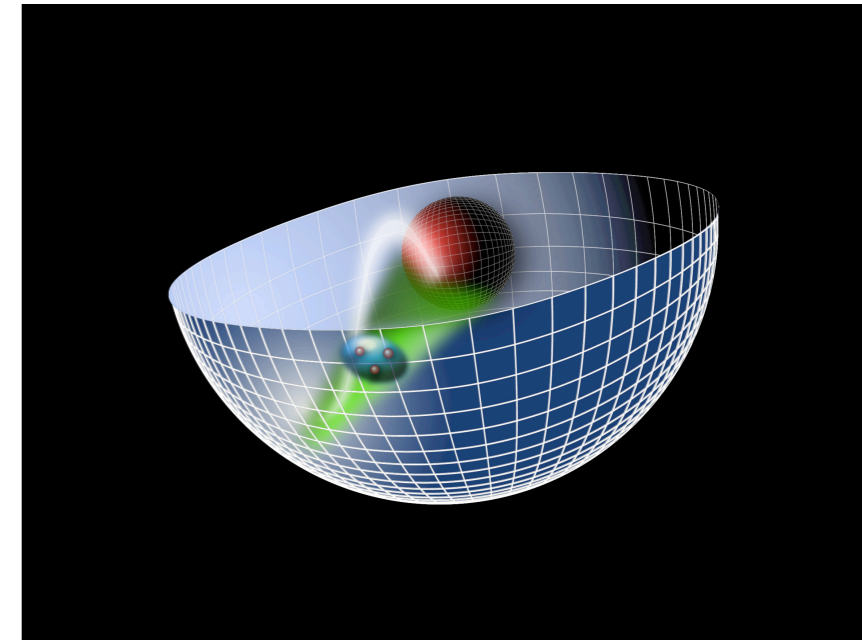
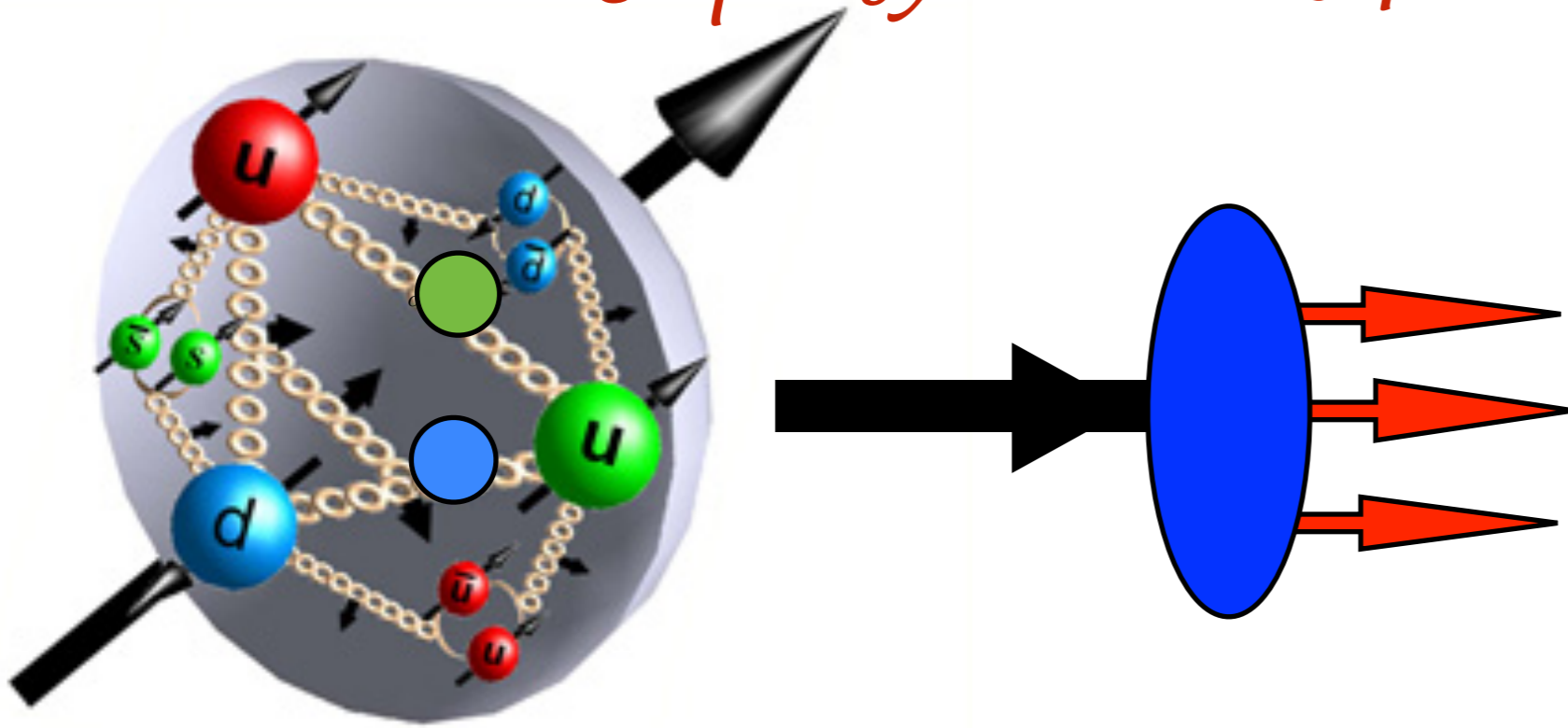


Light-Front Holographic QCD, Color Confinement, and Supersymmetric Features of QCD



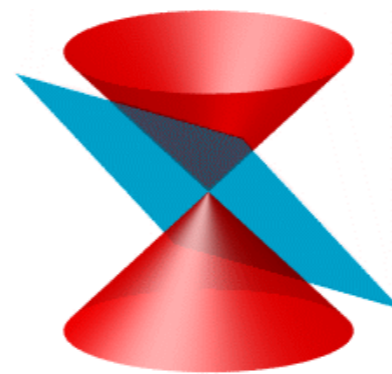
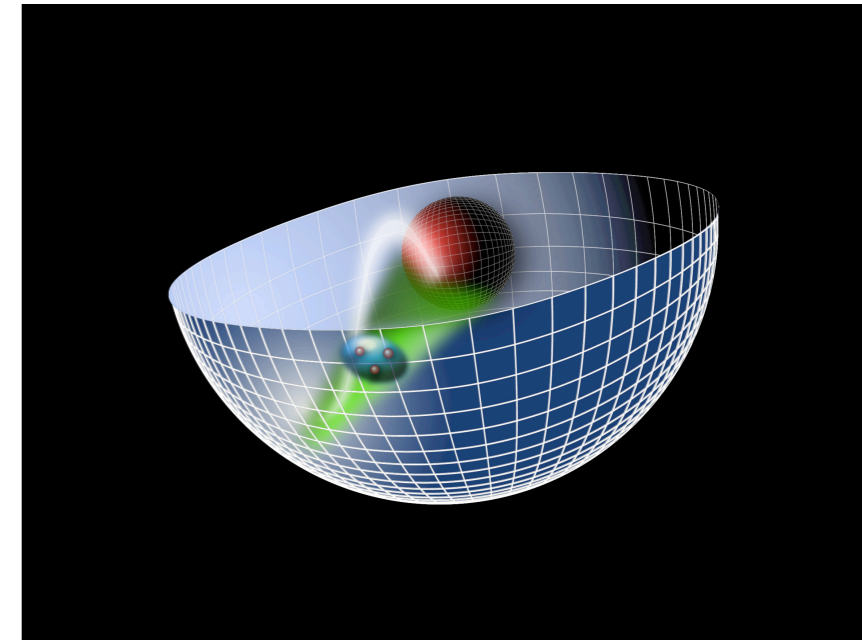
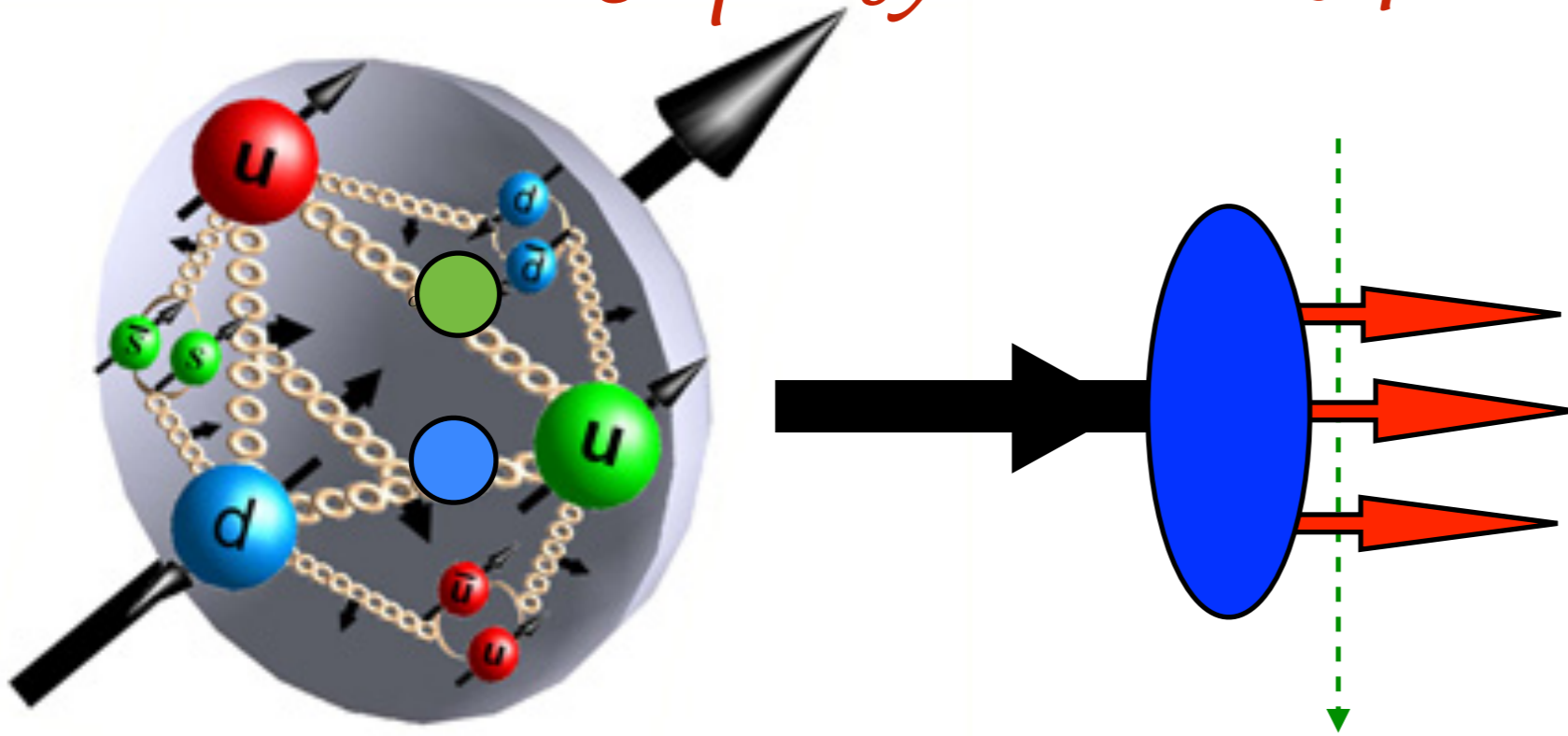
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Light-Front Holographic QCD, Color Confinement, and Supersymmetric Features of QCD



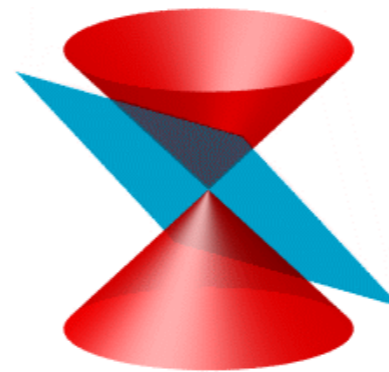
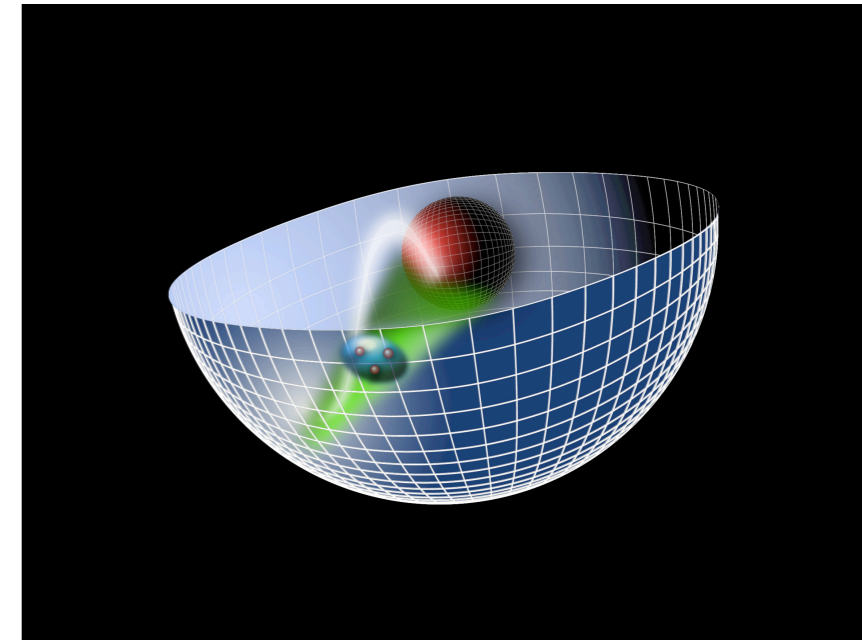
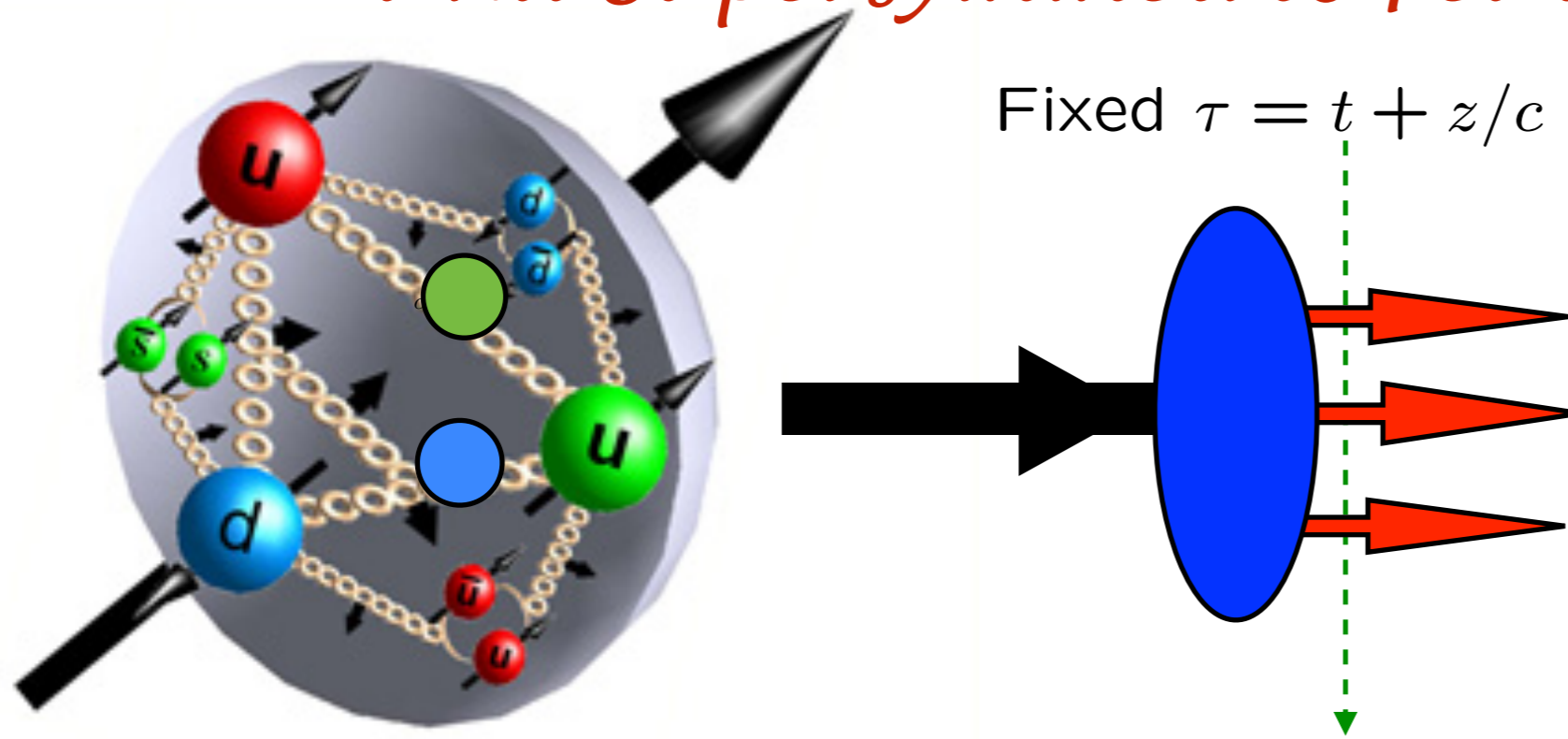
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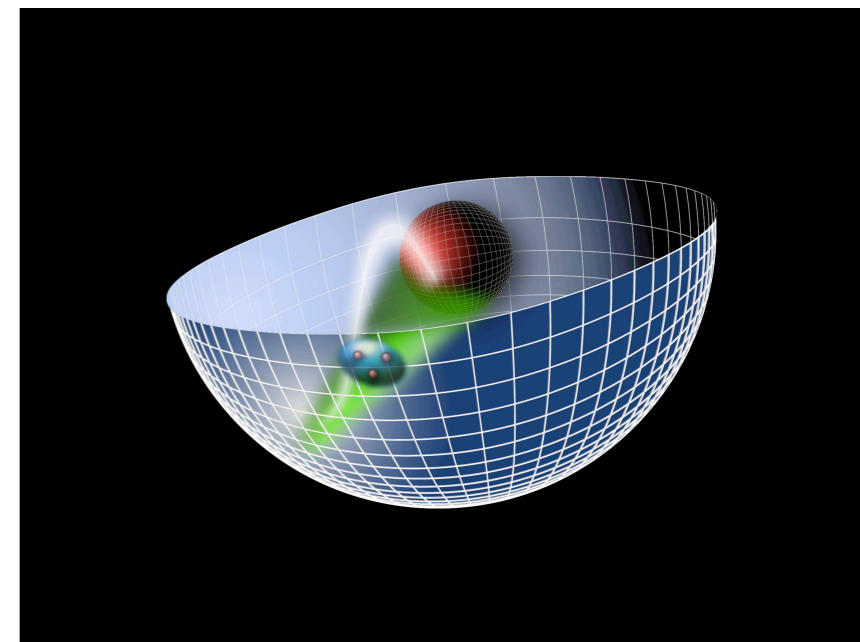
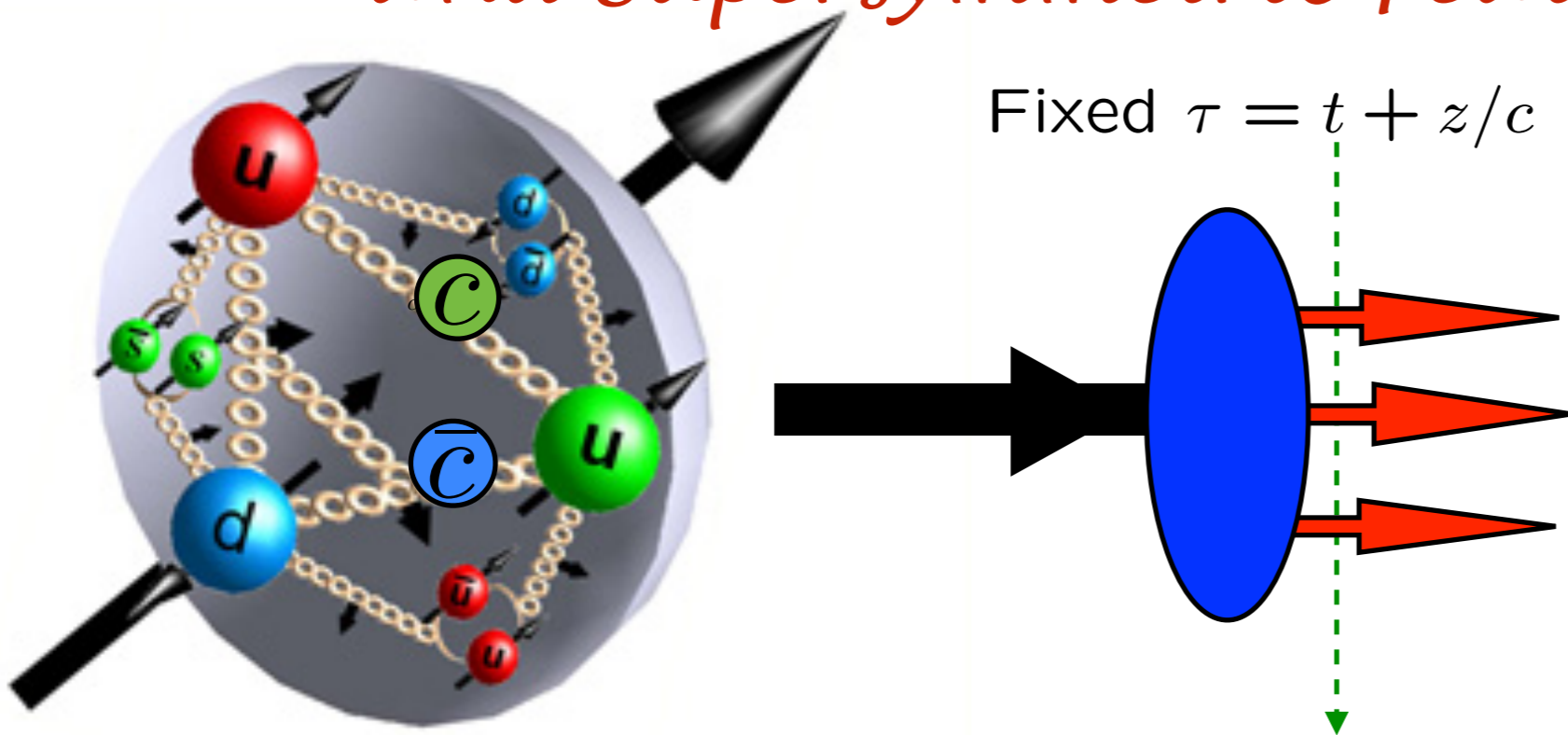
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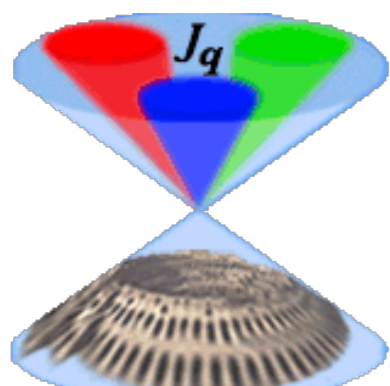
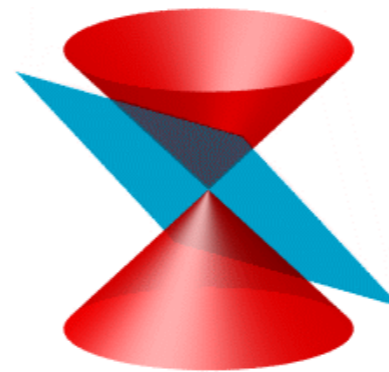
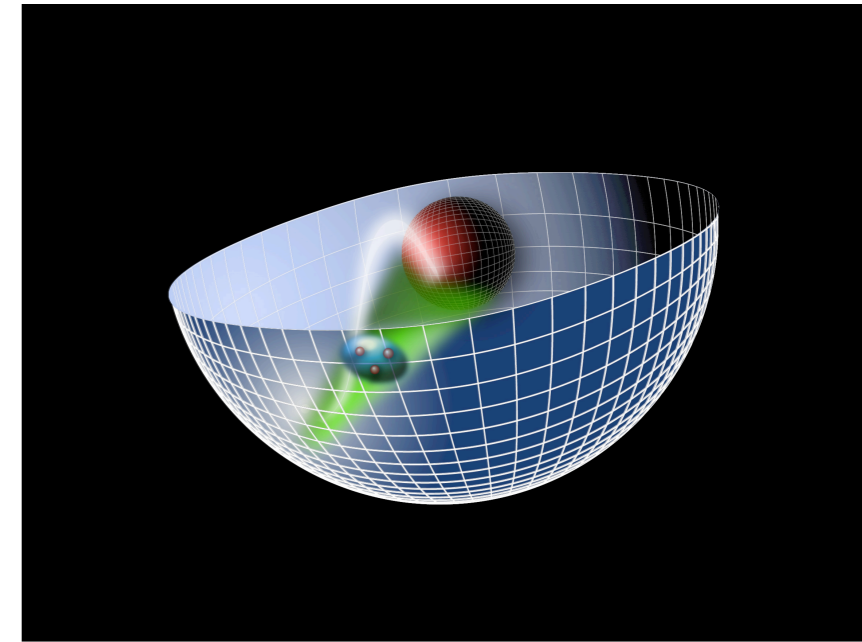
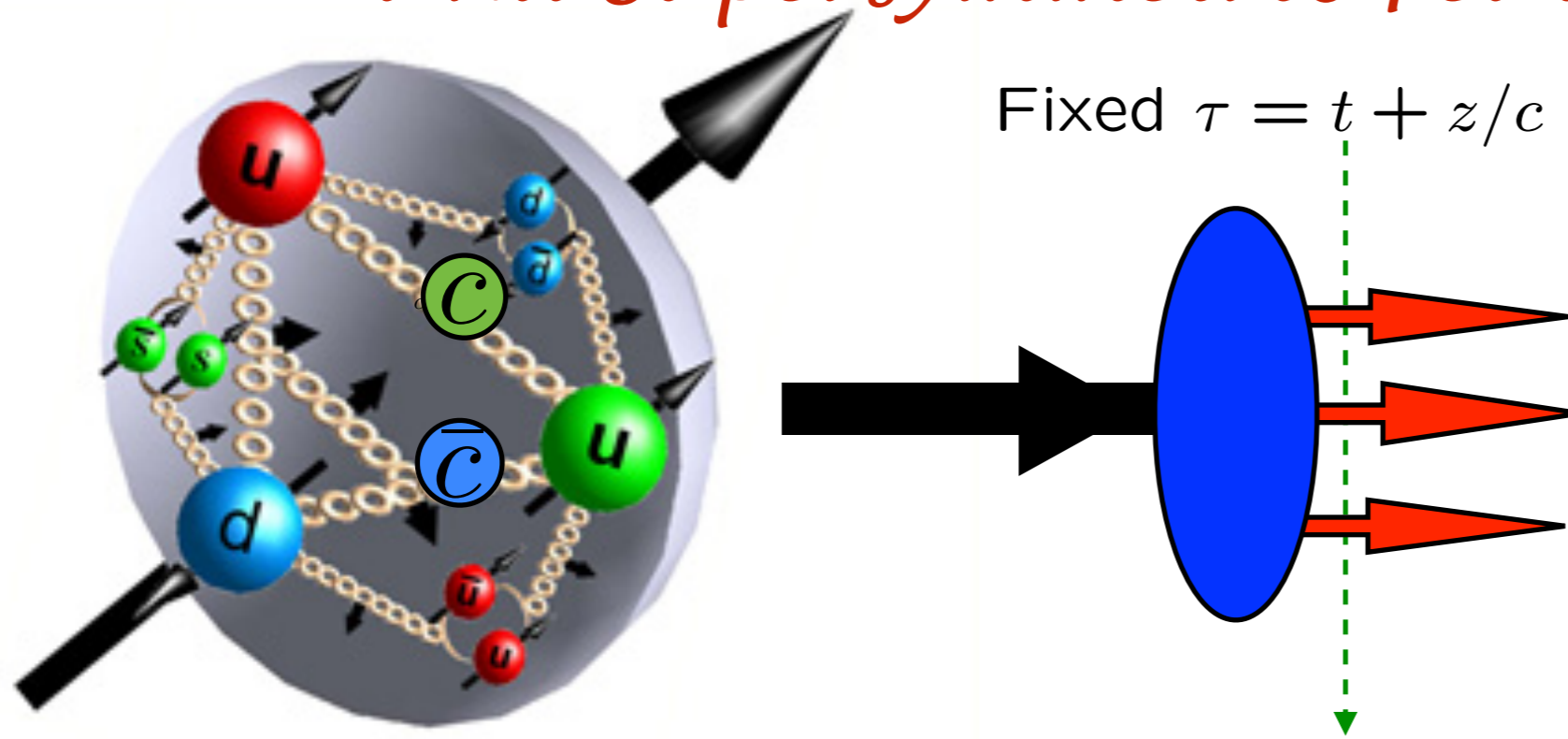
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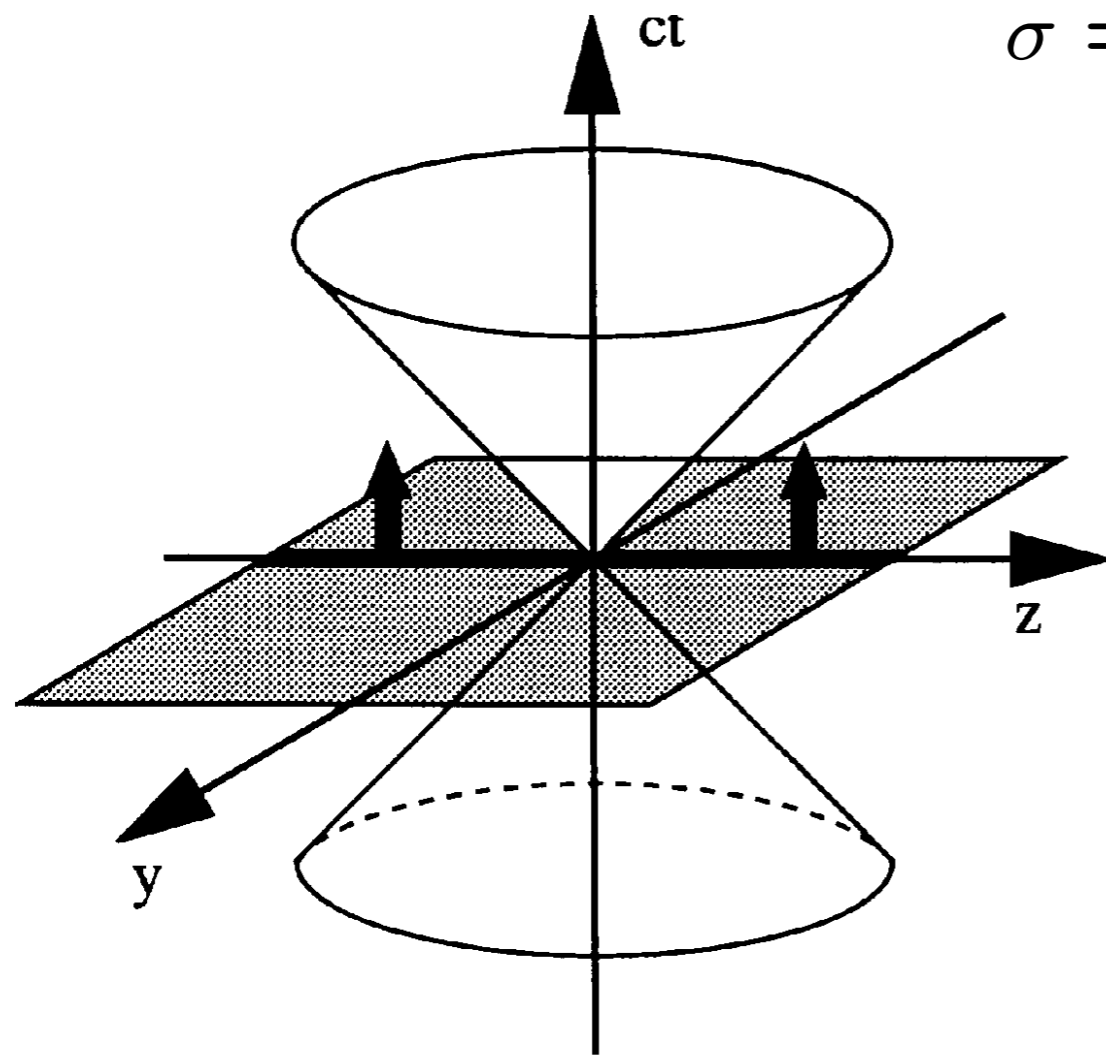
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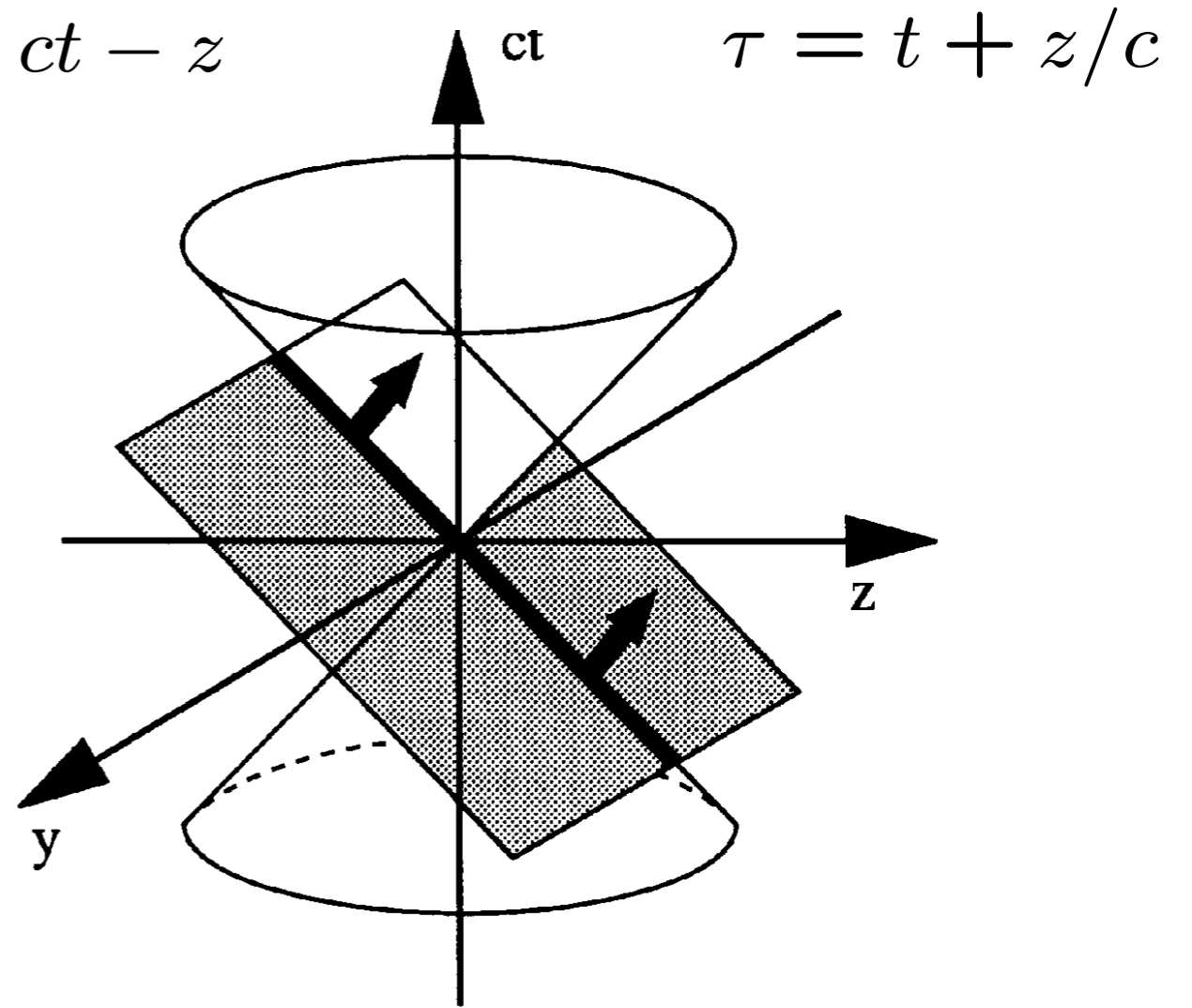


*Dirac's Amazing Idea:
The "Front Form"*



Instant Form

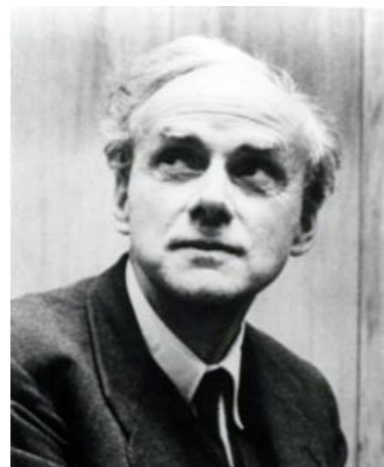
$$\sigma = ct - z$$



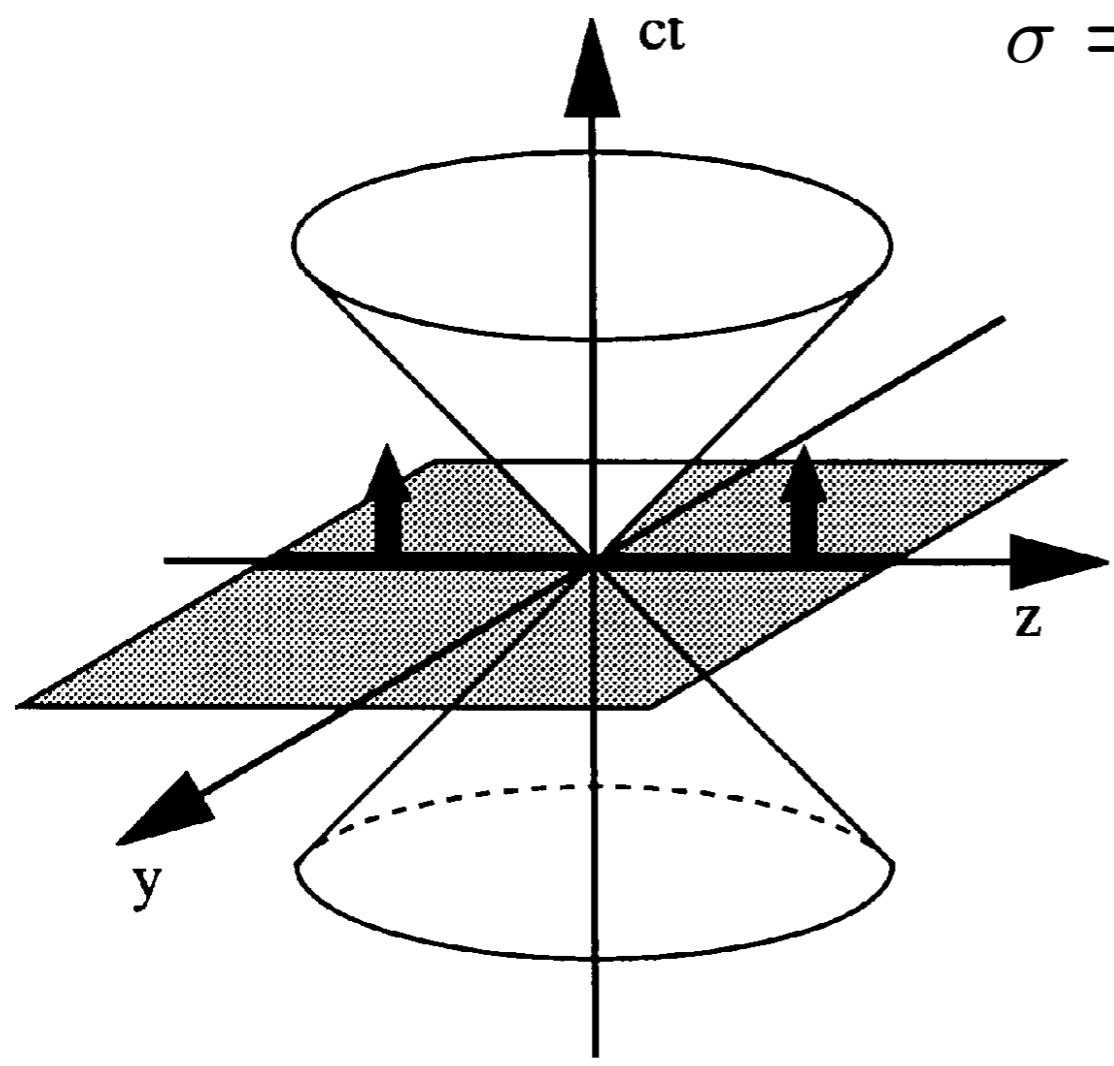
$$\tau = t + z/c$$

Front Form

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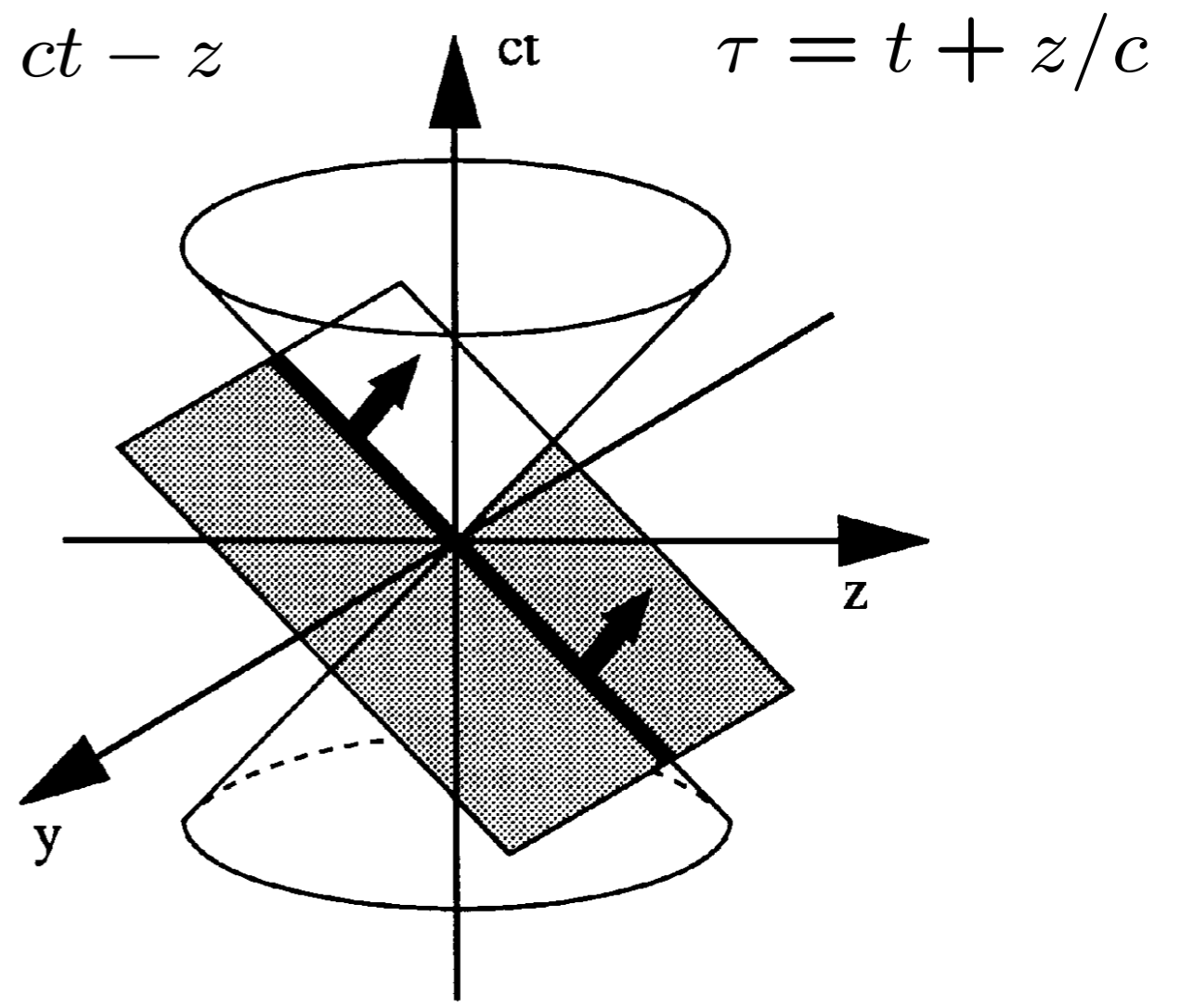


**P.A.M Dirac, Rev. Mod. Phys. 21,
392 (1949)**



Instant Form

$$\sigma = ct - z$$



Front Form

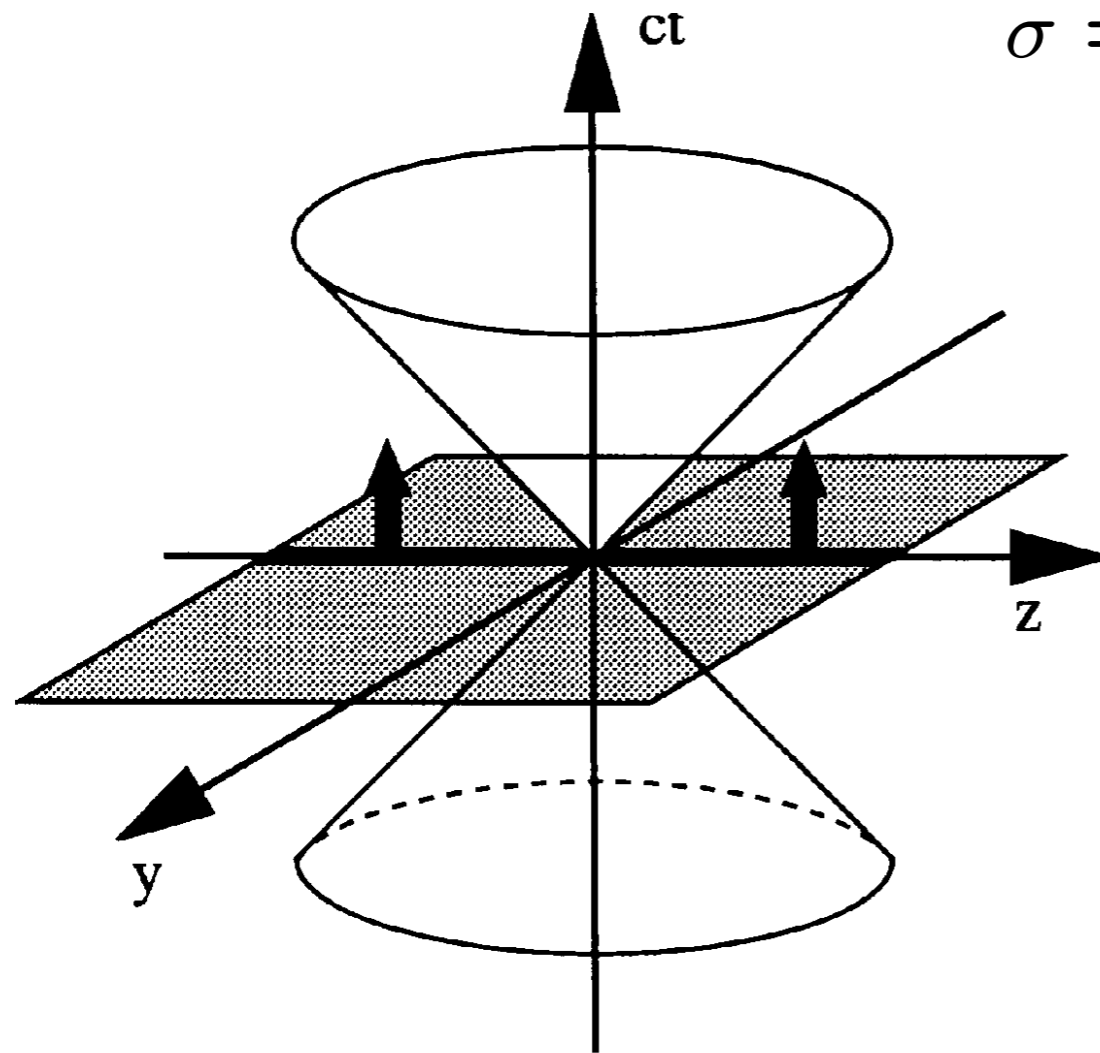
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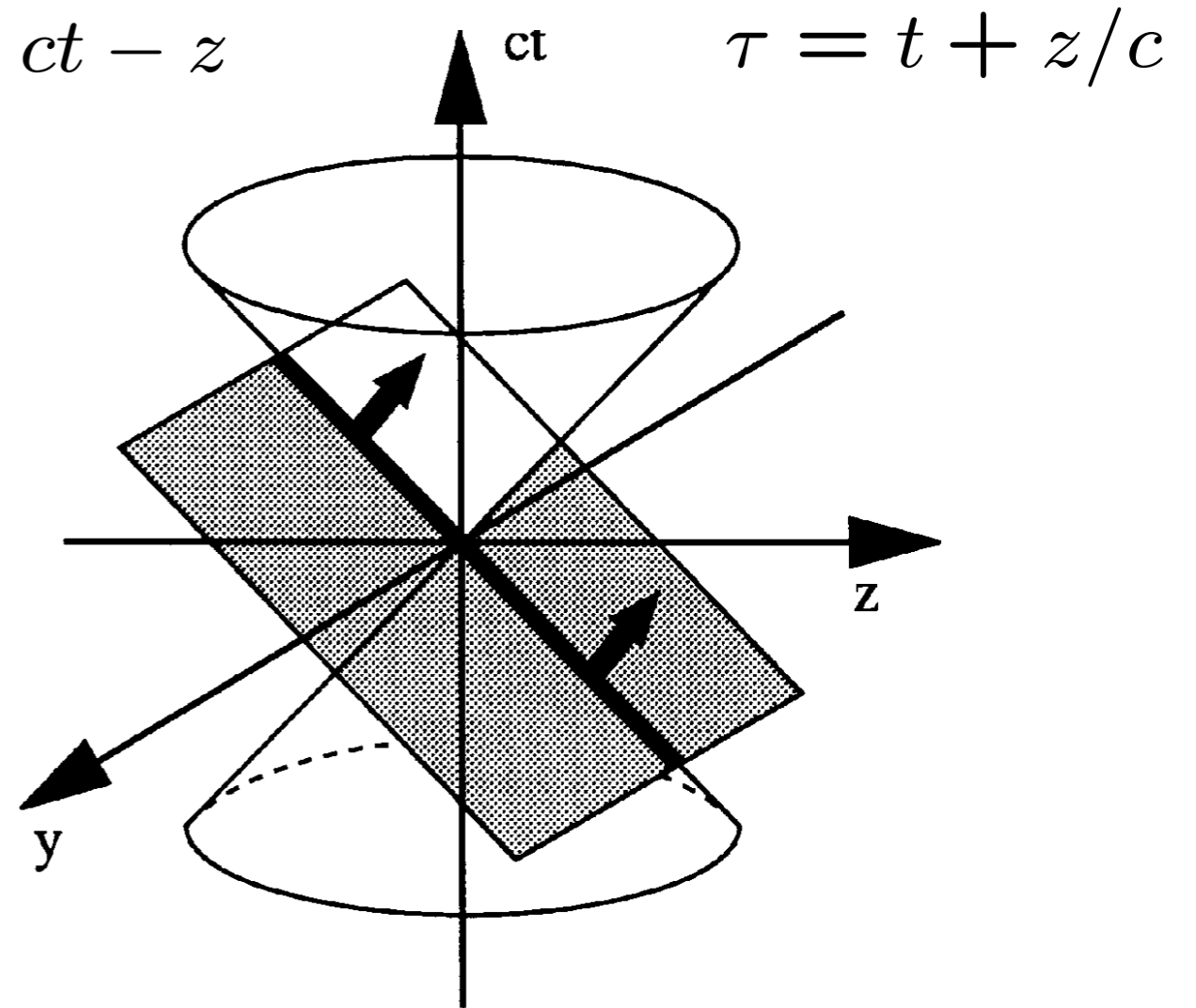
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**Evolve in
ordinary time**



Instant Form

$$\sigma = ct - z$$



Front Form

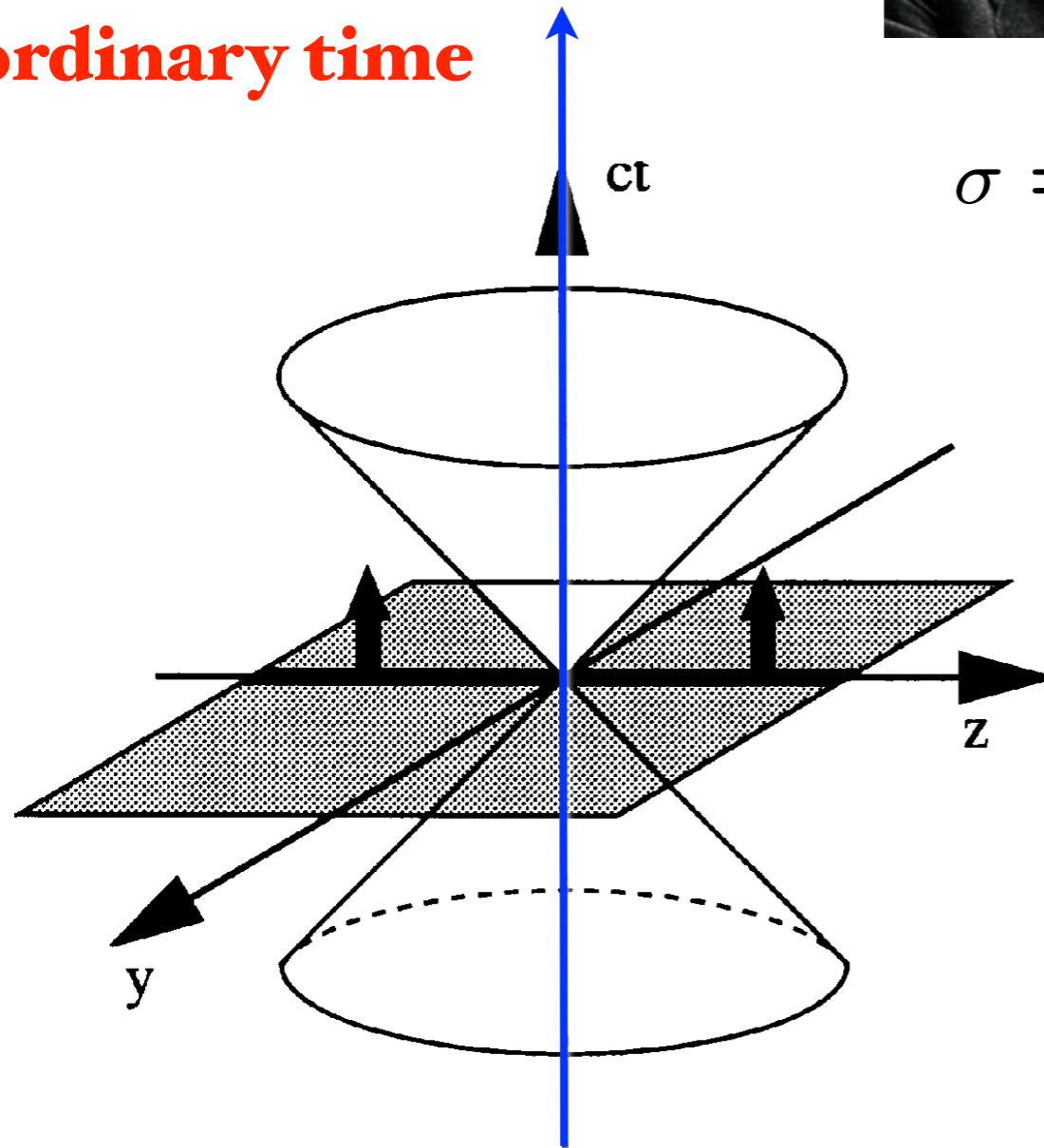
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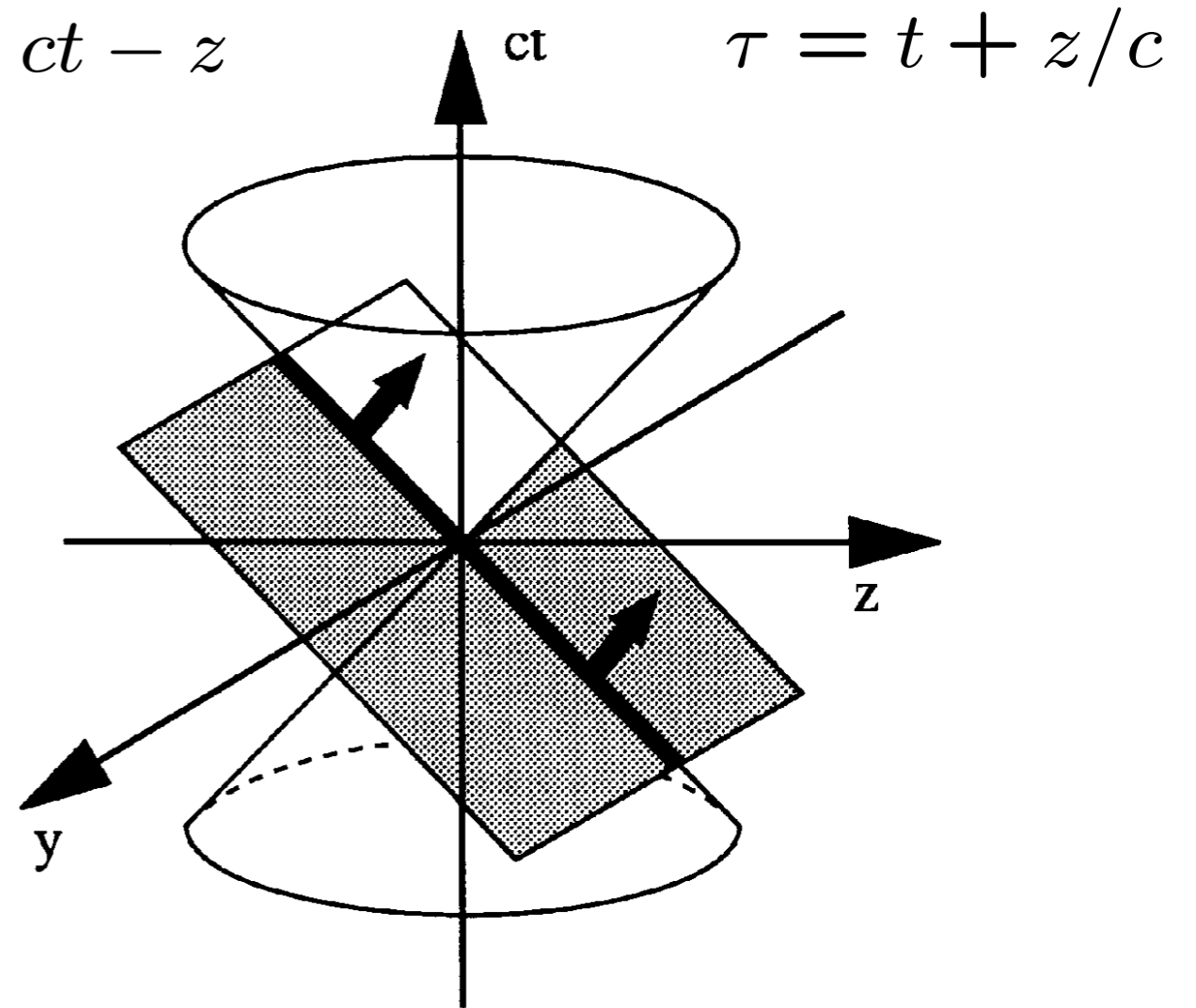
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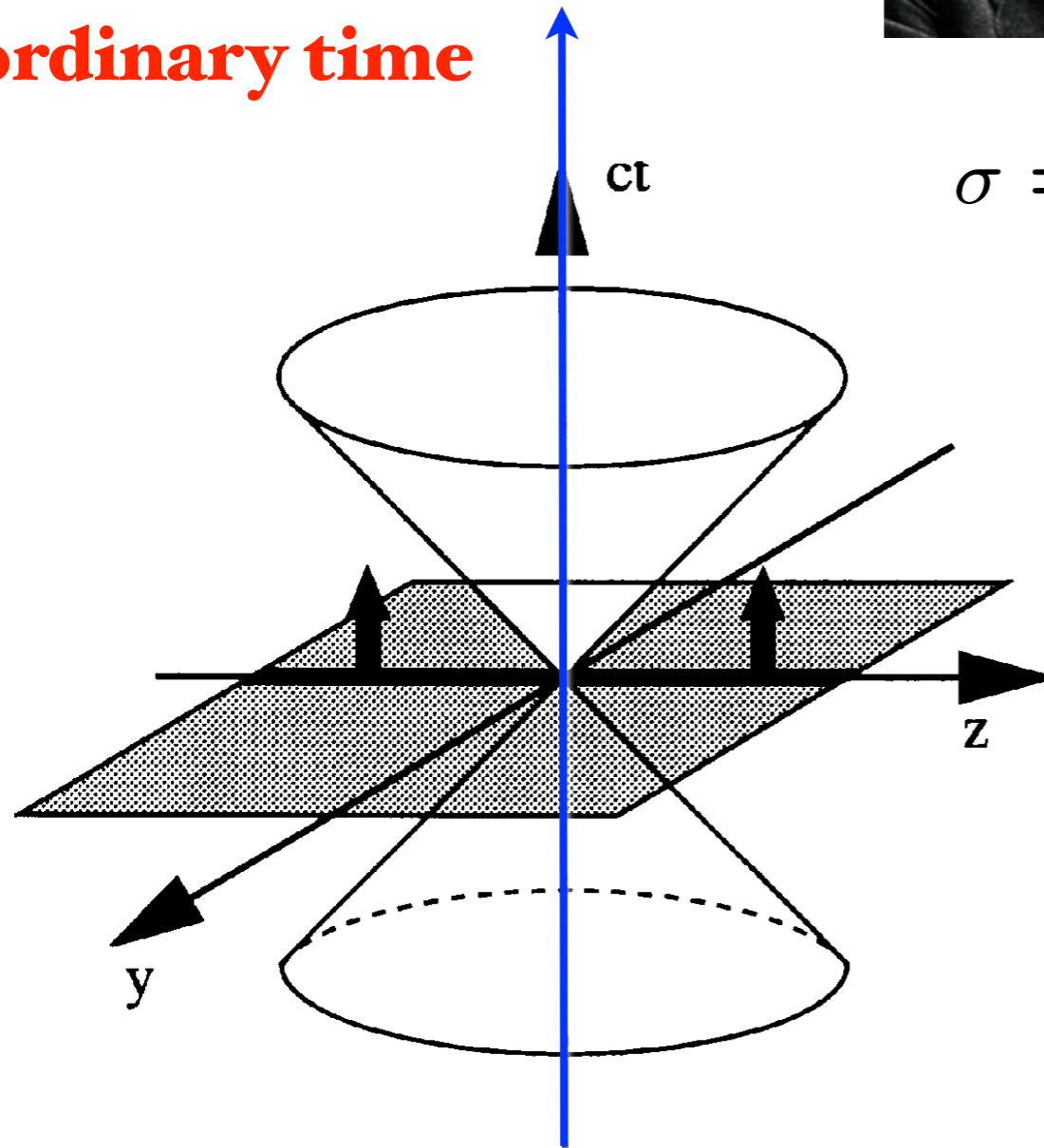
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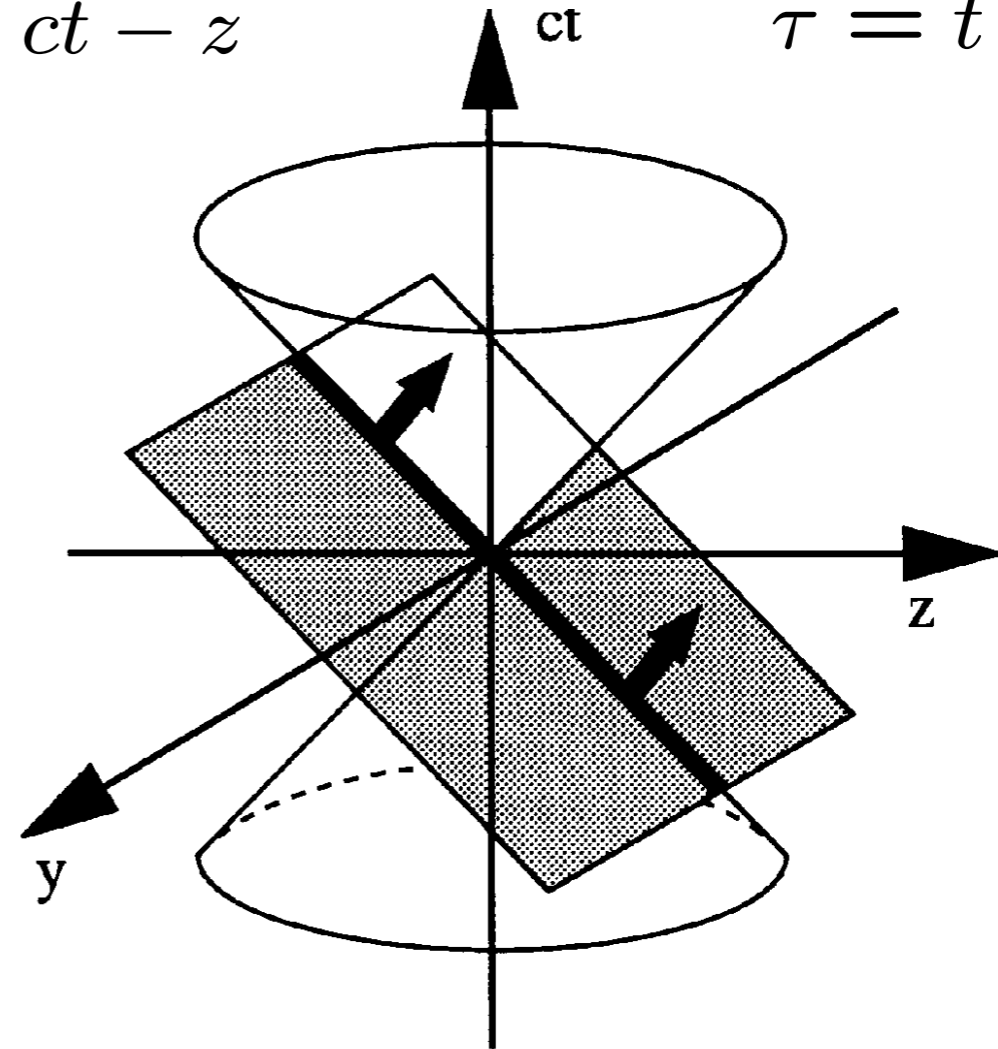


Instant Form

**Evolve in
light-front time!**

$$\sigma = ct - z$$

$$\tau = t + z/c$$



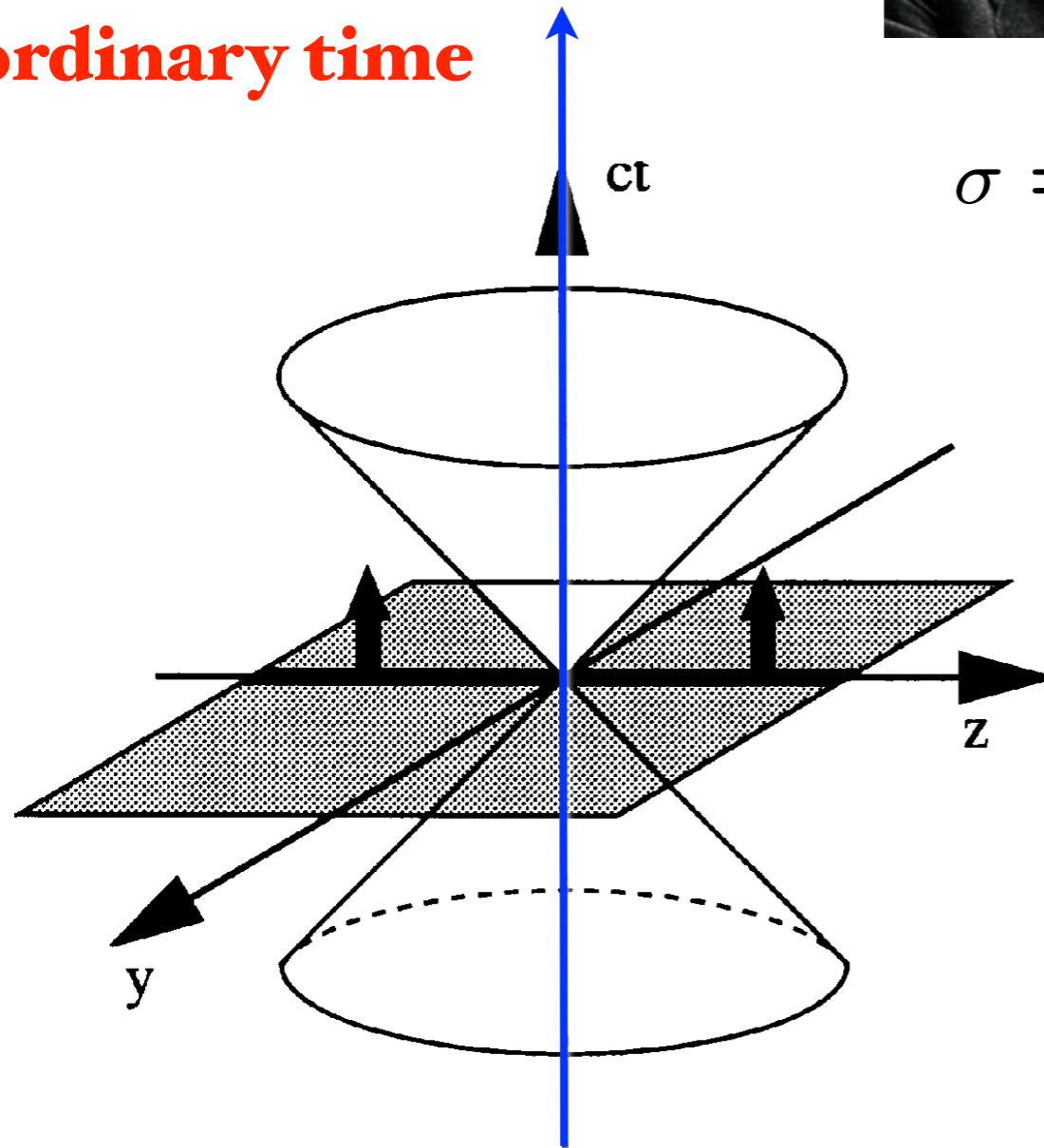
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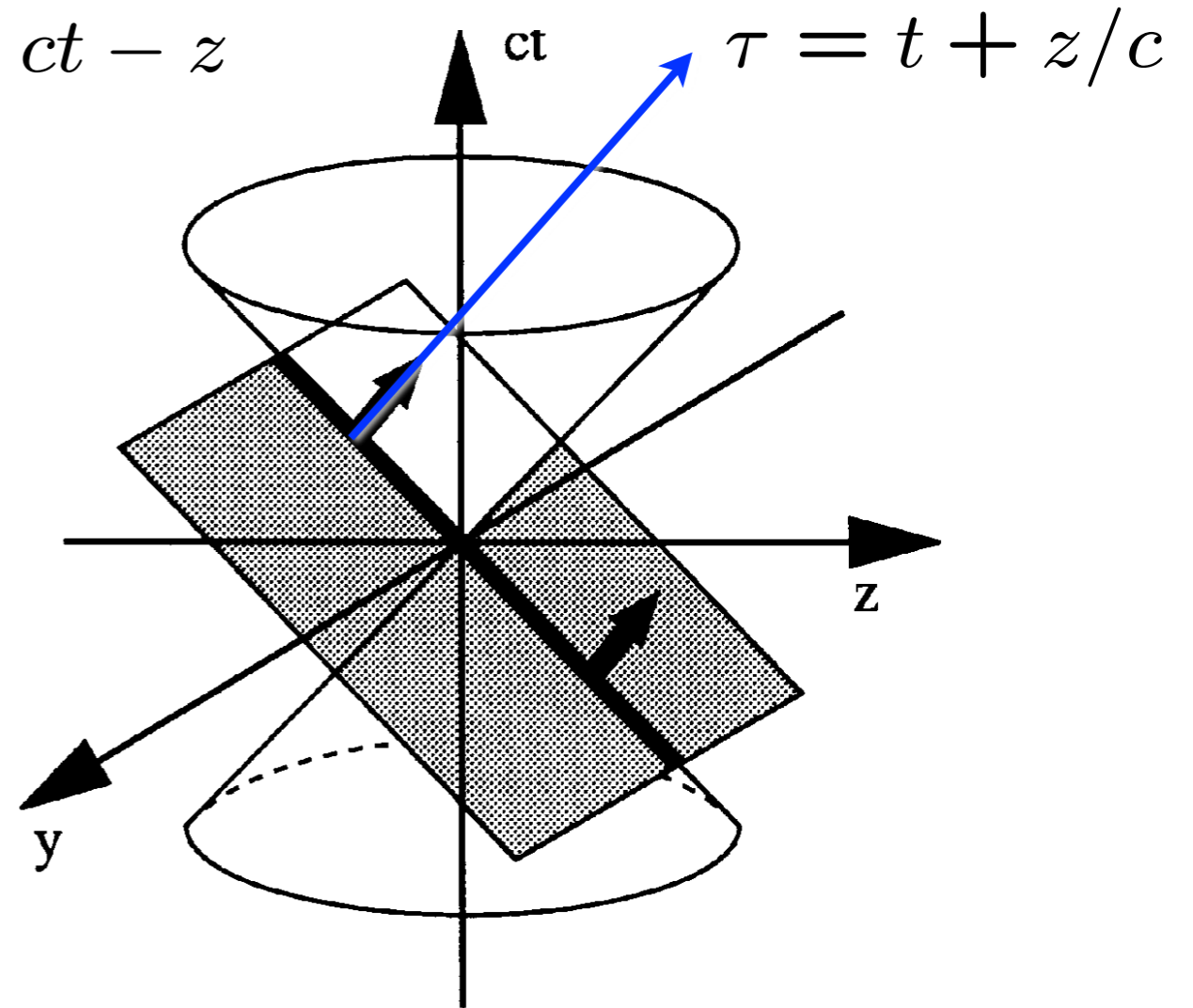
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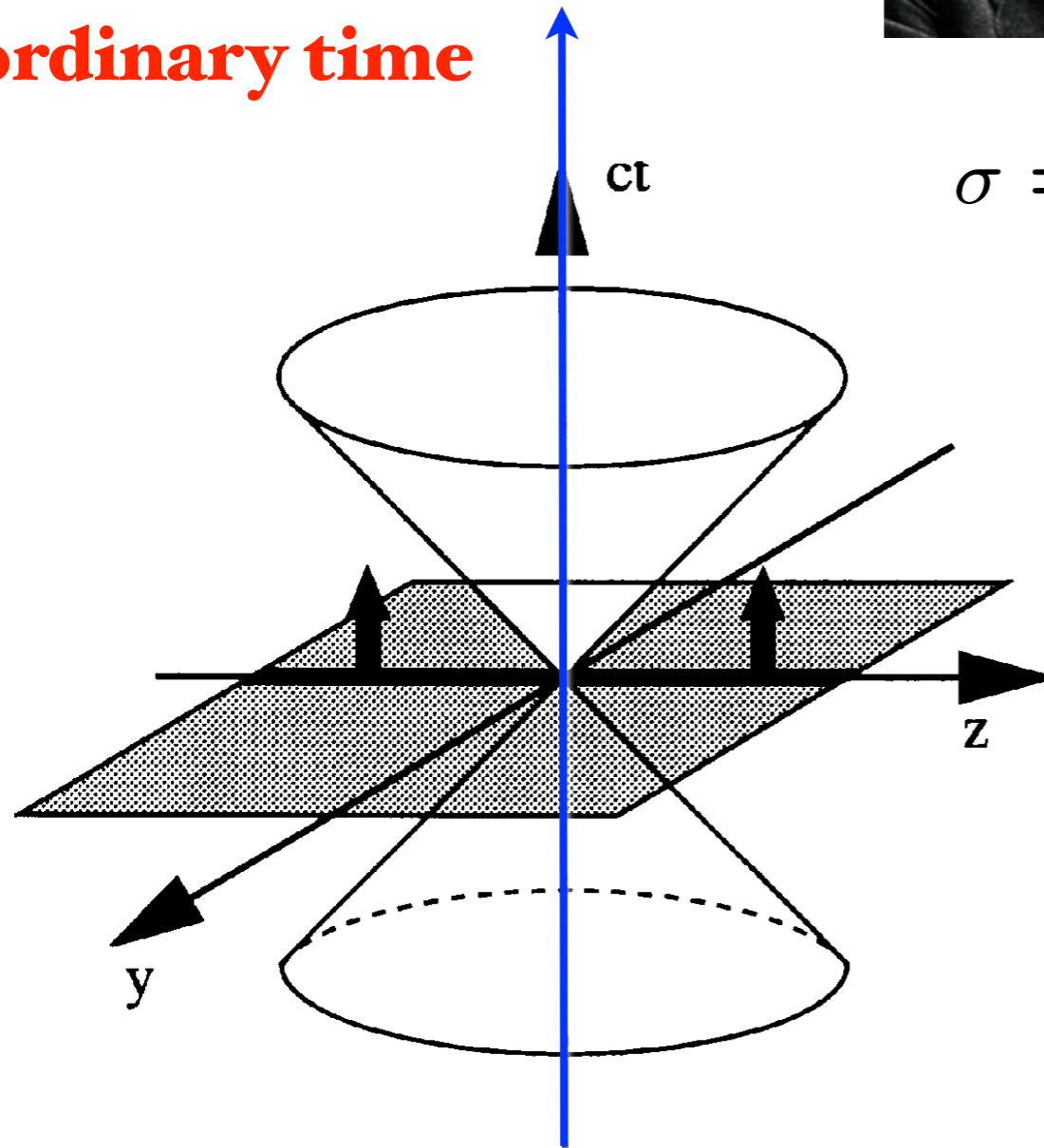
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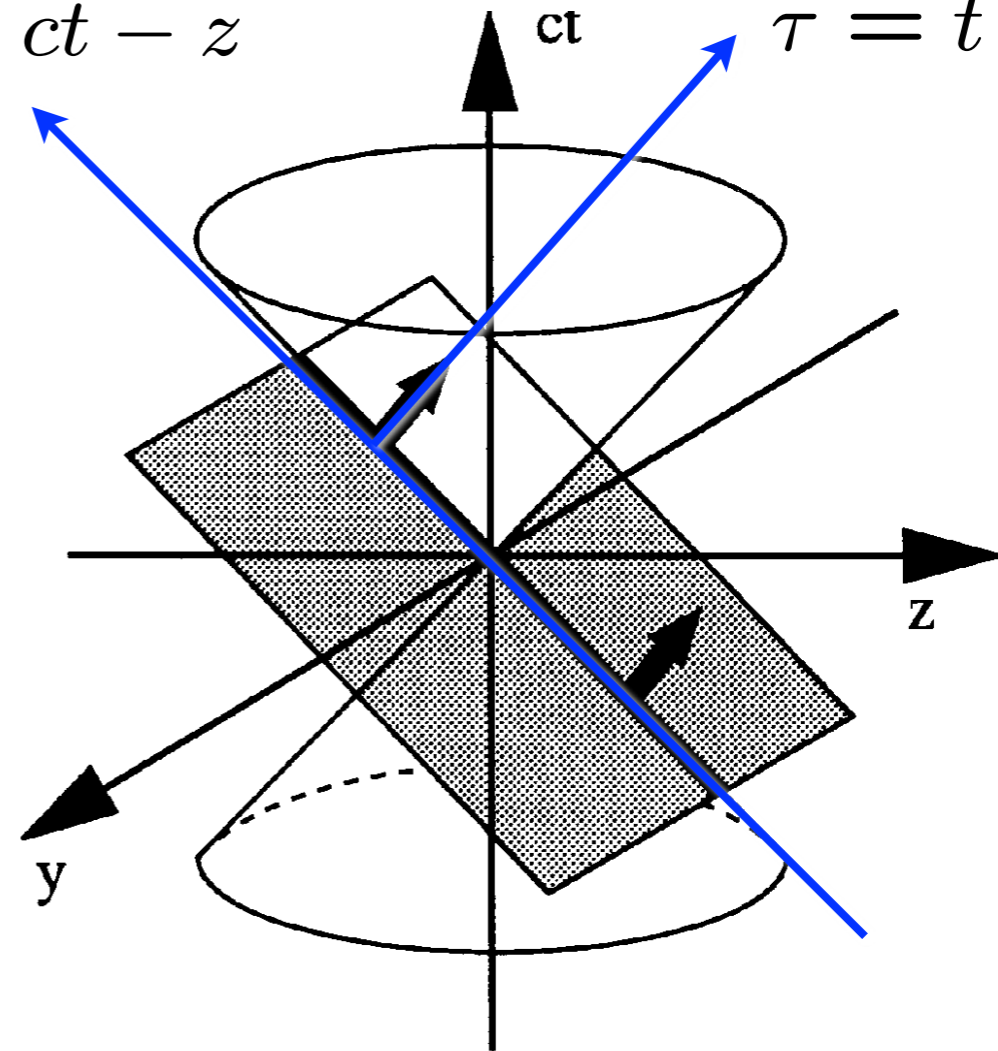


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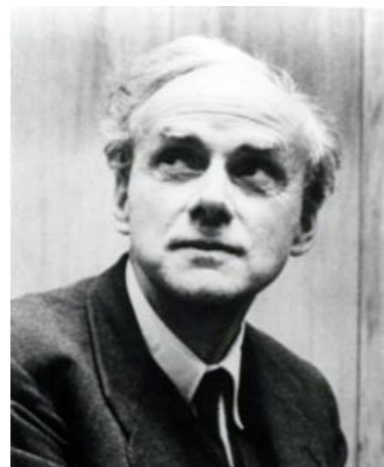
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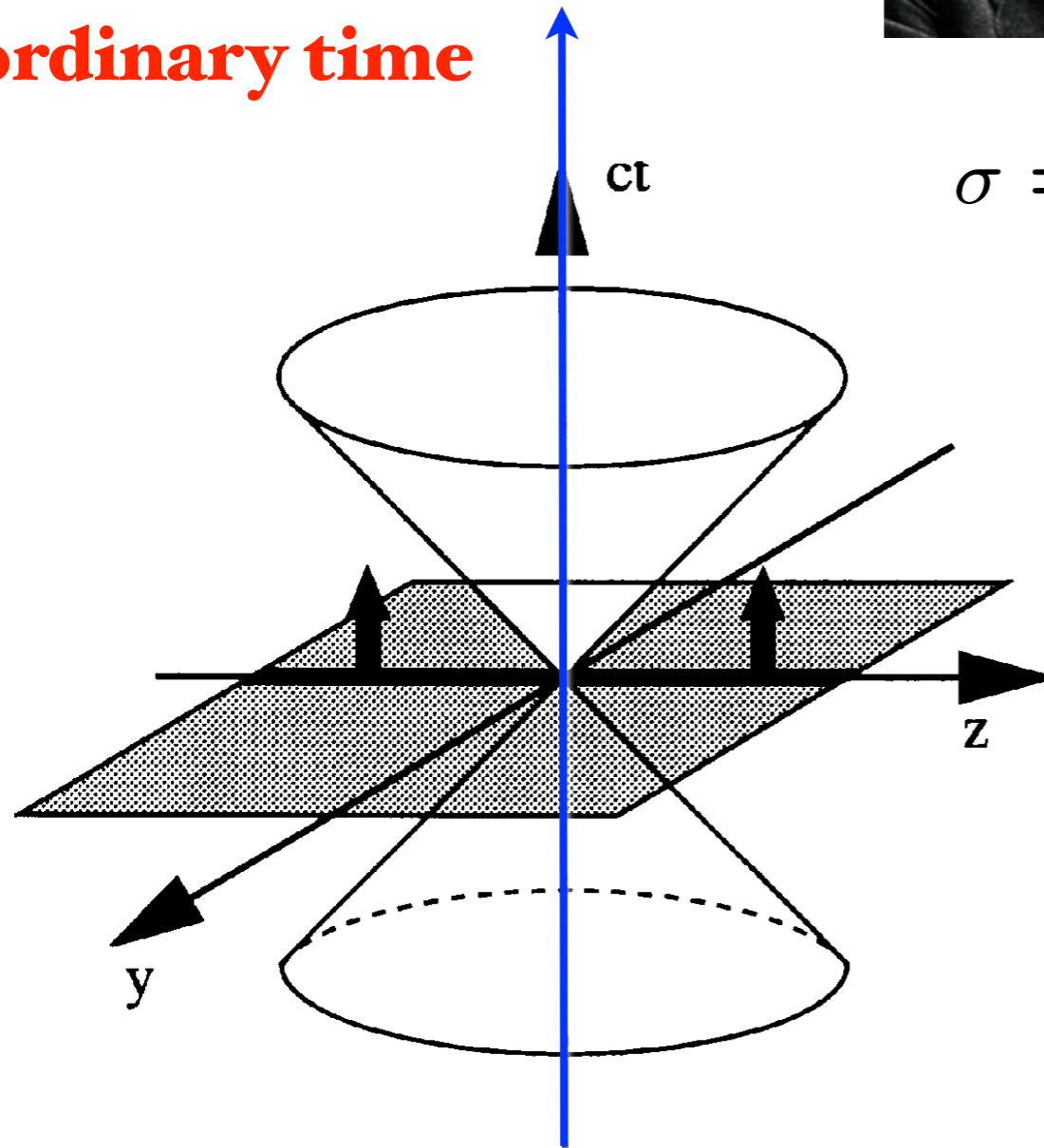
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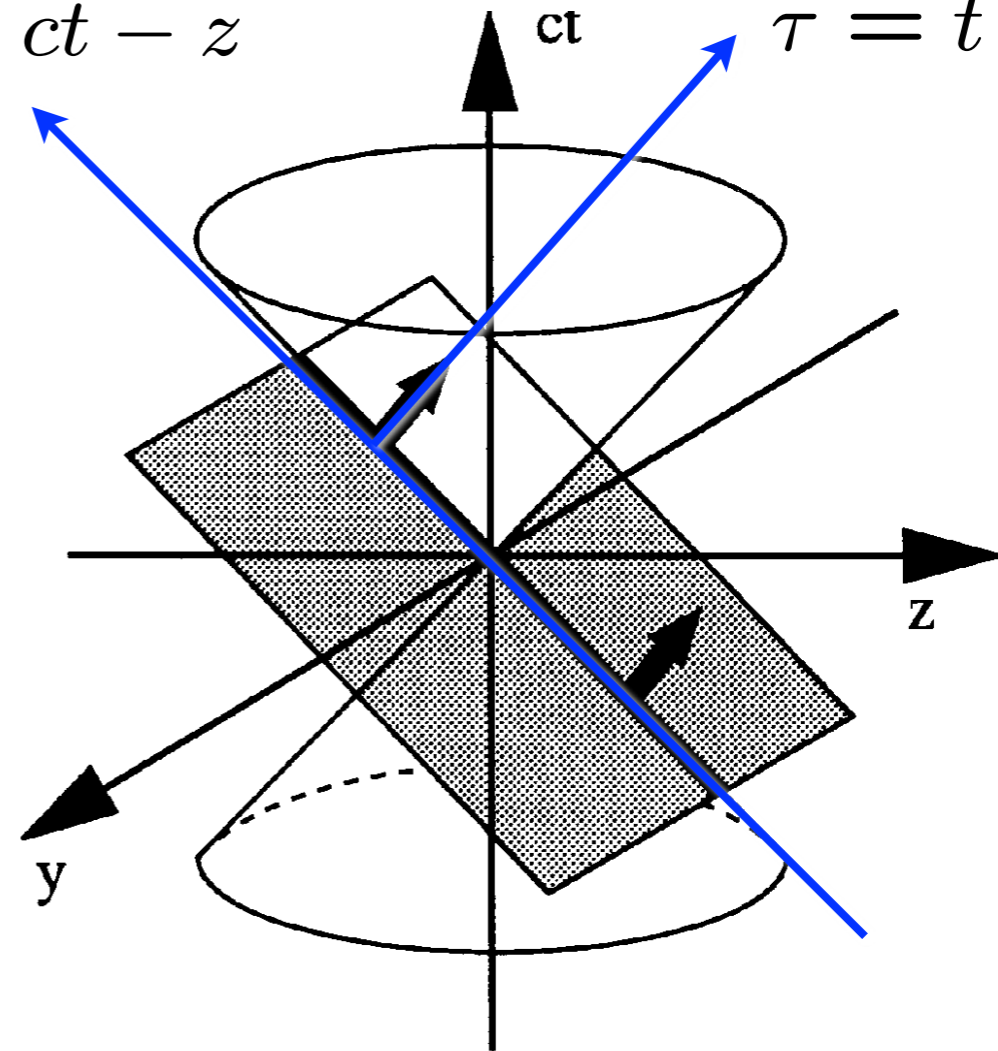


Instant Form

**Evolve in
light-front time!**

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$$\tau = t + z/c$$



Front Form

Boost Invariant!

