uniqueness theorem.

One fixes the binding energy, $B/m = 2 - M/m$, and the mass of the exchanged scalar, and looks for the eigenvalue (the coupling constant) and the eigenfunction (the Nakanishi weight function).

Comparison with the results from i) Carbonell-Karmanov (EPJA **27**, 1 (2006)) (valence w.f. based & covariant LF) and ii) Kusaka et al, (PRD **56**, 5071 (1997)) (uniqueness based &canonical approach).

Method of solution

Expansion on a basis for the z and γ variables

$$g_b^{(Ld)}(\gamma, z; \kappa^2) = \sum_{\ell=0}^{N_z} \sum_{j=0}^{N_g} A_{\ell j} \; F_\ell(z) \; G_j(\gamma)$$

- $F_\ell(z) = N_\ell (1 - z^2) C_{2\ell}^{(5/2)}(z)$, C =Gegenbauer polynomials
- $G_j(\gamma) = \sqrt{a} \; L_j(a\gamma) \; e^{-a\gamma/2}$, L =Laguerre polynomials
- Conditions $F_\ell(\pm 1) = 0$, $G_j(\gamma \to \infty) = 0$
- Chosen so that $\int_{-1}^{1} dz \; F_\ell(z) \; F_{\ell'}(z) = \delta_{\ell\ell'}$, $\int_0^\infty \; d\gamma \; G_j(\gamma) \; G_{j'}(\gamma) = \delta_{jj'}$

Orthonormality of the basis very useful from the numerical point of view!

Results

$\mu/m = 0.15$

B/m	α LF-V (CK)	α LF-V (FSV)	α LF-U (FSV)
0.01	0.5716	0.5716	0.5716
0.10	1.437	1.437	1.437
0.20	2.100	2.099	2.099
0.50	3.611	3.610	3.611
1.00	5.315	5.313	5.314

$\mu/m = 0.50$

B/m	α LF-V (CK)	α LF-V (FSV)	α LF-U (FSV)
0.01	1.440	1.440	1.440
0.10	2.498	2.498	2.498
0.20	3.251	3.251	3.251
0.50	4.901	4.901	4.901
1.00	6.712	6.711	6.711

Values of $\alpha = g^2/(16\pi m^2)$, obtained by solving the valence-based eigenequation (LF-V) and the uniqueness-based one (LF-U). Gegenbauer $\times$ Laguerre expansion of the Nakanishi wf

LF-V (CK): from Carbonell -Karmanov, EPJA **27**, 1 (2006) (spline expansion of the Nakanishi wf).

$\mu/m = 0.50$

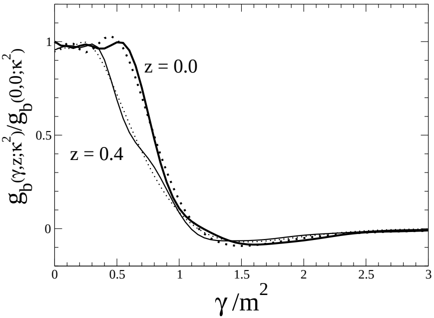
B/m	α C-U	α LF-U	α LF-V
0.002	1.211	1.216	1.216
0.02	1.624	1.623	1.623
0.20	3.252	3.251	3.251
0.40	4.416	4.415	4.416
0.80	6.096	6.094	6.094
1.20	7.206	7.204	7.204
1.60	7.850	7.849	7.849
2.00	8.062	8.061	8.061

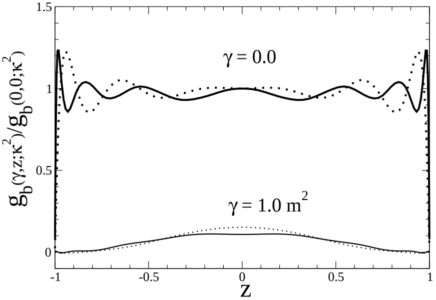
Values of $\alpha = g^2/(16\pi m^2)$, obtained by solving the valence-based eigenequation (LF-V) and the uniqueness-based one (LF-U). Gegenbauer $\times$ Laguerre expansion of the Nakanishi wf

C-U: from Kusaka,Simpson and Williams, PRD **56**, 5071 (1997), where uniqueness and canonical (not LF !) variables have been used and iteration method for solving the eigenequation.

A flash on the Nakanishi weight function $g_b(\gamma, z; \kappa^2)$

Just an example: $B/m = 1$ and $\mu/m = 0.5$ ($\kappa^2 = 4 - M^2$)





Valence Probabilities and LF Distributions

Once the Nakanishi weight functions is evaluated, one can straightforwardly obtain the BS amplitude and normalize it.