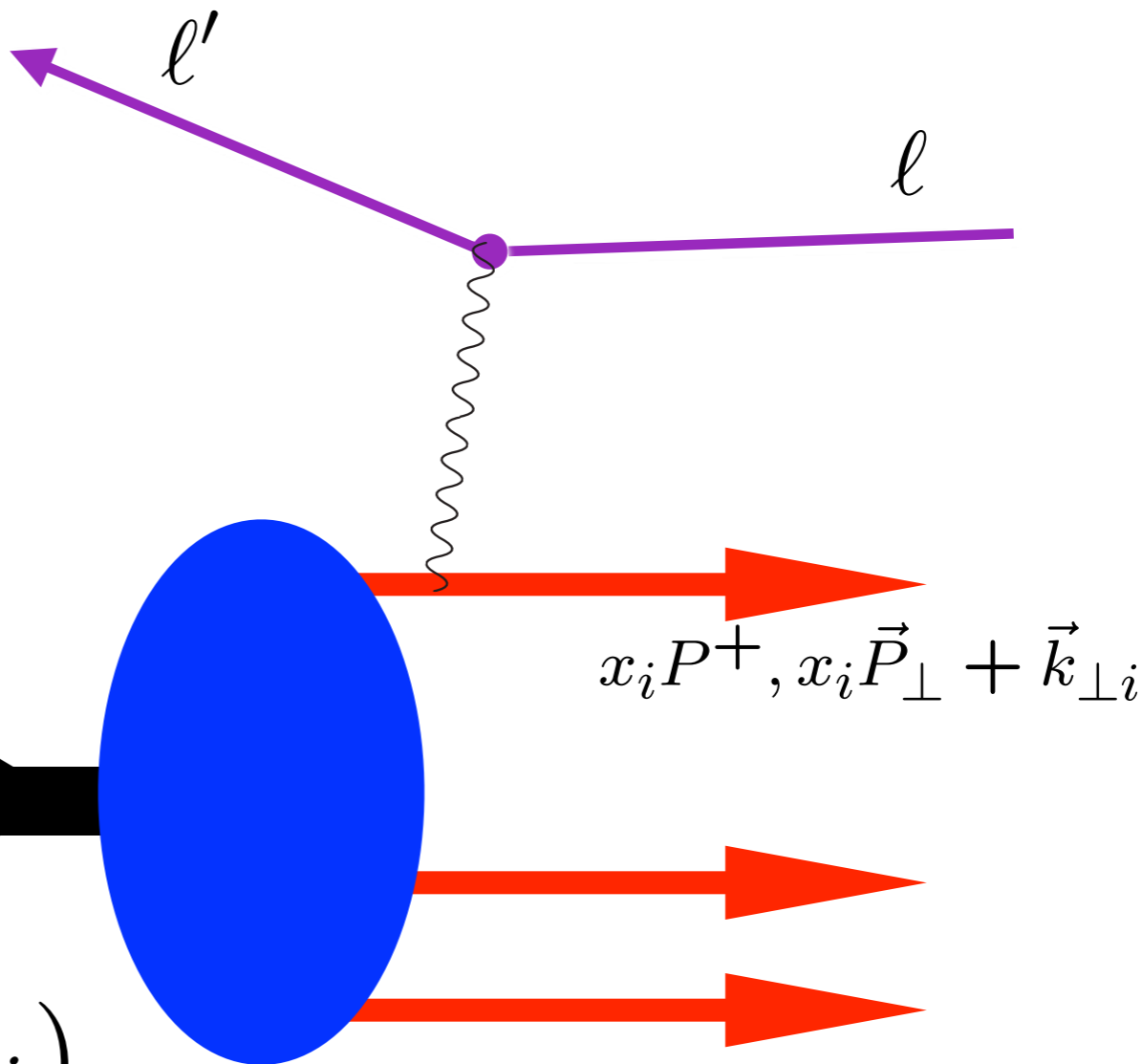


J. D. Bjorken

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

P^+, \vec{P}_\perp

$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

P^+, \vec{P}_\perp

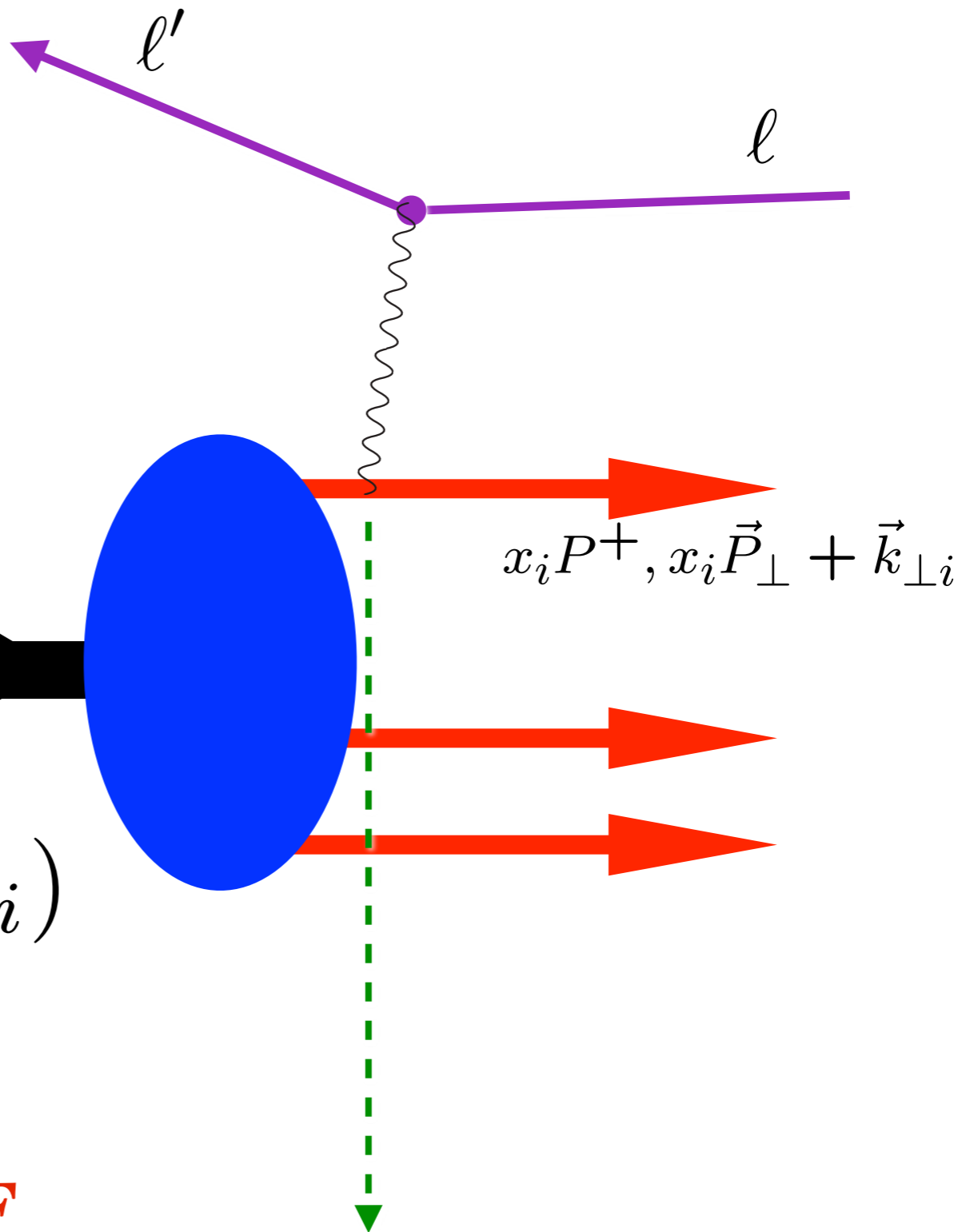
$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

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**Measurements of hadron LF
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Like a flash photograph

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P^+, \vec{P}_\perp

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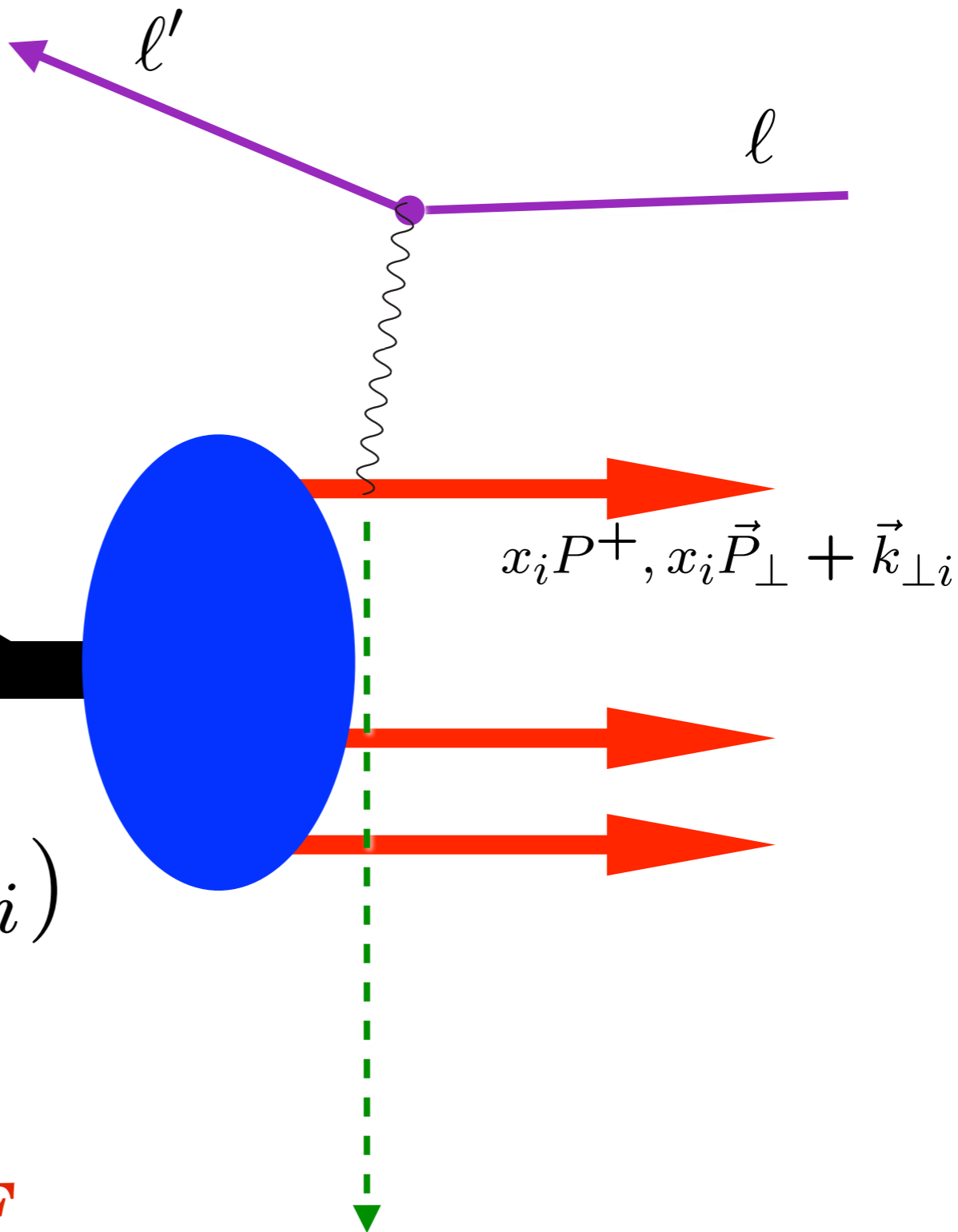
$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

Fixed $\tau = t + z/c$

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Measurements of hadron LF wavefunction are at fixed LF time

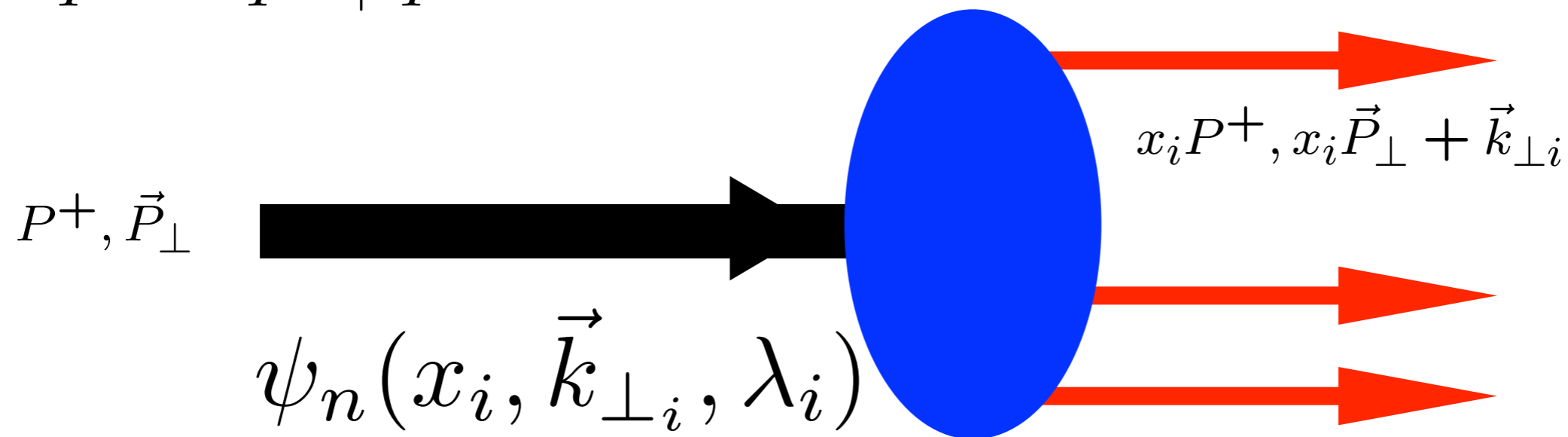
Like a flash photograph



Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

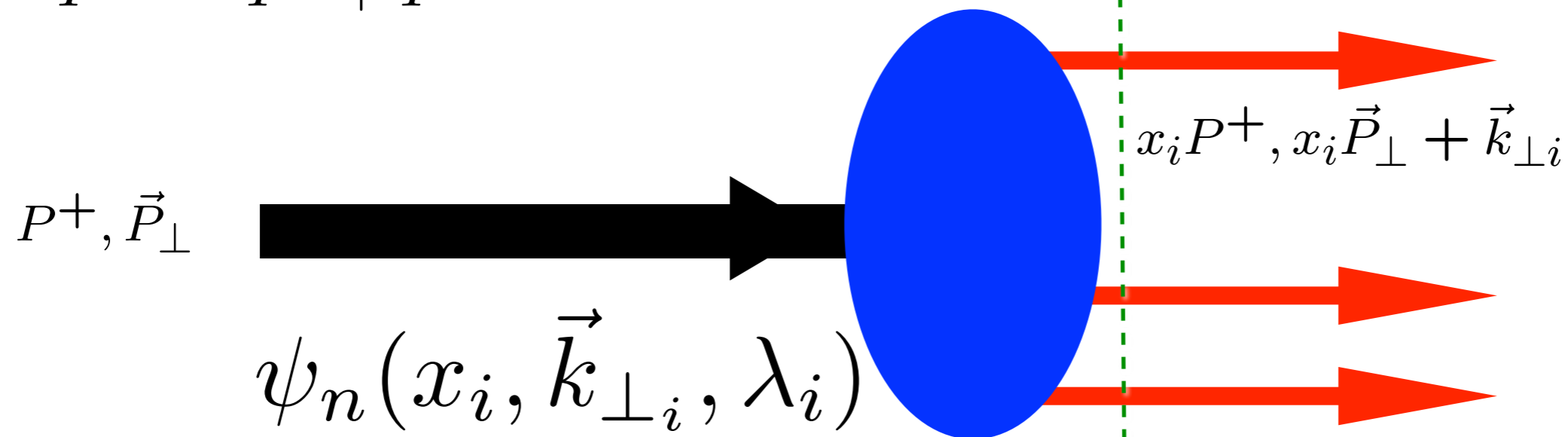


$$|p, J_z \rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i \rangle$$

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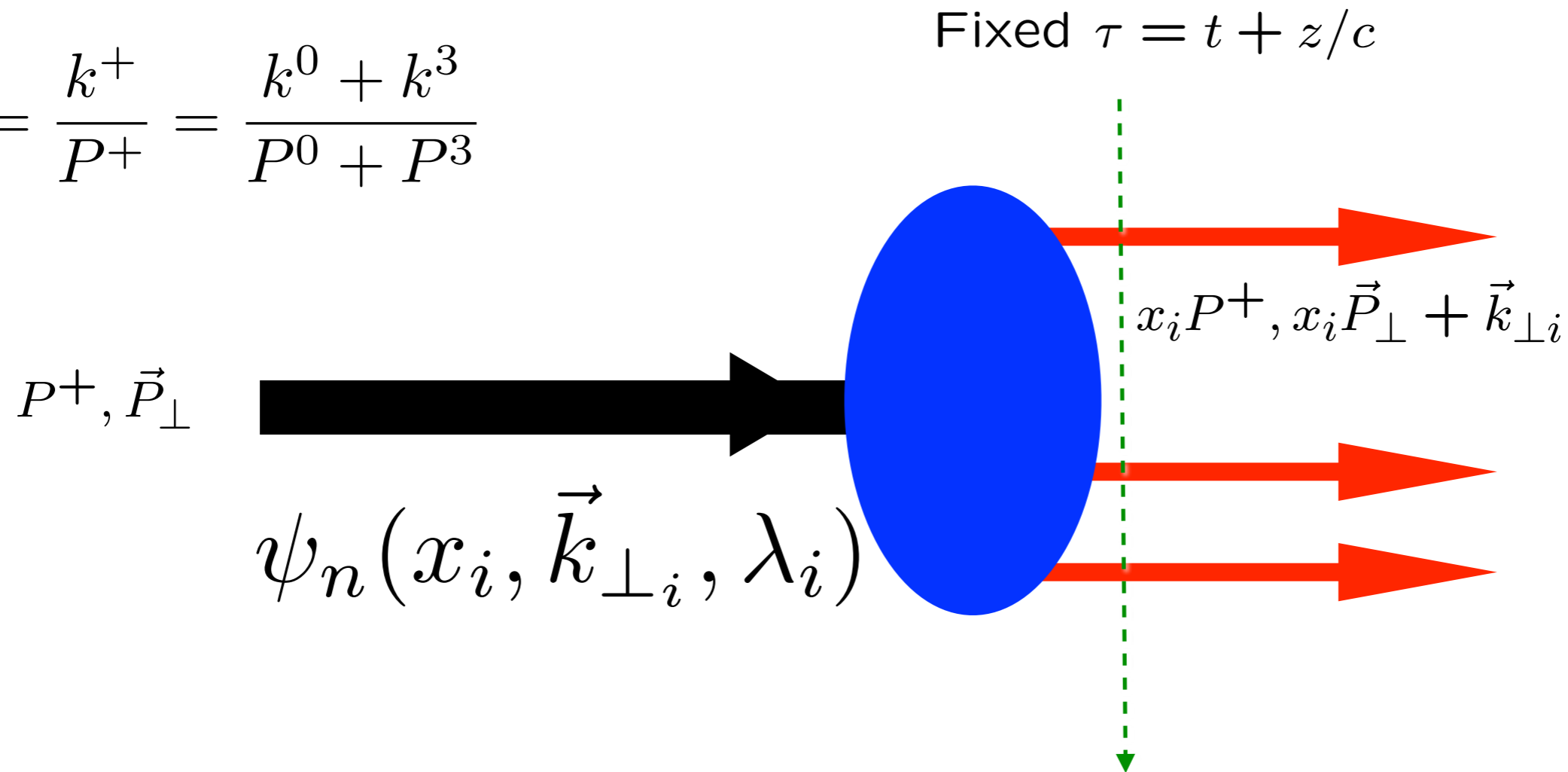


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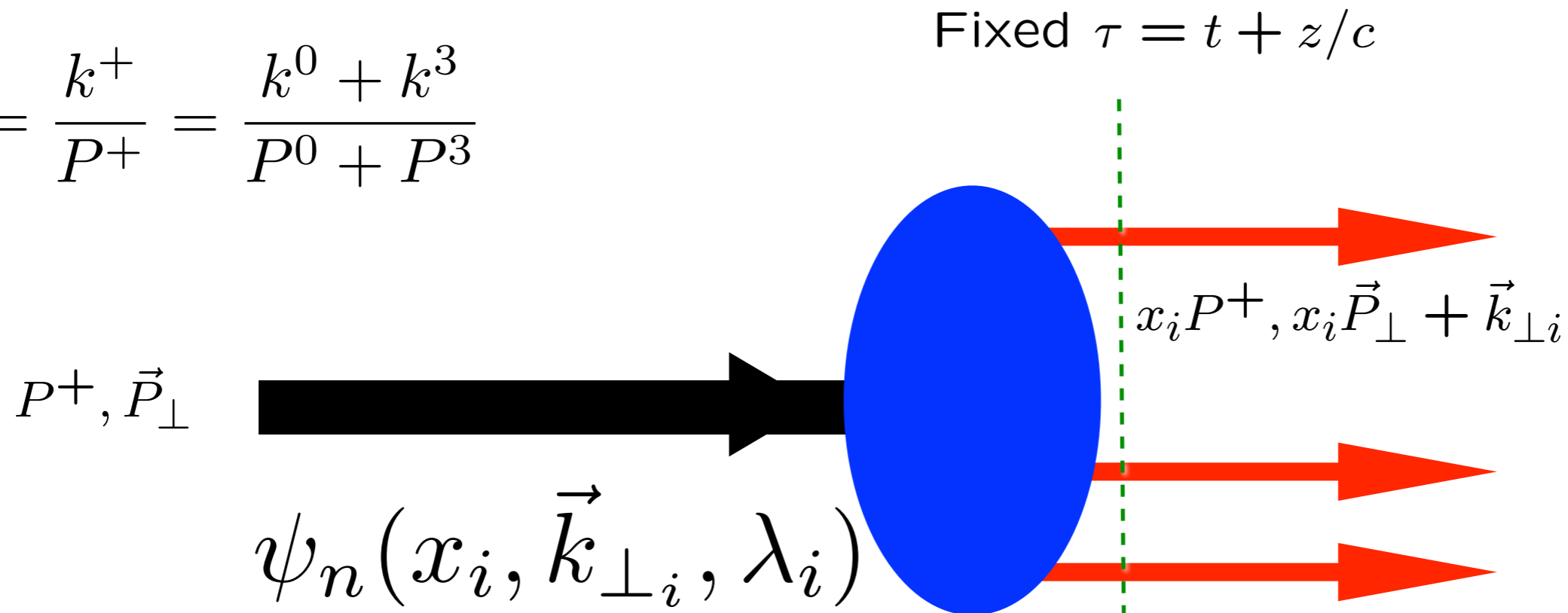


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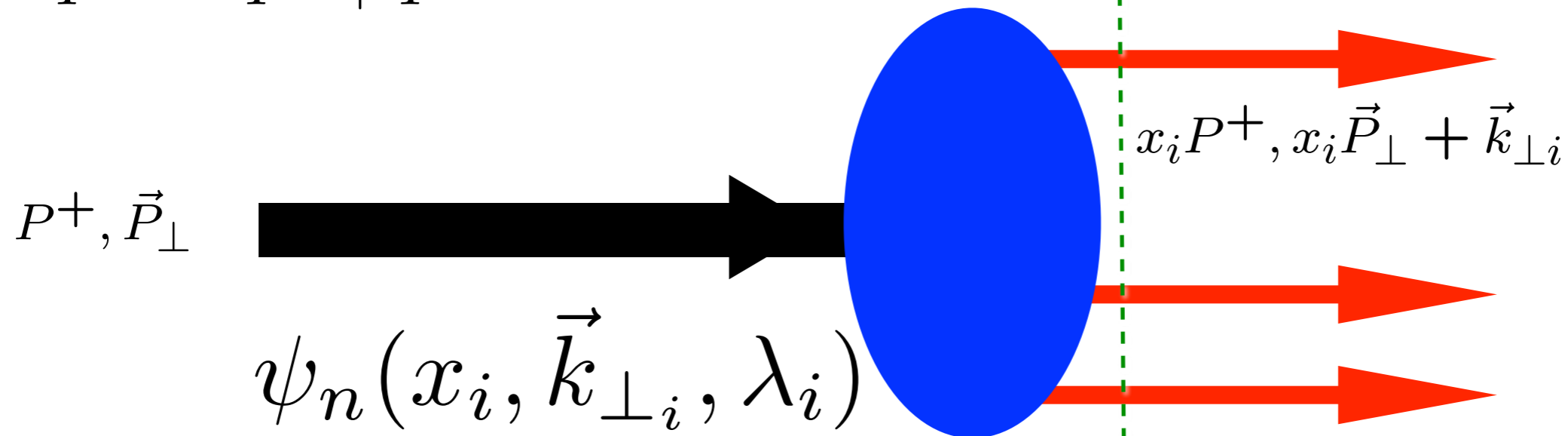
$$|p, J_z \rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i \rangle \quad \sum_i^n x_i = 1$$

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Eigenstate of LF Hamiltonian

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Fixed $\tau = t + z/c$



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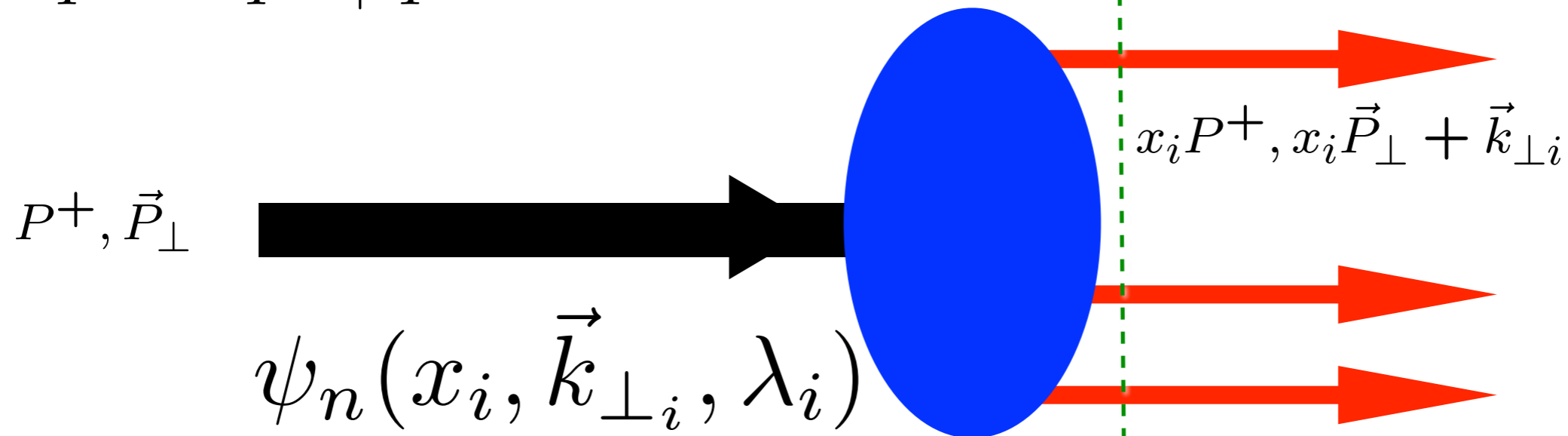
$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

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$$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

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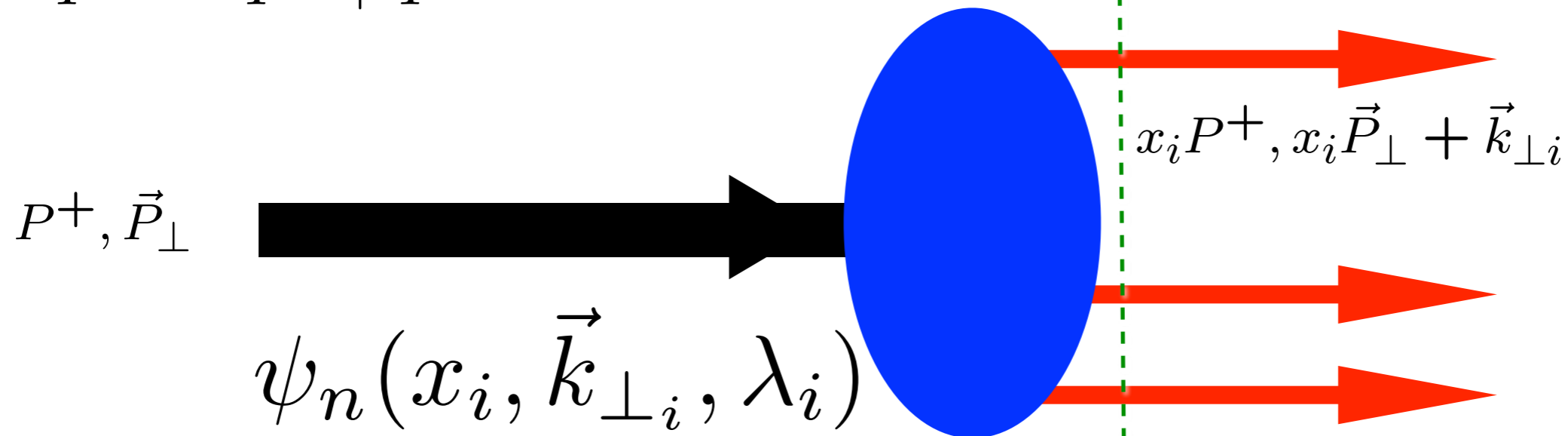
Invariant under boosts! Independent of P^μ

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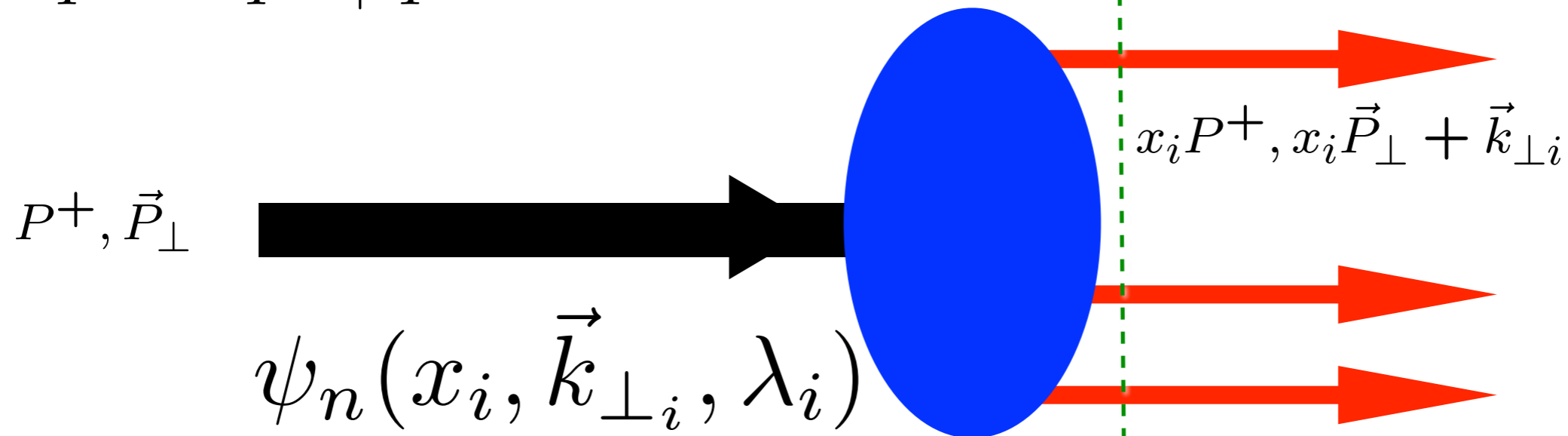
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Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

*Each element of
flash photograph
illuminated
at same LF time*

$$\tau = t + z/c$$



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*Each element of
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Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$



*Each element of
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Eigenstate -- independent of τ



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Causal, frame-independent

Evolve in LF time

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Eigenstate -- independent of τ

$$H_{LF} = P^+ P^- - \vec{P}_\perp^2$$



Each element of
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Causal, frame-independent

Evolve in LF time

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Eigenstate -- independent of τ

$$H_{LF} = P^+ P^- - \vec{P}_\perp^2$$

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$



Advantages of the Dirac's Front Form for Hadron Physics

- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent -- no boosts, no pancakes**
- **No dependence on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no condensates!**
- **Profound implications for Cosmological Constant**



LC2015

Frascati INFN

September 25, 2015

Light-Front Holographic QCD, Color Confinement,
and Supersymmetric Features of QCD

Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

QCD Lagrangian

gluon dynamics

quark kinetic energy +
quark-gluon dynamics

quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics quark kinetic energy +
quark-gluon dynamics quark mass term

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**Yang Mills Gauge Principle: Color
Rotation and Phase Invariance at
Every Point of Space and Time**

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**Yang Mills Gauge Principle: Color
Rotation and Phase Invariance at
Every Point of Space and Time**

**Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement**

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gluon dynamics quark kinetic energy + quark-gluon dynamics quark mass term

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Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

**Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement**

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics quark kinetic energy + quark-gluon dynamics quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if $m_q=0$

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

**Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement**

QCD Lagrangian

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QCD Mass Scale from Confinement not Explicit

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of
nonperturbative QCD!

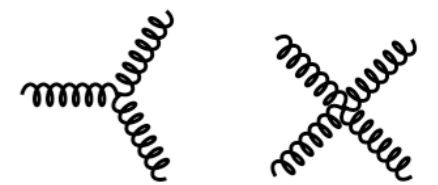
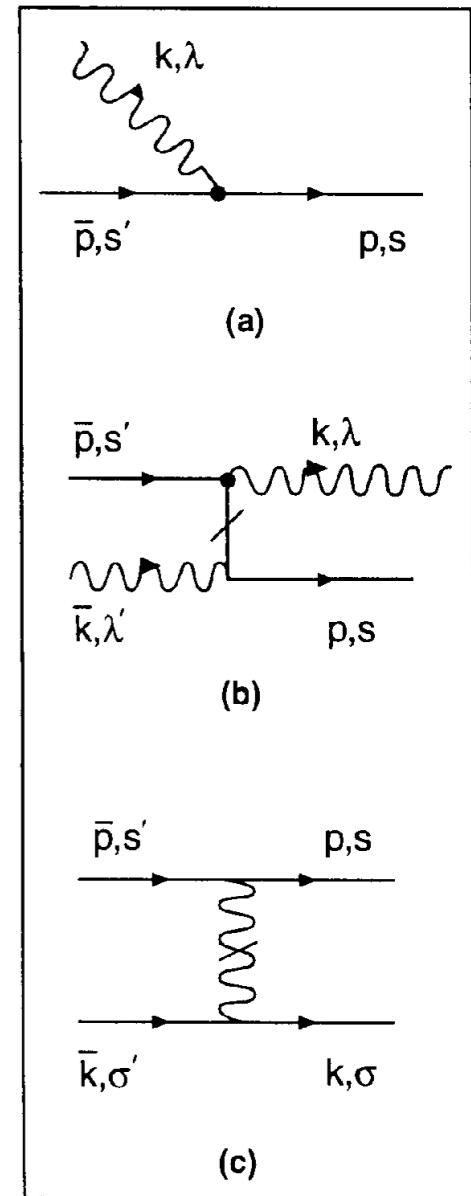
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$



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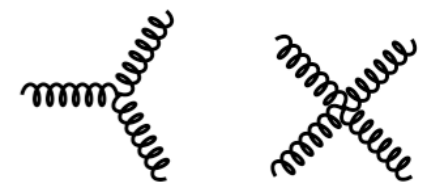
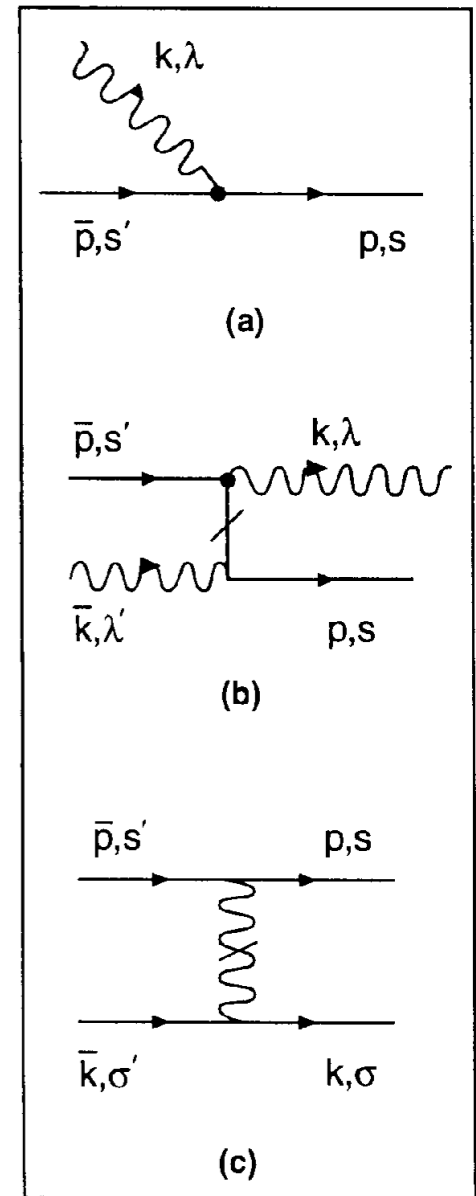
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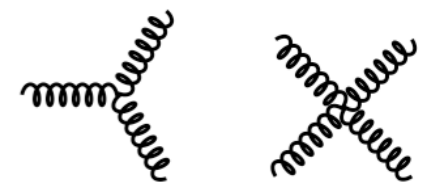
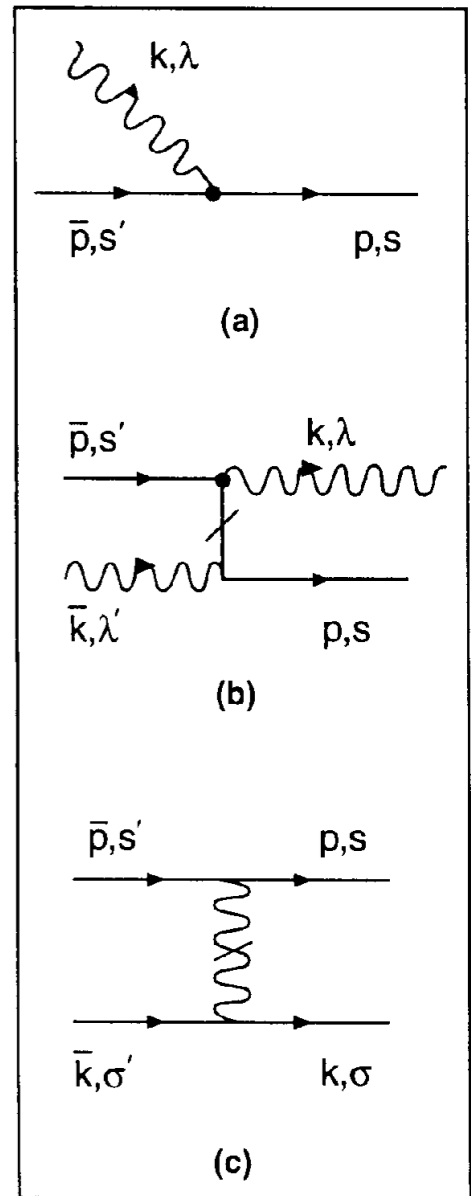
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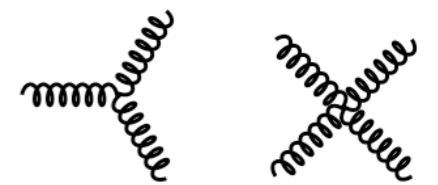
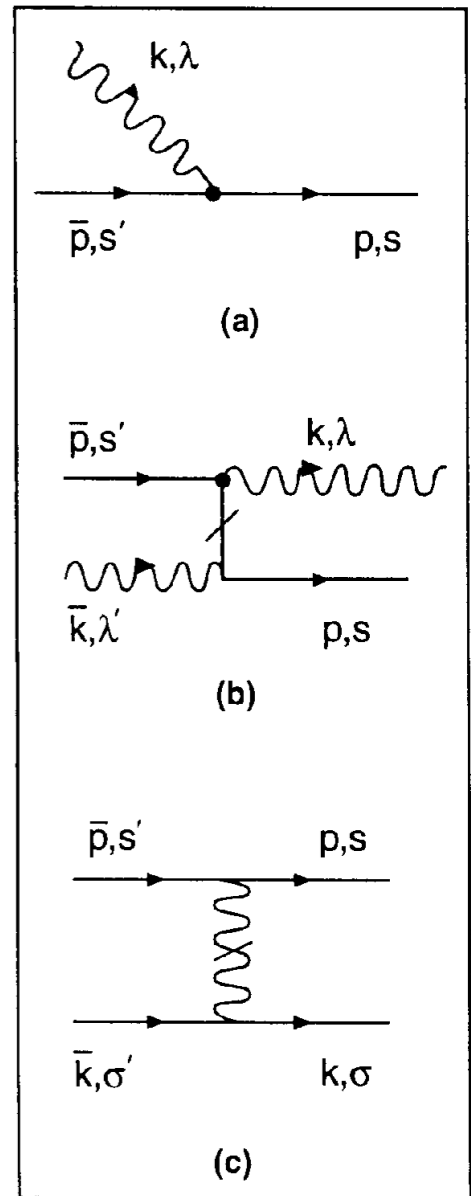
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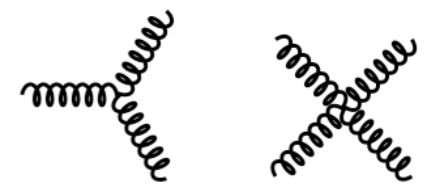
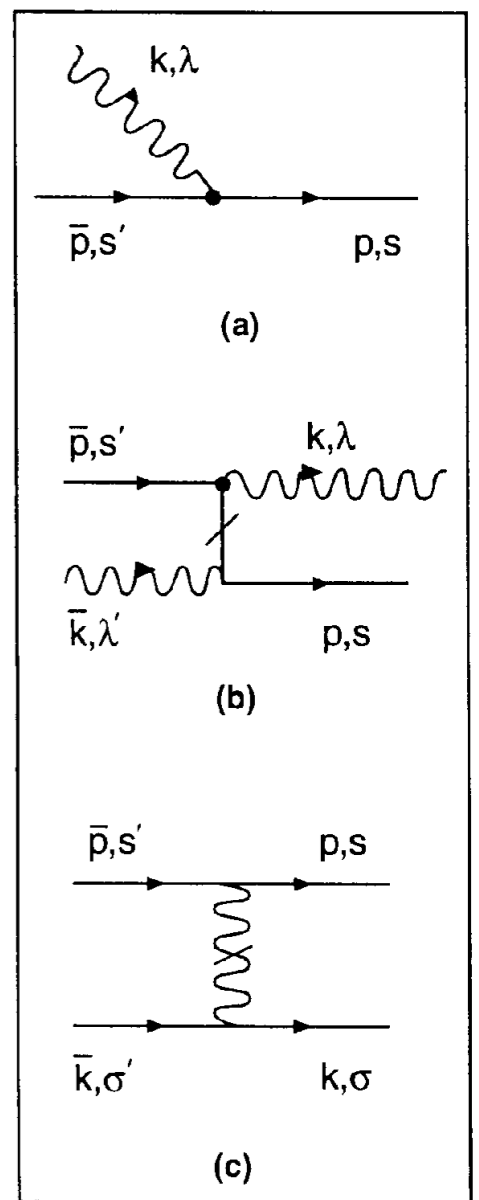
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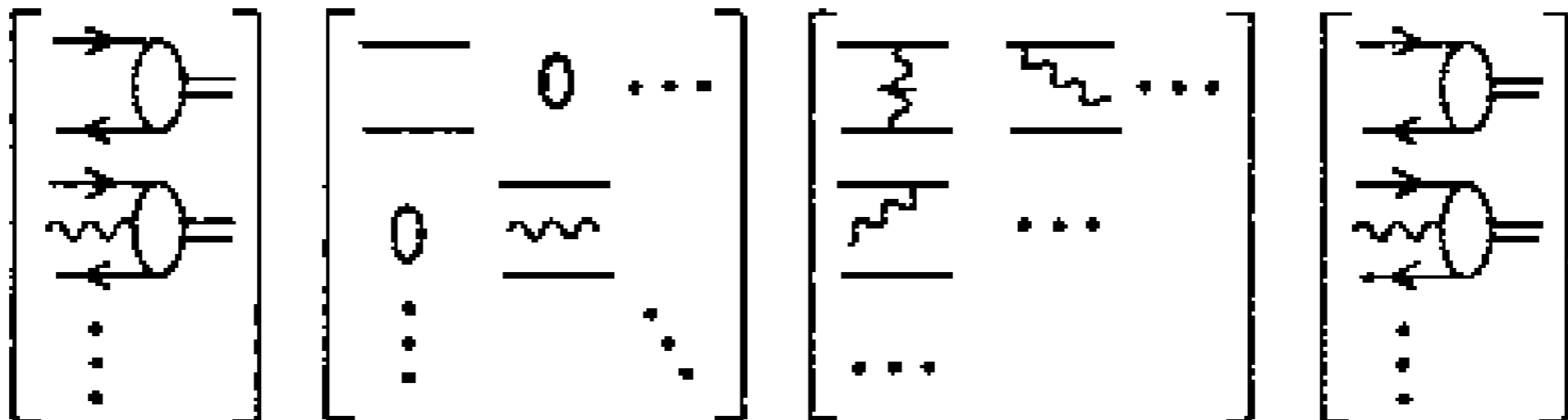
LFWFs: Off-shell in P- and invariant mass

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} \right) \begin{bmatrix} \psi_{q\bar{q}}/\pi \\ \psi_{q\bar{q}g}/\pi \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q} | V | q\bar{q} \rangle & \langle q\bar{q} | V | q\bar{q}g \rangle & \cdots \\ \langle q\bar{q}g | V | q\bar{q} \rangle & \langle q\bar{q}g | V | q\bar{q}g \rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}}/\pi \\ \psi_{q\bar{q}g}/\pi \\ \vdots \end{bmatrix}$$

$$A^+ = 0$$



Minkowski space; frame-independent; no fermion doubling; no ghosts

- *Light-Front Vacuum = vacuum of free Hamiltonian!*

Stan Brodsky

LC2015

Frascati INFN

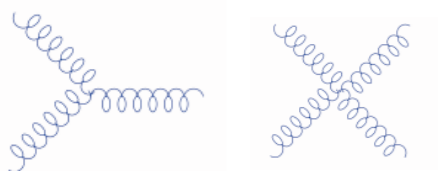
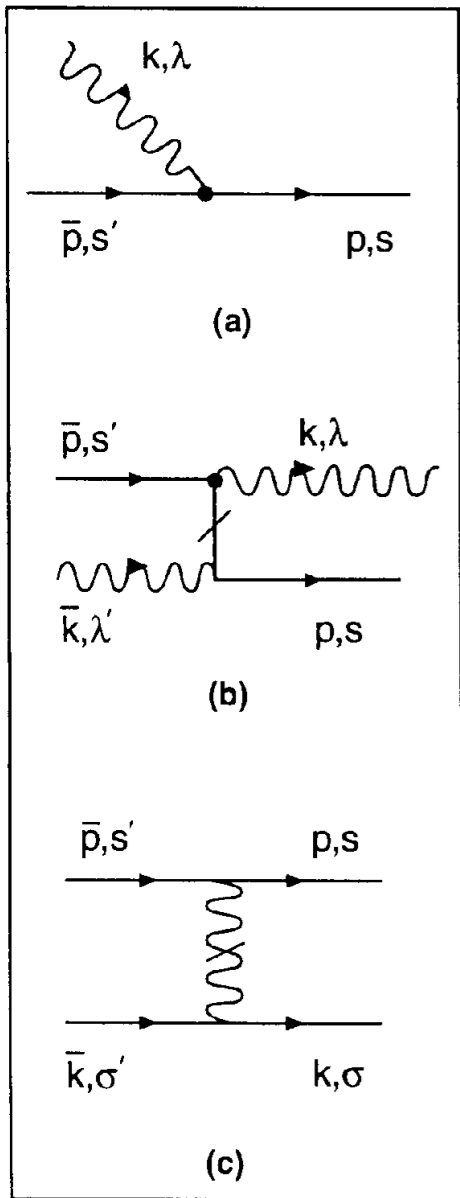
September 25, 2015

**Light-Front Holographic QCD, Color Confinement,
and Supersymmetric Features of QCD**

SLAC
NATIONAL ACCELERATOR LABORATORY

$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

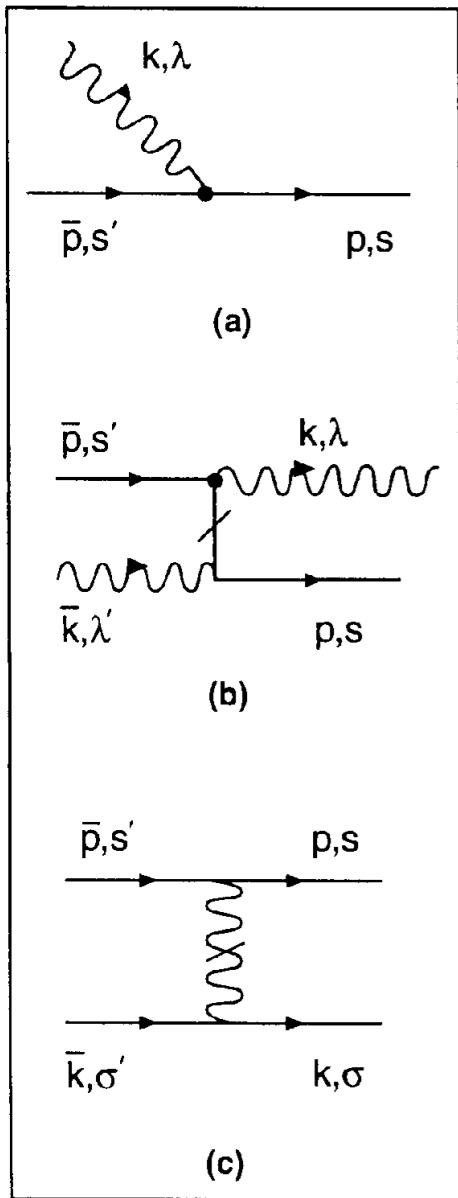
Hornbostel, Pauli, sjb



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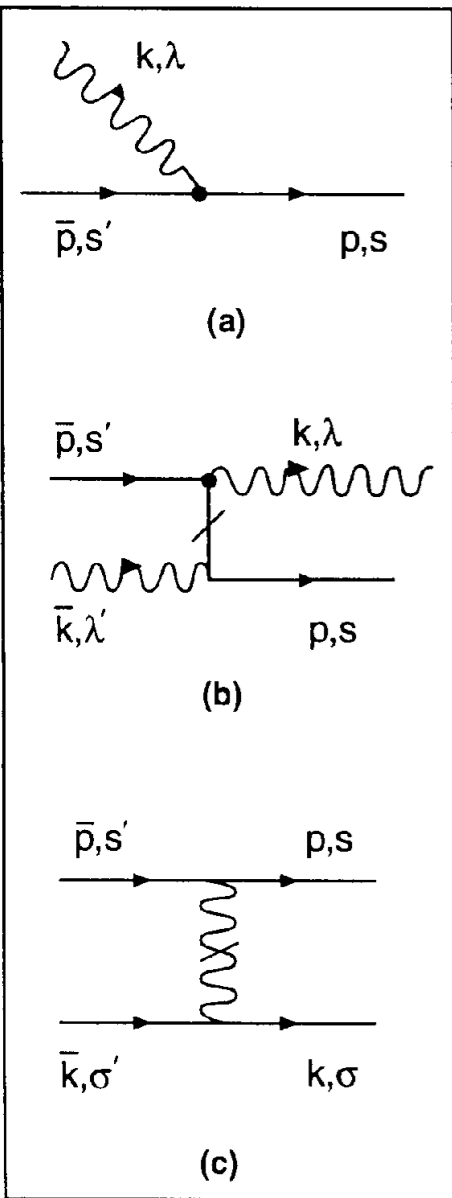


n	Sector	1 q \bar{q}	2 gg	3 q \bar{q} g	4 q \bar{q} q \bar{q}	5 gg g	6 q \bar{q} gg	7 q \bar{q} q \bar{q} g	8 q \bar{q} q \bar{q} q \bar{q}	9 gg gg	10 q \bar{q} gg g	11 q \bar{q} q \bar{q} gg	12 q \bar{q} q \bar{q} q \bar{q} g	13 q \bar{q} q \bar{q} q \bar{q} q \bar{q}
1	q \bar{q}				
2	gg			
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5	gg g
6	q \bar{q} gg								.				.	.
7	q \bar{q} q \bar{q} g
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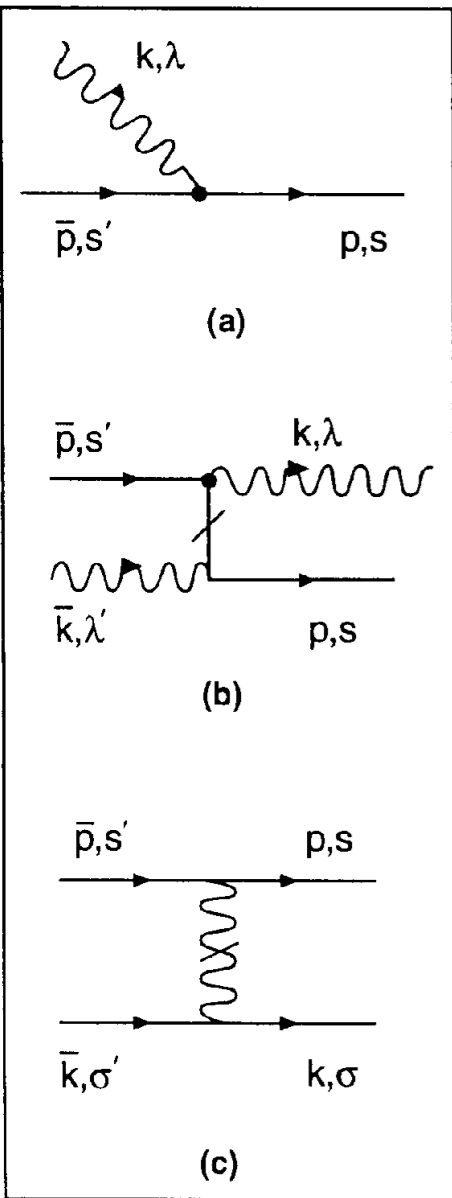
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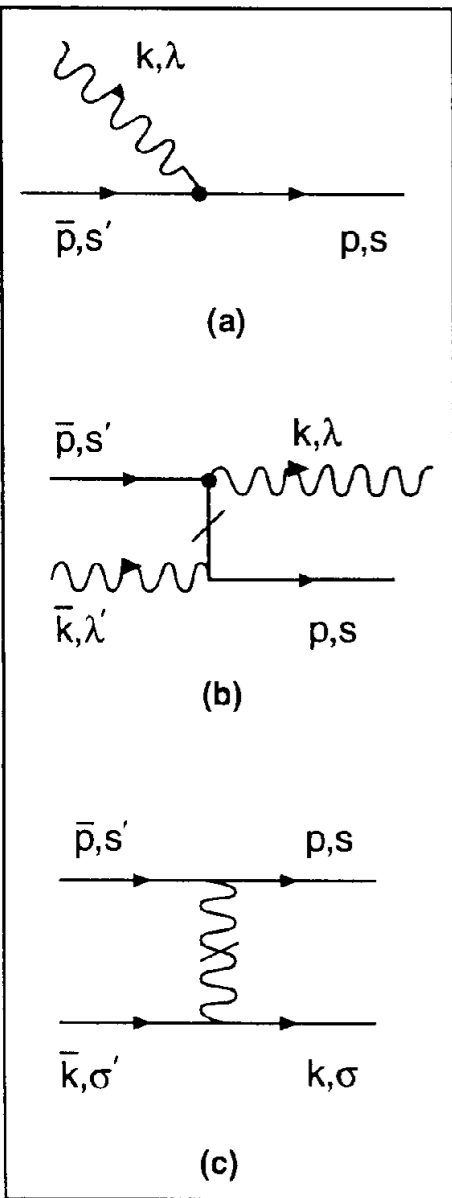
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Light-Front QCD
Heisenberg Equation

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Hornbostel, Pauli, sjb



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12	q \bar{q} q \bar{q} q \bar{q} g	[Diagram]	[Diagram]	.	.	[Diagram]	[Diagram]	[Diagram]
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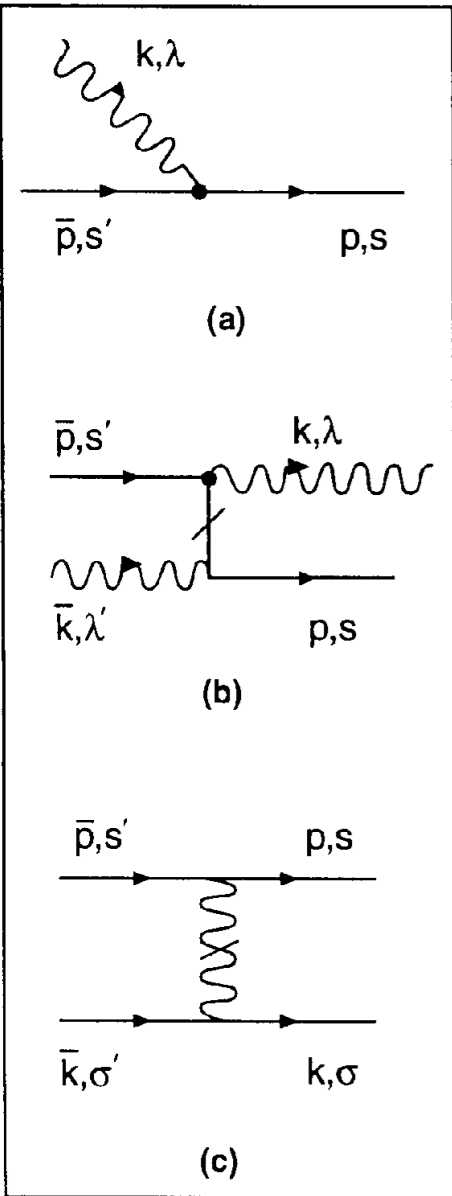
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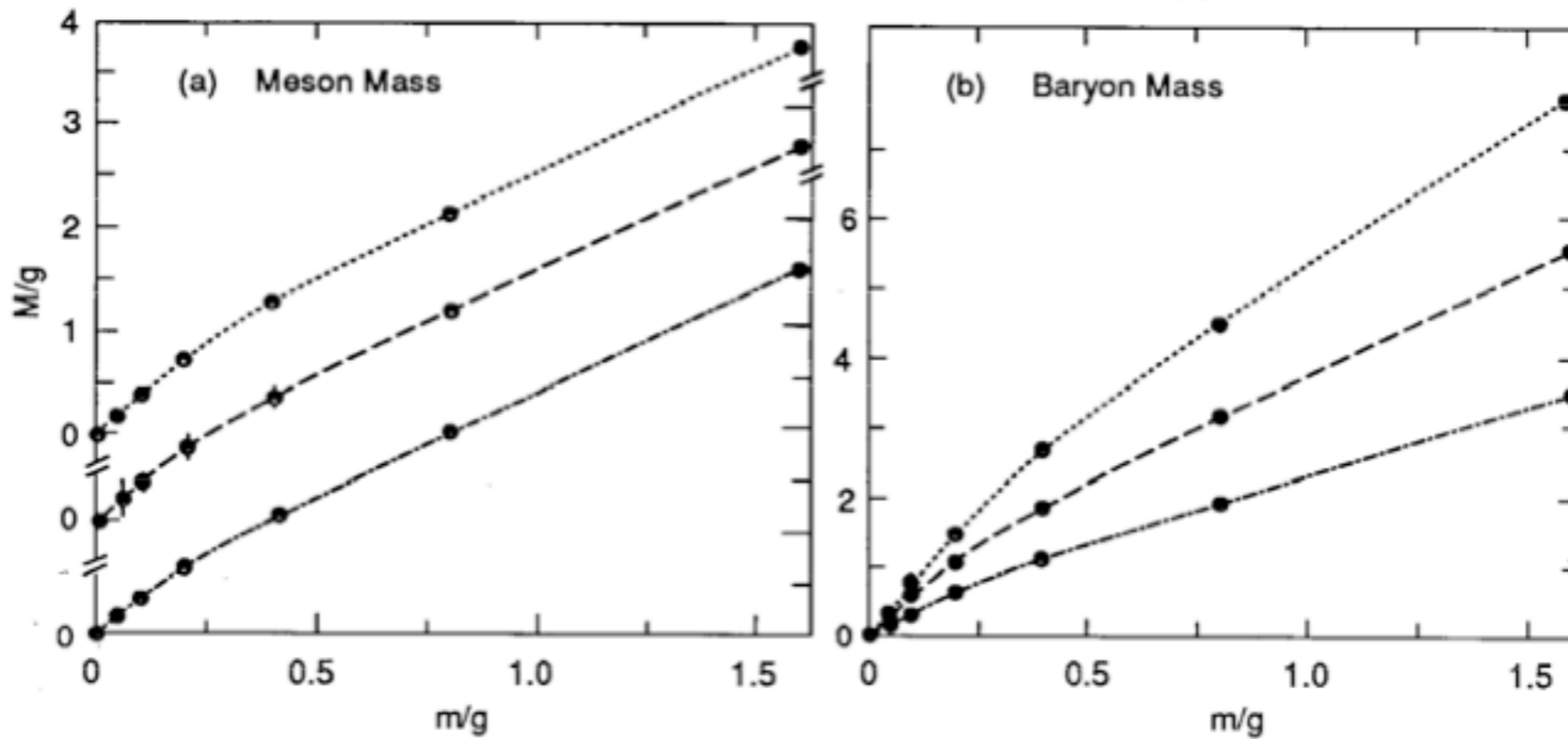
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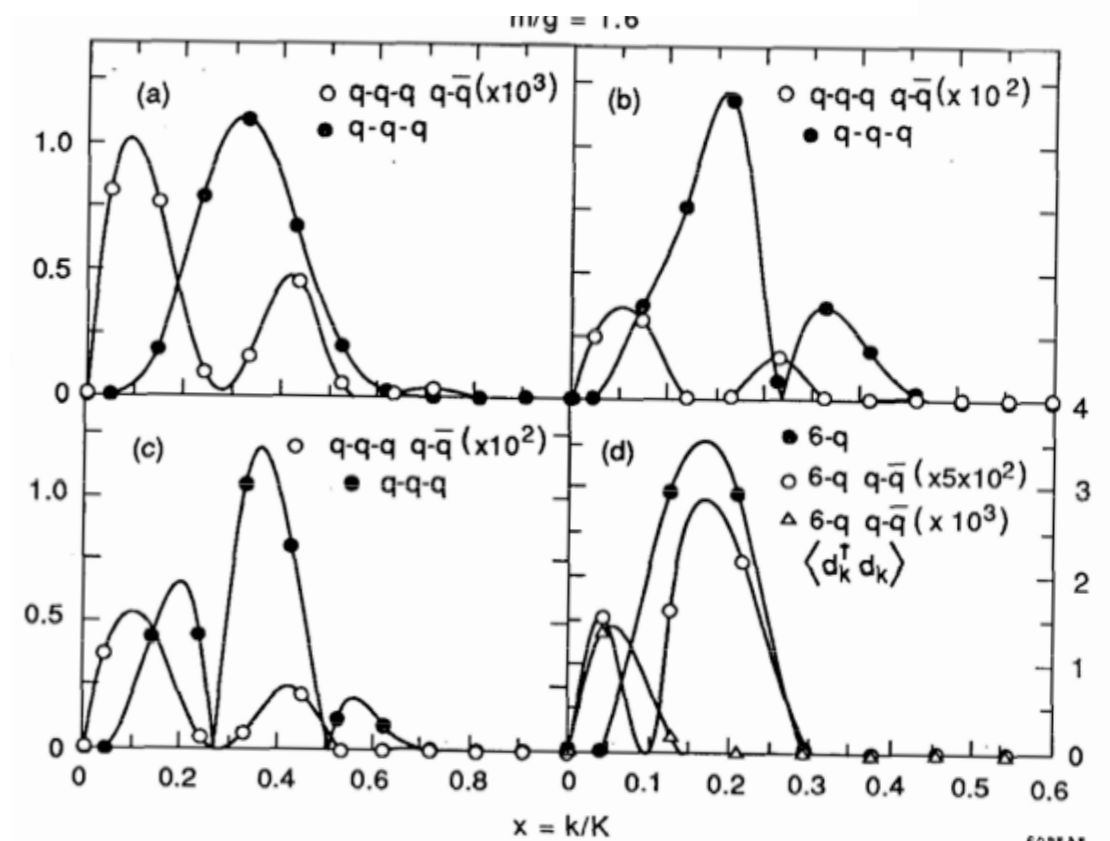
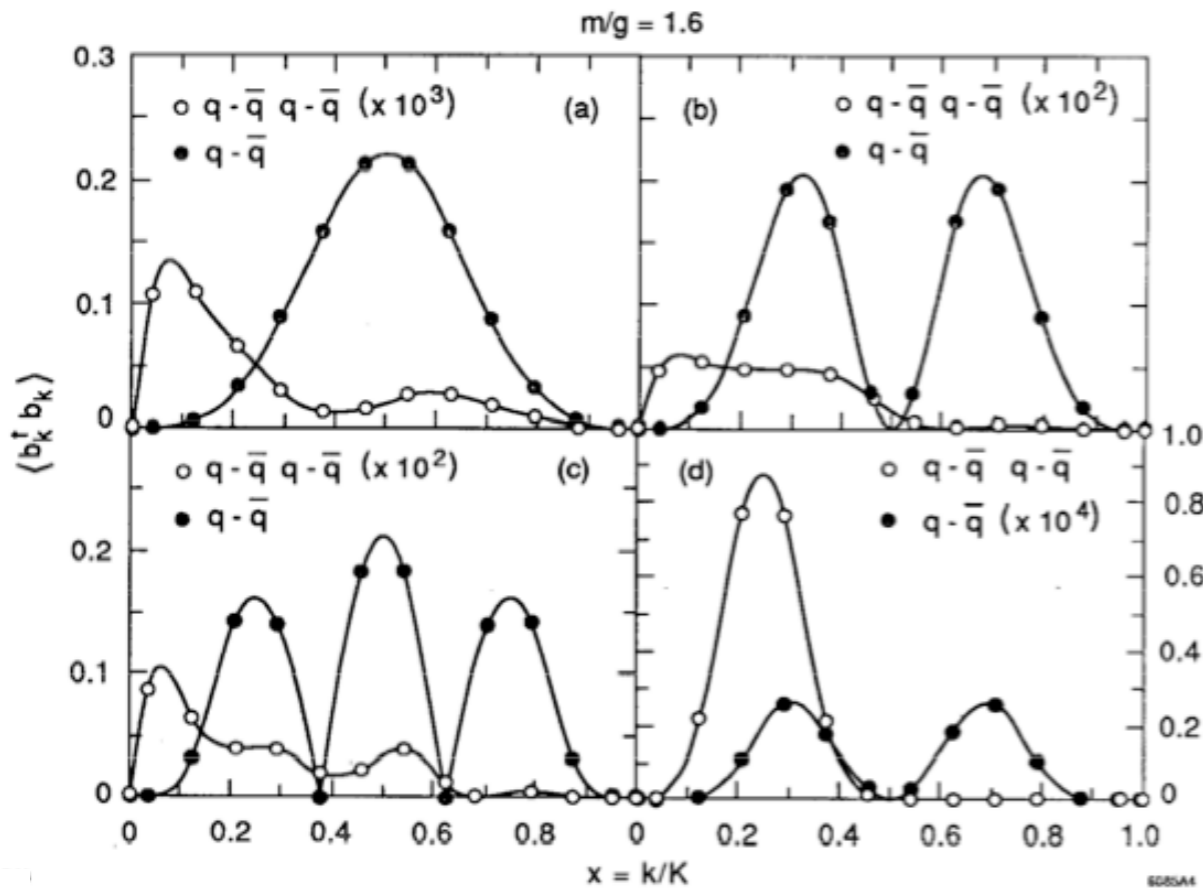


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Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



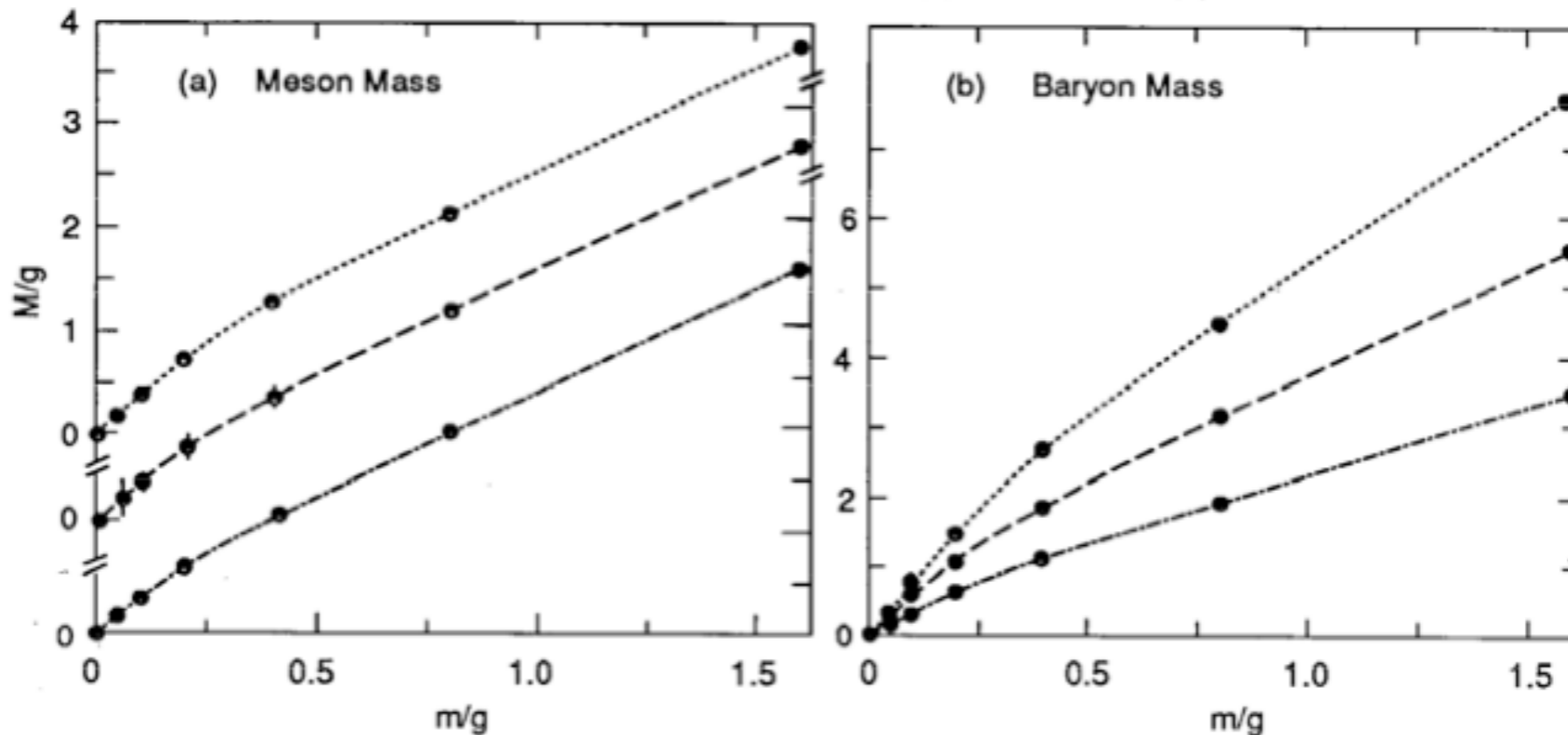
a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, $2K=24$. d) Eleventh

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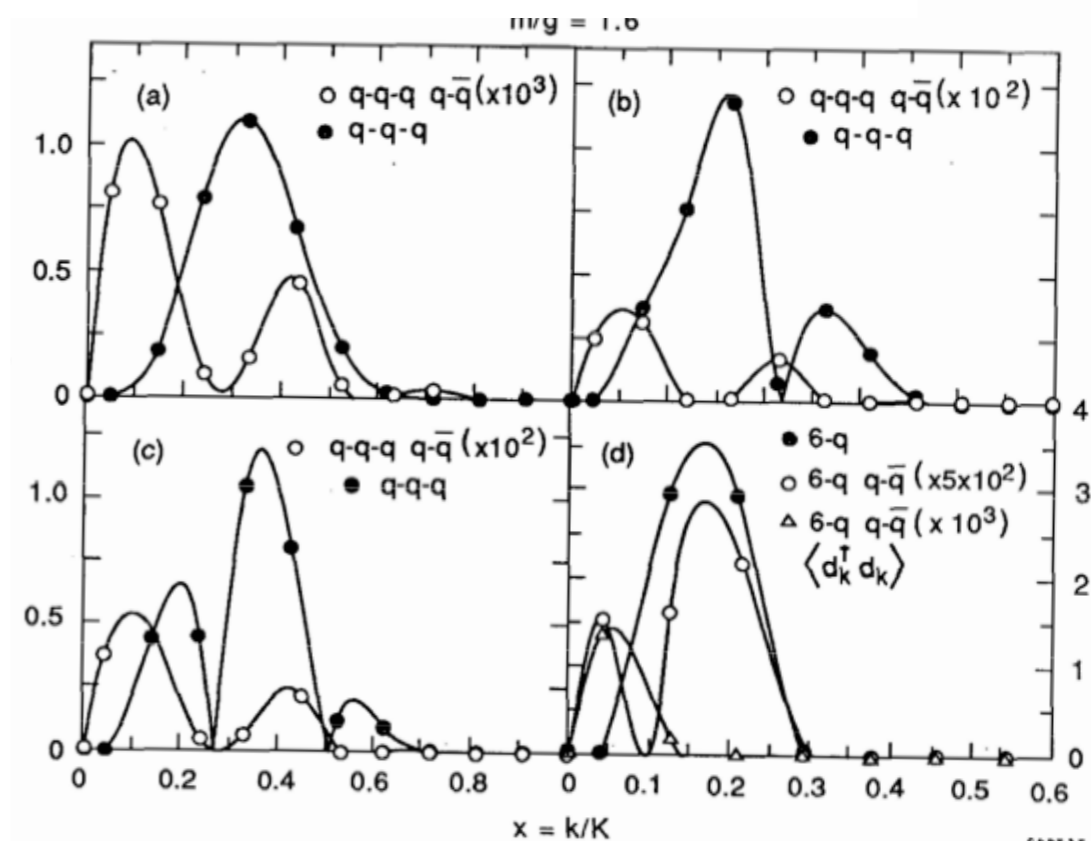
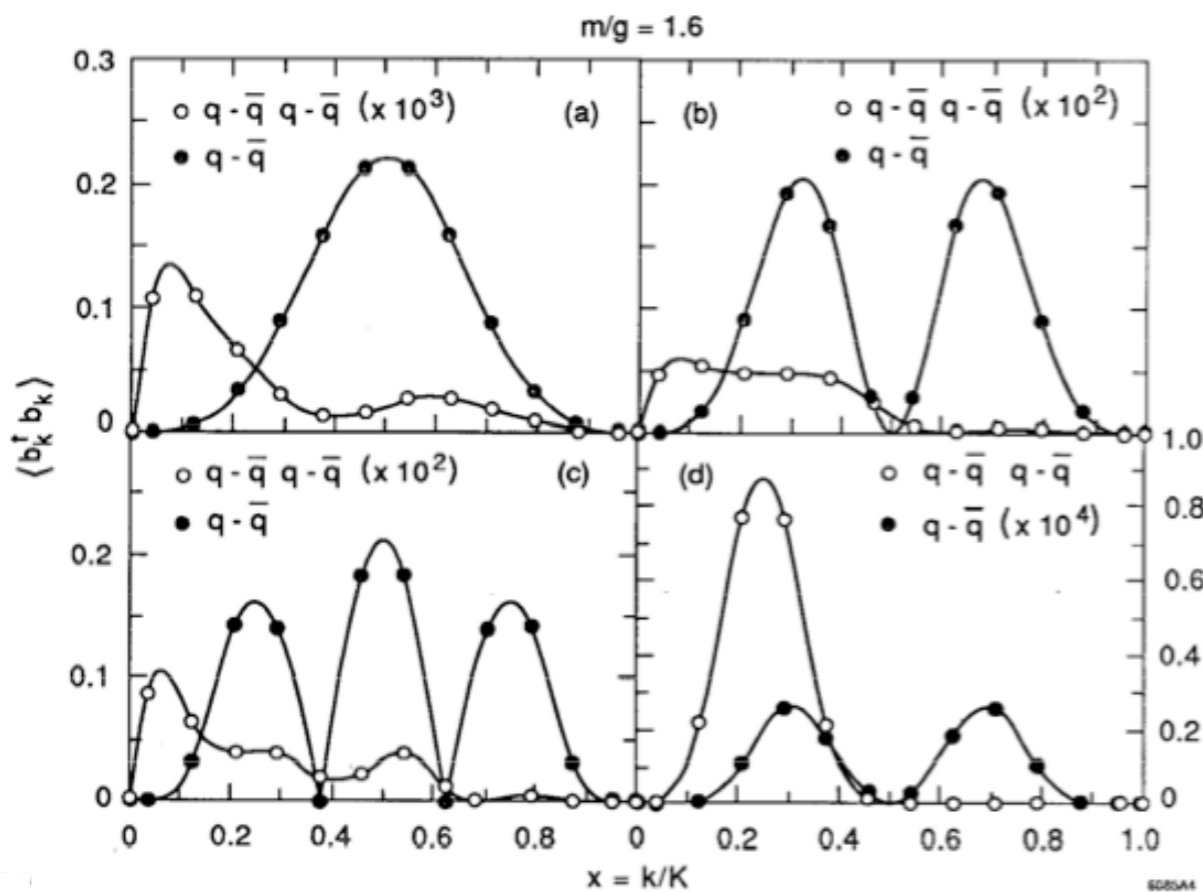
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state:

Hornbostel, Pauli, sjb

Light Front Theory

- Frame-Independent, causal, Minkowski space,
- DLCQ, BLFQ: No fermion doubling
- Equivalent to Bethe-Salpeter $\int dk^- \psi_{BS} = \psi_{LF}$
- Hadronization at the Amplitude Level
- Holographically Dual to AdS₅

This meeting:
New LF methods for solving QCD

- Light-Front Holography
- Basis Light-Front Quantization
- Polynomial Basis
- Iterated Resolvent
- Rigorous gauge-invariant renormalization
- True muonium
-

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

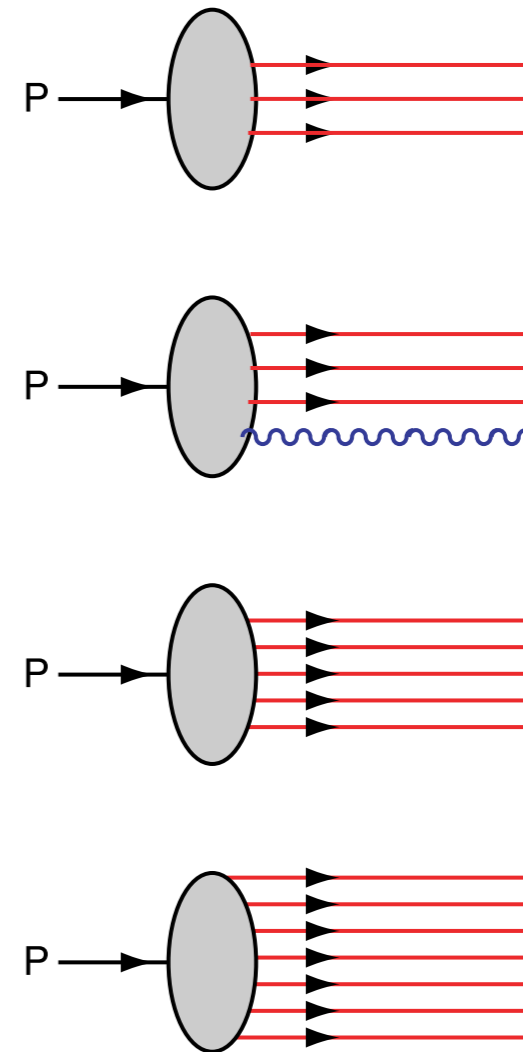
are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks
 $s(x), c(x), b(x)$ at high x !

$$\bar{s}(x) \neq s(x)$$

$$\bar{u}(x) \neq \bar{d}(x)$$



Mueller: gluon Fock states: BFKL Pomeron

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

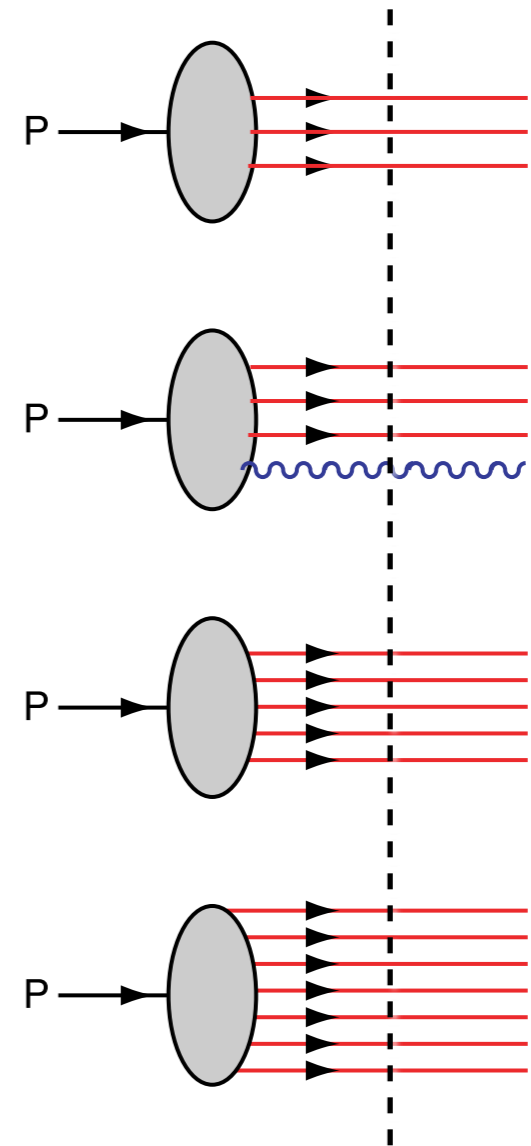
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$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks
 $s(x), c(x), b(x)$ at high x !

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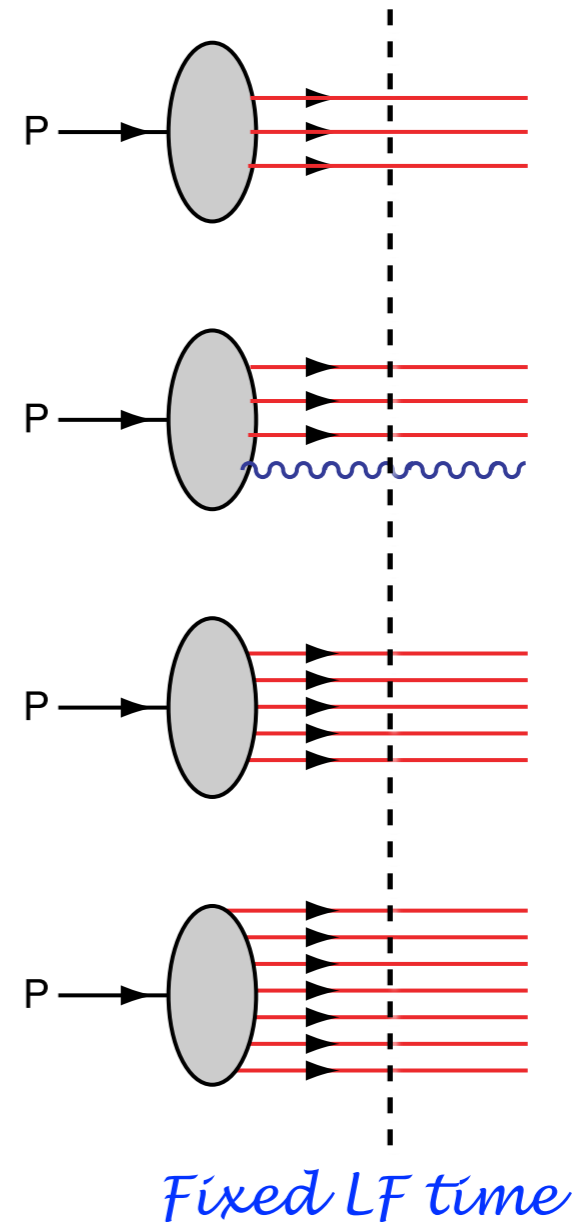
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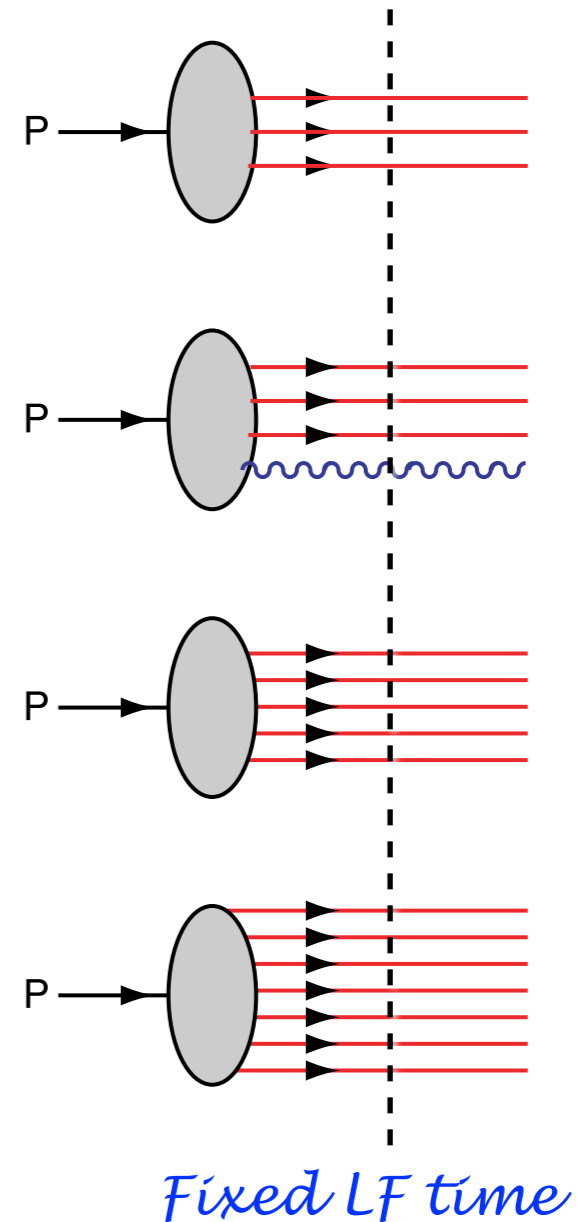
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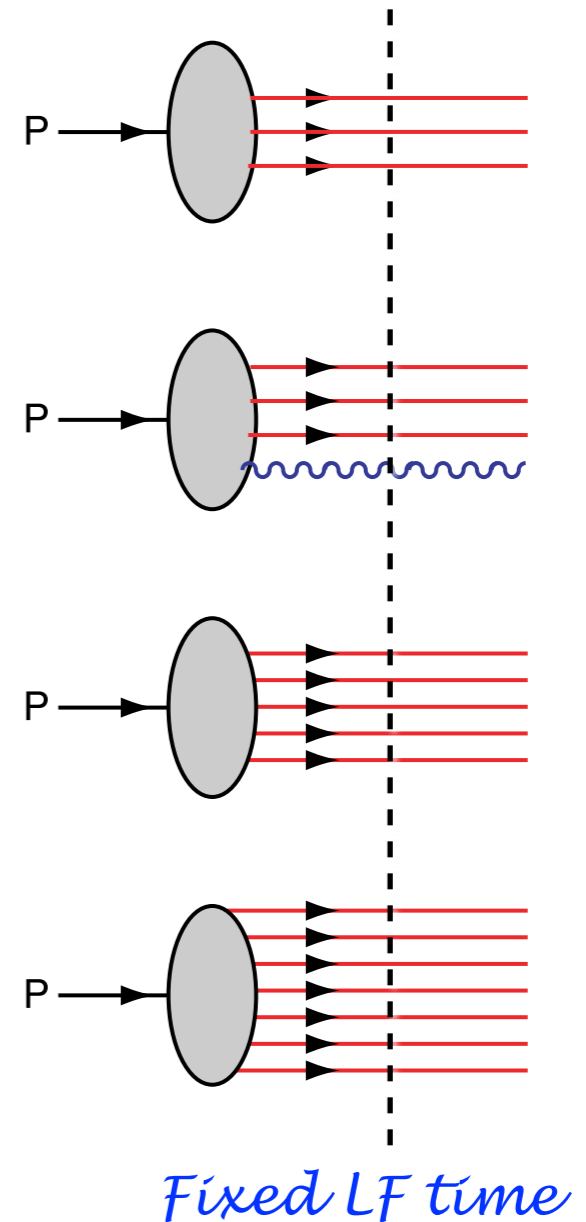
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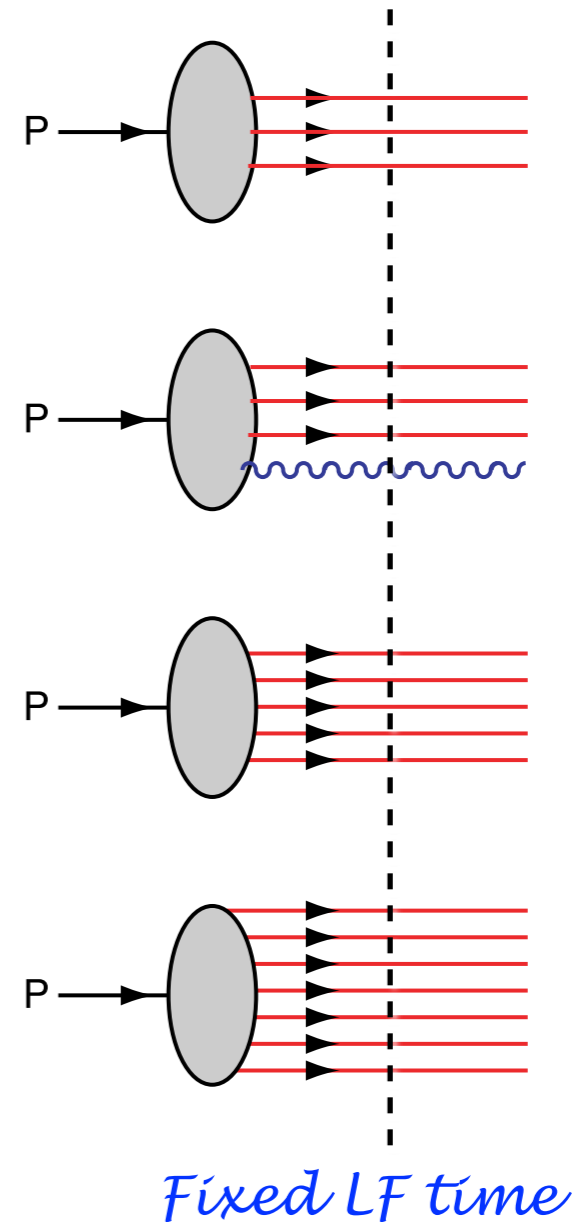
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Mueller: gluon Fock states:

BFKL Pomeron

Hidden Color

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

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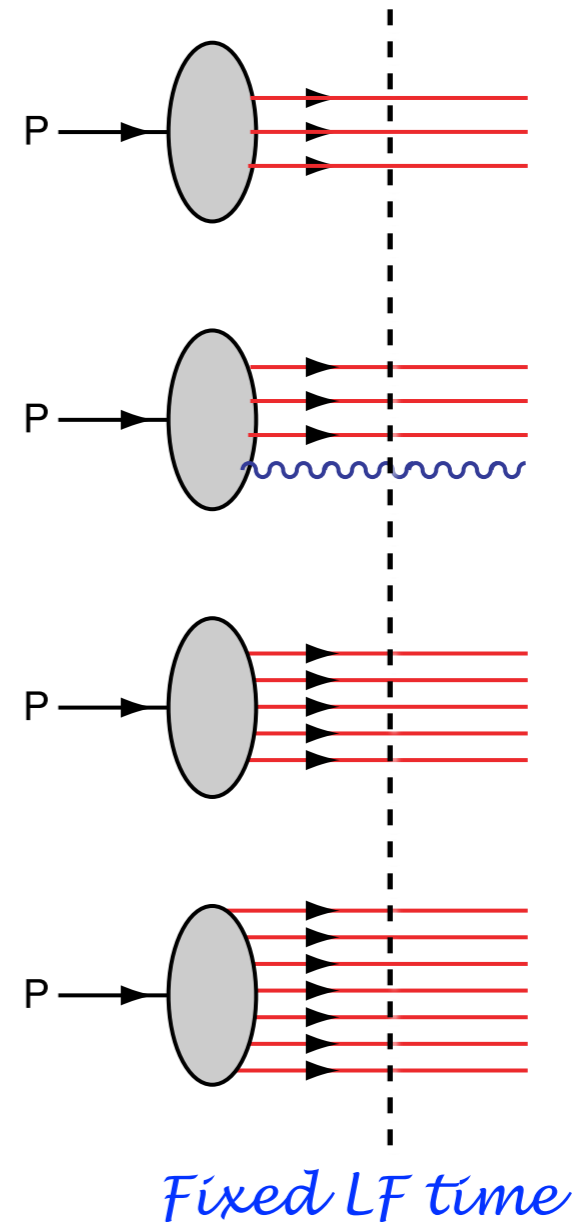
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Fixed LF time

Hidden Color

Mueller: gluon Fock states: BFKL Pomeron

Angular Momentum on the Light-Front

LC gauge

$A^+=0$

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

Glunon orbital angular momentum defined in physical lc gauge

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right) \quad n-1 \text{ orbital angular momenta}$$

Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->

Nonzero quark orbital angular momentum!

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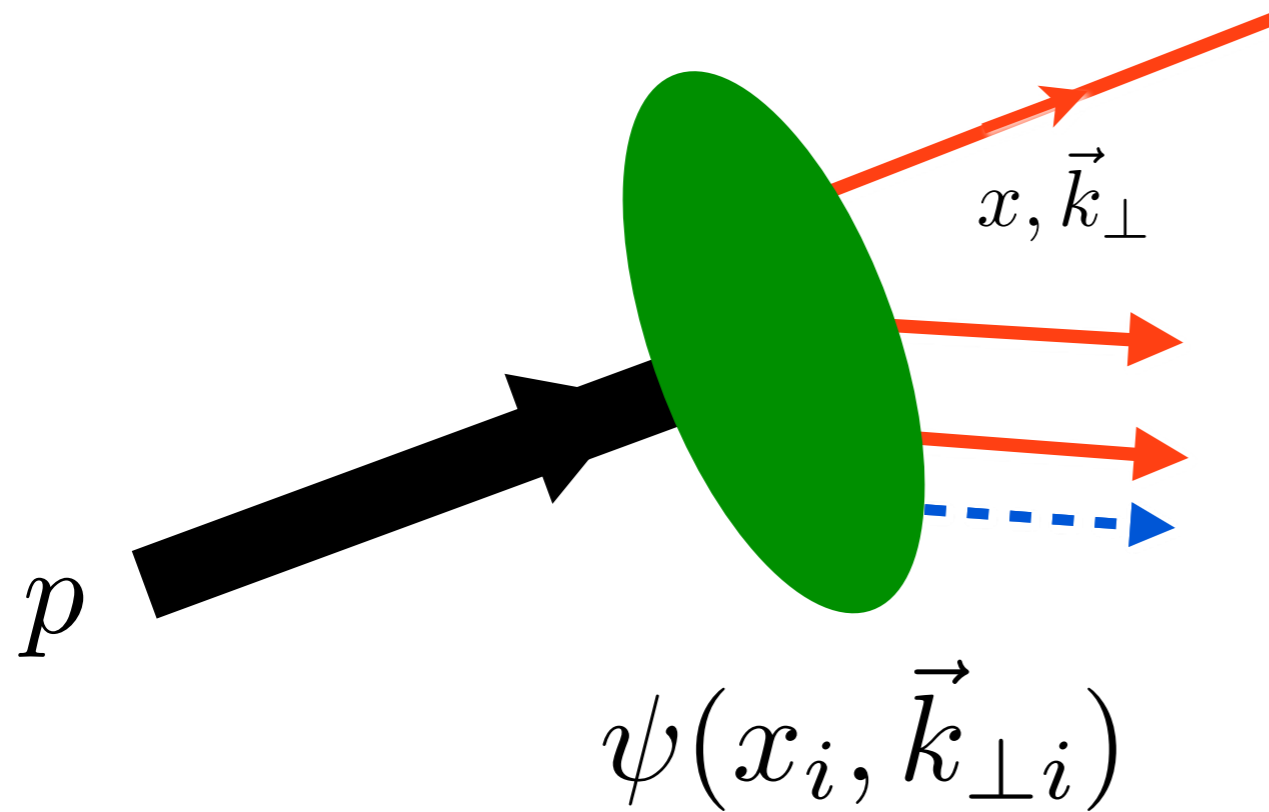
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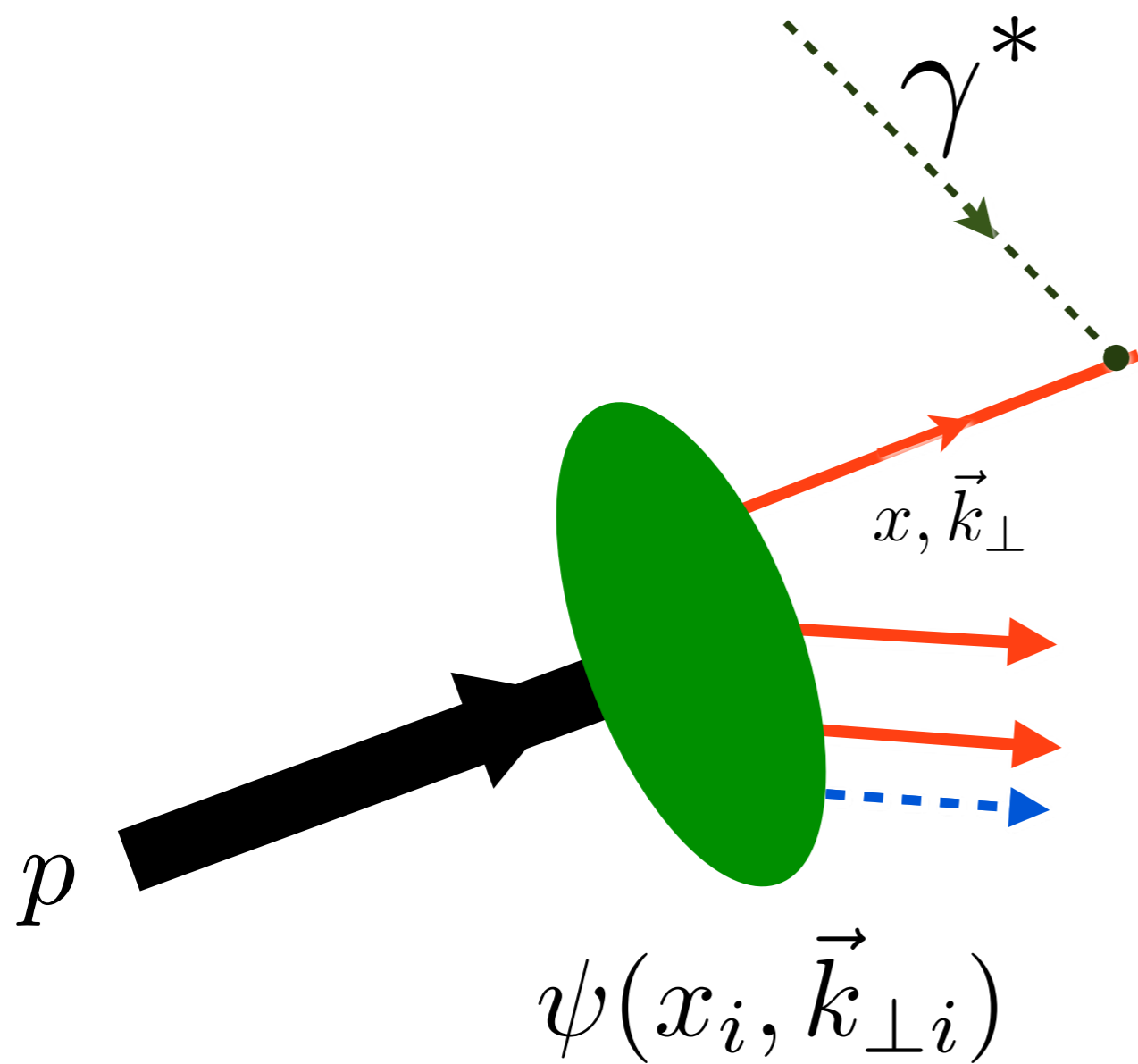
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**Drell & Yan, West
Exact LF formula!**

Interaction picture



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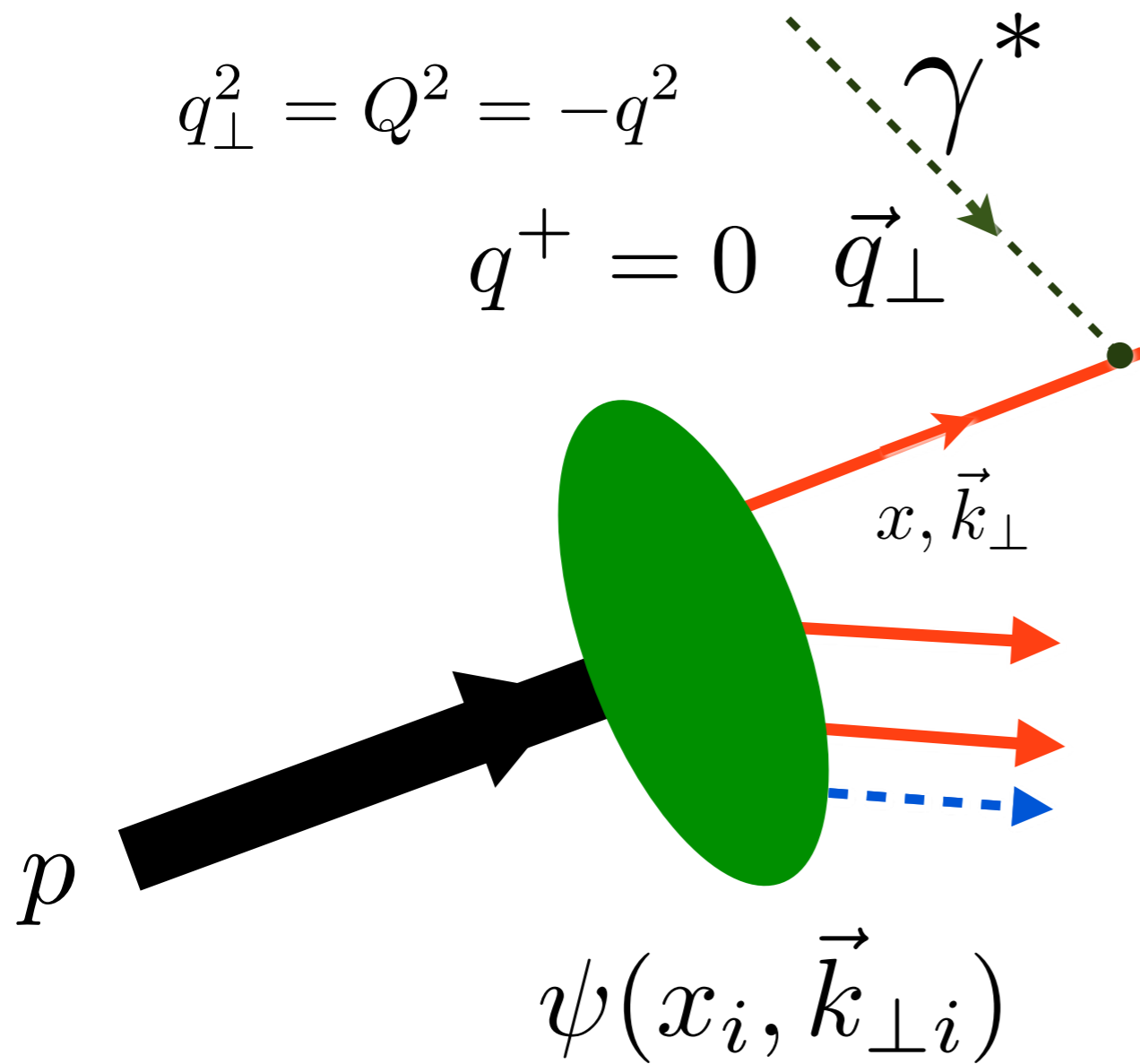
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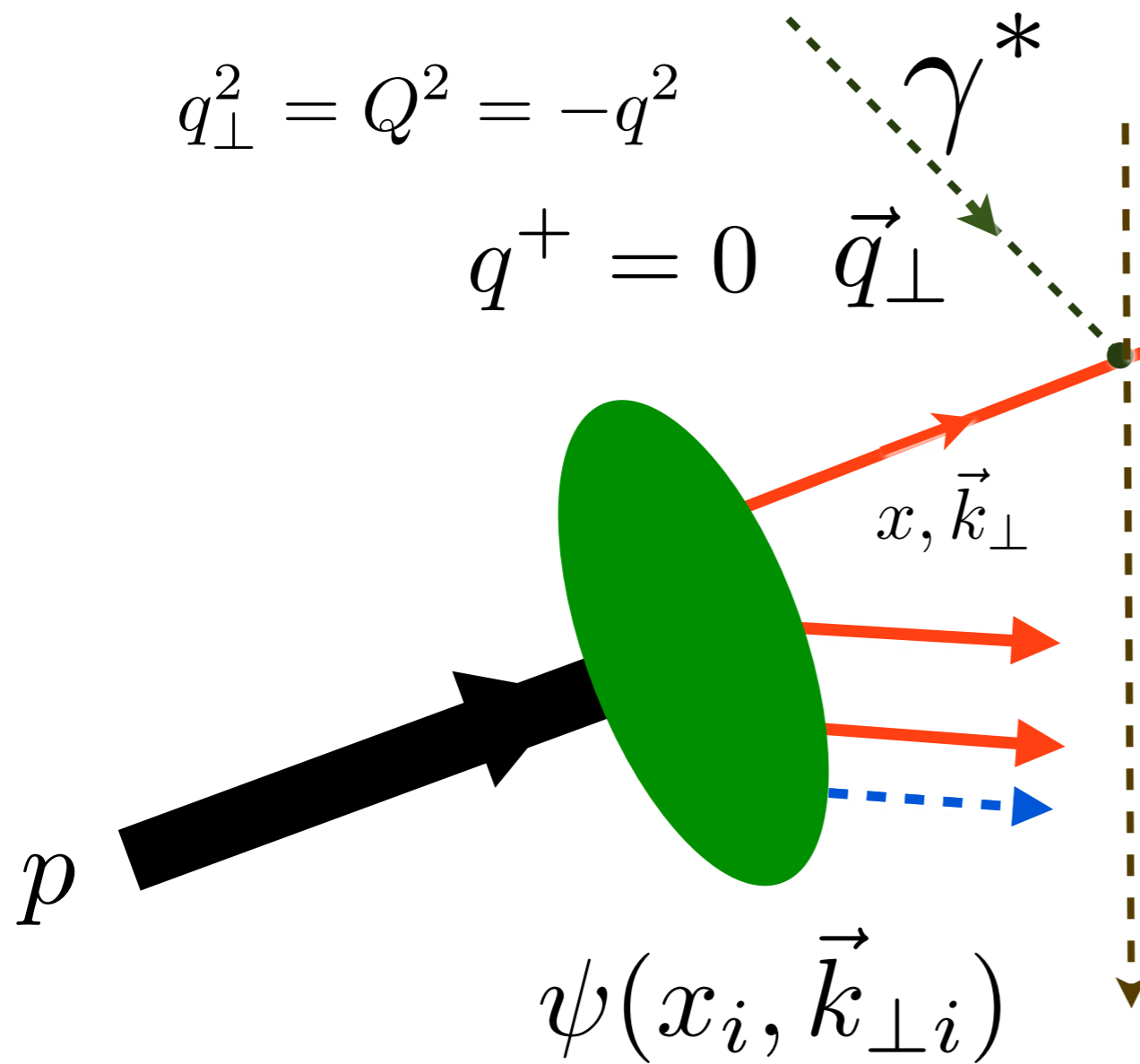
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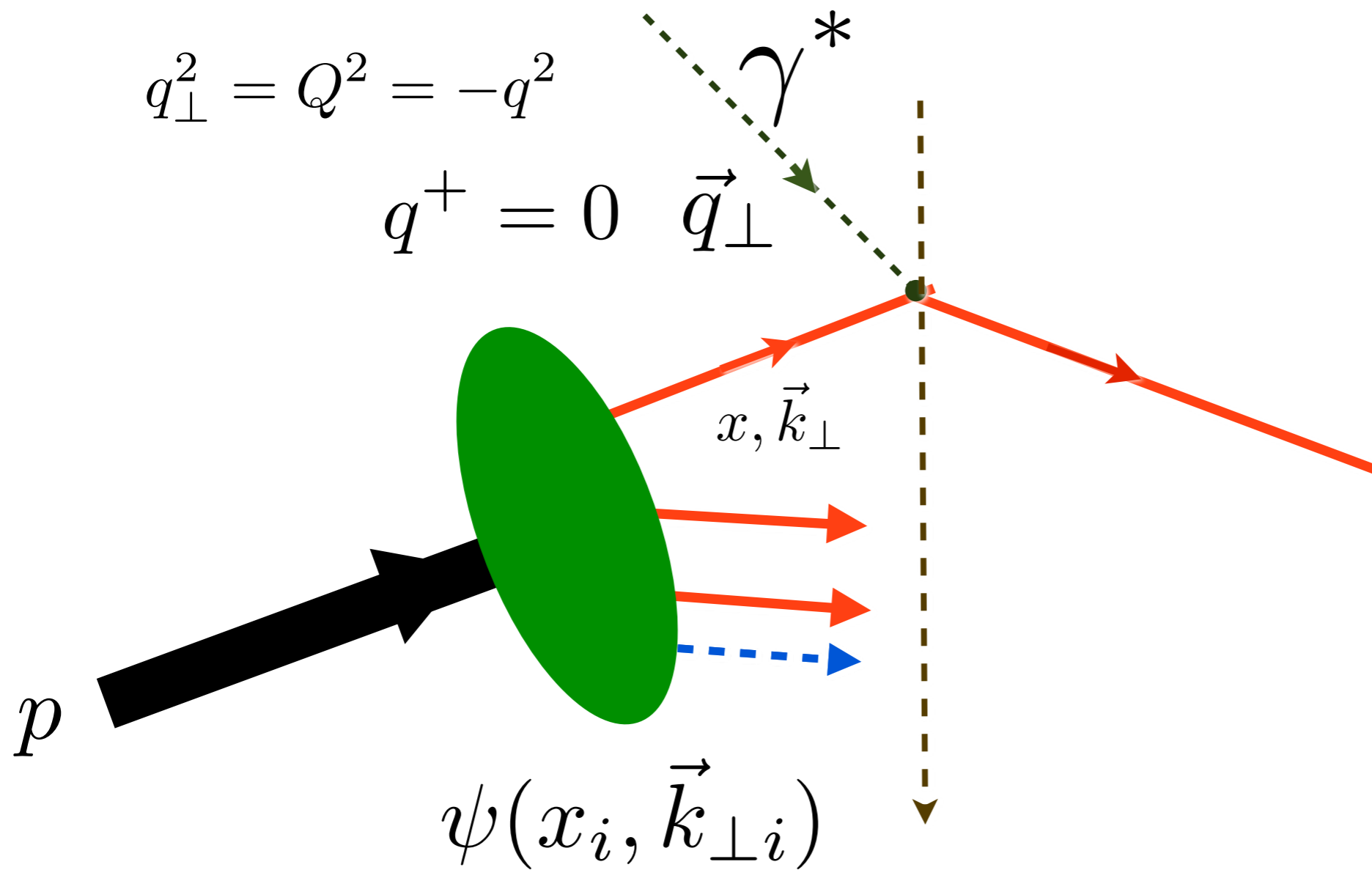
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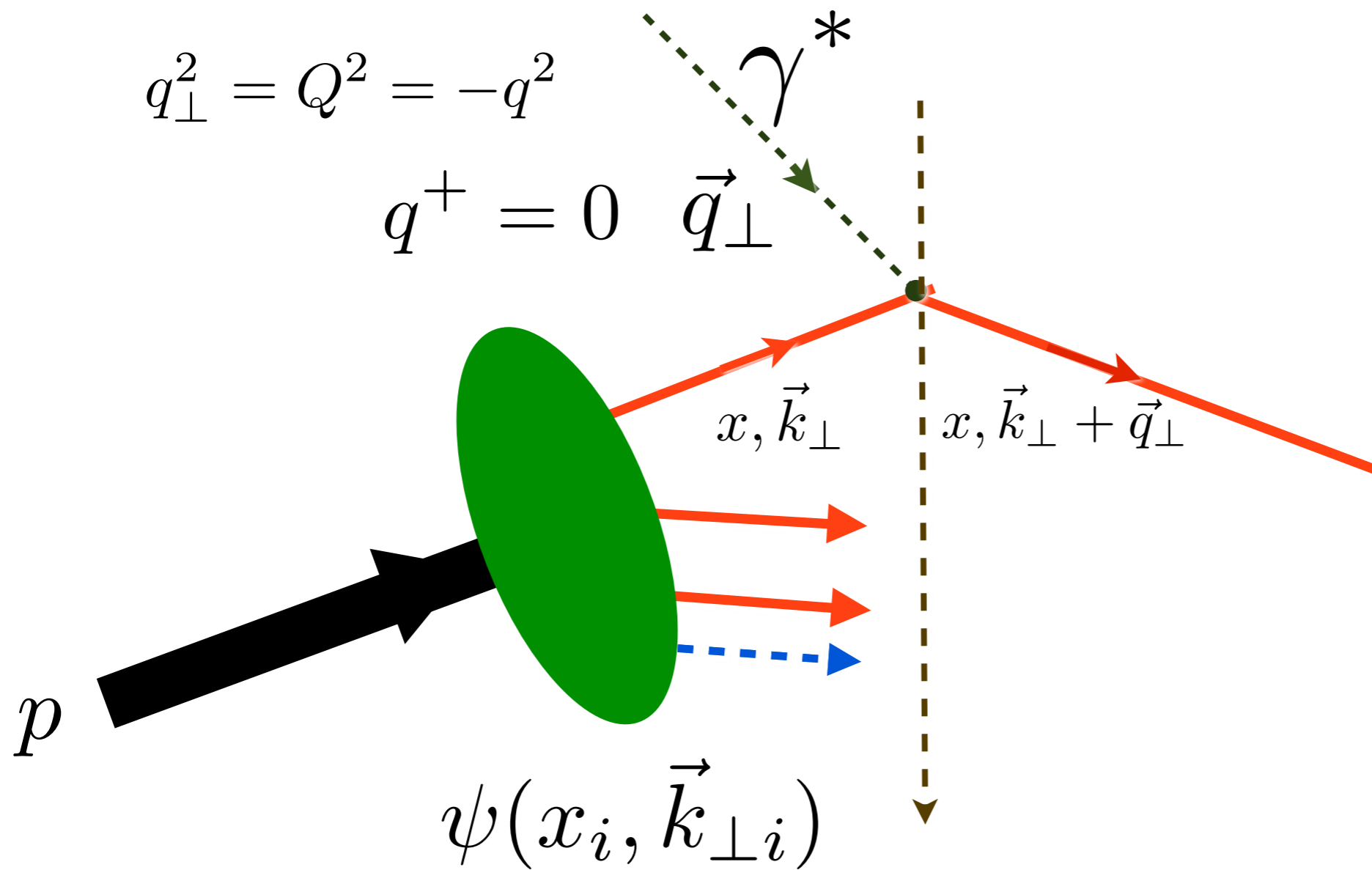
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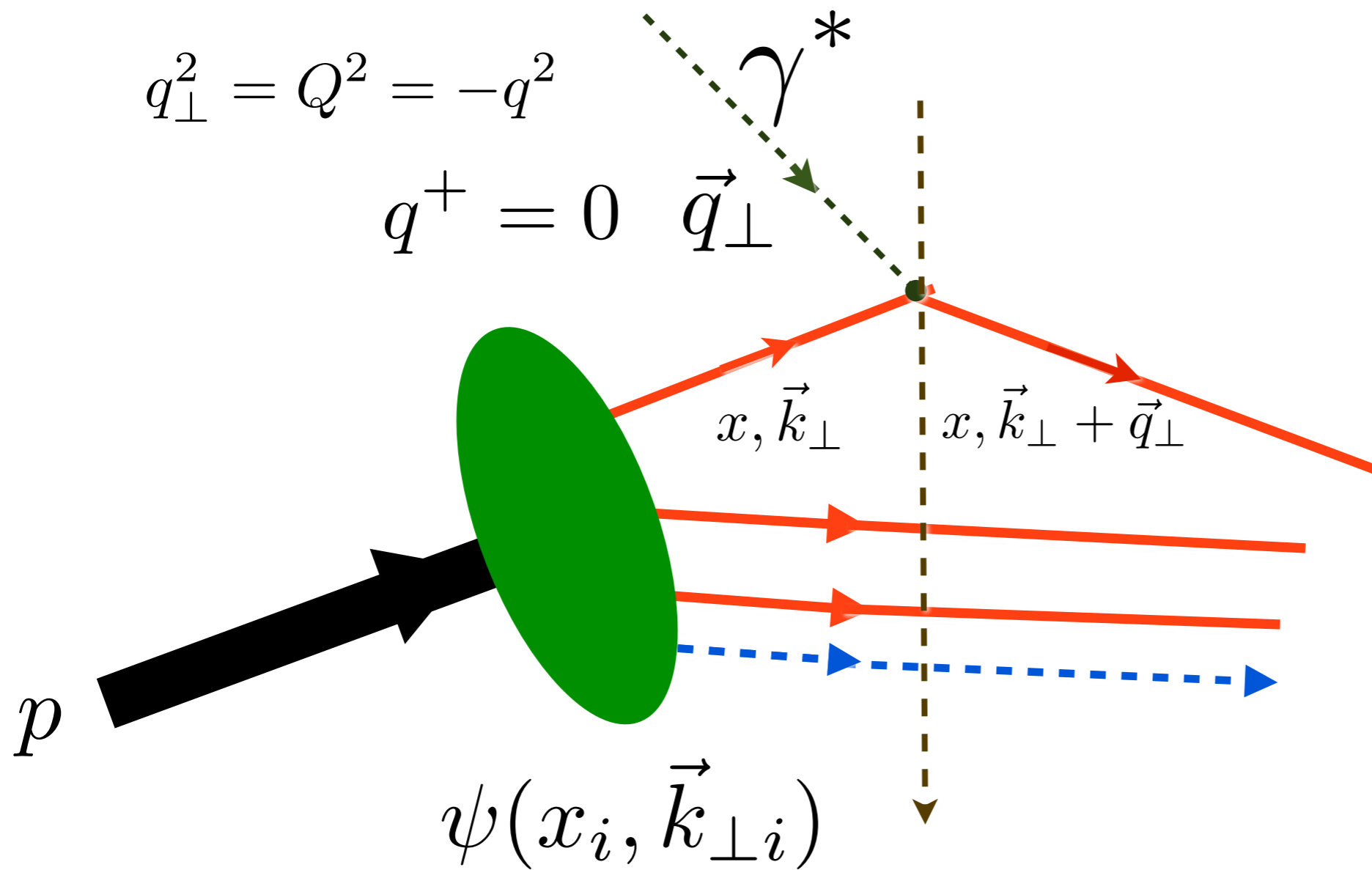
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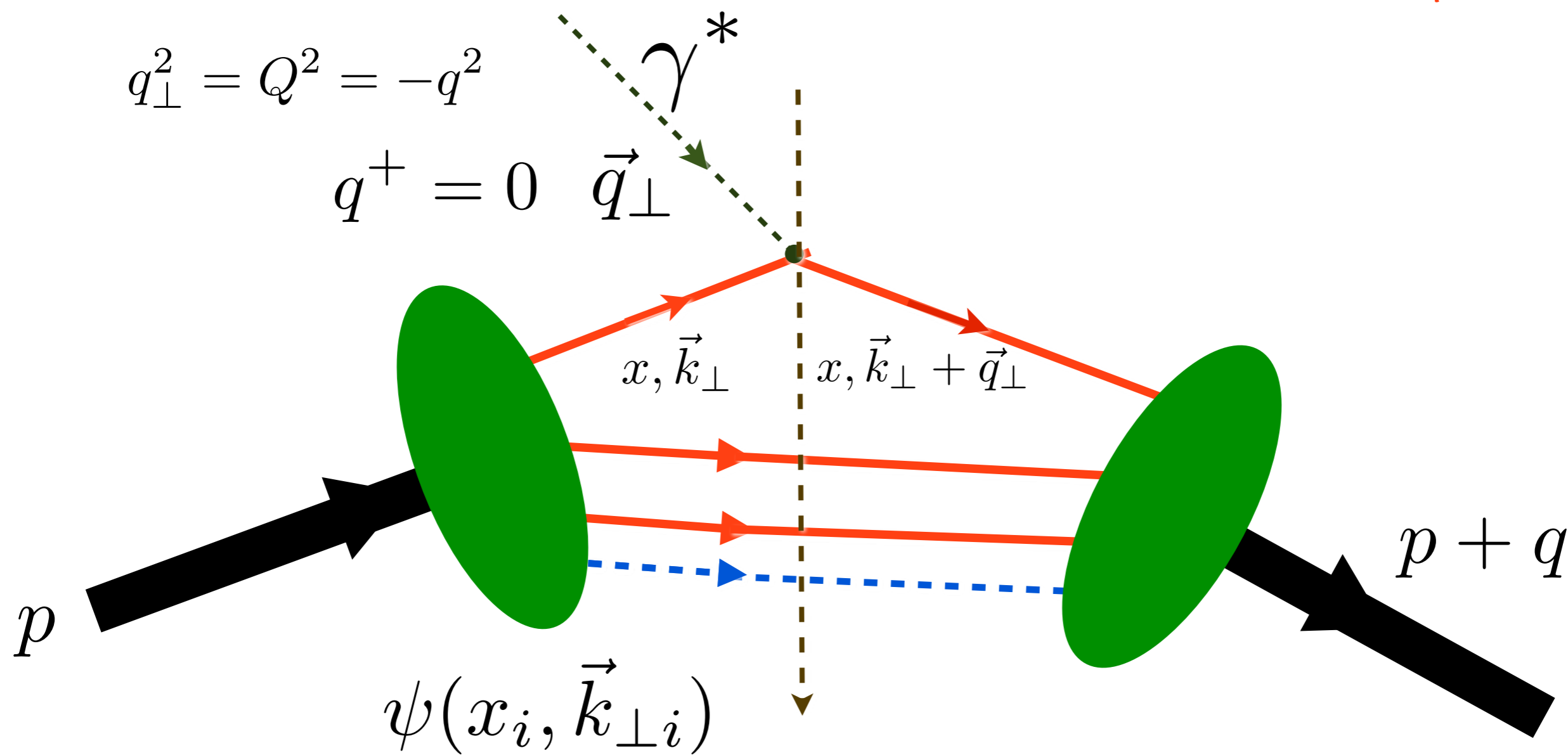
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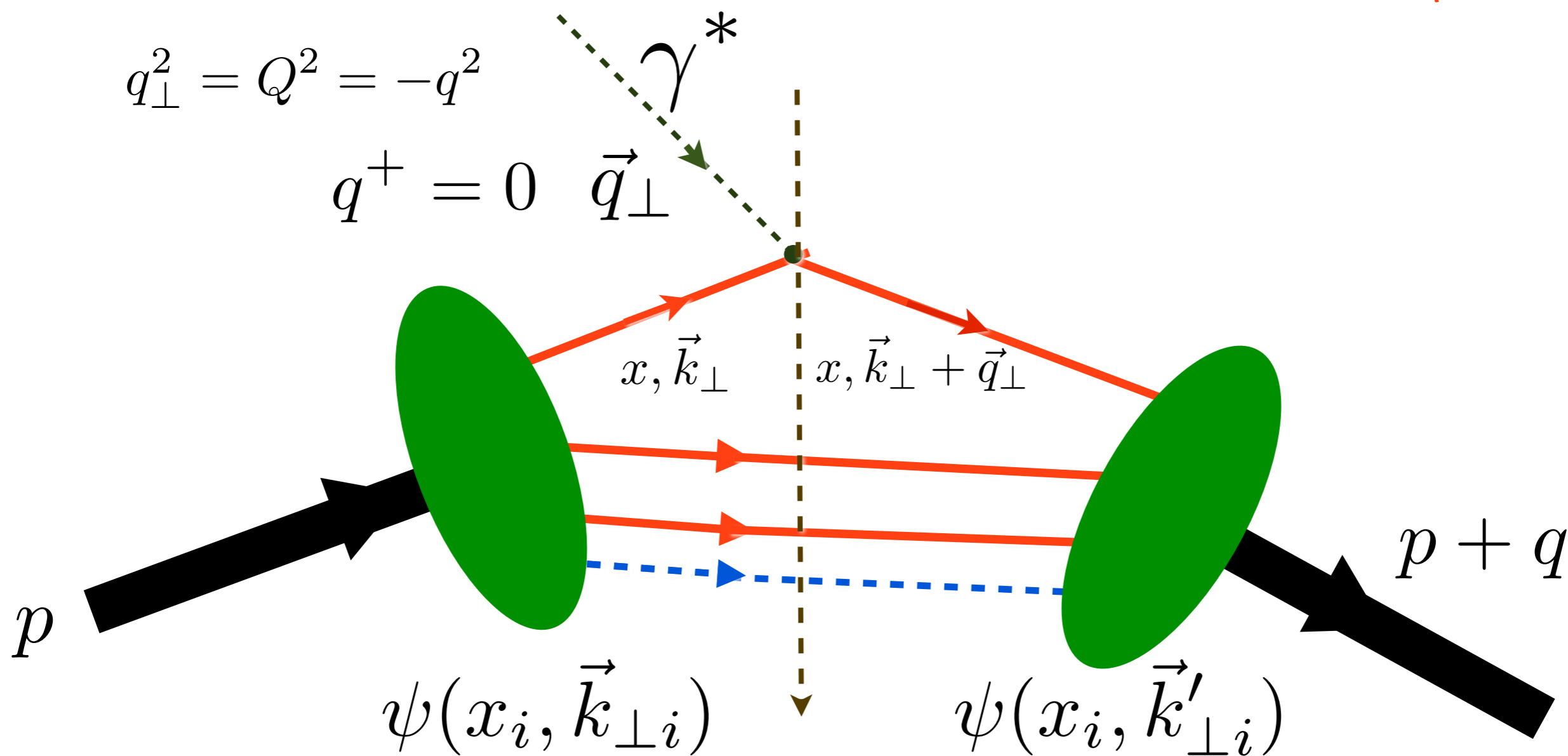
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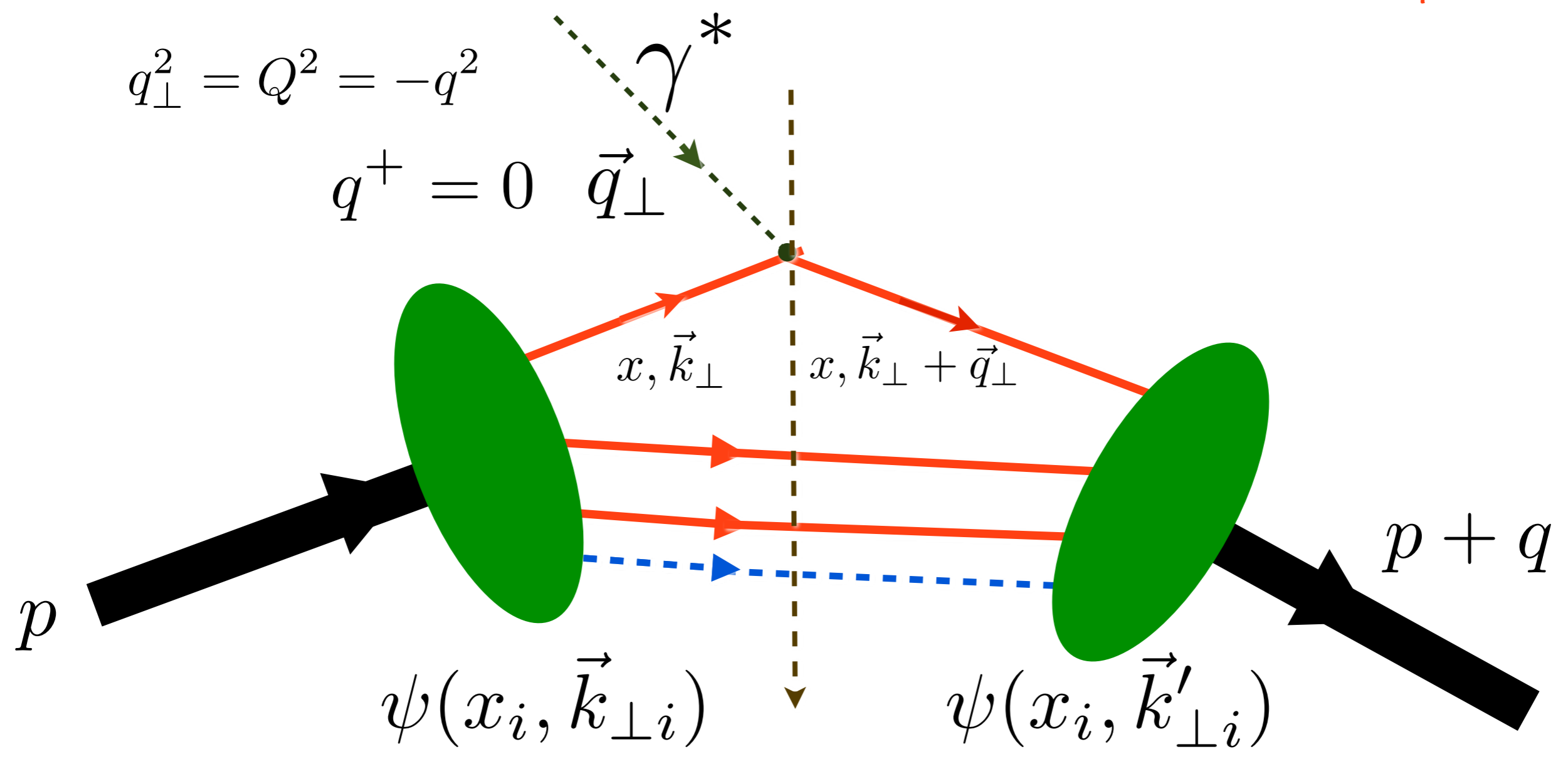
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$$q_{\perp}^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_{\perp}$$

$$x, \vec{k}_{\perp}$$

$$x, \vec{k}_{\perp} + \vec{q}_{\perp}$$

$$\psi(x_i, \vec{k}_{\perp i})$$

$$\psi(x_i, \vec{k}'_{\perp i})$$

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_{\perp}$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_{\perp}$

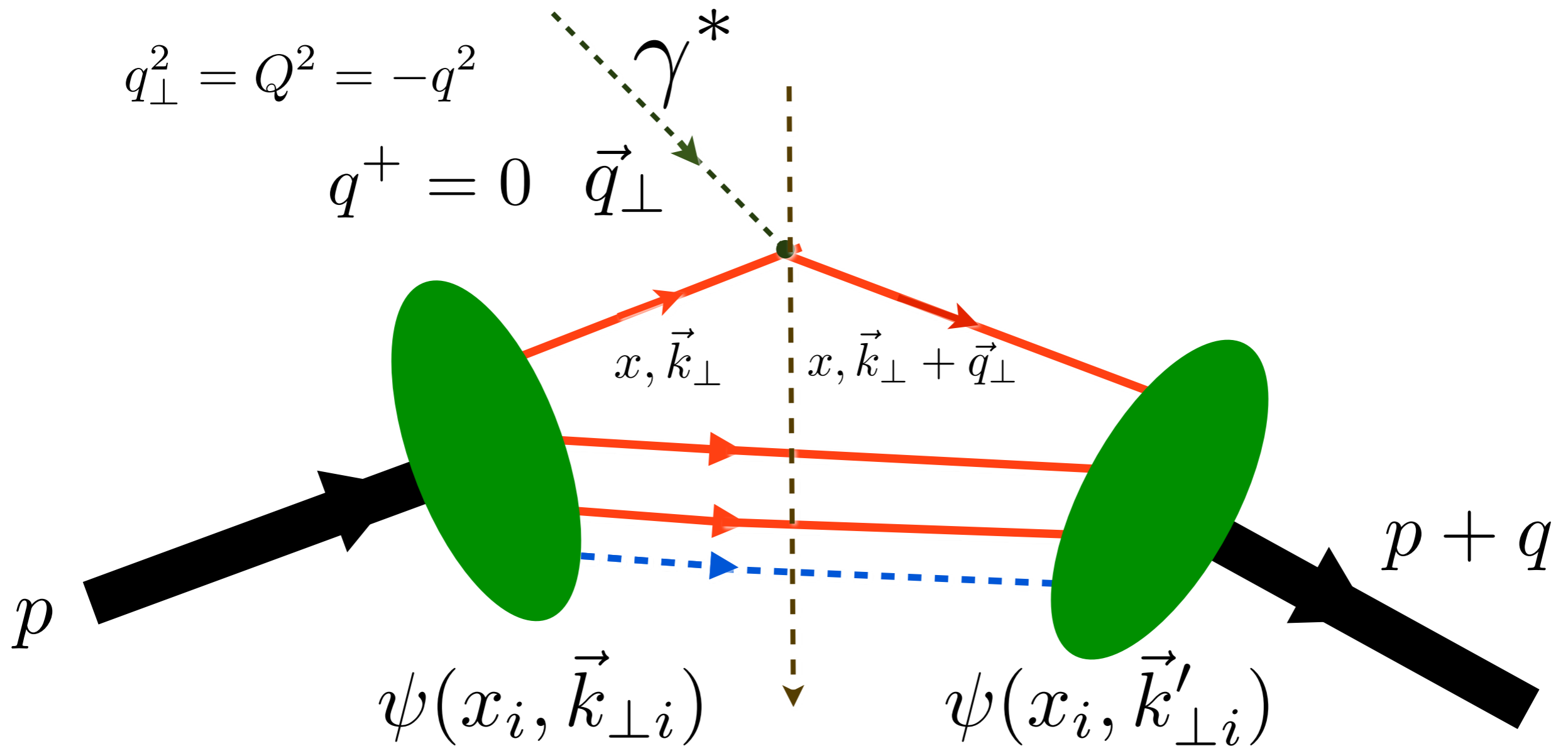
**Drell & Yan, West
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$$\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

Interaction picture

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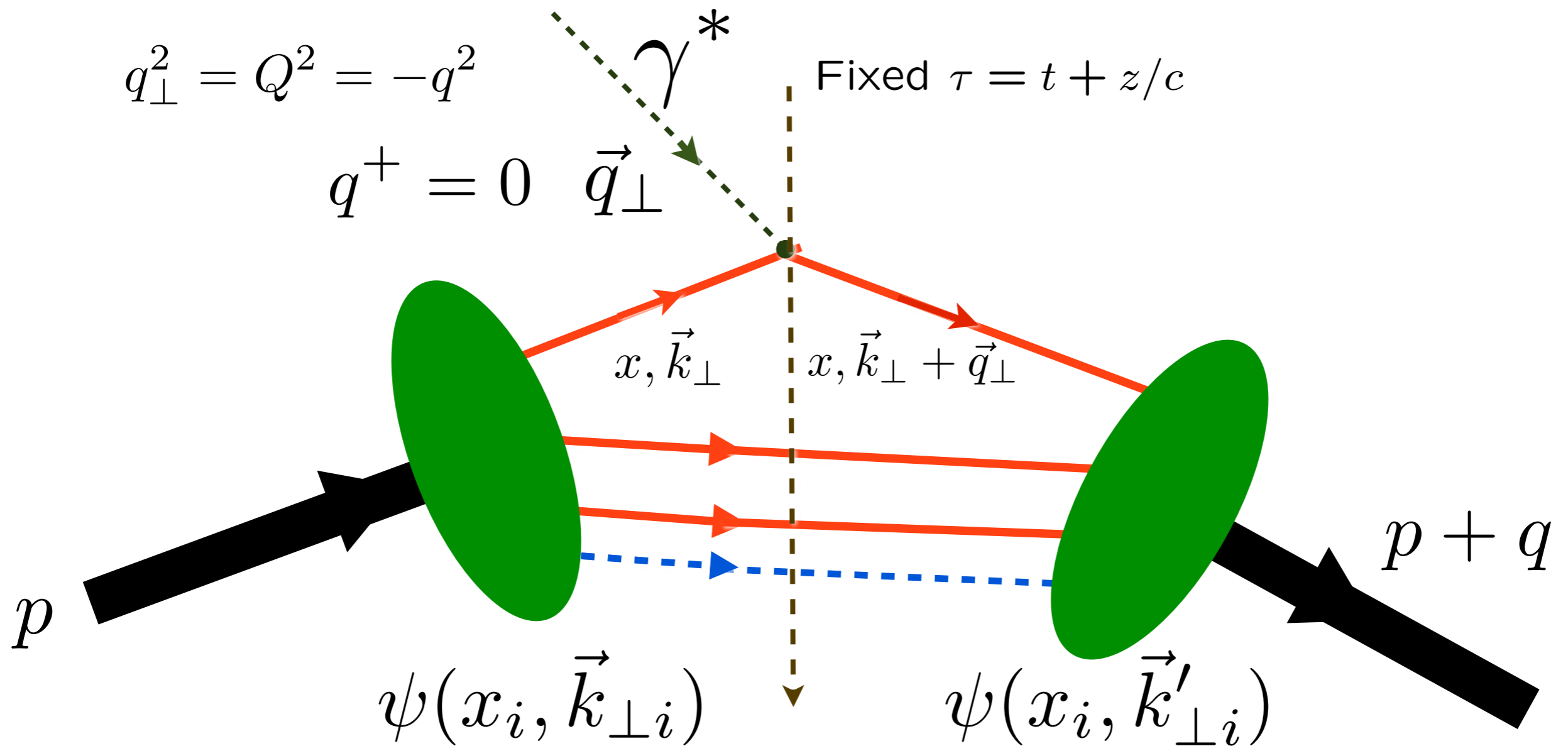
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Interaction picture



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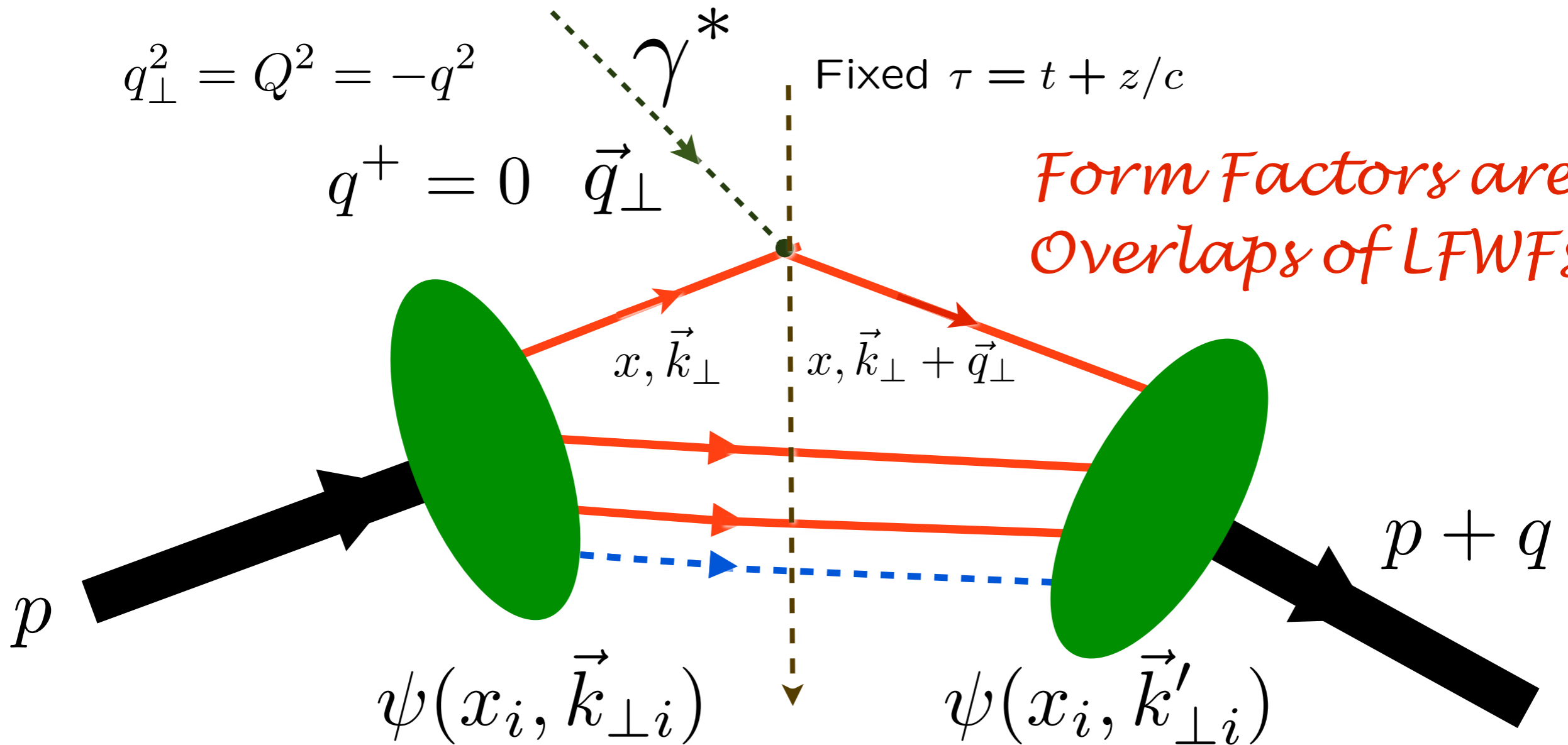
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$$q_{\perp}^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_{\perp}$$

Fixed $\tau = t + z/c$

Form Factors are Overlaps of LFWFs



$$\psi(x_i, \vec{k}_{\perp i})$$

$$\psi(x_i, \vec{k}'_{\perp i})$$

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Exact LF Formula for Pauli Form Factor

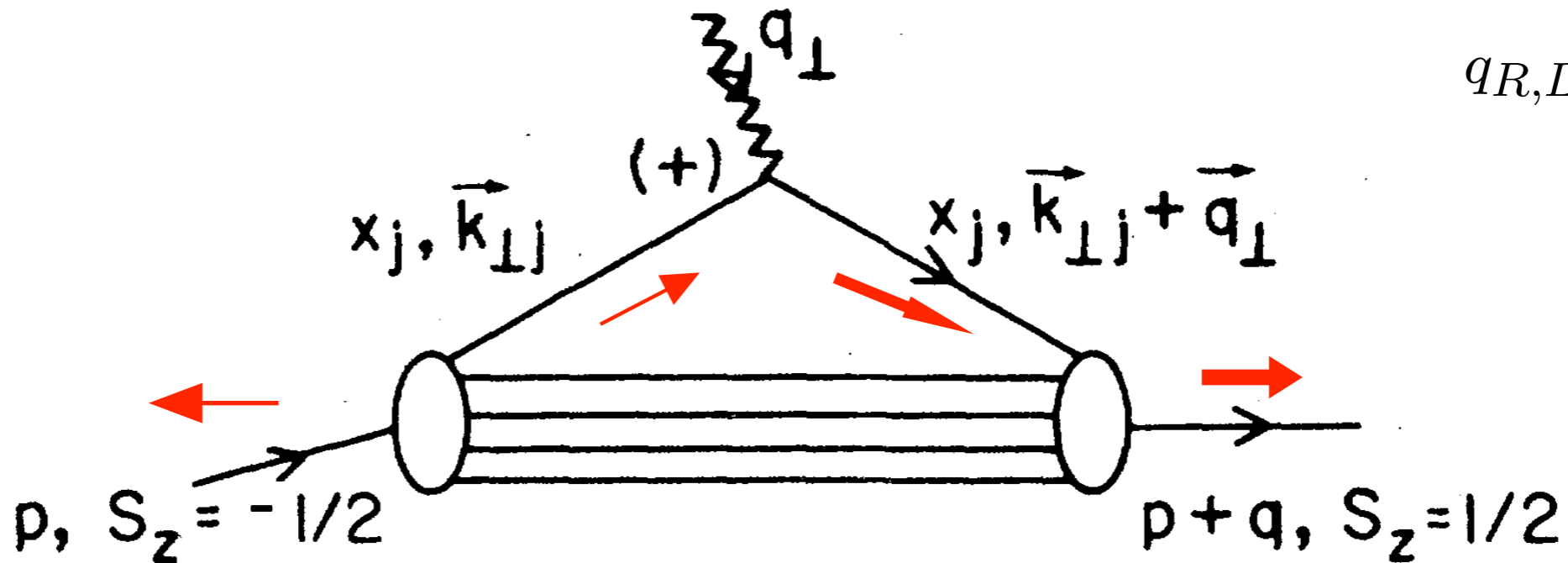
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

Drell, sjb

$$q_{R,L} = q^x \pm iq^y$$



Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

*Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum*

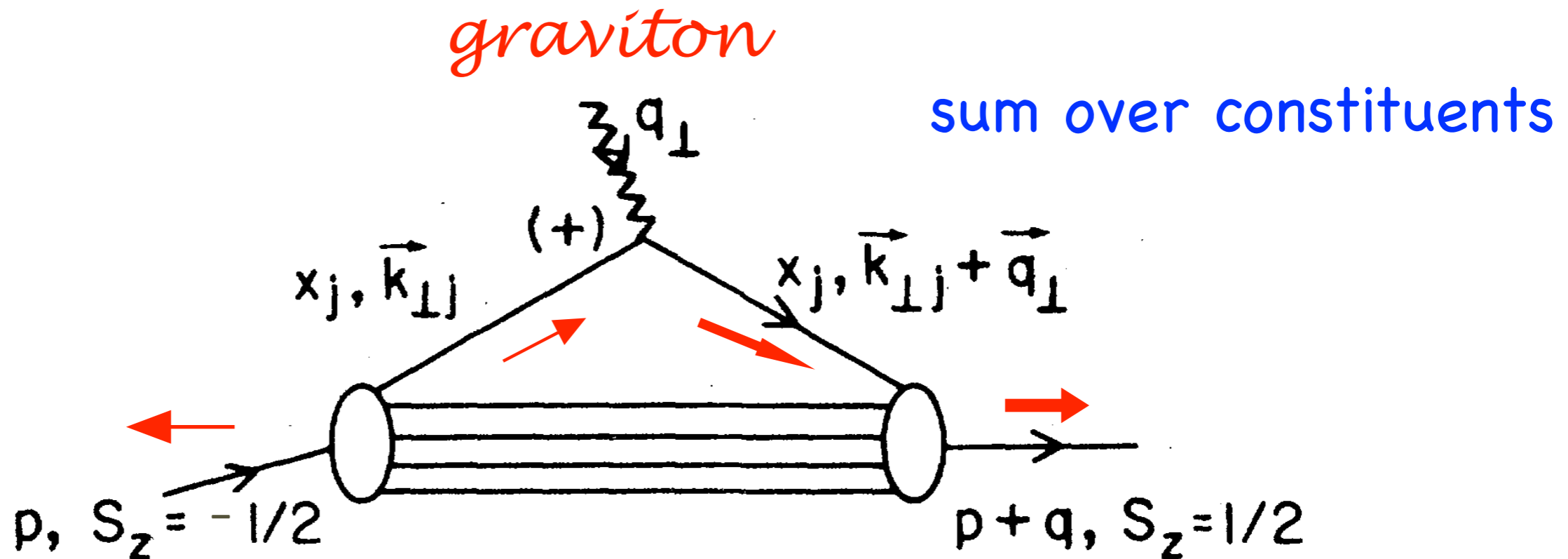
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Vanishing Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem



Hwang, Schmidt, sjb;
Holstein et al

$$B(0) = 0$$

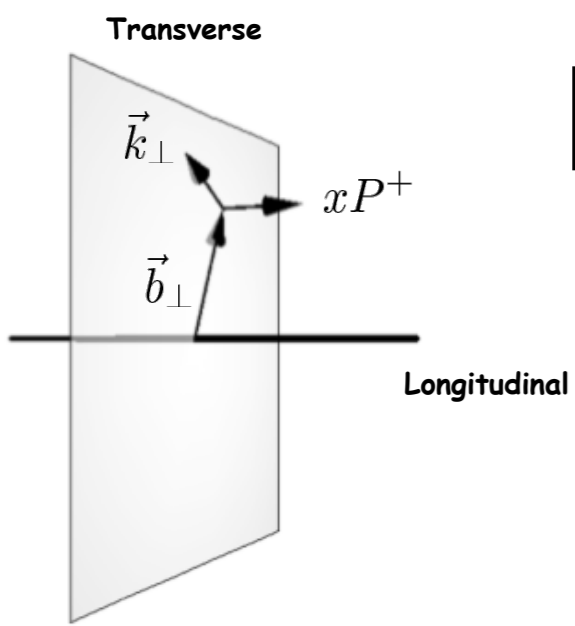
Each Fock State



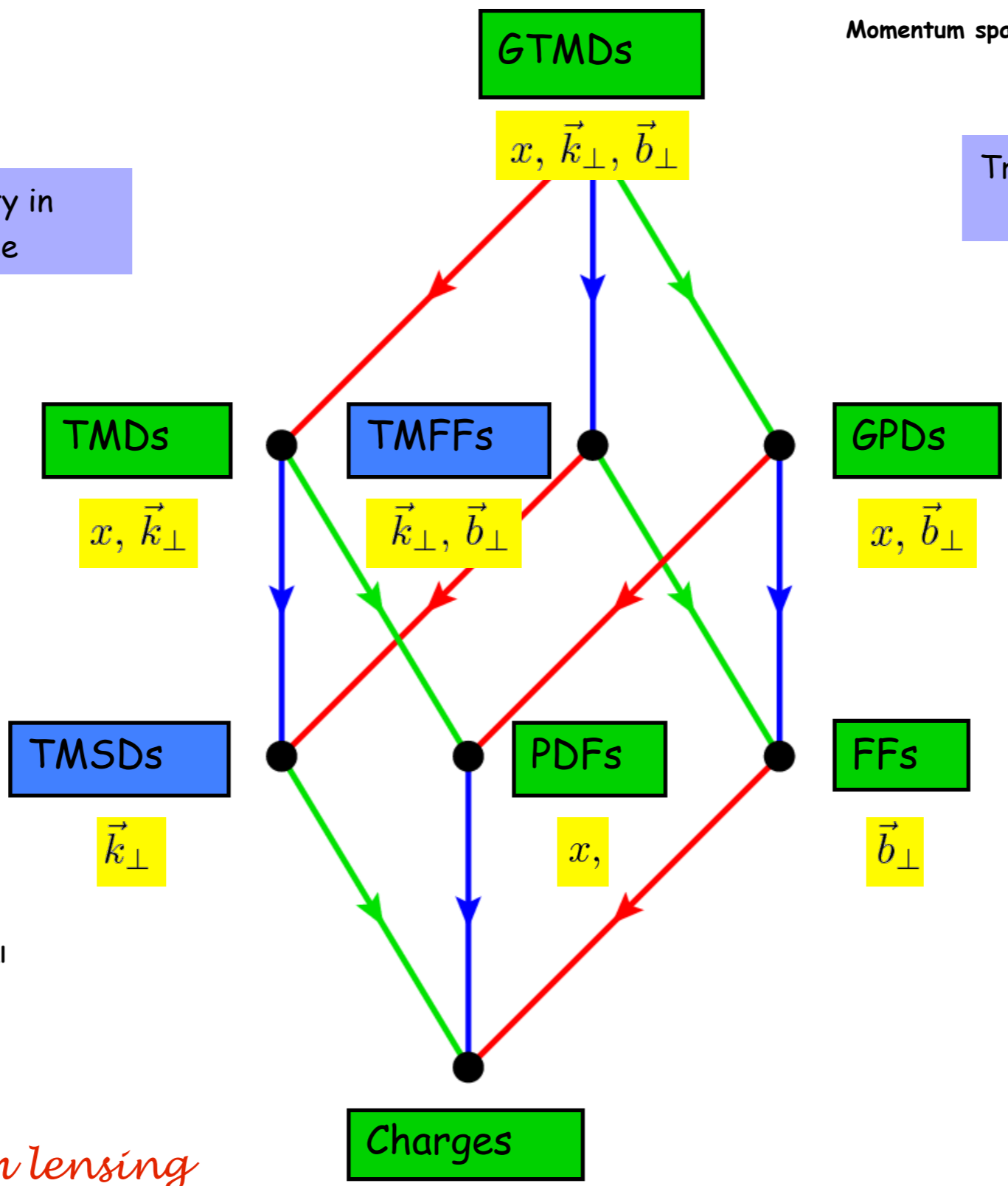
Transverse density in momentum space

Transverse density in position space

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space
 $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$



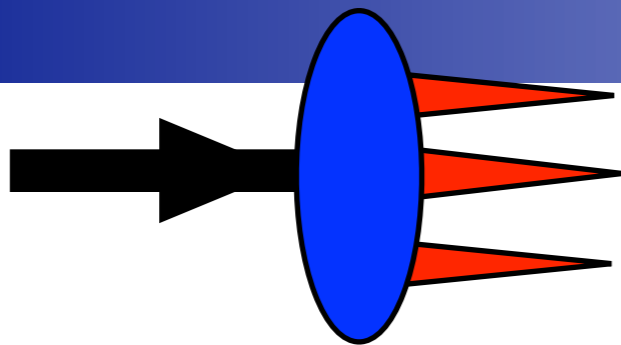
Sivers, T-odd from lensing



Lorce, Pasquini

Diehl, Hwang, sjb

- $\int d^2 b_\perp$
- $\int dx$
- $\int d^2 k_\perp$

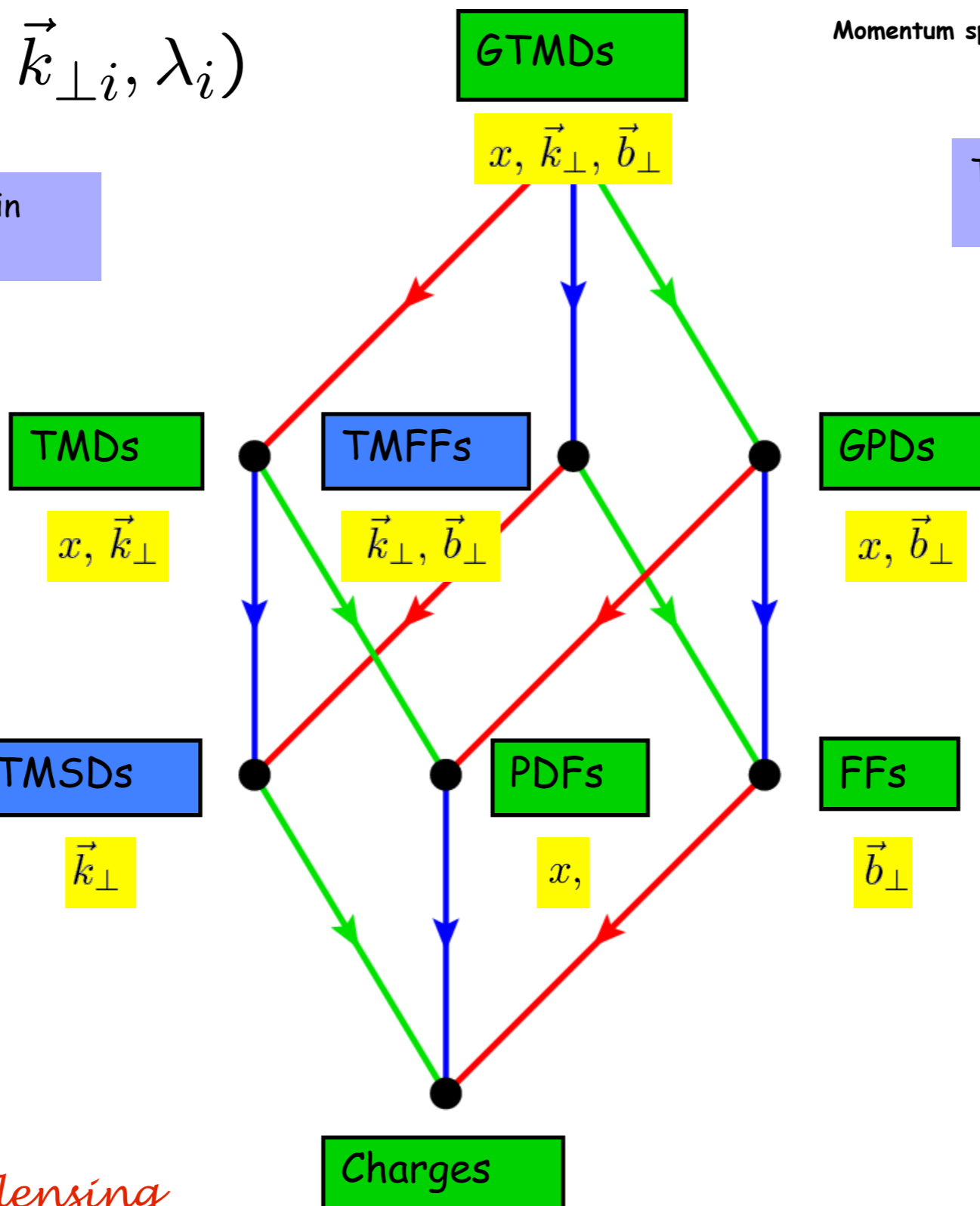


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
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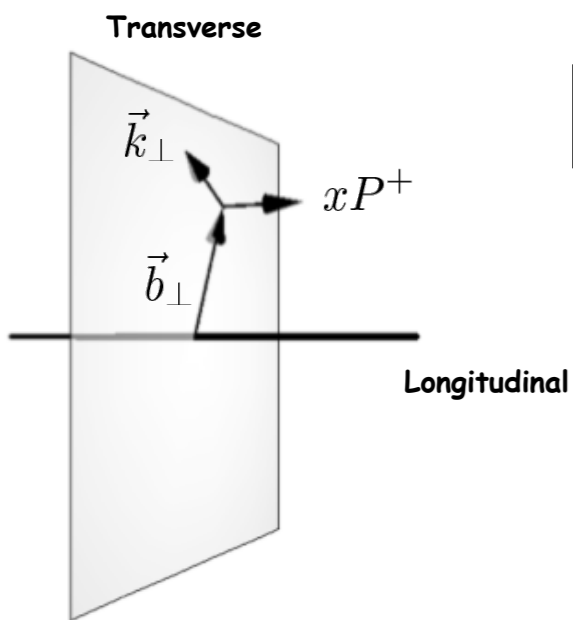
Transverse density in position space



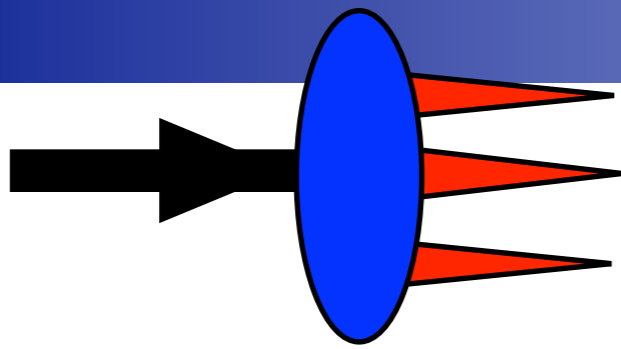
*Lorce,
Pasquini*

Diehl, Hwang, sjb

→ $\int d^2 b_{\perp}$
 → $\int dx$
 → $\int d^2 k_{\perp}$



Sivers, T-odd from lensing



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

GTMDs

$$x, \vec{k}_{\perp}, \vec{b}_{\perp}$$

Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$

Transverse density in position space

TMDs

$$x, \vec{k}_{\perp}$$

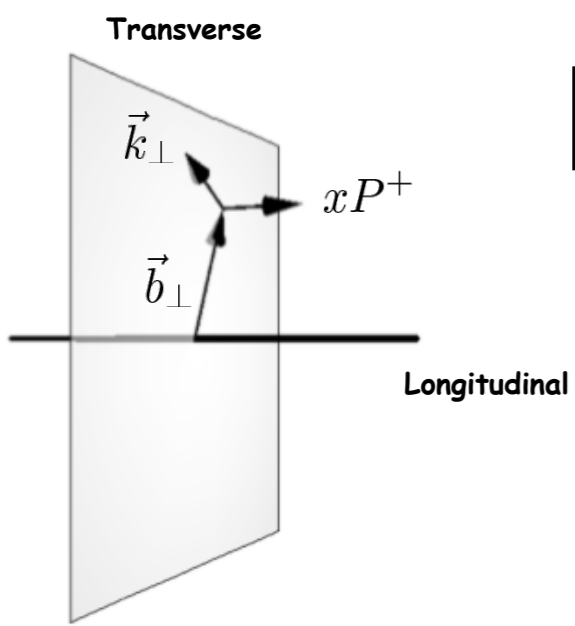
TMFFs

$$\vec{k}_{\perp}, \vec{b}_{\perp}$$

GPDs

$$x, \vec{b}_{\perp}$$

Lorce, Pasquini



TMSDs

$$\vec{k}_{\perp}$$

PDFs

$$x,$$

FFs

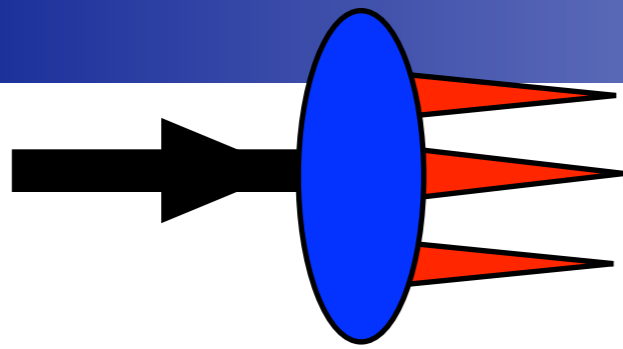
$$\vec{b}_{\perp}$$

Diehl, Hwang, sjb

Charges

- $\int d^2 b_{\perp}$
- $\int dx$
- $\int d^2 k_{\perp}$

Sivers, T-odd from lensing

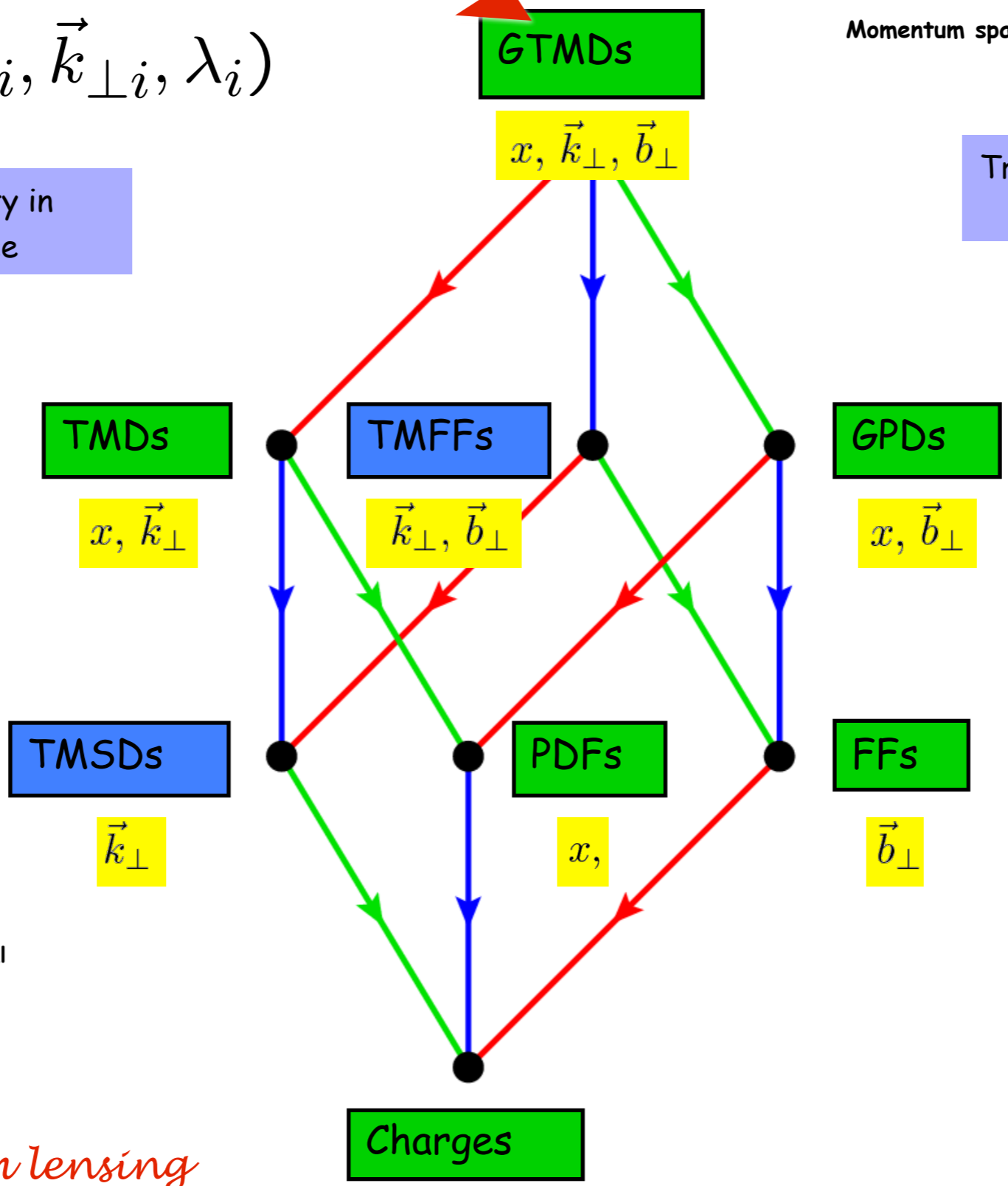


• *Light Front Wavefunctions:*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

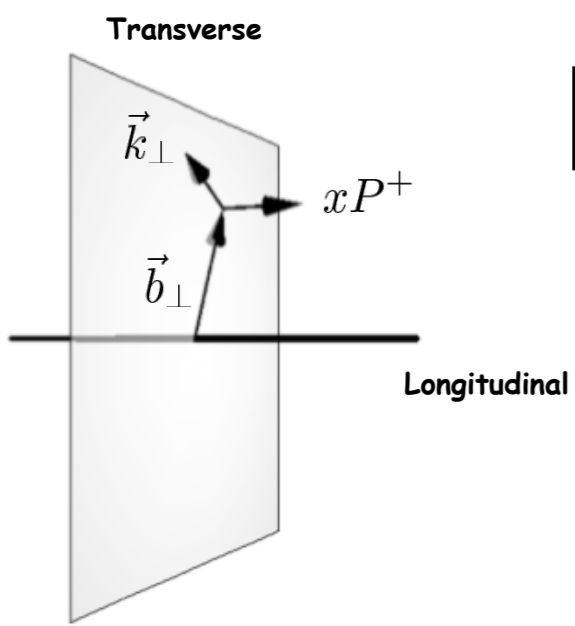
Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
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 Transverse density in position space



Lorce, Pasquini

Diehl, Hwang, sjb

- $\int d^2 b_{\perp}$
- $\int dx$
- $\int d^2 k_{\perp}$



Sivers, T-odd from lensing

$$|\psi_p(P^+, \vec{P}_\perp)\rangle = \sum_n \prod_{i=1}^n \frac{dx_i d^2\vec{k}_{\perp i}}{\sqrt{x_i} 16\pi^3} 16\pi^3 \delta\left(1 - \sum_{i=1}^n x_i\right) \delta^{(2)}\left(\sum_{i=1}^n \vec{k}_{\perp i}\right) \\ \times \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle.$$

$$q_{\lambda_q/\Lambda_p}(x, \Lambda) = \sum_{n, q_a} \int \prod_{j=1}^n dx_j d^2\vec{k}_{\perp j} \sum_{\lambda_i} |\psi_{n/H}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 \\ \times \delta\left(1 - \sum_i x_i\right) \delta^{(2)}\left(\sum_i \vec{k}_{\perp i}\right) \delta(x - x_q) \delta_{\lambda_a \lambda_q} \Theta(\Lambda^2 - \mathcal{M}_n^2),$$

Obeys DGLAP Evolution ***Defines quark distributions***

Connection to Bethe-Salpeter:

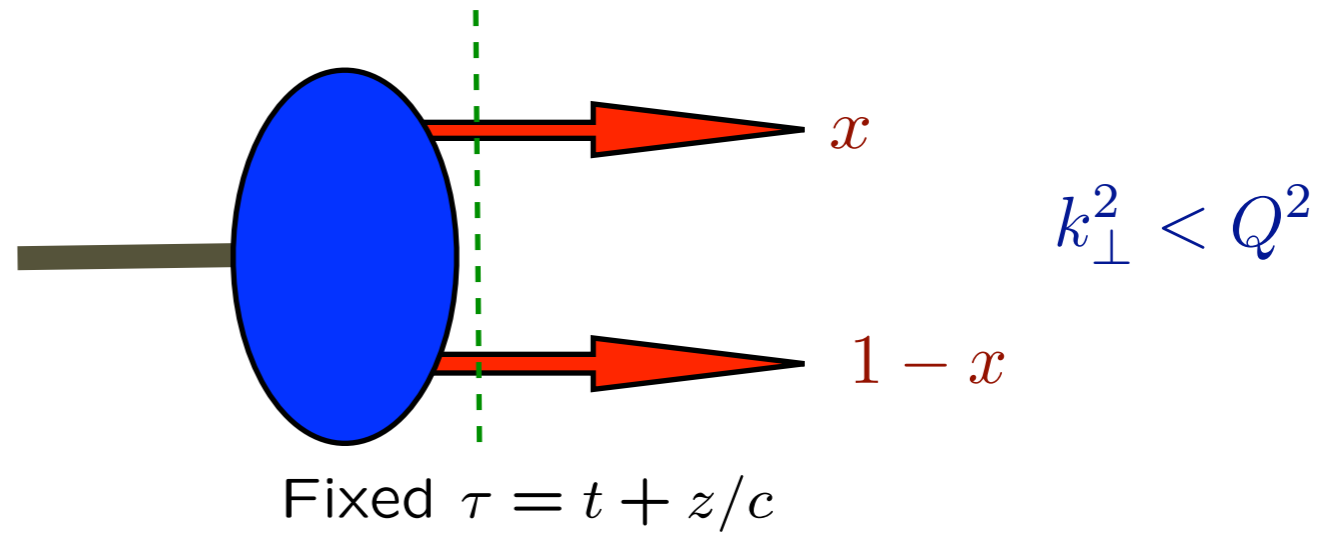
$$\int dk^- \Psi_{BS}(k, P) \rightarrow \psi_{LF}(x, \vec{k}_\perp) \quad \Psi_{BS}(x, P)|_{x^+=0}$$

Hadron Distribution Amplitudes

$$A^+ = 0$$

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

$$\sum_i x_i = 1$$



- Fundamental **gauge invariant** non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

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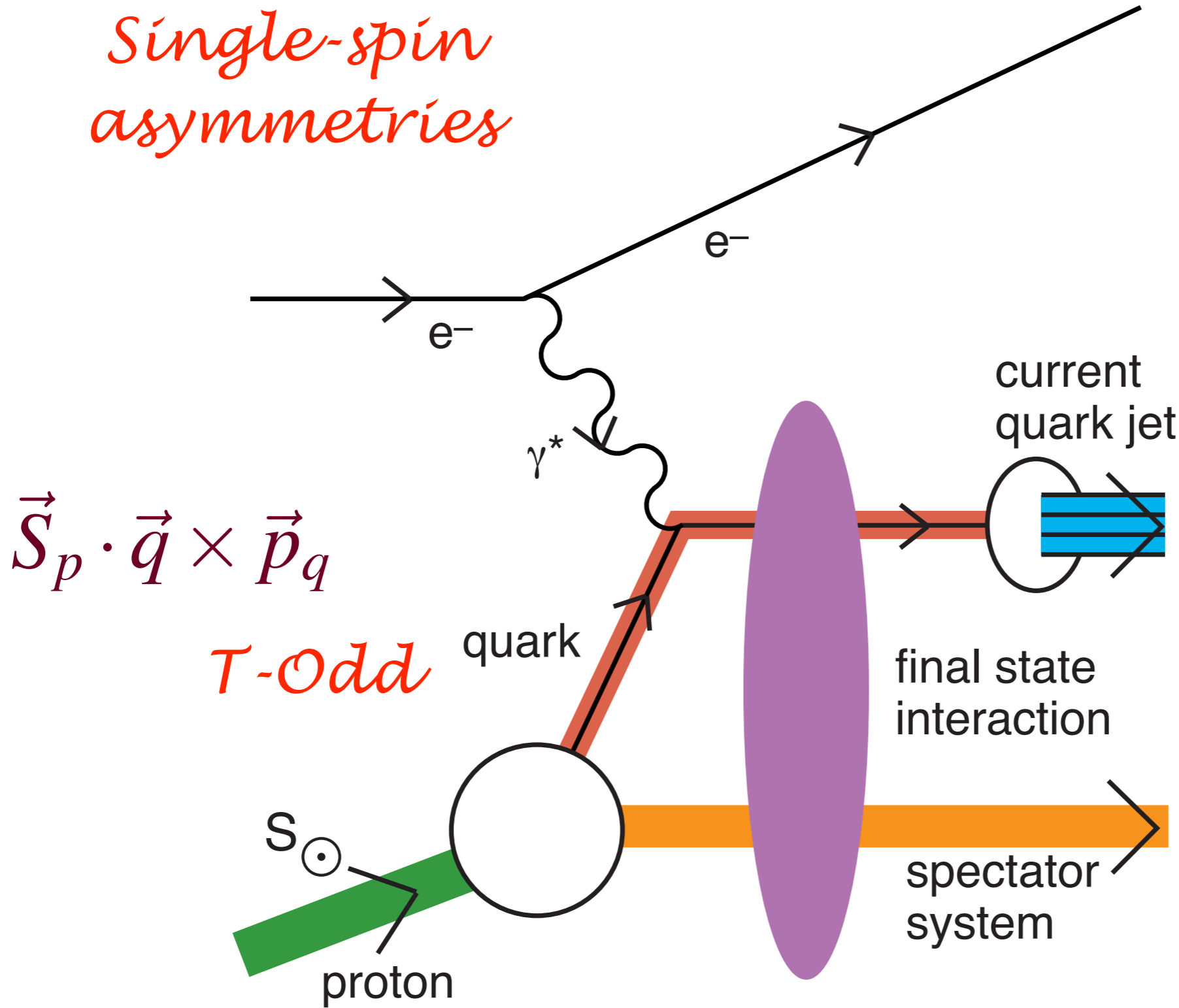
September 25, 2015

Light-Front Holographic QCD, Color Confinement,
and Supersymmetric Features of QCD

Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

Single-spin asymmetries



Hwang, Schmidt,
sjb

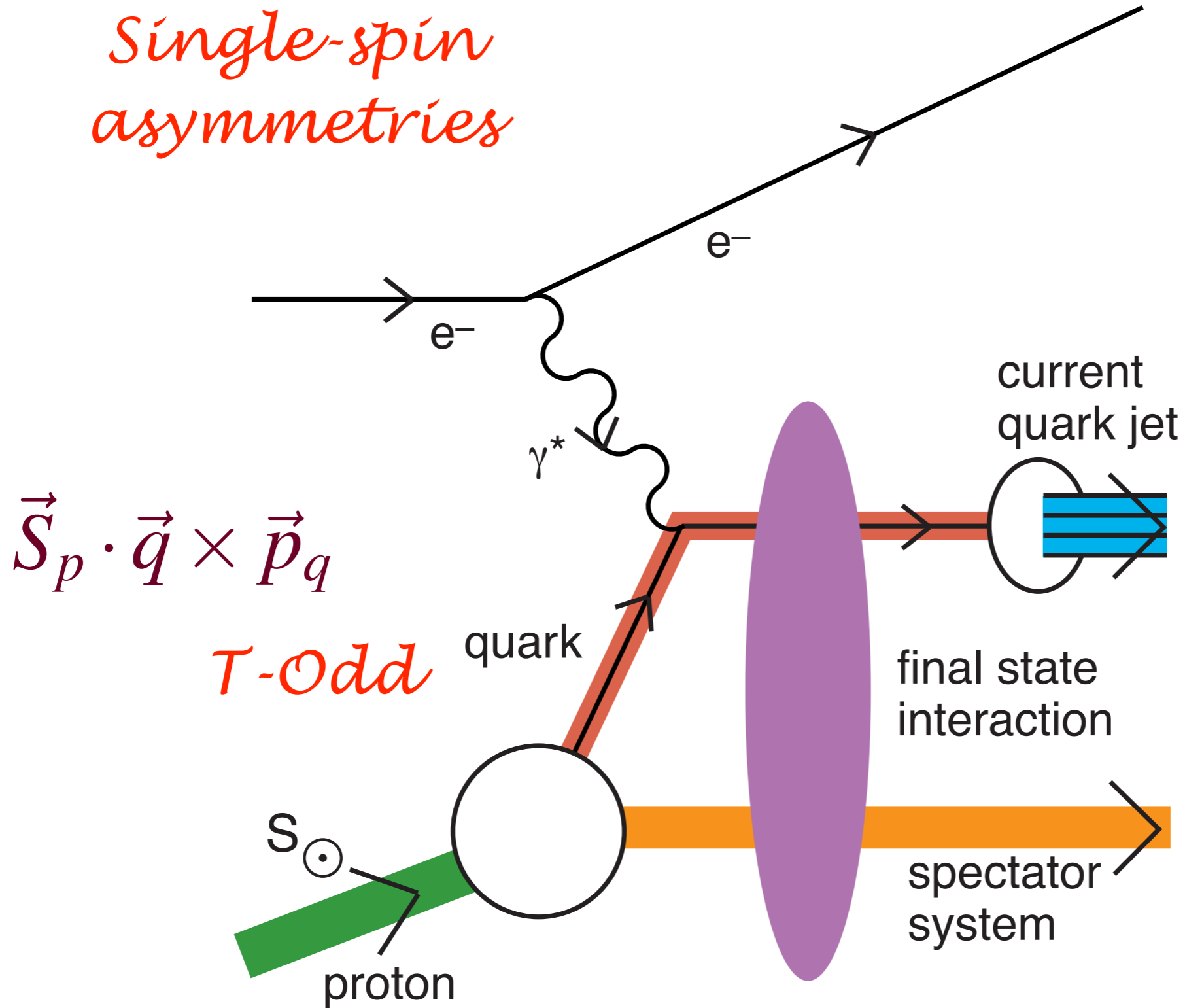
Collins, Burkardt, Ji,
Yuan. Pasquini, ...

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

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$$\vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

T-Odd

quark

current quark jet

final state interaction

spectator system

S_p

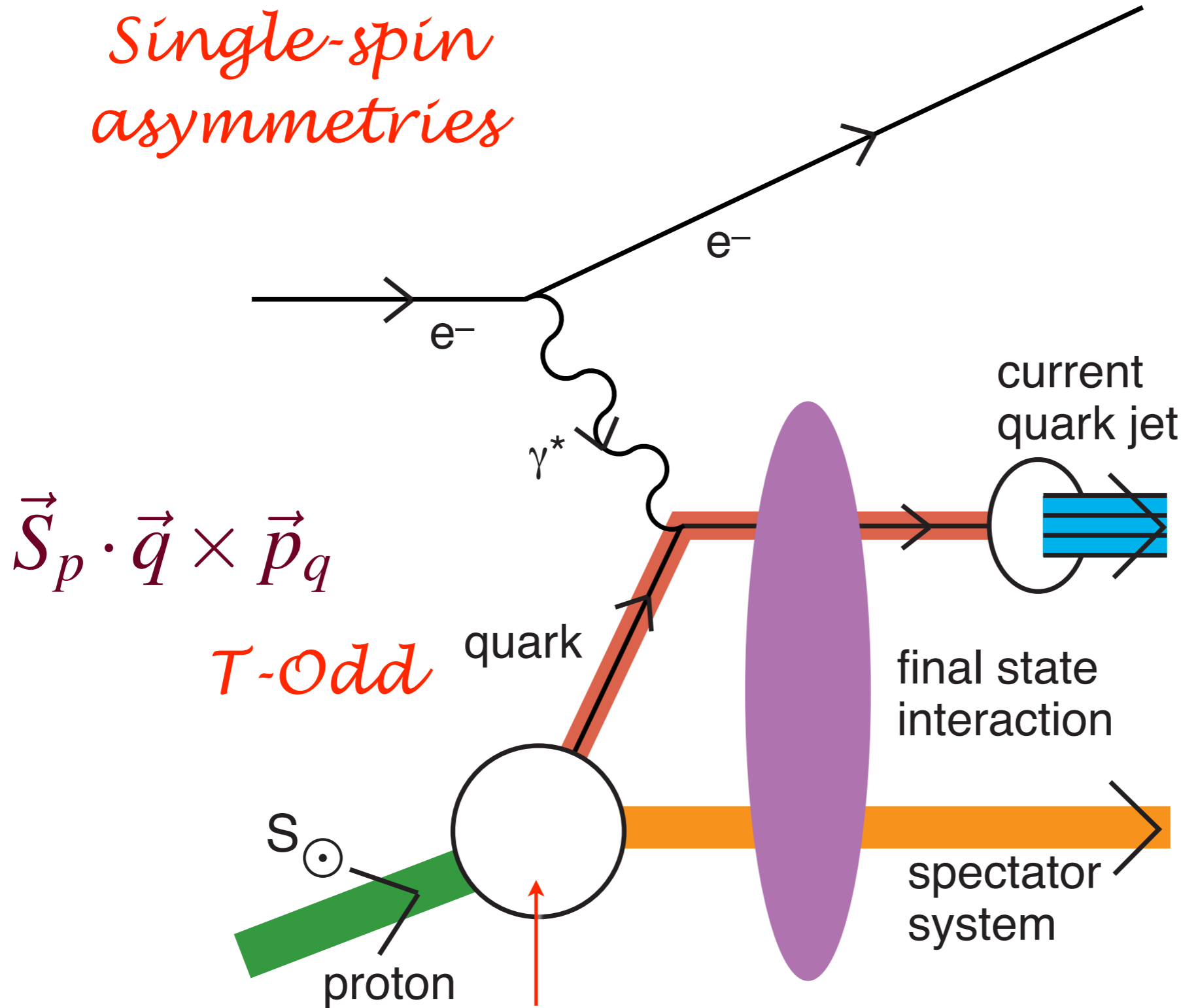
proton

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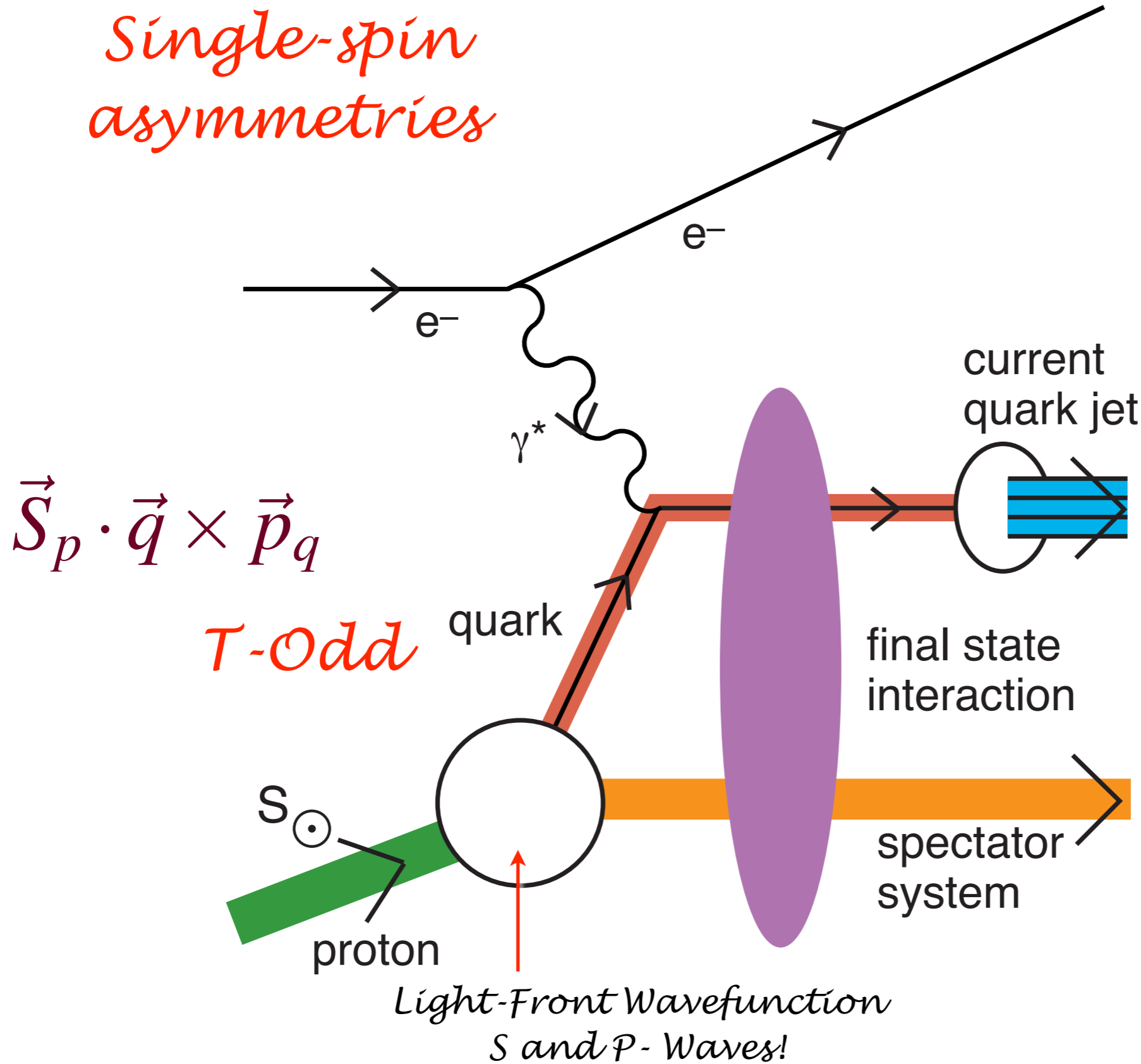
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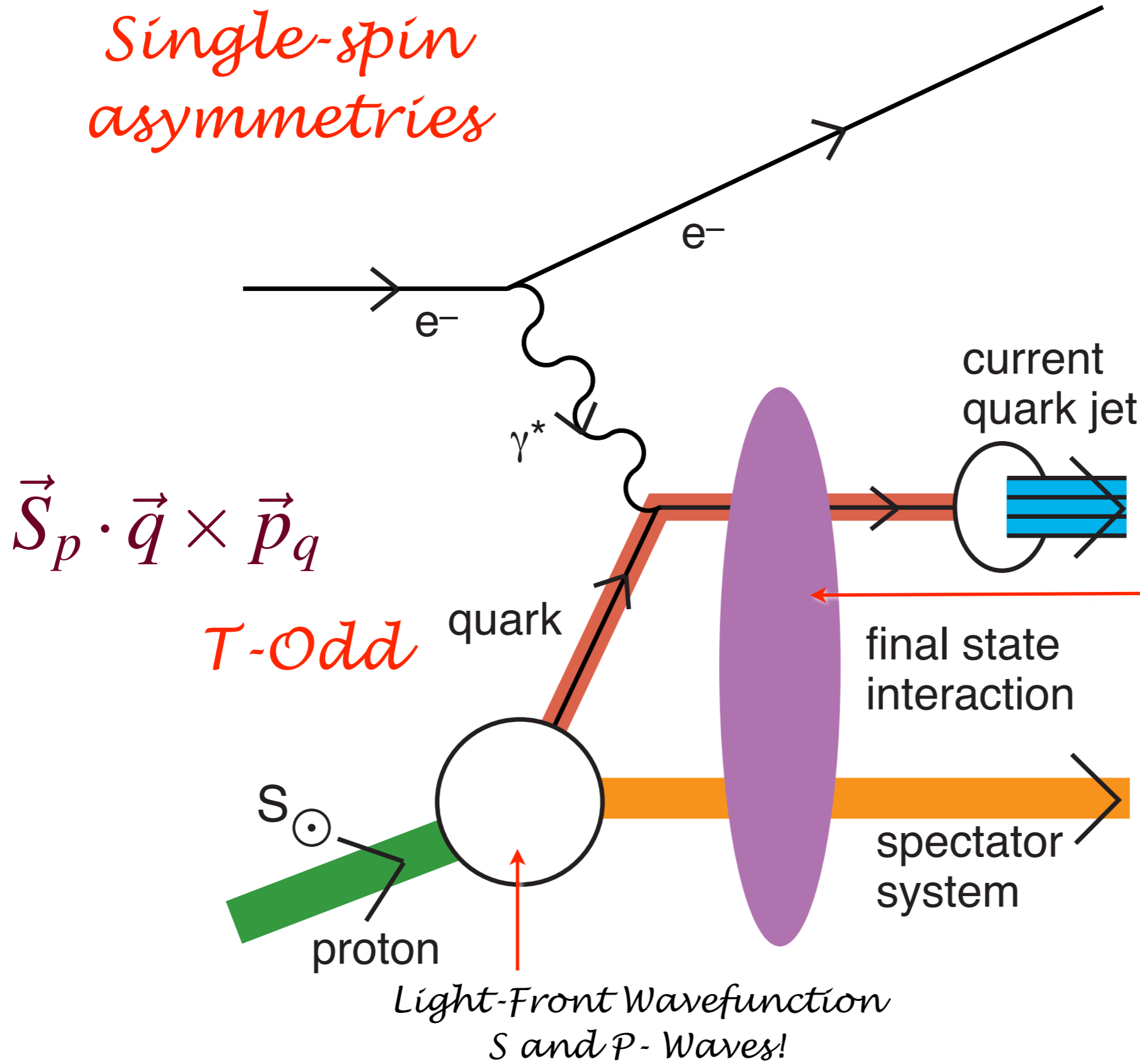
Light-Front Wavefunction
S and P-Waves!

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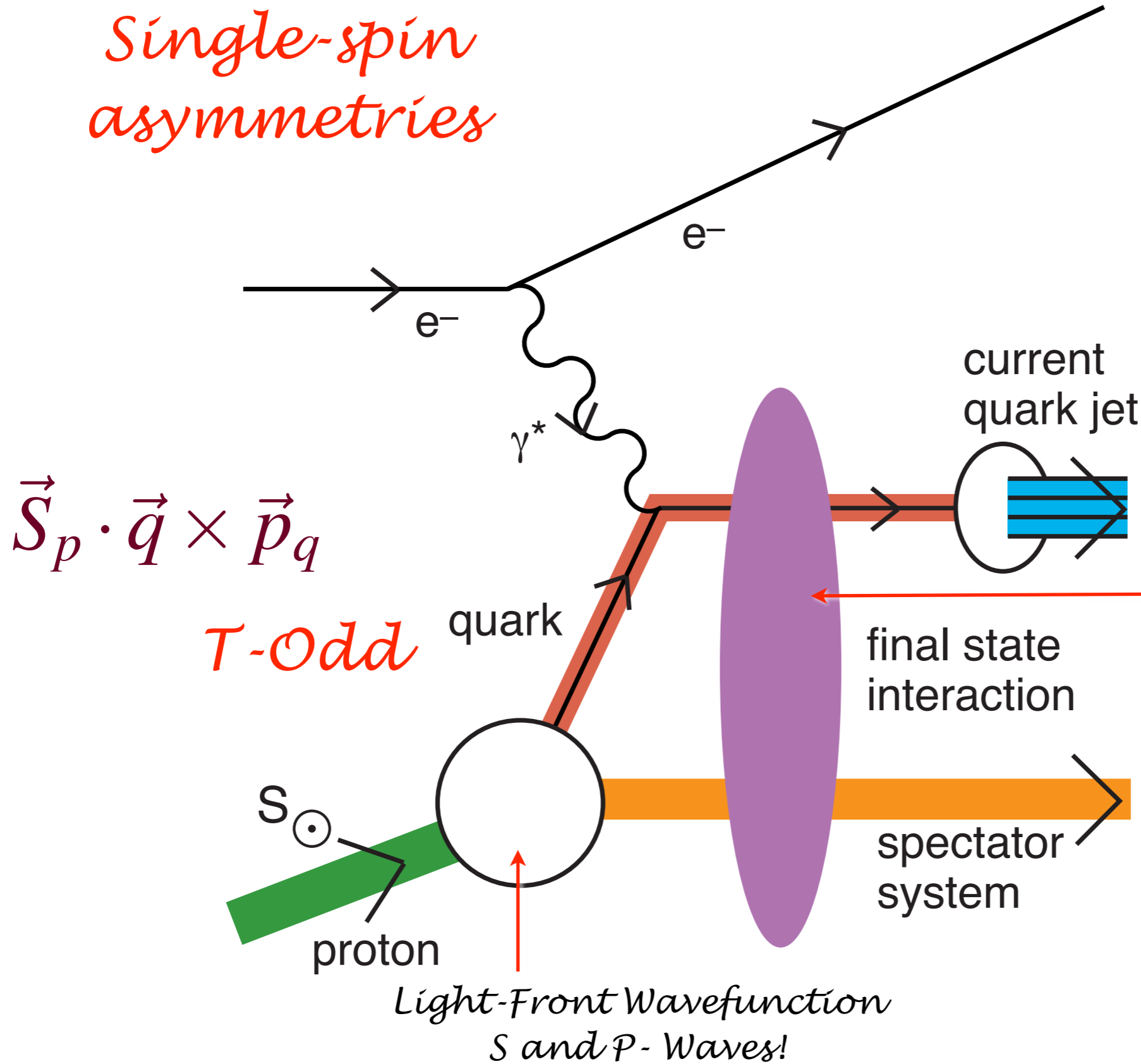
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QCD S- and P-Coulomb Phases --Wilson Line

“Lensing Effect”



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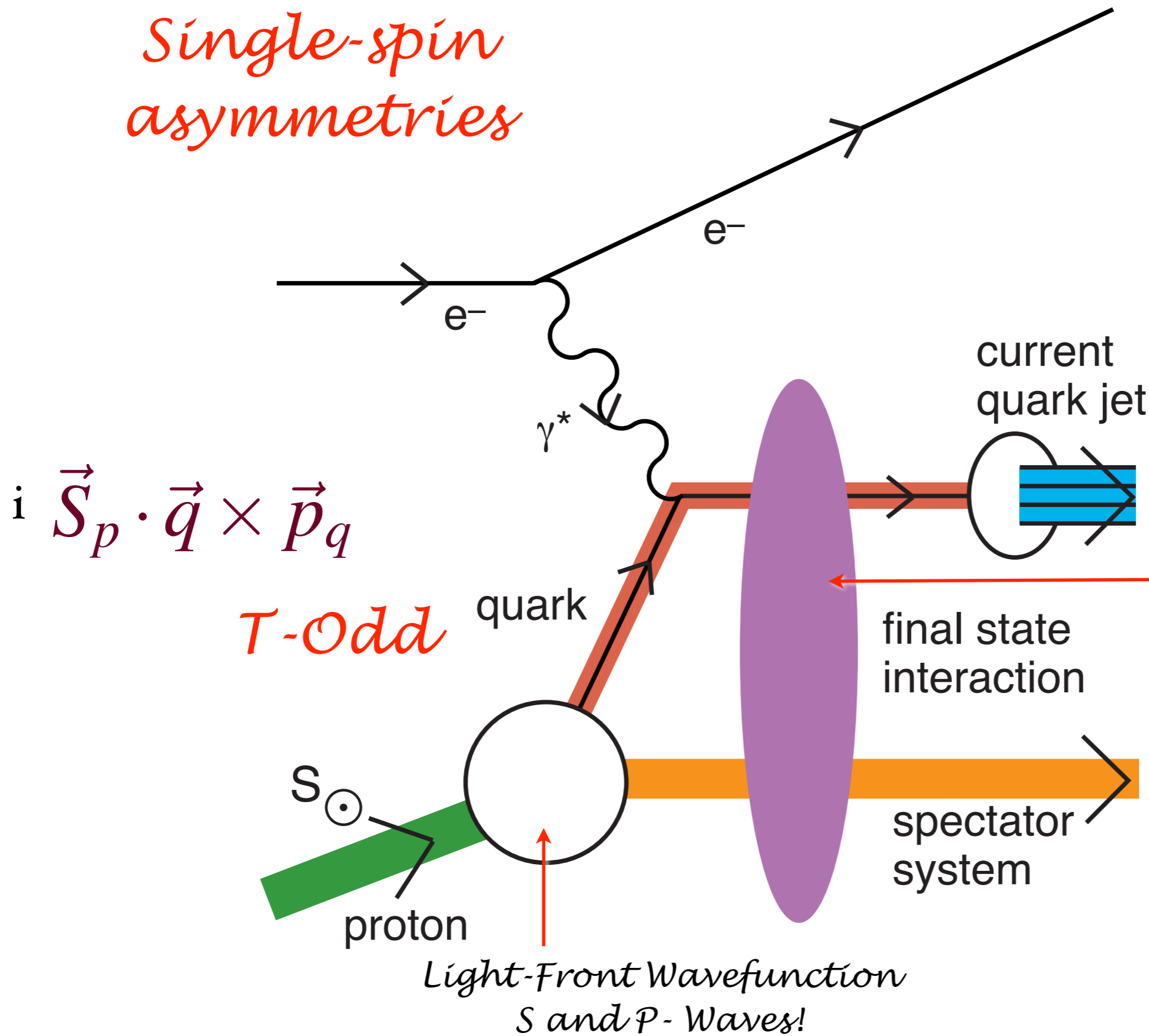
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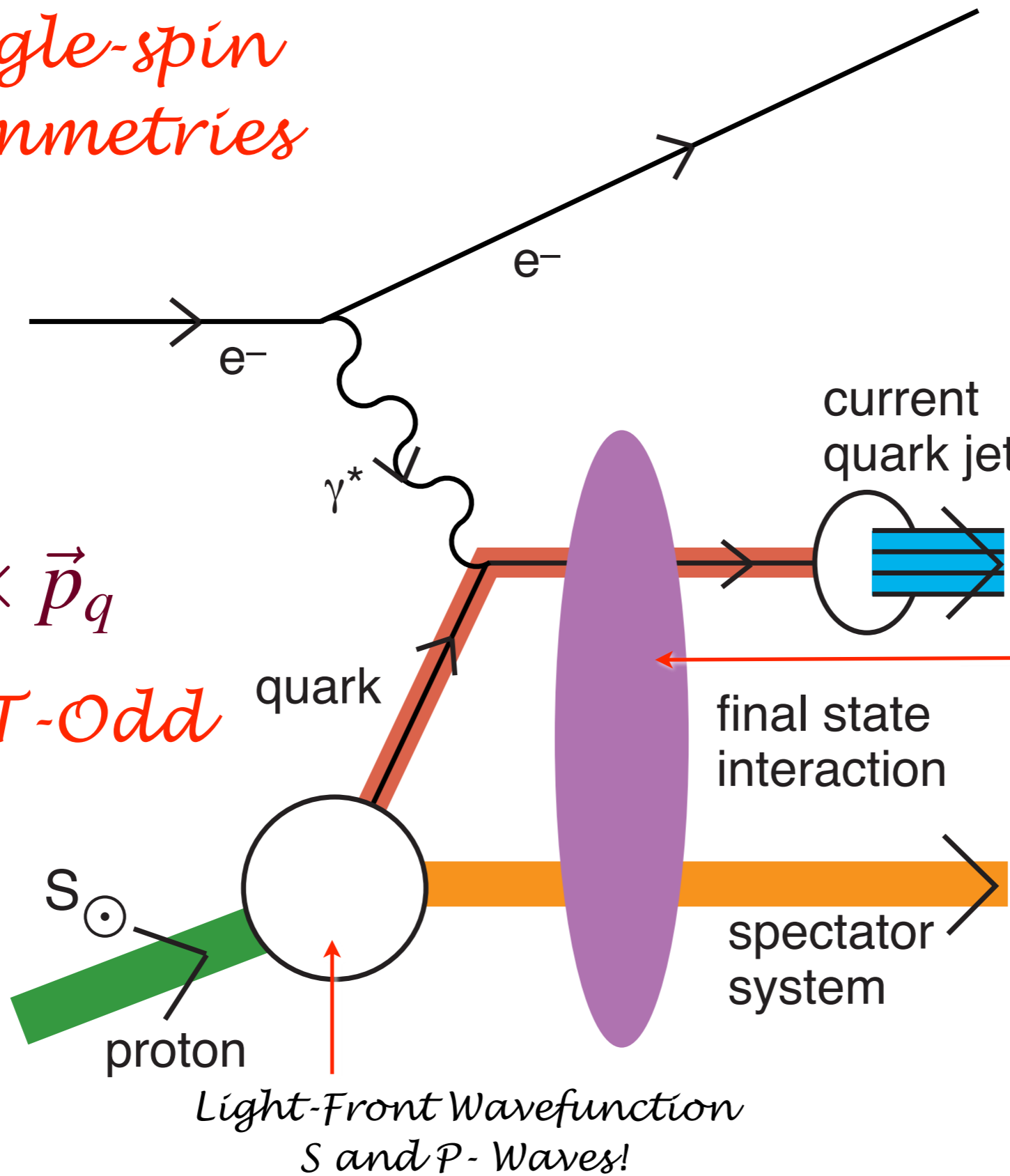
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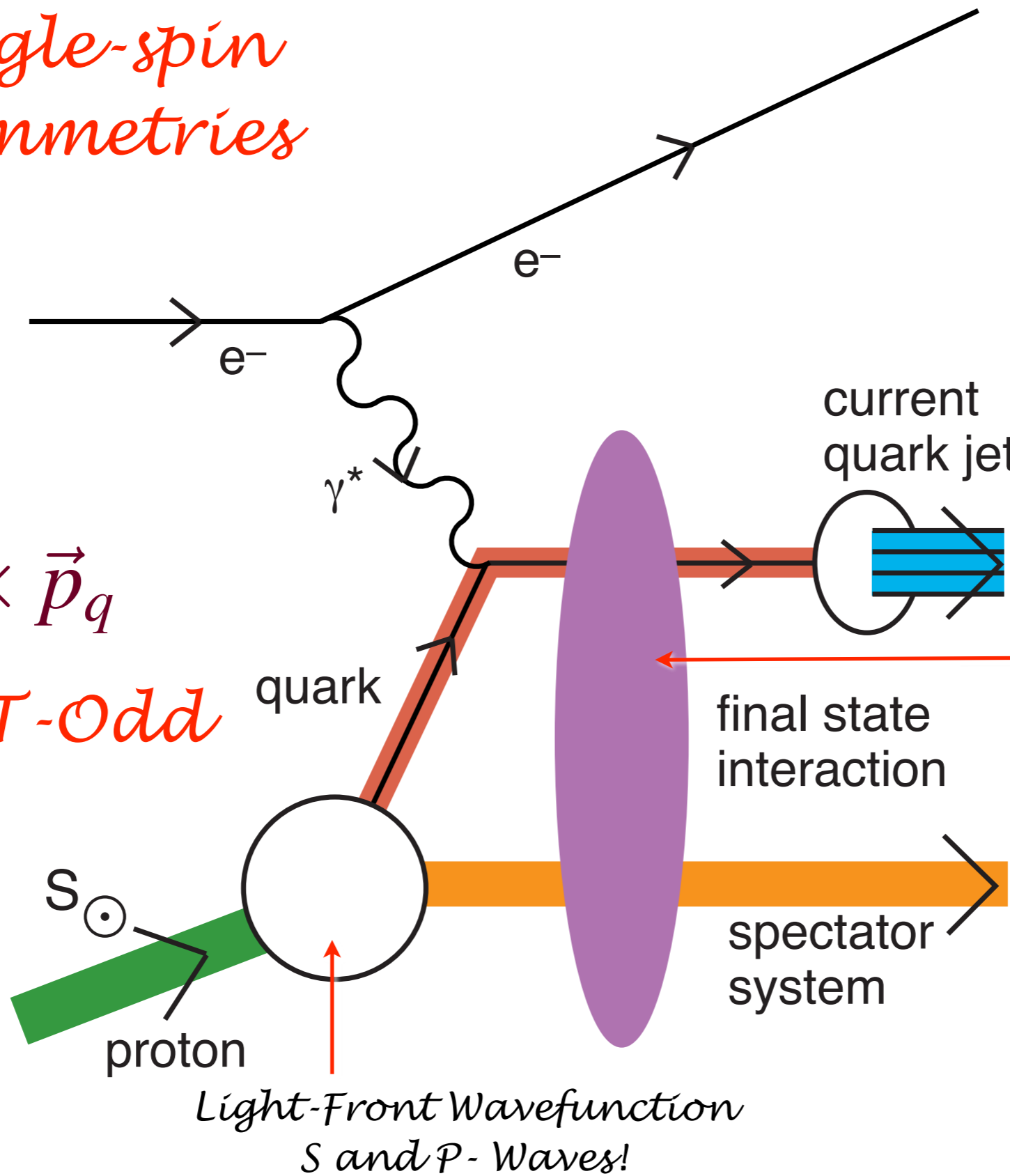
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S and P-Waves!*

Single-spin asymmetries



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*Leading-Twist
Rescattering
Violates pQCD
Factorization!*

Single-spin asymmetries

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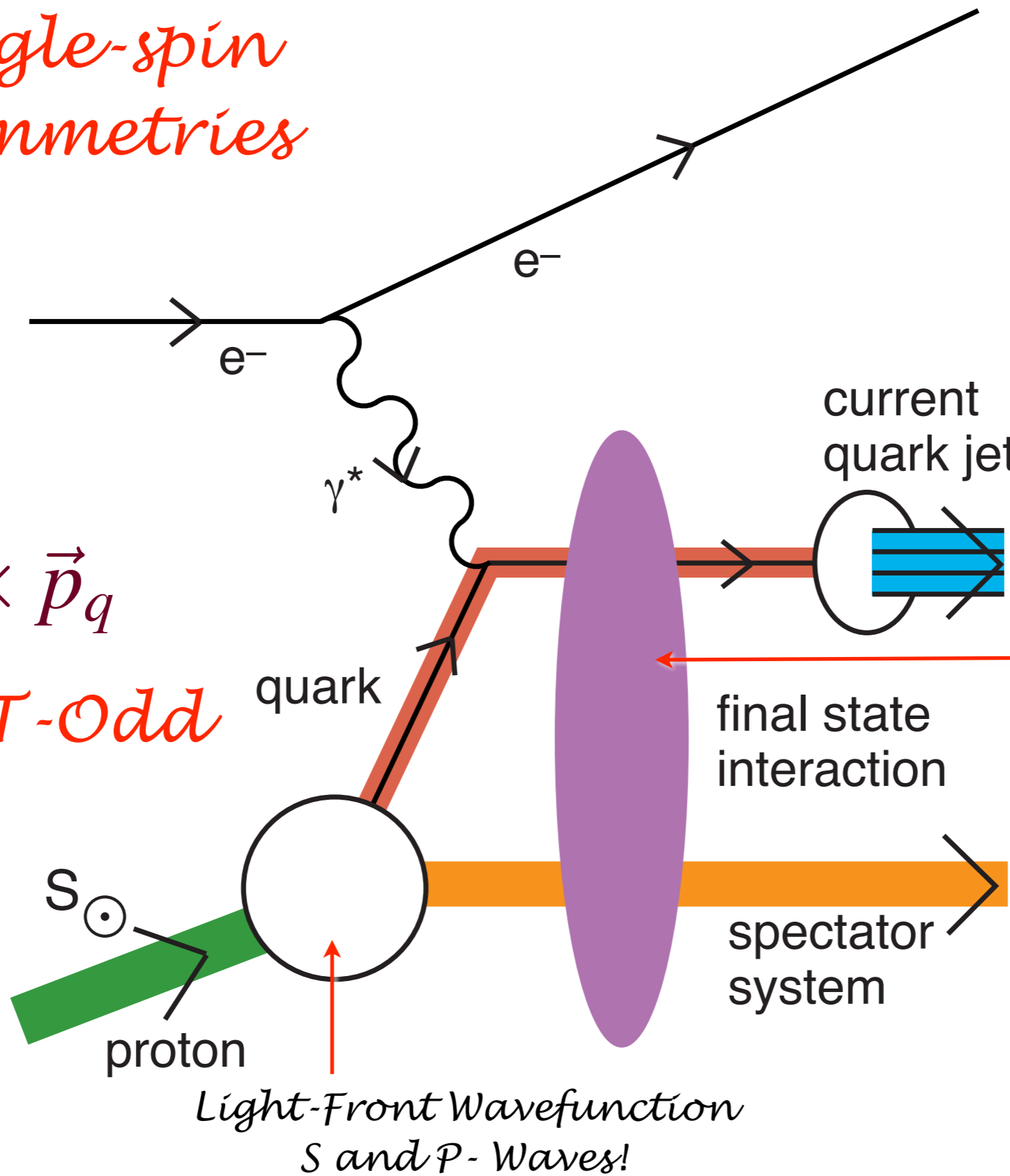
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Sign reversal in DY!