Then, the probability of the the valence wave function, $\psi_{n=2}(\xi, k_\perp)$, results properly determined and one can also calculate the LF distributions, relevant in Hadron Physics

Valence wave function

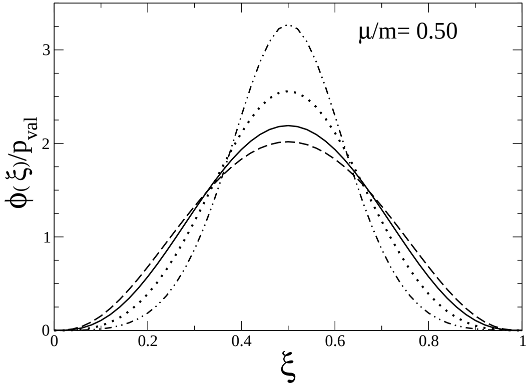
$$\psi_{n=2}(\xi, k_\perp) = \frac{p^+}{\sqrt{2}}\, \xi\, (1-\xi) \int \frac{dk^-}{2\pi} \Phi_b(k,p)$$

$\mu/m = 0.50$

B/m	α	P_{val}
0.001	1.167	0.98
0.01	1.440	0.96
0.10	2.498	0.87
0.20	3.251	0.83
0.50	4.900	0.77
1.00	6.711	0.74
2.00	8.061	0.72

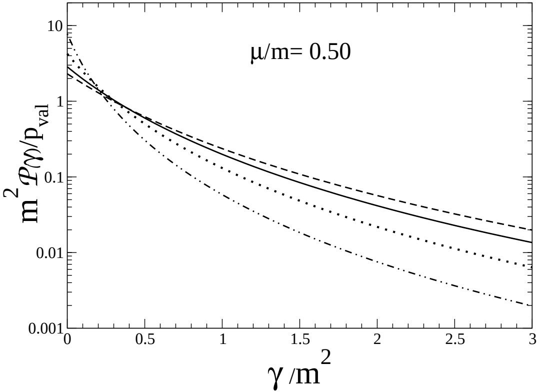
$P_{val} \rightarrow 1$ for $B \rightarrow 0$!

NO sizable difference between LF-V and LF-U results !!



The longitudinal LF-distribution, $\phi(\xi) = \int dk_\perp^2 \ |\psi_{n=2}(\xi, k_\perp)|^2$, vs the longitudinal-momentum fraction $\xi = k^+/M$. Dash-double-dotted line: $B/m = 0.20$. Dotted line: $B/m = 0.50$. Solid line: $B/m = 1.0$. Dashed line: $B/m = 2.0$. N.B. $\int_0^1 d\xi \ \phi(\xi) = P_{val}$

NO sizable difference between LF-V and LF-U results !!



The transverse LF-distribution $\mathcal{P}(\gamma) = \int d\xi \; |\psi_{n=2}(\xi, k_\perp)|^2$ vs the adimensional variable γ/m^2 ($\gamma = k_\perp^2$). Dash-double-dotted line: $B/m = 0.20$. Dotted line: $B/m = 0.50$. Solid line: $B/m = 1.0$. Dashed line: $B/m = 2.0$. N.B.$\int_0^\infty d\gamma \; \mathcal{P}(\gamma) = P_{val}$.

Solution of the BSE at zero-energy scattering in ladder approximation – now $\kappa = 0$

Light front projection of the inhomogeneous BSE "LF-V" version

$$\int_0^\infty d\gamma' \, \frac{g_0^{(Ld)}(\gamma', z)}{(\gamma + \gamma' + z^2 m^2)^2} = I_{Ld}(\gamma, z) + \int_0^\infty d\gamma' \int_{-1}^{+1} dz' \; V_0(\gamma, z, \gamma', z') g_0^{(Ld)}(\gamma', z')$$

Or applying uniqueness ... "LF-U" version

$$g_0^{(Ld)}(\gamma, z) = \frac{g^2}{\mu^2}\left[\theta\left(1 - |z| - \frac{\gamma}{\mu^2}\right)\right] + \int_0^\infty d\gamma' \int_{-1}^{+1} dz' \; \mathcal{V}_0(\gamma, z, \gamma', z') g_0^{(Ld)}(\gamma', z')$$