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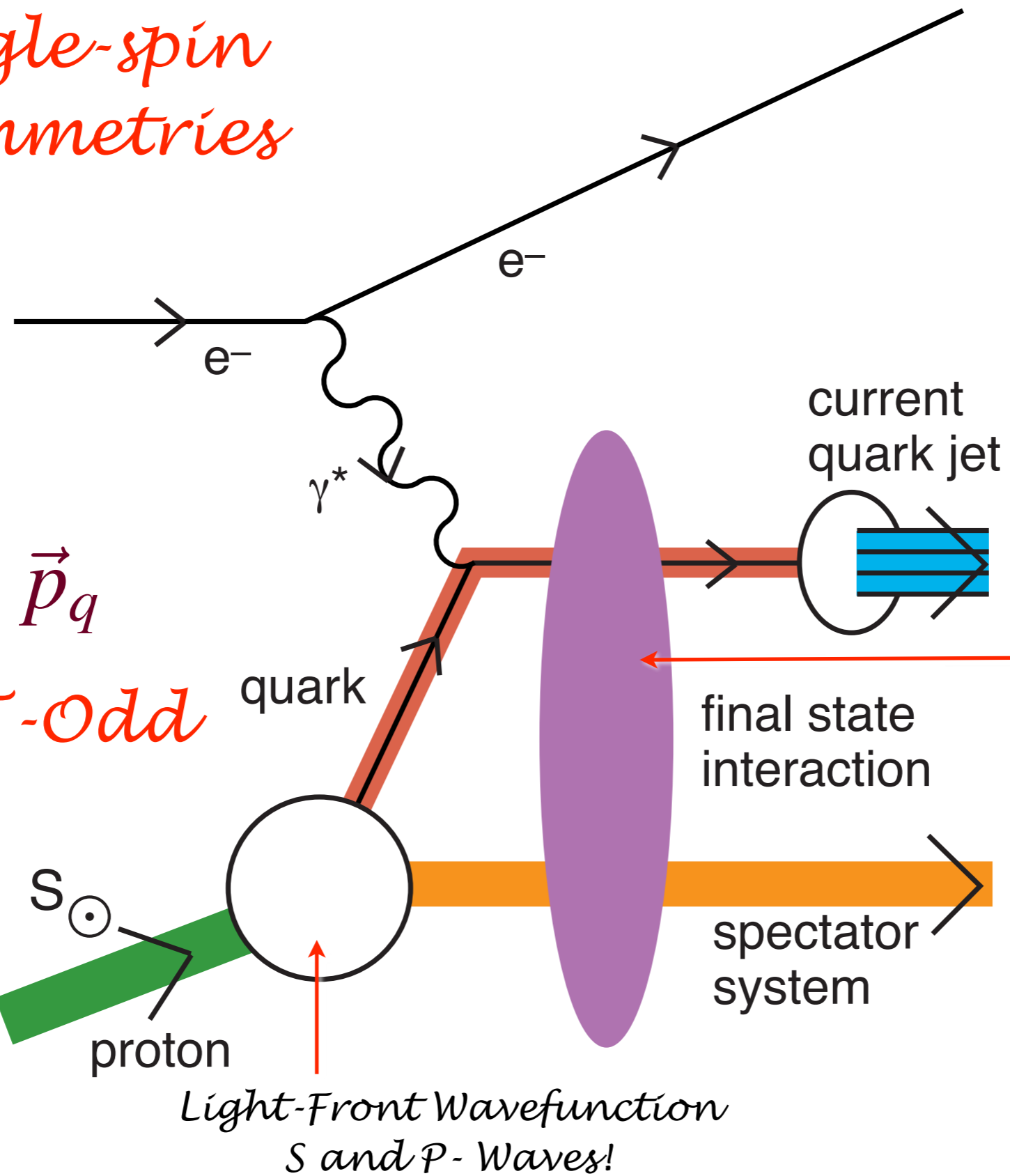
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Pseudo-T-Odd

**QED:
Lensing
involves soft
scales**

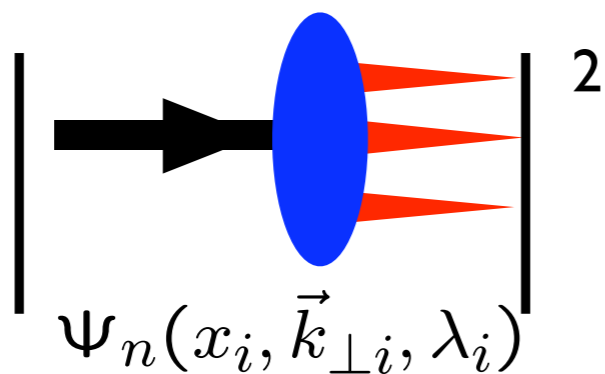
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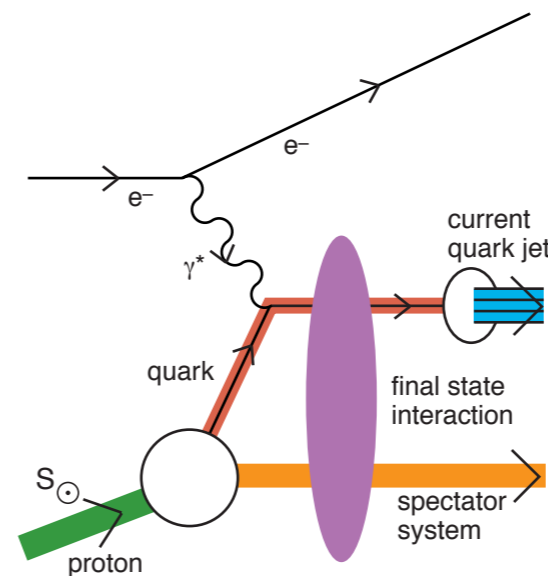
Static

- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and J^z
- DGLAP Evolution; mod. at large x
- No Diffractive DIS



Dynamic

- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Sum Rules Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon Diffractive DIS



**Hwang,
Schmidt, sjb,
Mulders, Boer
Qiu, Sterman
Collins, Qiu**

**Pasquini, Lorce, Xi
Yuan, sjb**

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**
- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, modulo 'lensing' from ISIs, FSIs**
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- **Hadron Physics without LFWFs is like Biology without DNA!**

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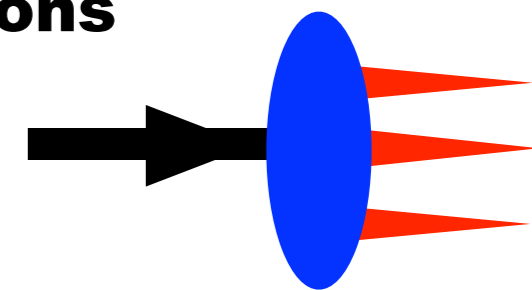
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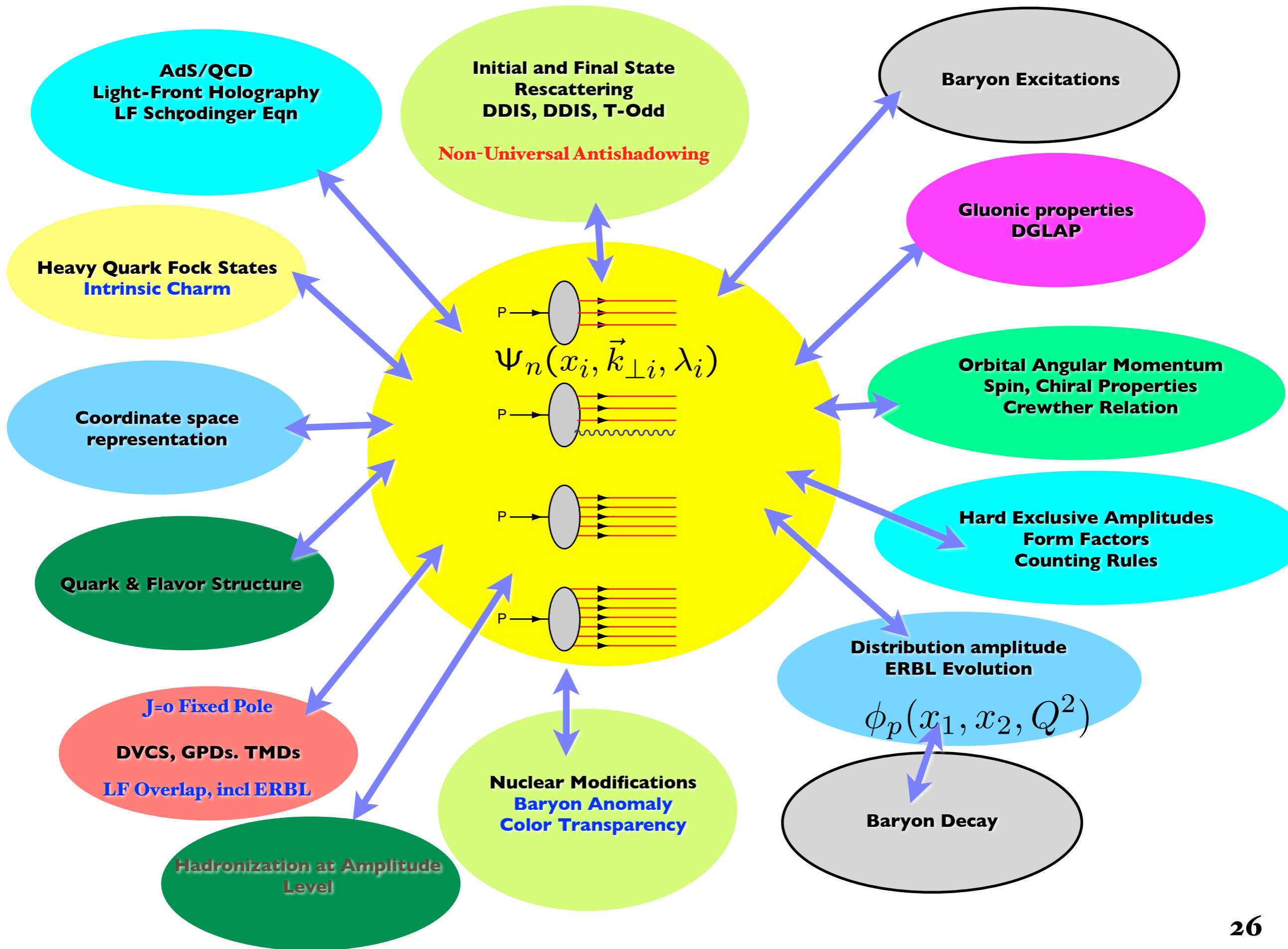
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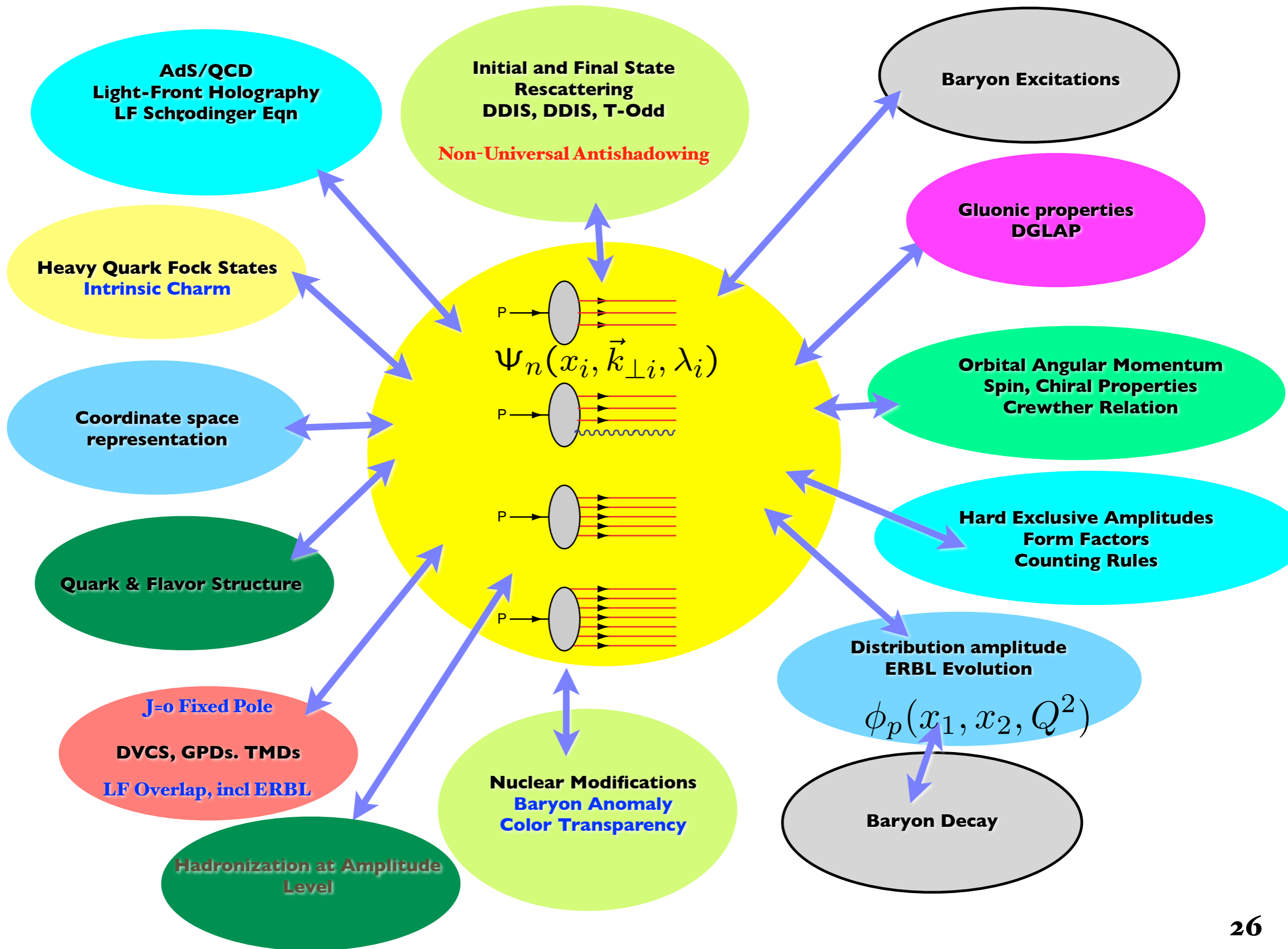


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

QCD and the LF Hadron Wavefunctions



QCD and the LF Hadron Wavefunctions



QCD Lagrangian

gluon dynamics

quark kinetic energy +
quark-gluon dynamics

quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

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Rotation and Phase Invariance at
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QCD Mass Scale from Confinement not Explicit

Atomic Physics from First Principles

\mathcal{L}_{QED} →

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

$$\left[-\frac{\Delta^2}{2m_{red}} + V_{eff}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{red}} \frac{d^2}{dr^2} + \frac{1}{2m_{red}} \frac{\ell(\ell+1)}{r^2} + V_{eff}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Semiclassical first approximation to QED --> Bohr Spectrum

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Light-Front QCD

\mathcal{L}_{QCD} →

$$H_{QCD}^{LF}$$

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

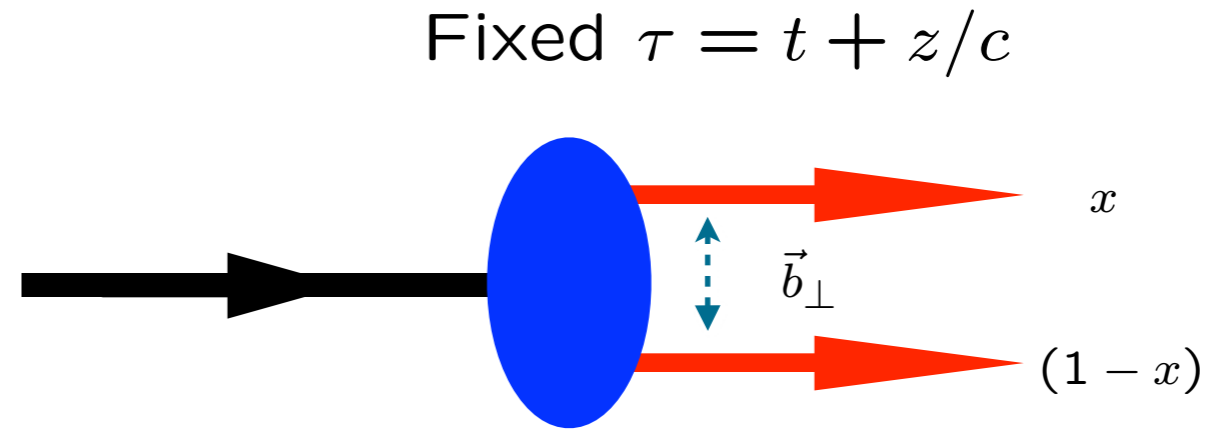
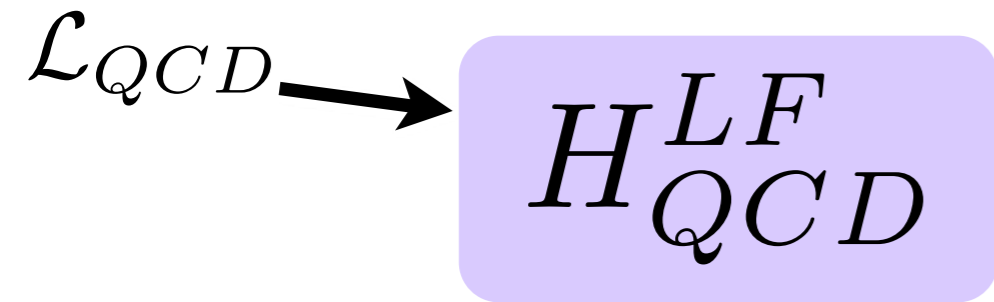
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Azimuthal Basis

ζ, ϕ

Light-Front QCD



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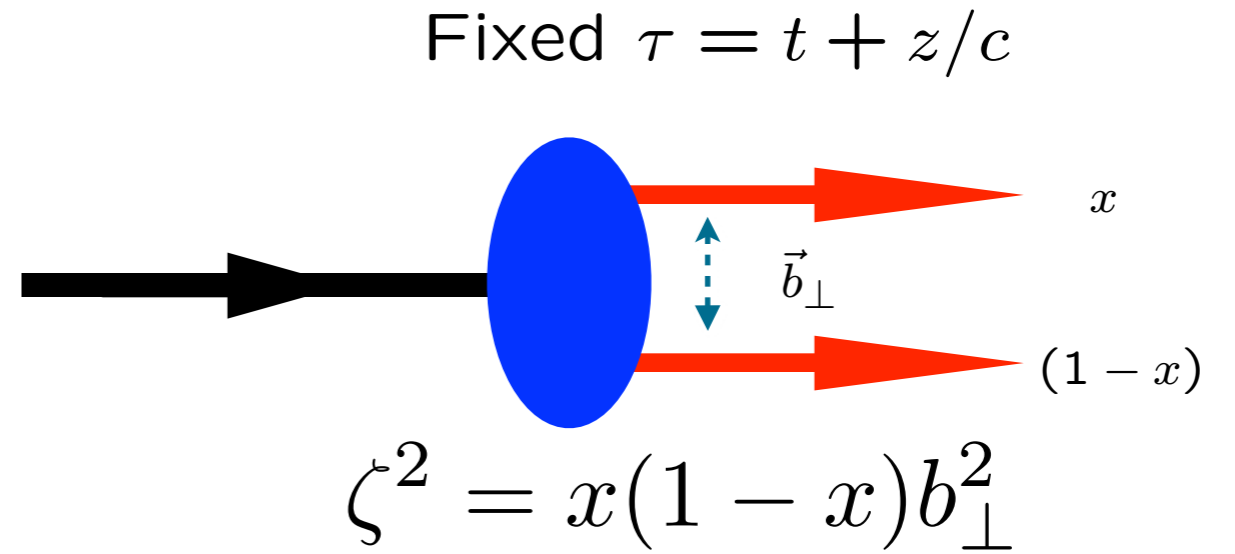
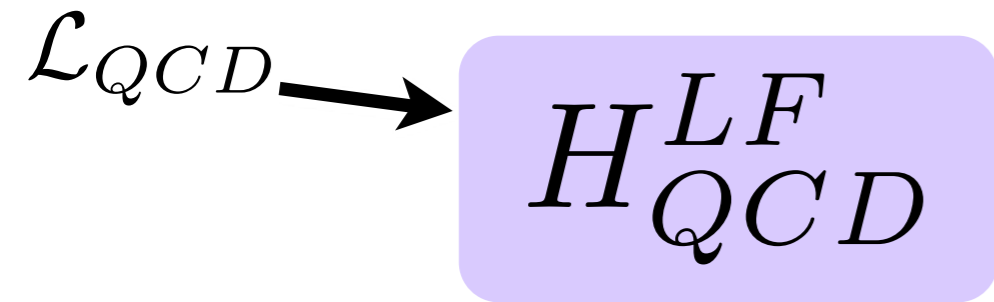
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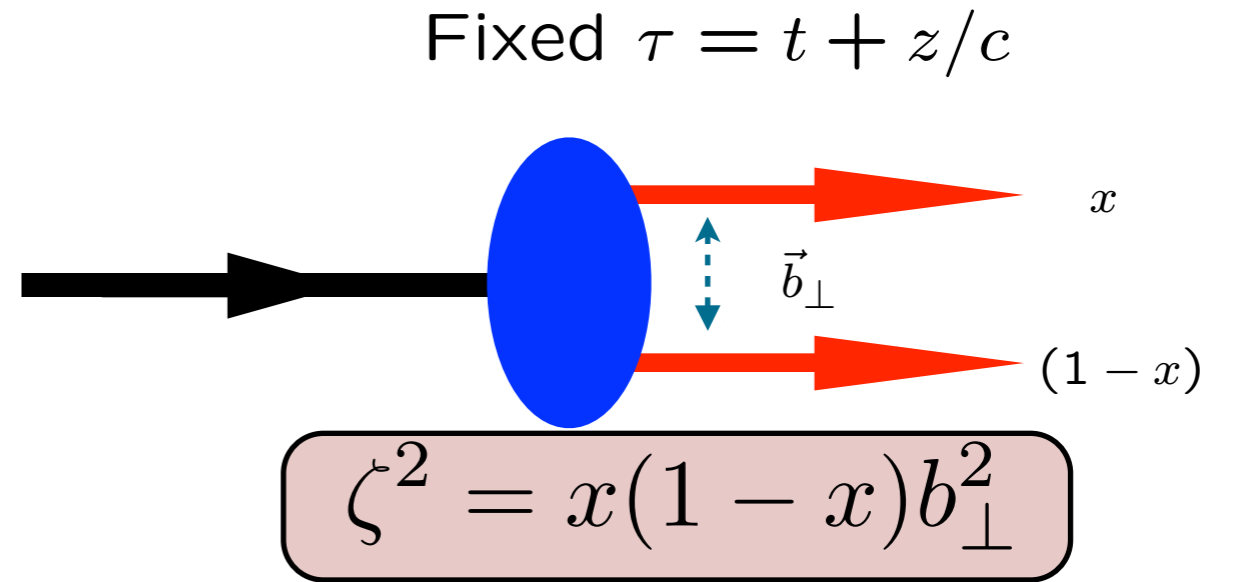
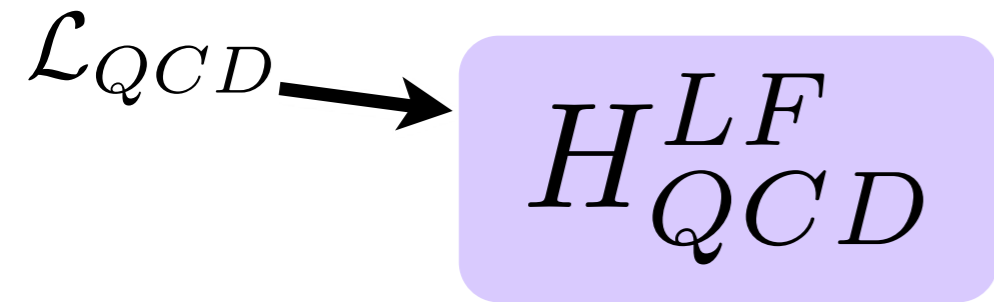
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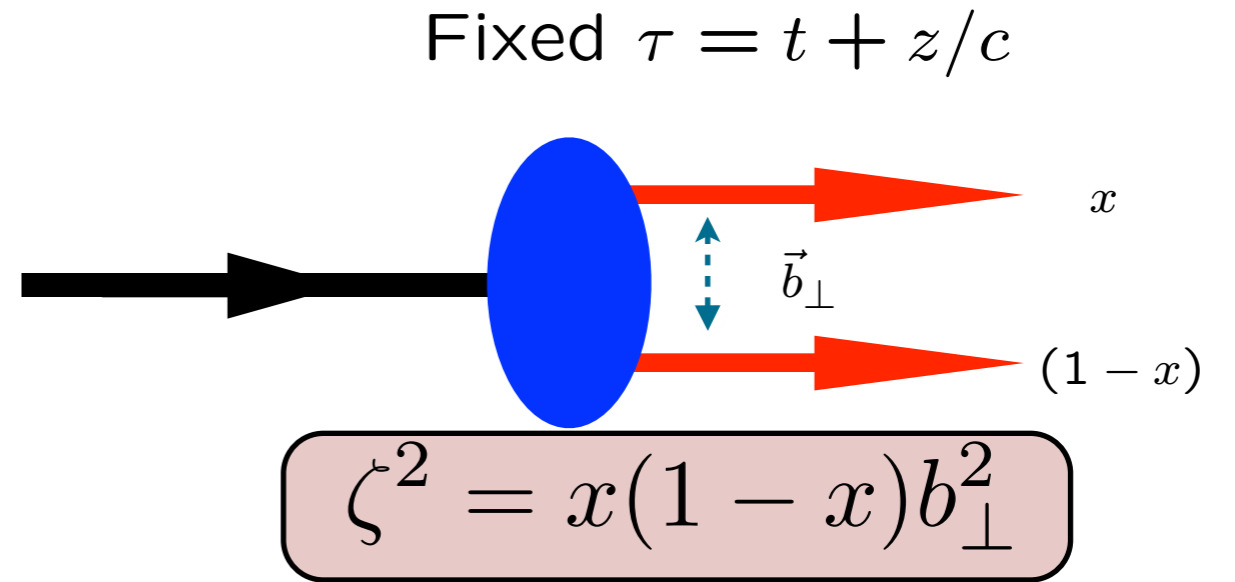
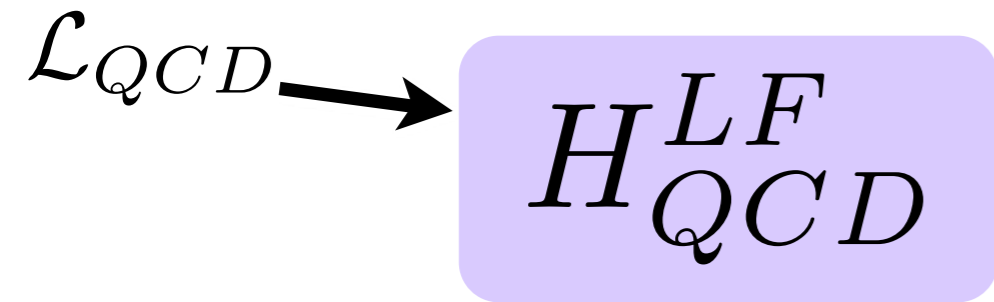
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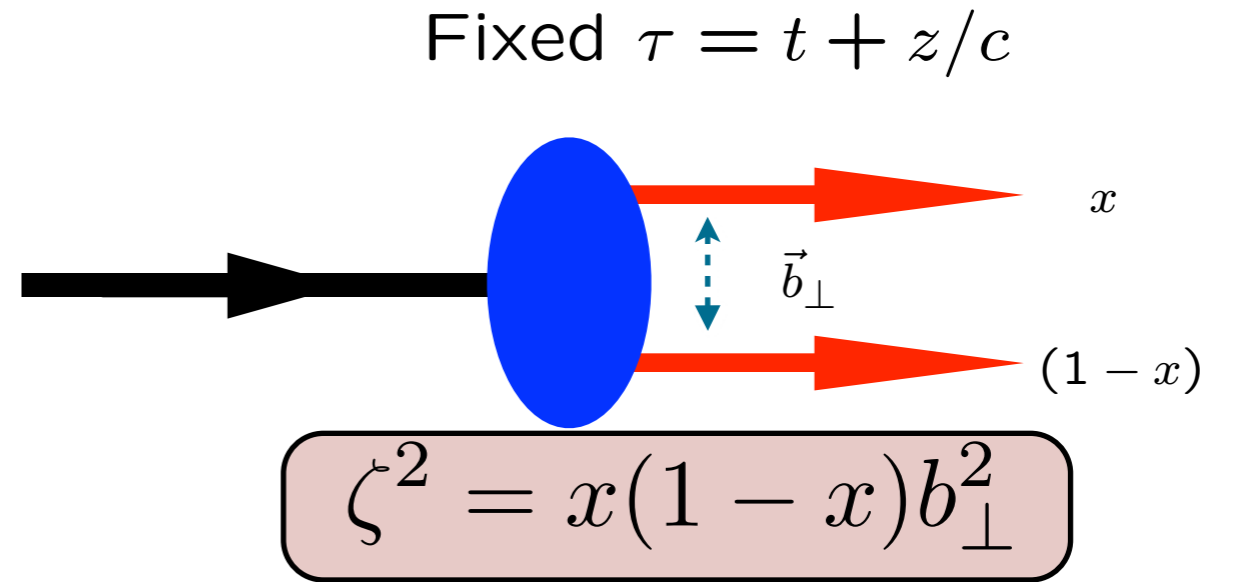
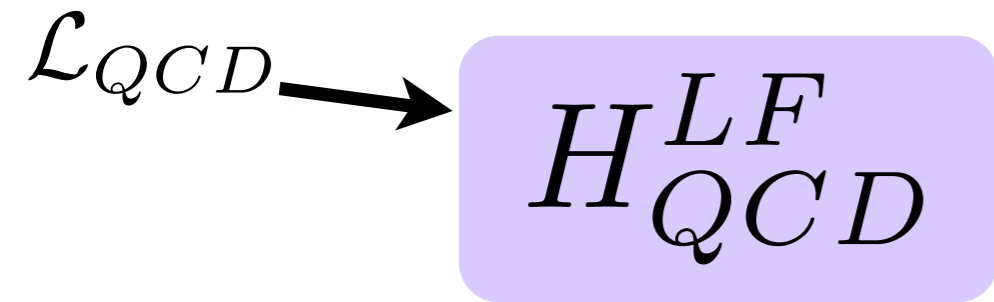
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AdS/QCD:

Light-Front QCD



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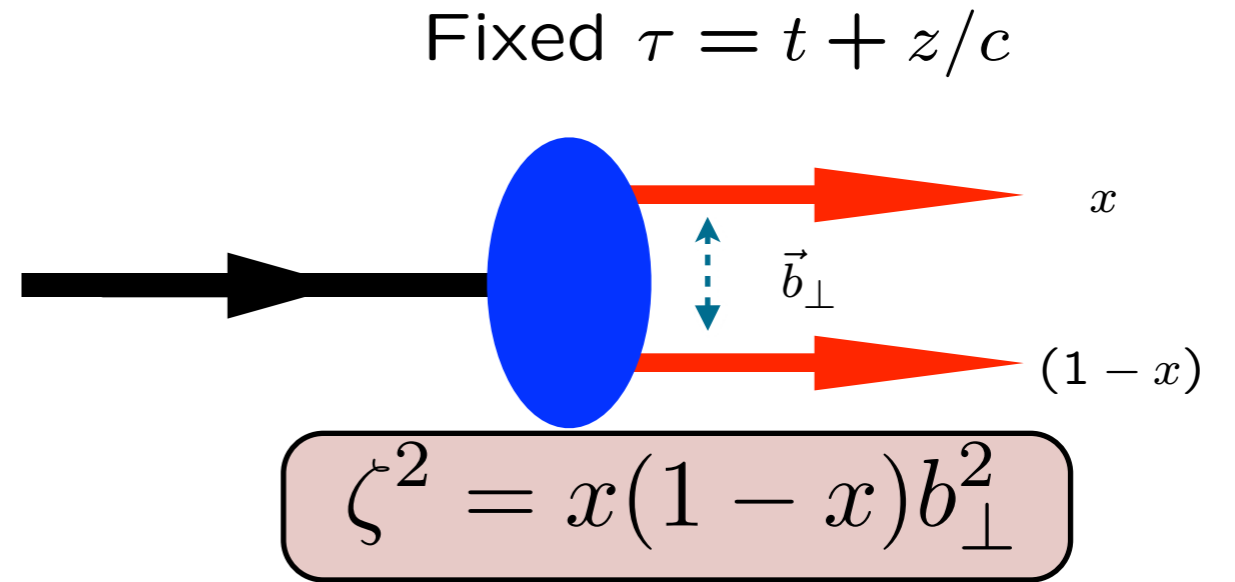
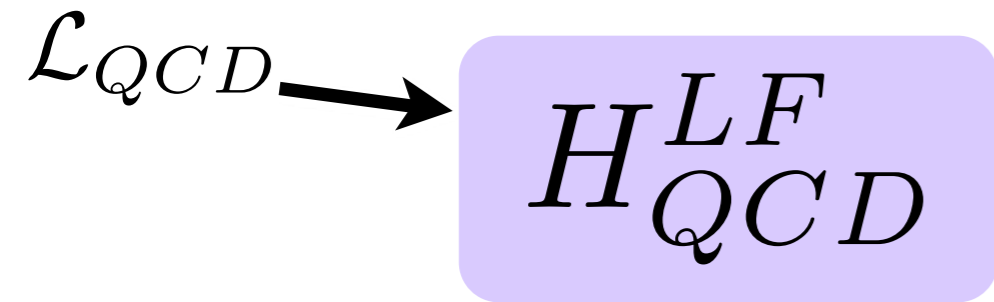
$$\zeta, \phi$$

AdS/QCD:

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

de Tèramond, Dosch, sjb

Light-Front QCD



$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_{\perp}^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_{\perp}) = M^2 \psi_{LF}(x, \vec{k}_{\perp})$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1 + 4L^2}{4\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

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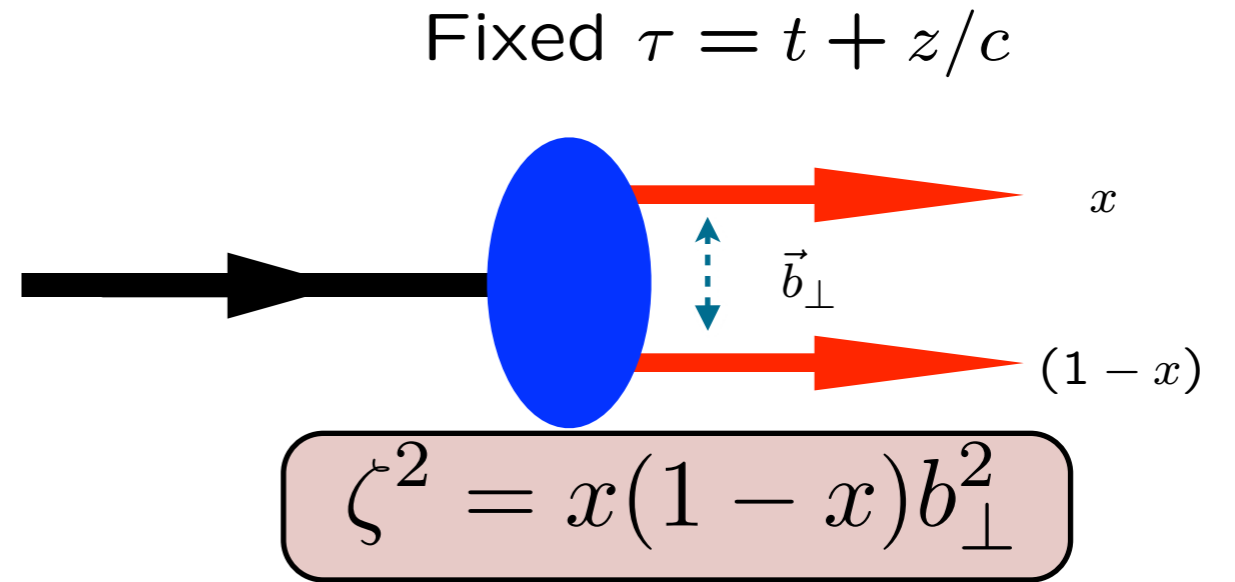
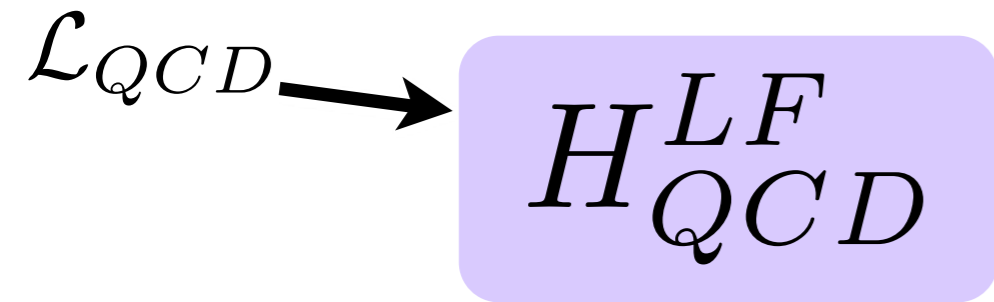
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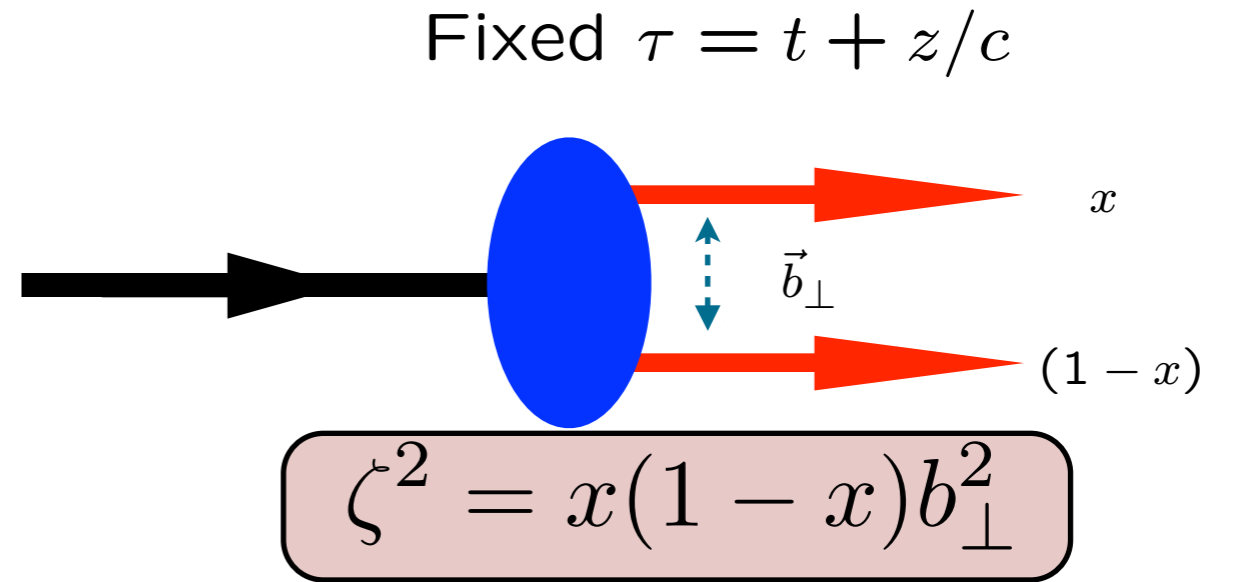
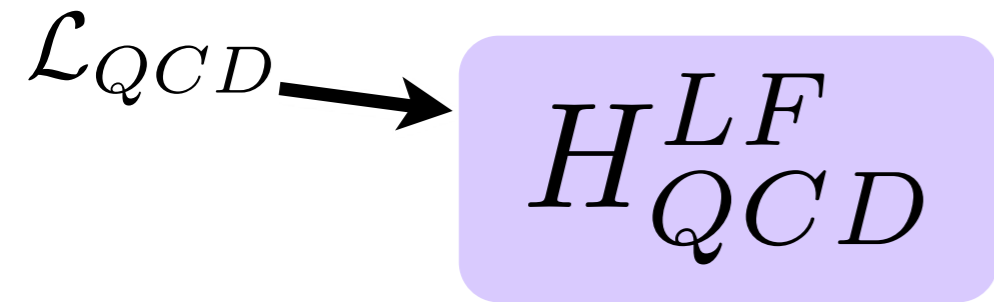
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Semiclassical first approximation to QCD

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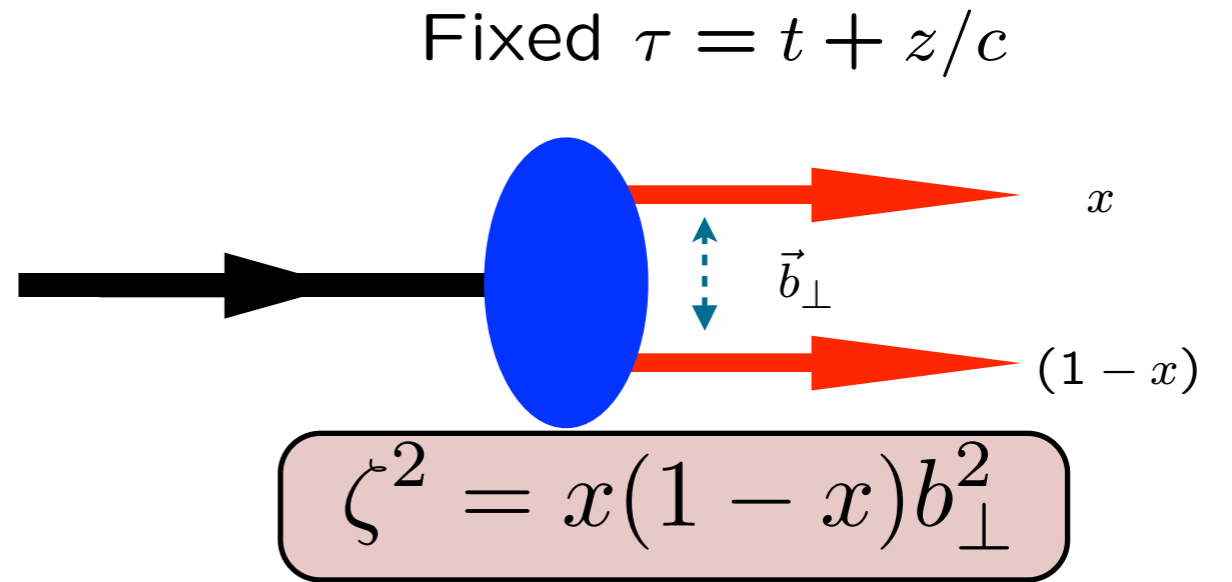
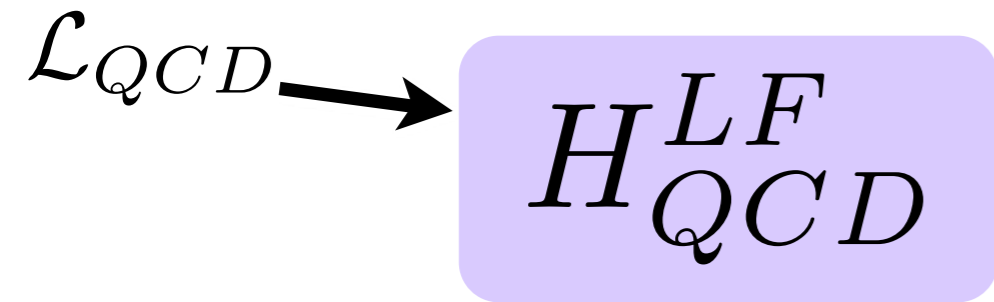
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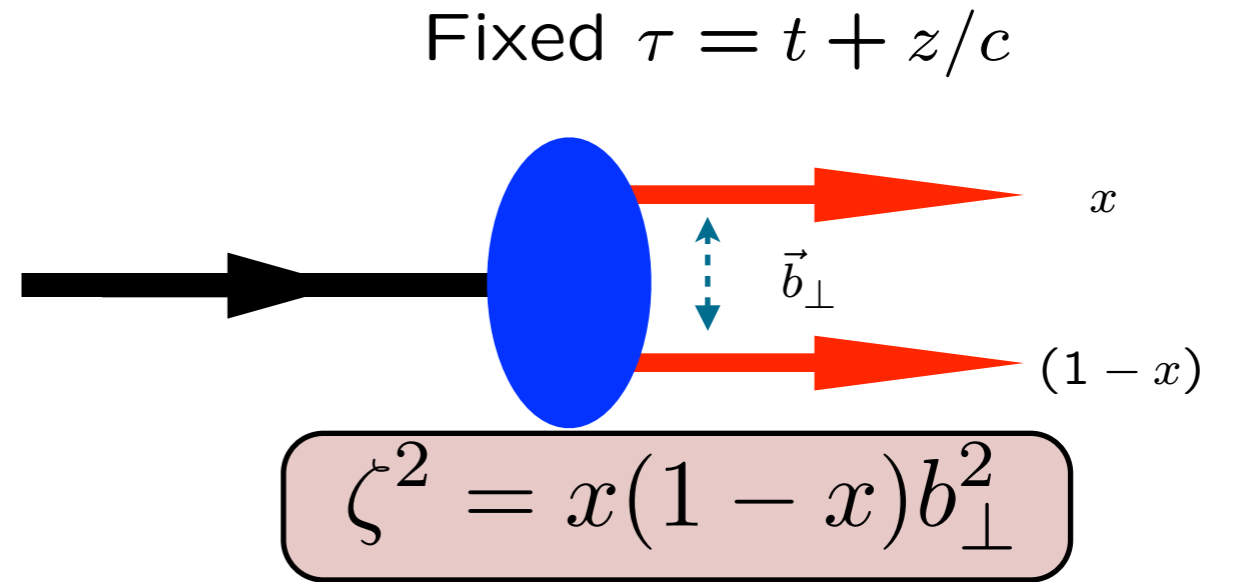
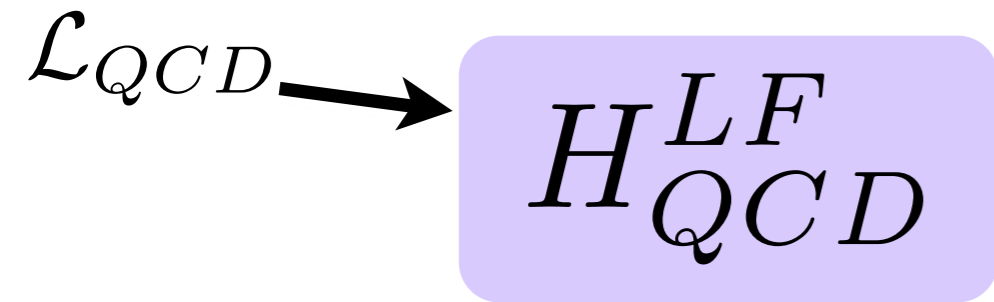
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Sums an infinite # diagrams

de Tèramond, Dosch, sjb

Light-Front QCD



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Coupled Fock states

Eliminate higher Fock states and retarded interactions

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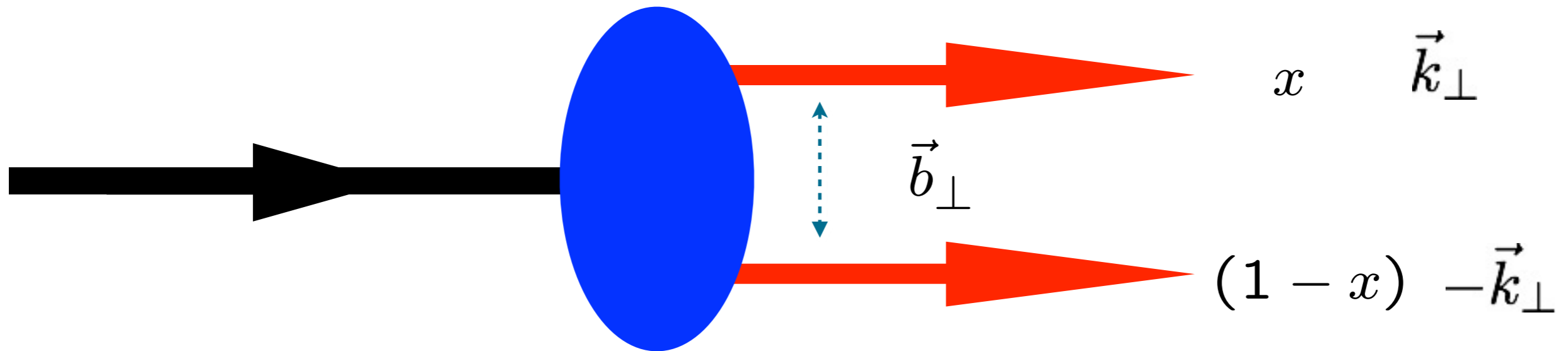
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Fixed $\tau = t + z/c$



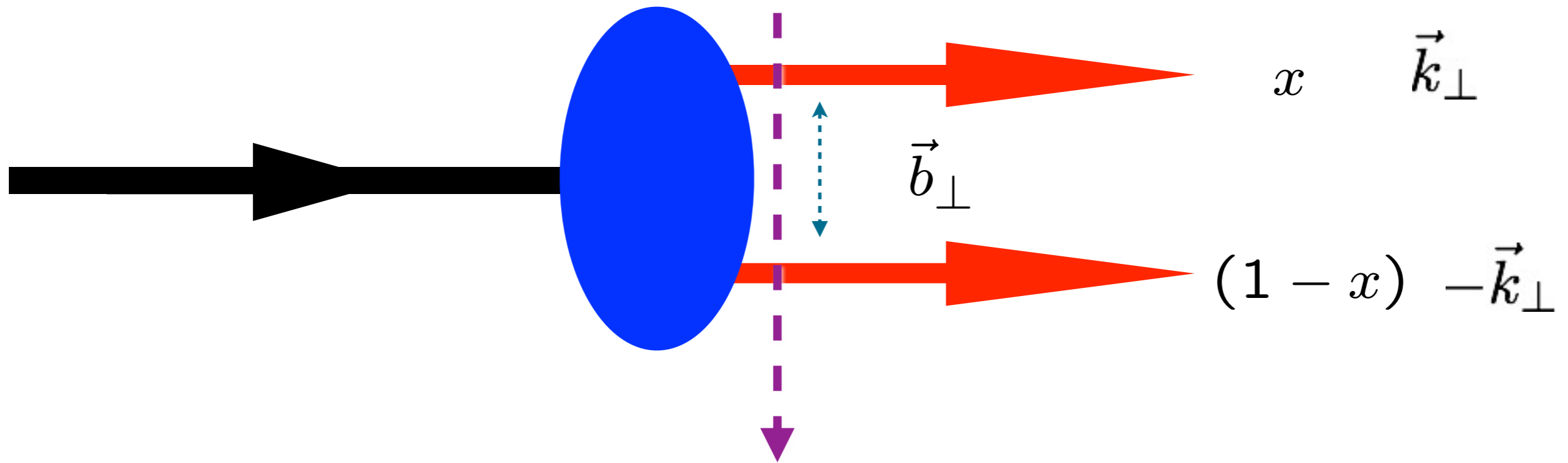
$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}_{q+\bar{q}}^2$$

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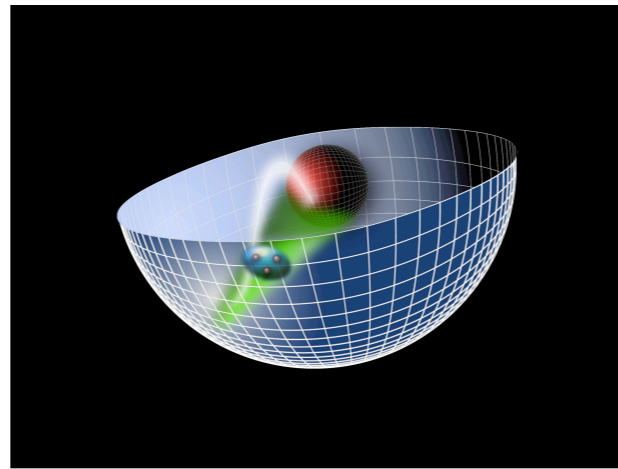
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*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



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$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Light-Front Schrödinger Equation

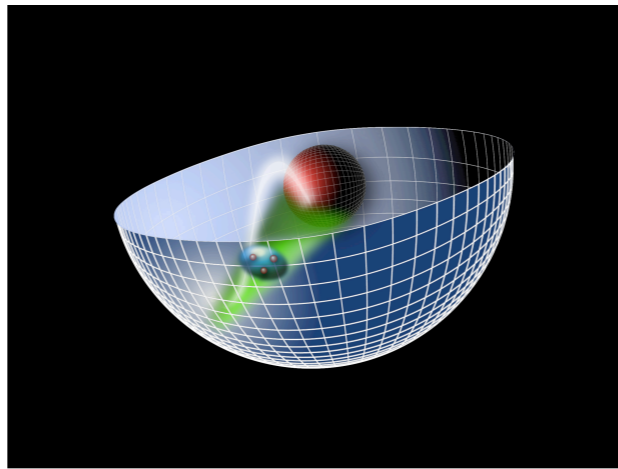


Confinement scale: $\kappa \simeq 0.5 \text{ GeV}$
 $1/\kappa \simeq 1/3 \text{ fm}$

● **Fubini, Rabinovici:**

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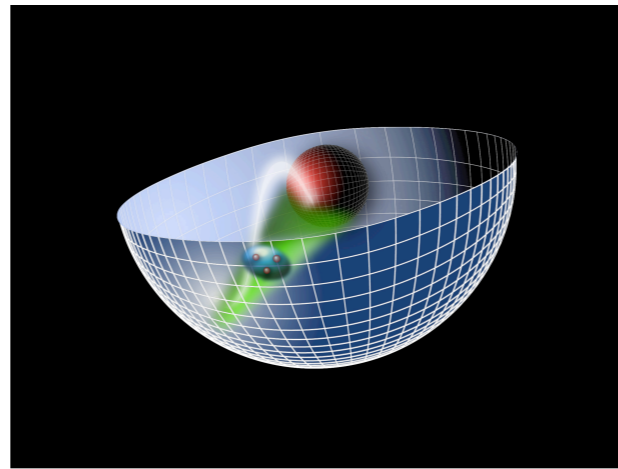
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*Preserves Conformal Symmetry
of the action*

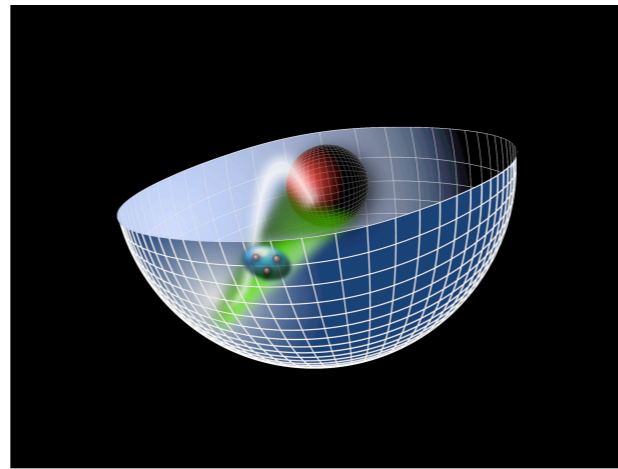
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***Unique
Confinement Potential!***

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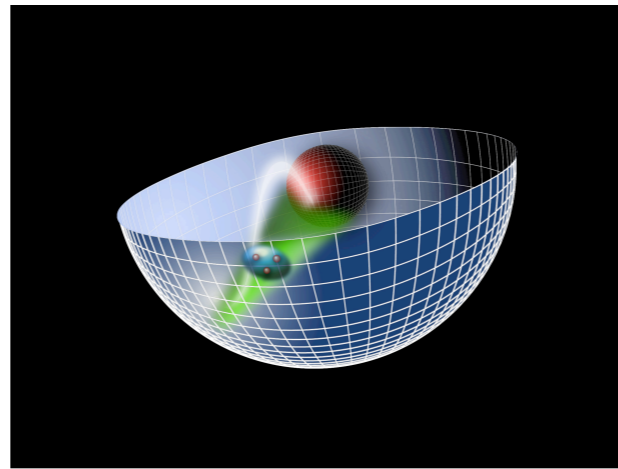
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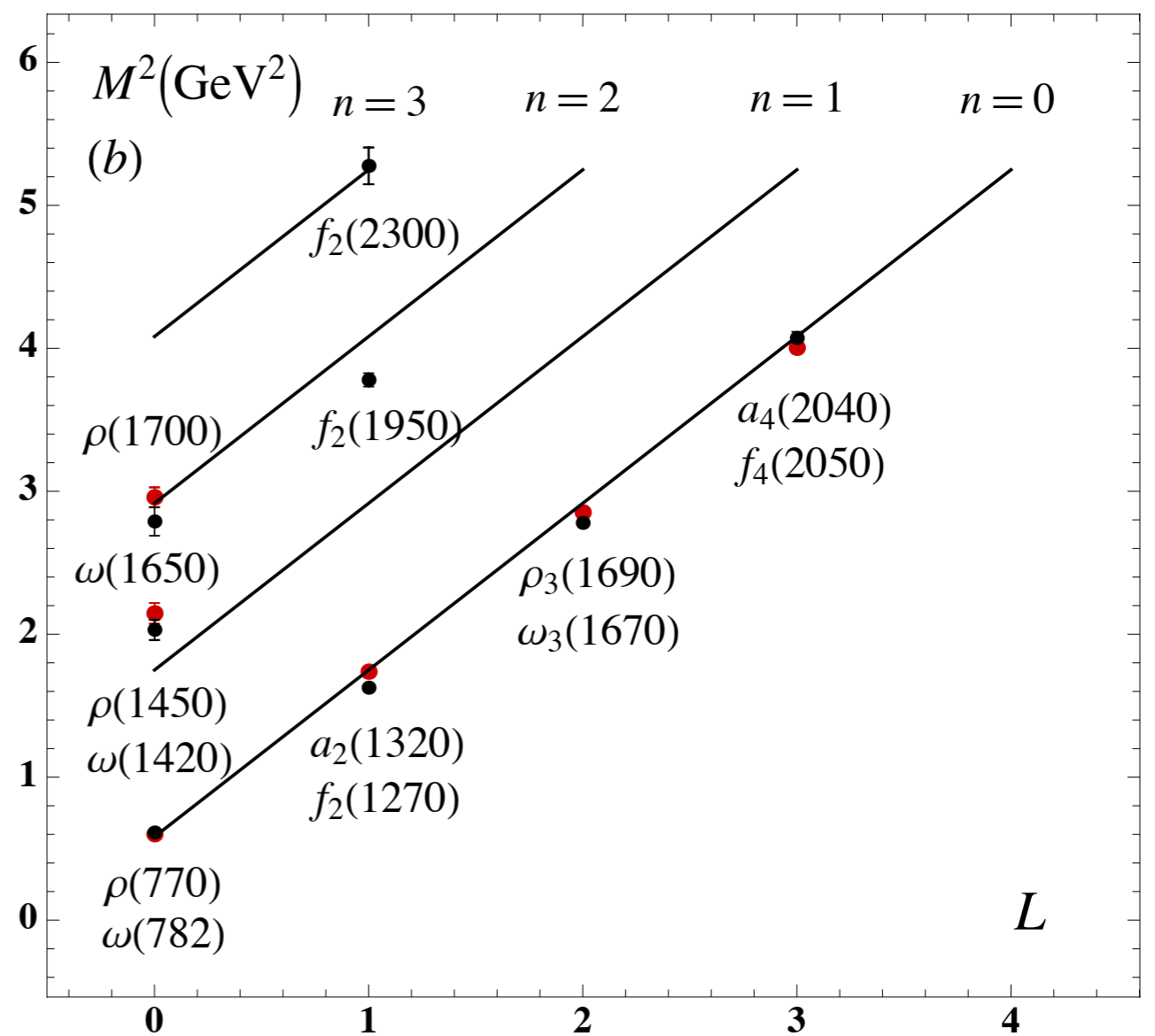
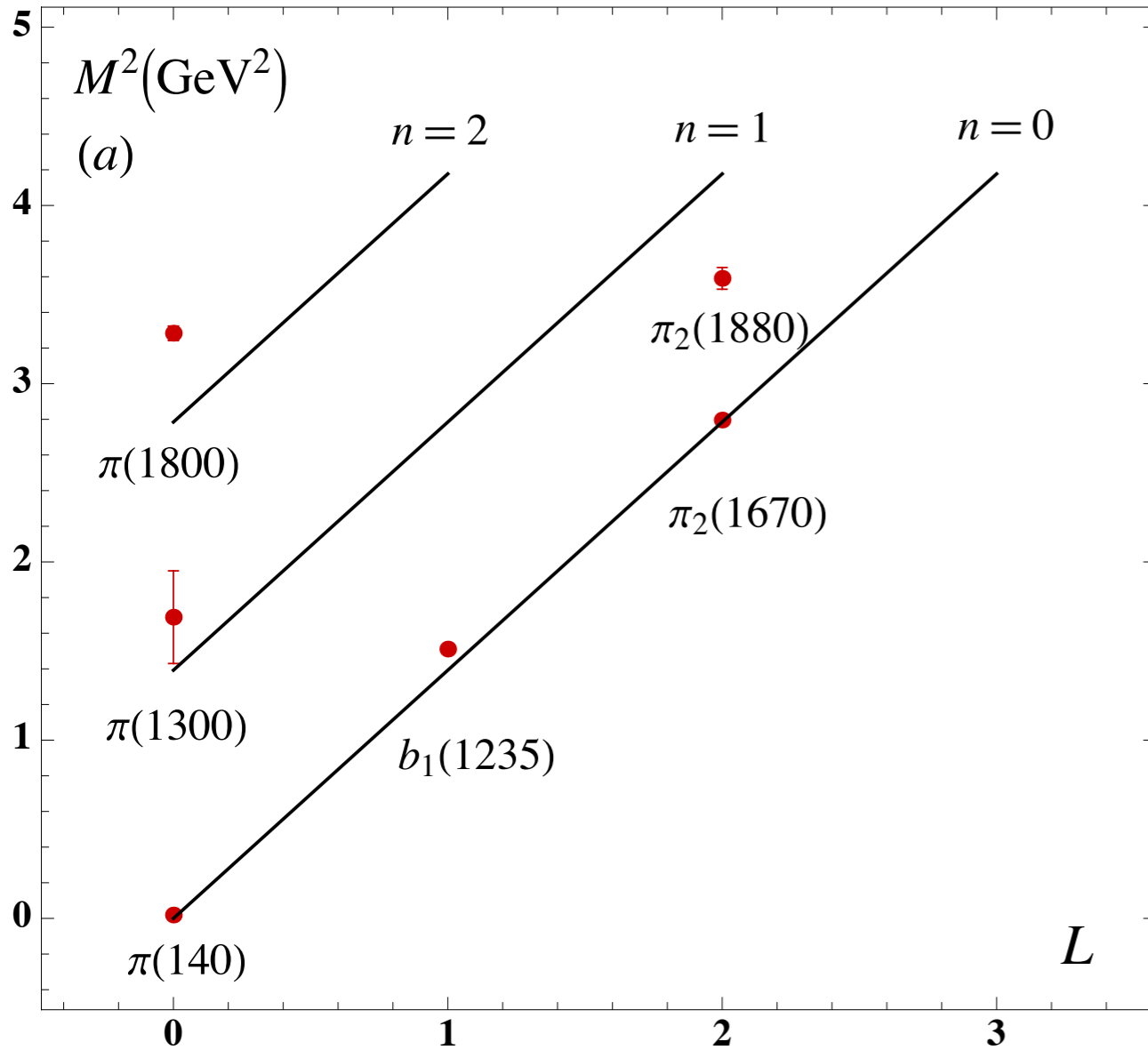
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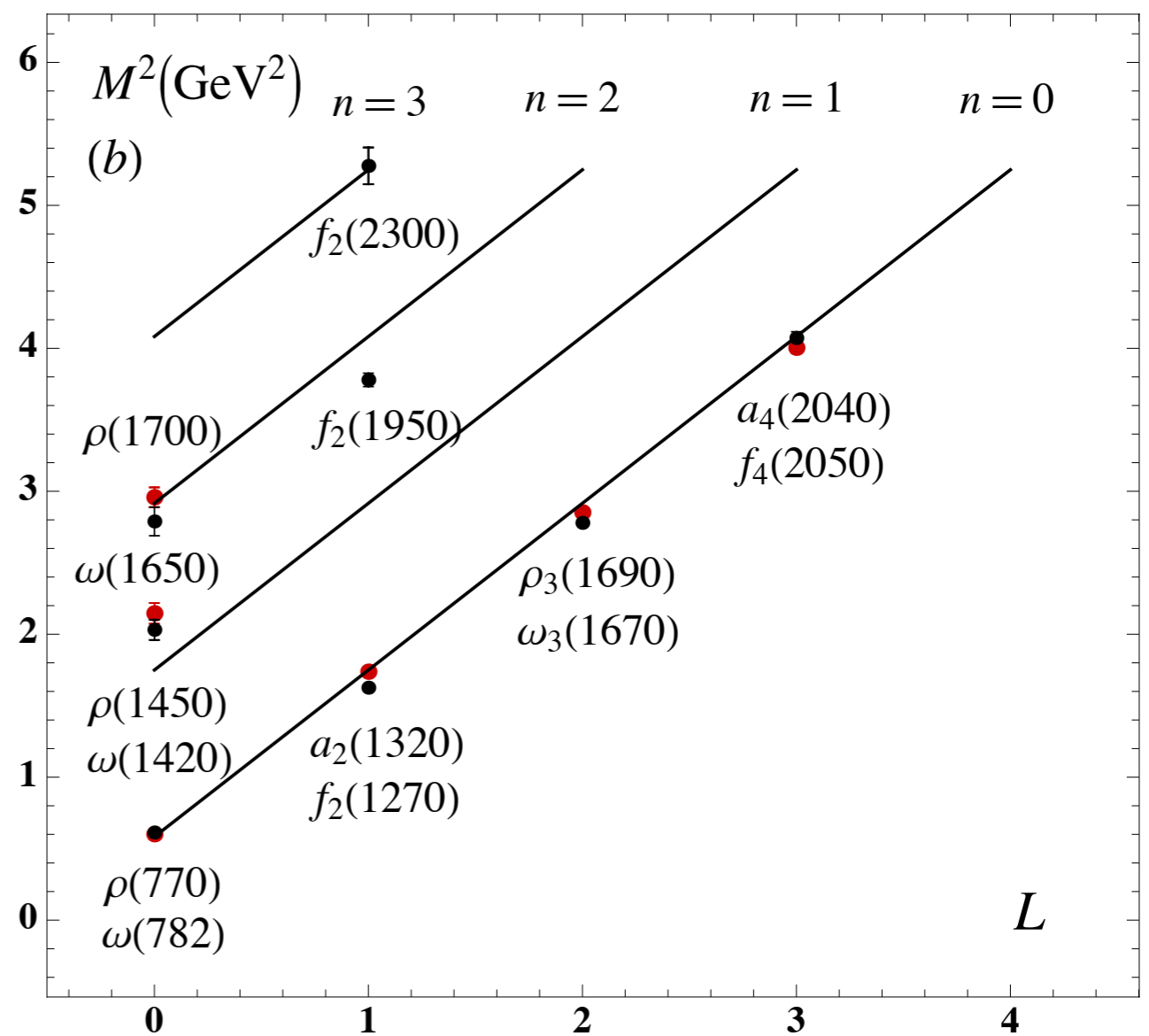
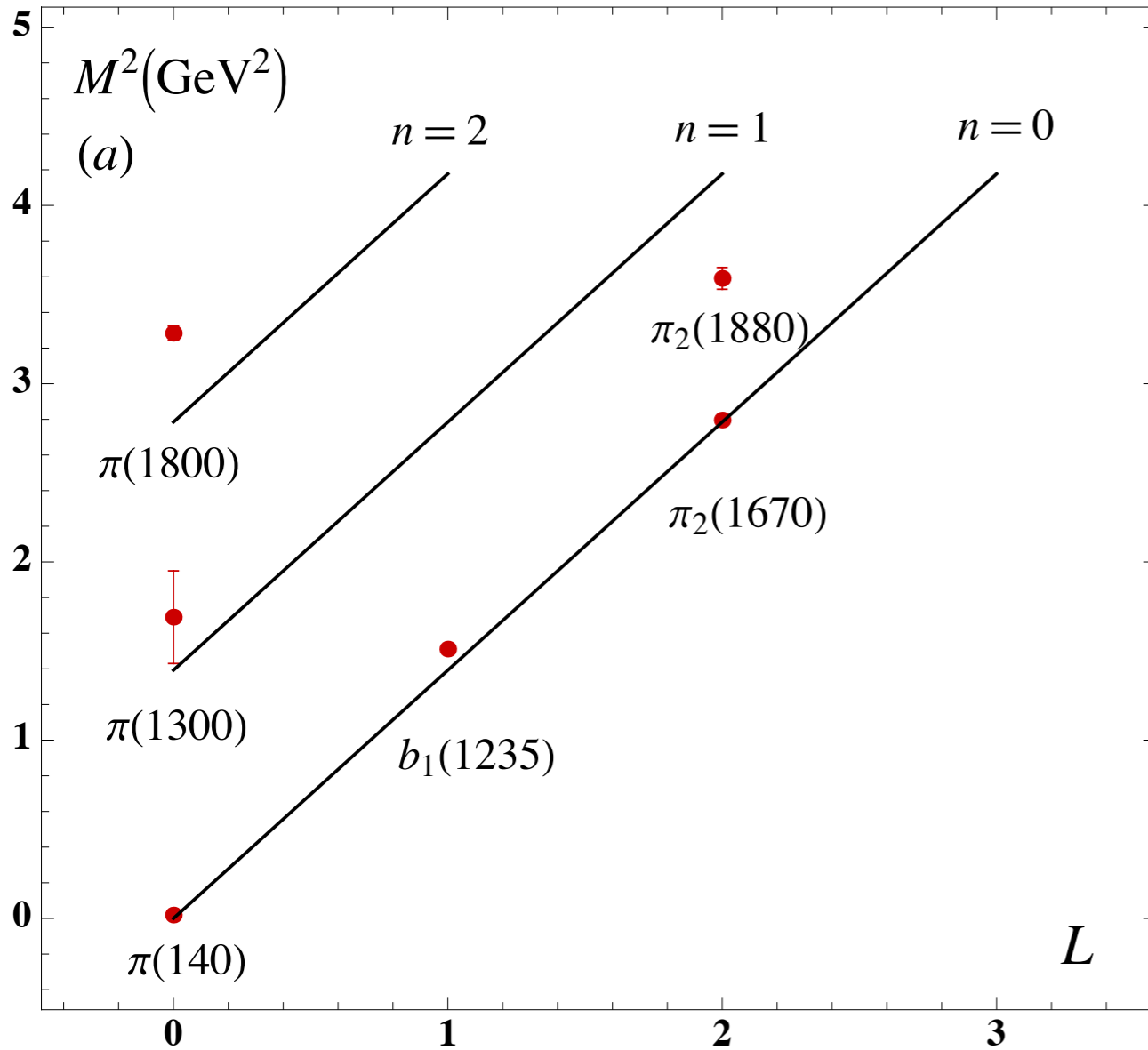
***Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!***

$$m_u = m_d = 0$$

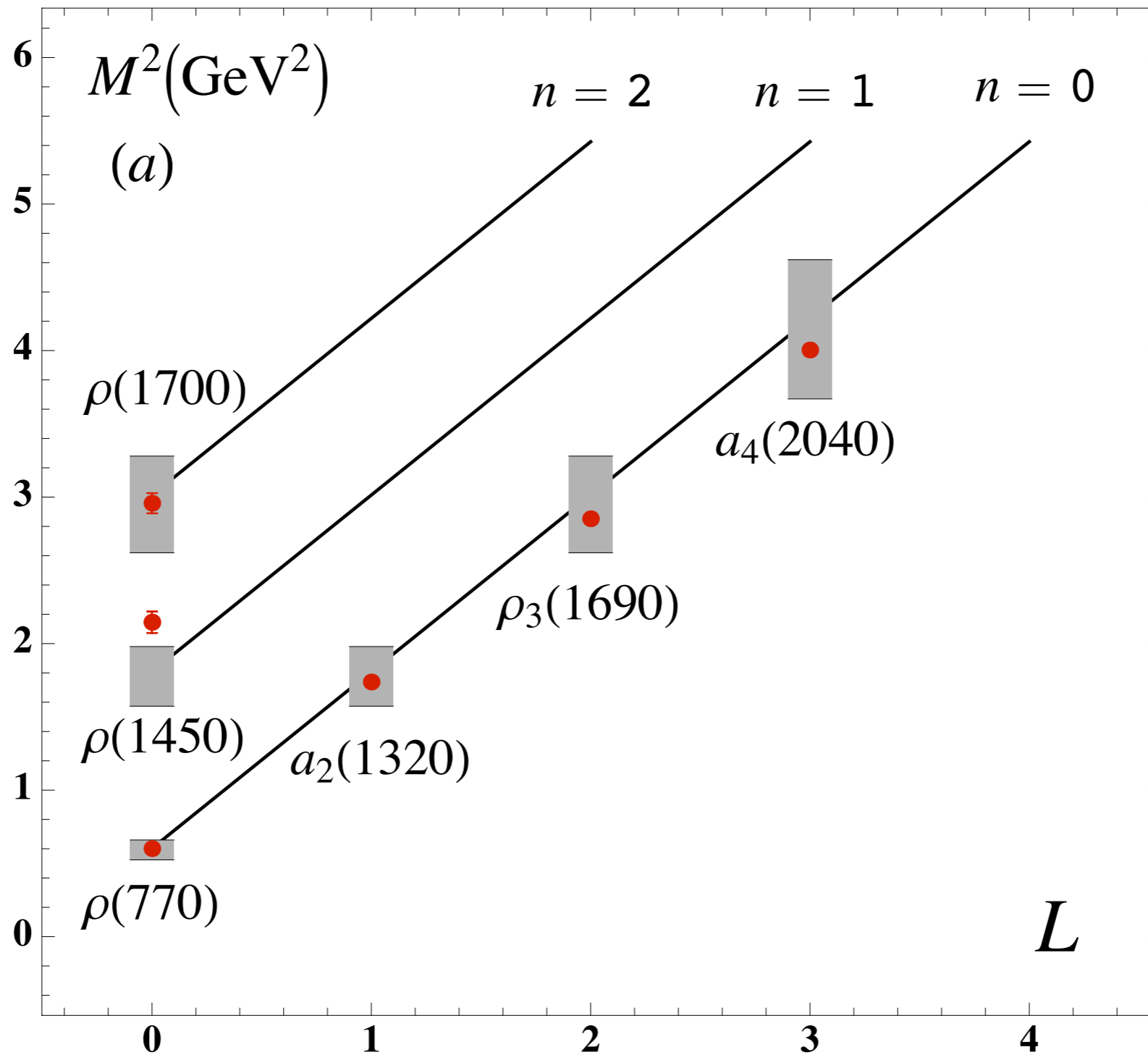


$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

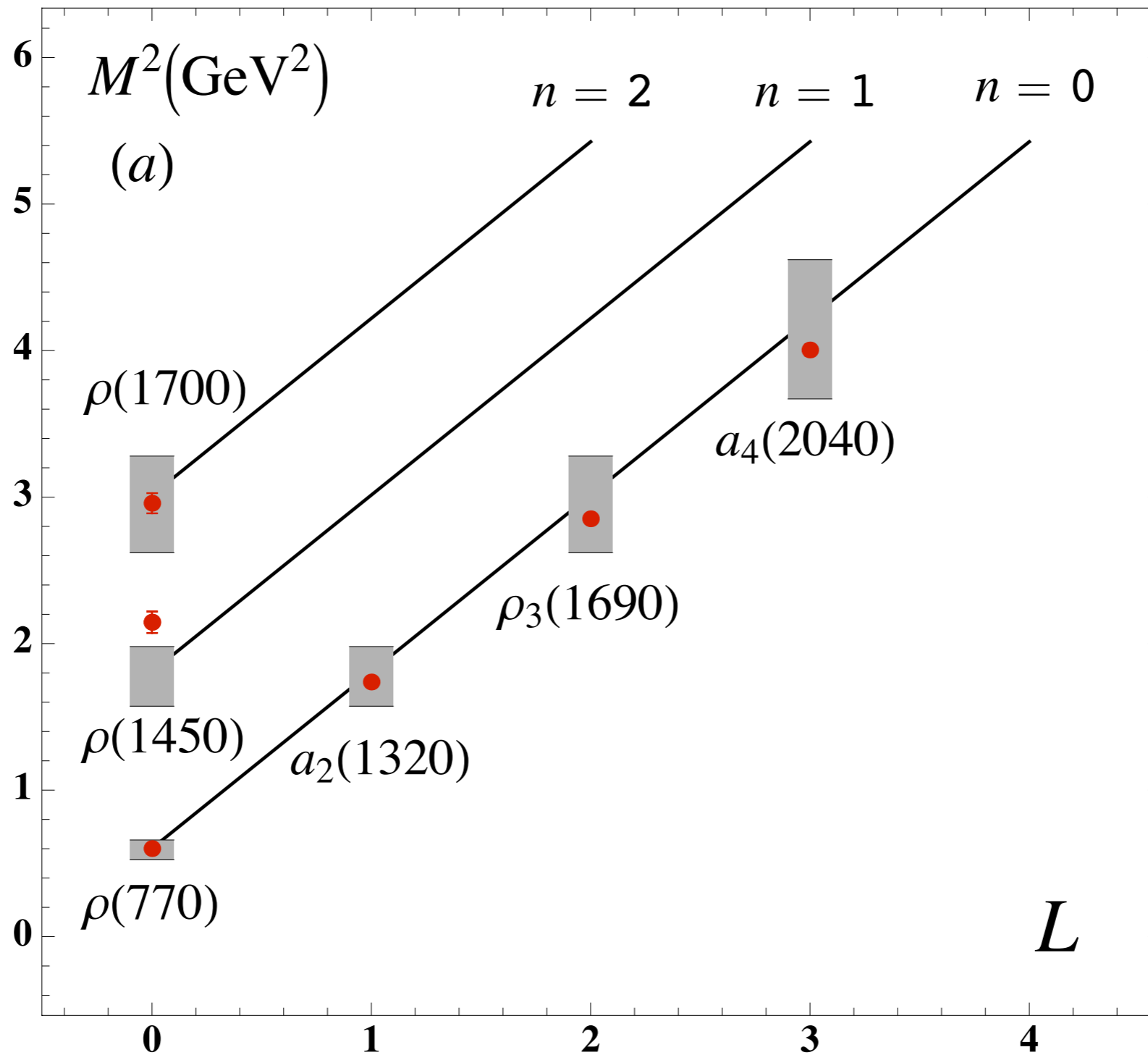
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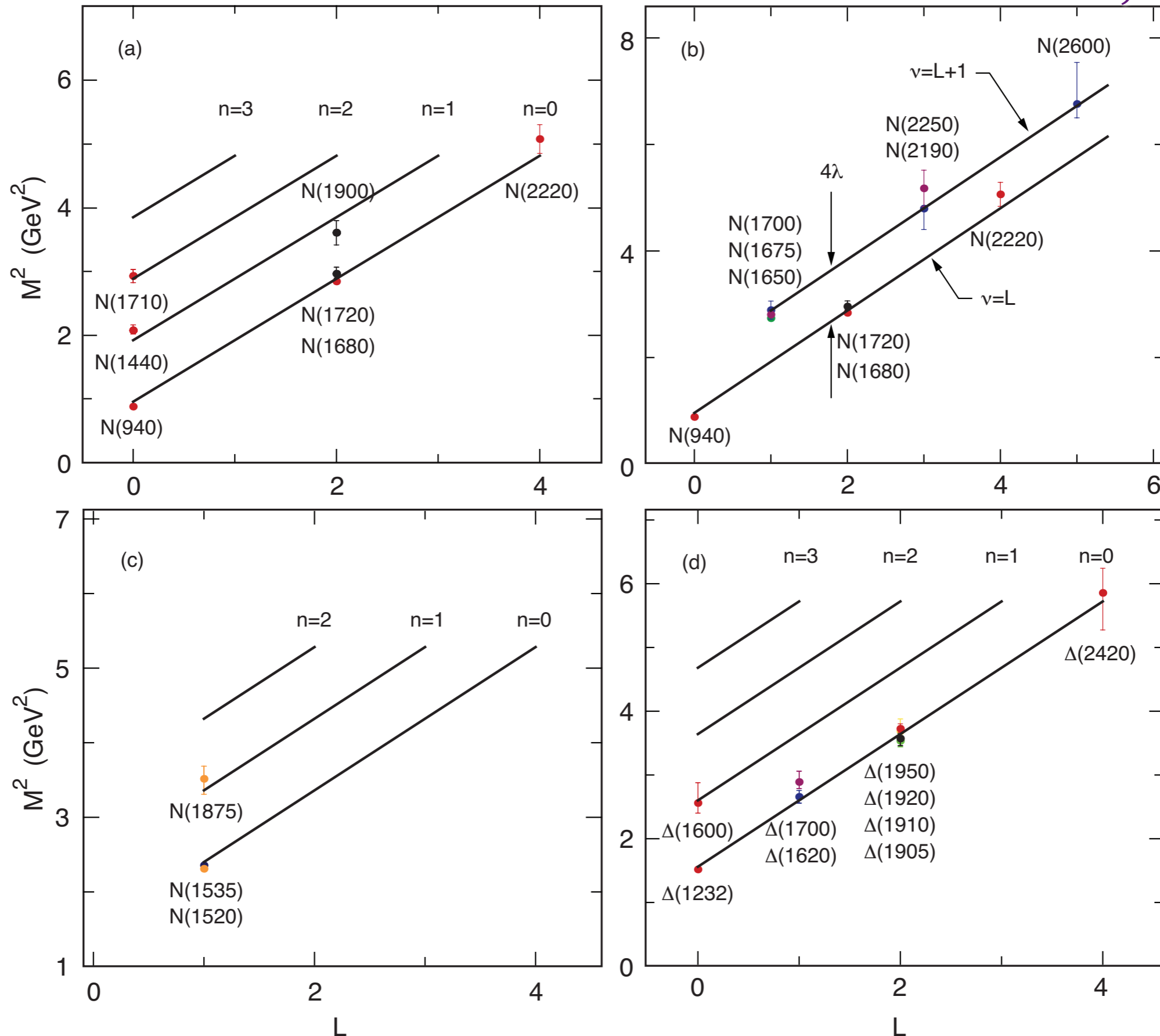
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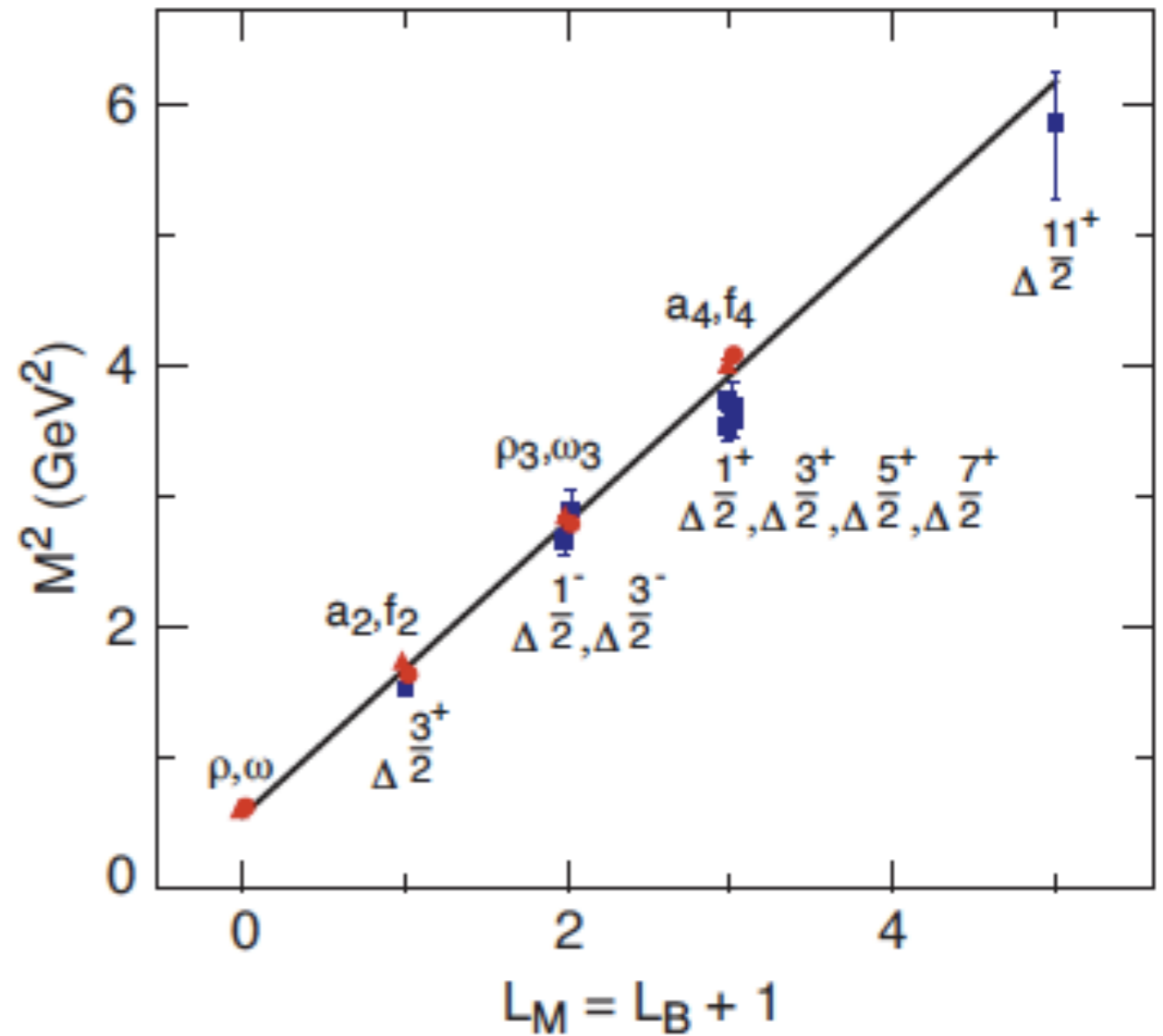
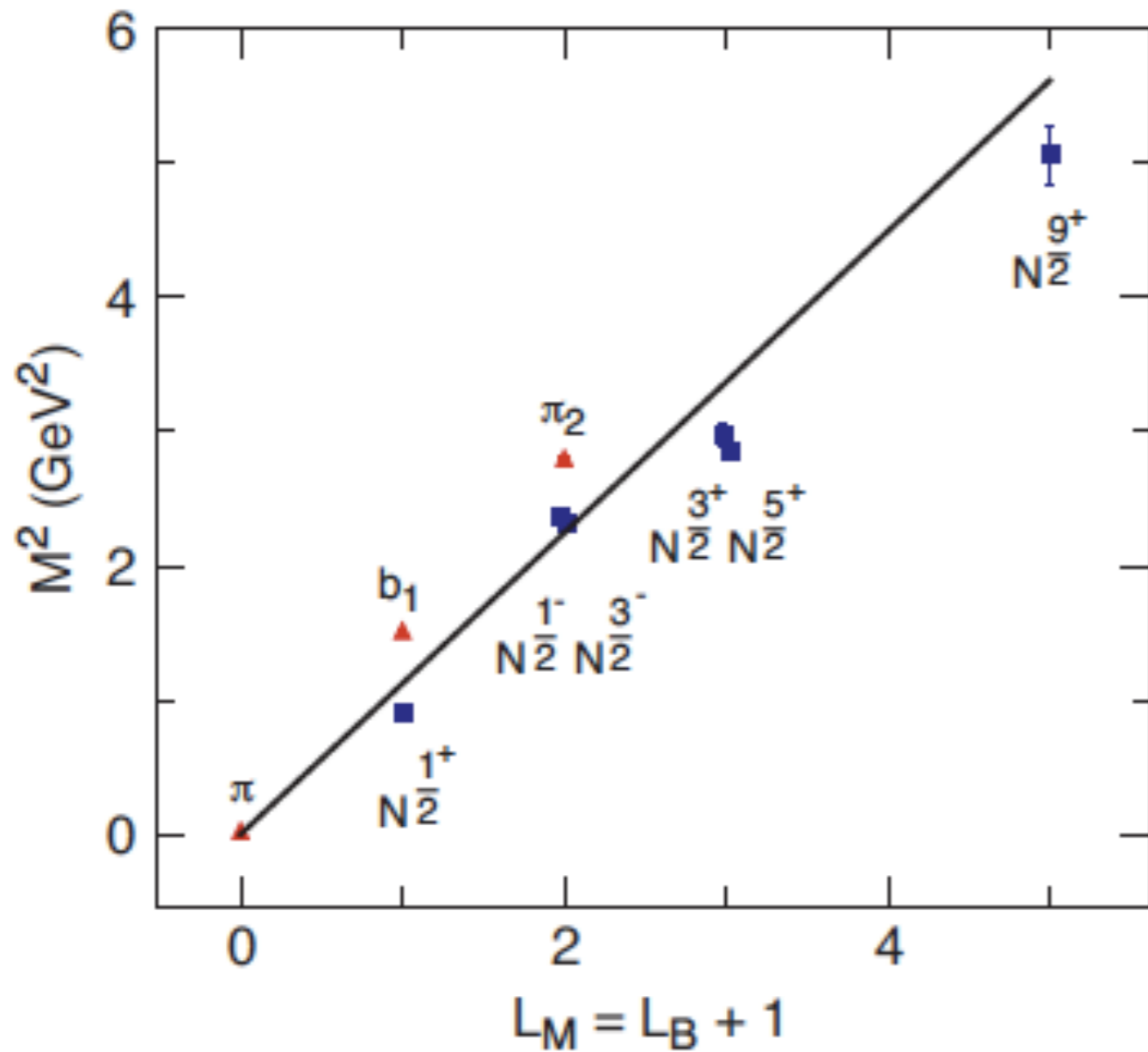
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Baryon orbital and radial excitations for $\kappa = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)



Superconformal Meson-Nucleon Partners

$$\kappa = 530 \text{ MeV}$$

AdS/QCD and Light-Front Holography

- Single Scale κ ; Only ratios predicted
- Spectroscopy, LFWFs, and Dynamics
- LF Schrödinger Equation — Analogous to Schrödinger Equation for Atomic Physics
- QCD Running Couplings
- Matching Scale Q_0

Some Features of AdS/QCD

- **Regge spectroscopy—same slope in n, L for mesons, baryons**
- **Chiral features for $m_q=0$: $m_\pi=0$, chiral-invariant proton**
- **Hadronic LFWFs**
- **Counting Rules**
- **Connection between hadron masses and $\Lambda_{\overline{MS}}$**

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

Need a First Approximation to QCD

*Comparable in simplicity to
Schrödinger Theory in Atomic Physics*

Relativistic, Frame-Independent, Color-Confining

Goal: An analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **Confinement in QCD -- What is the analytic form of the confining interaction?**
- **What sets the QCD mass scale?**
- **QCD Running Coupling at all scales**
- **Hadron Spectroscopy-Regge Trajectories**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **Constituent Counting Rules**
- **Hadronization at the Amplitude Level**
- **Insights into QCD Condensates**
- **Chiral Symmetry**
- **Systematically improvable**

LC2015

Frascati INFN

September 25, 2015

**Light-Front Holographic QCD, Color Confinement,
and Supersymmetric Features of QCD**

Stan Brodsky



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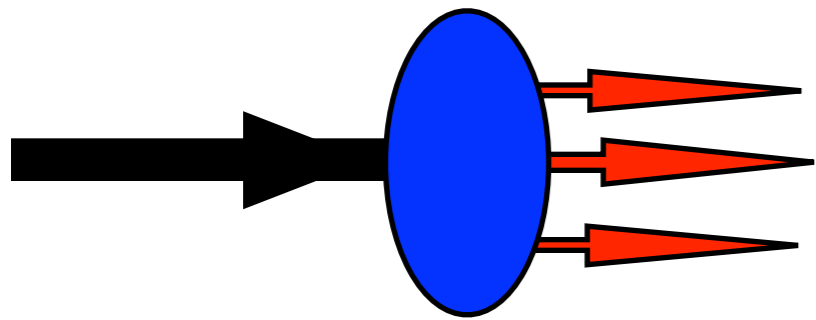
Stan Brodsky



Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

Direct connection to QCD Lagrangian

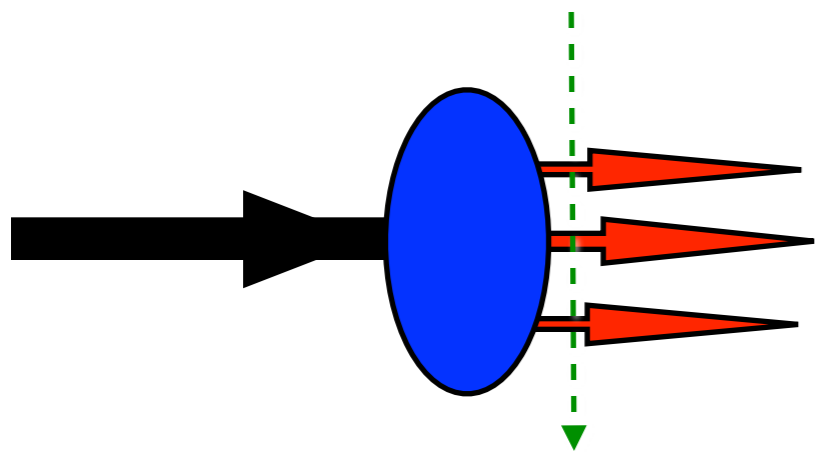
Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

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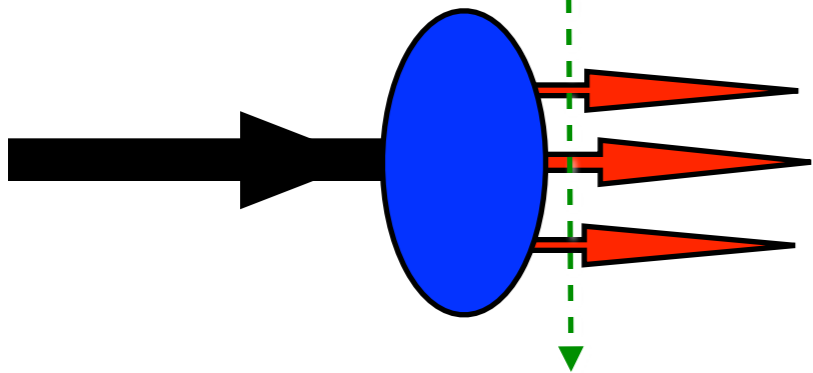
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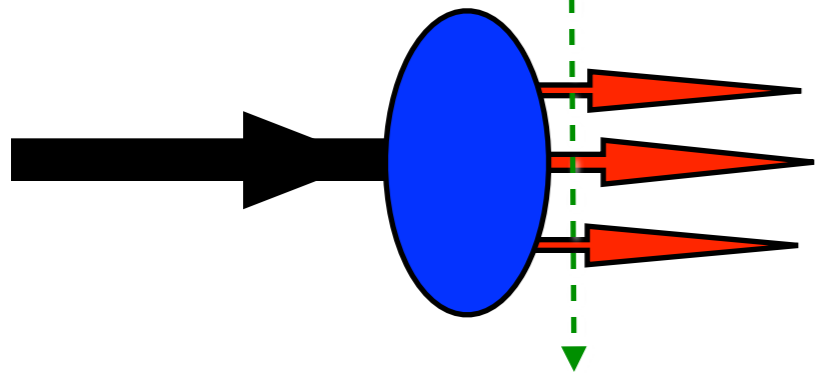
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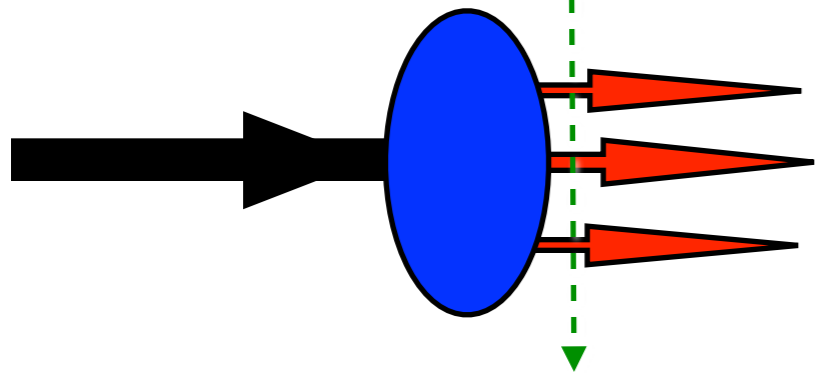
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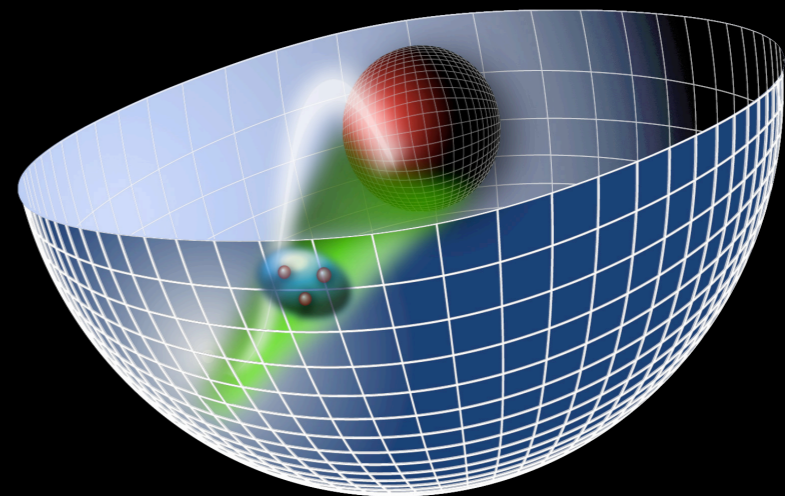
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Light-Front Holography and Non-Perturbative QCD

Goal:

***Use AdS/QCD duality to construct
a first approximation to QCD***

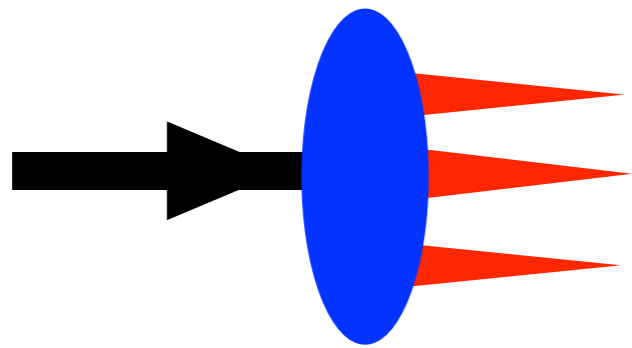
*Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc*



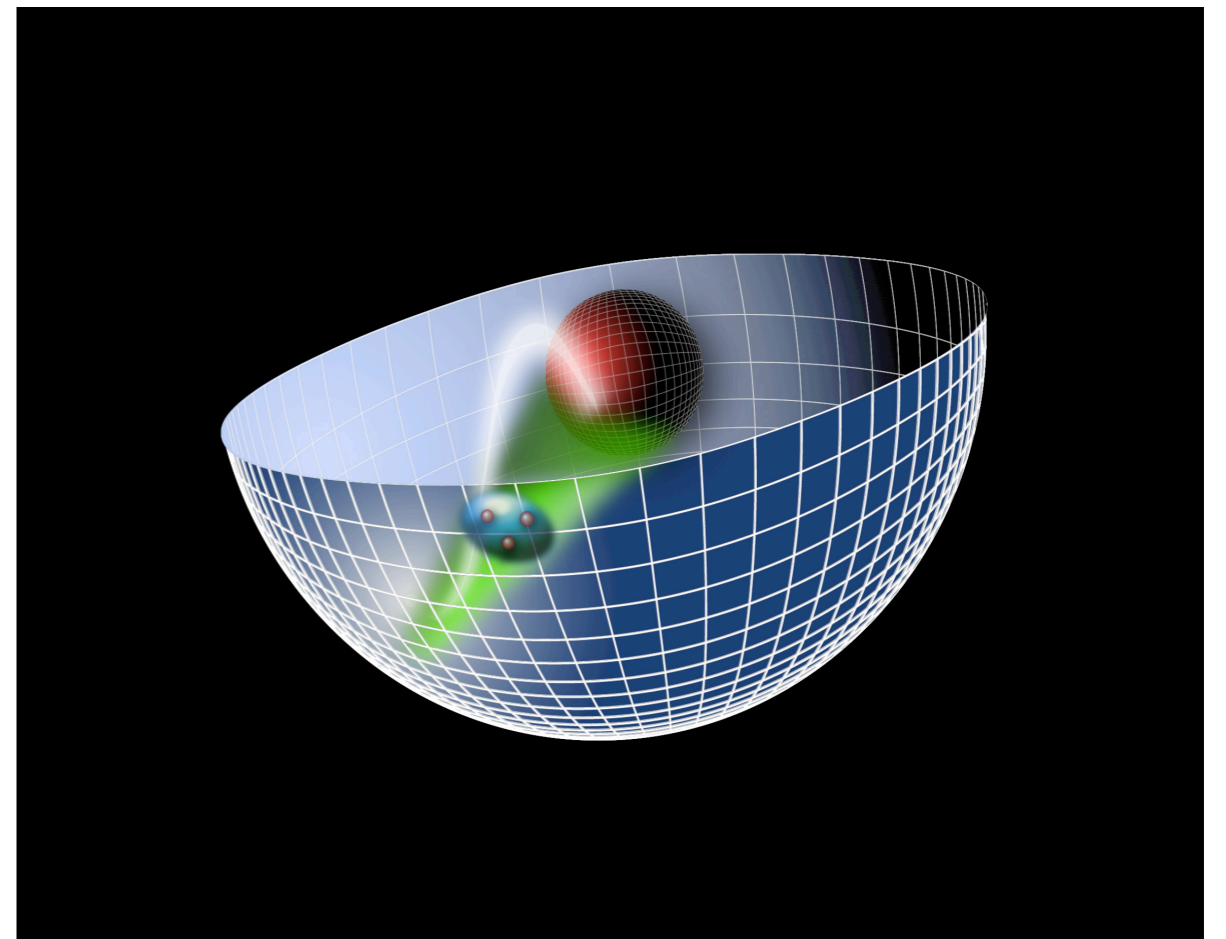
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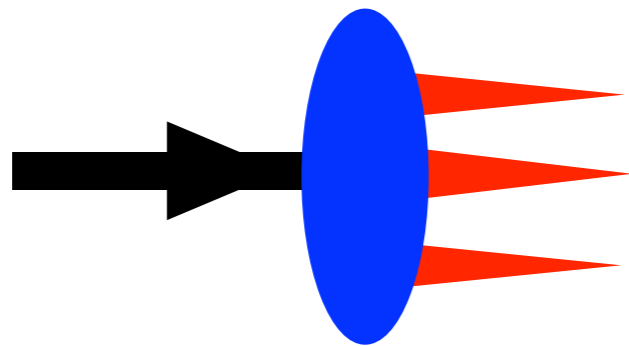


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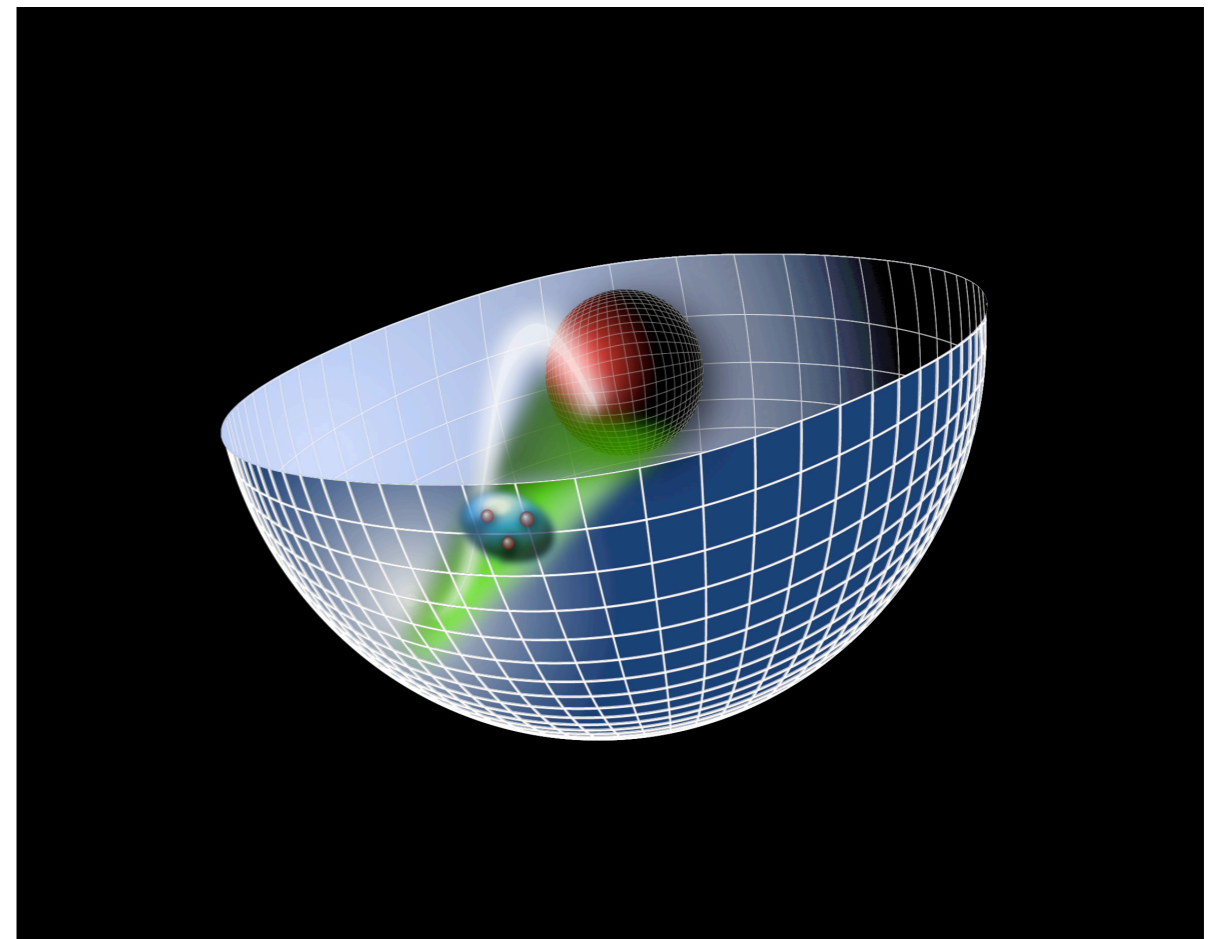
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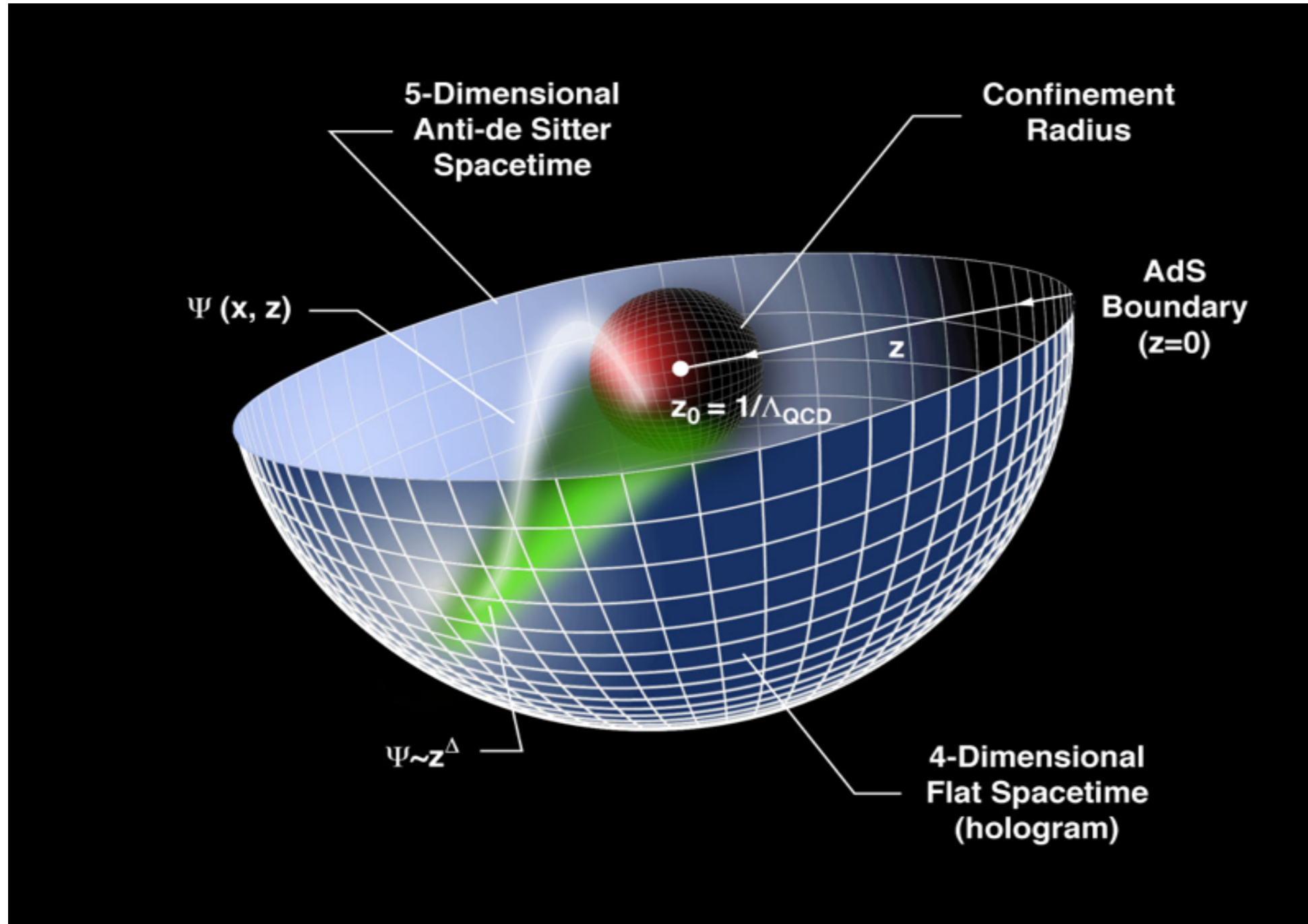


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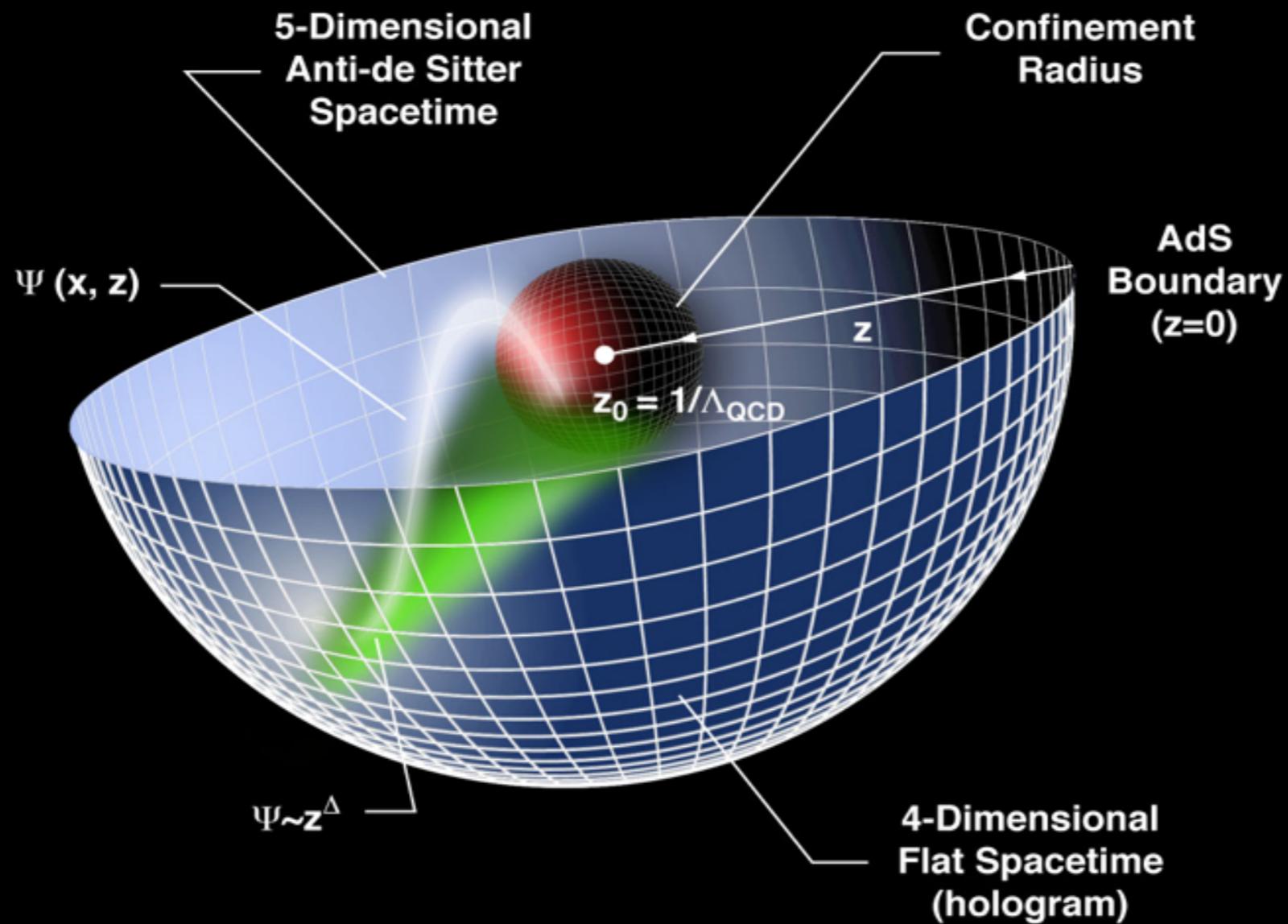


in collaboration with Guy de Teramond and H. Guenter Dosch

Applications of AdS/CFT to QCD

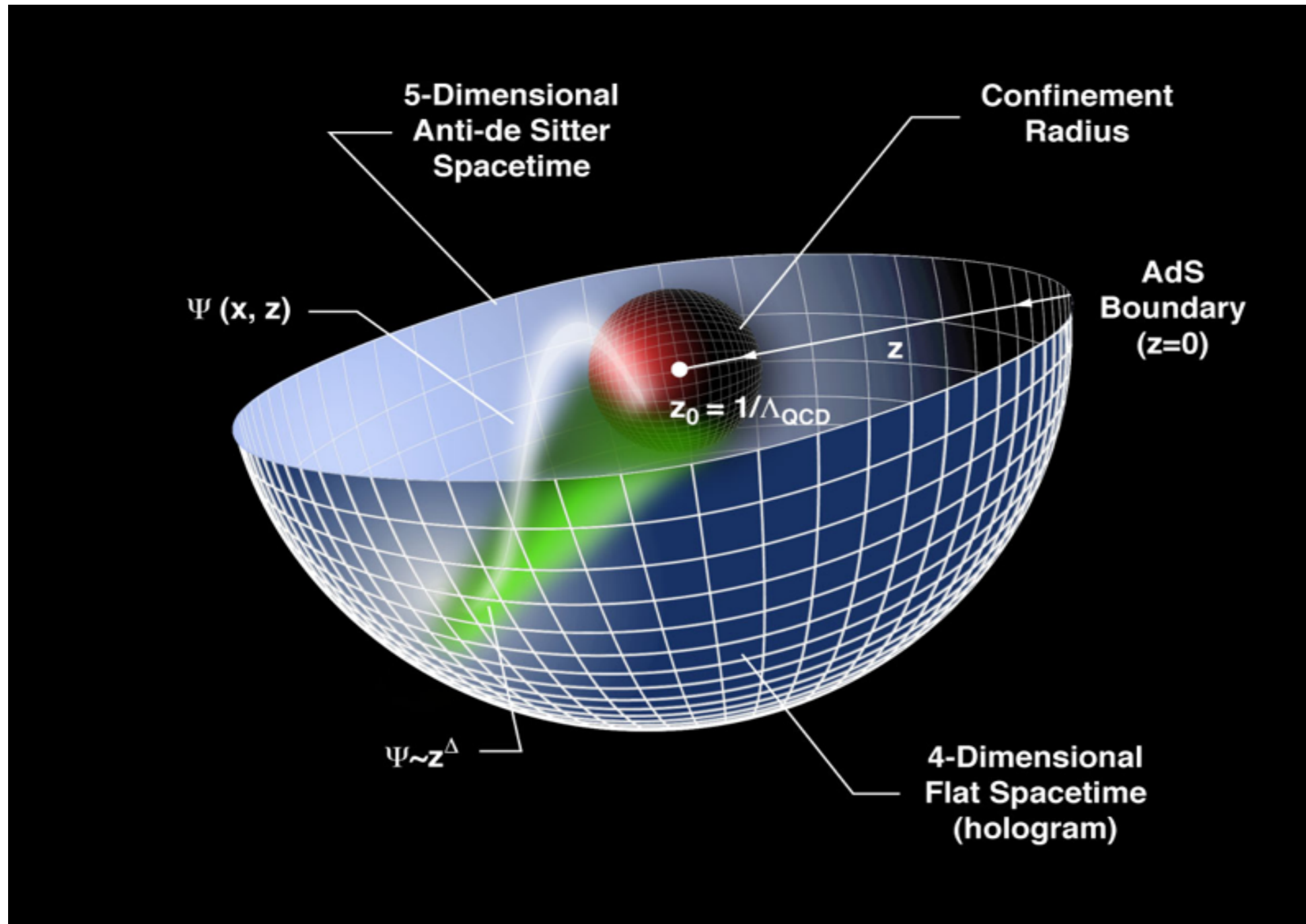


Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond and H. Guenter Dosch

AdS/CFT

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$


$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

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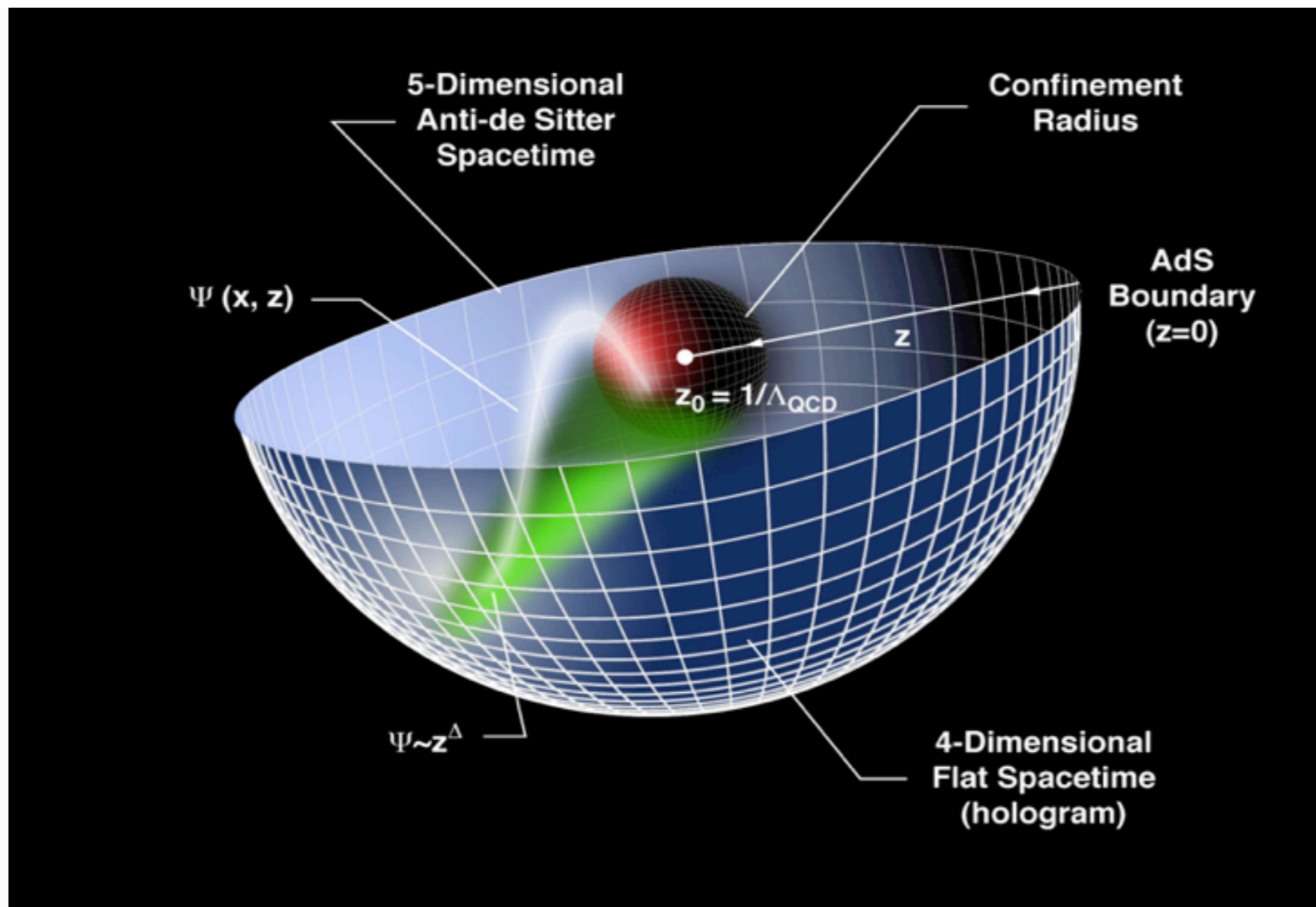
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Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) **Polchinski and Strassler (2001)**.
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) **Karch, Katz, Son and Stephanov (2006)**.

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS₅ as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

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Positive-sign dilaton

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*Derived from variation of Action for Dilaton-Modified
AdS₅*

Identical to Light-Front Bound State Equation!

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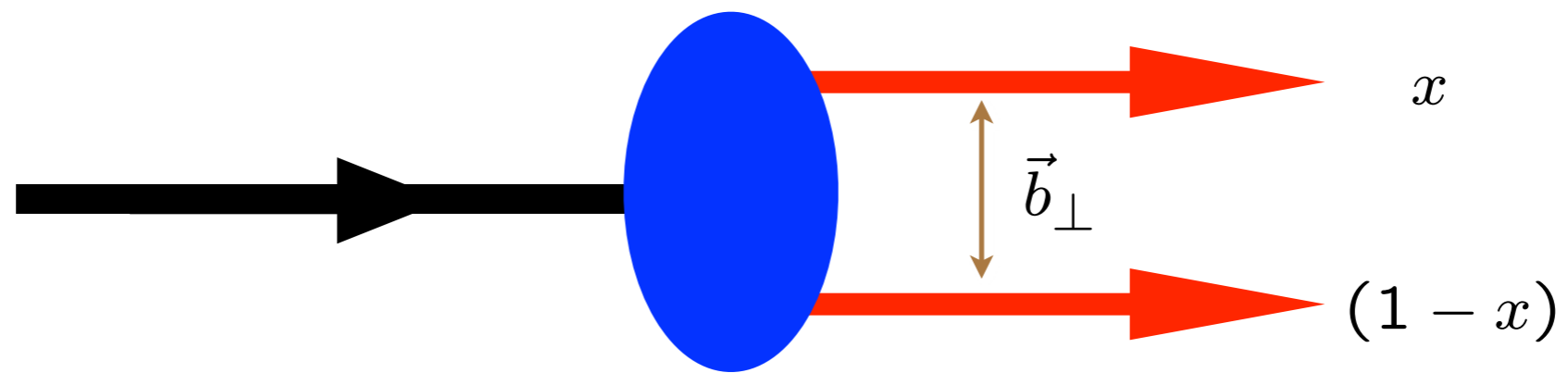
Identical to Light-Front Bound State Equation!

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

$LF(3+1) \longleftrightarrow AdS_5$

de Teramond, sjb

Light-Front Holographic Dictionary



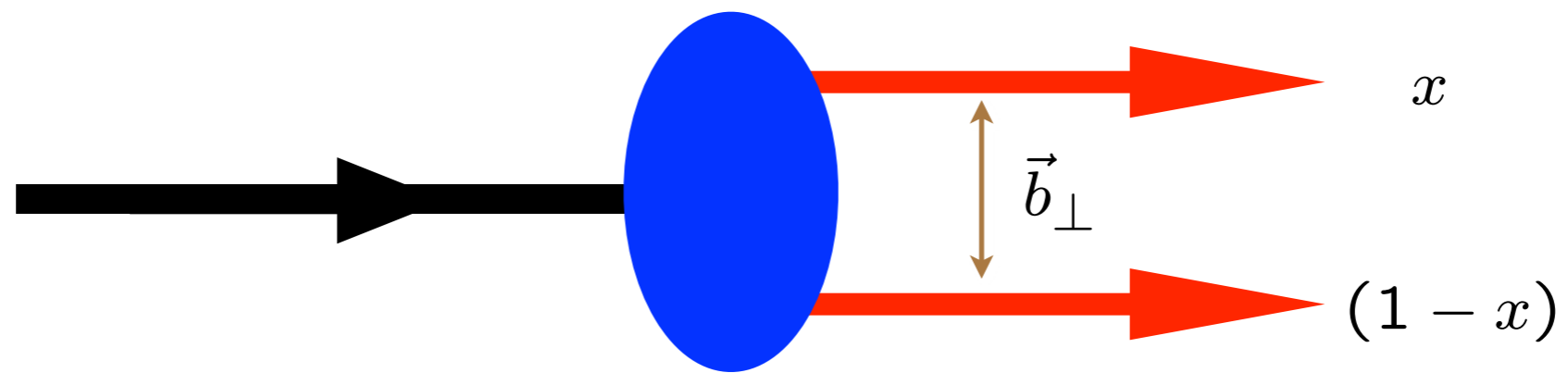
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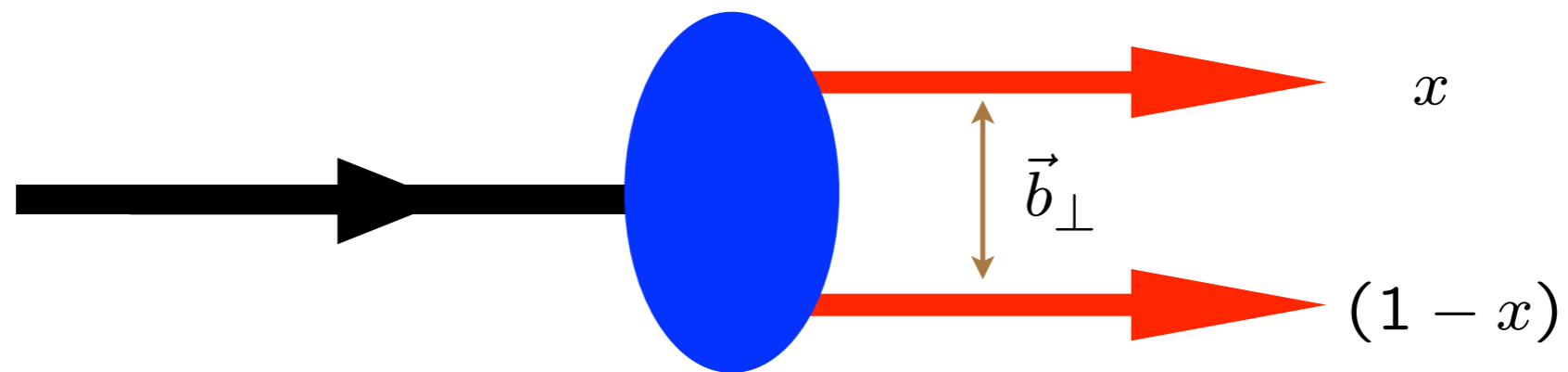
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Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp)$$



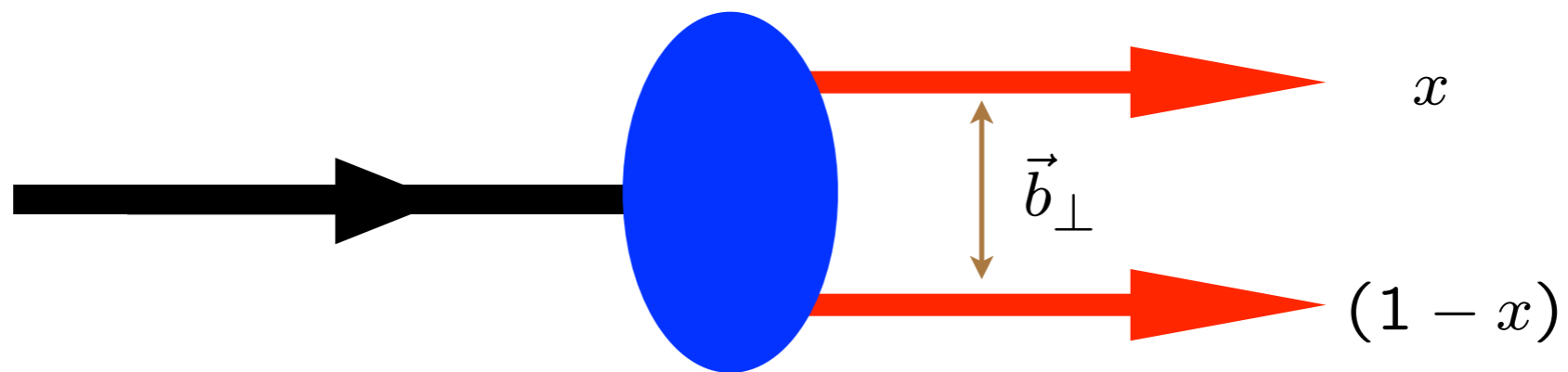
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Light-Front Holographic Dictionary

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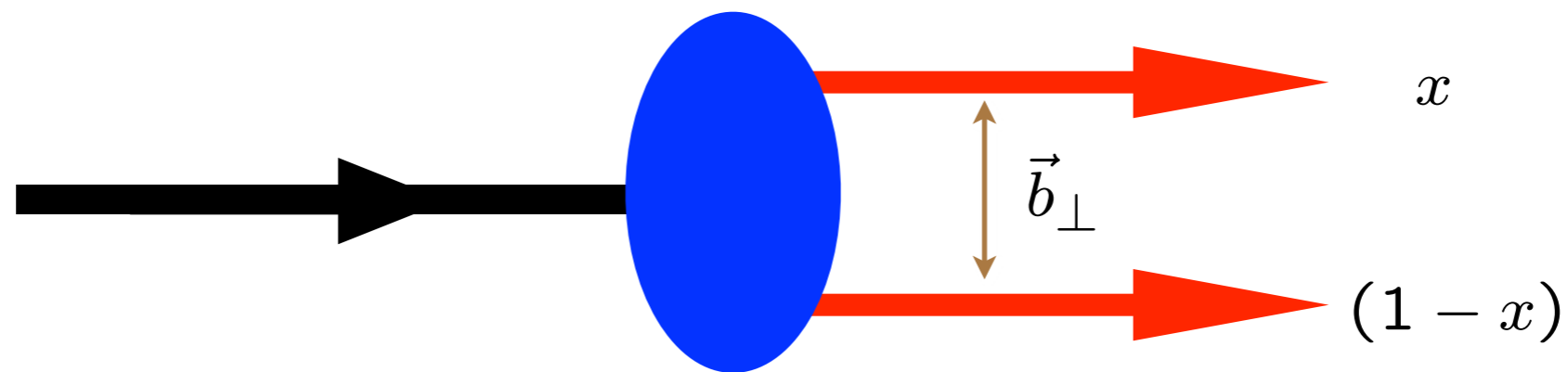
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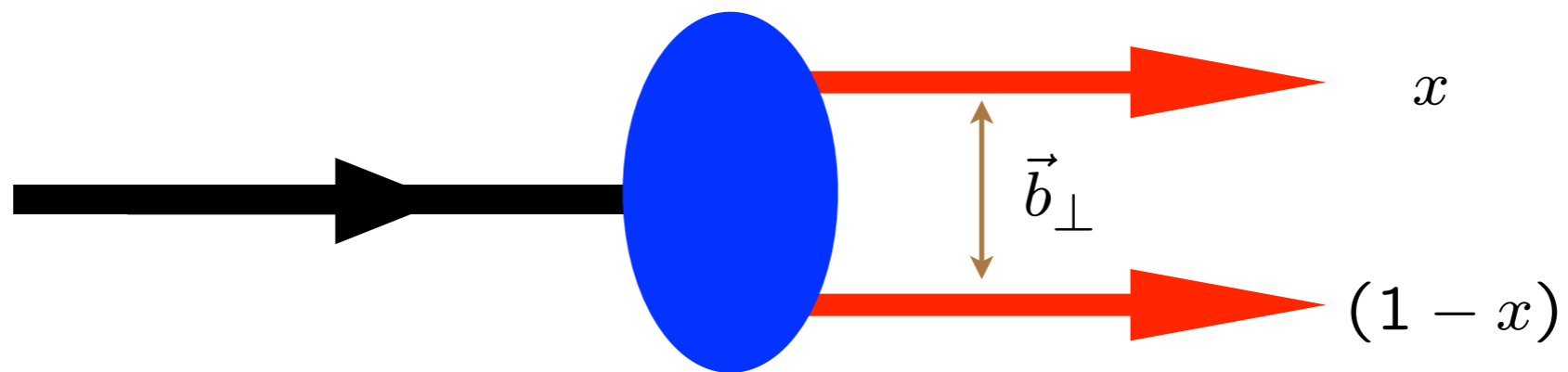
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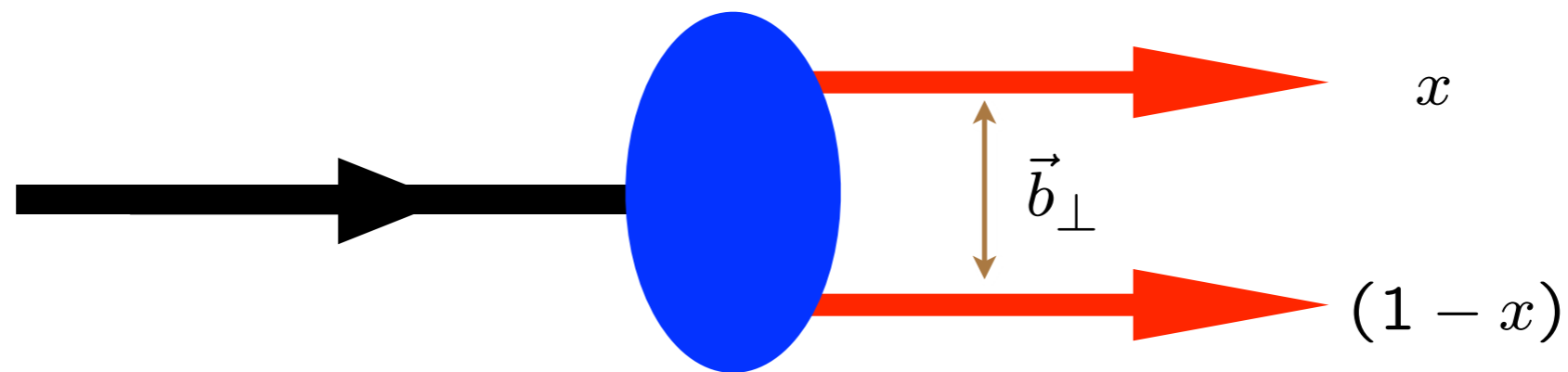
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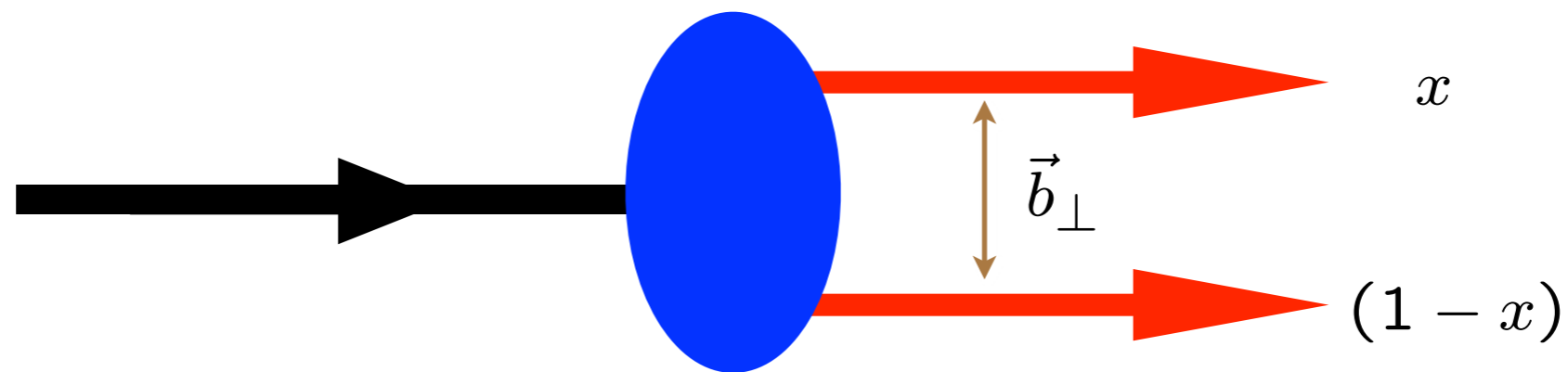
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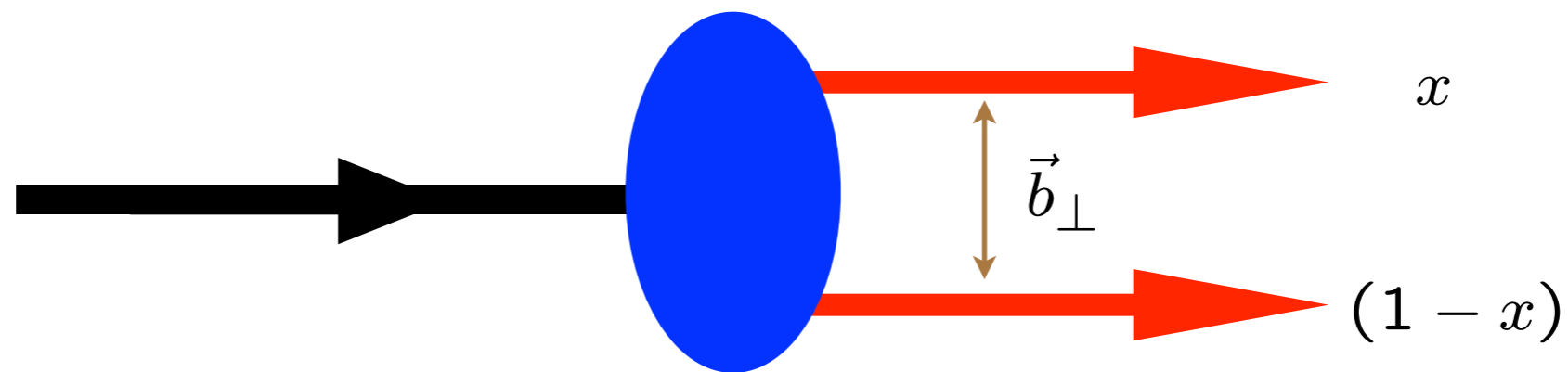
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Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

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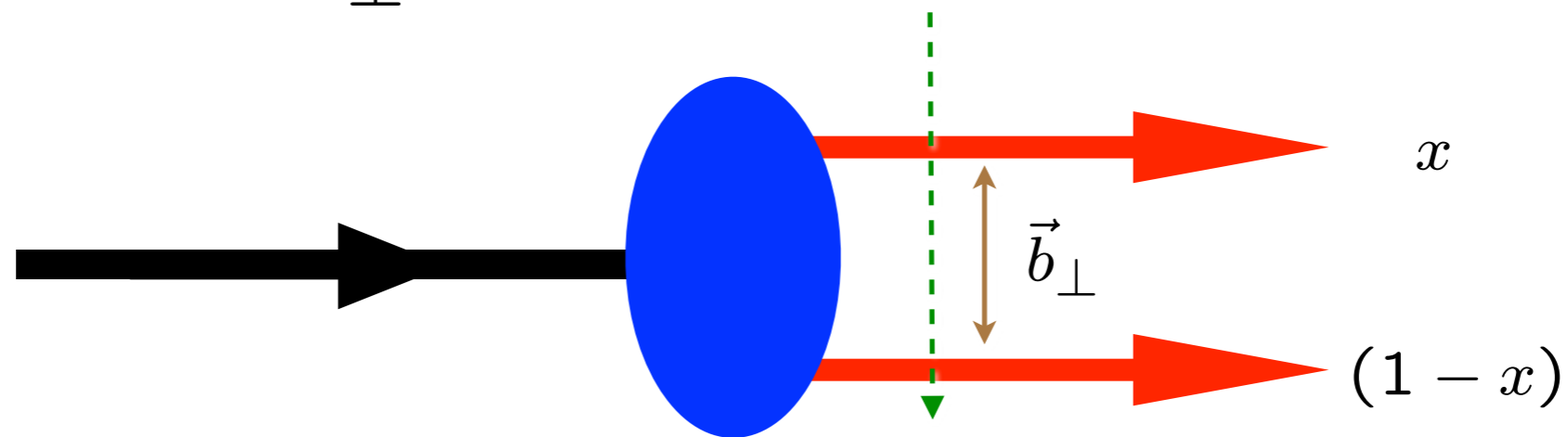
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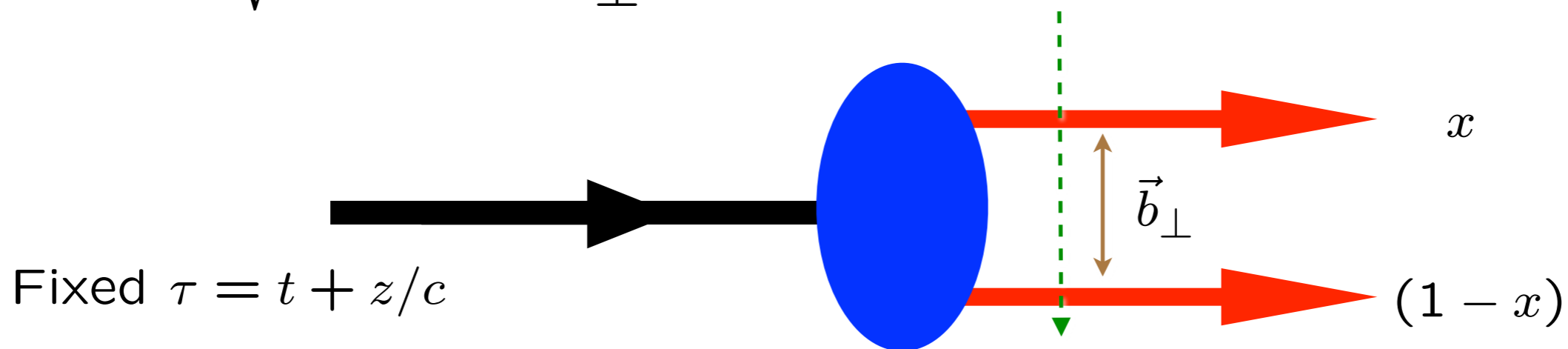
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General-Spin Hadrons

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

we find the LF wave equation

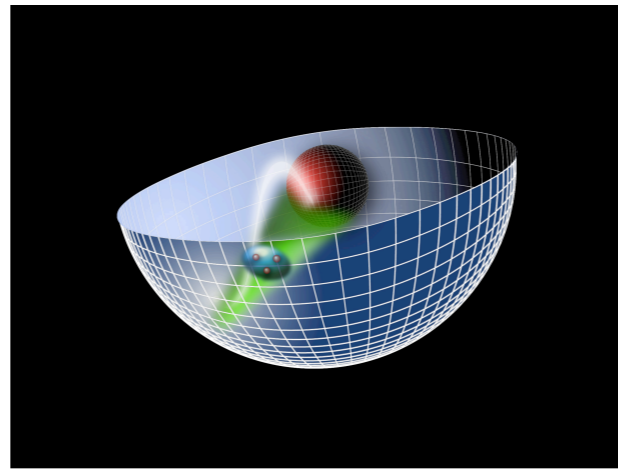
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$



with $(\mu R)^2 = -(2 - J)^2 + L^2$

*AdS/QCD
Soft-Wall Model*

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Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Light-Front Schrödinger Equation



Confinement scale:

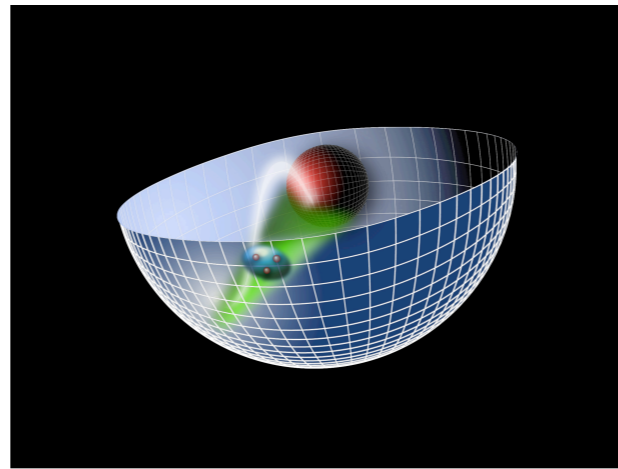
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$$1/\kappa \simeq 1/3 \text{ fm}$$

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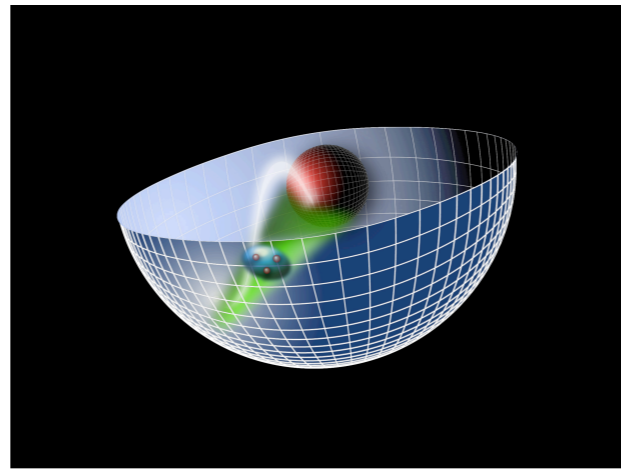
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$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

*Preserves Conformal Symmetry
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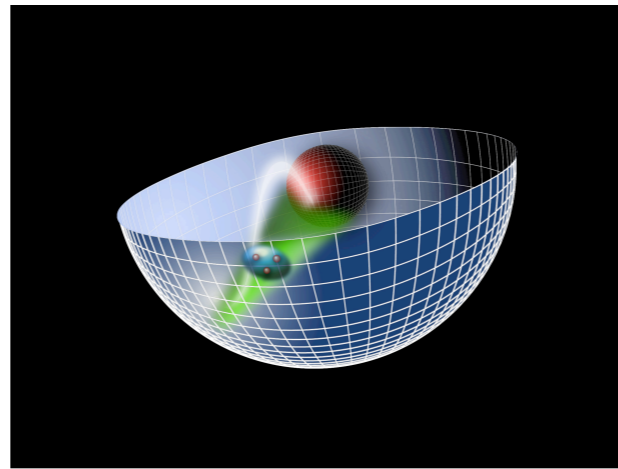
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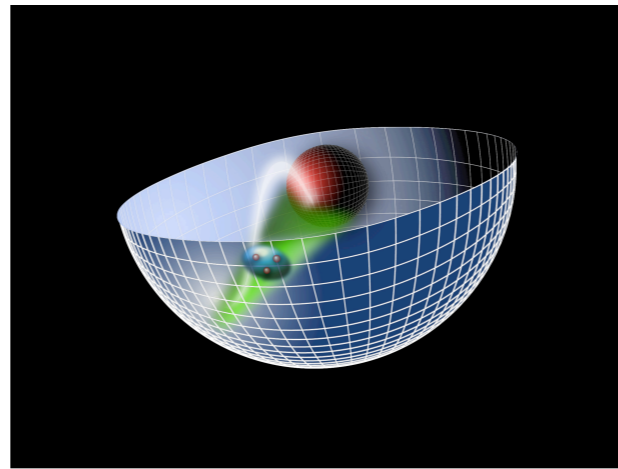
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● **de Alfaro, Fubini, Furlan:**

● **Fubini, Rabinovici:**

***Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!***

Meson Spectrum in Soft Wall Model

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

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Pion: Negative term for $J=0$ cancels positive terms from LFKE and potential

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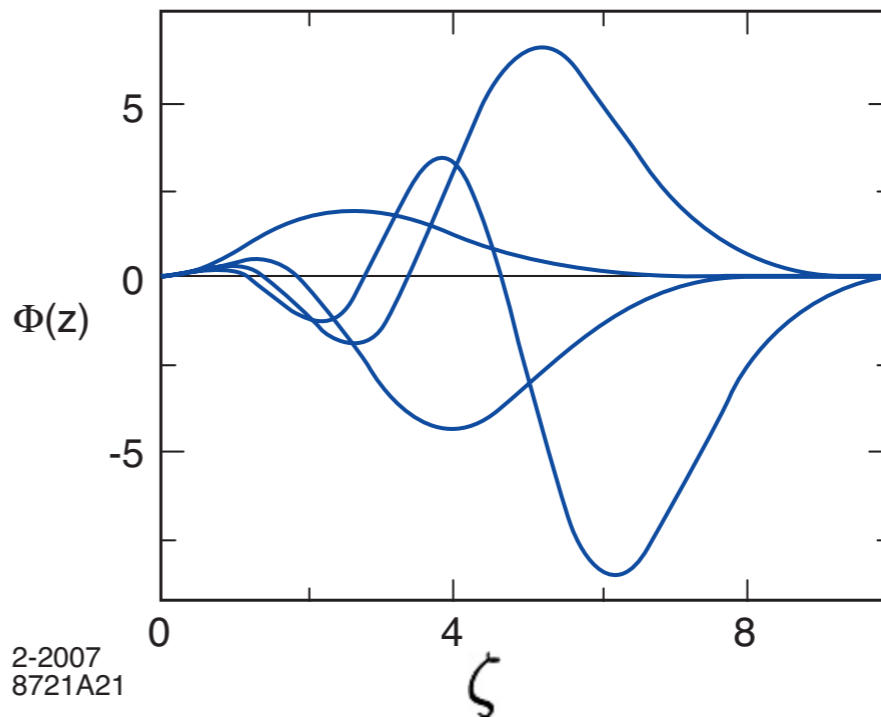
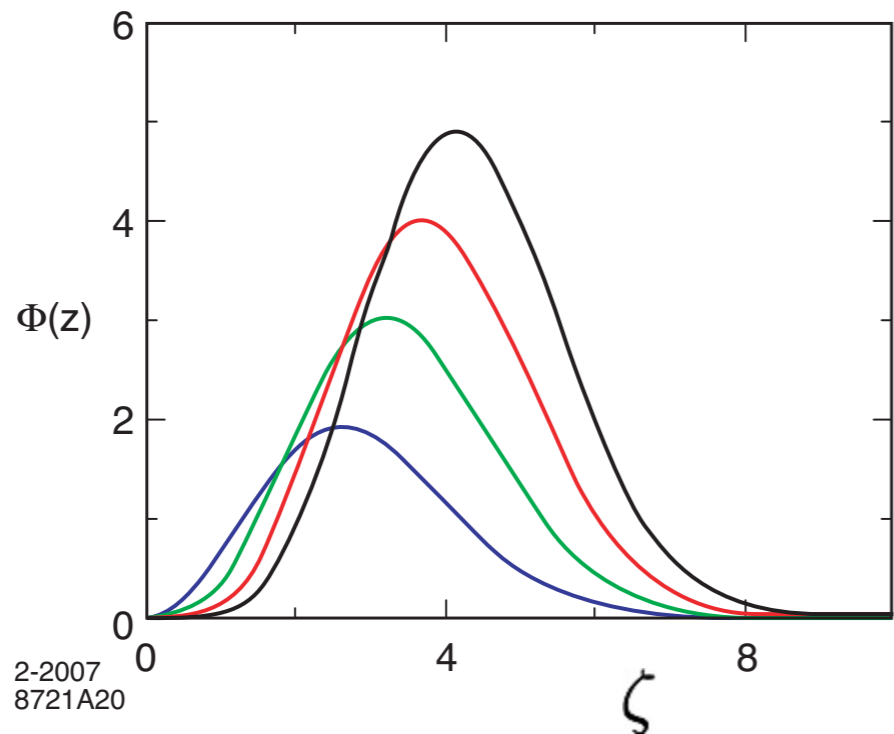
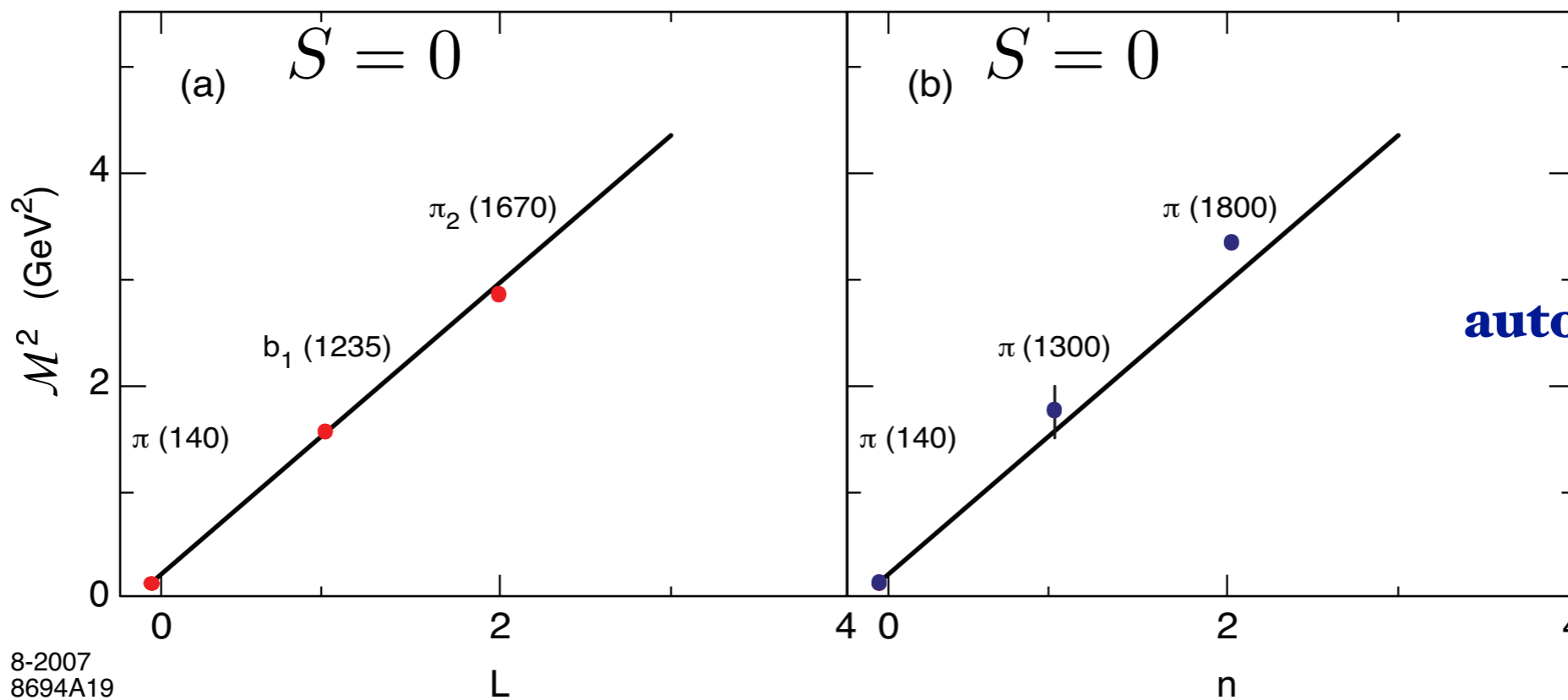


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

*Soft Wall
Model*



**Pion mass
automatically zero!**

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

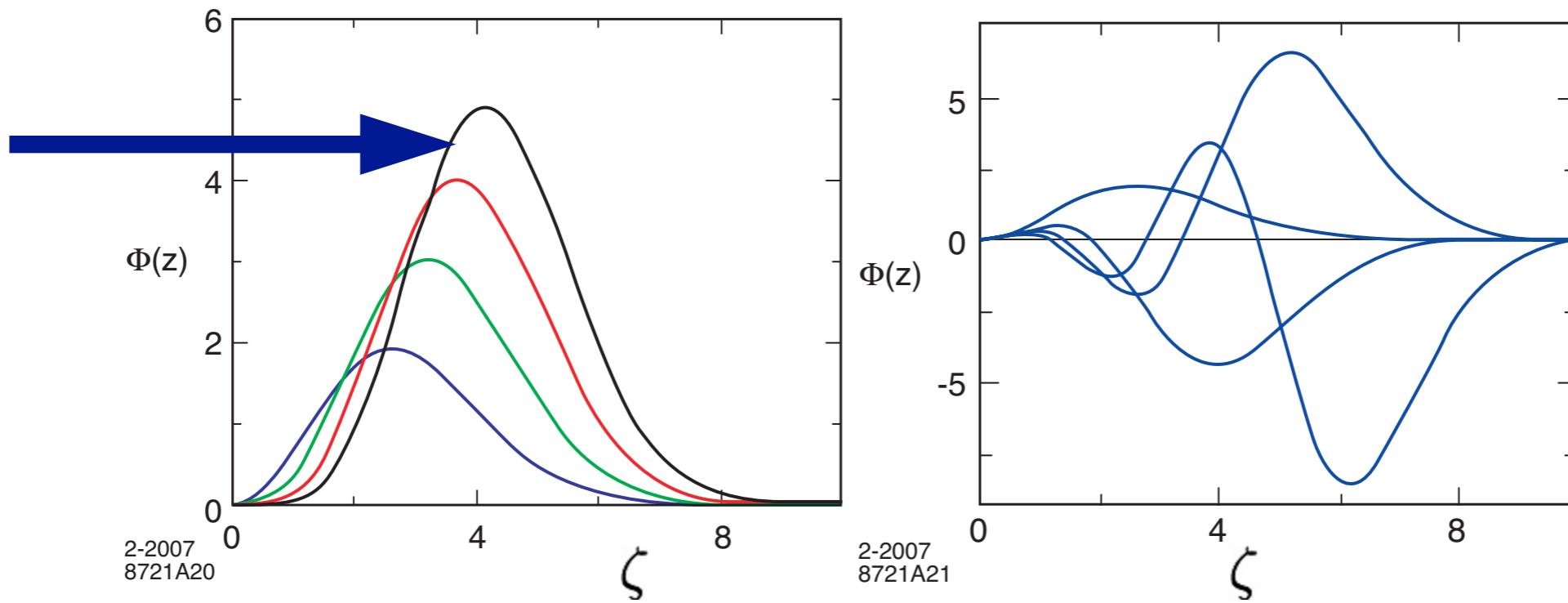
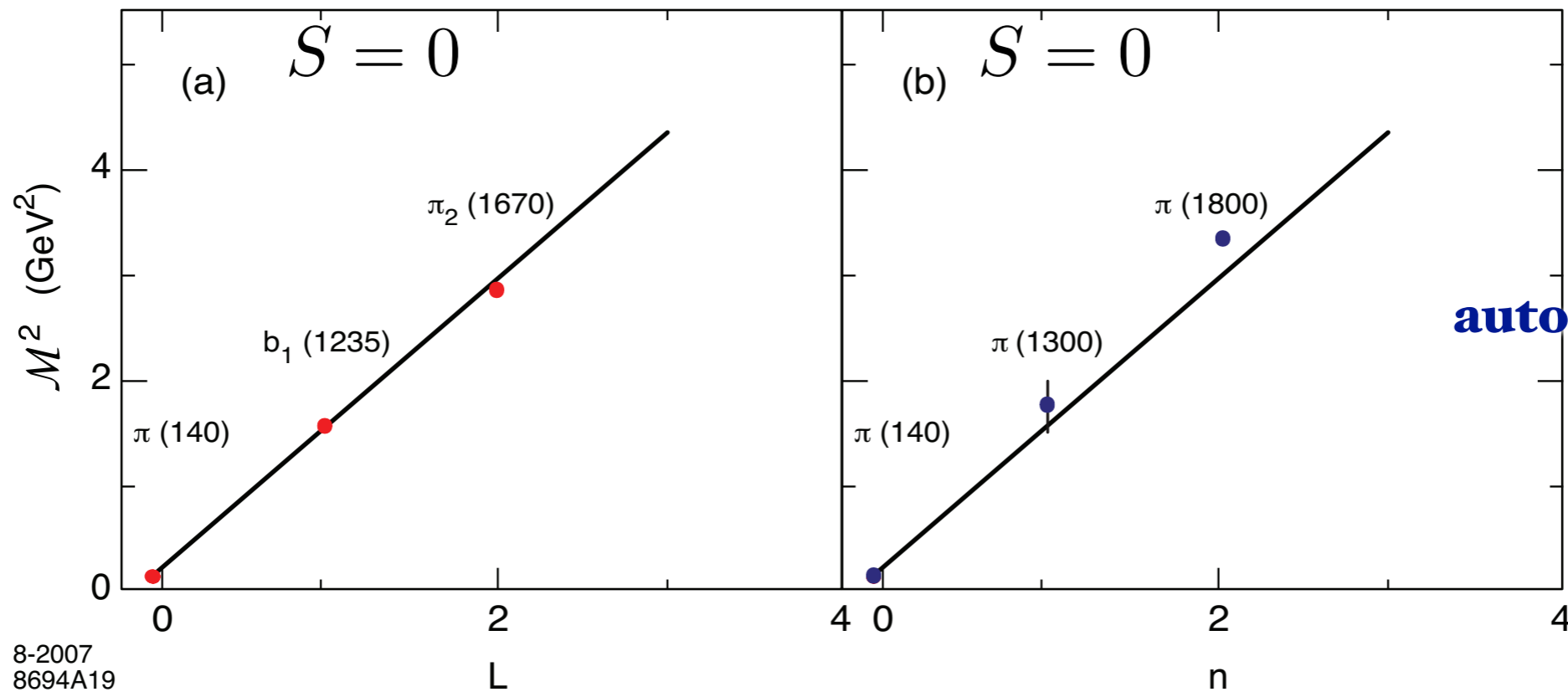


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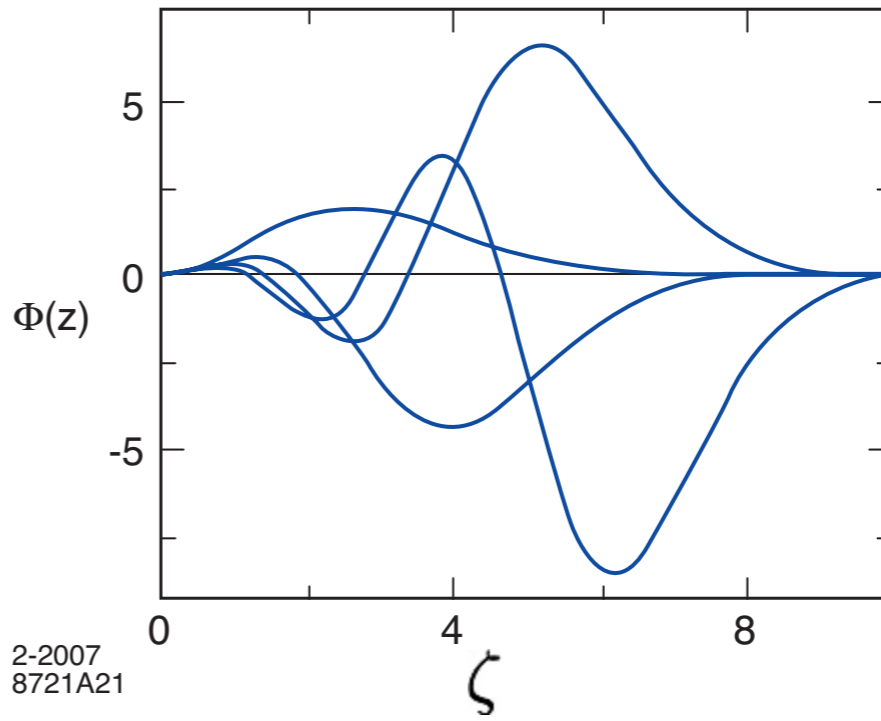
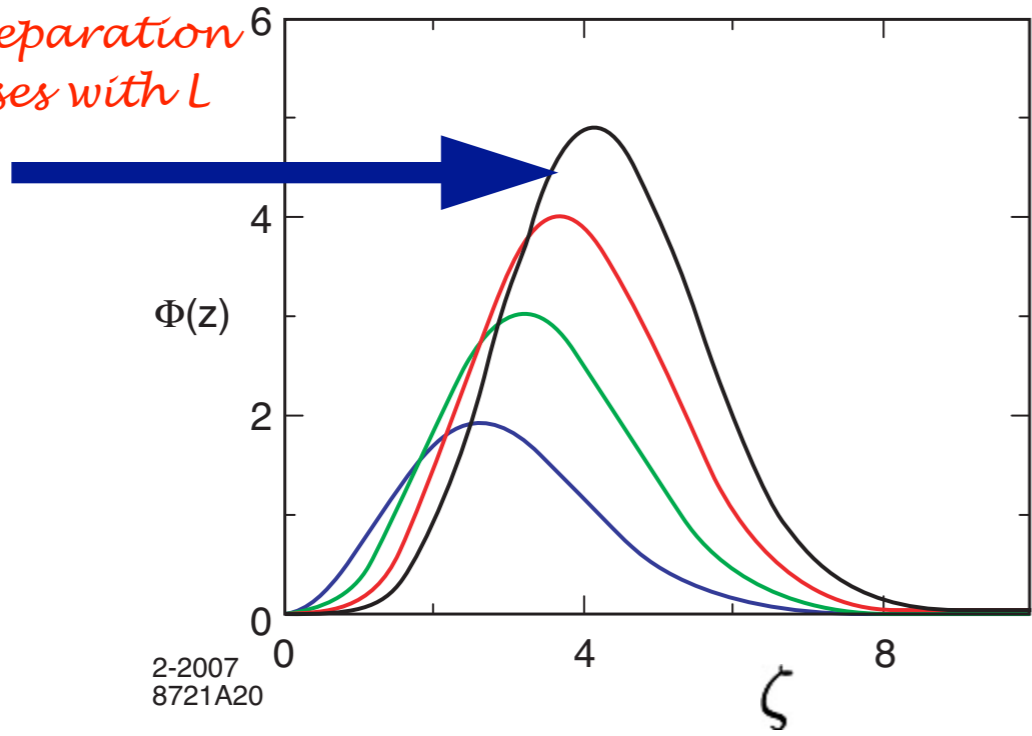


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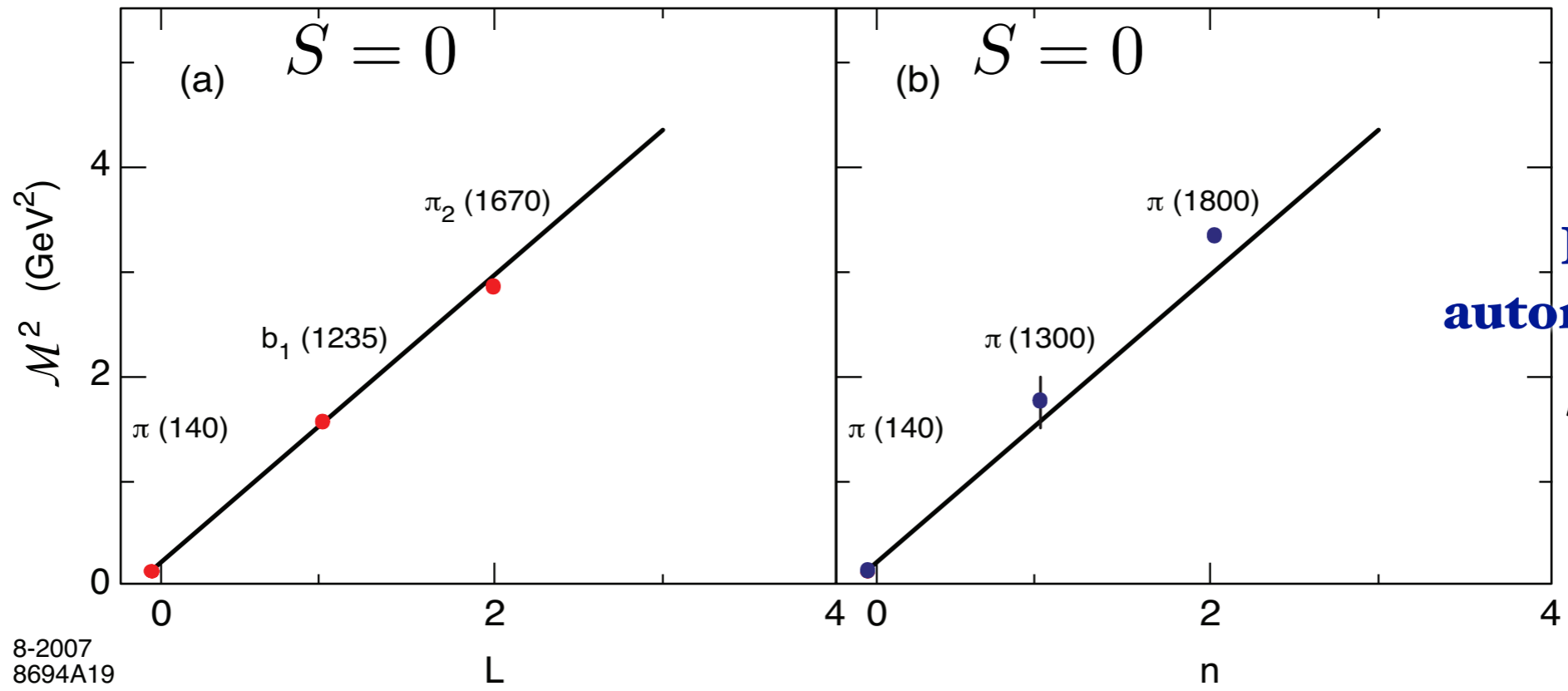


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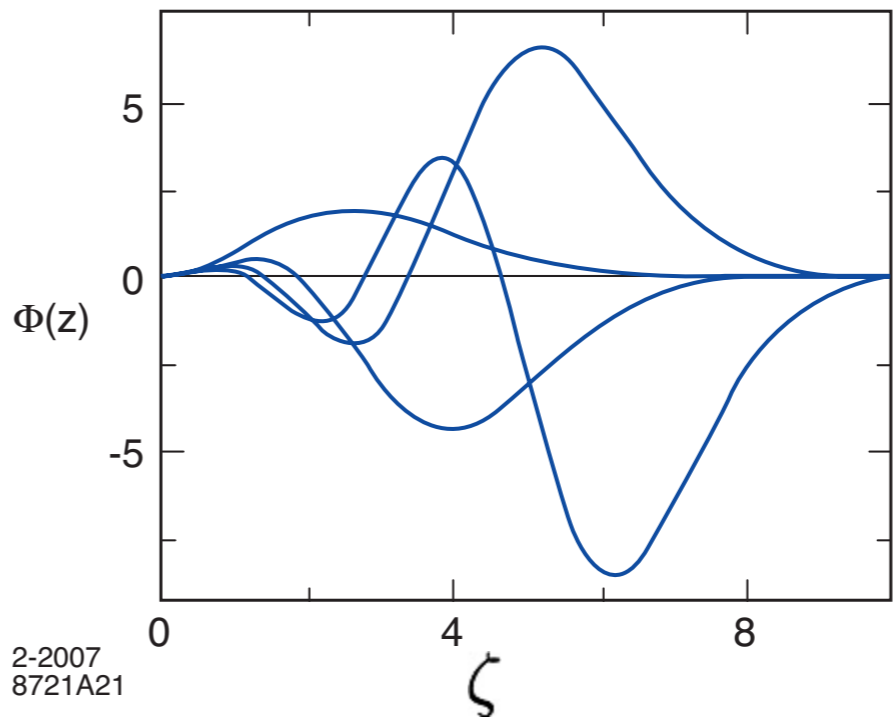
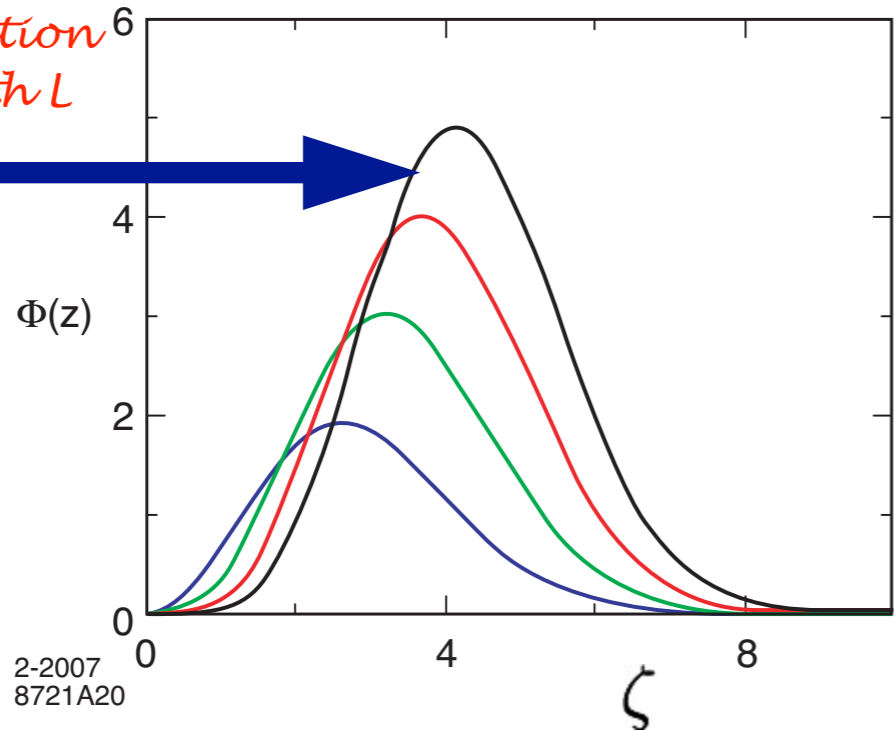
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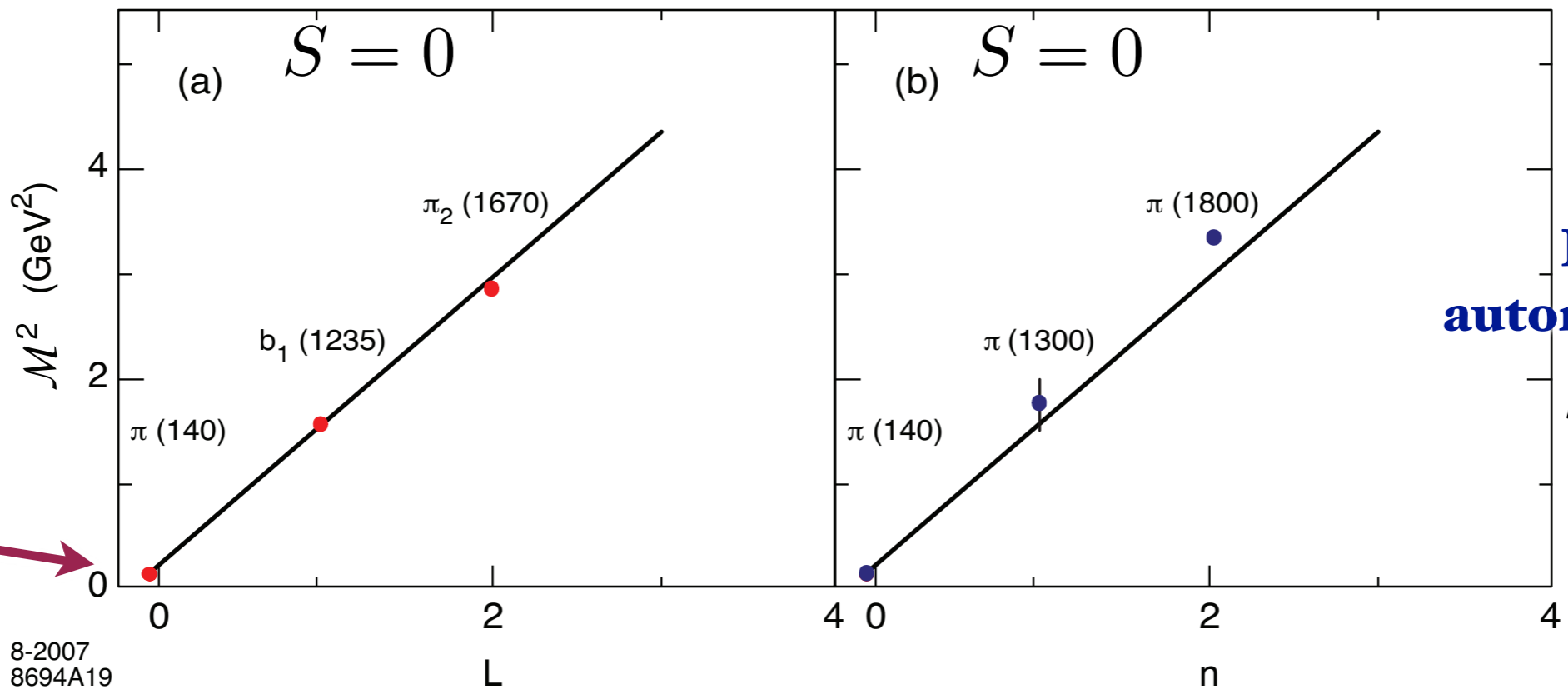


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Soft Wall Model



8-2007
8694A19

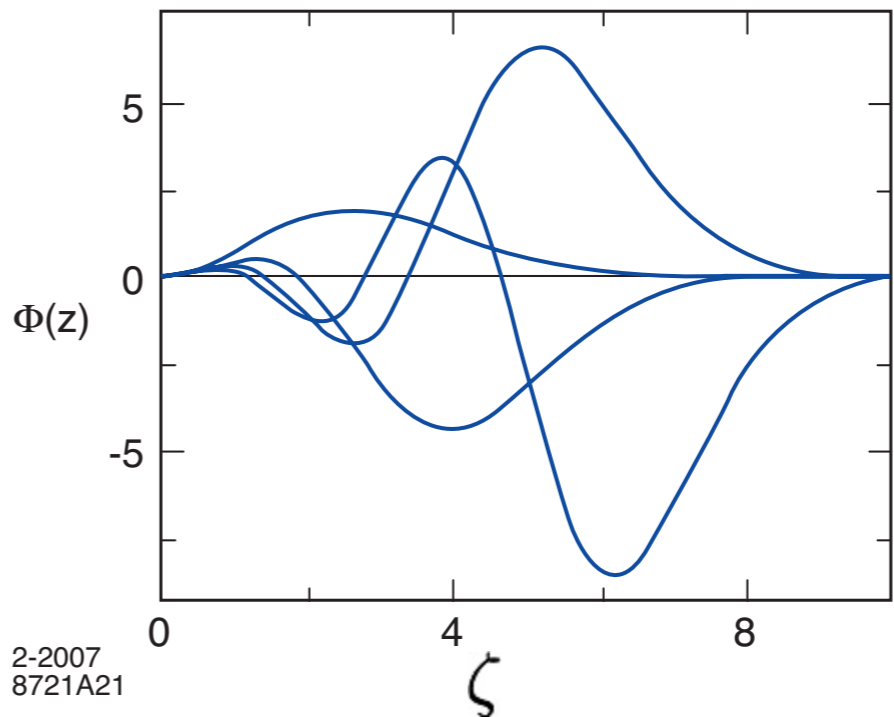
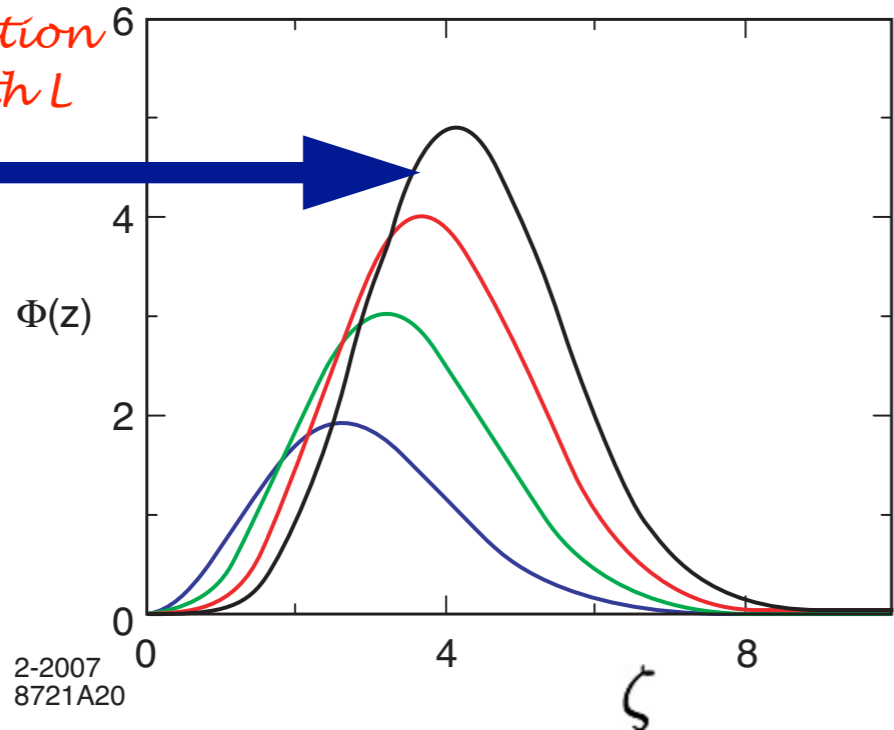
Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Quark separation increases with L



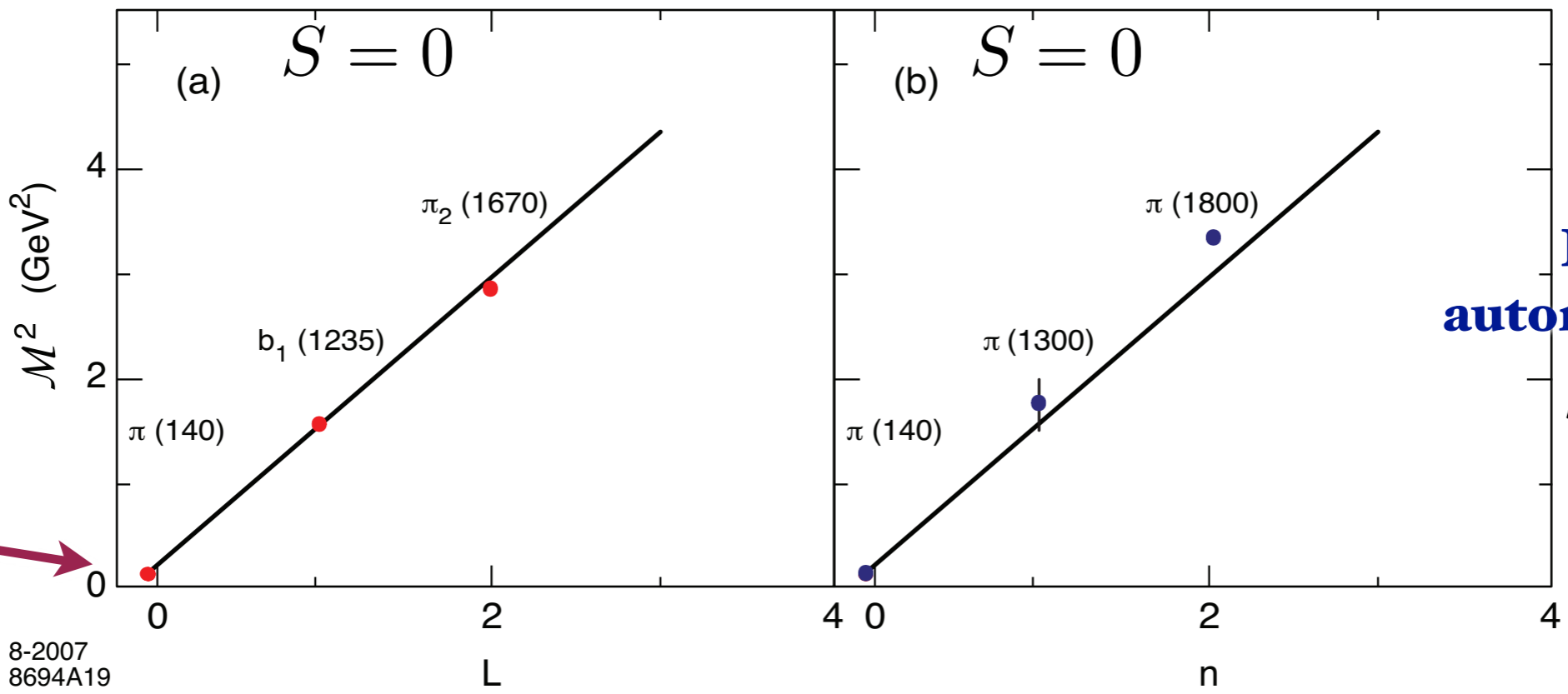
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8721A20

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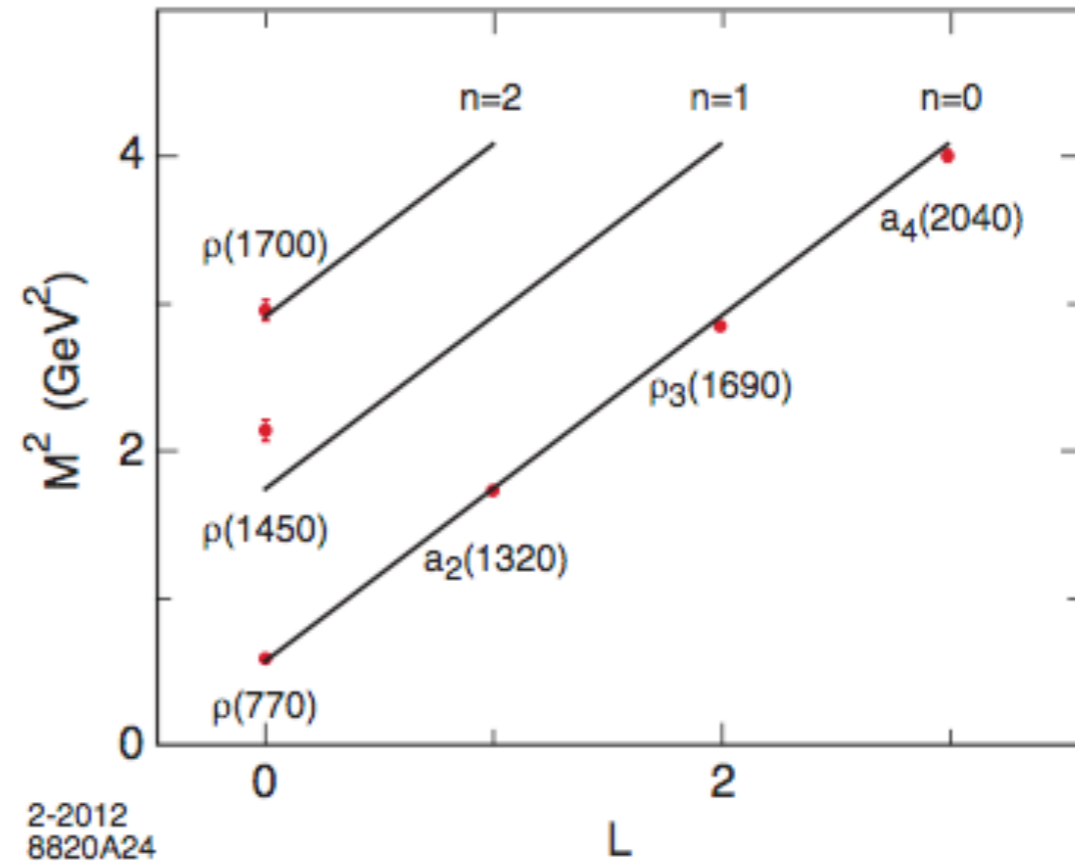
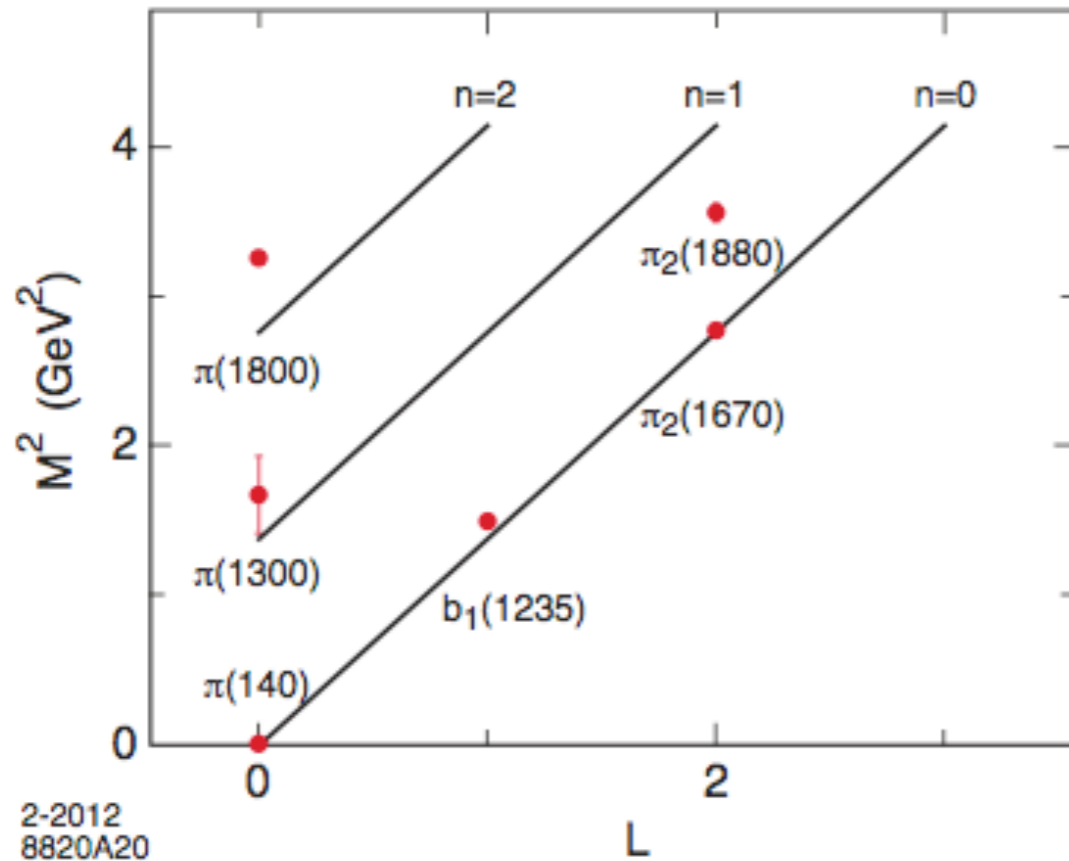
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$I=1$ orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ -meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a -states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

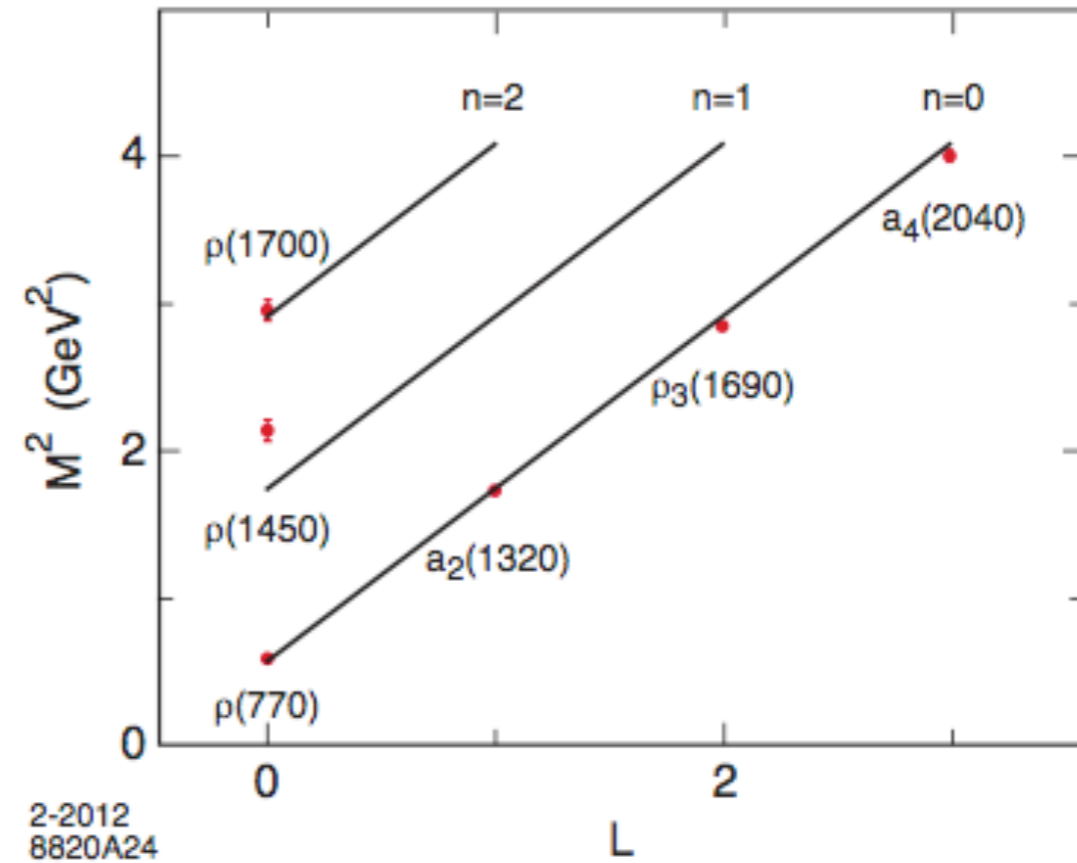
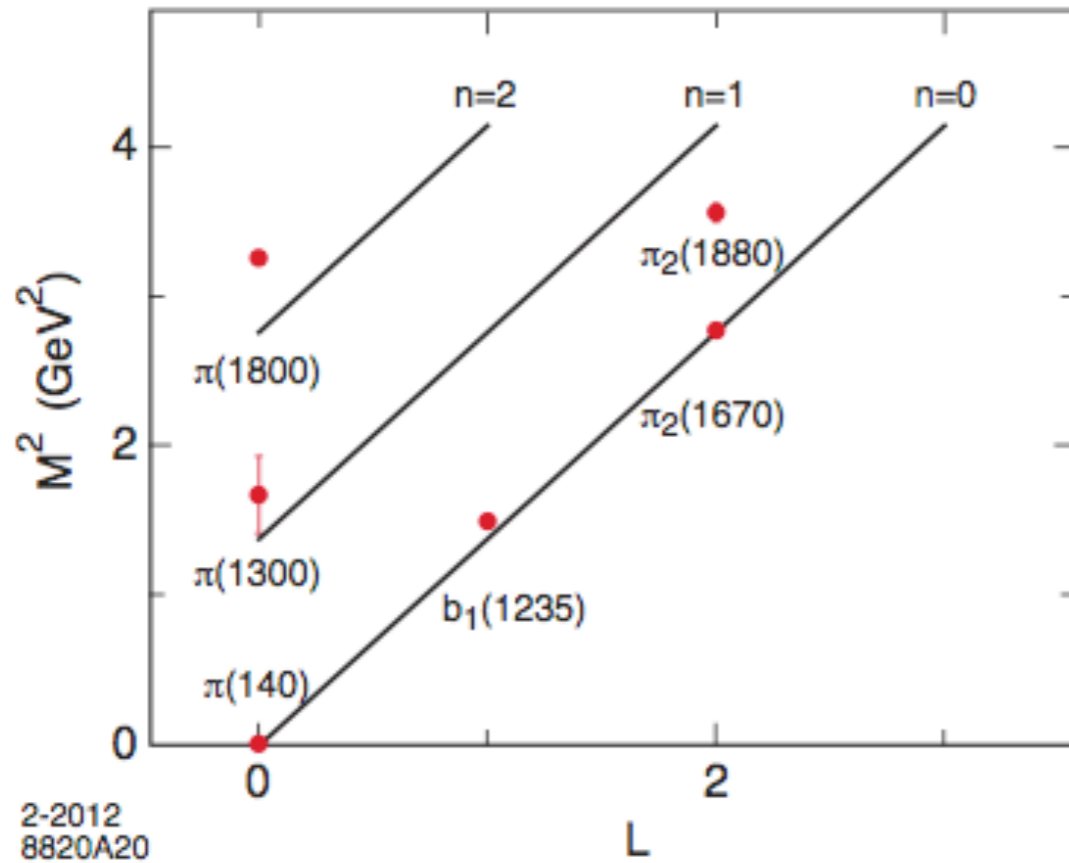
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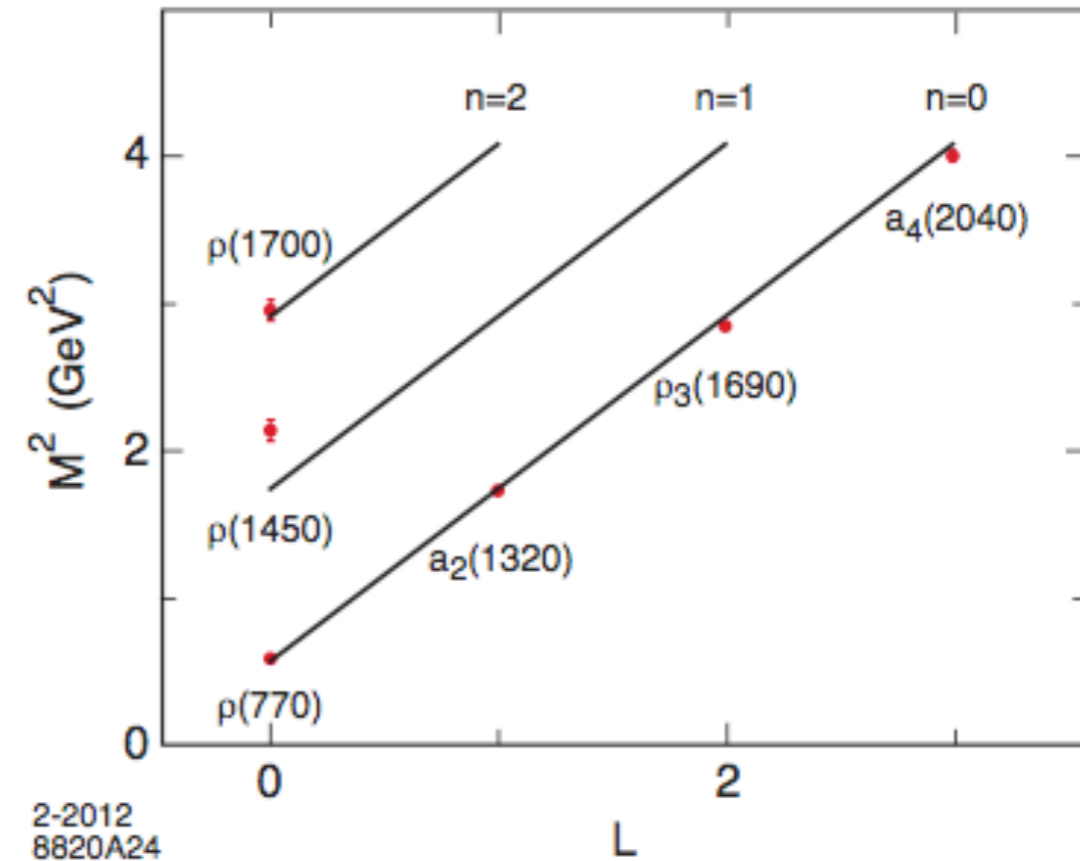
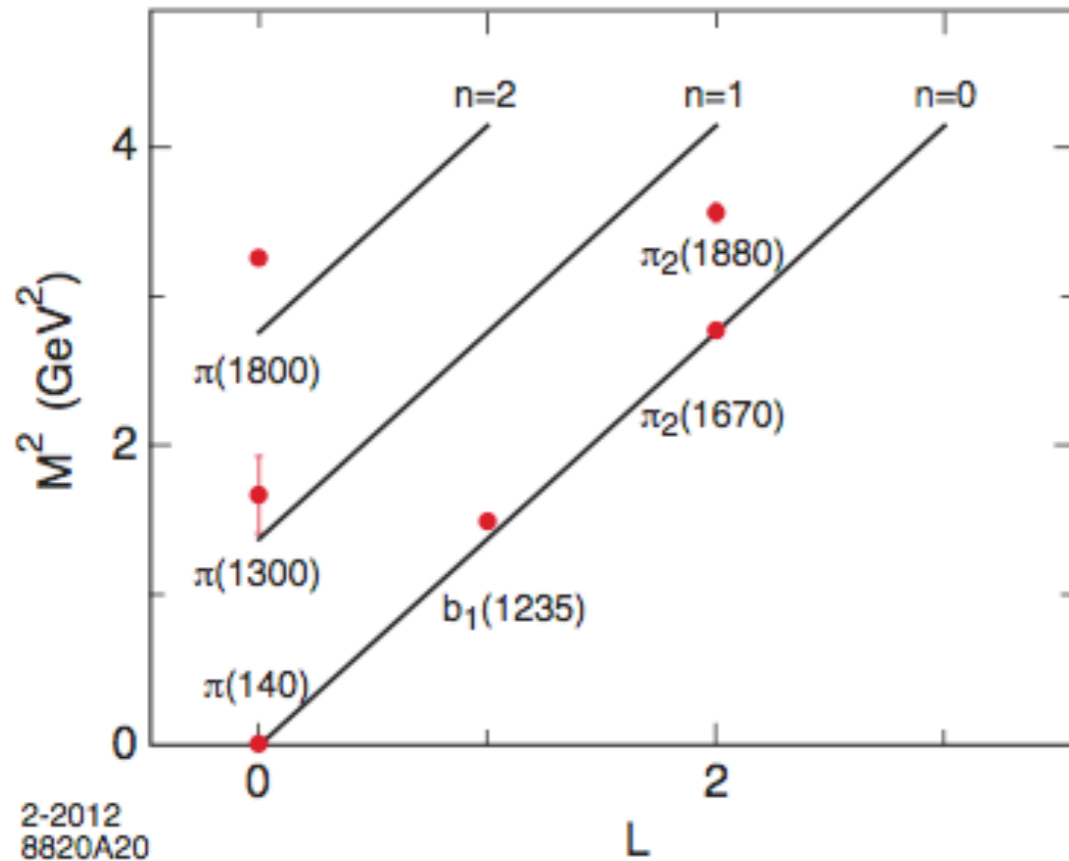
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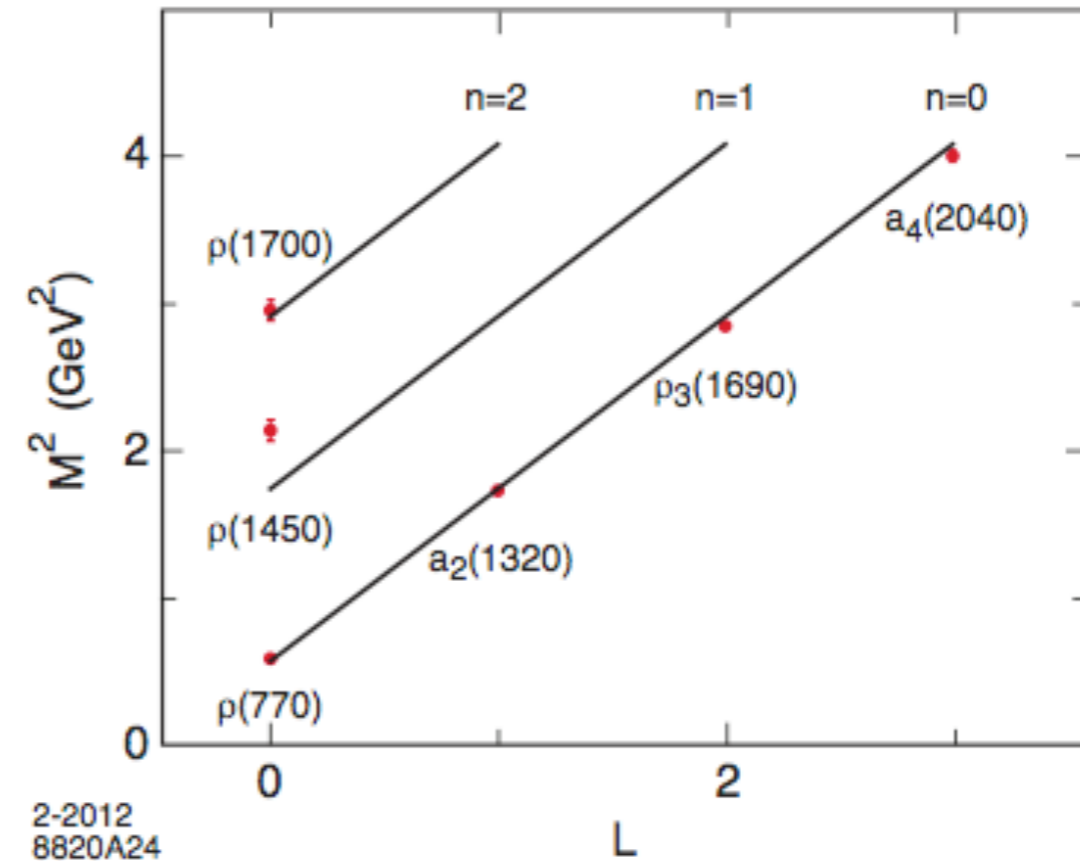
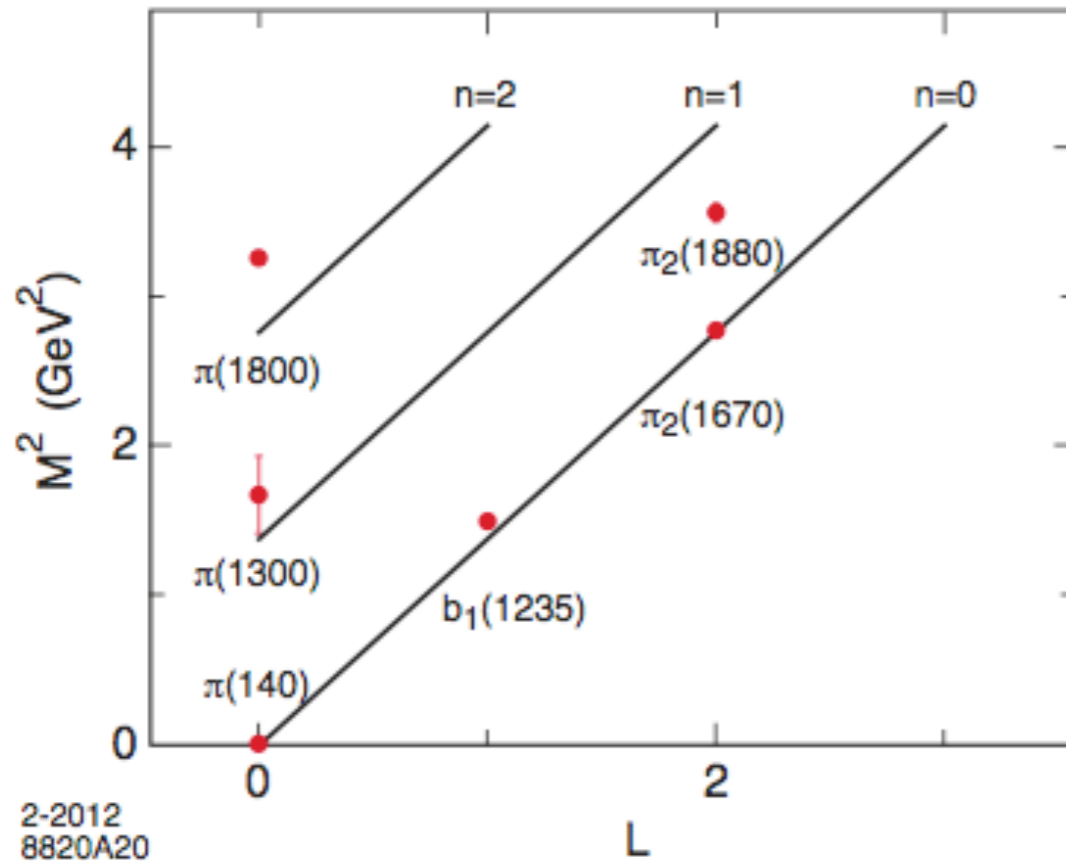
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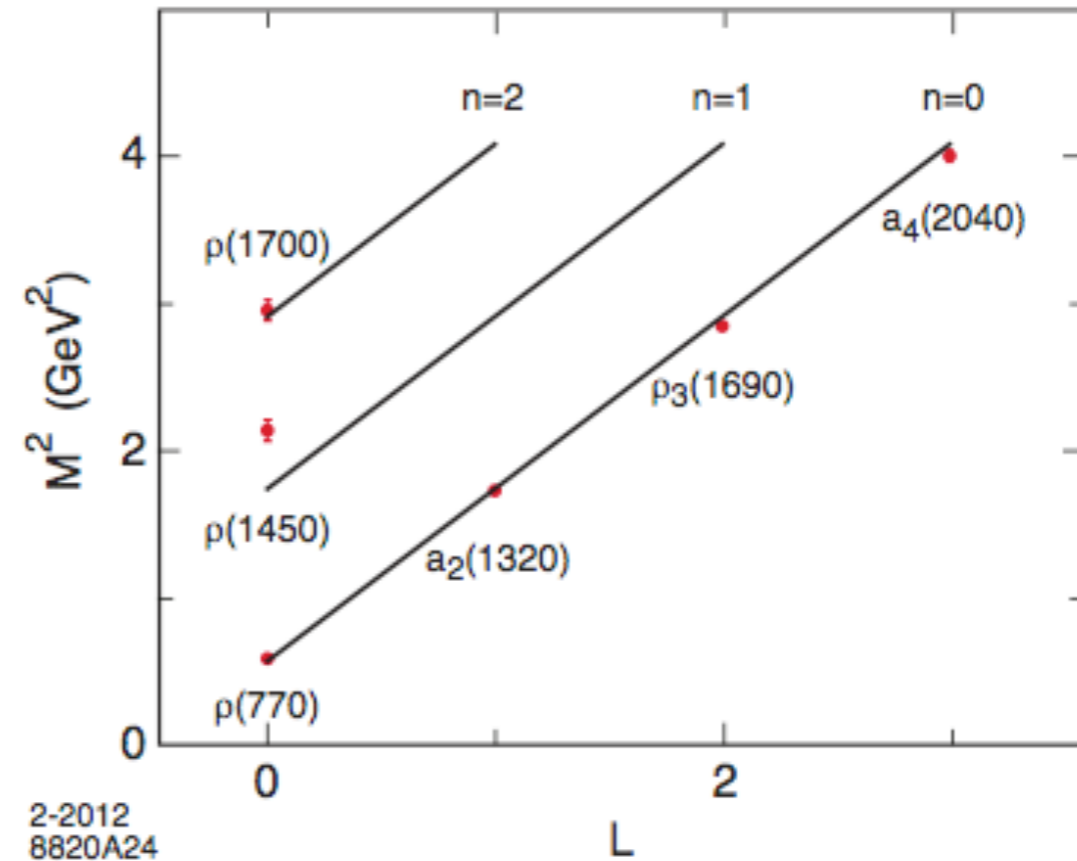
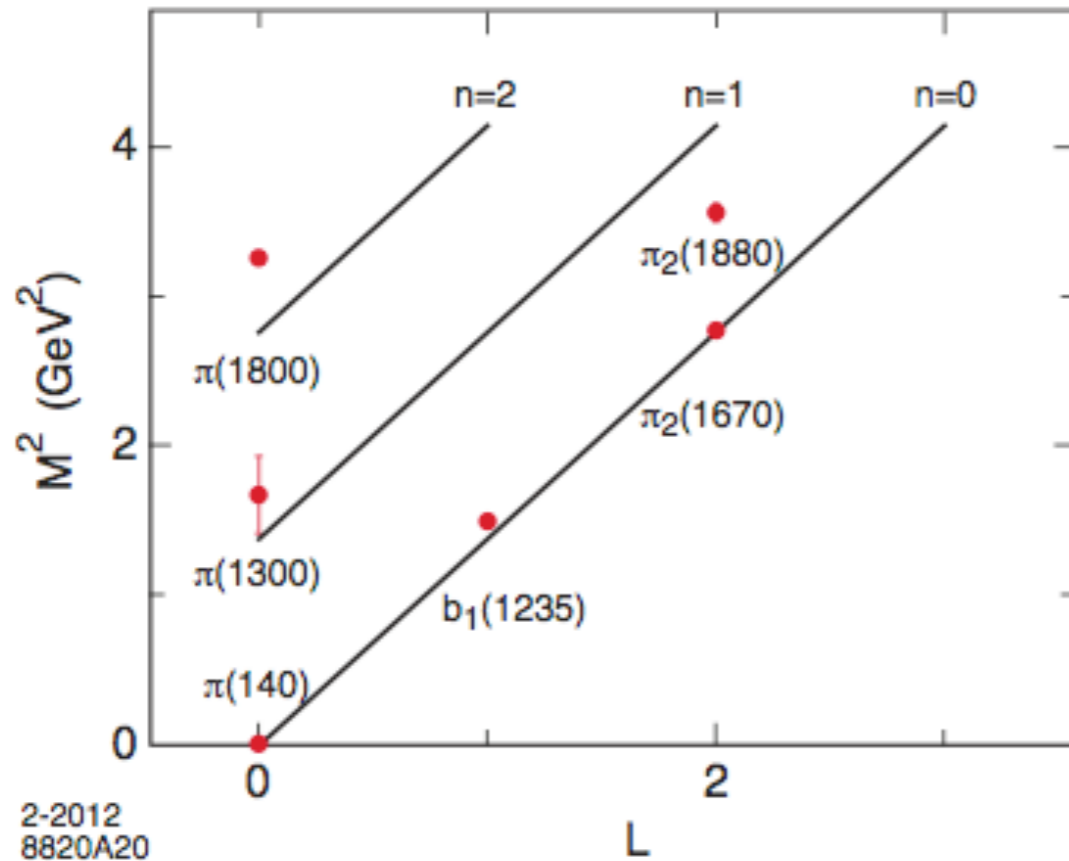
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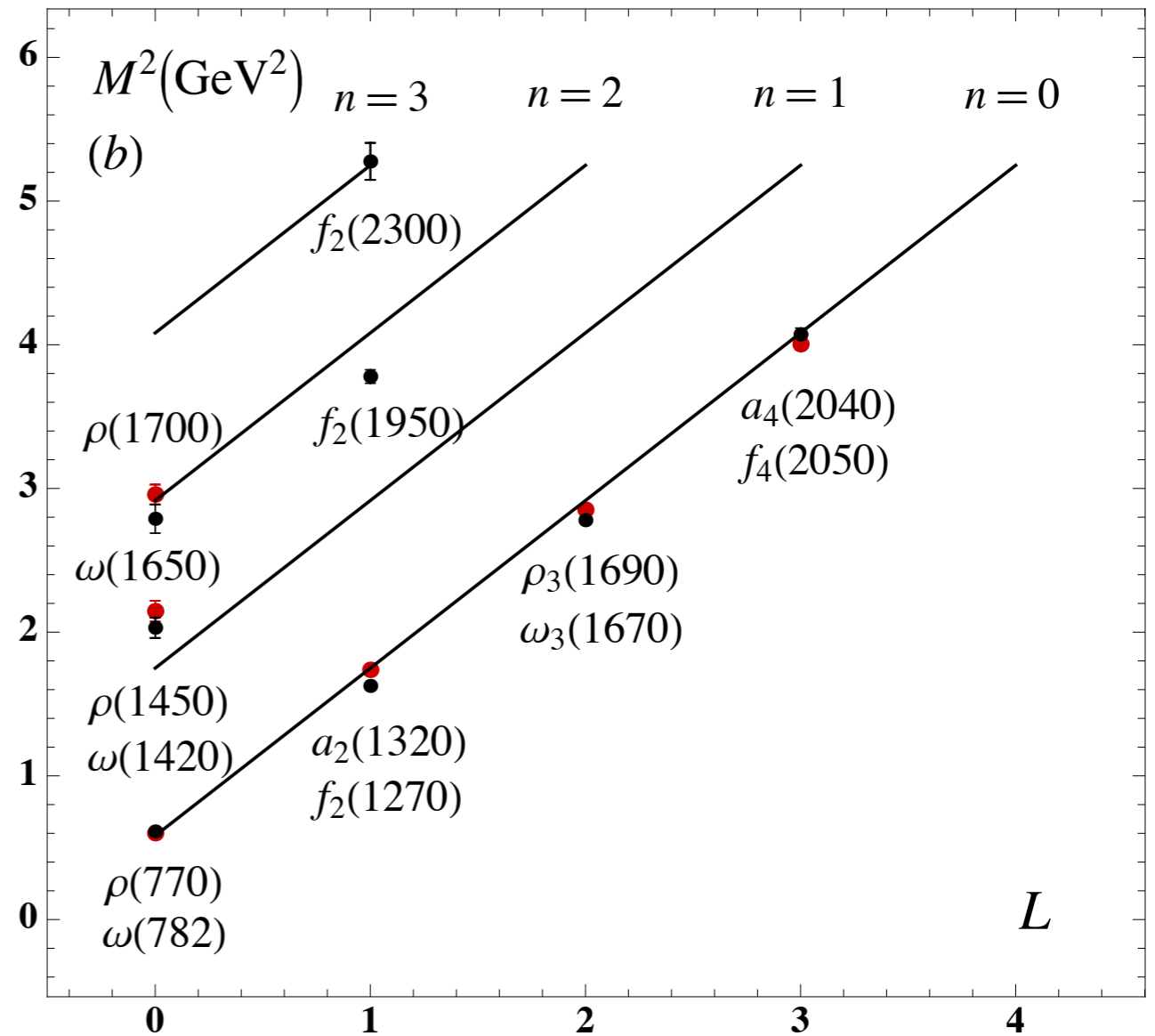
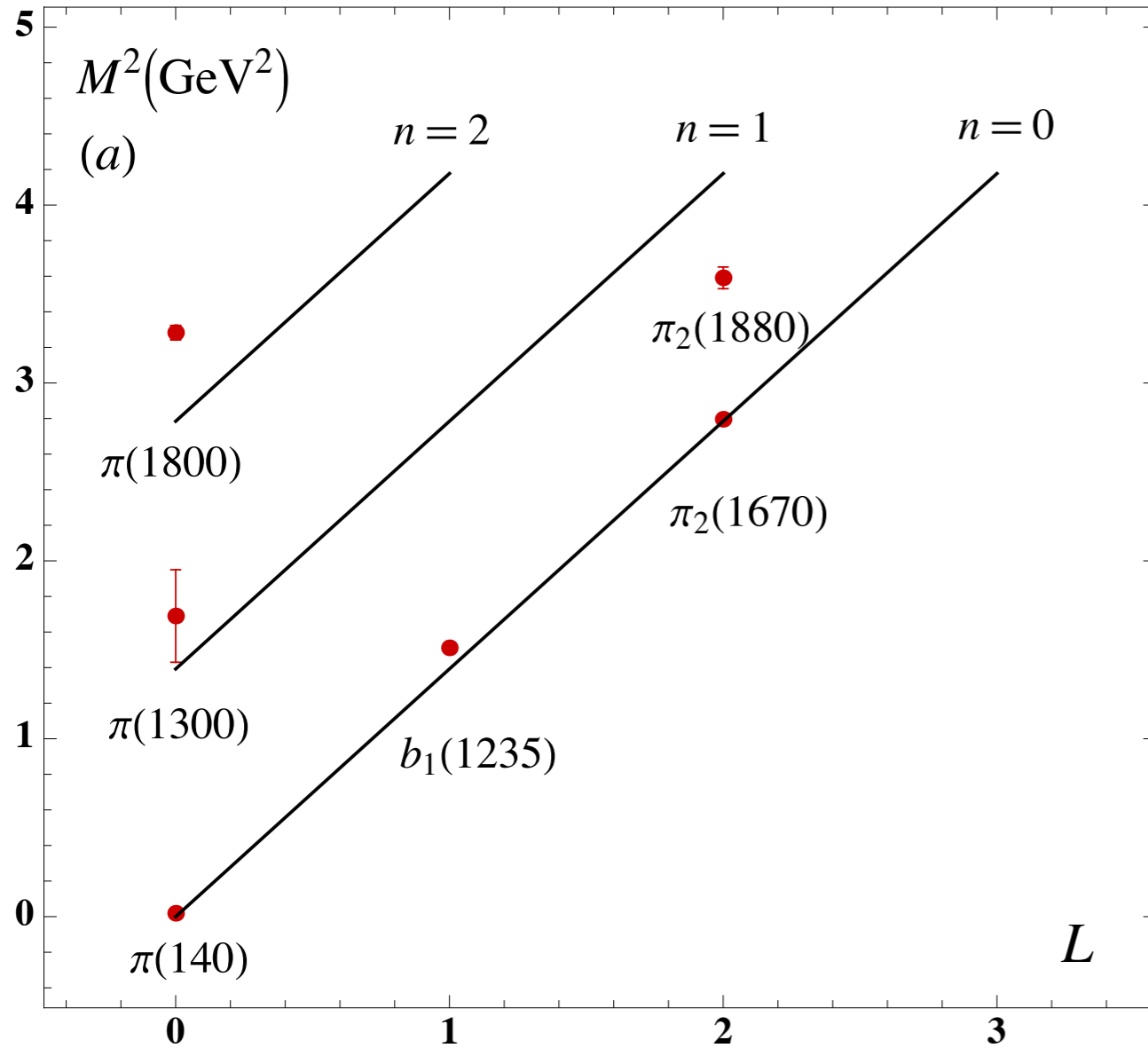
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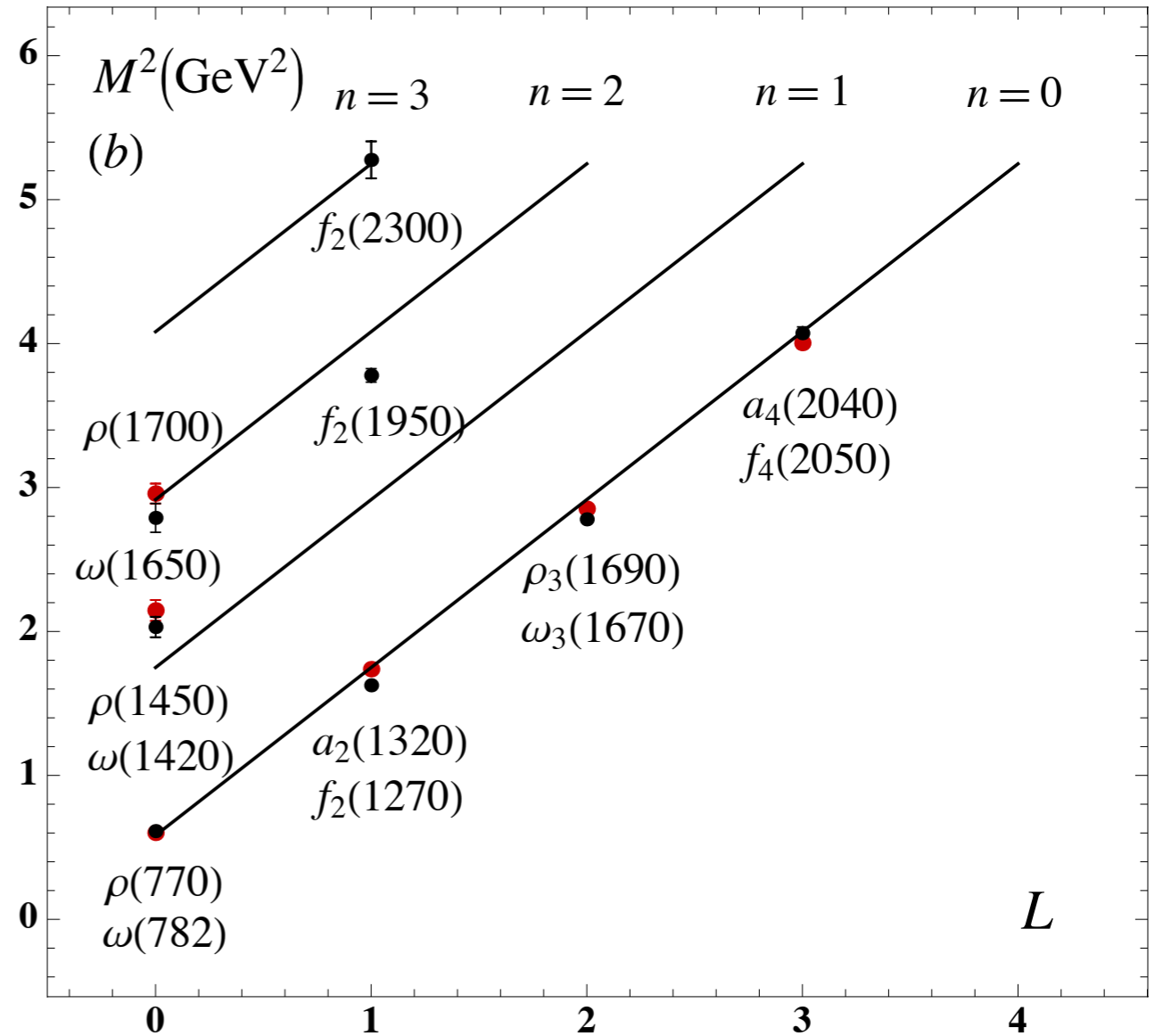
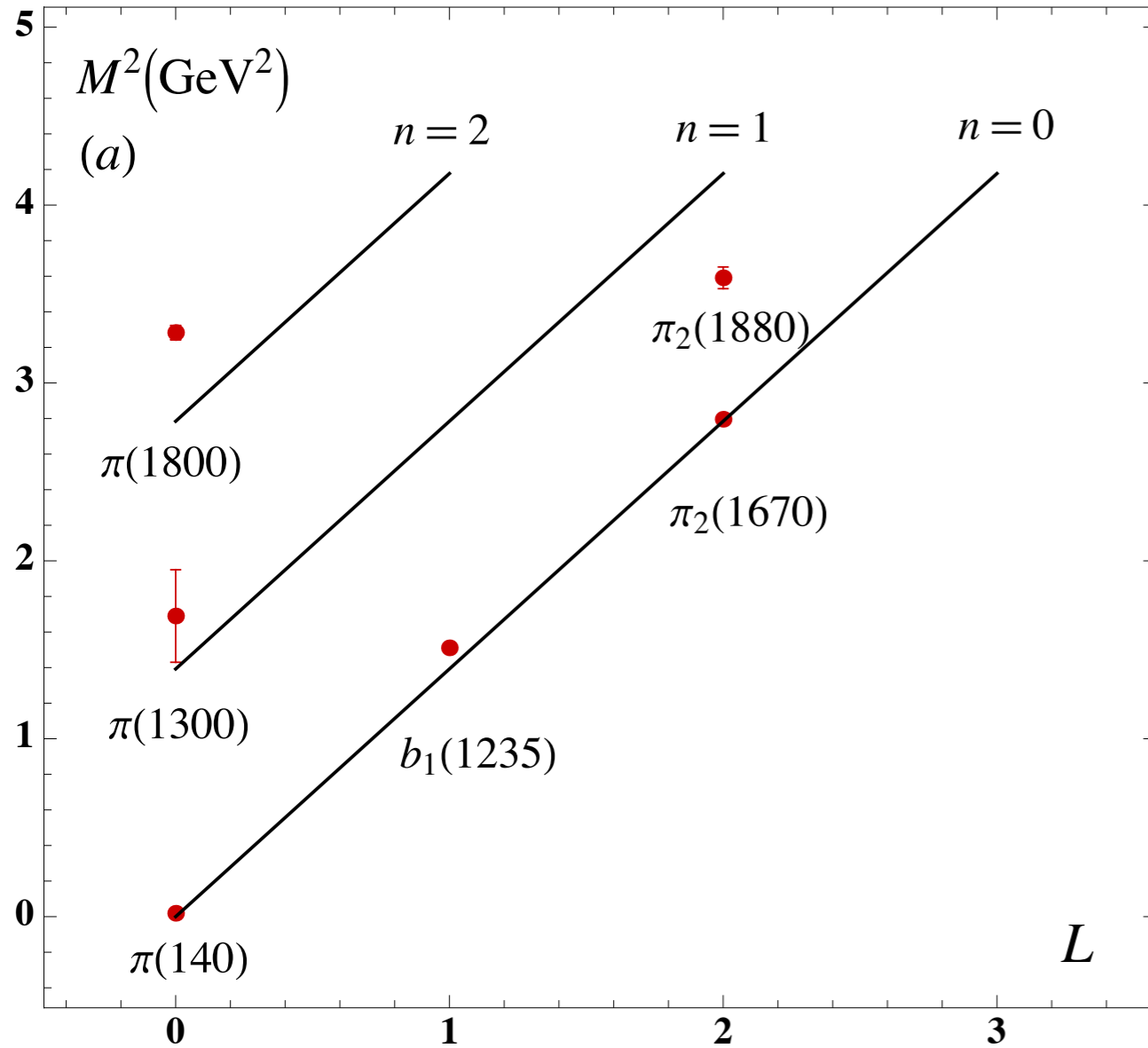
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- Results easily extended to light quarks masses (Ex: K -mesons)
- First order perturbation in the quark masses

$$\Delta M^2 = \langle \psi | \sum_a m_a^2 / x_a | \psi \rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x, \zeta) \sim \sqrt{x(1-x)} e^{-\frac{1}{2\lambda} \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right)} e^{-\frac{1}{2} \lambda \zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA
[J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