- In ladder approximation V_0 and $\mathcal{V}_0 \sim \alpha$
- $V_0(\gamma, z, \gamma', z')$ is a regular function $\rightarrow$ no problems
- $\mathcal{V}_0(\gamma, z, \gamma', z')$ has "mild" (integrable) singularities $\sim 1/\sqrt{\gamma}$
- the second equations shows that $g_0^{(Ld)}$ is discontinuous
- more severe for $\mu \rightarrow 0$ and large α

The scattering lenght and the Nakanishi function

Calculation of the scattering length a

$$ma = \alpha \frac{m^2}{\mu^2} - \frac{\alpha}{2(4\pi)^2 m^2} \int_0^\infty d\gamma \int_{-1}^{+1} dz\; g_0^{(Ld)}(\gamma, z) I(\gamma/m^2, z, \mu/m)$$

Using the "LF-U" version, it can be shown that for $0 \le \gamma \le \mu^2(1 - |z|)$ and $\mu \le 2m$

$$g_0^{(Ld)}(\gamma, z) = -16\pi m a$$

New expansion $(t \equiv 1 - |z| - \frac{\gamma}{\mu^2})$

$$g_0^{(Ld)}(\gamma, z) = \beta\theta(t) + \theta(-t) \sum_{\ell,j} A_{\ell,j} F_\ell(z) G_\ell(\gamma)$$

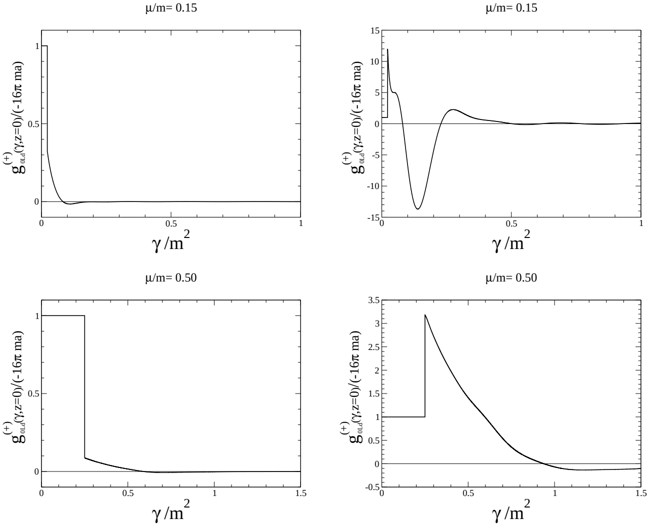
β and $A_{\ell,j}$ unknown parameters computed solving either the LF-V or LF-U equation
$[\beta \to -16\pi m a]$

After a discussion with J. Carbonell in Sao Jose' in 2013

Comparison with Carbonell & Karmanov [PLB**727**, 319 (2013)]

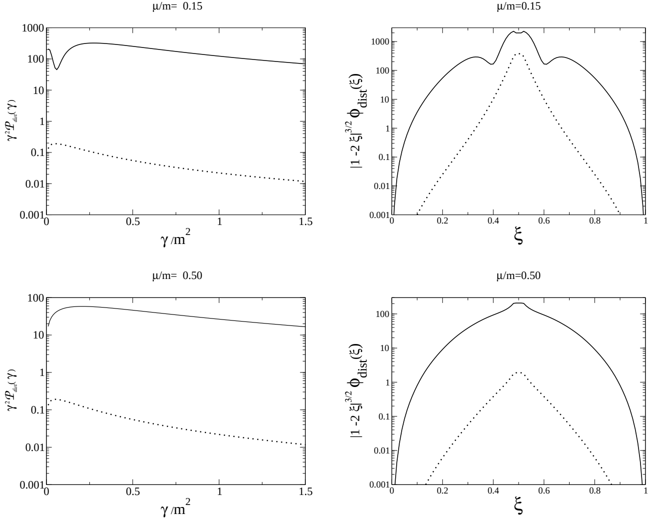
α	$\mu/m = 0.50$ $m = 1$				α	$\mu/m = 0.15$ $m = 1$			
	a_{CK}	a_{LF-V}	a_{LF-U}	a^{BA}		a_{CK}	a_{LF-V}	a_{LF-U}	a^{BA}
0.01	-0.0403	-0.0403	-0.0403	-0.04	0.01	-0.460	-0.459	-0.459	-0.444
0.10	-0.438	-0.438	-0.438	-0.40	0.10	-6.92	-6.66	-6.66	-4.44
0.50	-3.66	-3.66	-3.66	-2.00	0.50	17.7	17.2	17.2	-22.2
0.90	-24.7	-24.7	-24.8	-3.60	0.90	-4.57	-1.13	-1.13	-40.0
1.00	-103.0	-103.2	-103.0	-4.00	1.00	-28.1	-7.89	-7.89	-44.4
1.10	62.0	61.9	61.8	-4.40	1.10	900.	-19.4	-19.4	-48.9
1.50	11.0	11.0	11.0	-6.00	1.50	24.7	66.9	66.9	-66.7
2.00	6.34	6.34	6.34	-8.00	2.00	17.4	23.2	23.2	-88.9
2.50	4.54	4.53	4.53	-10.00	2.50	14.4	12.3	12.2	-111.0