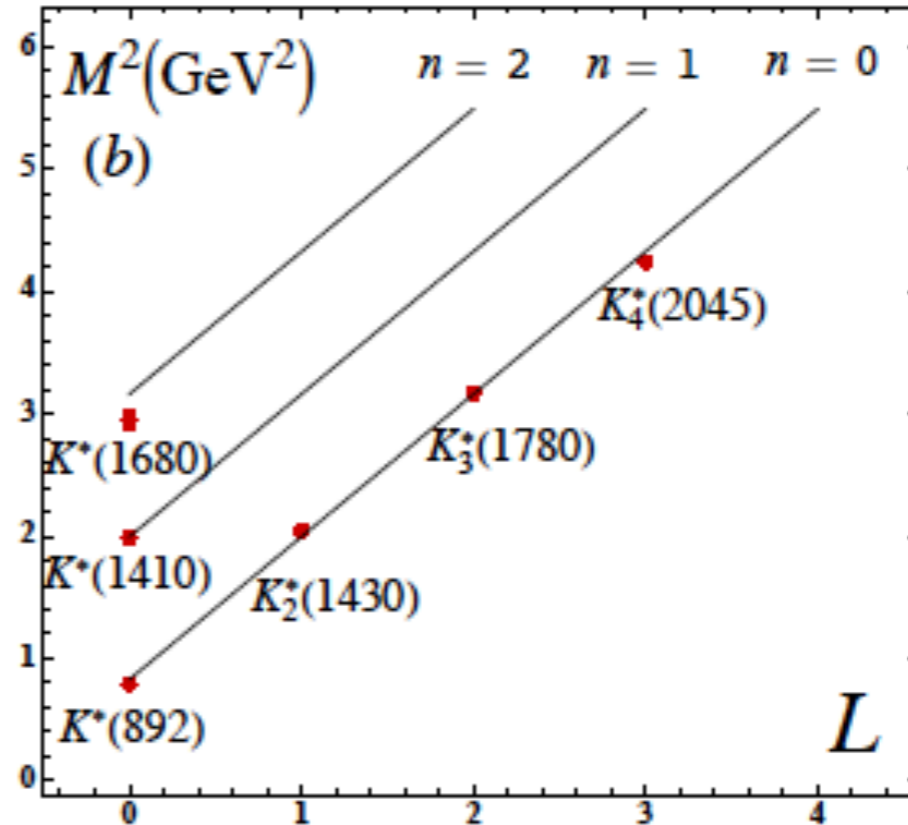
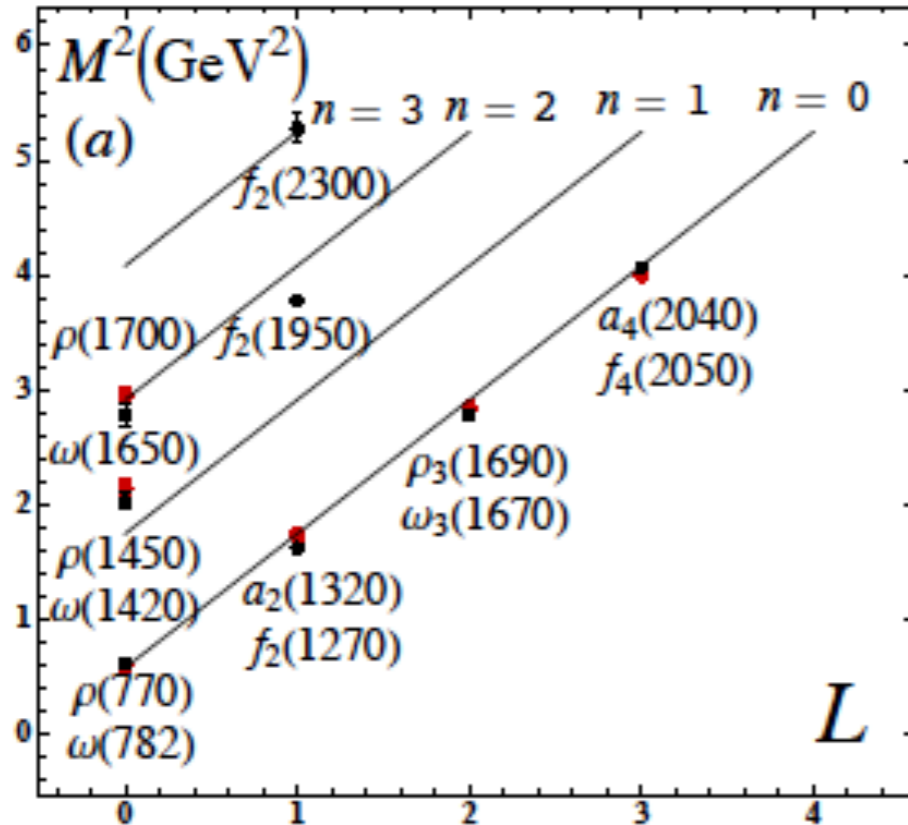
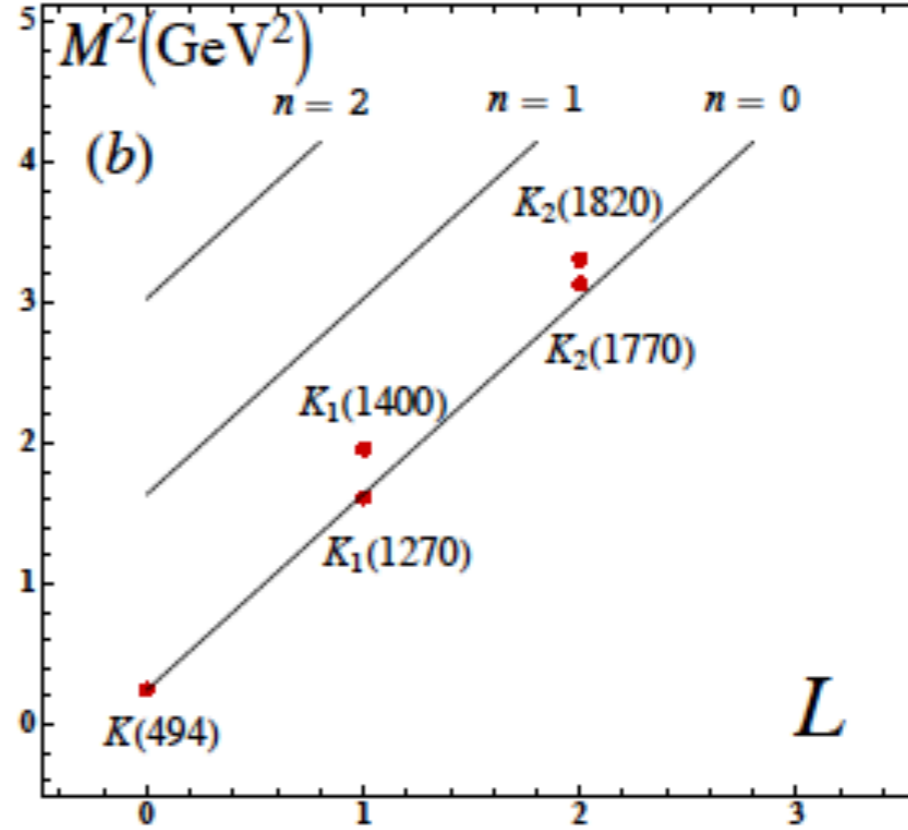
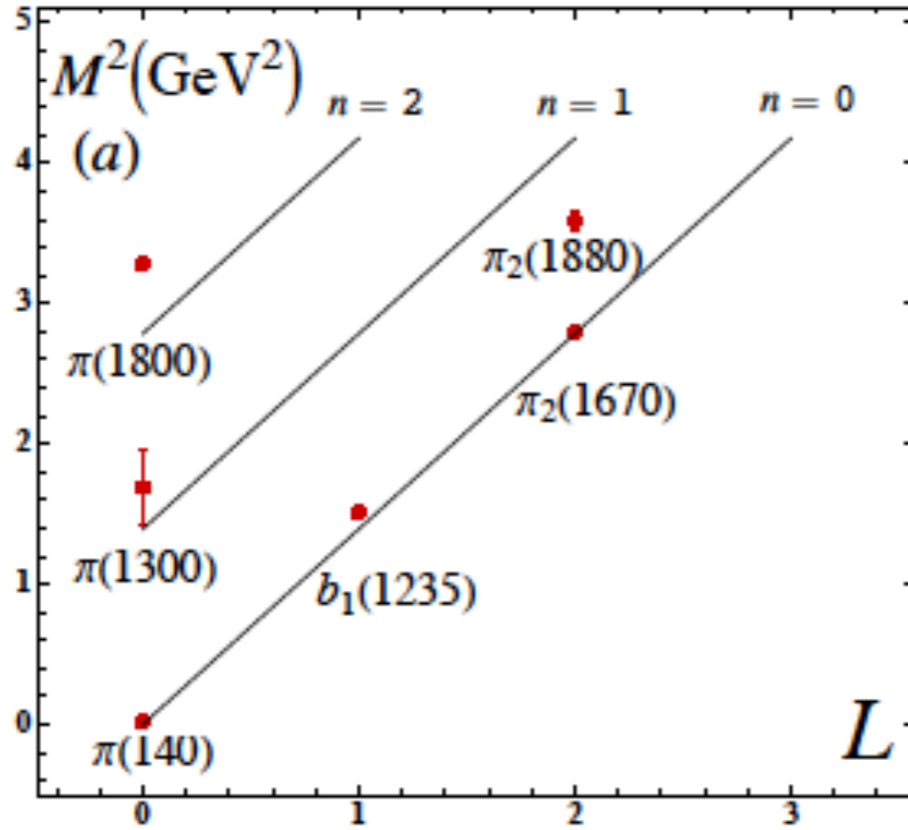
- For the K^*

$$M_{n,L,S}^2 = M_{K^\pm}^2 + 4\lambda \left(n + \frac{J+L}{2} \right)$$

- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

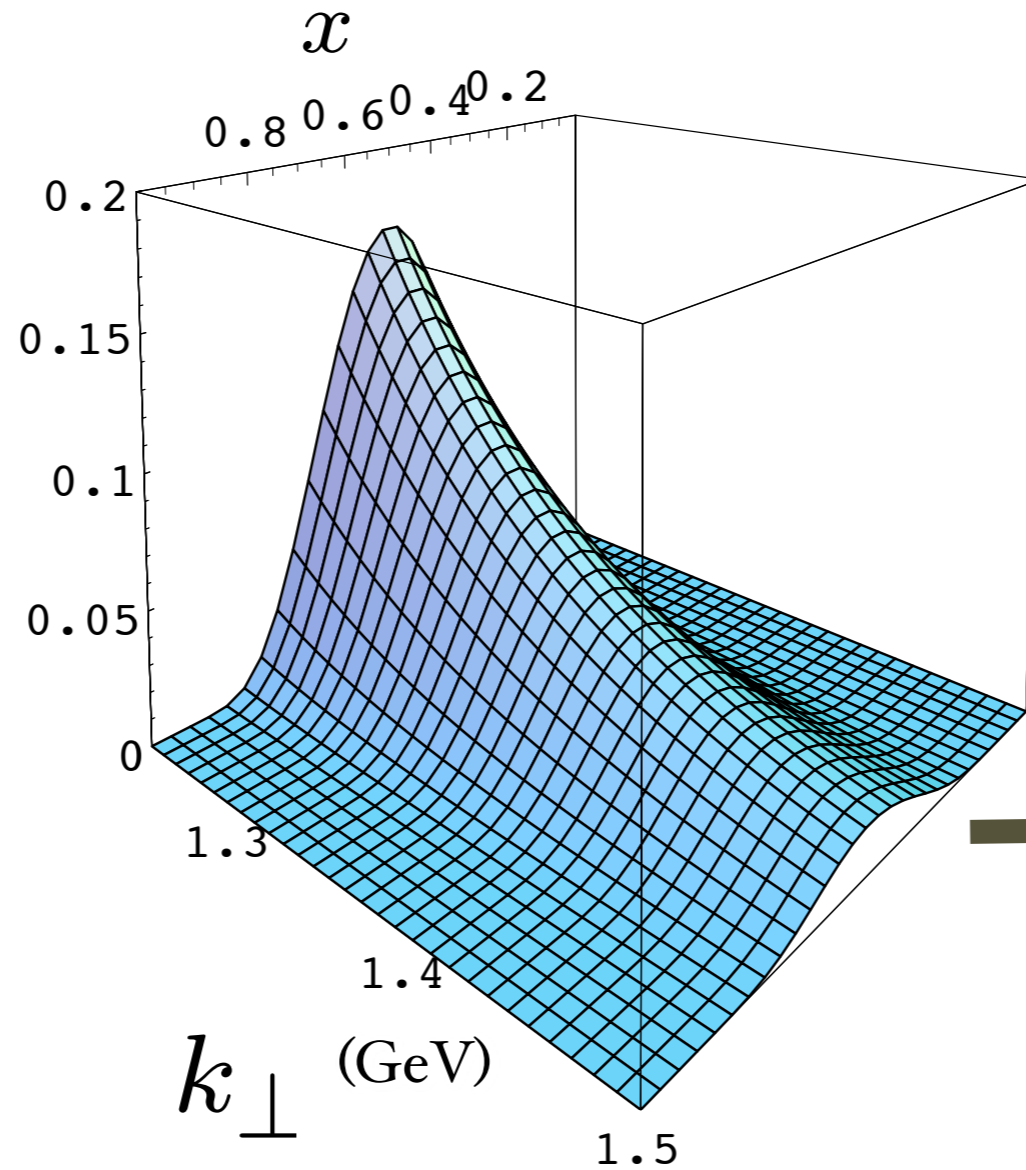
$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



Prediction from AdS/QCD: Meson LFWF

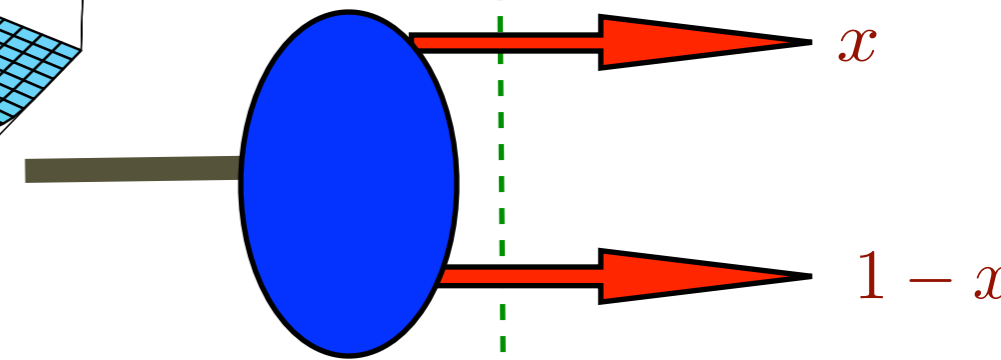
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_{\perp}^2)$$



de Teramond,
Cao, sjb

“Soft Wall”
model



massless quarks

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_{\pi}(x) = \frac{4}{\sqrt{3}\pi} f_{\pi} \sqrt{x(1-x)}$$

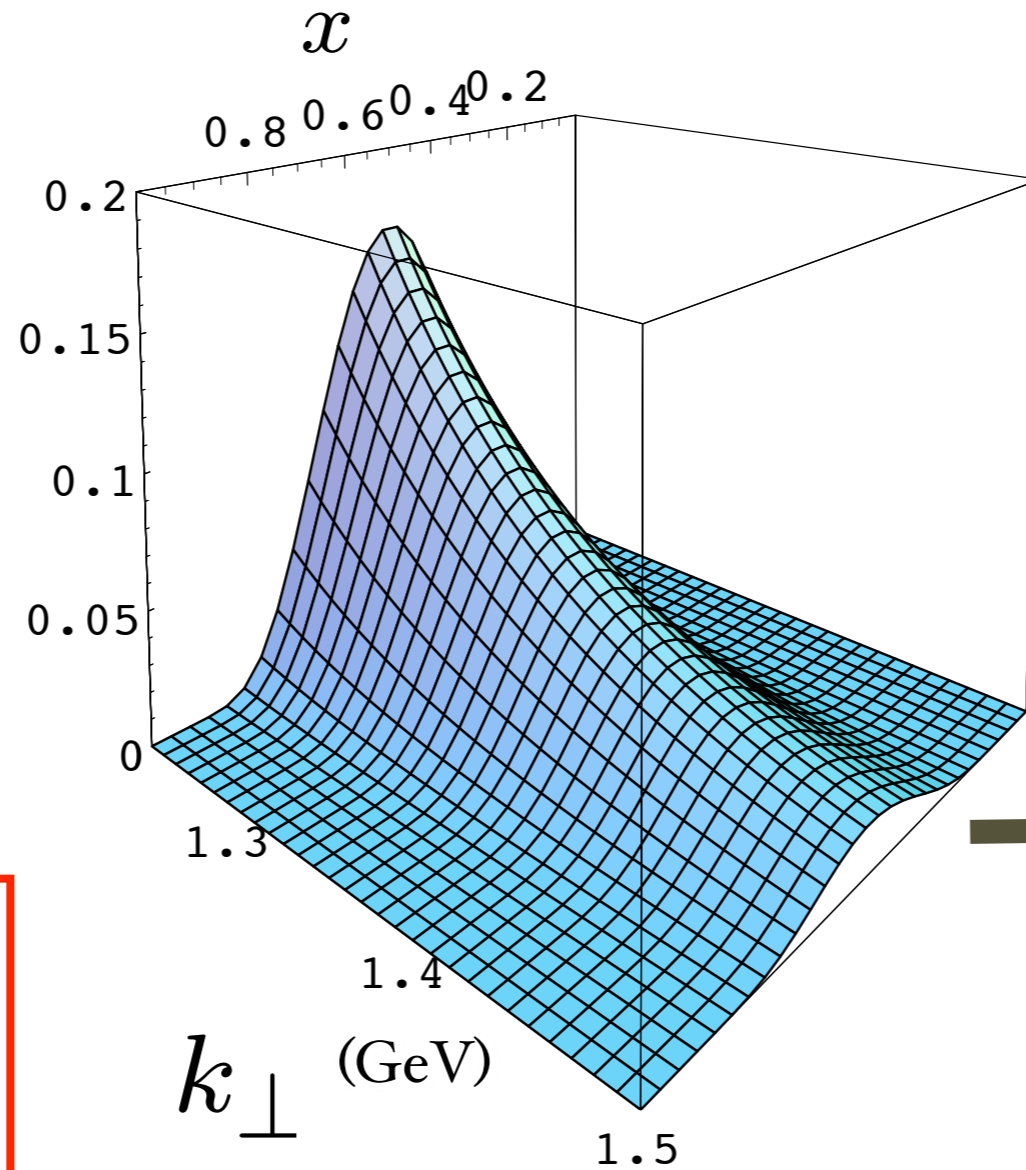
$$f_{\pi} = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE!

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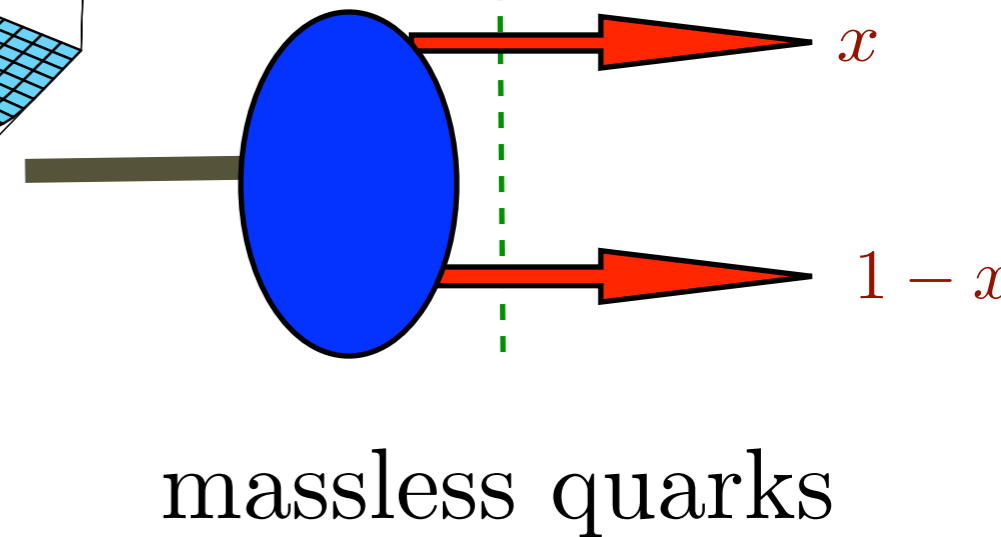
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Note coupling

$$k_{\perp}^2, x$$

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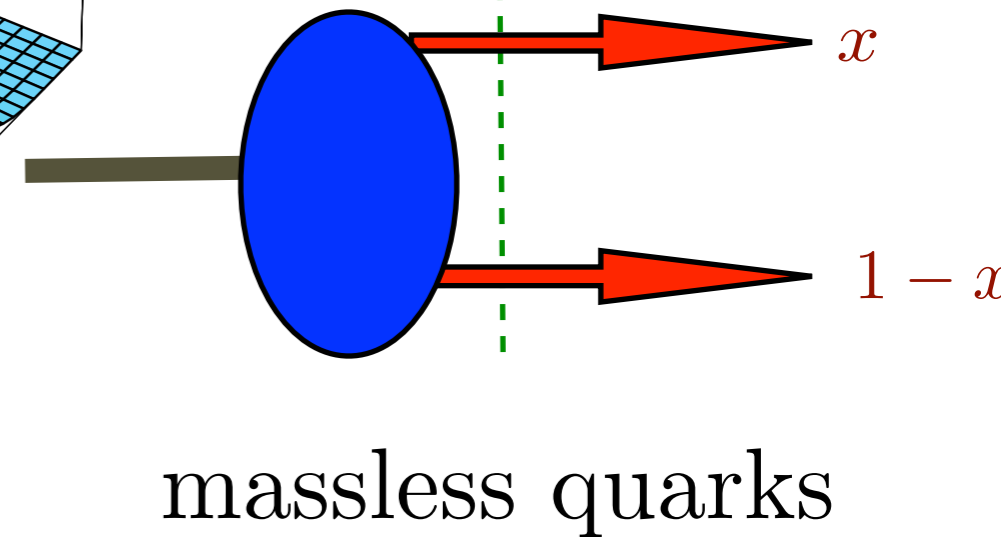
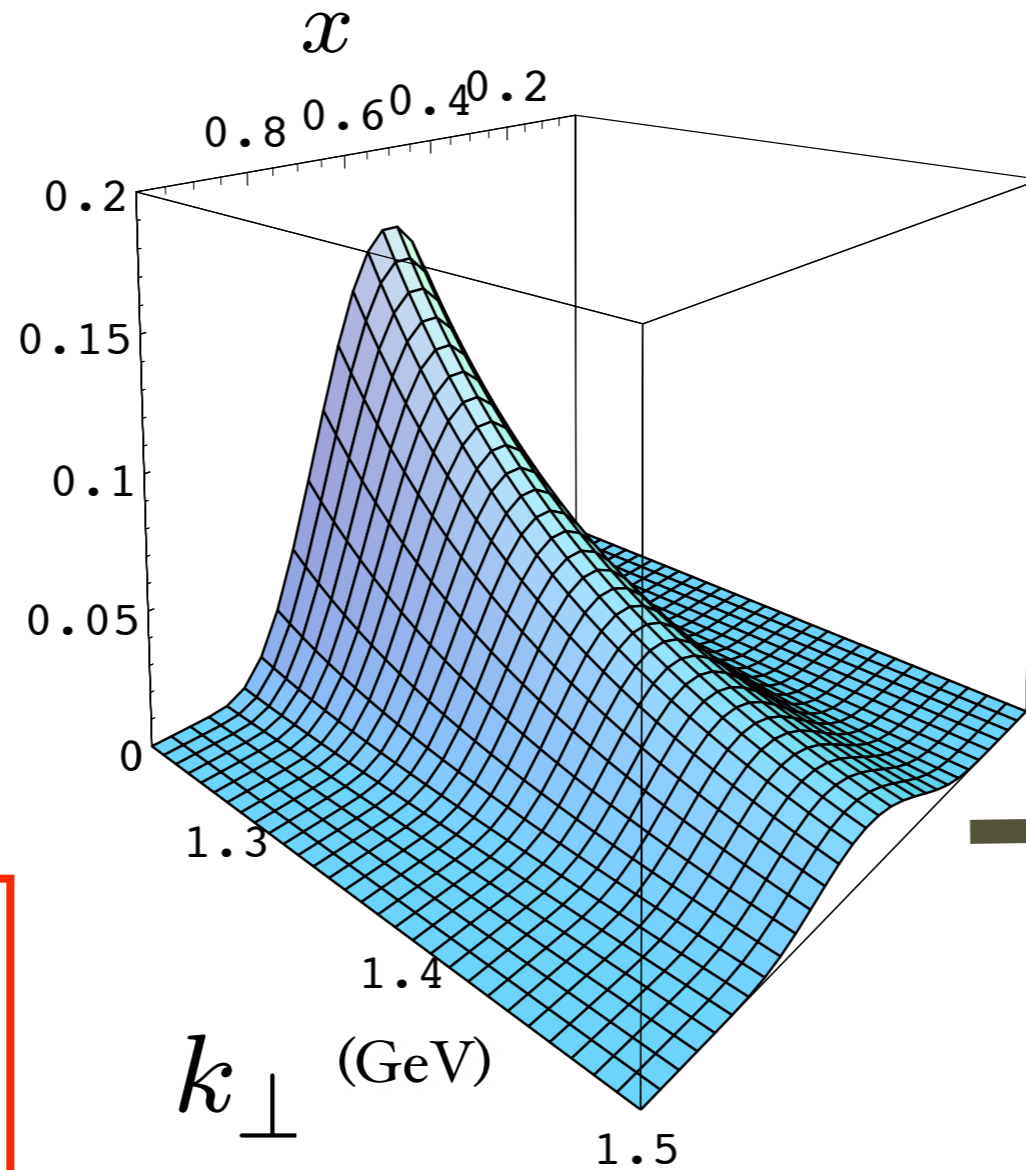
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Provides Connection of Confinement to Hadron Structure

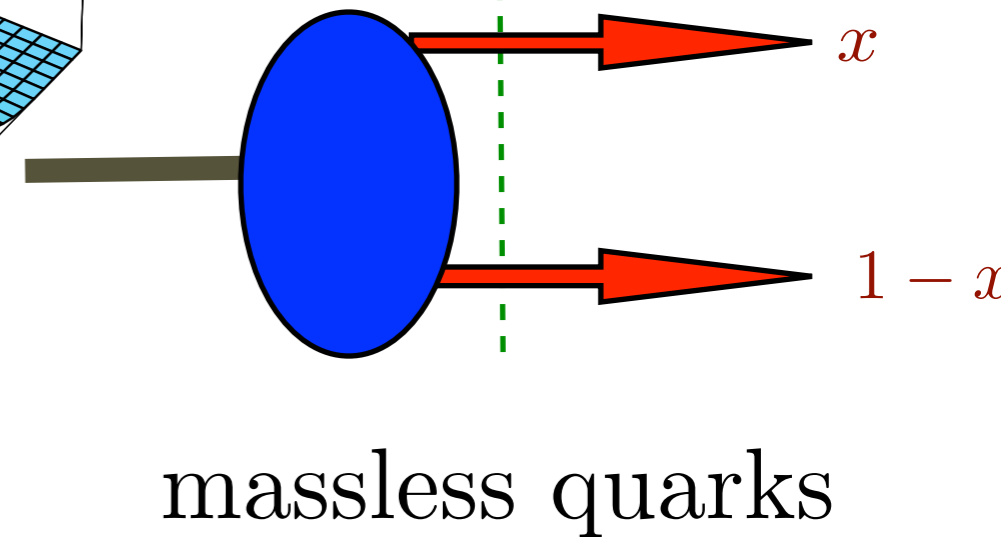
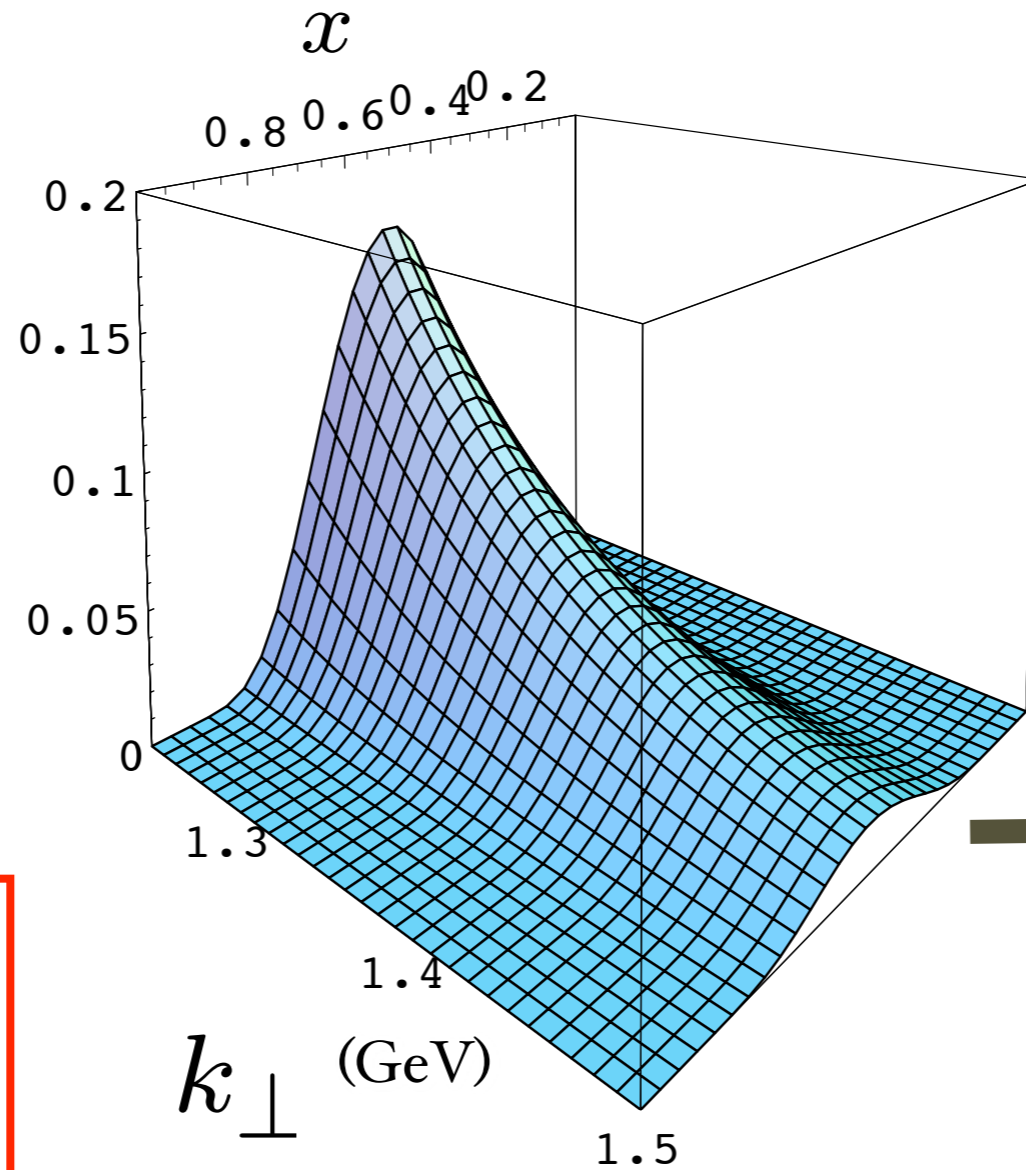
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Provides Connection of Confinement to Hadron Structure

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

J. R. Forshaw*

*Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester,
Oxford Road, Manchester M13 9PL, United Kingdom*

R. Sandapen†

Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A3E9, Canada
(Received 5 April 2012; published 20 August 2012)

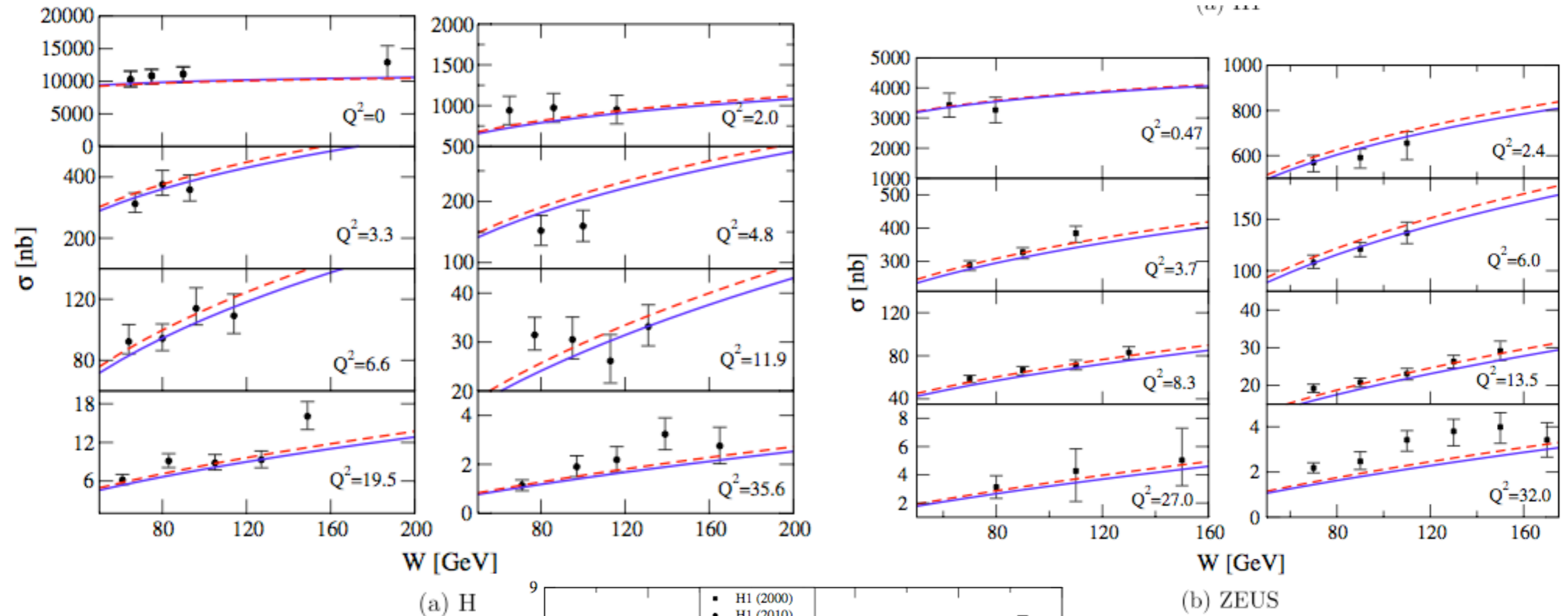
We show that anti-de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ -meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron-proton collider.

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**See also Ferreira
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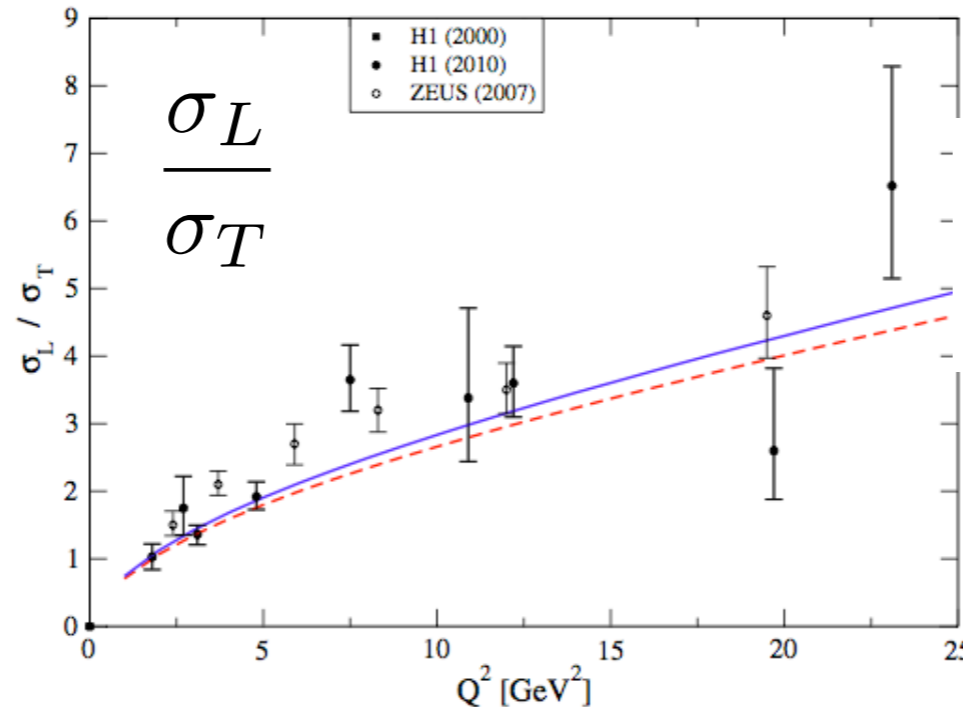
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AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



**J. R. Forshaw,
R. Sandapen**

$$\gamma^* p \rightarrow \rho^0 p'$$



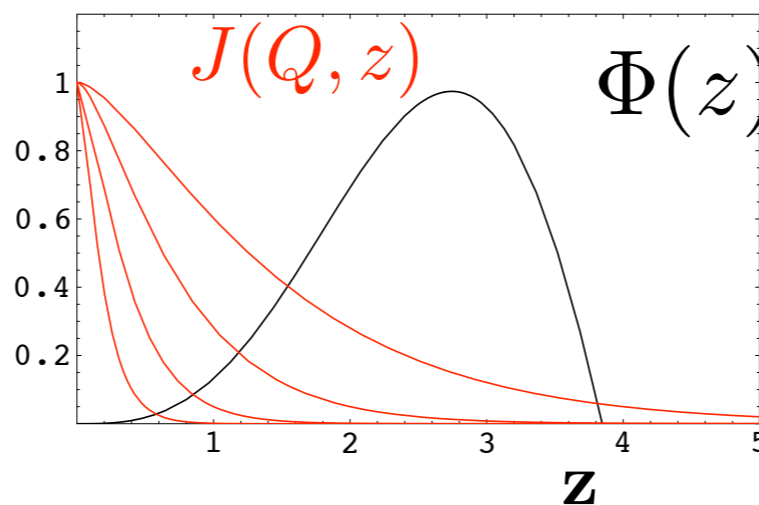
$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right)$$

**See also Ferreira
and Dosch**

Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$



**Polchinski, Strassler
de Teramond, sjb**

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

**Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance**

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$.

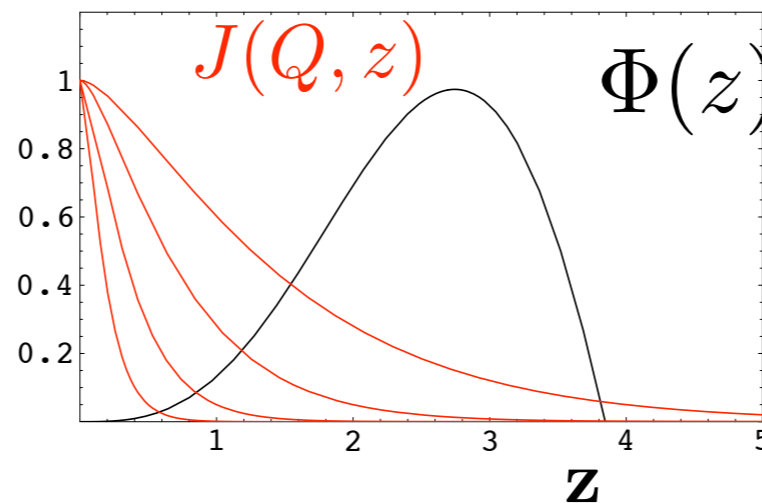
Twist $\tau = n + L$

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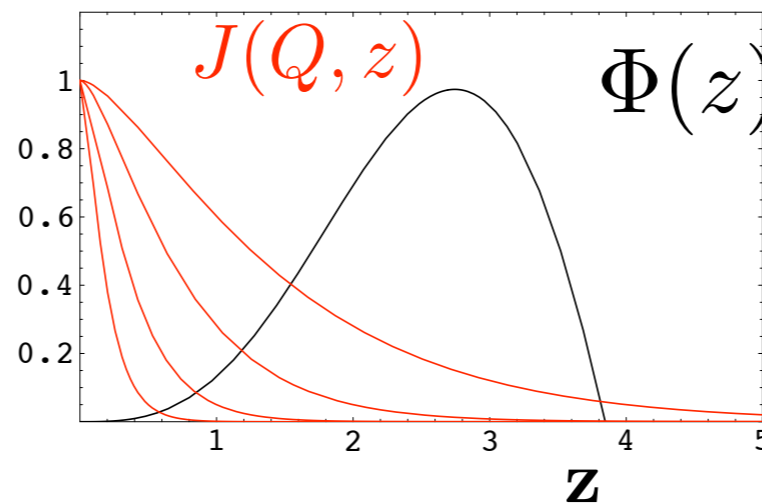
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High Q^2
from
small $z \sim 1/Q$



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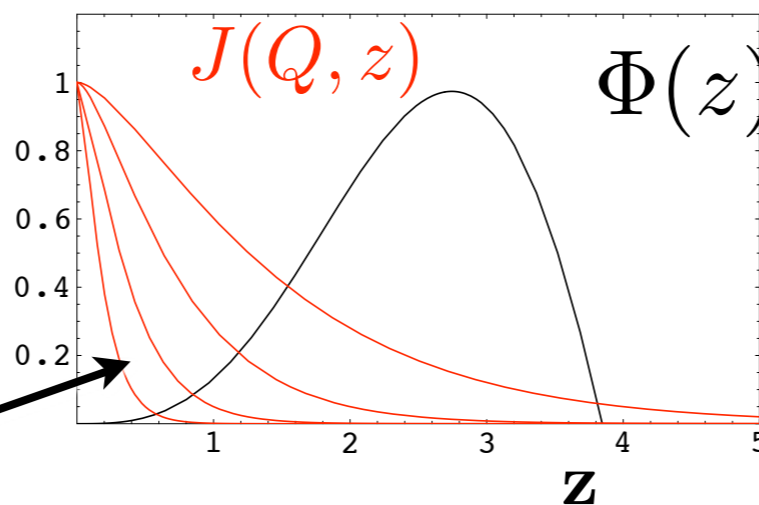
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Holographic Mapping of AdS Modes to QCD LFWFs

*Drell-Yan-West: Form Factors are
Convolution of LFWFs*

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

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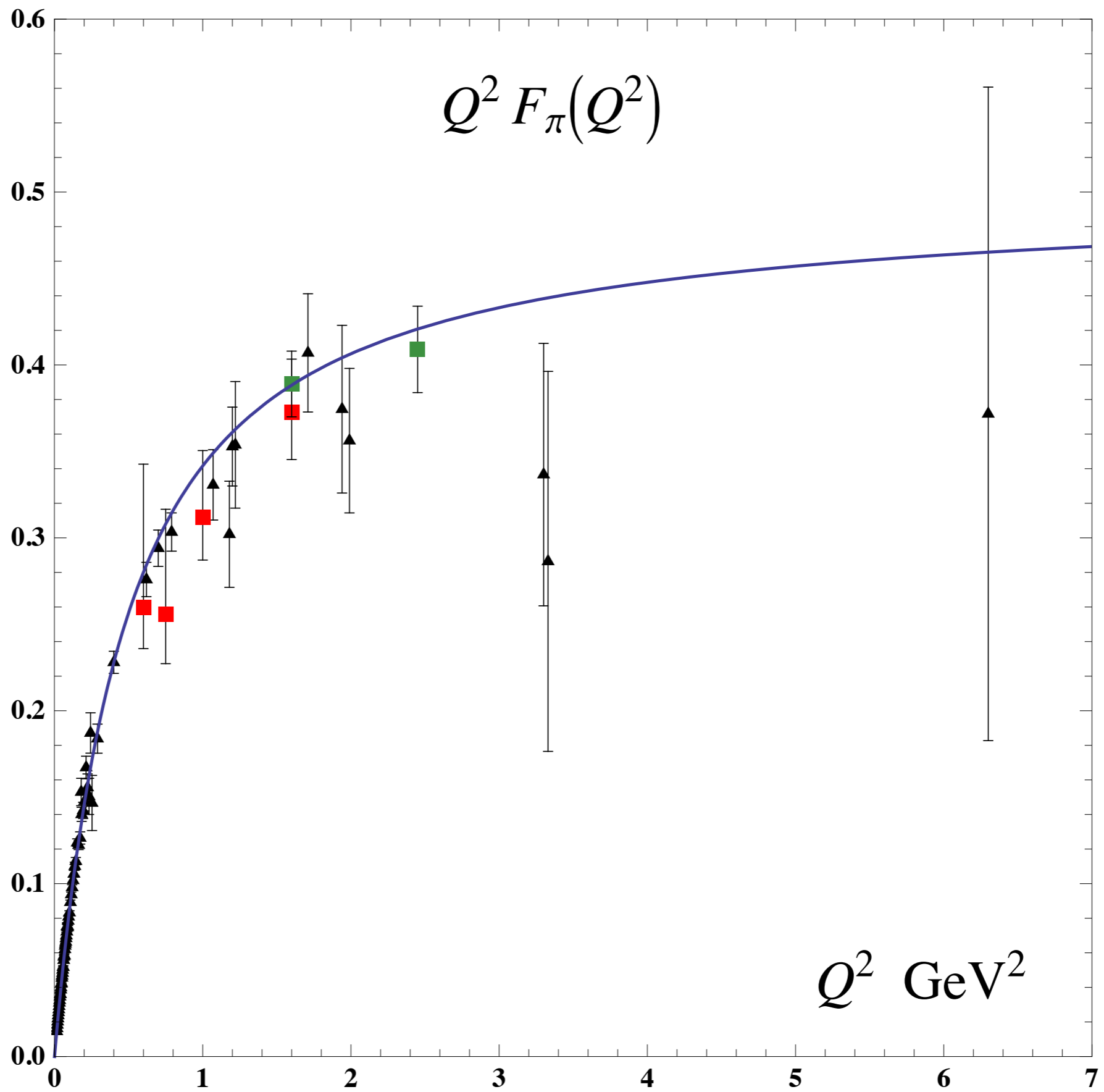
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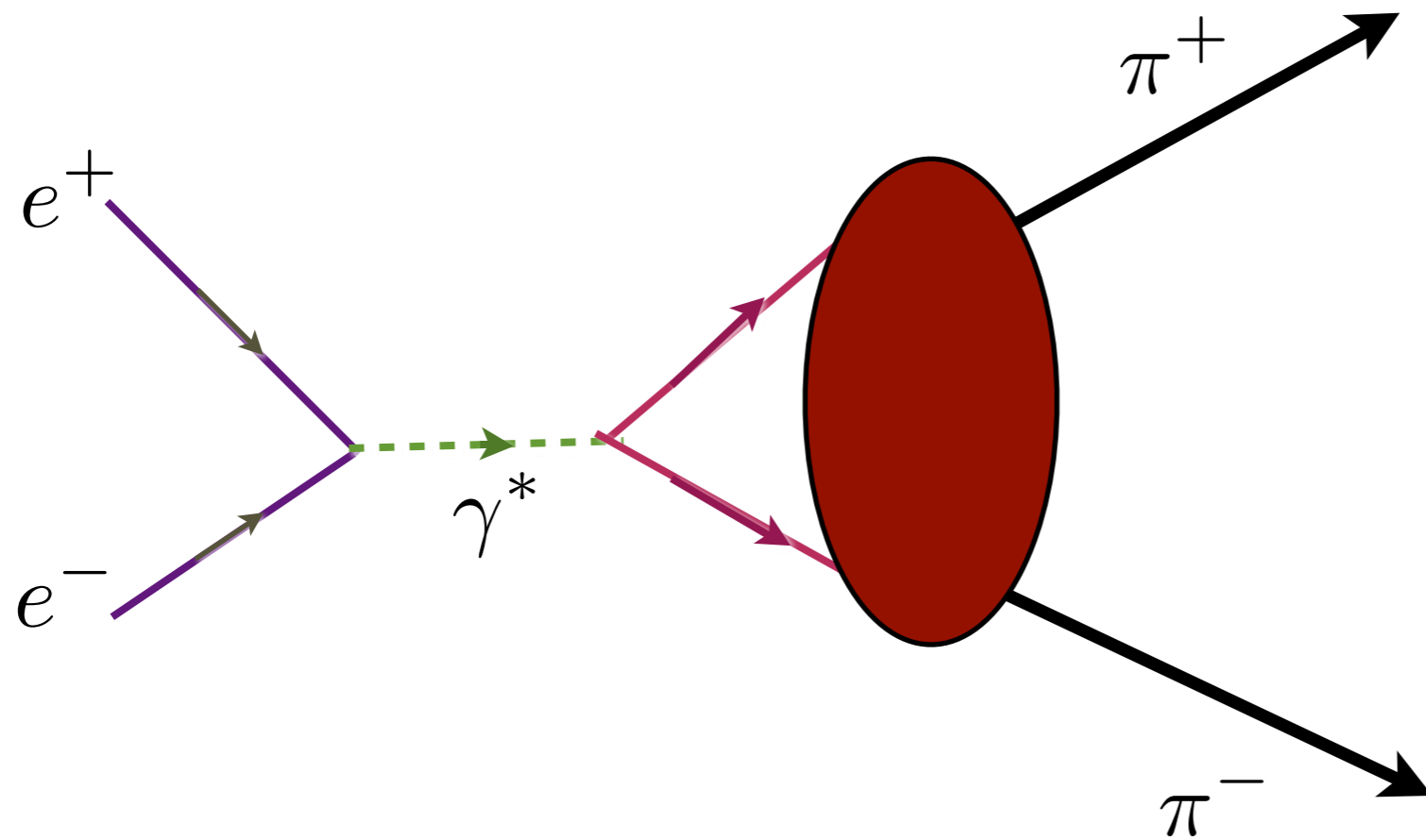
Uniqueness

de Tèramond, Dosch, sjb

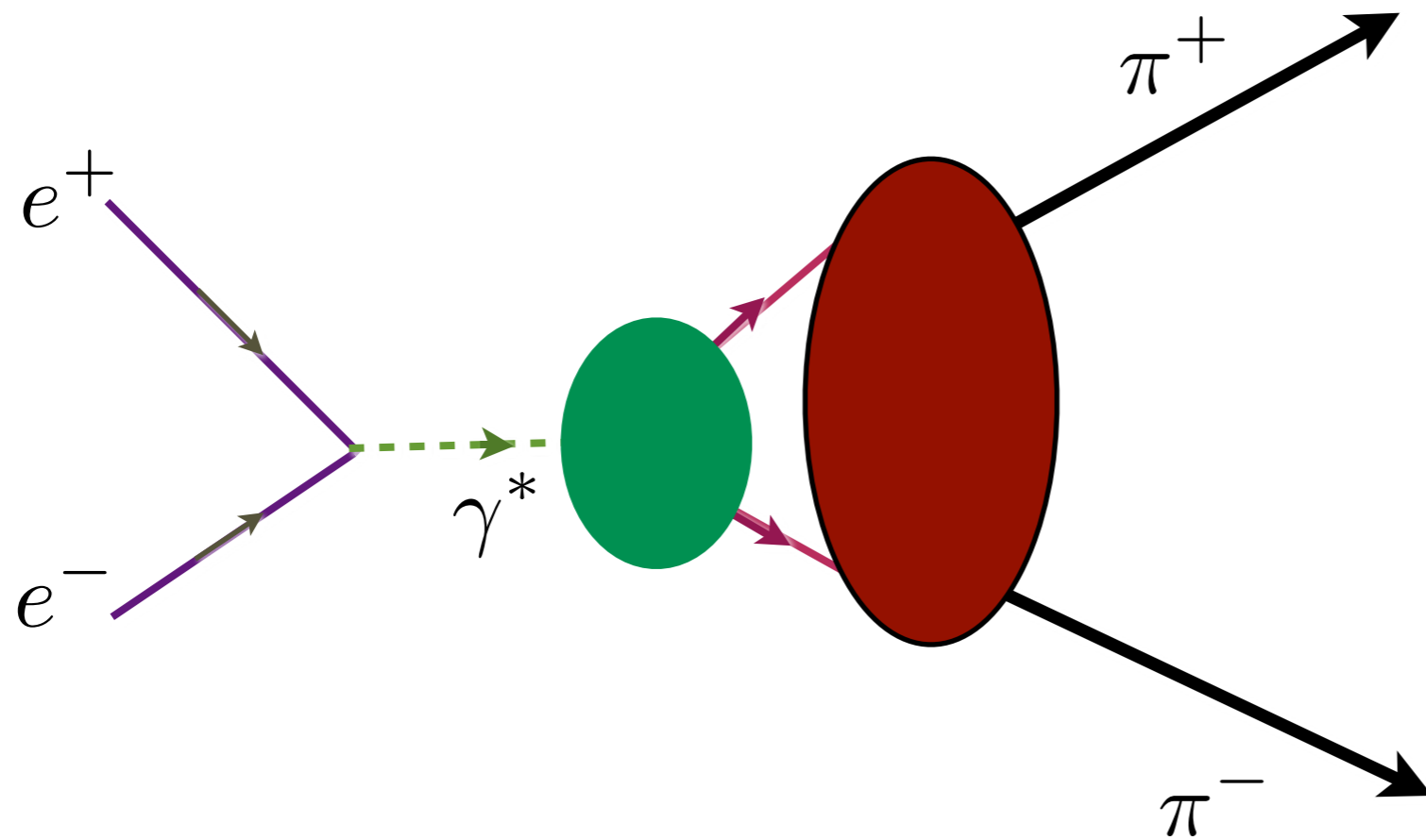
$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \quad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L : same slope!**
- **Massless pion in chiral limit! No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, Furlan, Fubini,
Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976)
569**

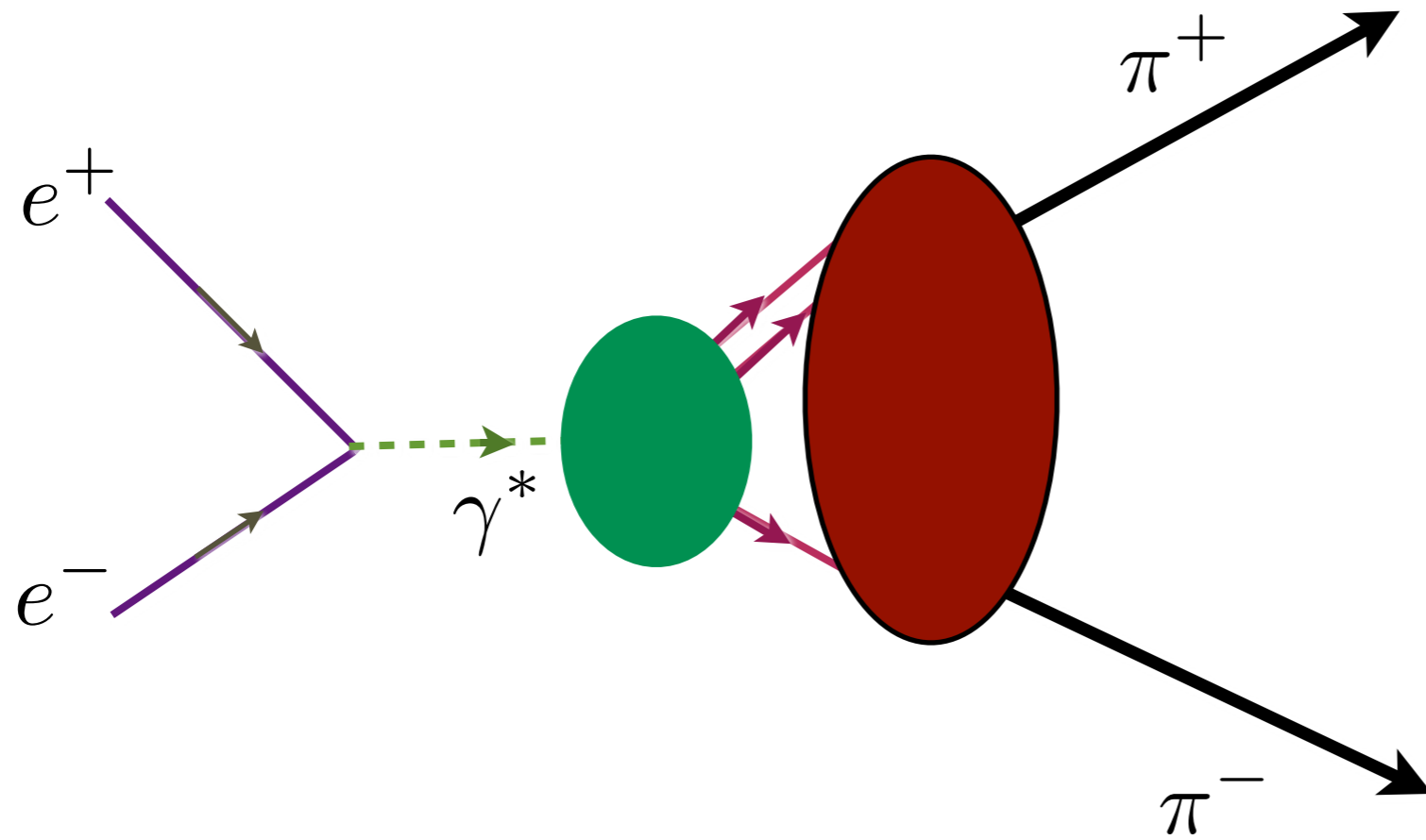
Dressed soft-wall current brings in higher Fock states and more vector meson poles



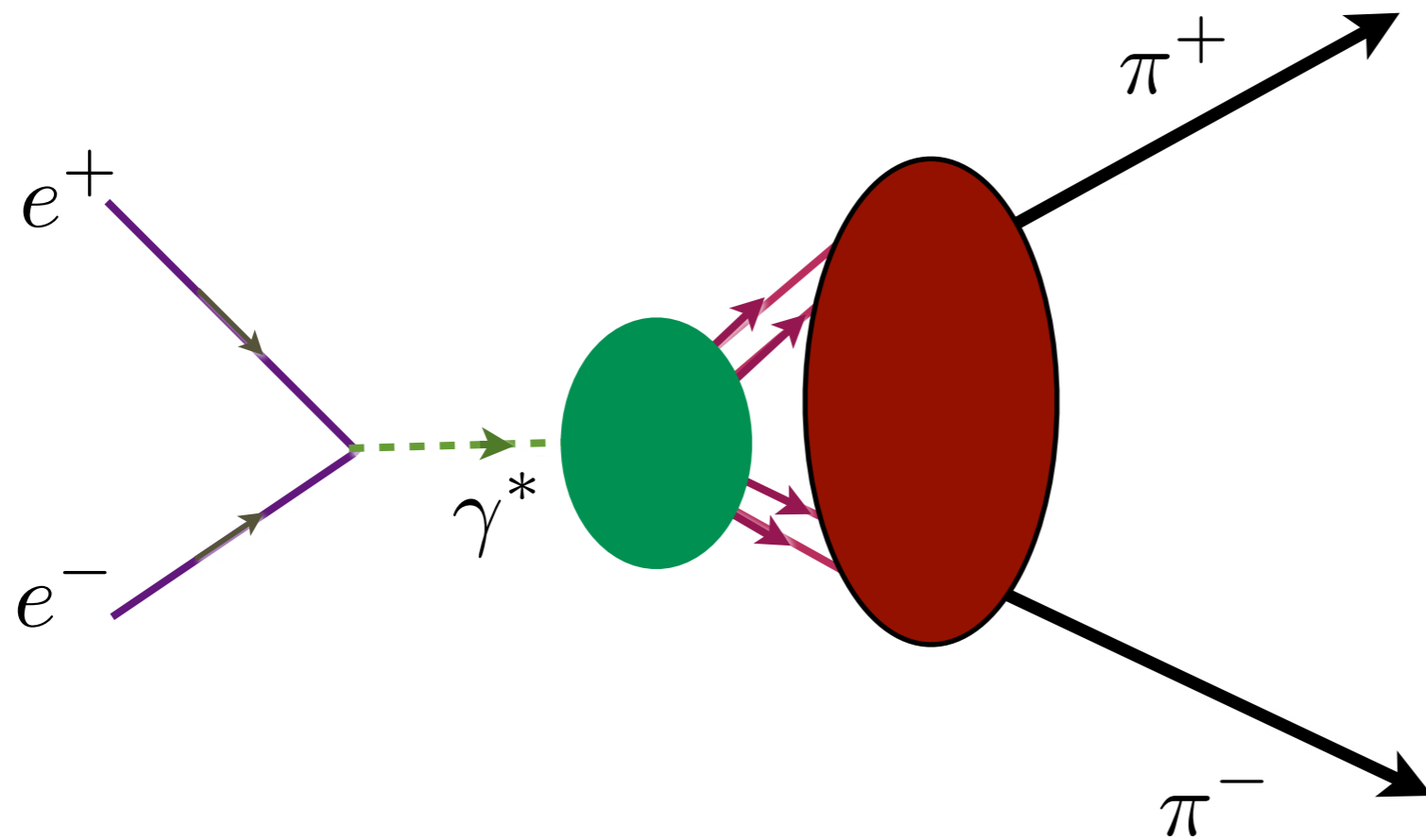
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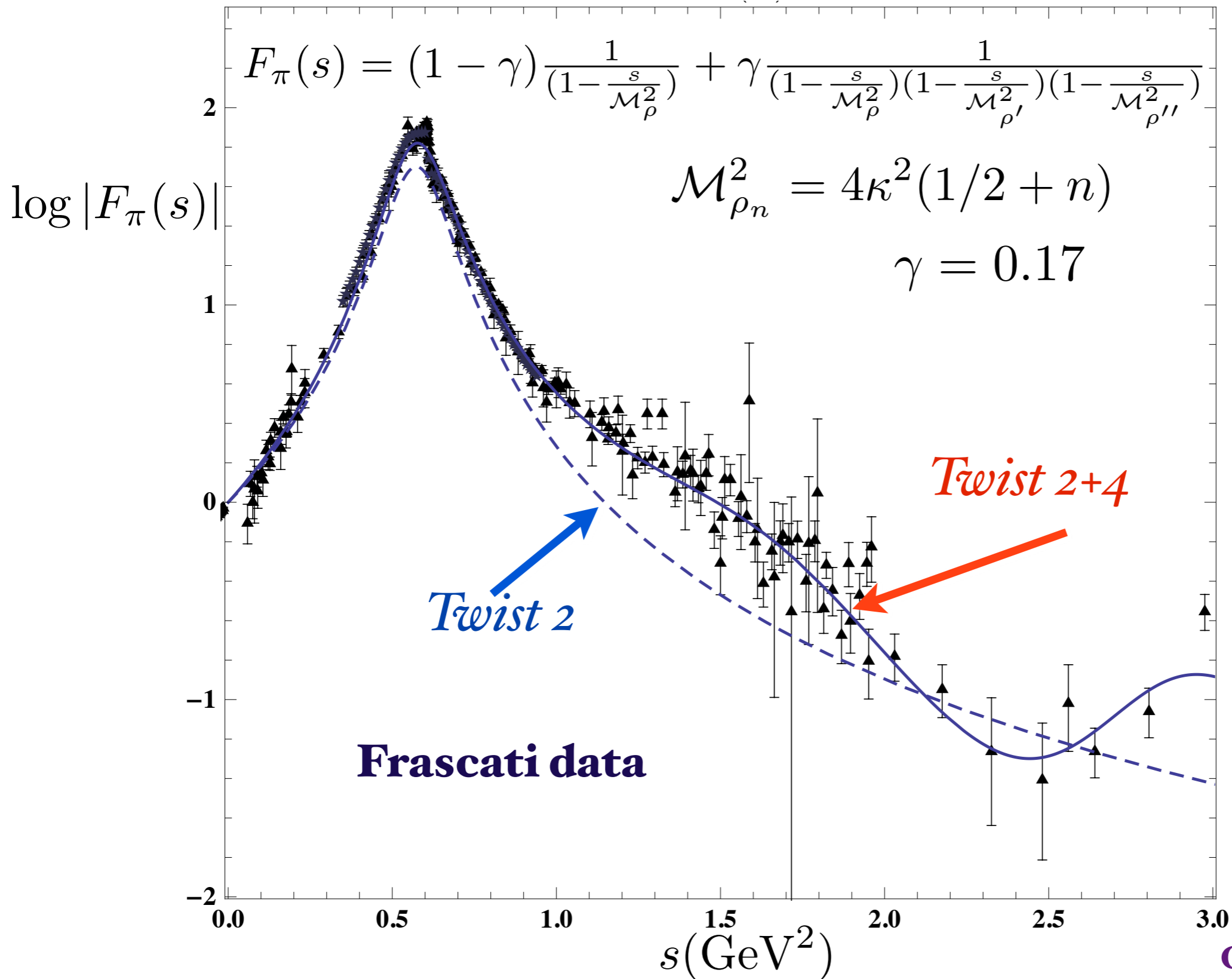
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Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography

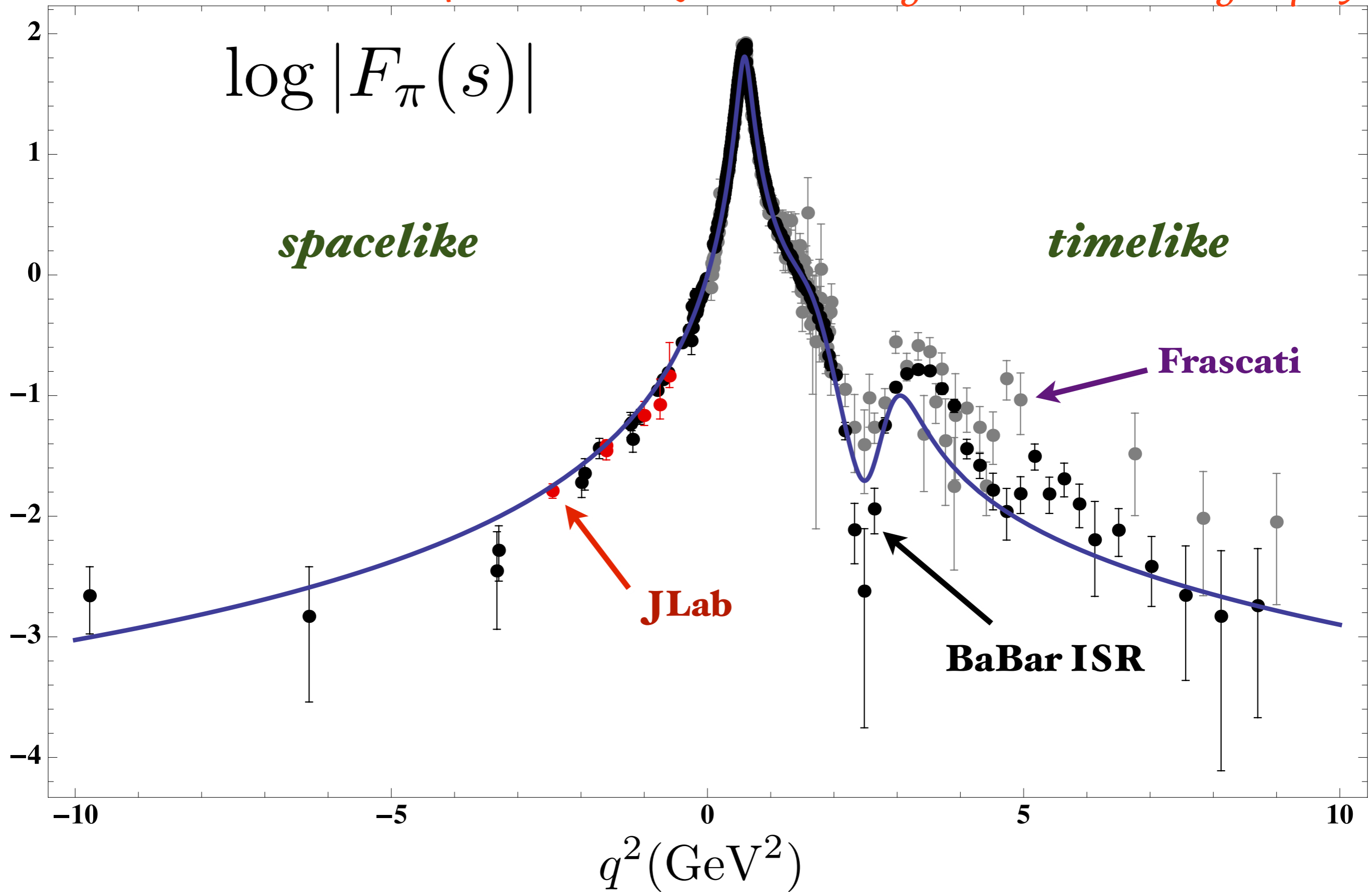


**Prescription for
Timelike poles :**

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

**14% four-quark
probability**

Pion Form Factor from AdS/QCD and Light-Front Holography



Remarkable Features of Light-Front Schrödinger Equation

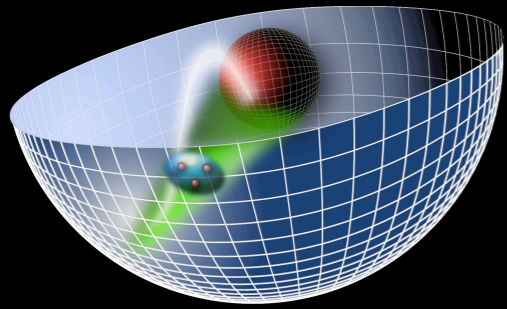
- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

$$\phi(z)$$

AdS₅: Conformal Template for QCD

- *Light-Front Holography*



**Duality of AdS₅ with LF
Hamiltonian Theory**

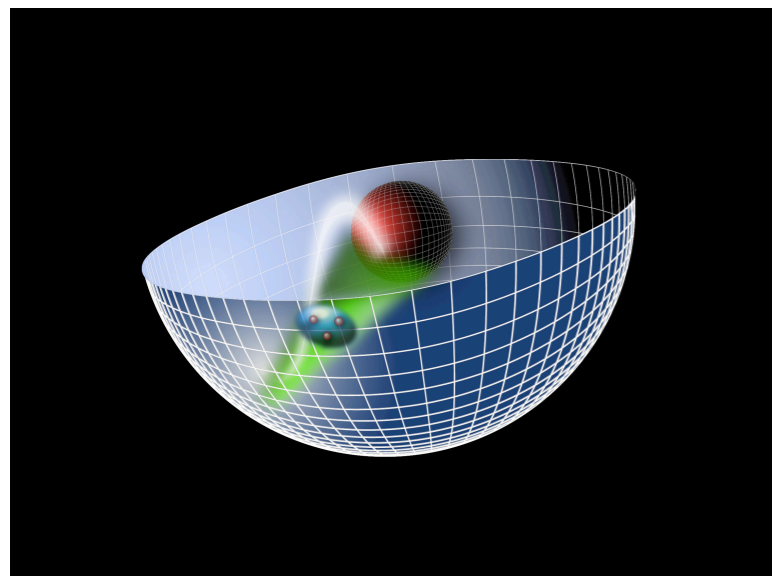
- *Light Front Wavefunctions:*

***Light-Front Schrödinger Equation
Spectroscopy and Dynamics***

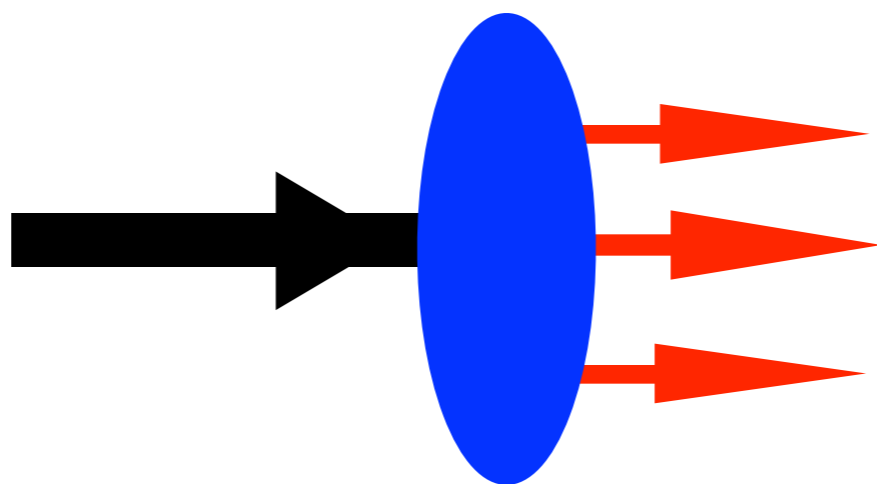
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**Duality of AdS₅ with LF
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$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

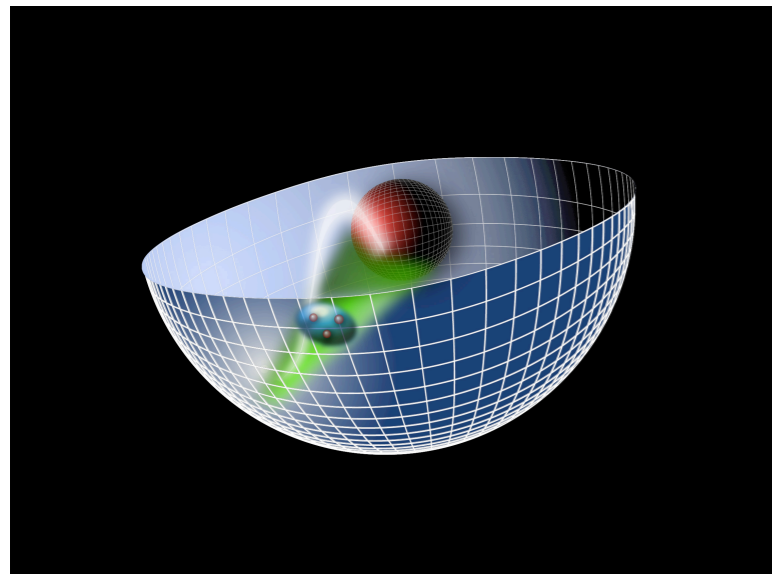
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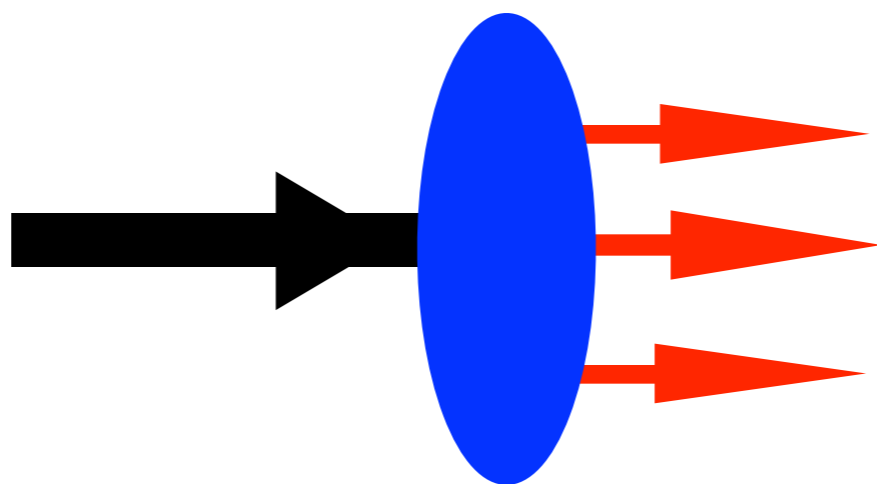
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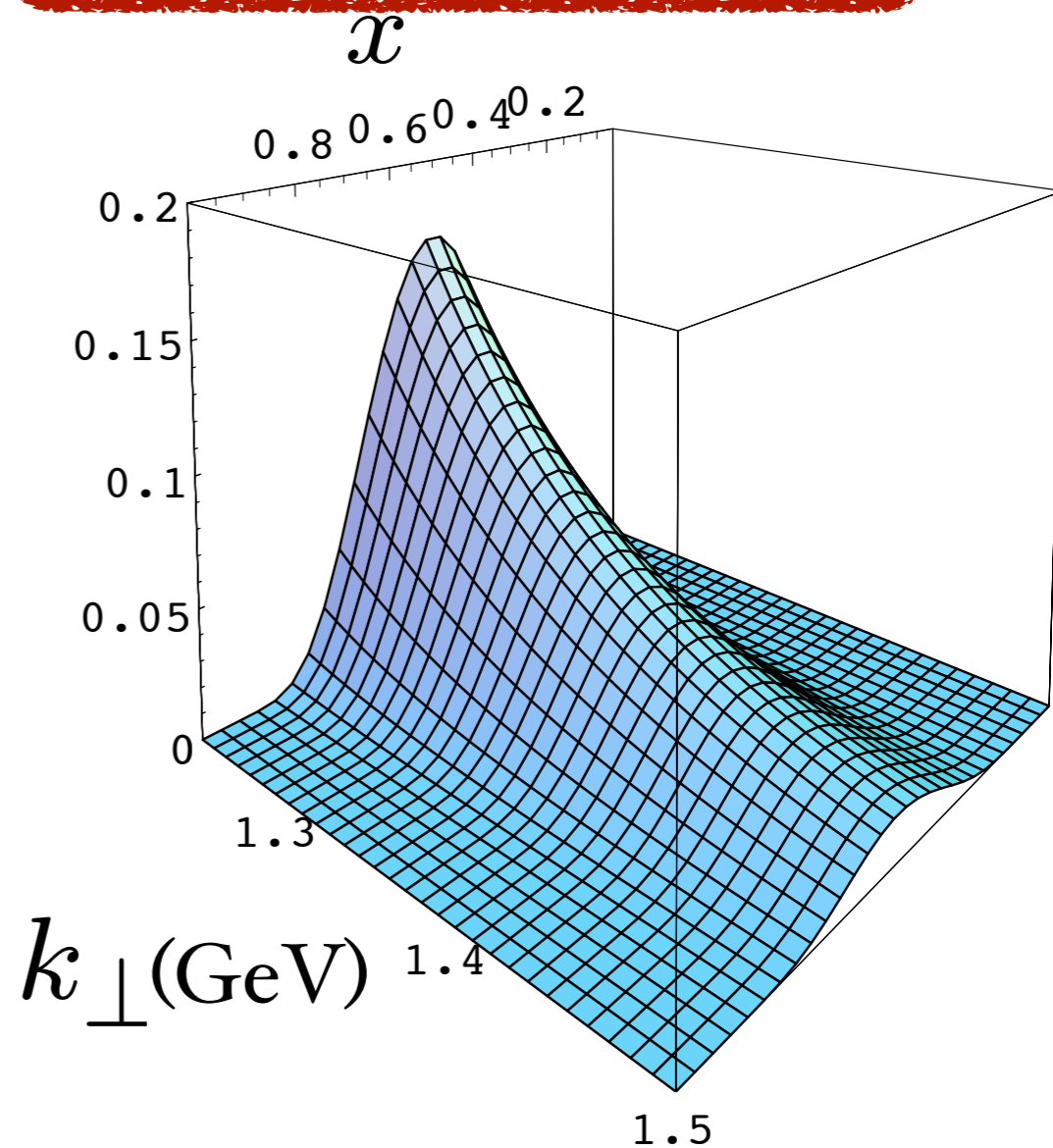
Duality of AdS₅ with LF Hamiltonian Theory



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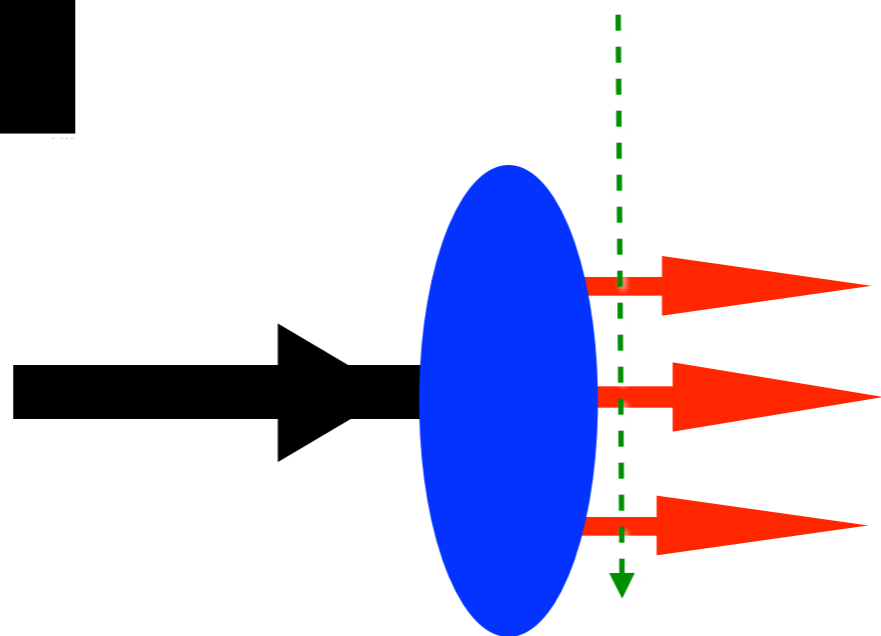
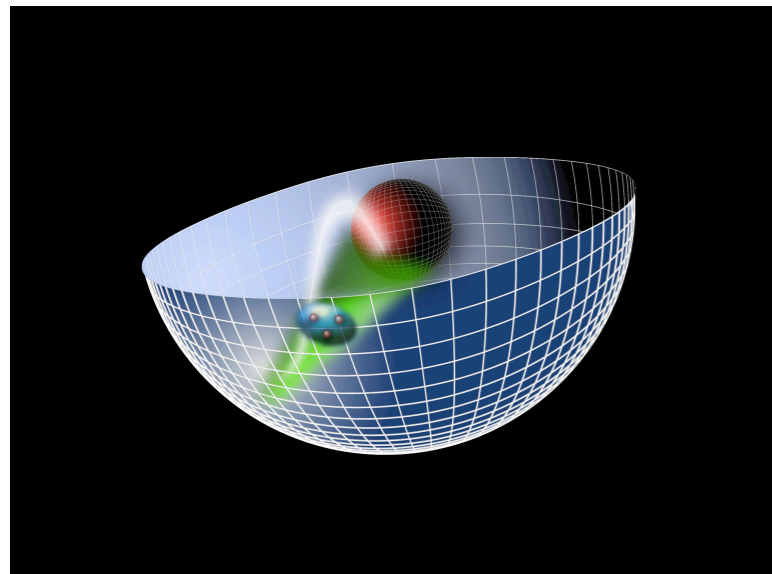
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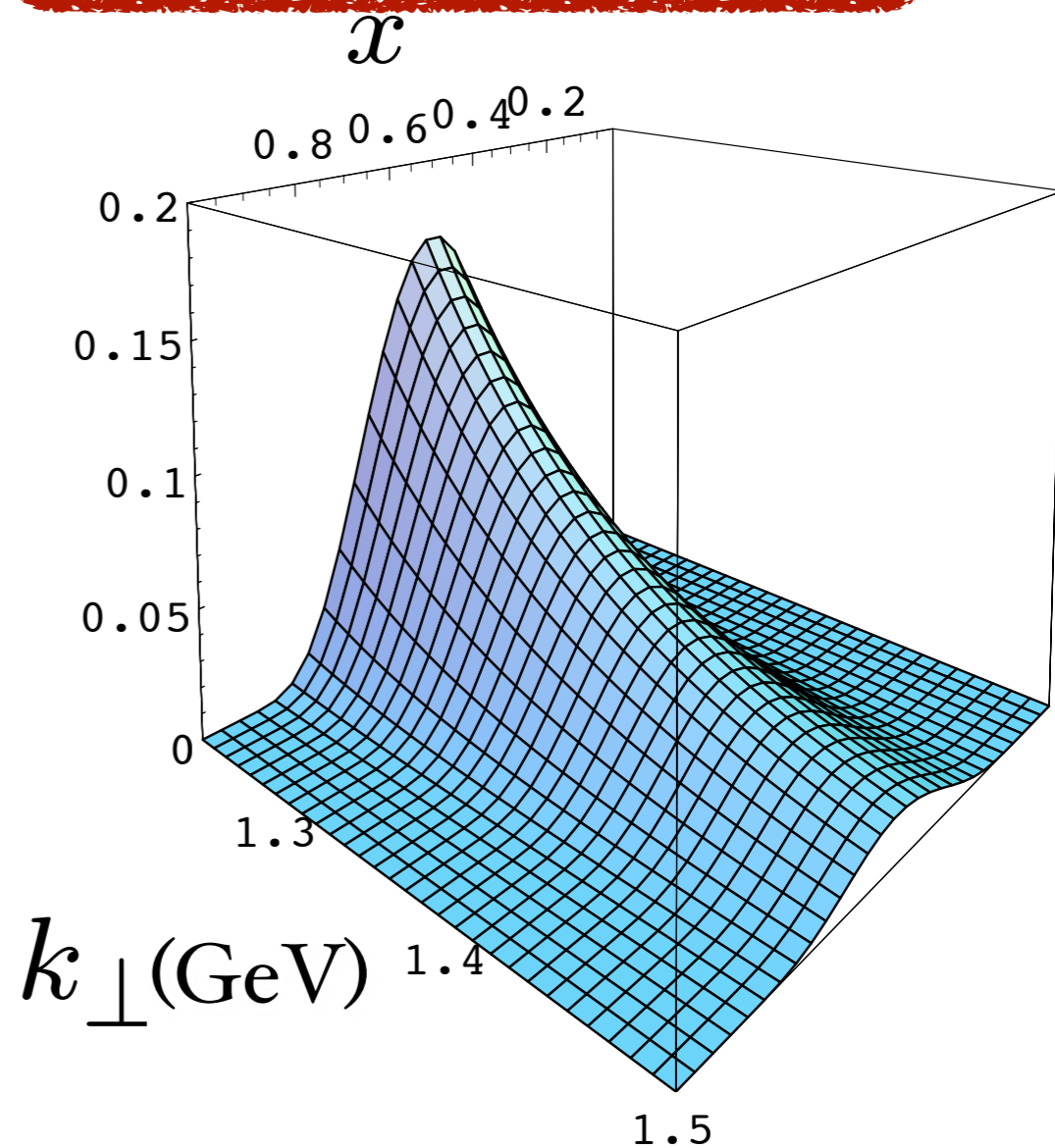
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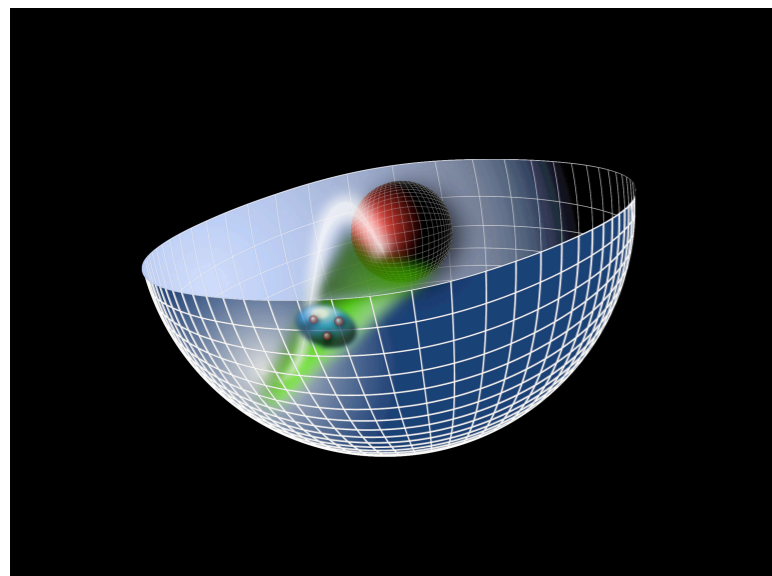
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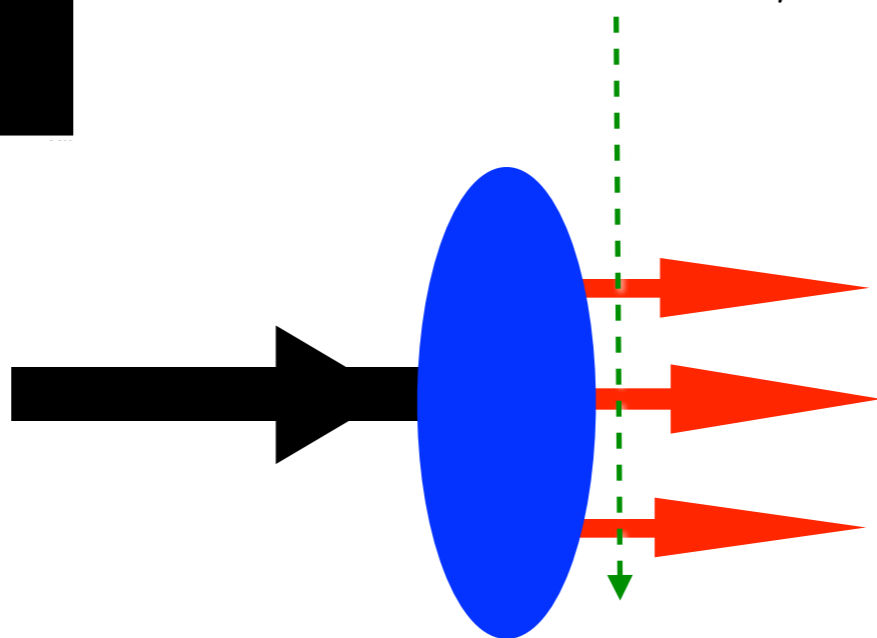
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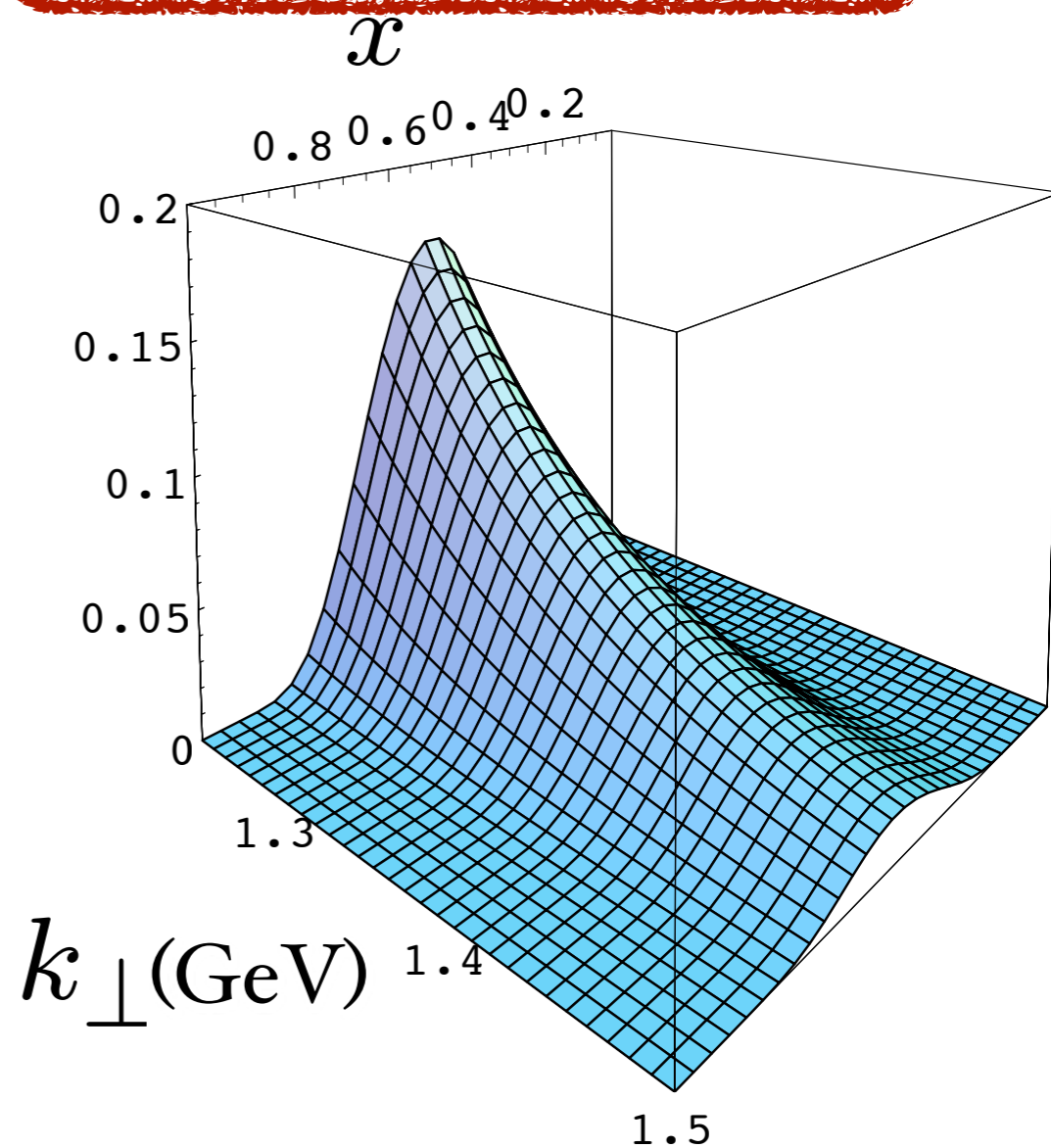


Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Duality of AdS₅ with LF Hamiltonian Theory



- *Light Front Wavefunctions:*

**Light-Front Schrödinger Equation
Spectroscopy and Dynamics**

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

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Classical Chiral Lagrangian is Conformally Invariant

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Where does the QCD Mass Scale Λ_{QCD} come from?

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How does color confinement arise?

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● **de Alfaro, Fubini, Furlan:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Unique confinement potential!

● **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)\rangle = i\frac{\partial}{\partial\tau}|\psi(\tau)\rangle$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

● **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

What determines the QCD mass scale Λ_{QCD} ?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK \quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

LC2015

Frascati INFN

September 25, 2015

Light-Front Holographic QCD, Color Confinement,
and Supersymmetric Features of QCD

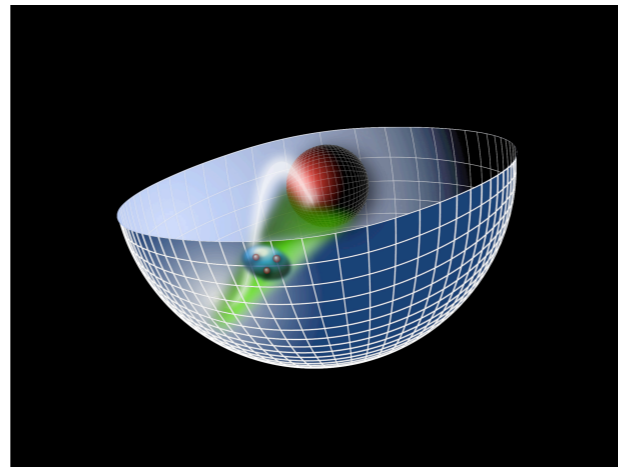
Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan \left(\frac{2tw + v}{\sqrt{4uw - v^2}} \right),$$

- **Identify with difference of LF time $\Delta x^+ / P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**



*AdS/QCD
Soft-Wall Model*

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Light-Front Schrödinger Equation

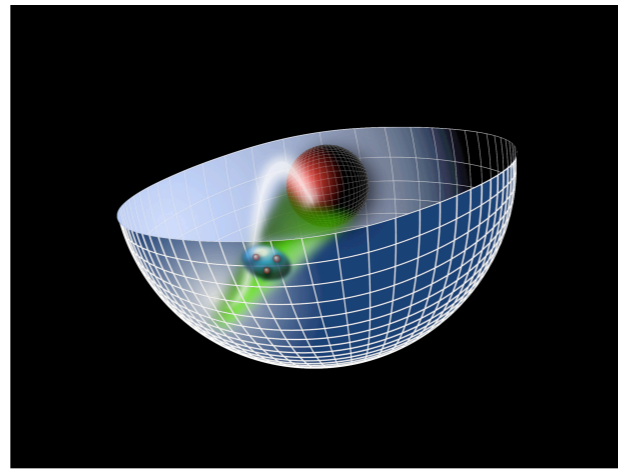


Confinement scale:

$$\kappa \simeq 0.6 \text{ GeV}$$

$$1/\kappa \simeq 1/3 \text{ fm}$$

*AdS/QCD
Soft-Wall Model*



Light-Front Holography

$$\zeta^2 = x(1-x)b_{\perp}^2.$$

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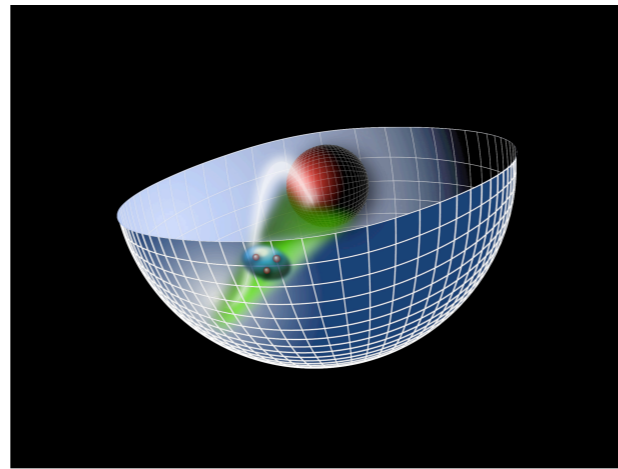
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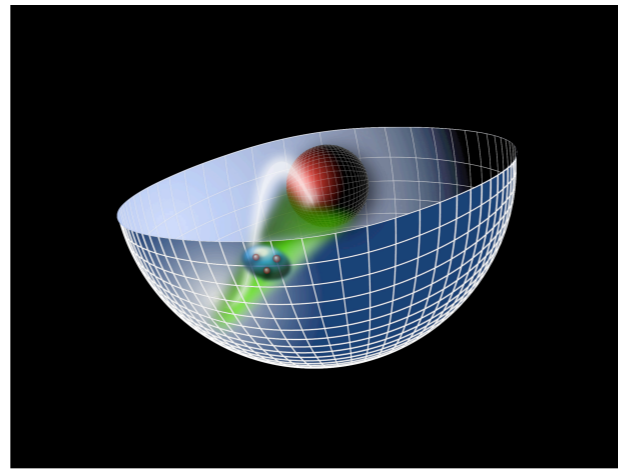
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*Conformal Symmetry
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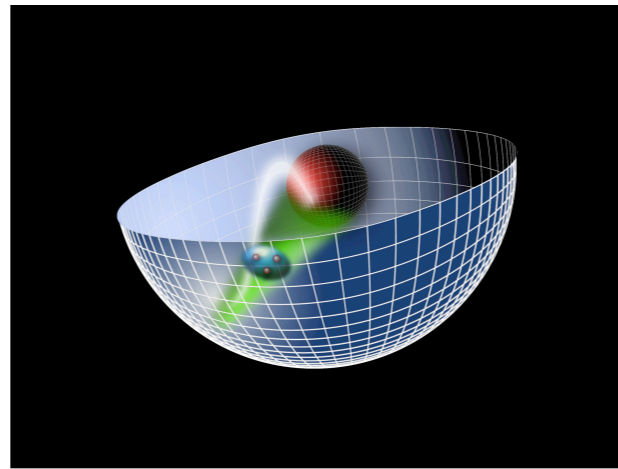
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***Unique
Confinement Potential!
Conformal Symmetry
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***Unique
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Conformal Symmetry
of the action***

● **de Alfaro, Fubini, Furlan:**

**Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!**

Interpretation of Mass Scale \mathcal{K}

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of \mathcal{K}
- Value of \mathcal{K} itself not determined -- place holder
- Need external constraint such as f_π
- “Zero-Parameter” Model

1+1

$$\{\psi, \psi^+\} = 1$$

*two anti-commuting
fermionic operators*

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Realization as Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \quad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

1+1

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$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

Superconformal Algebra

Baryon Equation

Consider $R_w = Q + wS$; w : dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \quad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right)$$

$$\text{Identify } f - \frac{1}{2} = L_B, \quad w = \kappa^2$$

$$\text{Eigenvalue of } G: M^2(n, L) = 4\kappa^2(n + L_B + 1)$$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+$$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

LF Holography

Baryon Equation

Superconformal Algebra

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Same κ !

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Same κ !

S=0, I=I Meson is superpartner of S=1/2, I=I Baryon

LF Holography

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Meson Equation

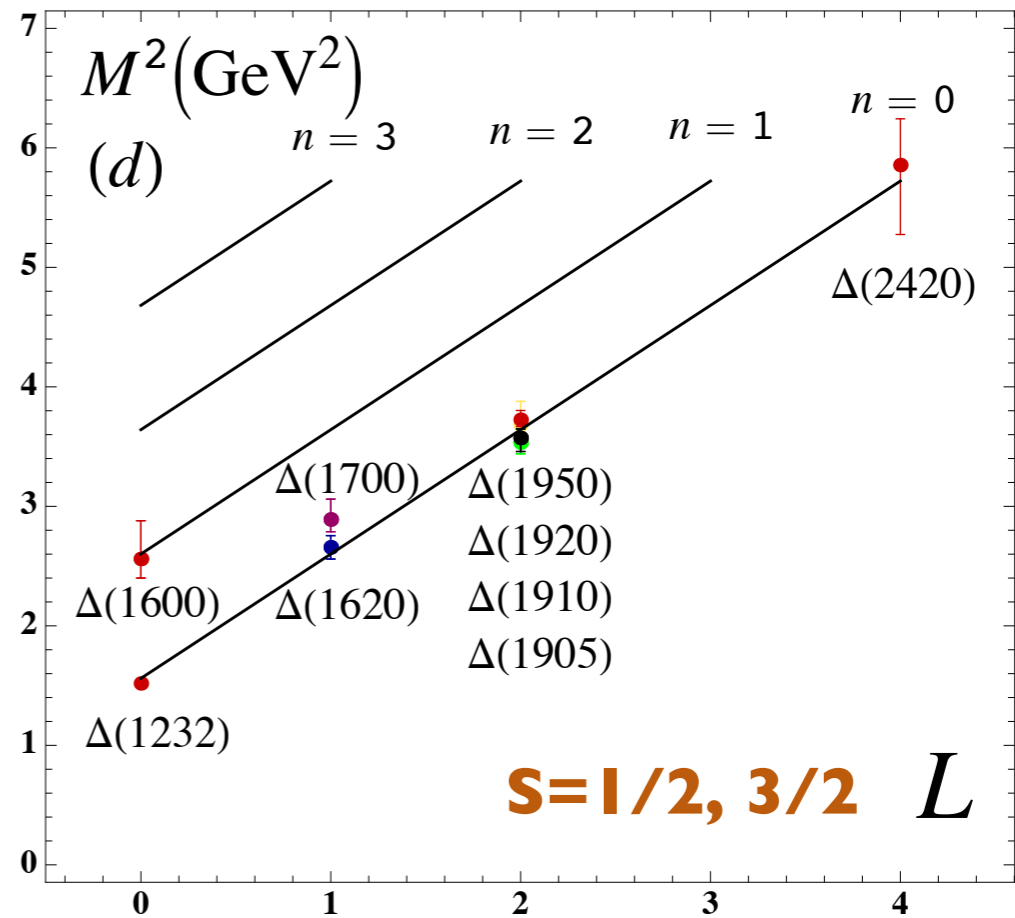
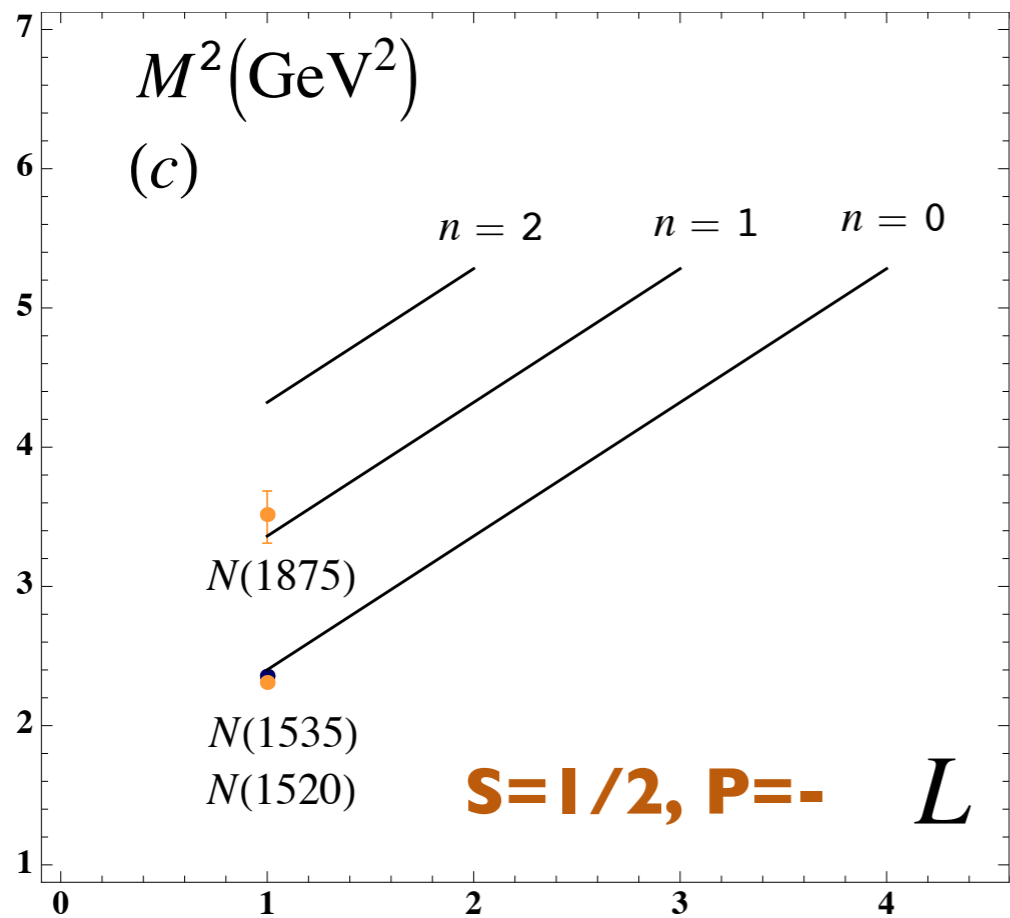
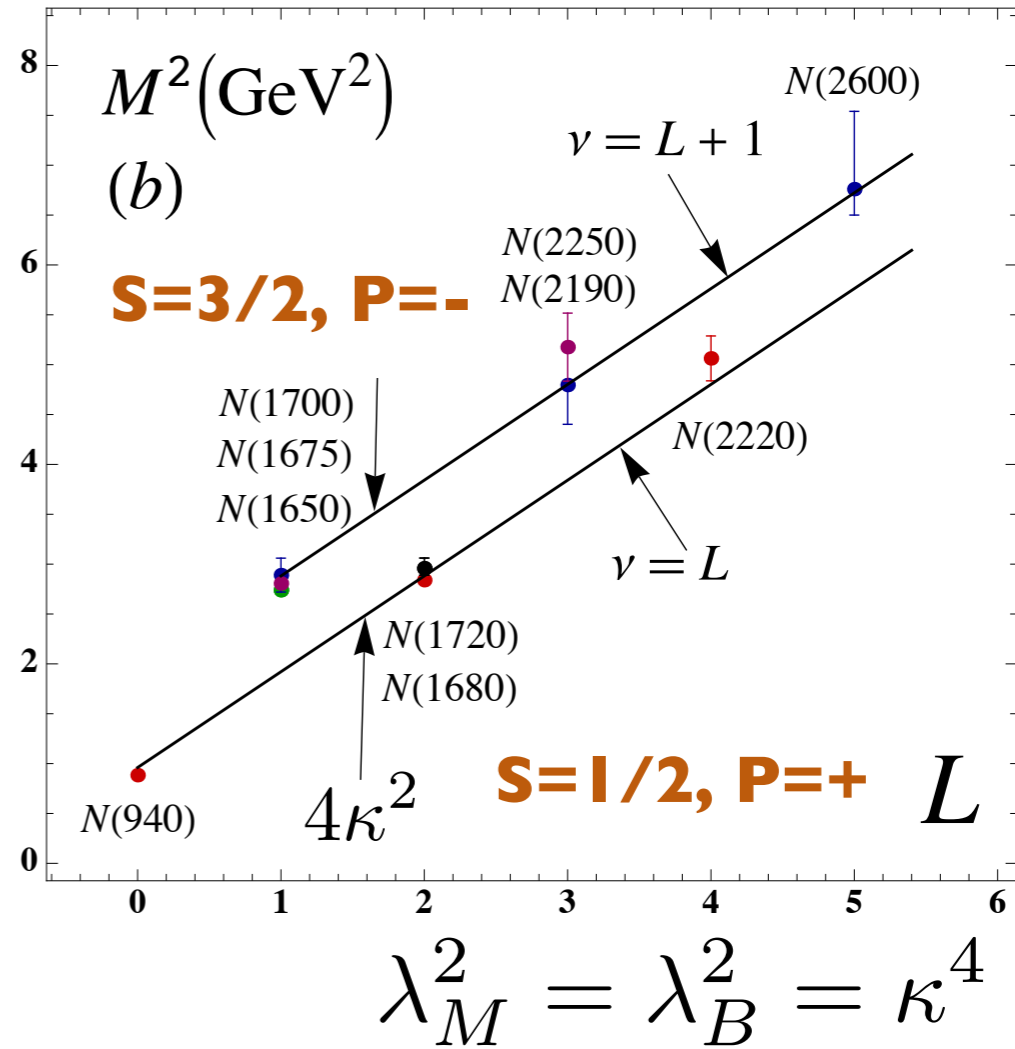
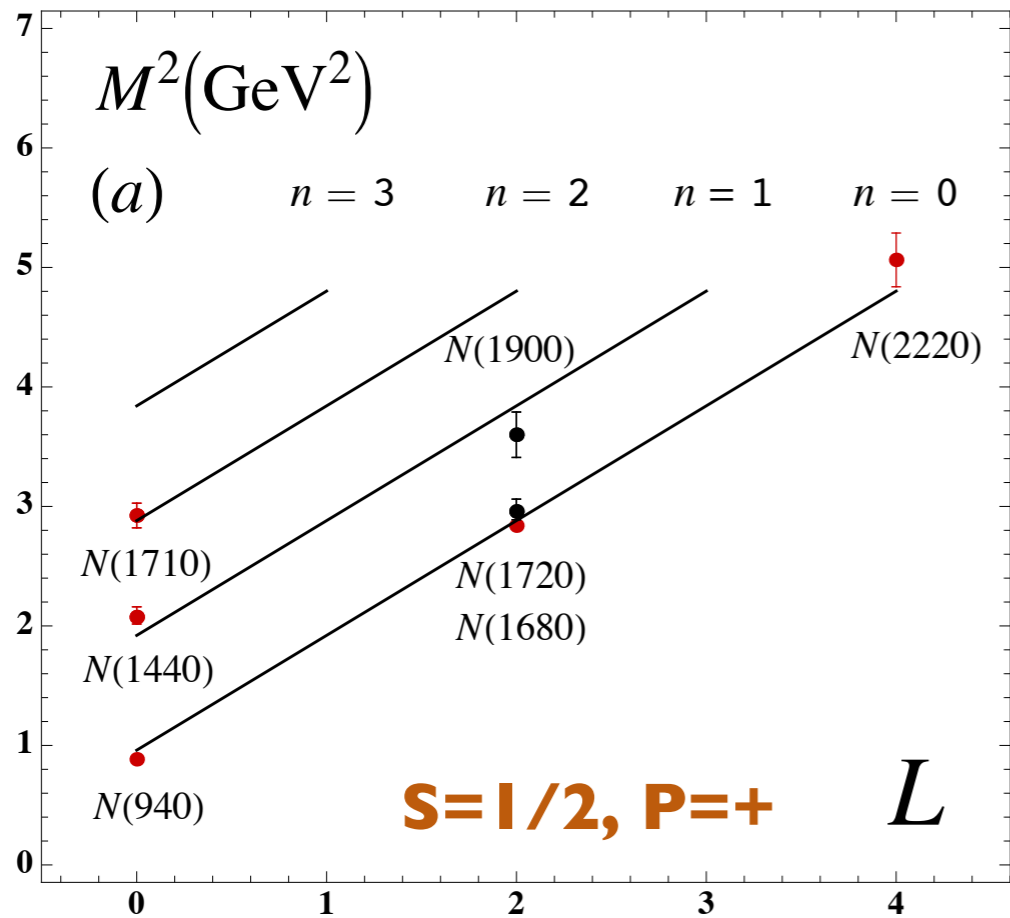
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Same κ !

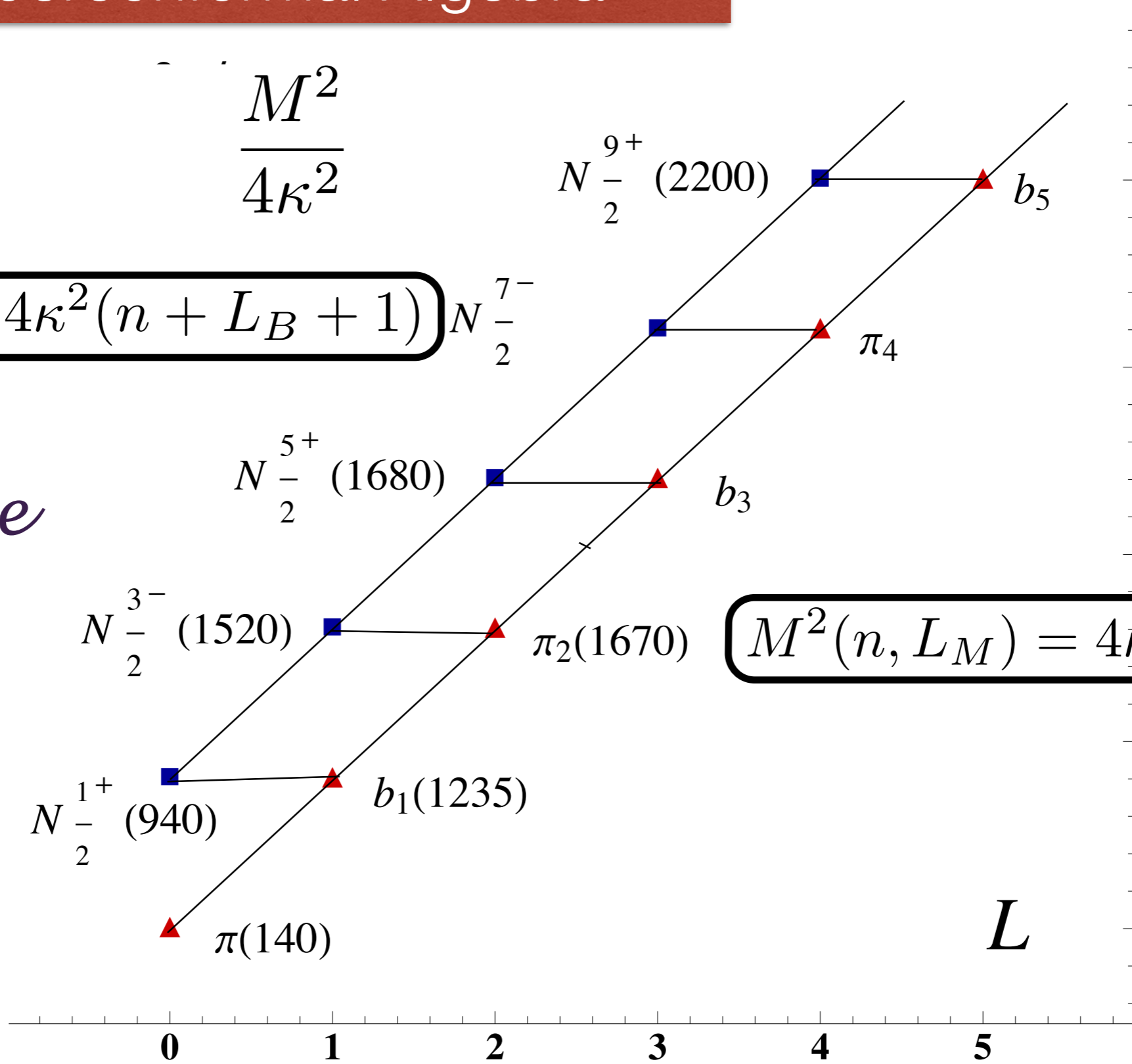
S=0, I=I Meson is superpartner of S=1/2, I=I Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

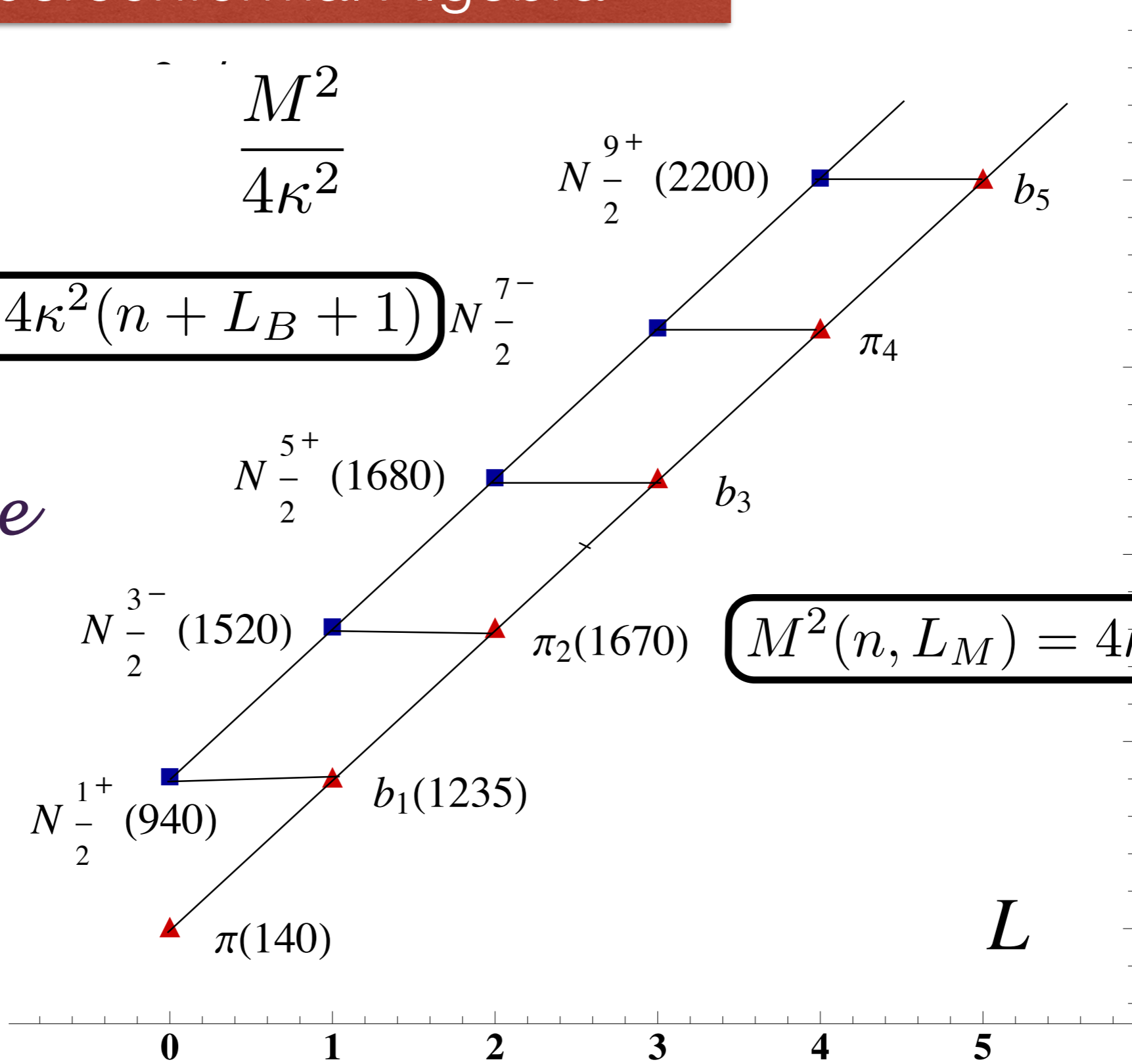


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

**Meson-Baryon
Mass Degeneracy
for $L_M=L_B+1$**

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

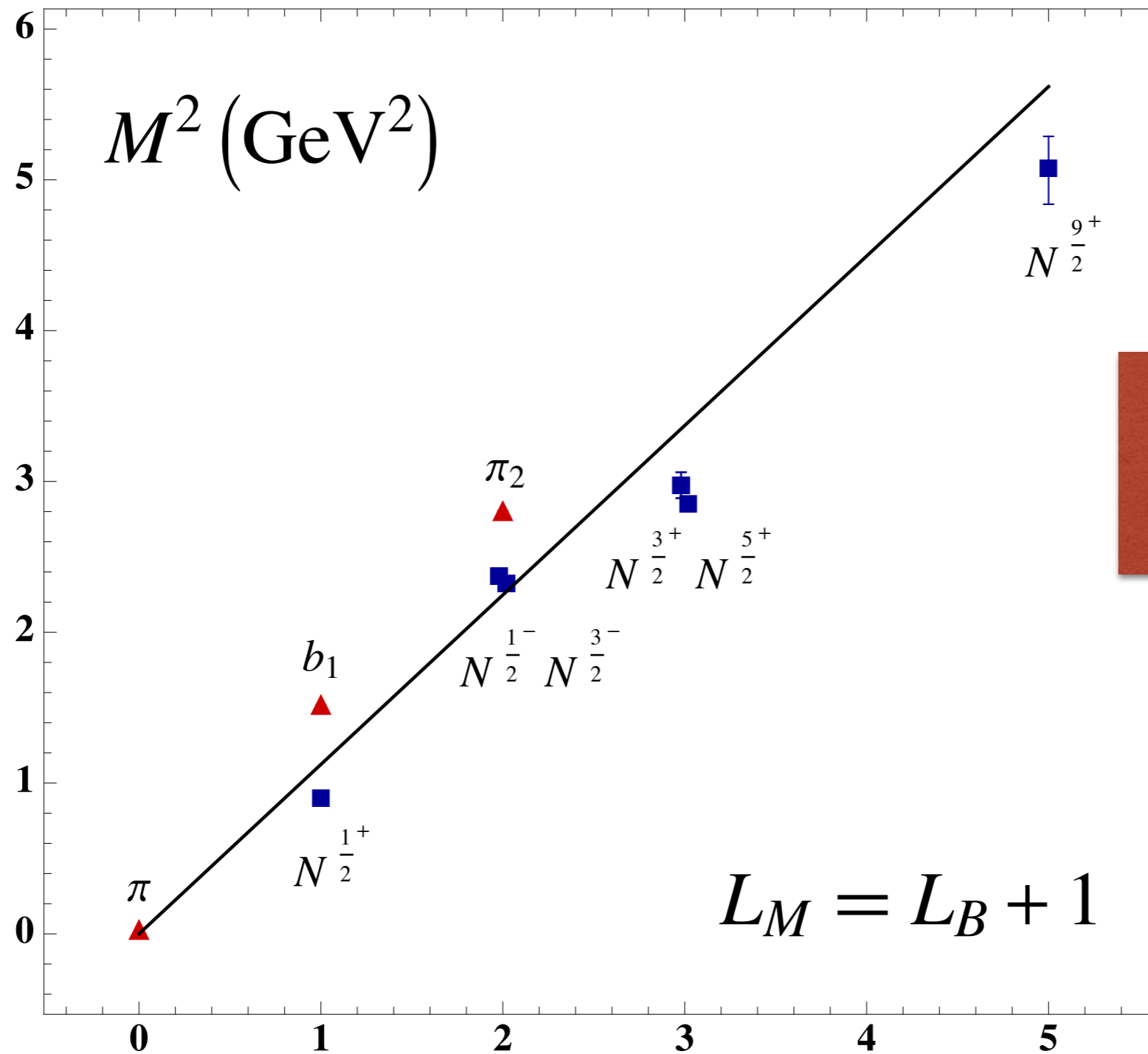


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

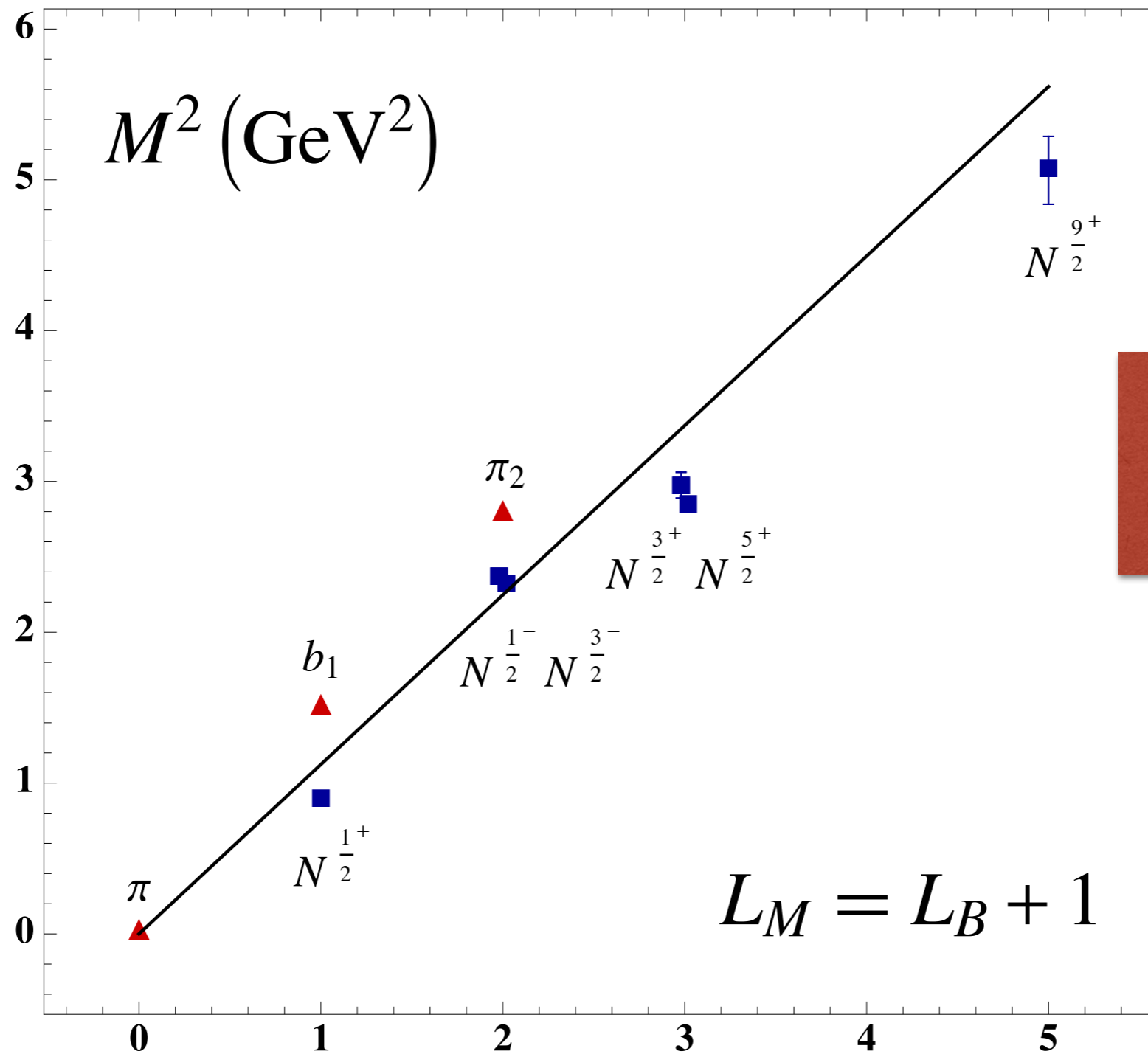
**Meson-Baryon
Mass Degeneracy
for $L_M=L_B+1$**

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



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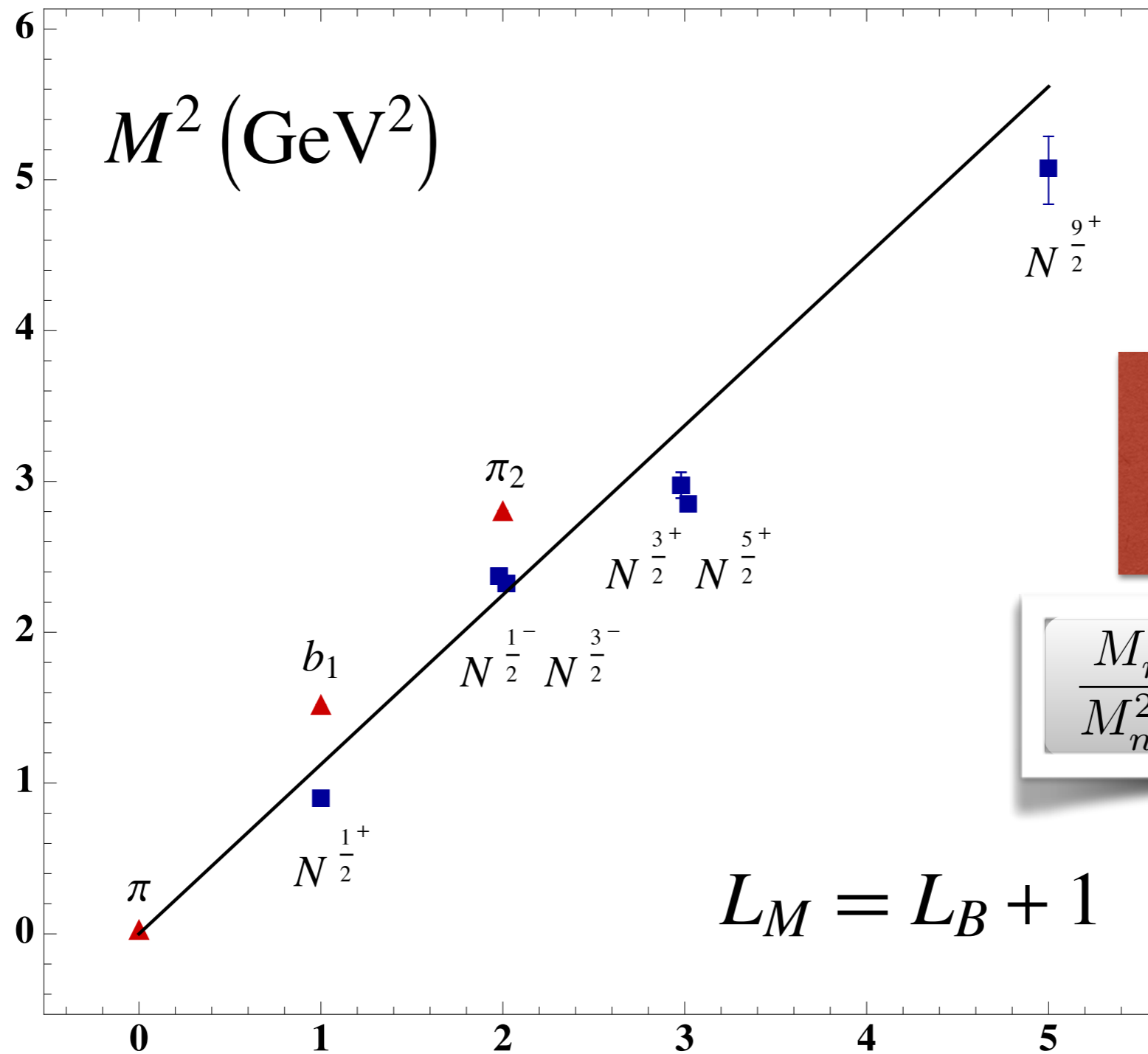
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$S=0, I=I$ Meson is superpartner of $S=1/2, I=I$ Baryon

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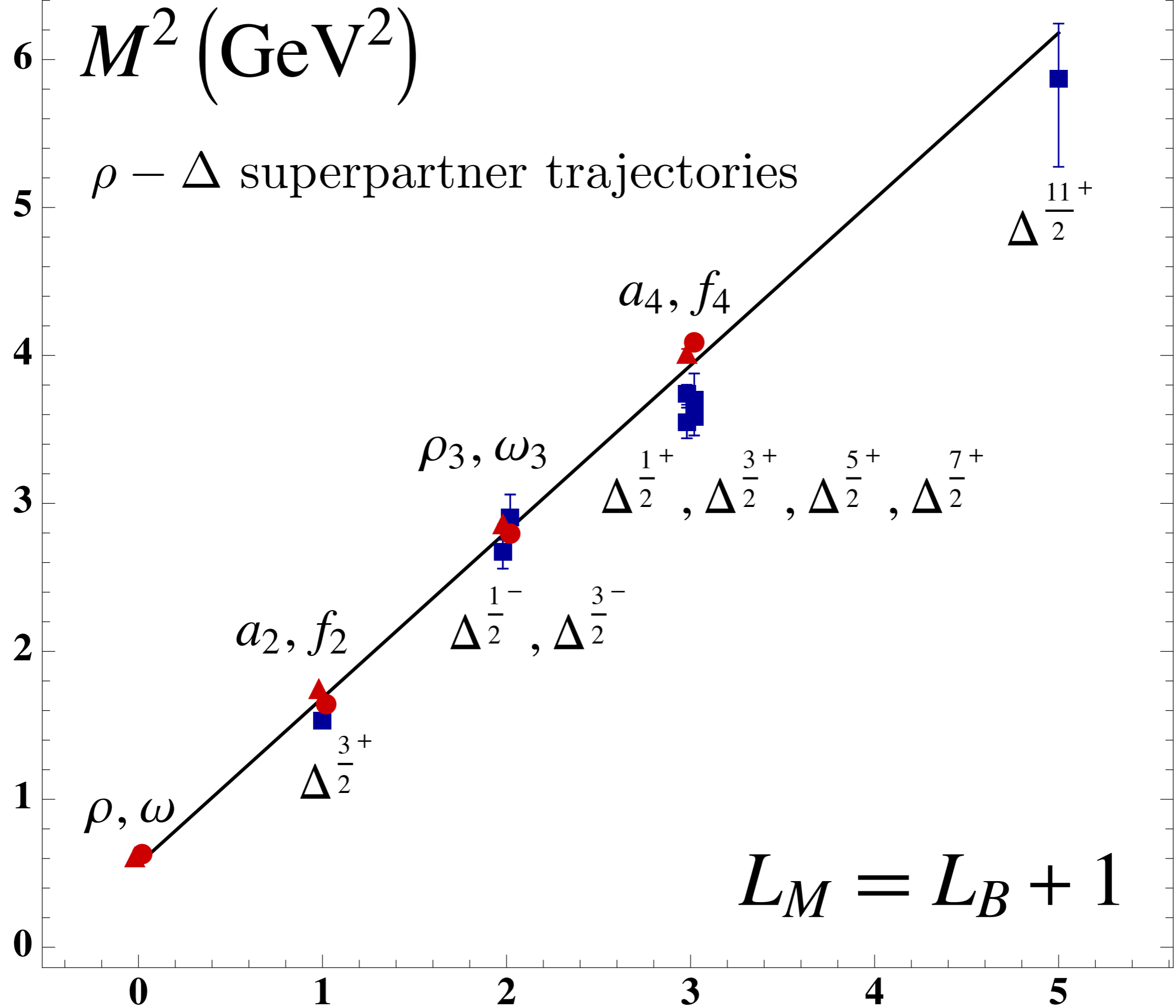
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M^2 (GeV²)

$\rho - \Delta$ superpartner trajectories



Features of Supersymmetric Equations

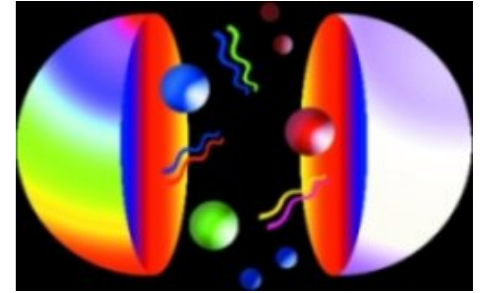
- $J = L + S$ baryon simultaneously satisfies both equations of G with L , $L + 1$ for same mass eigenvalue
- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $S^z = \pm 1/2$
- Baryon spin carried by quark orbital angular momentum: $\langle J^z \rangle = L^z + 1/2$
- Mass-degenerate meson “superpartner” with $L_M = L_B + 1$. *“Shifted meson-baryon Duality”*

Meson and baryon have same κ !

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

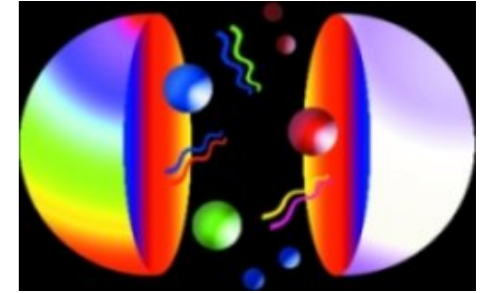
- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Fermionic Modes and Baryon Spectrum

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- Normalization

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*Chiral Symmetry
of Eigenstate!*

- Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$
 $J^z = +1/2 : \langle L^z \rangle = 1/2, \langle S_q^z \rangle = 0$
- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z=0$.**

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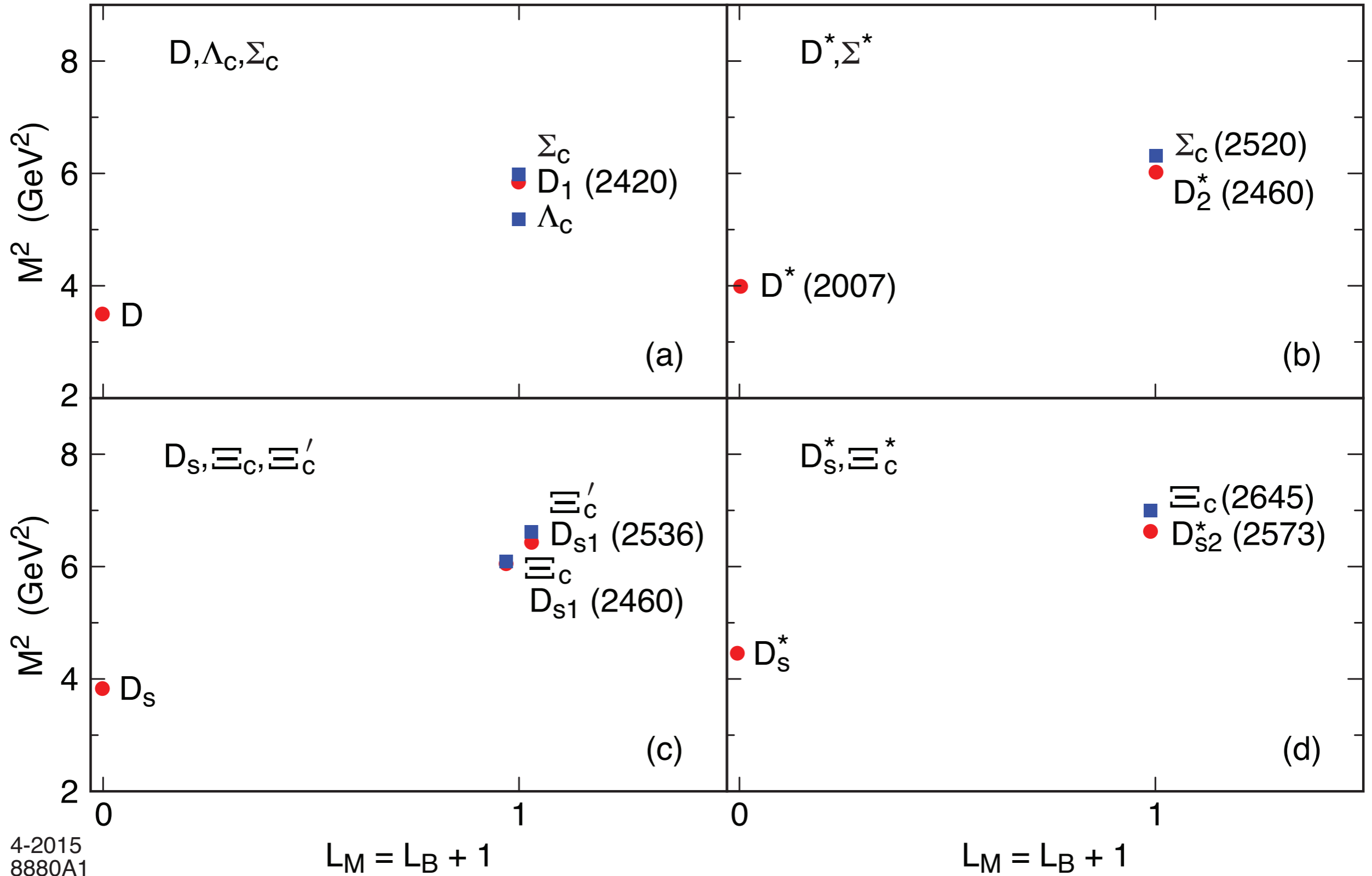
No mass-degenerate parity partners!

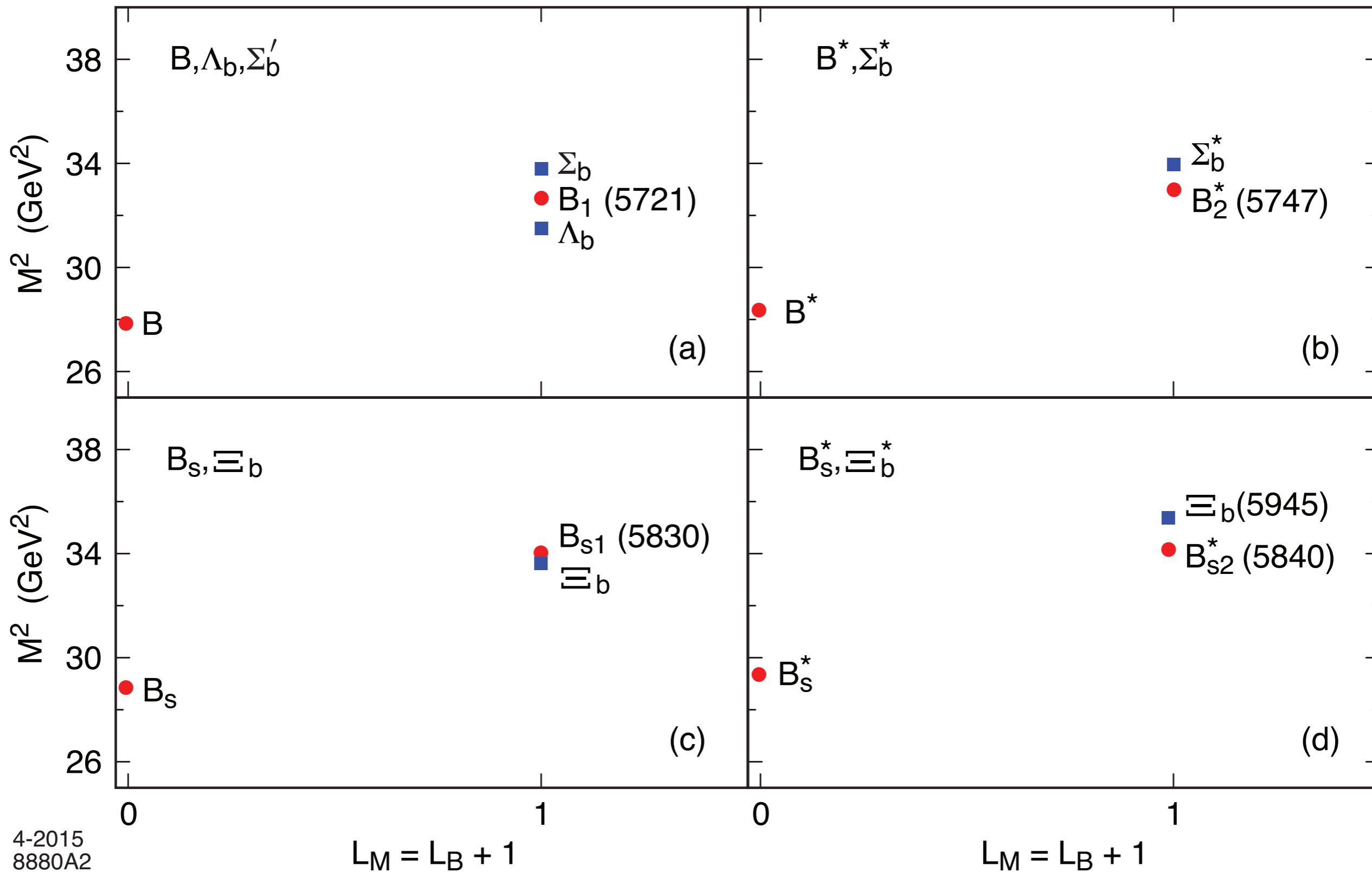
Supersymmetry Across the Light and Heavy-Light Hadronic Spectrum

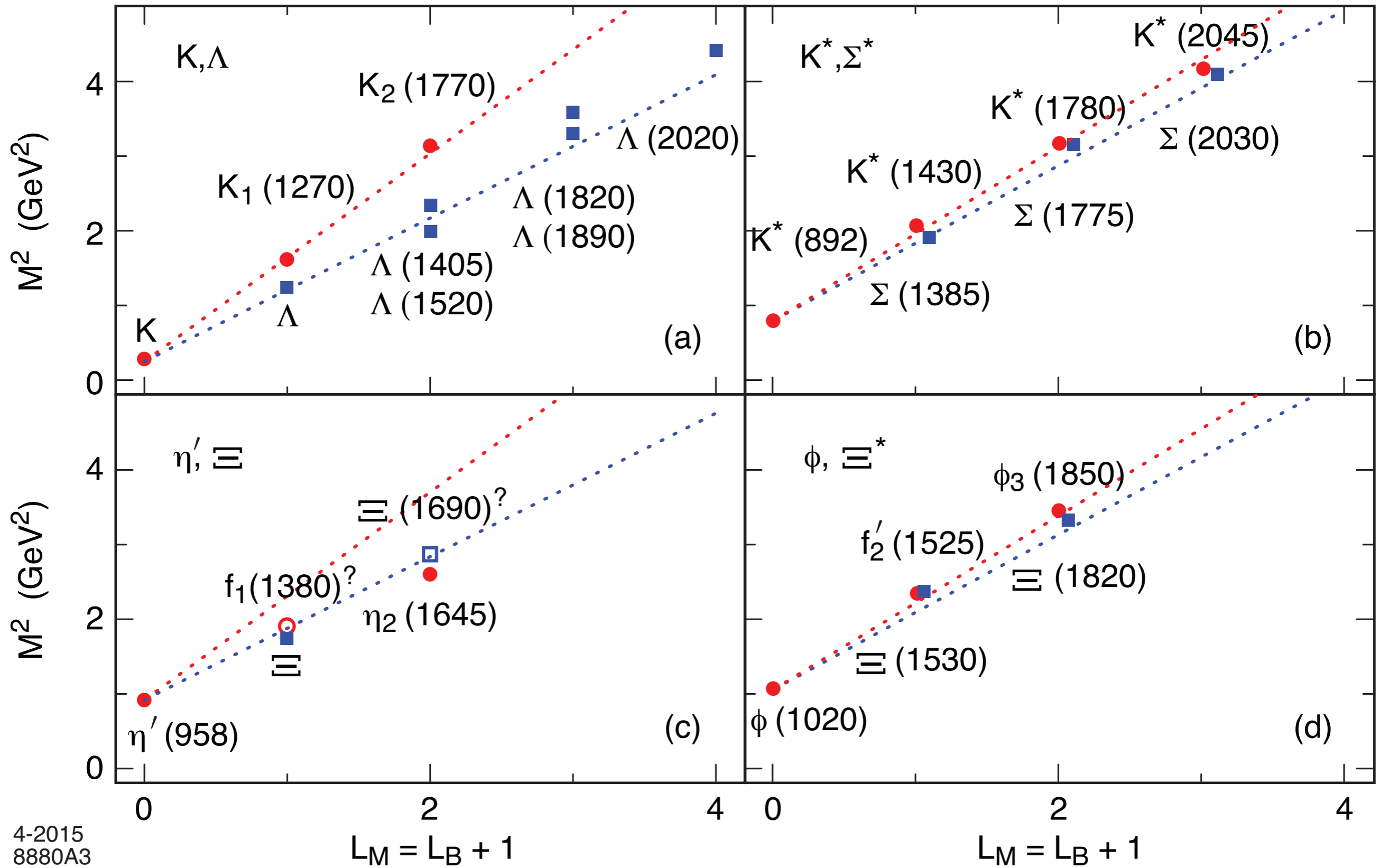
Dosch, de Teramond, sjb

Relativistic light-front bound-state equations for mesons and baryons can be constructed in the chiral limit from the supercharges of a superconformal algebra which connect baryon and meson spectra. Quark masses break the conformal invariance, but the basic underlying supersymmetric mechanism, which transforms meson and baryon wave functions into each other, still holds and gives remarkable connections across the entire spectrum of light and heavy-light hadrons. We also briefly examine the consequences of extending the supersymmetric relations to double-heavy mesons and baryons.

Dosch, de Teramond, sjb







Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

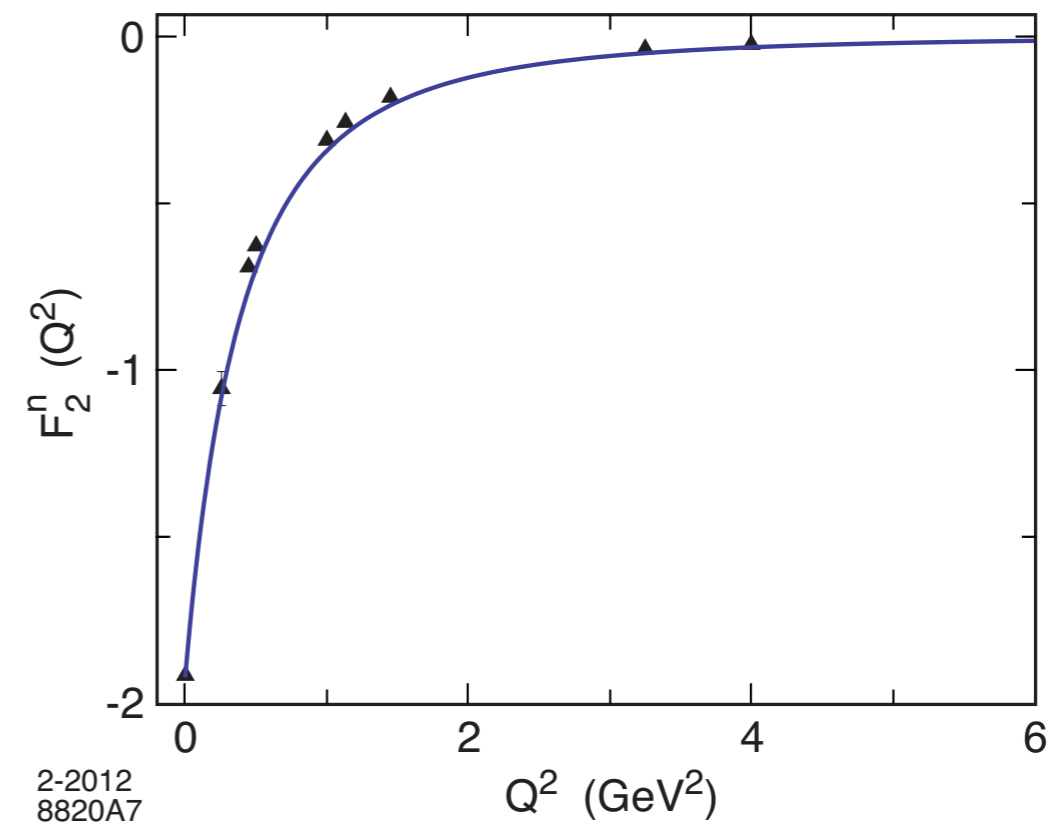
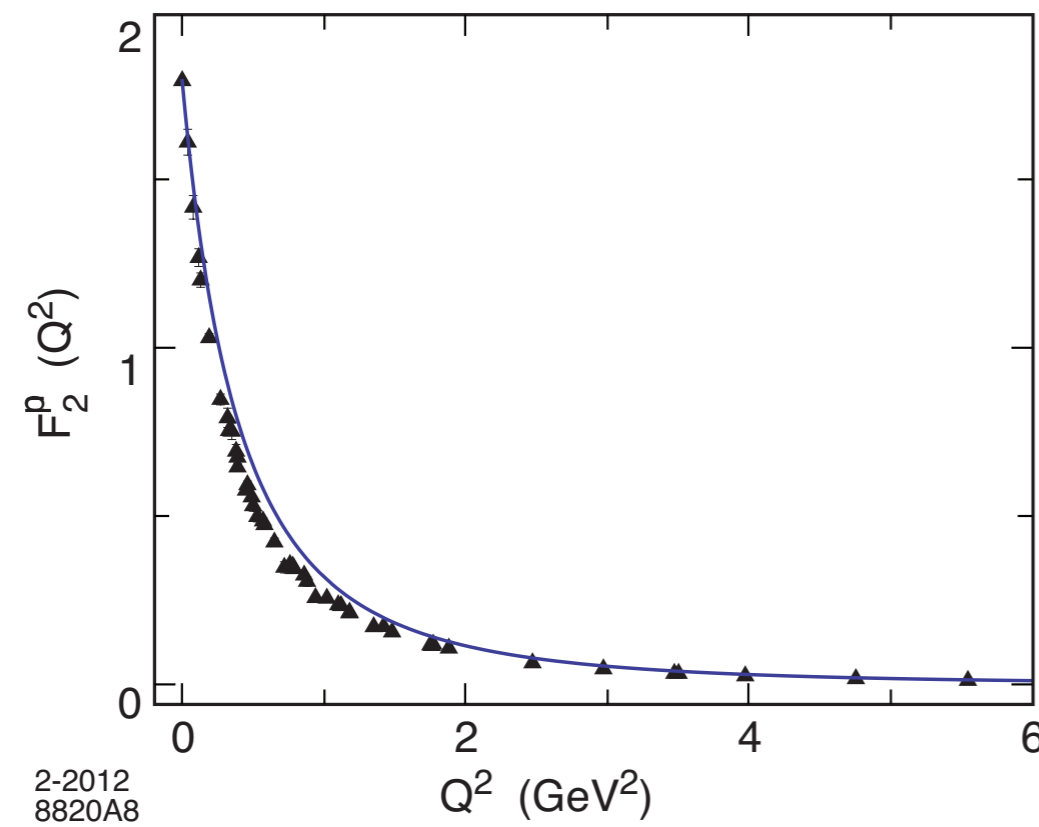
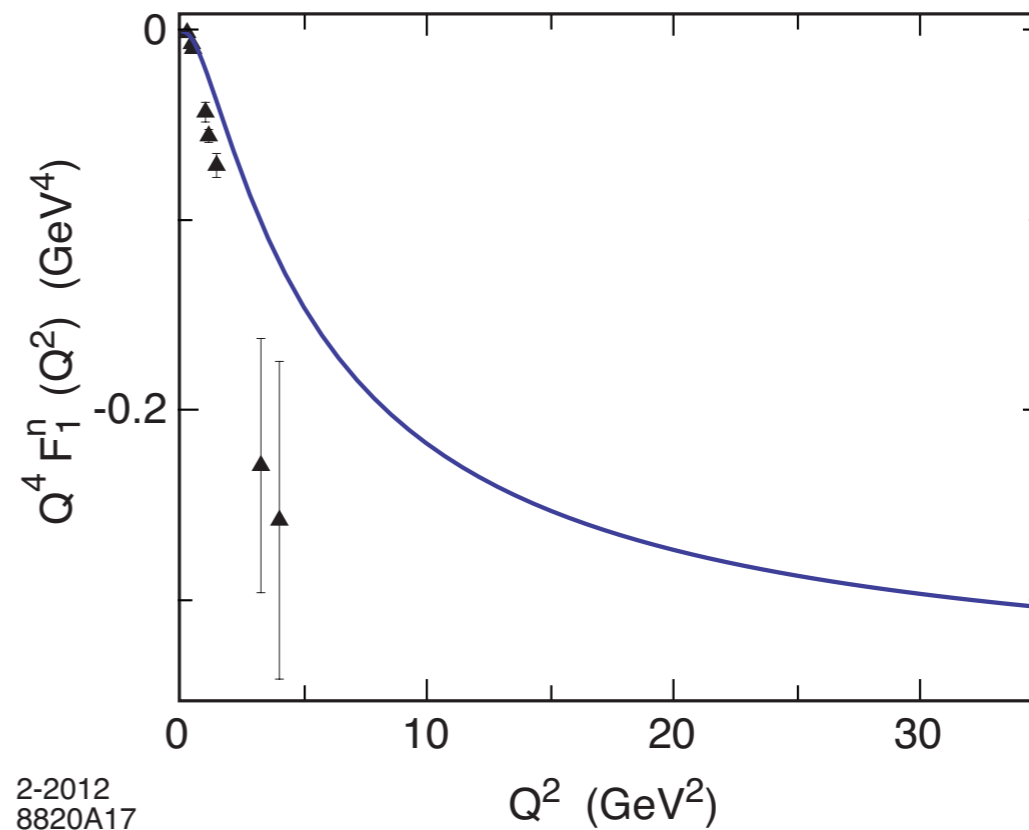
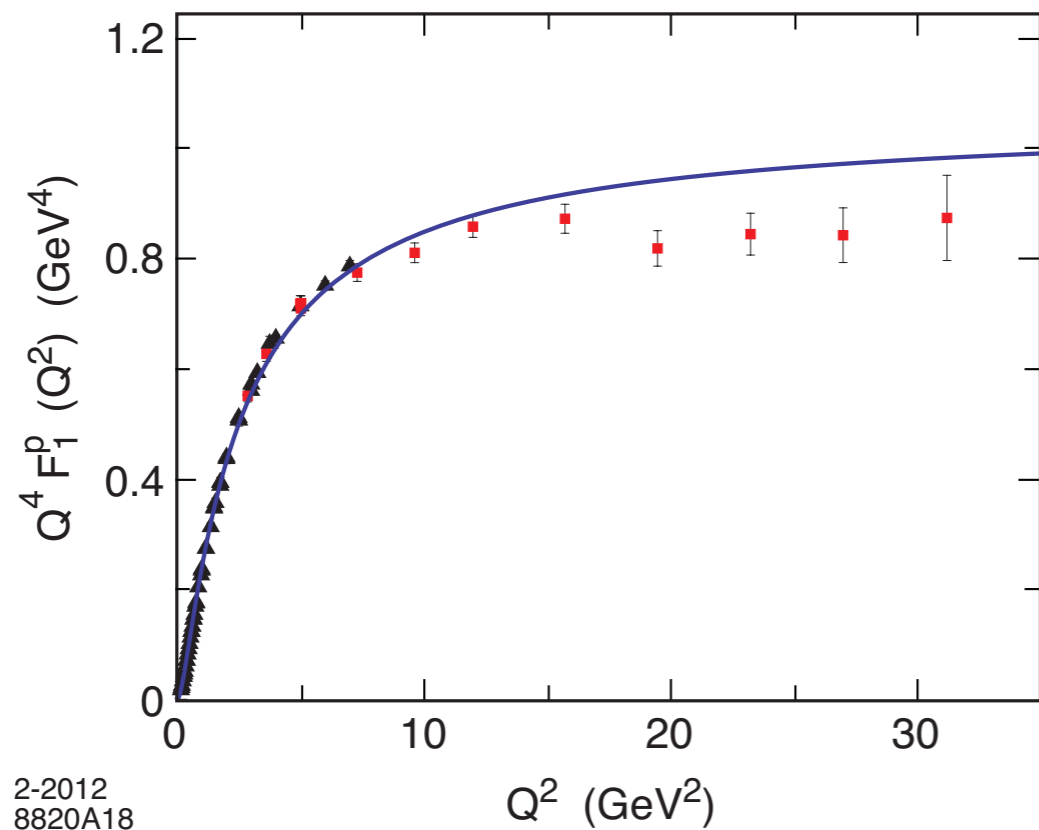
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

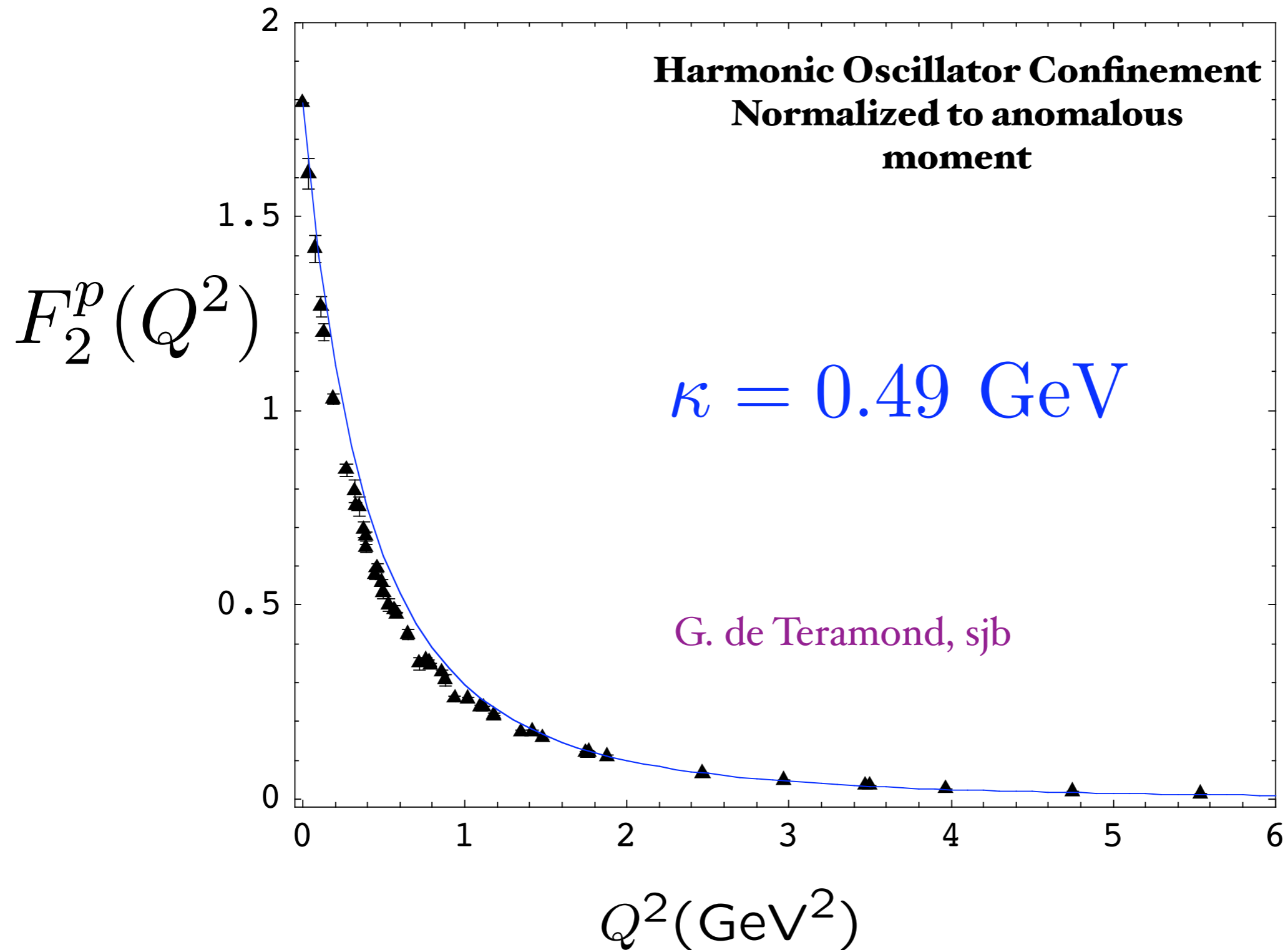
where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

Using $SU(6)$ flavor symmetry and normalization to static quantities



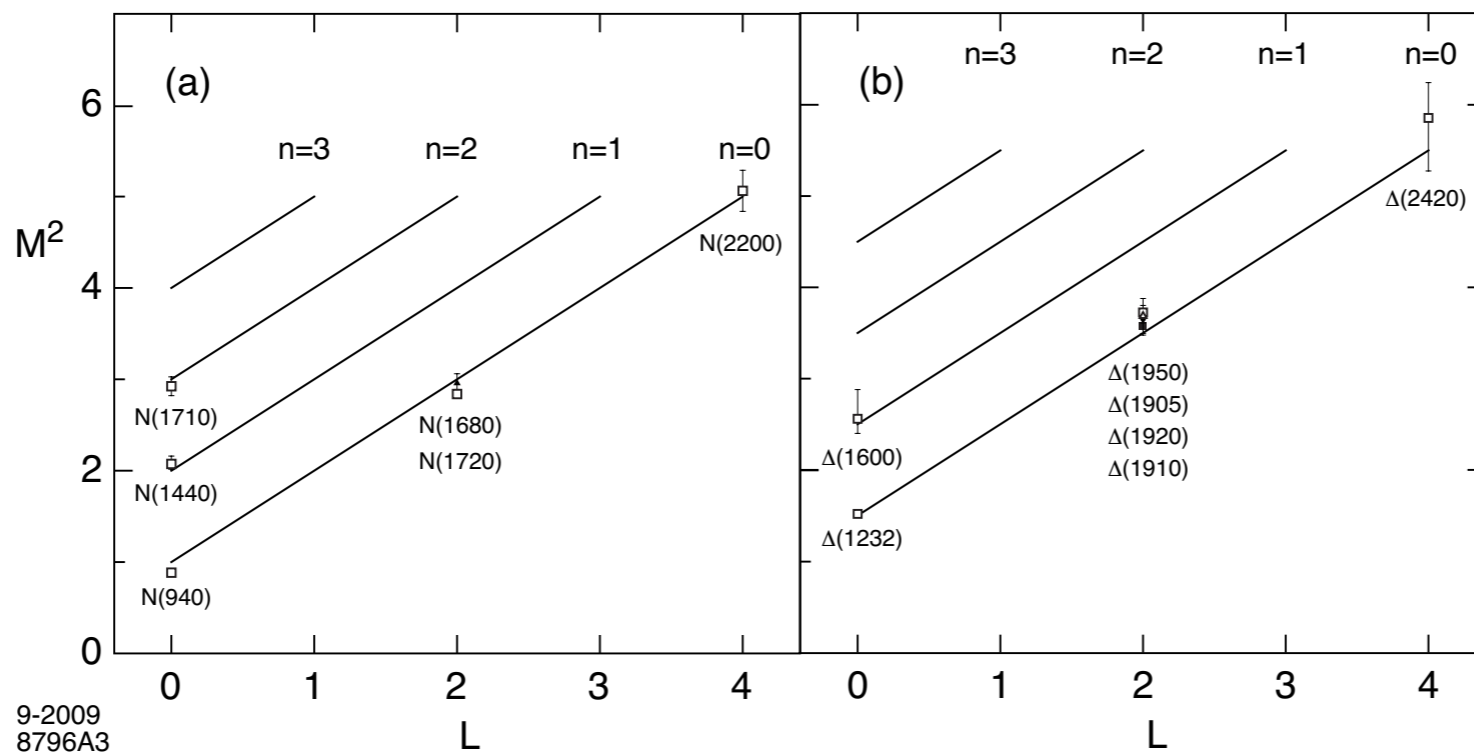
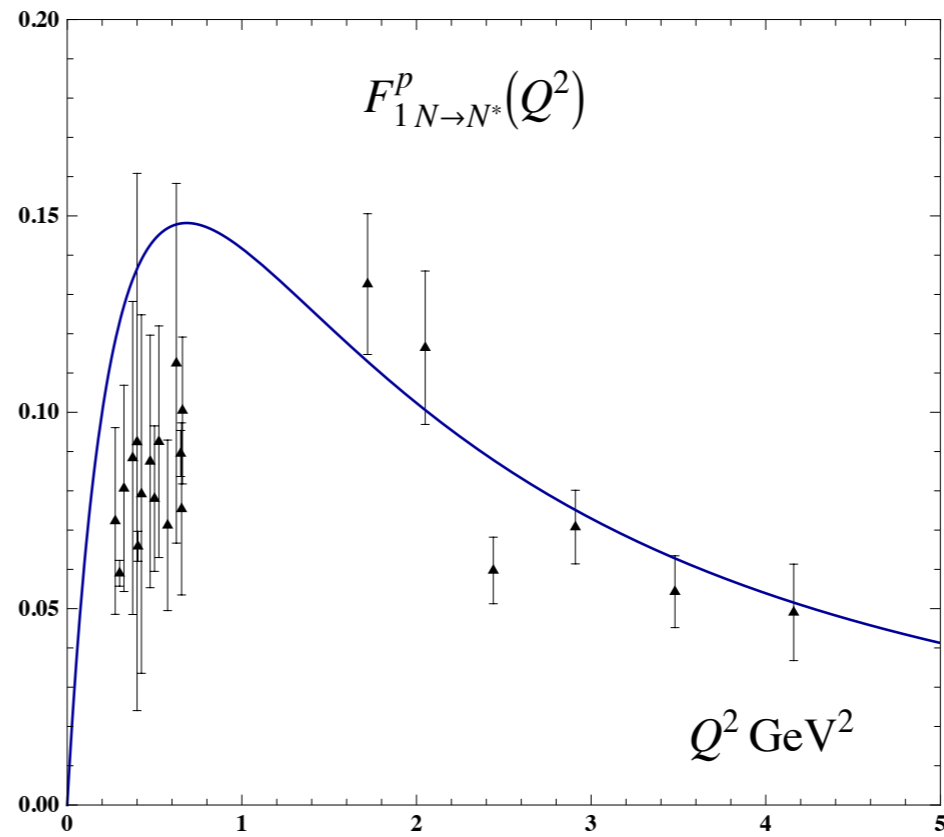
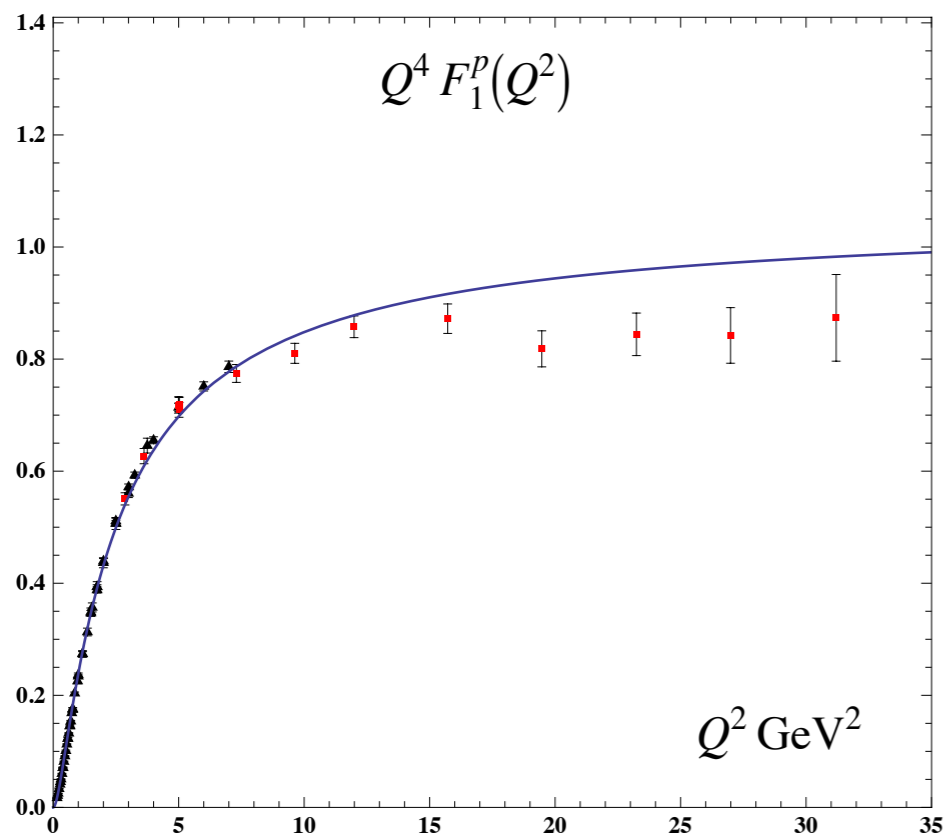
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Excited Baryons in Holographic QCD

G. de Teramond & sjb



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \rightarrow N^*(1440)$: $\Psi_+^{n=0,L=0} \rightarrow \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1N \rightarrow N^*}^p(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q, z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $(F_{1N \rightarrow N^*}^p(0) = 0, \quad V(Q=0, z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

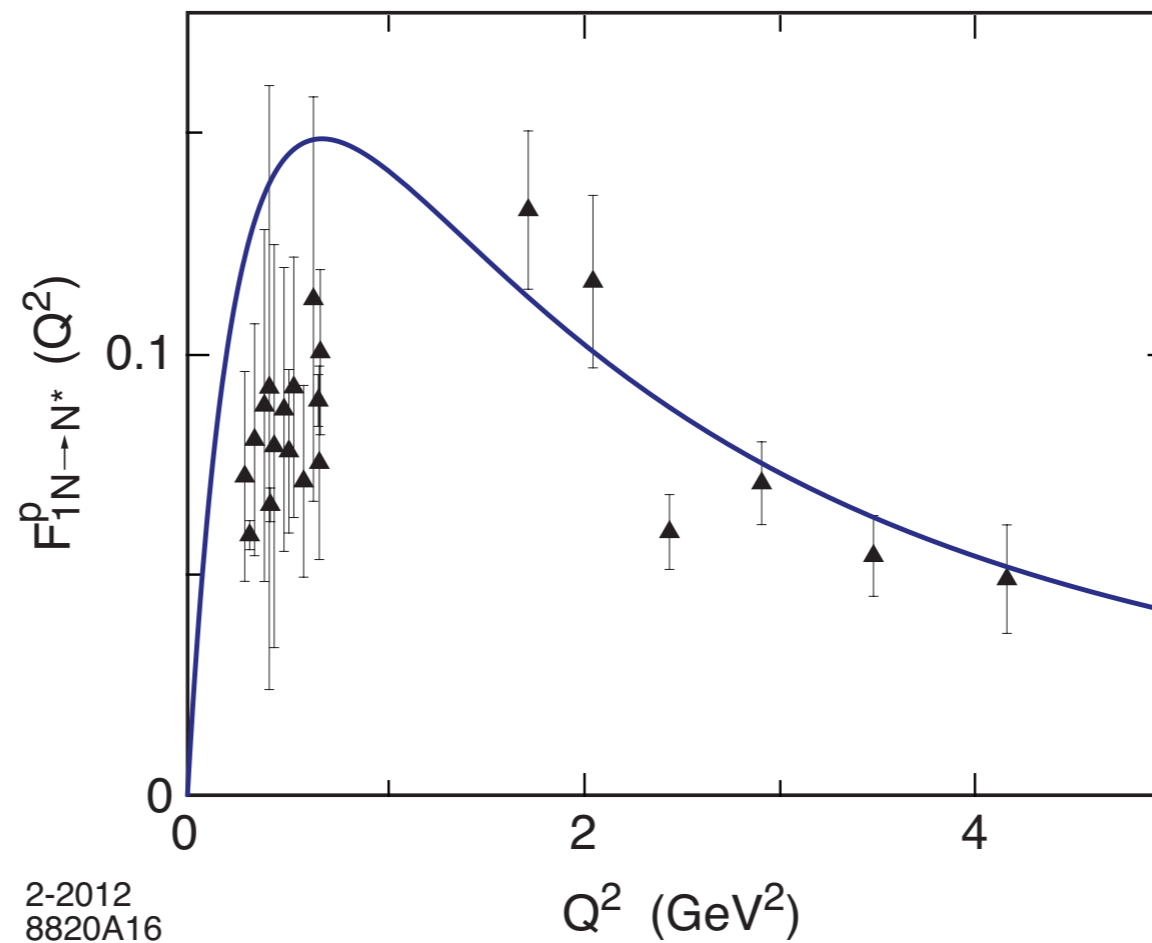
with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$

de Teramond, sjb

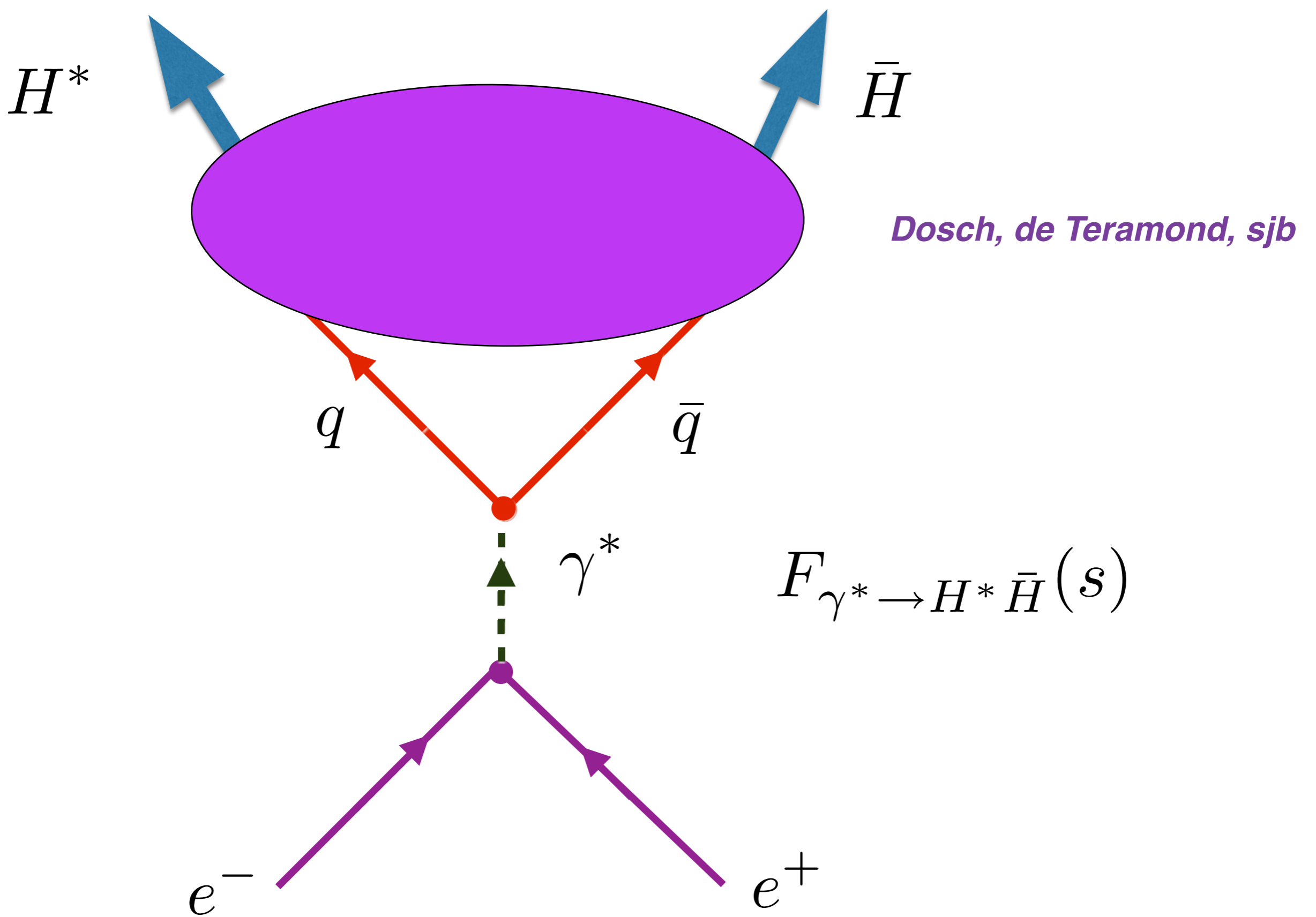
Consistent with counting rule, twist 3

Nucleon Transition Form Factors

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab



Prediction from Super Conformal AdS/QCD:
 Same Form Factors for $H=M$ and $H=B$ if $L_M=L_B+1$

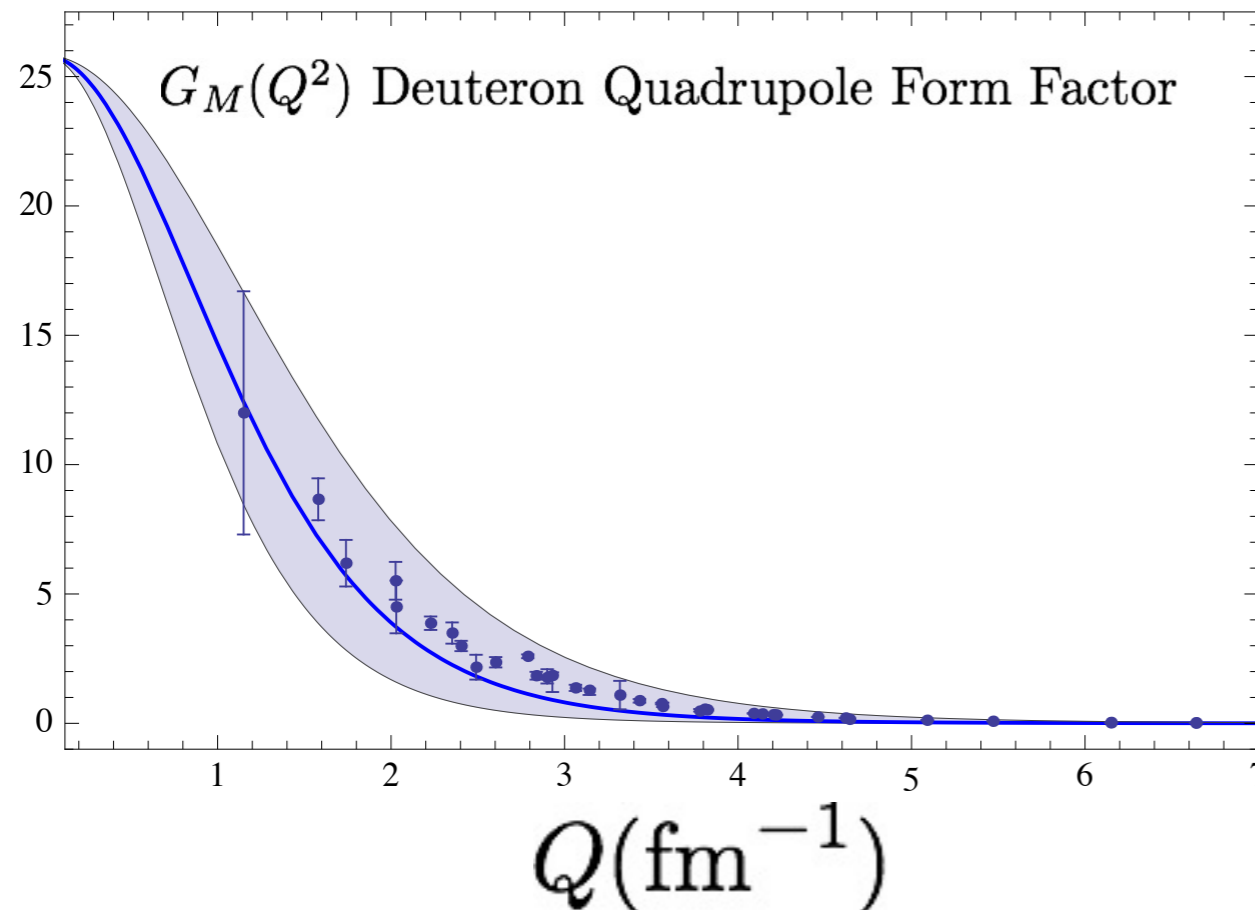
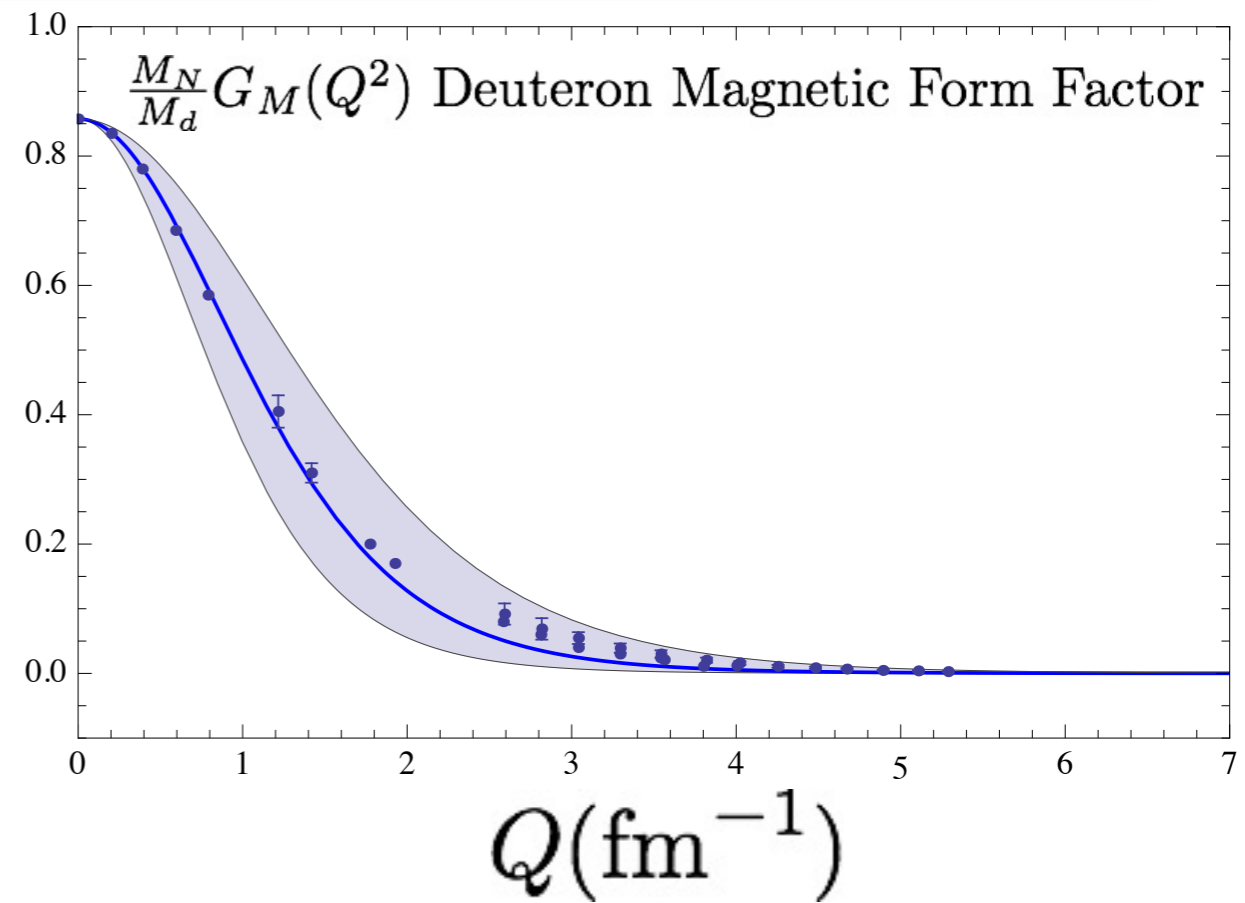
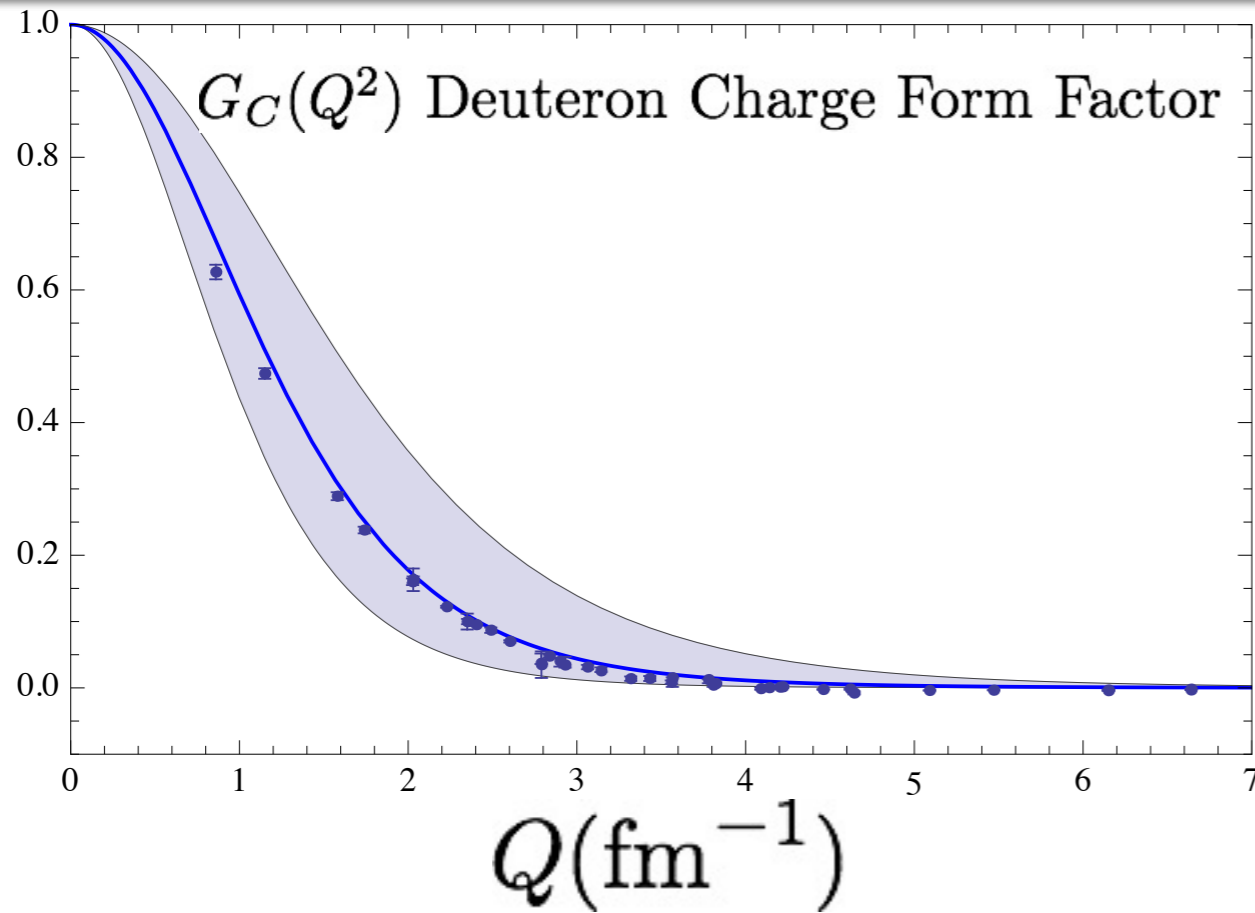
Nuclear physics in soft-wall AdS/QCD: Deuteron electromagnetic form factors

Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt, Alfredo Vega

We present a high-quality description of the deuteron electromagnetic form factors in a soft-wall AdS/QCD approach. We first propose an effective action describing the dynamics of the deuteron in the presence of an external vector field. Based on this action the deuteron electromagnetic form factors are calculated, displaying the correct $1/Q^{10}$ power scaling for large Q^2 values. This finding is consistent with quark counting rules and the earlier observation that this result holds in confining gauge/gravity duals. The Q^2 dependence of the deuteron form factors is defined by a single and universal scale parameter K , which is fixed from data.

arXiv:1501.02738 [hep-ph]

Application of Light-Front Holography to the Deuteron Form Factors



Thomas Gutsche, Valery E. Lyubovitskij,
Ivan Schmidt, and Alfredo Vega

<http://arxiv.org/abs/1501.02738v3>

Consistent with quark counting rules
Ji, Lepage, sjb

Bjorken sum rule defines effective charge

$$\alpha_{g1}(Q^2)$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large Q^2**
- **Computable at large Q^2 in any pQCD scheme**
- **Universal β_0, β_1**

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

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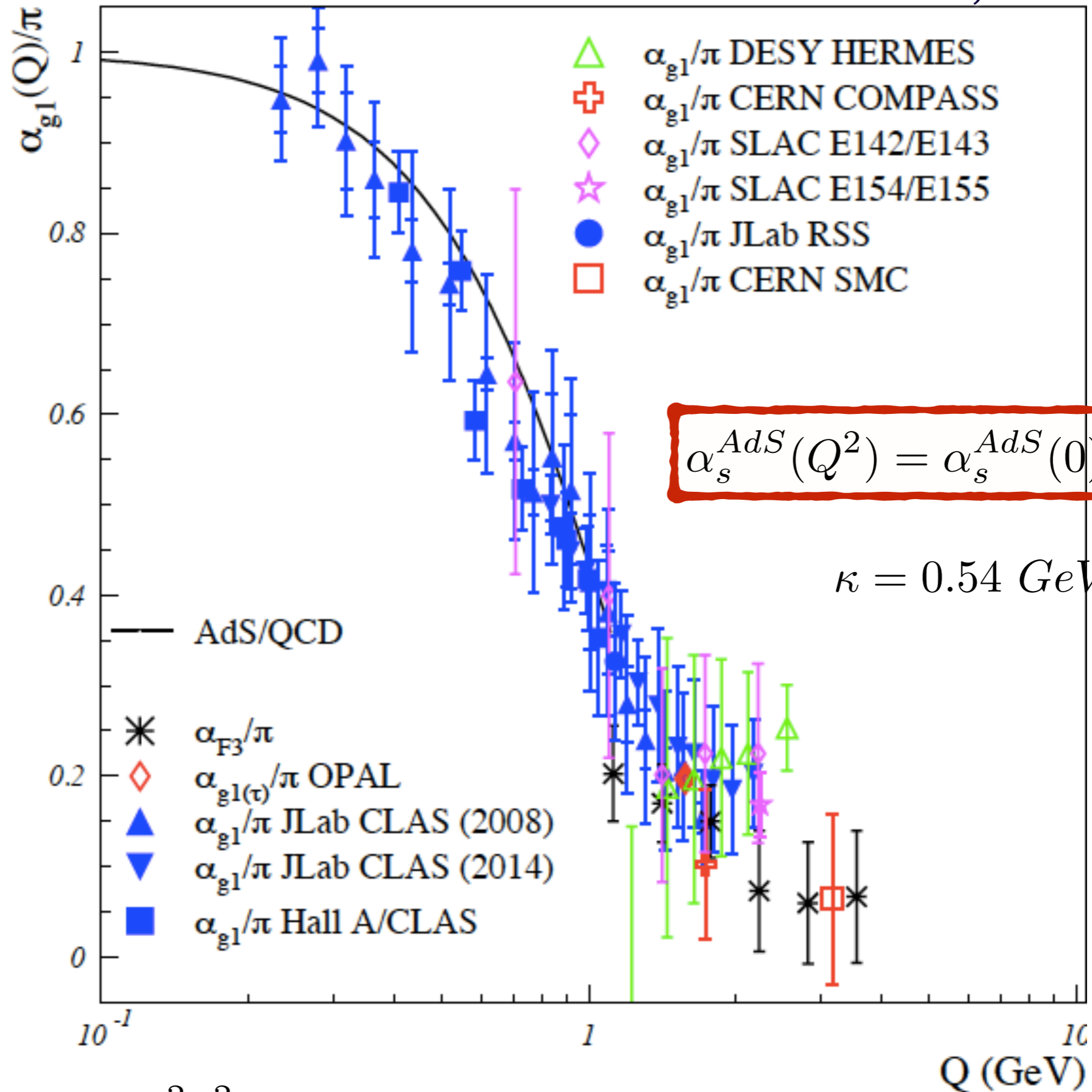
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where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement



$$e^\varphi = e^{+\kappa^2 z^2}$$

AdS/QCD dilaton captures the higher twist corrections to effective charges for $Q < 1$ GeV

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa_1$$

Deur, de Teramond, sjb

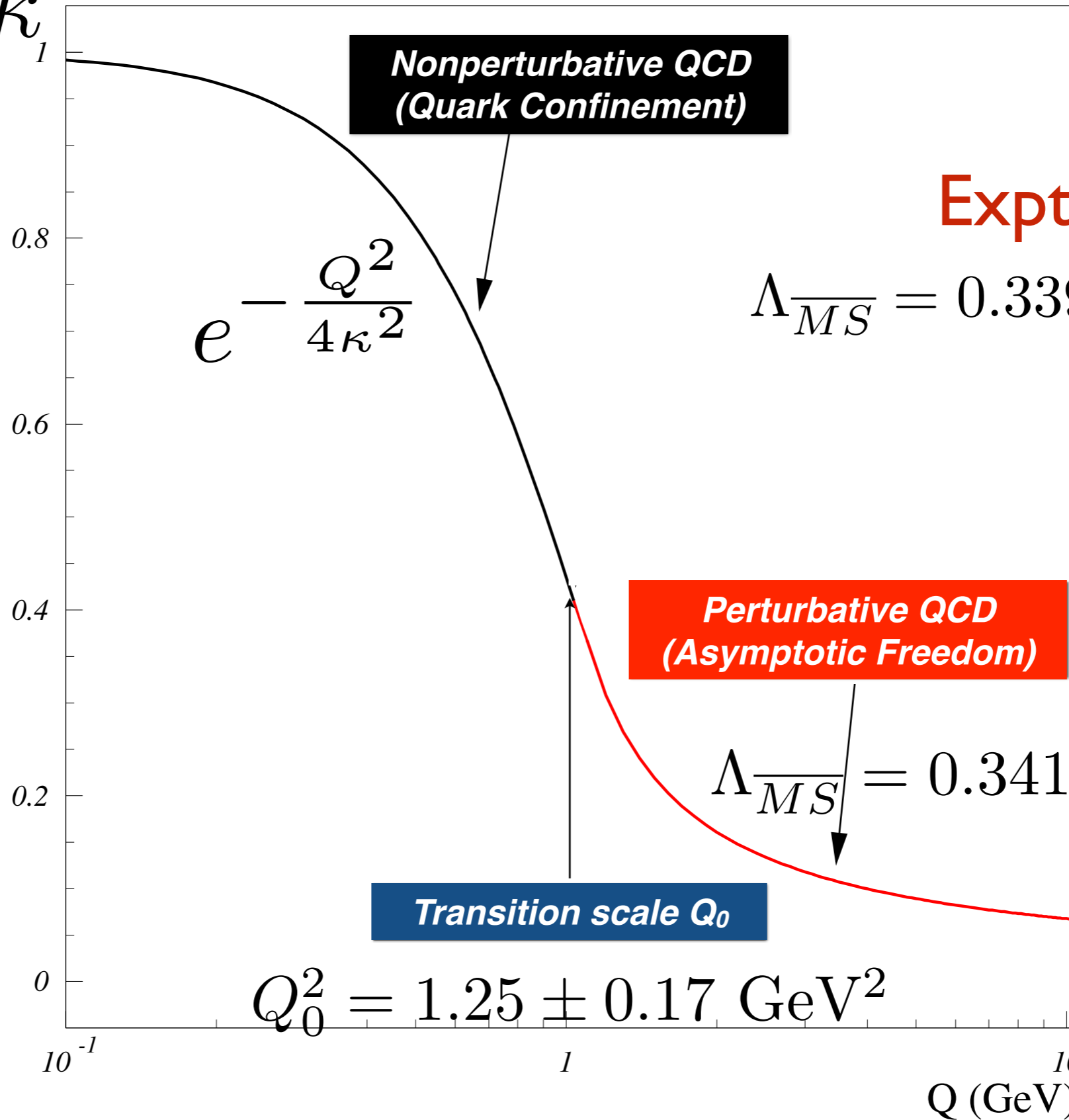
All-Scale QCD Coupling

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$

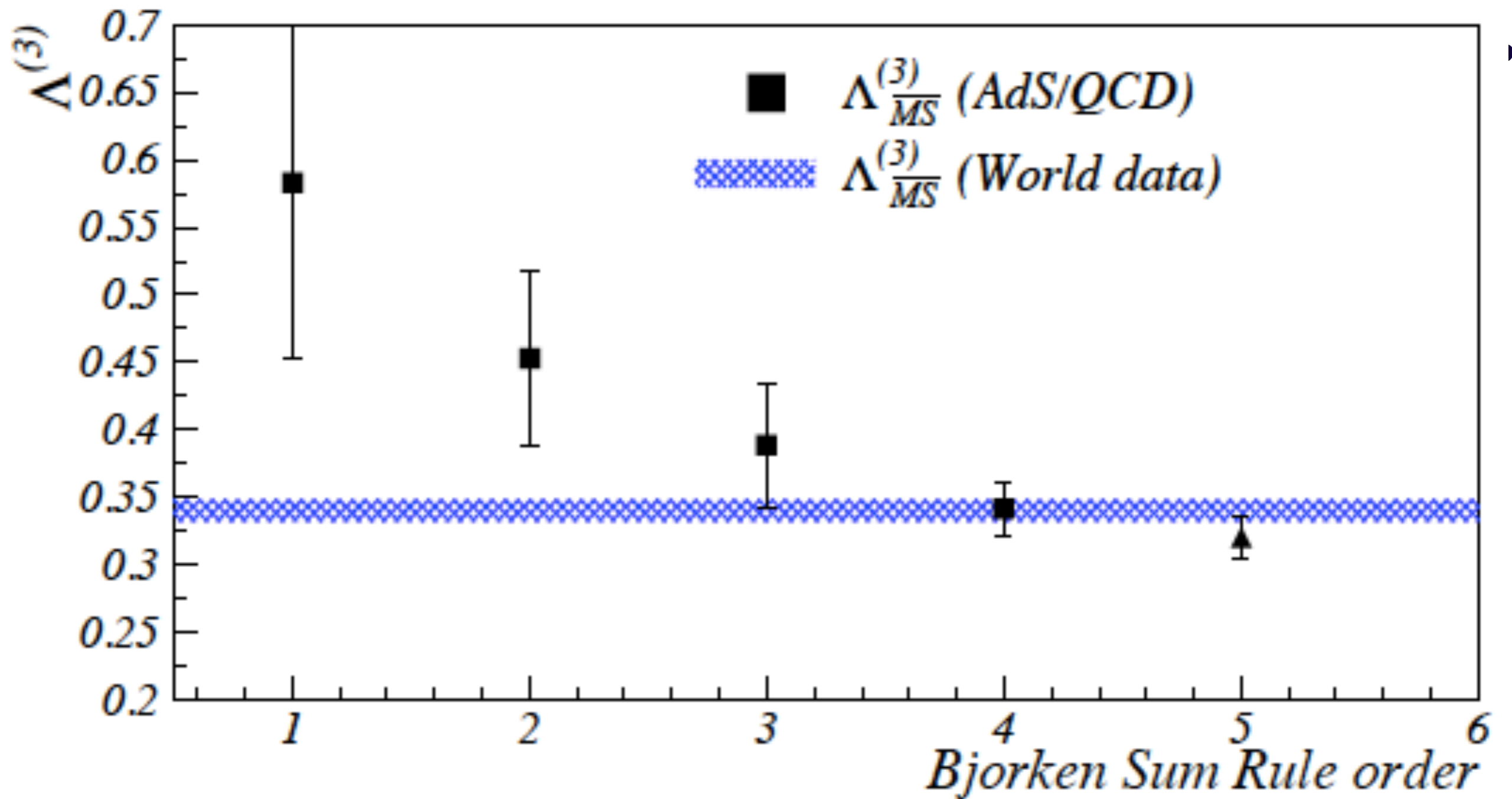
$$e^{-\frac{Q^2}{4\kappa^2}}$$

Expt:

$$\Lambda_{\overline{MS}} = 0.3339 \pm 0.016 \text{ GeV}$$



$$\lambda \equiv \kappa^2$$

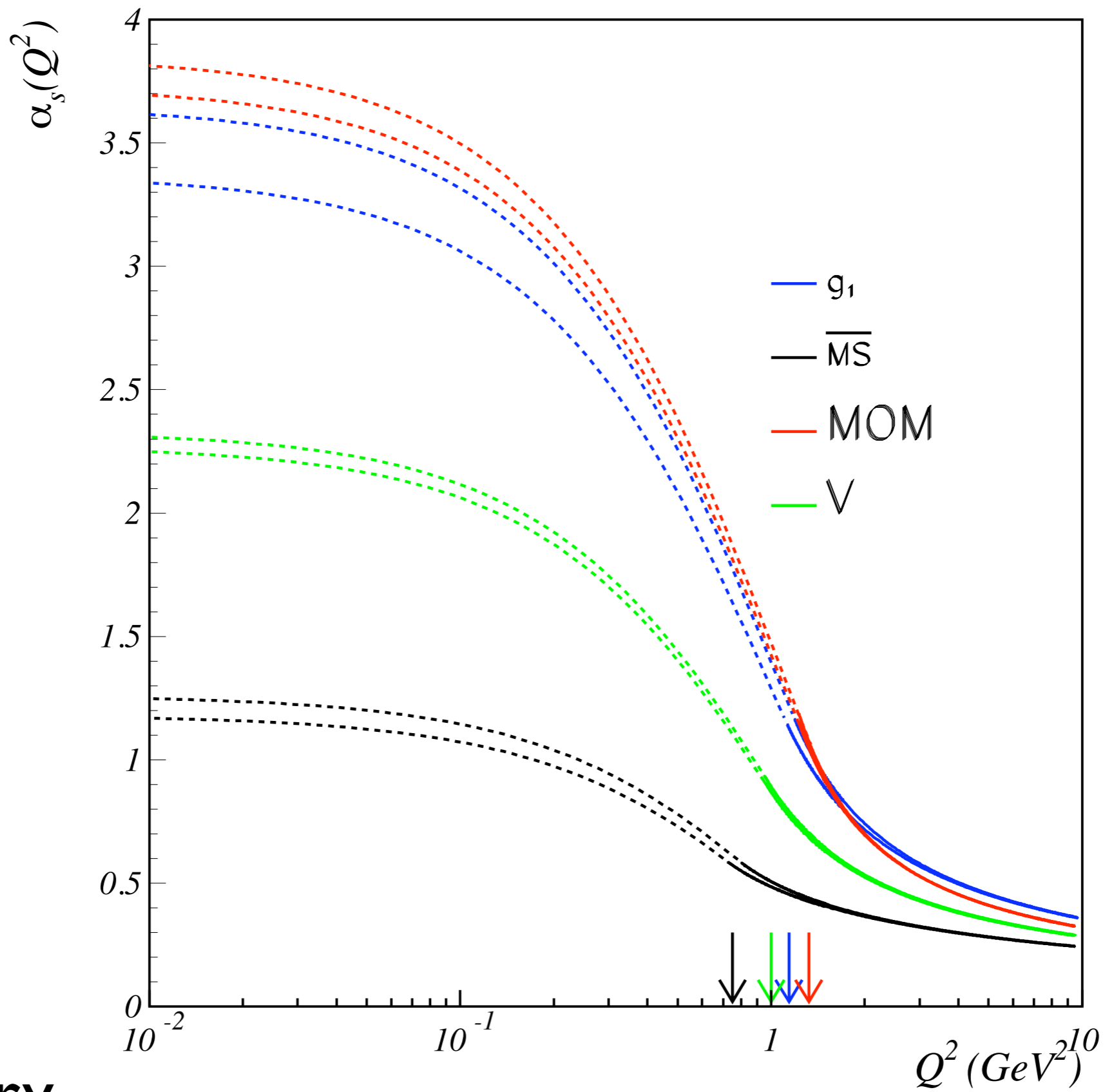


$$\Lambda_{\overline{MS}} = 0.341 \text{ GeV} = 0.440 m_\rho = 0.622 \kappa$$

Connect $\Lambda_{\overline{MS}}$ to hadron masses!