Nakanishi functions

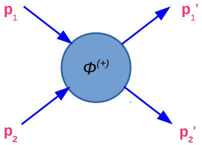


LF distributions

Solid line: $\alpha = 2.5$; dotted line: $\alpha = 0.1$



Positive energies



- $p^\mu = p_1^\mu + p_2^\mu$
- $k_i^\mu = (p_1^\mu - p_2^\mu)/2$
- $k^\mu = (p_1'^\mu - p_2'^\mu)/2$
- CM frame $\mathbf{p}_1 + \mathbf{p}_2 = 0,\ k_i^\mu \equiv (0, \mathbf{p}_s)$
- $s = (p_1 + p_2)^2 \equiv M^2 = p_s^2$

$$\Phi^{(+)}(k,p,k_i) = (2\pi)^4 \delta^{(4)}(k - k_i) + G_0^{(12)}(k,p) \int \frac{d^4k'}{(2\pi)^4} i\, \mathcal{K}(k,k',p)\Phi^{(+)}(k',p,k_i)$$

$G_0^{(12)}$ free two-particle Green's function & $\mathcal{K}$ kernel
No self-energy insertions and vertex corrections:

$$G_0^{(12)}(k,p) = \frac{i}{(\frac{p}{2}+k)^2 - m^2 + i\epsilon} \frac{i}{(\frac{p}{2}-k)^2 - m^2 + i\epsilon}$$

$$\mathcal{K}^{(Ld)}(k,k',p) = \underbrace{\frac{(-ig)^2}{(k-k')^2 - \mu^2 + i\epsilon}}_{\text{Ladder approximation}}$$

Nakanishi PTIR

$$\Phi^{(+)}(k,p,k_i) = (2\pi)^4\delta^{(4)}(k-k_i) + i\int_{-1}^{1}dz'\int_{-1}^{1}dz''\int_{-\infty}^{\infty}d\gamma' \times \frac{g^{(+)}(\gamma',z',z'',z_i)}{\left[\gamma'+m^2-\frac{1}{4}M^2-k^2-p\cdot k\, z''-2k\cdot k_i\, z'-i\epsilon\right]^3}$$

Note the lower integration limit of the γ' variable

We have shown that for $E \leq 0$ the integration limit shrinks to 0 [FSV, EPJC (2015)]

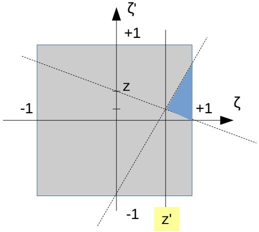
- $\mathbf{p}_s$ along z
- $z_i = -2k_i^+/M = -p_s/\sqrt{p_s^2+m^2}$
- $\kappa = -p_s^2$
- $\gamma_a(z') = z'(2\kappa^2 - \mu^2)$
- $\gamma = k_\perp^2$
- $z = -2k^+/M$

Symmetry under the exchange $1 \leftrightarrow 2$

$$g^{(+)}(\gamma',z',z'',z_i) = g^{(+)}(\gamma',z',-z'',-z_i)$$

The equation for $g^{(+)}$ (from uniqueness)

$$
\begin{aligned}
g^{(+)}(\gamma, z', z, z_i) \quad = \quad & g^2\theta(-z')\delta(\gamma - \gamma_a(z')) \times \\
& \Big[\theta(z - z_i)\theta\Big(1 - z + z'(1 - z_i)\Big) + \theta(z_i - z)\theta\Big(1 + z + z'(1 + z_i)\Big)\Big] \\
+ \quad & \frac{g^2}{2(4\pi)^2} \int_{-\infty}^{+\infty} d\gamma' \int_{-1}^{+1} d\zeta \int_{-1}^{+1} d\zeta' \; g^{(+)}(\gamma', \zeta, \zeta', z_i) \\
& \times \frac{\partial}{\partial\gamma} \Big[\theta(\zeta' - z - z_i\zeta)\mathcal{V}^{(+)}(\gamma, z', z, z_i, \gamma', \zeta, \zeta') \\
& \qquad + \theta(z + z_i\zeta - \zeta')\mathcal{V}^{(+)}(\gamma, z', -z, -z_i, \gamma', \zeta, -\zeta')\Big]
\end{aligned}
$$