Experiment: $M_\rho = 0.7753 \pm 0.0003 \text{ GeV}$

Unification Predictions in Various Schemes



Preliminary

Deur,
de Teramond, sjb

Unification Scale Q_0

- *Matches perturbative to nonperturbative QCD*
- *Use for ERBL, DGLAP*
- *Hadronization at amplitude level*
- *BLFQ transition scale*
- *Use Principle of Maximum Conformality (PMC) to make scheme-independent predictions without renormalization scale ambiguity*

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\text{init}})$

Choose μ_R^{init} ; arbitrary initial renormalization scale

Identify $\{\beta_i^R\}$ – terms using n_f – terms
through the PMC – BLM correspondence principle

Shift scale of α_s to μ_R^{PMC} to eliminate $\{\beta_i^R\}$ – terms

Conformal Series

Result is independent of μ_R^{init} and scheme at fixed order

PMC/BLM

No renormalization scale ambiguity!

*Result is independent of
Renormalization scheme
and initial scale!*

QED Scale Setting at $N_C=0$

**Eliminates unnecessary
systematic uncertainty**

Scale fixed at each order

**δ -Scheme automatically
identifies β -terms!**

*Xing-Gang Wu, Martin Mojaza
Leonardo di Giustino, SJB*

Stan Brodsky



Principle of Maximum Conformality

LC2015

Frascati INFN

September 25, 2015

**Light-Front Holographic QCD, Color Confinement,
and Supersymmetric Features of QCD**

Interpretation of Mass Scale \mathcal{K}

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of \mathcal{K}
- Value of \mathcal{K} itself not determined -- place holder
- Need external constraint such as f_π

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

“In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Similarly for m_ρ .

Put in precise terms, if you publish a paper with a formula giving m_ρ/m_P in terms of pure numbers such as 2 and π , the field theory community will hail you as a conquering hero who has solved QCD exactly.”

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$$(m_q = 0)$$

$$m_\pi = 0$$

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$$\begin{aligned} (m_q = 0) \\ m_\pi = 0 \end{aligned}$$

$$\frac{m_\rho}{m_P} = \frac{1}{\sqrt{2}}$$

$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

Remarkable Features of Light-Front Schrödinger Equation

- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

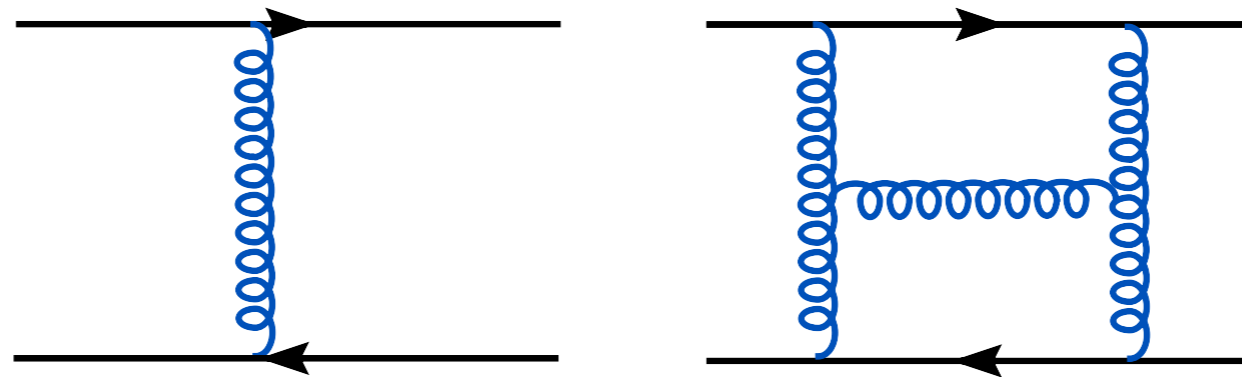
- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
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Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \left[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2 + (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \right]$$

Smirnov, Smirnov, Steinhauser, 2010

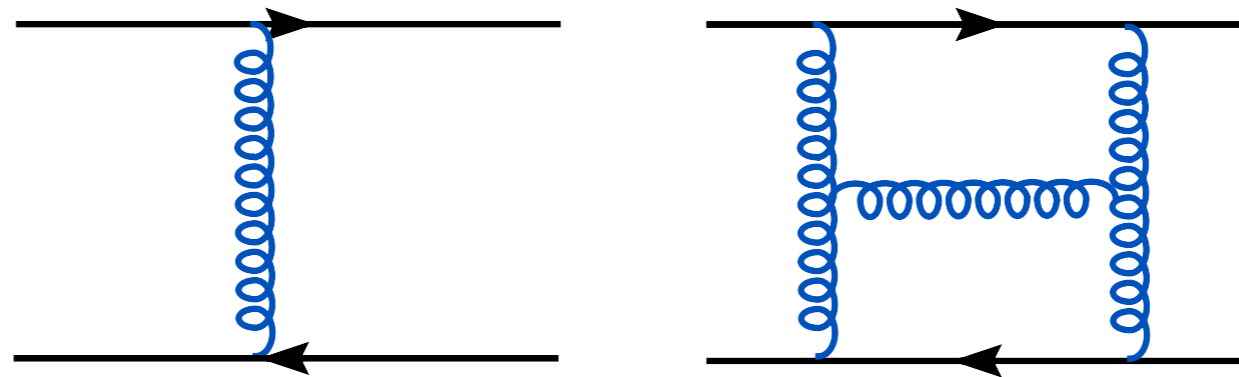


*Confinement eliminates IR divergences
Self-consistent mass scale κ*

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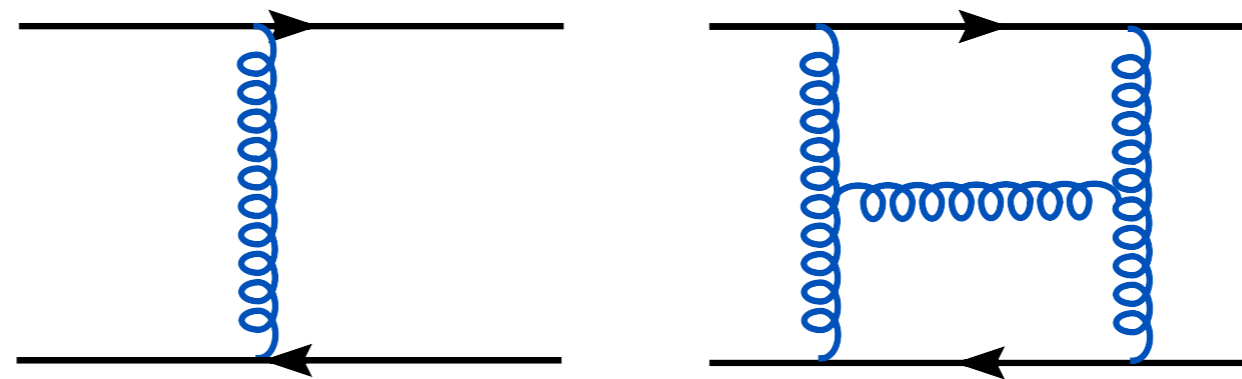


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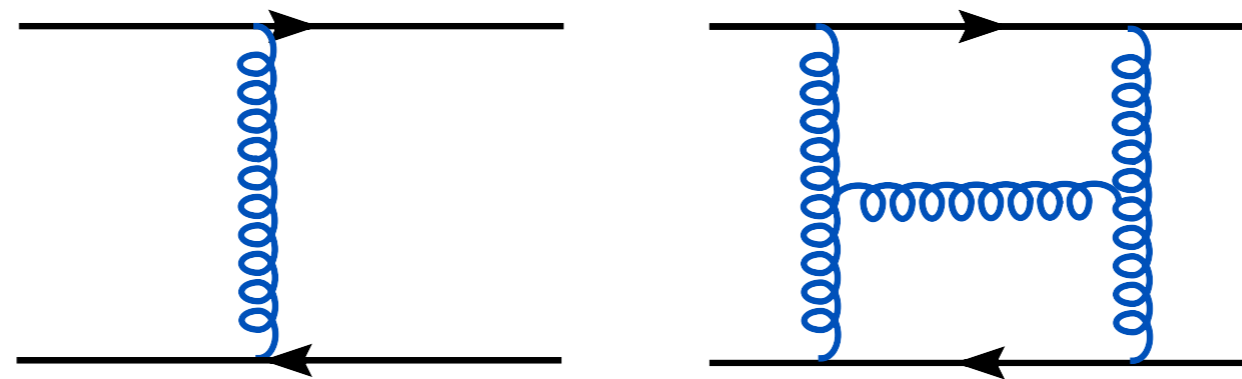
$\log \kappa^2 \zeta^2$

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Smirnov, Smirnov, Steinhauser, 2010



$\log \kappa^2 \zeta^2$

Summation of H graphs: confining potential

*Confinement eliminates IR divergences
Self-consistent mass scale κ*

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks



Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

Connection to the Linear Instant-Form Potential

- Compare invariant mass in the instant-form in the hadron center-of-mass system $\mathbf{P} = 0$,

$$M_{q\bar{q}}^2 = 4m_q^2 + 4\mathbf{p}^2$$

with the invariant mass in the front-form in the constituent rest frame, $\mathbf{k}_q + \mathbf{k}_{\bar{q}} = 0$

$$M_{q\bar{q}}^2 = \frac{\mathbf{k}_{\perp}^2 + m_q^2}{x(1-x)}$$

obtain

$$U = V^2 + 2\sqrt{\mathbf{p}^2 + m_q^2}V + 2V\sqrt{\mathbf{p}^2 + m_q^2}$$

where $\mathbf{p}_{\perp}^2 = \frac{\mathbf{k}_{\perp}^2}{4x(1-x)}$, $p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$, and V is the effective potential in the instant-form

- For small quark masses a linear instant-form potential V implies a harmonic front-form potential U and thus linear Regge trajectories

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

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September 25, 2015

Light-Front Holographic QCD, Color Confinement,
and Supersymmetric Features of QCD

Stan Brodsky



AdS/QCD and Light-Front Holography

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

- **Zero mass pion for $m_q = 0$ ($n=J=L=0$)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2 : Dimensional counting**
 $[Q^2]^{n-1} F(Q^2) \rightarrow \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$

Features of AdS/QCD

- **Color confining potential $\kappa^4 \zeta^2$ and universal mass scale from dilaton**
$$e^{\phi(z)} = e^{\kappa^2 z^2} \quad \alpha_s(Q^2) \propto \exp -Q^2/4\kappa^2$$
- **Dimensional transmutation** $\Lambda_{\overline{MS}} \leftrightarrow \kappa \leftrightarrow m_H$
- **Chiral Action remains conformally invariant despite mass scale** *DAFF*
- **Light-Front Holography: Duality of AdS and frame-independent LF QCD**
- **Reproduces observed Regge spectroscopy — same slope in n, L, and J for mesons and baryons**
- **Massless pion for massless quark**
- **Supersymmetric meson-baryon dynamics and spectroscopy:**
 $L_M = L_B + I$
- **Dynamics: LFWFs, Form Factors, GPDs**

*Superconformal Algebra
Fubini and Rabinovici*

Light and heavy mesons in a soft-wall holographic model

Valery E. Lyubovitskij^{*1†}, Tanja Branz¹, Thomas Gutsche¹, Ivan Schmidt², Alfredo Vega²

¹ *Institut für Theoretische Physik, Universität Tübingen,
Kepler Center for Astro and Particle Physics,
Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

² *Departamento de Física y Centro Científico Tecnológico de Valparaíso (CCTVal),
Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

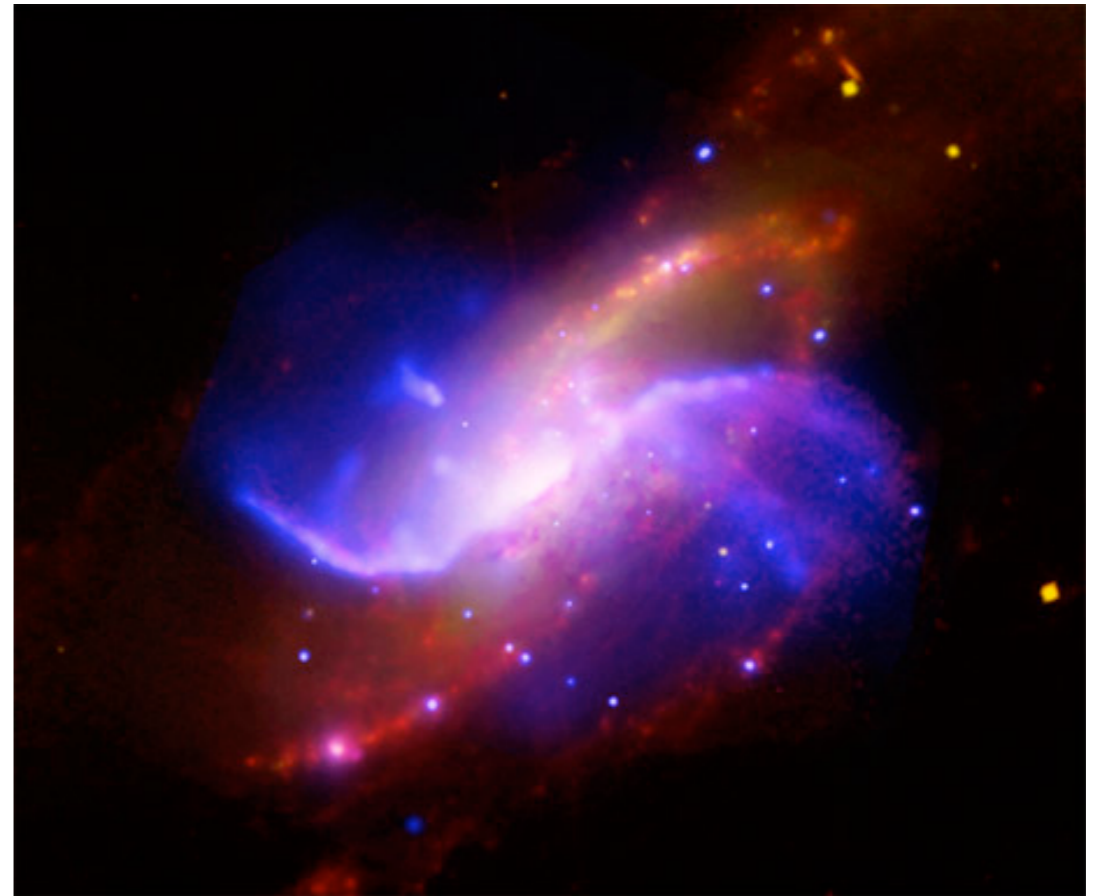
We study the spectrum and decay constants of light and heavy mesons in a soft-wall holographic approach, using the correspondence of string theory in Anti-de Sitter space and conformal field theory in physical space-time.

Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffraction dissociation of pion and proton to jets**
- **Identify the factorization Scale for ERBL, DGLAP evolution: Q_0**
- **Compute Tetraquark Spectroscopy Sequentially**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization**

*We view the universe
as light reaches us
along the light-front
at fixed*

$$\tau = t + z/c$$



Front Form Vacuum Describes the Empty, Causal Universe

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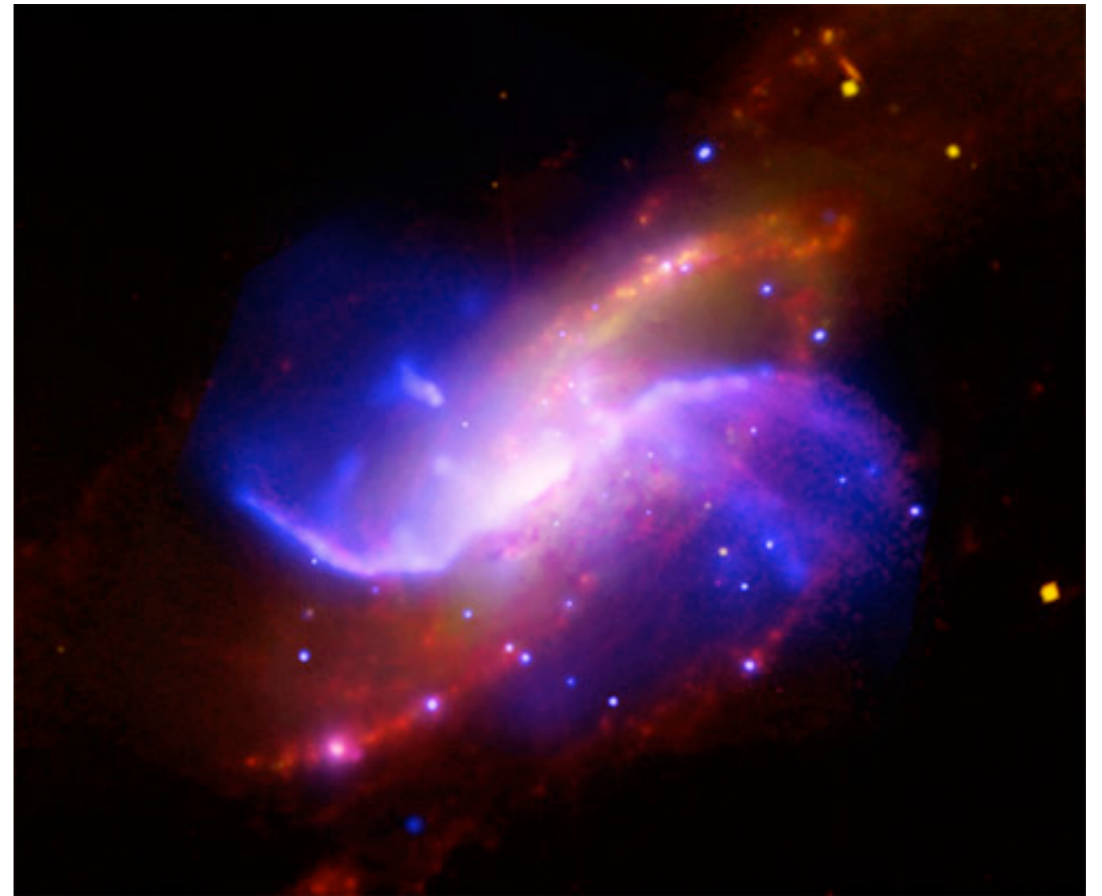
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Front Form Vacuum Describes the Empty, Causal Universe

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu*

$$(\Omega_{\Lambda})_{QCD} \sim 10^{45}$$

$$(\Omega_{\Lambda})_{EW} \sim 10^{56}$$

$$\Omega_{\Lambda} = 0.76(\text{expt})$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

“One of the gravest puzzles of theoretical physics”

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Elements of the solution:

(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD “Condensates”

(C) Higgs Light-Front Zero Mode

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0\rangle = E_0|\psi_0\rangle, E_0 = \min\{E_i\}$$

*Eigenstate defined at one time t over all space;
Acausal! Frame-Dependent*

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

$$H_{LF}|\psi_0\rangle_{LF} = M_0^2|\psi_0\rangle_{LF}, M_0^2 = 0.$$

*Frame-independent eigenstate at fixed LF time $\tau = t+z/c$
within causal horizon*

Frame-independent description of the causal physical universe!

Light-Front vacuum can simulate empty universe

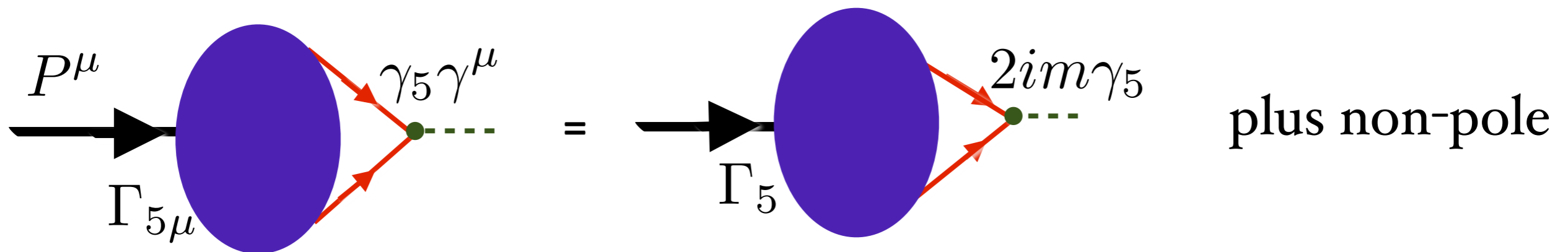
Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state $M=0$.
- Trivial up to $k^+=0$ zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: “In-hadron” condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD

Ward-Takahashi Identity for axial current

$$P^\mu \Gamma_{5\mu}(k, P) + 2im\Gamma_5(k, P) = S^{-1}(k + P/2)i\gamma_5 + i\gamma_5 S^{-1}(k - P/2)$$

$$S^{-1}(\ell) = i\gamma \cdot \ell A(\ell^2) + B(\ell^2) \quad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



Identify pion pole at $P^2 = m_\pi^2$

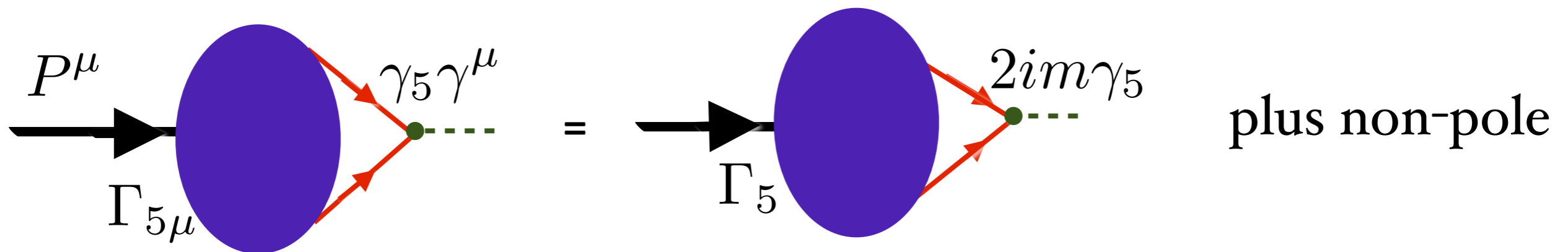
$$P^\mu \langle 0 | \bar{q} \gamma_5 \gamma^\mu q | \pi \rangle = 2m \langle 0 | \bar{q} i \gamma_5 q | \pi \rangle$$

$$f_\pi m_\pi^2 = -(m_u + m_d) \rho_\pi$$

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Light-front formulation of the standard model

Prem P. Srivastava*

*Instituto de Física, Universidade do Estado de Rio de Janeiro, RJ 20550, Brazil,
Theoretical Physics Department, Fermilab, Batavia, Illinois 60510,
and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309*

Stanley J. Brodsky[†]

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(Received 20 February 2002; published 20 August 2002)

Light-front (LF) quantization in the light-cone (LC) gauge is used to construct a renormalizable theory of the standard model. The framework derived earlier for QCD is extended to the Glashow-Weinberg-Salam (GWS) model of electroweak interaction theory. The Lorentz condition is automatically satisfied in LF-quantized QCD in the LC gauge for the free massless gauge field. In the GWS model, with the spontaneous symmetry breaking present, we find that the 't Hooft condition accompanies the LC gauge condition corresponding to the massive vector boson. The two transverse polarization vectors for the massive vector boson may be chosen to be the same as found in QCD. The nontransverse and linearly independent third polarization vector is found to be parallel to the gauge direction. The corresponding sum over polarizations in the standard model, indicated by $K_{\mu\nu}(k)$, has several simplifying properties similar to the polarization sum $D_{\mu\nu}(k)$ in QCD. The framework is unitary and ghost free (except for the ghosts at $k^+ = 0$ associated with the light-cone gauge prescription). The massive gauge field propagator has well-behaved asymptotic behavior. The interaction Hamiltonian of electroweak theory can be expressed in a form resembling that of covariant theory, plus additional instantaneous interactions which can be treated systematically. The LF formulation also provides a transparent discussion of the Goldstone boson (or electroweak) equivalence theorem, as the illustrations show.

Light-front formulation of the standard model

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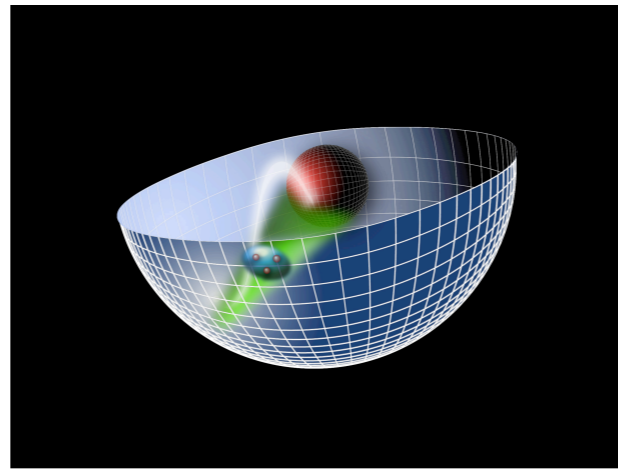
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Standard Model on the Light-Front

- Same phenomenological predictions
- Higgs field has three components
- Real part creates Higgs particle
- Imaginary part (Goldstone) become longitudinal components of W, Z
- *Higgs VEV of instant form becomes $k^+=0$ LF zero mode!*
- Analogous to a background static classical Zeeman or Stark Fields
- Zero contribution to T^{μ}_{μ} ; zero coupling to gravity

*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Light-Front Schrödinger Equation



Confinement scale:

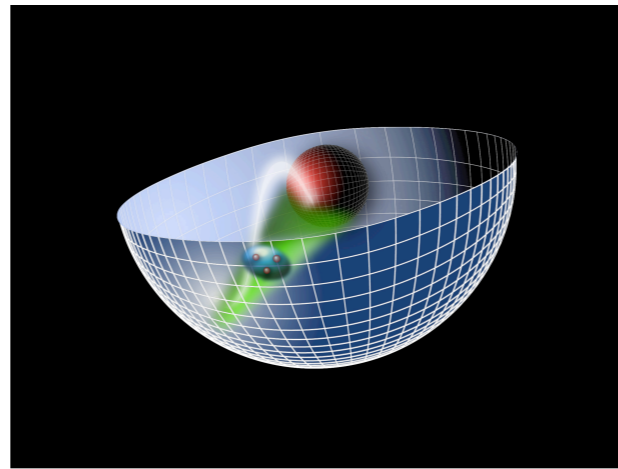
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● **Fubini, Rabinovici:**

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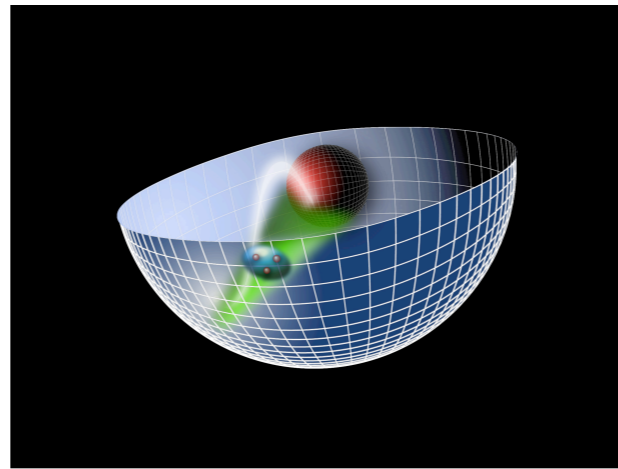
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*Preserves Conformal Symmetry
of the action*

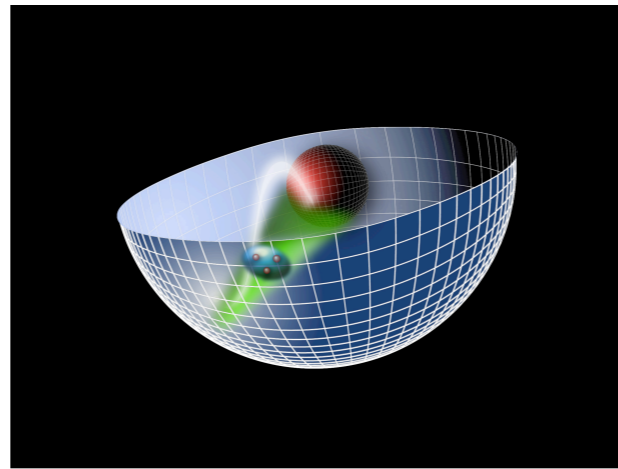
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***Unique
Confinement Potential!***

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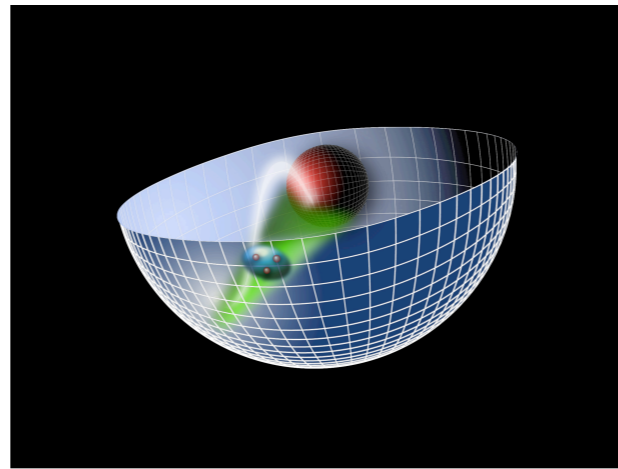
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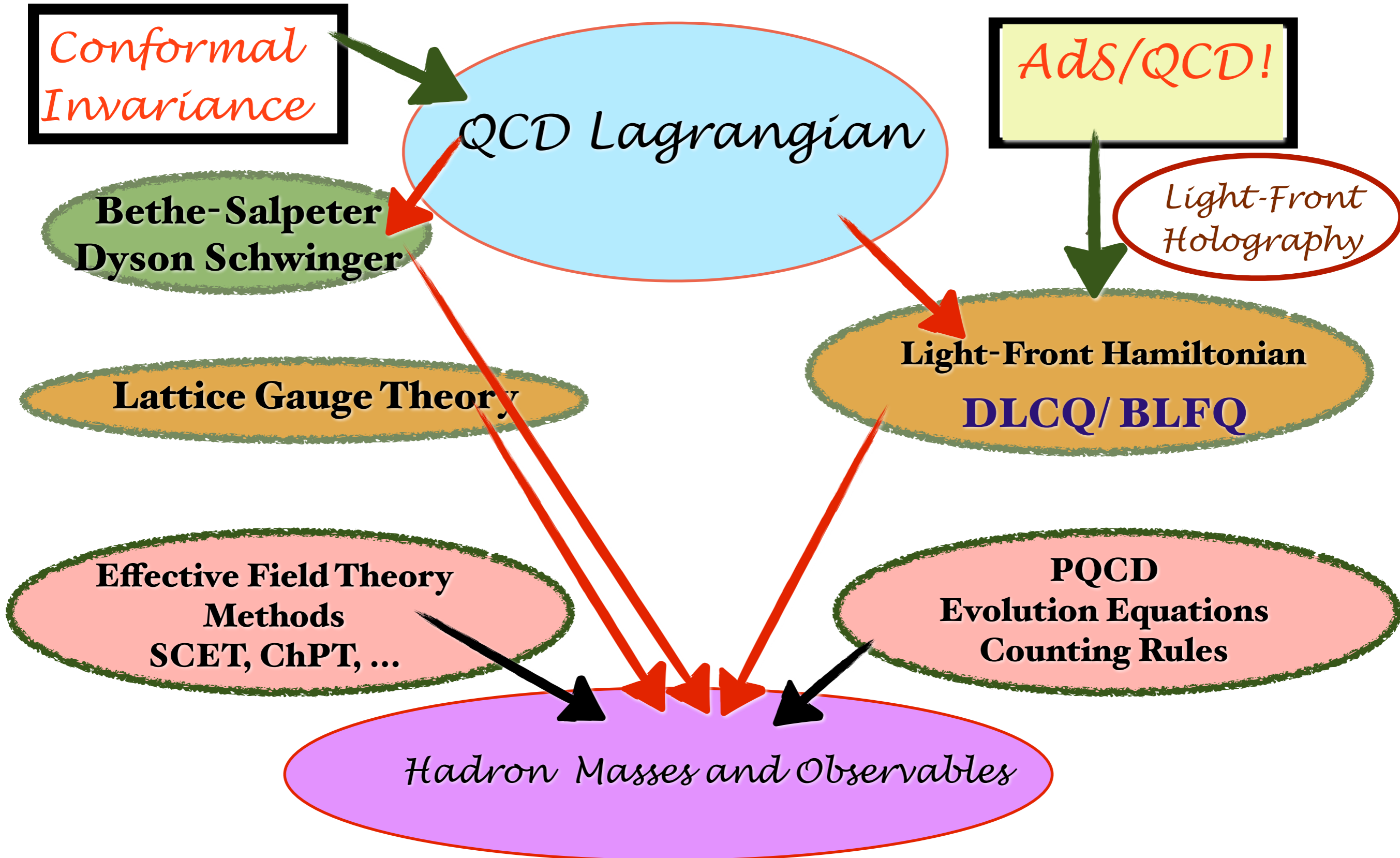
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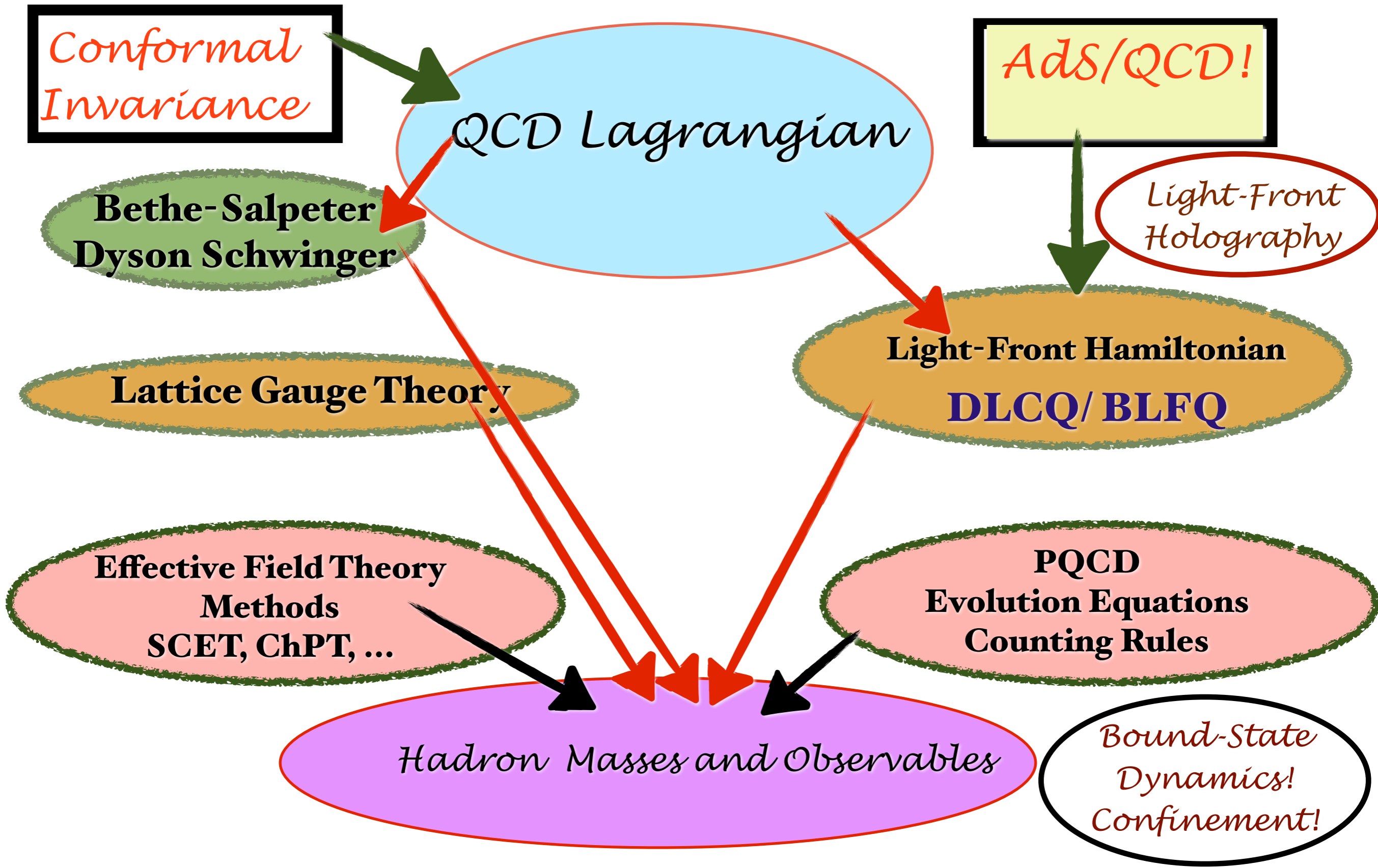
***Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!***

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Predict Hadron Properties from First Principles!

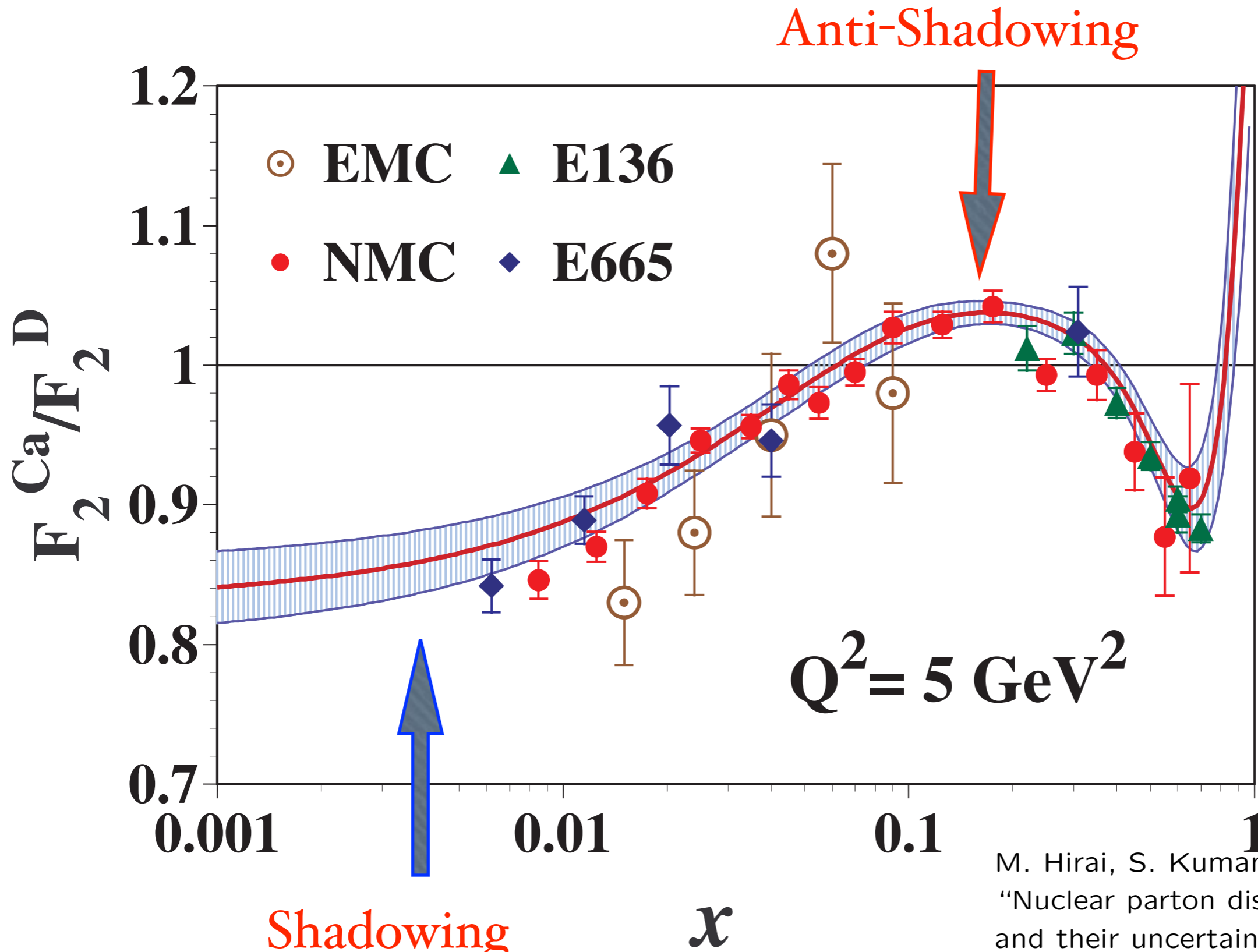


Predict Hadron Properties from First Principles!



QCD Myths

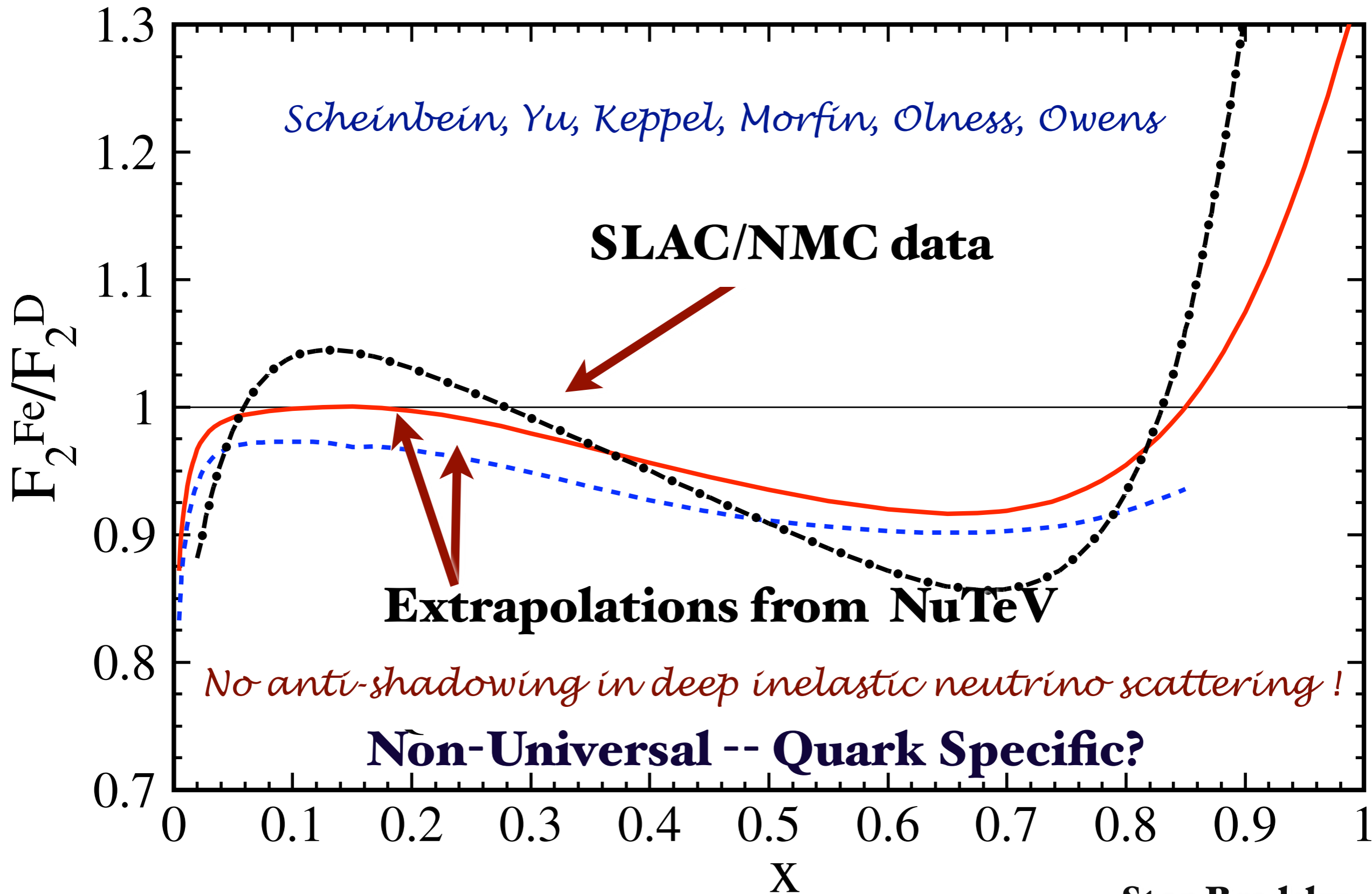
- **Anti-Shadowing is Universal**
- **ISI and FSI are higher twist effects and universal**
- **High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!**
- **Heavy quarks only from gluon splitting**
- **Renormalization scale cannot be fixed**
- **QCD condensates are vacuum effects**
- **QCD gives 10^{42} to the cosmological constant**



M. Hirai, S. Kumano and T. H. Nagai,
 "Nuclear parton distribution functions
 and their uncertainties,"
 Phys. Rev. C **70**, 044905 (2004)
 [arXiv:hep-ph/0404093]

Stan Brodsky

$$Q^2 = 5 \text{ GeV}^2$$



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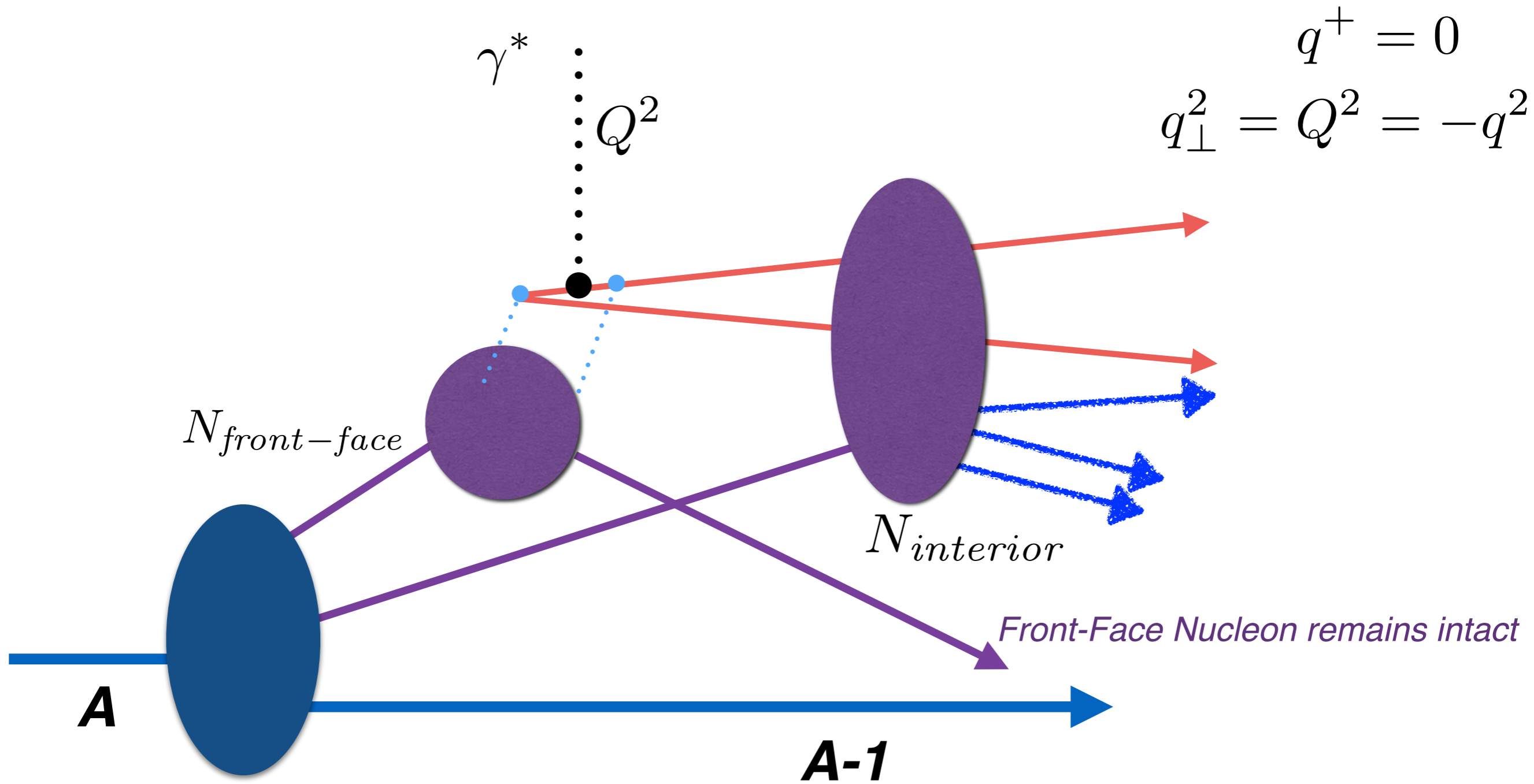
September 25, 2015

Light-Front Holographic QCD, Color Confinement,
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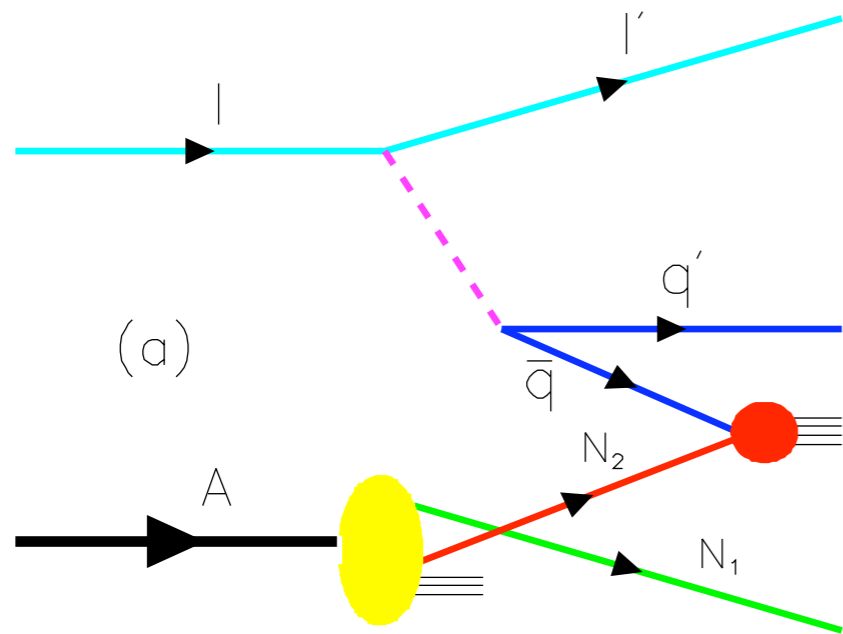
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Two-Step Process in the $q^+ = 0$ Parton Model Frame

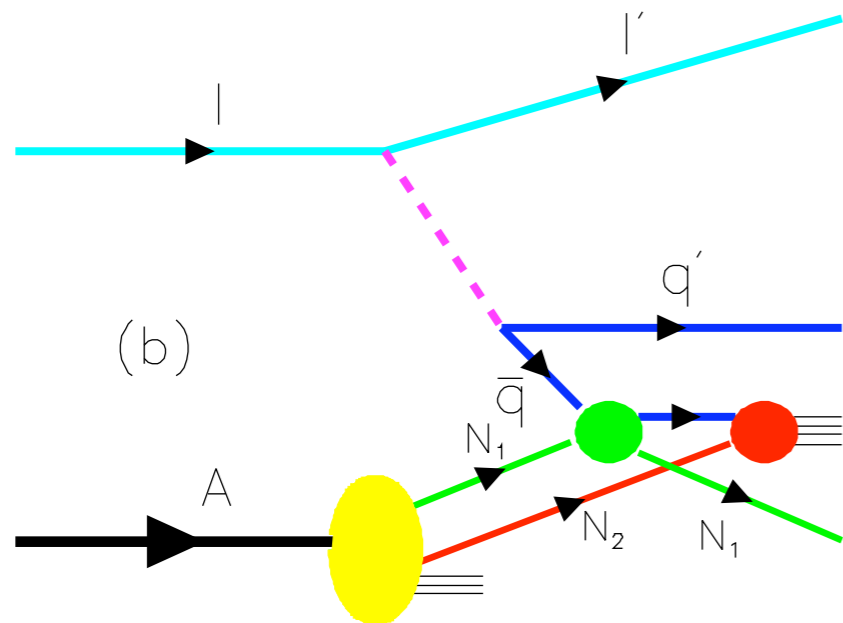
Illustrates the LF time sequence



The one-step and two-step processes in DIS on a nucleus.

(a)

Coherence at small Bjorken x_B :
 $1/Mx_B = 2\nu/Q^2 \geq L_A$.

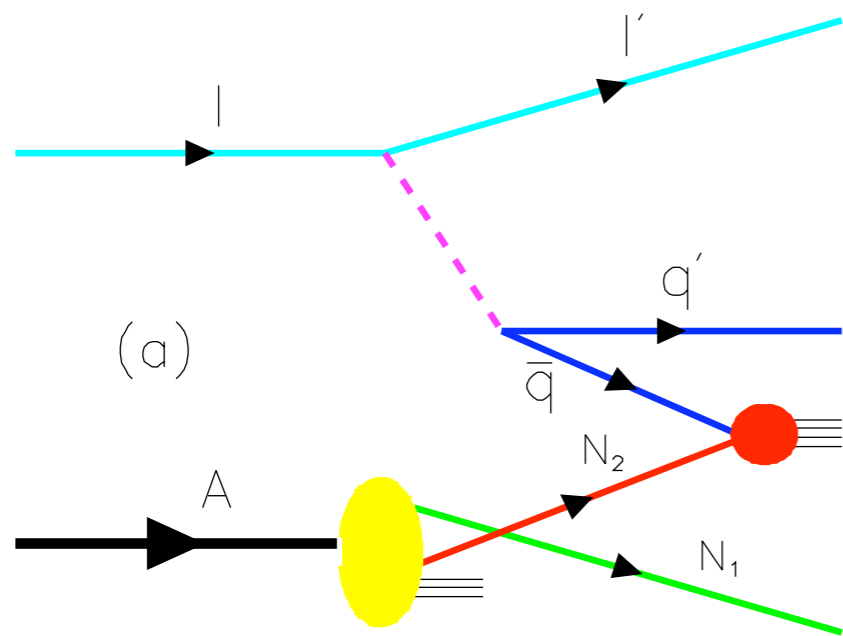


(b)

If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the \bar{q} flux reaching N_2 .

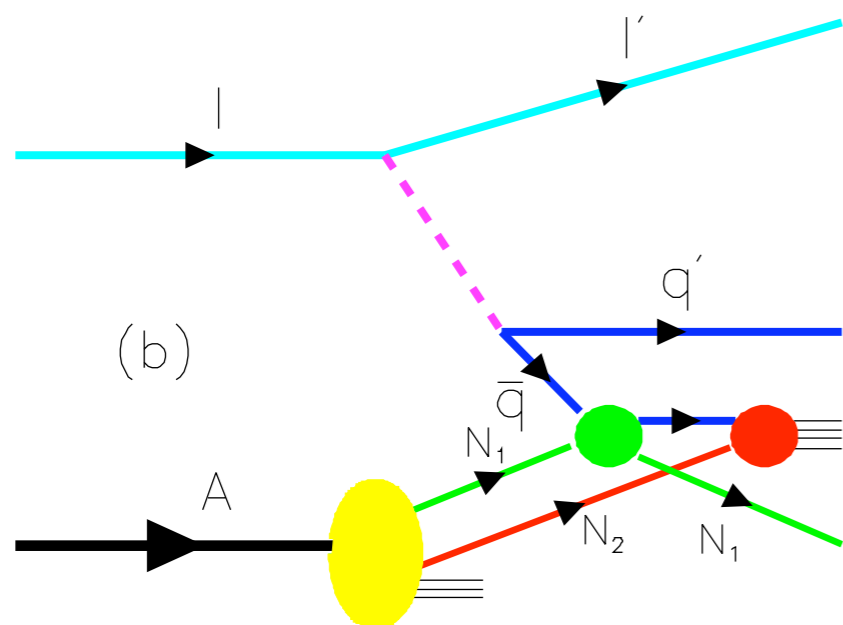
Diffraction via Pomeron gives destructive interference!

Shadowing



The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken x_B :
 $1/Mx_B = 2\nu/Q^2 \geq L_A$.



If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the \bar{q} flux reaching N_2 .

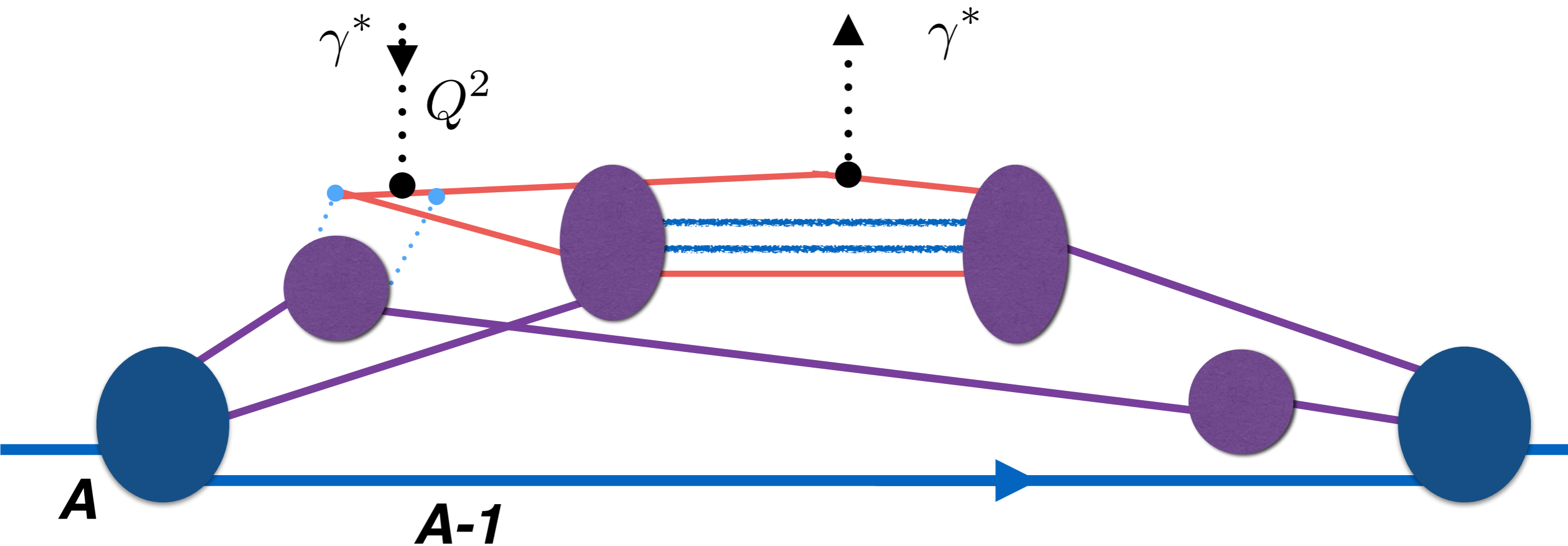
→ Shadowing of the DIS nuclear structure functions.

Diffraction via Pomeron gives destructive interference!

Shadowing

Illustrates the LF time sequence

$$q^+ = 0 \quad q_{\perp}^2 = Q^2 = -q^2$$



Front-Face Nucleon struck

Front-Face Nucleon not struck

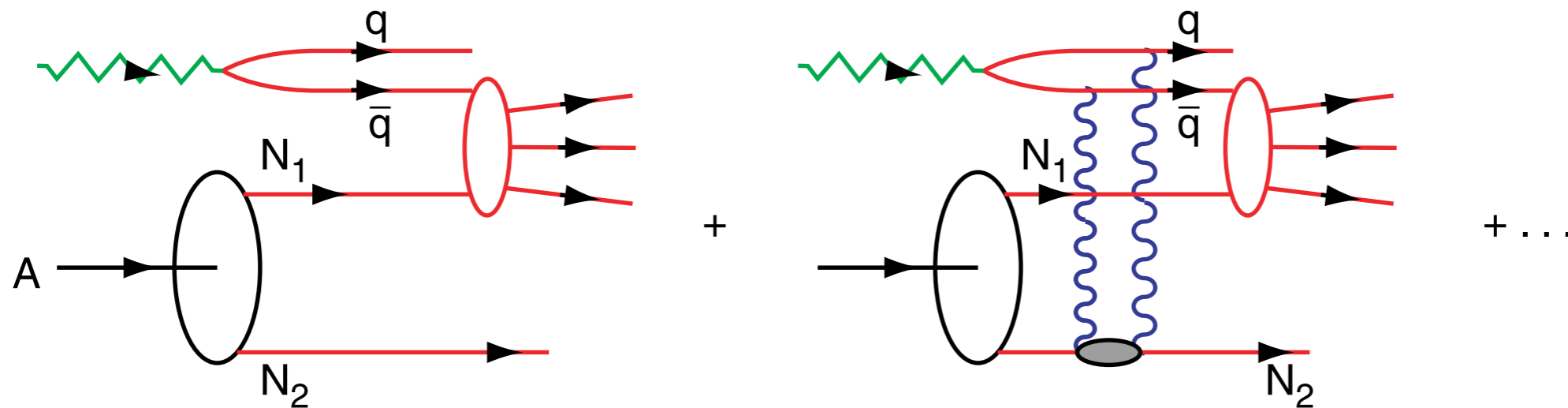
One-Step / Two-Step Interference

Study Double Virtual Compton Scattering $\gamma^* A \rightarrow \gamma^* A$

Cannot reduce to matrix element of local operator

Sum Rules not proven

Nuclear Shadowing in QCD



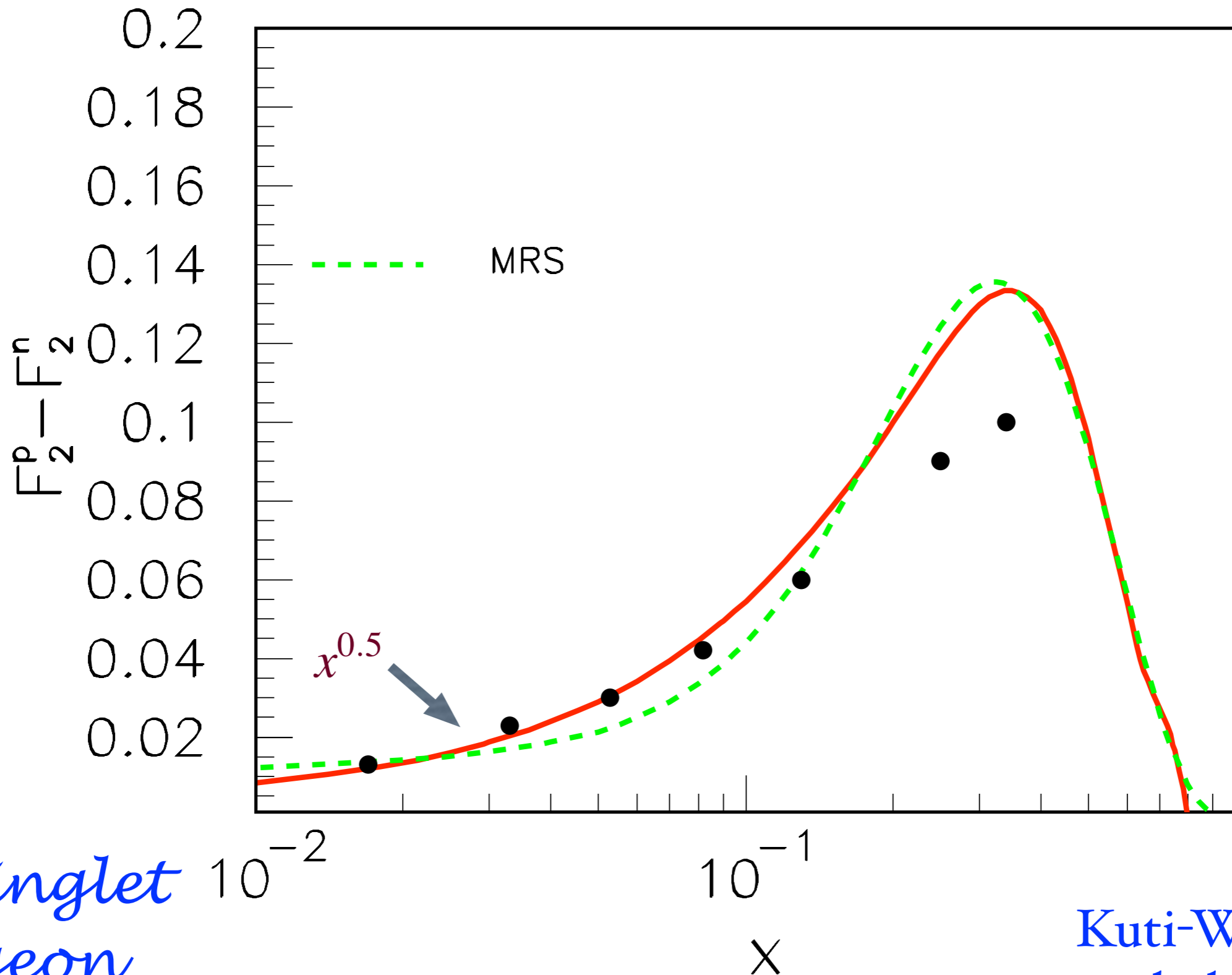
Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus

Diffraction via Reggeon gives constructive interference!

Anti-shadowing not universal



*Non-singlet
Reggeon
Exchange*

*Kuti-Weisskopf
behavior*

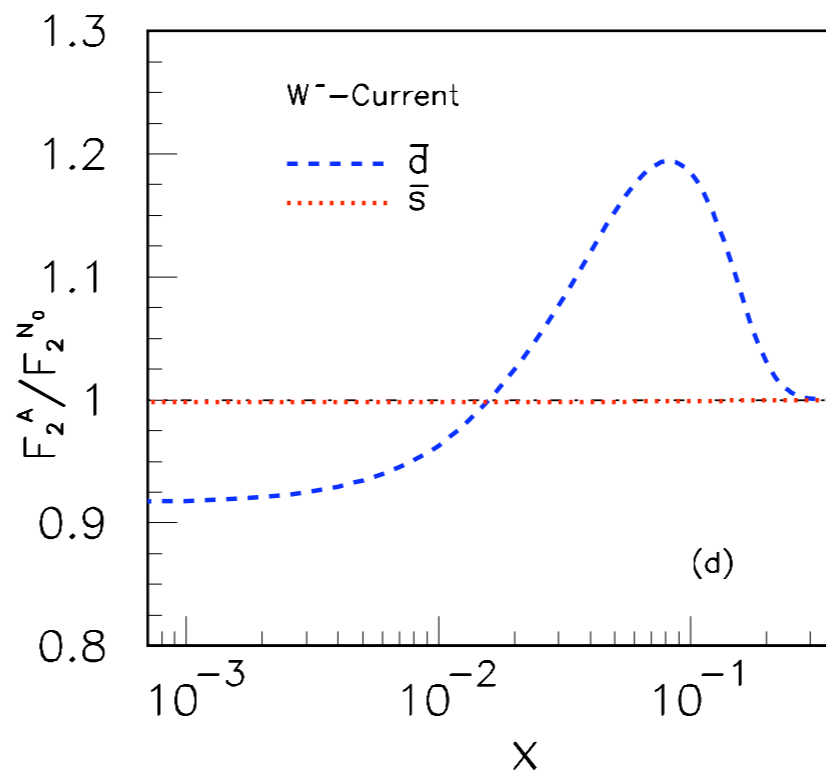
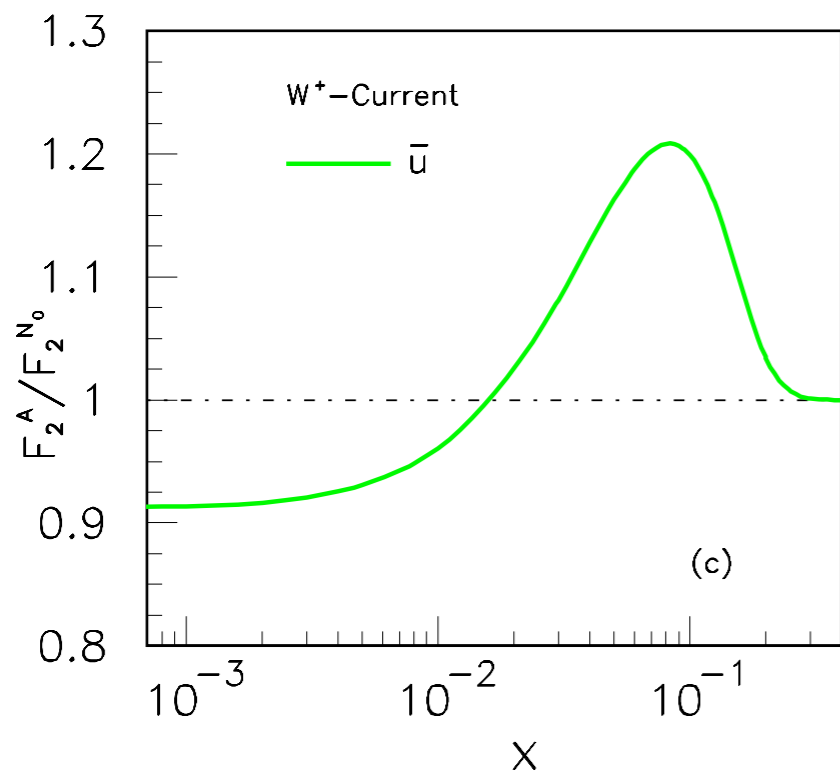
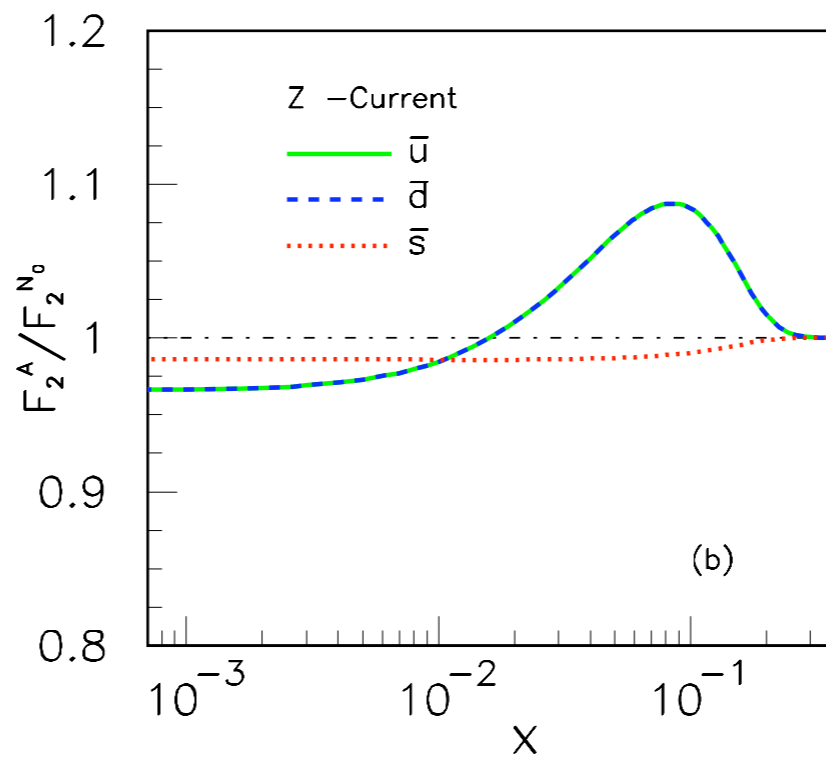
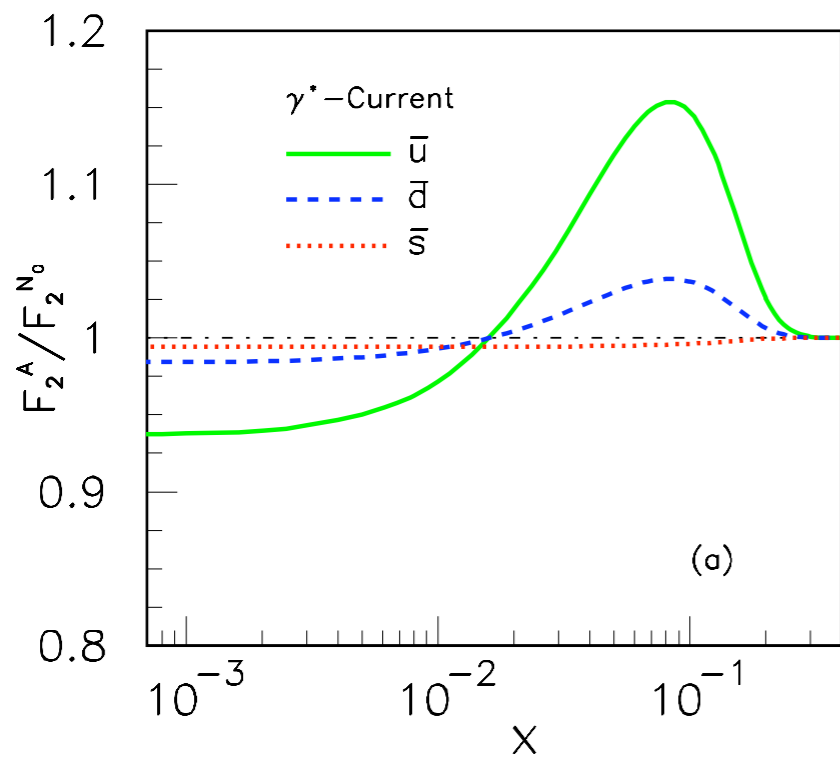
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September 25, 2015**

**Light-Front Holographic QCD, Color Confinement,
and Supersymmetric Features of QCD**

Stan Brodsky



Schmidt, Yang; sjb



Modifies
NuTeV extraction of
 $\sin^2 \theta_W$

Test in flavor-tagged
DIS at the EIC

Nuclear Antishadowing not universal !

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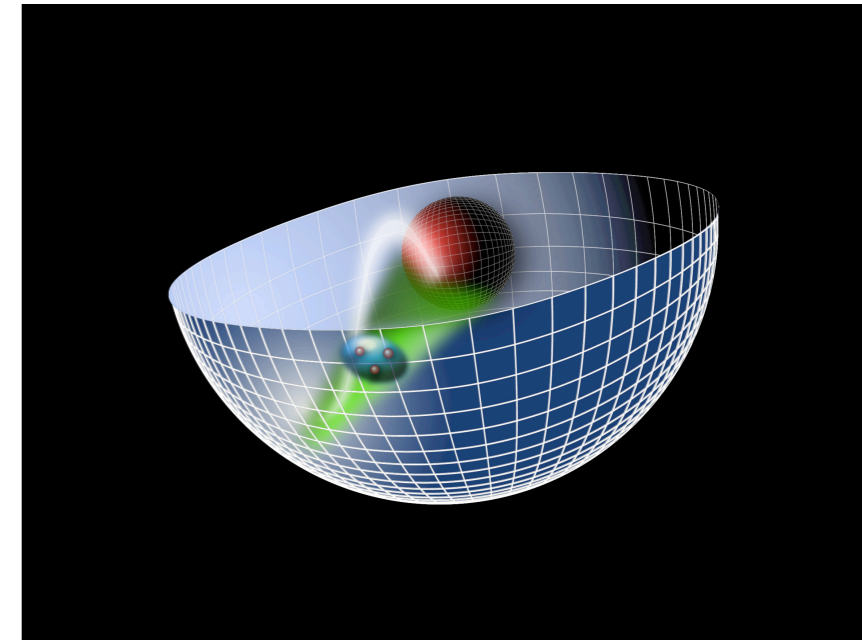
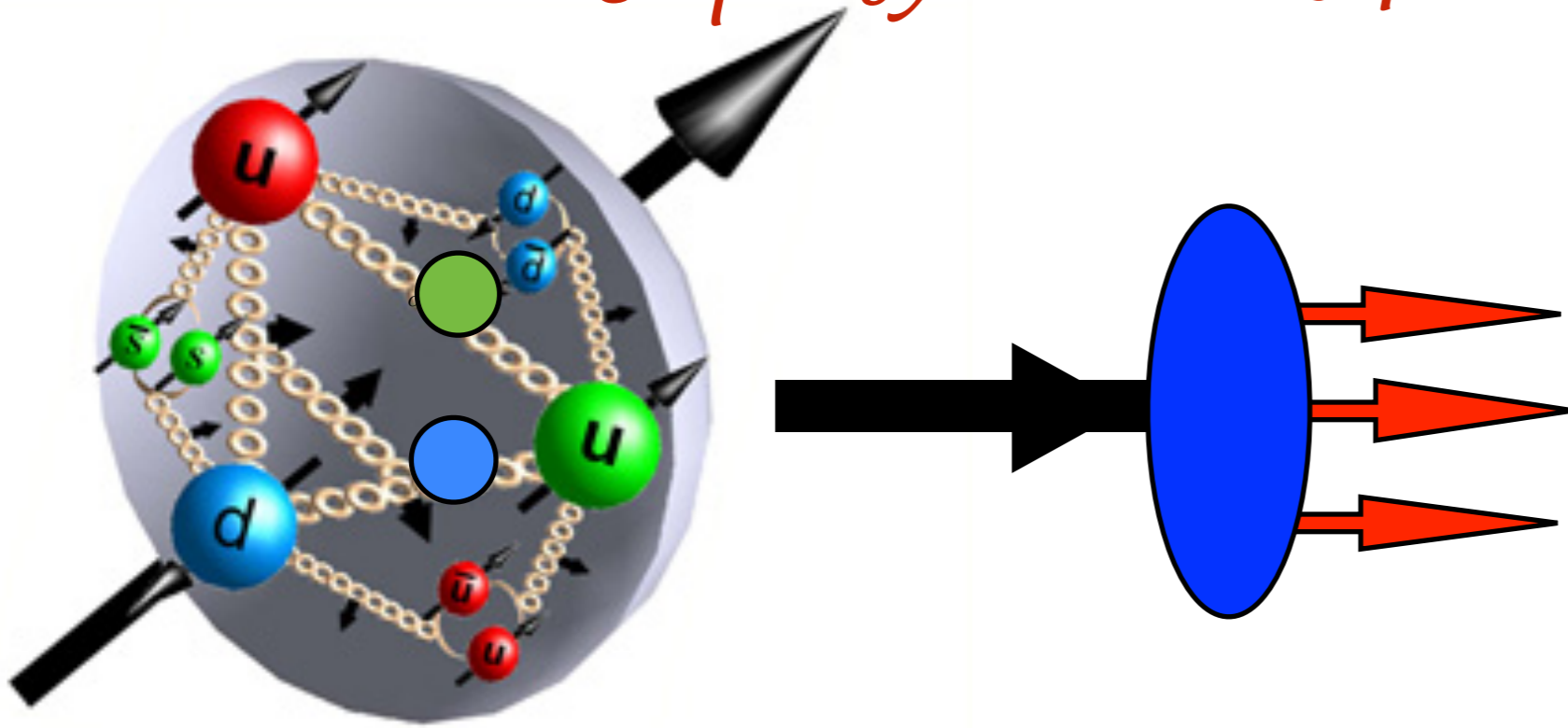
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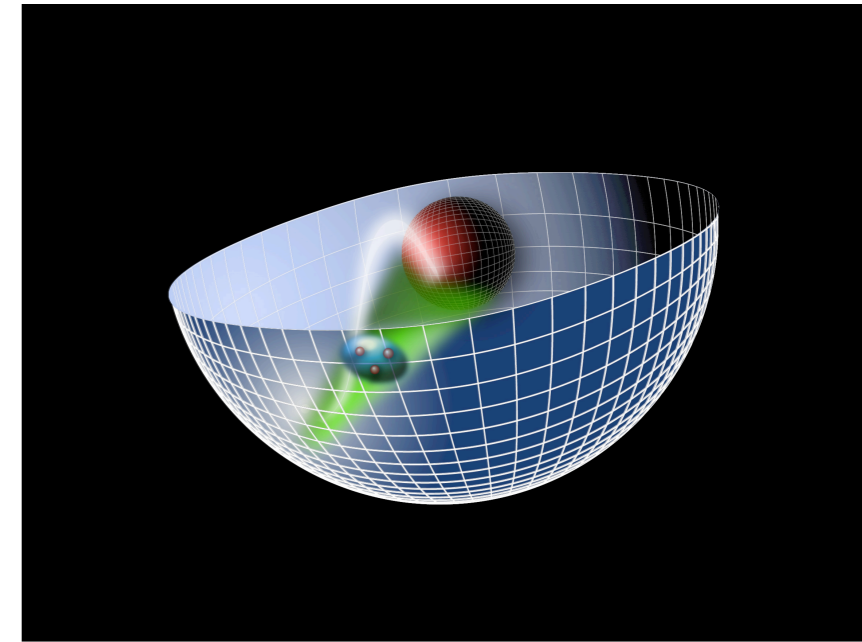
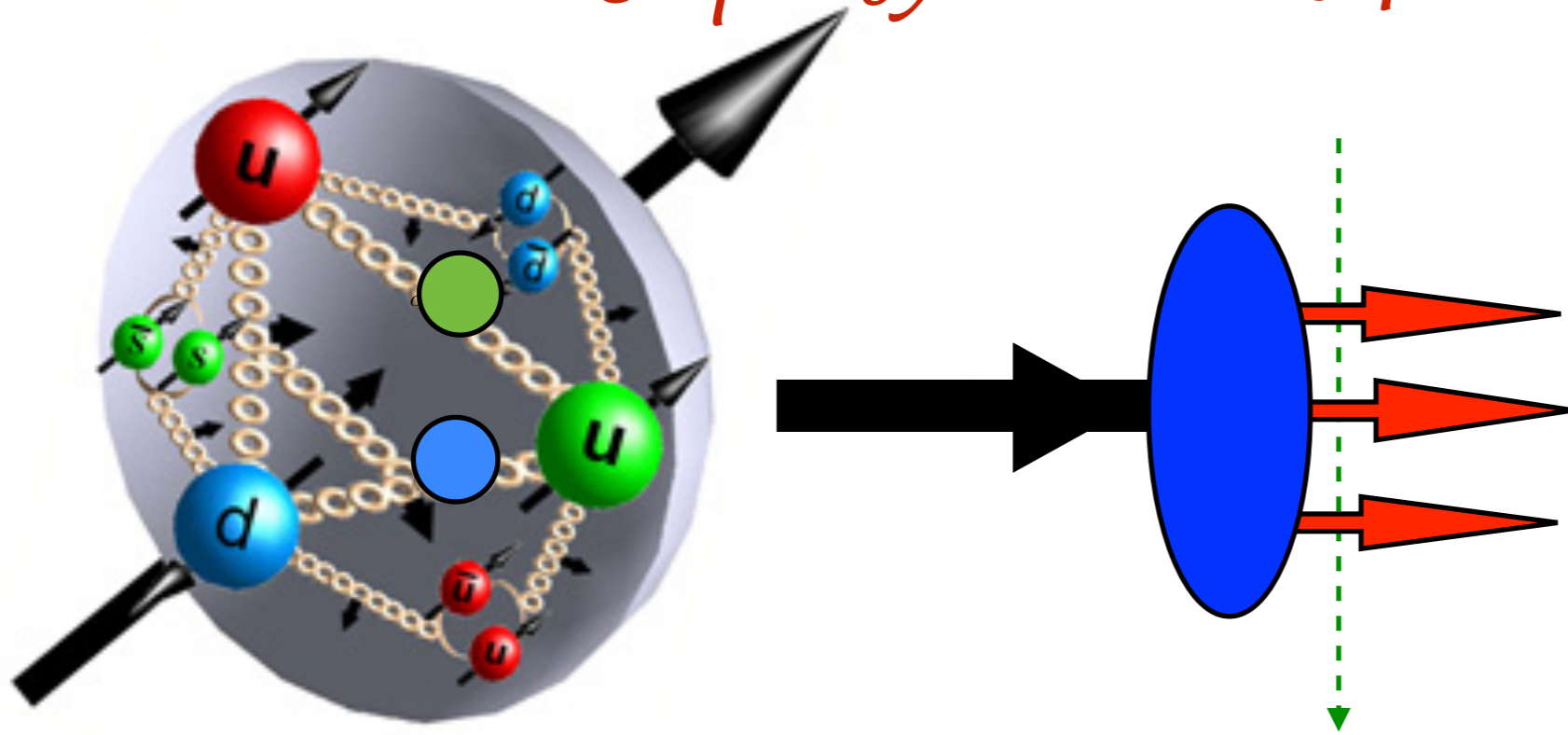
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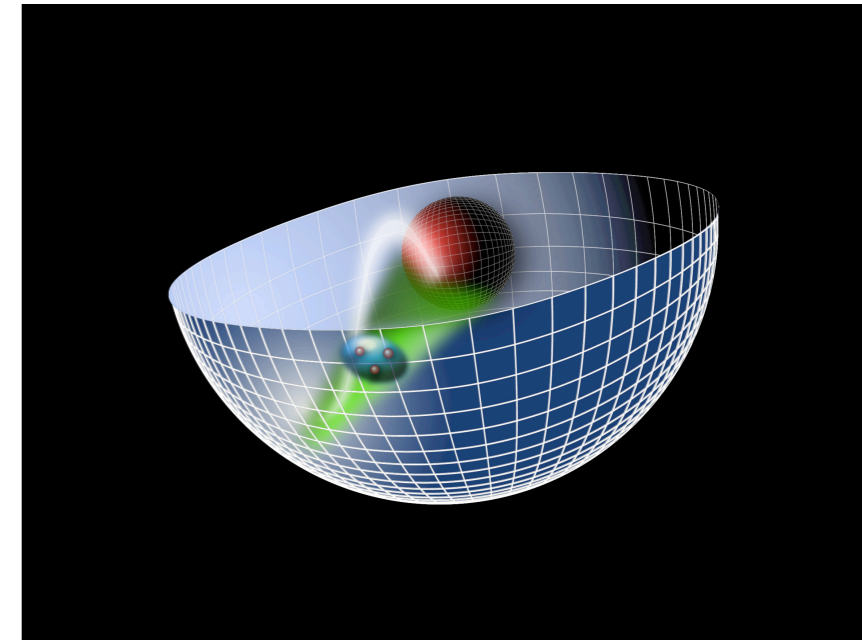
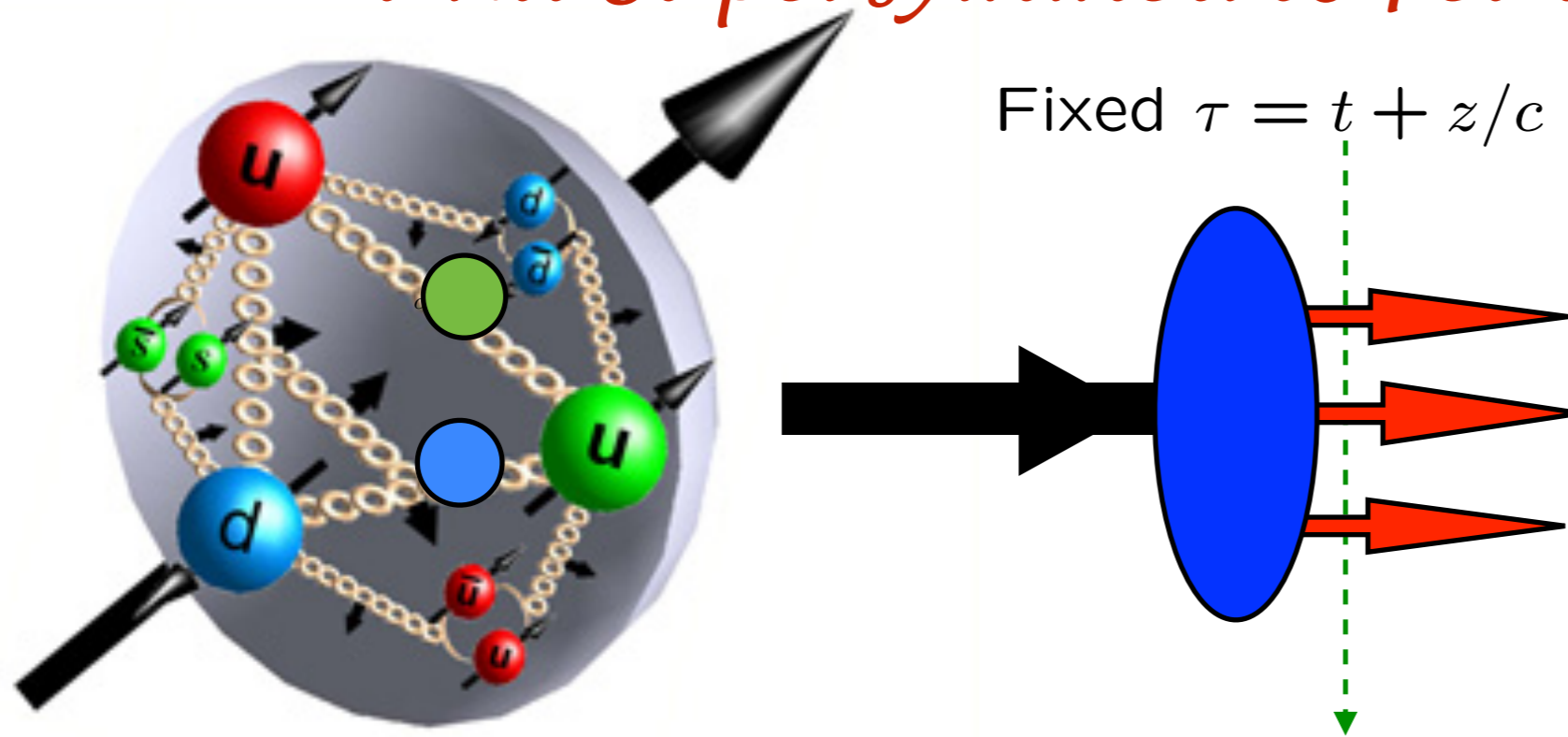


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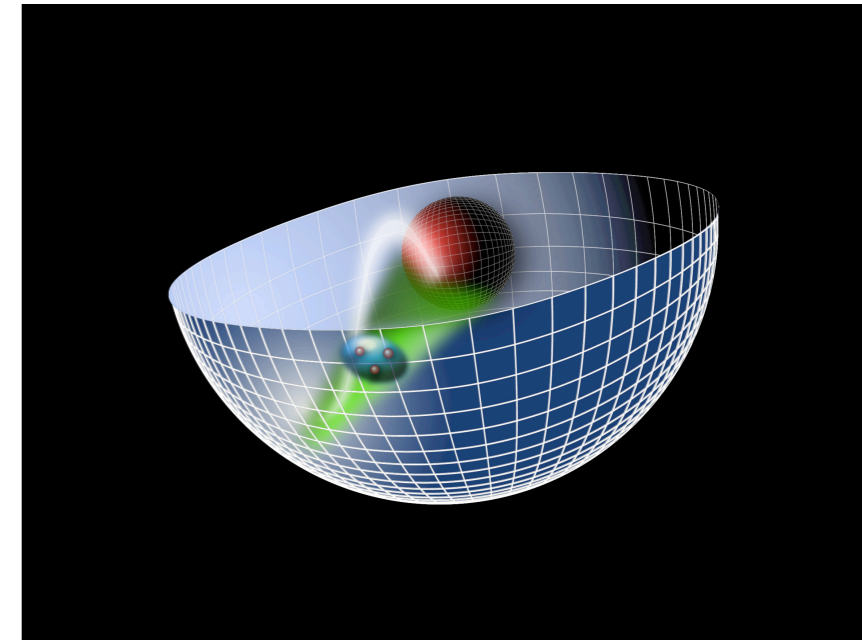
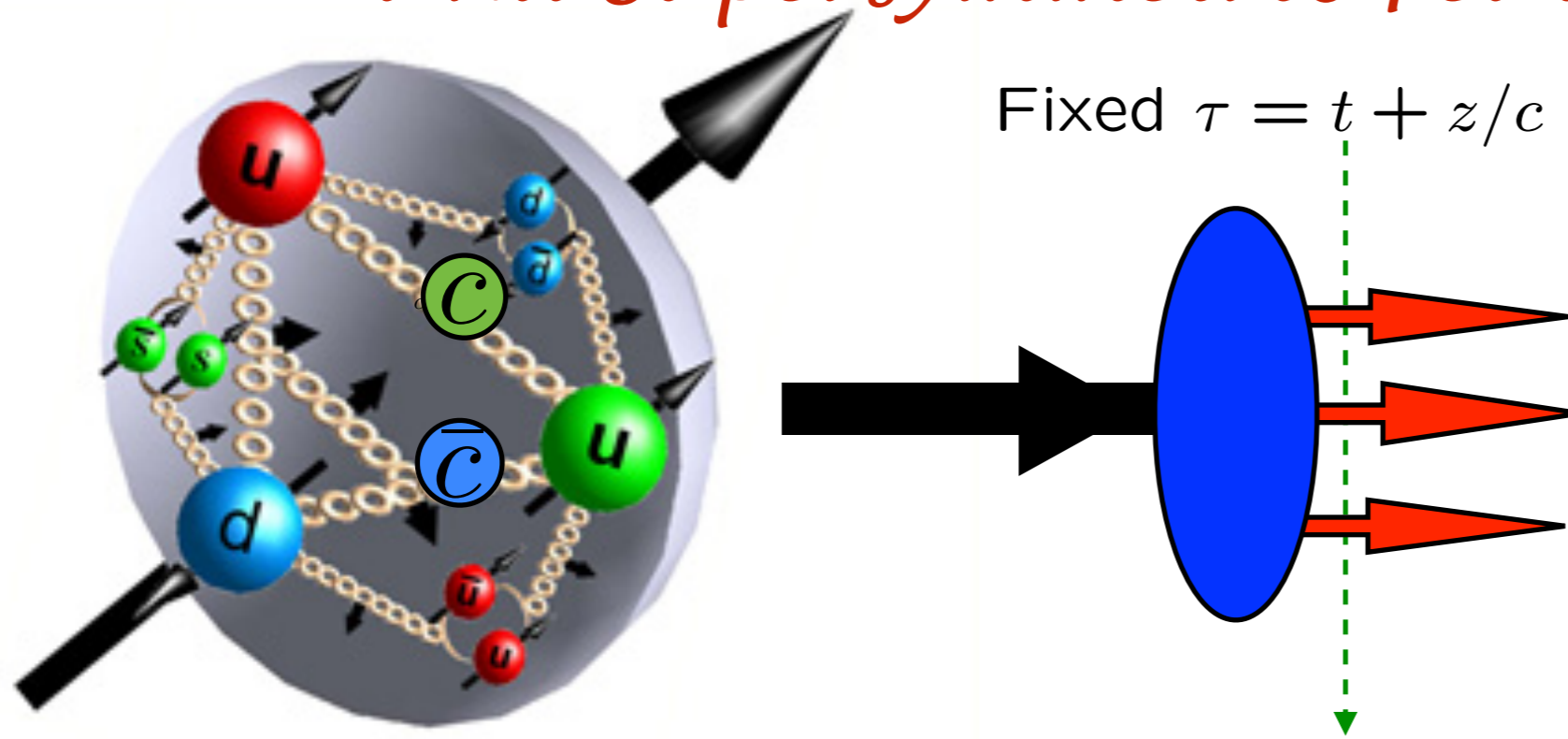
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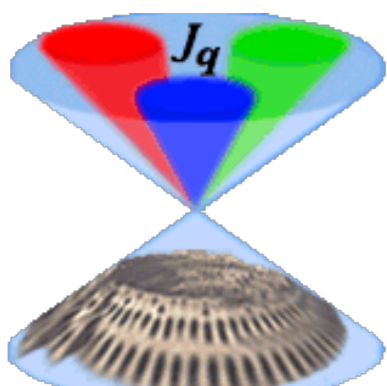
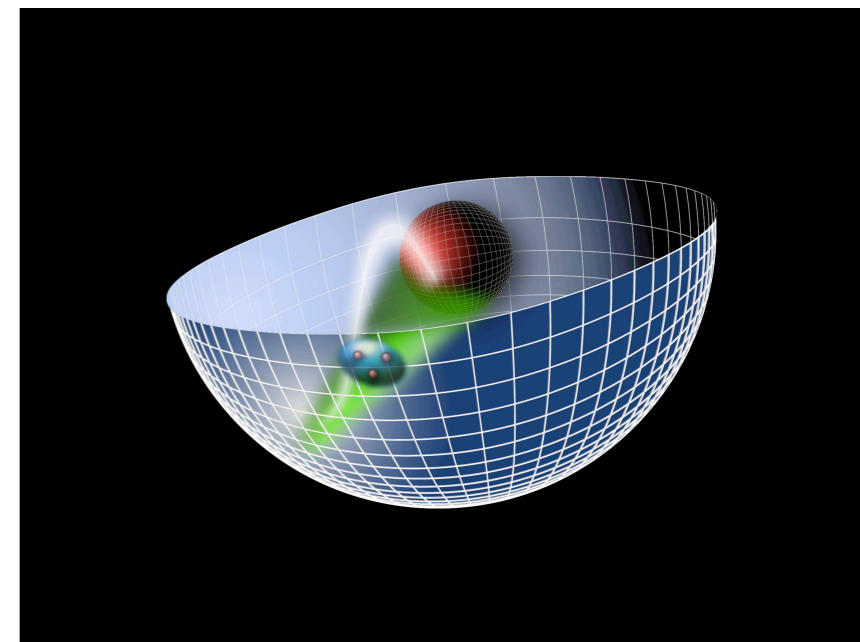
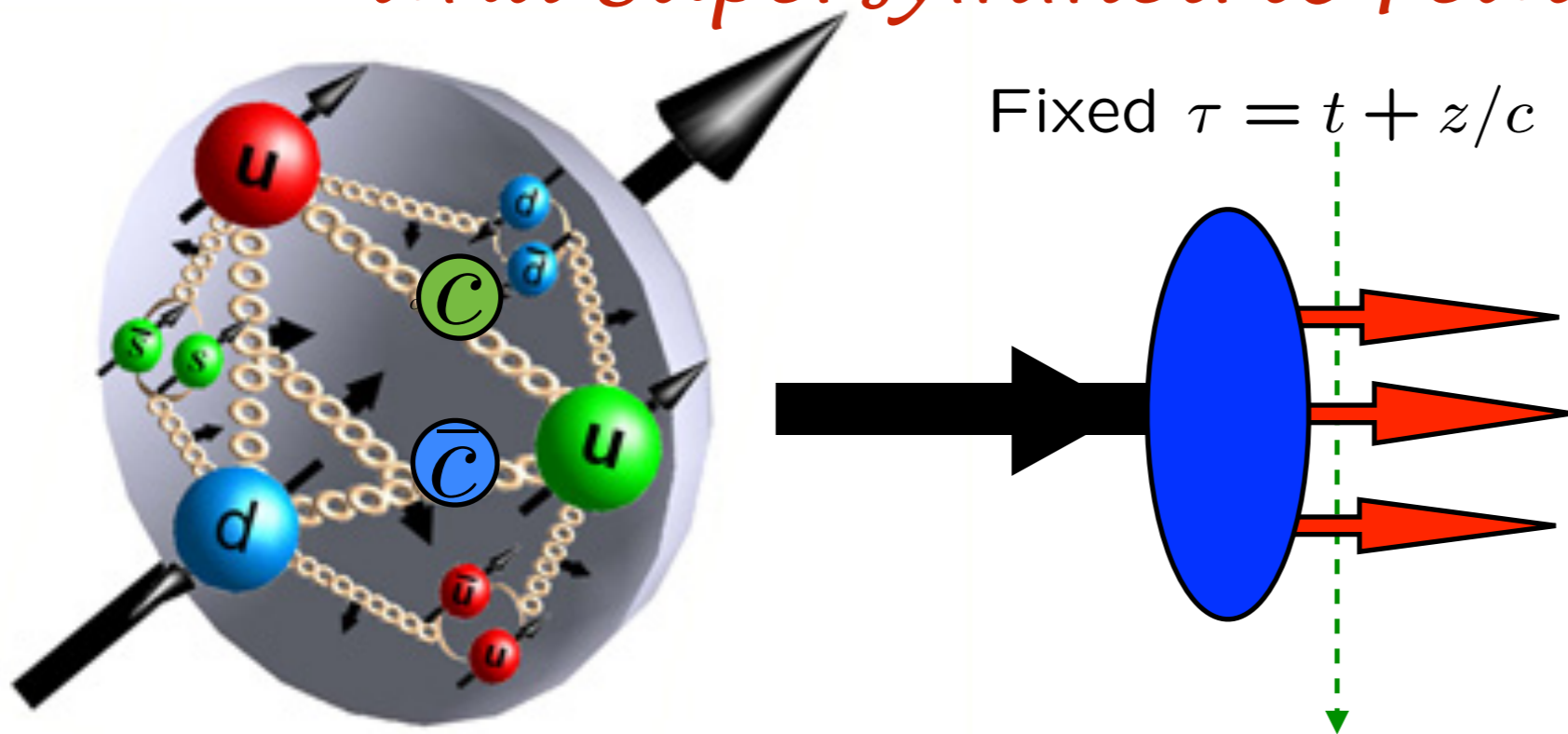
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SEPTEMBER 21 - 25, 2015

Topics

- Hadron Physics in present and future facilities
- AdS/CFT - Theory and applications
- Few-body problems on the Light Cone
- Relativistic models of nuclear and hadronic structures
- Non perturbative methods in quantum field theory
- Light-front field theory in QCD and QED
- Lattice gauge theories

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designed by Piero Gatta

Thanks for a Wonderful LC2015!

Barbara Pasquini
Giovanni Salmè

INFN Frascati National Laboratories



LC2016: Lisbon, Portugal