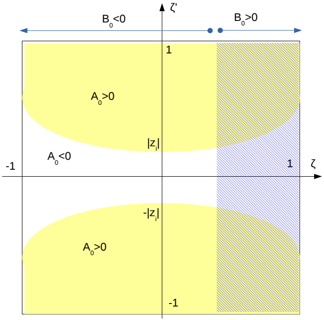


In progress: study of the region of integrations for the ζ, ζ' variable

$\leftarrow$ example for the case $z' > 0$

Scattering function

$$f^{el}_{(Ld)}(s,\theta) = \frac{2\alpha\ m^2}{\sqrt{s}}\left[\frac{1}{2p_s^2(1-\cos\theta)+\mu^2-i\epsilon}\right.$$
$$+\frac{1}{2(4\pi)^2}\int_0^\infty d\gamma'\int_{-1}^{1}d\zeta\int_{-1}^{1}d\zeta' g^{(+)}_{(Ld)}(\gamma',\zeta,\zeta',z_i)$$
$$\left.\times\int_0^\infty dy\ \frac{y^2}{\left[\Big(A_0(\zeta,\zeta')+\gamma'\Big)y^2+\Big(B_0(\zeta)+\gamma'\Big)y+\mu^2-i\epsilon\right]^2}\right]$$



$$A_0 = (\zeta')^2(p_s^2+m^2)-p_s^s(1+\zeta^2)$$
$$B_0 = \mu^2+2\zeta p_s^2\cos\theta$$

Perform first the γ' int.

$$F(\gamma_c) = \int_0^\infty d\gamma'\ \frac{f(\gamma')}{(\gamma'-\gamma_c-i\epsilon)^2}$$
$$\gamma_c = -\frac{A_0y^2+B_0y+\mu^2}{y^2+y}$$

Study of $F(\gamma_c)$

- If $\gamma_c < 0$ no singularities
- If $\gamma_c > 0$:

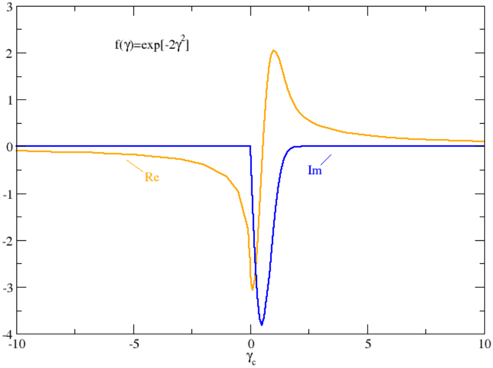
$$F(\gamma_c) = i\pi f'(\gamma_c) + \text{non} - \text{singular}$$

- For $\gamma_c \to 0$?

$$F(\gamma_c) = -\frac{f(0)}{\gamma_c + i\epsilon} + \widetilde{F}(\gamma_c)$$

$\widetilde{F}(\gamma_c)$ is a regular function

test with $f(\gamma) = exp(-2\gamma^2)$
$\Im(\widetilde{F}(\gamma_c)) = 0$ for $\gamma_c < 0$
$\Im(\widetilde{F}(\gamma_c)) = \pi f'(\gamma_c)$ for $\gamma_c > 0$



Conclusions & Perspectives

- The cross-fertilization between the Light-Front framework and the Nakanishi PTIR paves the path toward a new class of non perturbative calculations, within a rigorous field-theoretical framework (the Bethe-Salpeter Equation in Minkowski space).
- The LF framework has well-known advantages in performing analytical integrations, that within the canonical approach appear highly non trivial.
- Our numerical investigations, performed in ladder approximation at the present stage, confirm both the robusteness of the Nakanishi Ansatz for the BS amplitude and the Uniqueness Theorem. Morever, we extended the numerical analysis of an actual dynamical model to the valence probability and the LF distributions, of great relevance for Hadron Physics.
- Very good results also for the scattering length, in particular using the Uniqueness Theorem (analytical expression of the discontinuity).
- Calculations in progress for energy > 0. Singularities seem to be under control.