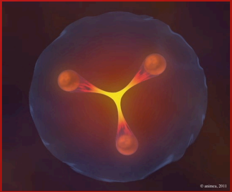
DE LA RECHERCHE À L'INDUSTRIE



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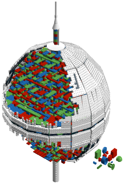
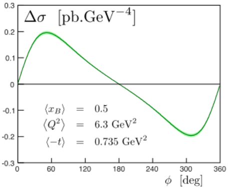
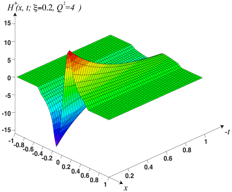
Sketching the pion's valence-quark Generalized Parton Distribution





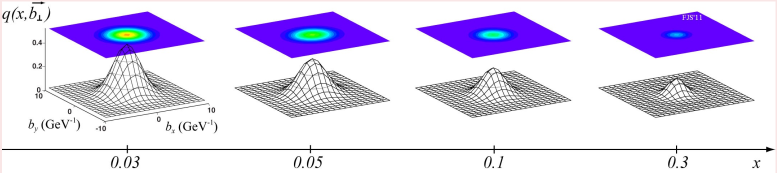


Light Cone 2015 | **Hervé MOUTARDE**

Sept. 24$^{\text{th}}$, 2015

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in the nucleon.
- Insights on:
  - **Spin** structure,
  - **Energy-momentum** structure.
- **Probabilistic interpretation** of Fourier transform of $\mathrm{GPD}(x, \xi = 0, t)$ in **transverse plane**.

Transverse plane density (Goloskokov and Kroll model)



Overview.

Development of a new GPD model in the Dyson-Schwinger and Bethe-Salpeter framework.

- Important topic for several **past, existing and future** experiments: H1, ZEUS, HERMES, CLAS, CLAS12, JLab Hall A, COMPASS, EIC, ...
- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- **Recent applications** of the Dyson-Schwinger and Bethe-Salpeter framework to **hadron structure studies**.

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- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- **Recent applications** of the Dyson-Schwinger and Bethe-Salpeter framework to **hadron structure studies**.

- Here develop **pion GPD model** for simplicity.
- No planned experiment on pion GPDs but existing proposal of DVCS on a virtual pion.

Amrath *et al.*, Eur. Phys. J. **C58**, 179 (2008)

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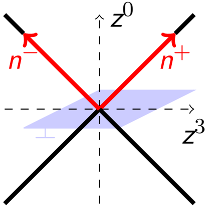
GPDs: Theoretical Framework

$$H_\pi^q(x, \xi, t) = \frac{1}{2} \int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- From **isospin symmetry**, all the information about pion GPD is encoded in $H^u_{\pi^+}$ and $H^d_{\pi^+}$.
- Further constraint from **charge conjugation**: $H^u_{\pi^+}(x, \xi, t) = -H^d_{\pi^+}(-x, \xi, t)$.

- PDF **forward limit**

$$H^q(x, 0, 0) = q(x)$$

- PDF **forward limit**
- Form factor **sum rule**

$$\int_{-1}^{+1} dx\, H^q(x, \xi, t) = F_1^q(t)$$

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality**

$$\int_{-1}^{+1} dx\, x^n H^q(x, \xi, t) = \text{polynomial in } \xi$$

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality**
- **Positivity**

$$H^q(x, \xi, t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality**
- **Positivity**
- H^q is an **even function** of ξ from time-reversal invariance.

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- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.

Properties.

Generalization of form factors and Parton Distribution Functions.

- PDF **forward limit**
- Form factor **sum rule**
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- H^q has support $x \in [-1, +1]$.
- **Soft pion theorem** (pion target)

$$H^q(x, \xi = 1, t = 0) = \frac{1}{2}\phi_\pi^q\left(\frac{1+x}{2}\right)$$

- PDF **forward limit**
- Form factor **sum rule**
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Numerous theoretical constraints on GPDs.

- There is no known GPD parameterization **relying only on first principles**.
- Modeling becomes a key issue.

- A function satisfying a polynomiality property is the **Radon transform** of another function.
- Representation of GPD in terms of **Double Distributions**:

$$H^q(x,\xi,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x-\beta-\alpha\xi)\big(F^q(\beta,\alpha,t) + \xi\, G^q(\beta,\alpha,t)\big)$$

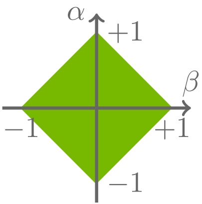
Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Radyushkin, Phys. Rev. **D59**, 014030 (1999)
Radysuhkin, Phys. Lett. **B449**, 81 (1999)

- Support property: $x \in [-1,+1]$.
- Discrete symmetries: F^q is α-even and G^q is α-odd.

- Define Double Distributions F^q and G^q as matrix elements of **twist-2 quark operators**:

$$\left\langle P+\frac{\Delta}{2}\middle|\bar{q}(0)\gamma^{\{\mu}i\overleftrightarrow{D}^{\mu_1}\ldots i\overleftrightarrow{D}^{\mu_m\}}q(0)\middle|P-\frac{\Delta}{2}\right\rangle = \sum_{k=0}^{m}\binom{m}{k}$$

$$\left[F^q_{mk}(t)2P^{\{\mu}-G^q_{mk}(t)\Delta^{\{\mu}\right]P^{\mu_1}\ldots P^{\mu_{m-k}}\left(-\frac{\Delta}{2}\right)^{\mu_{m-k+1}}\ldots\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$



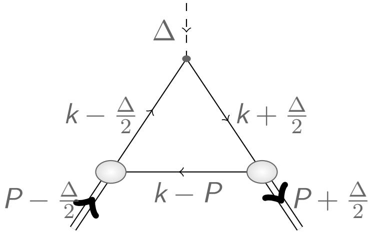
with

$$F^q_{mk} = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\alpha^k\beta^{m-k}F^q(\beta,\alpha)$$

$$G^q_{mk} = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\alpha^k\beta^{m-k}G^q(\beta,\alpha)$$

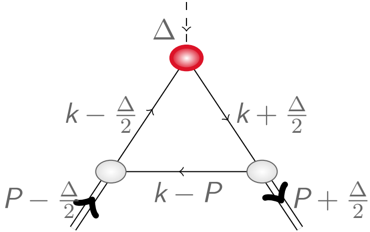
GPDs in the Dyson-Schwinger and Bethe-Salpeter Approach

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD *H*.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$

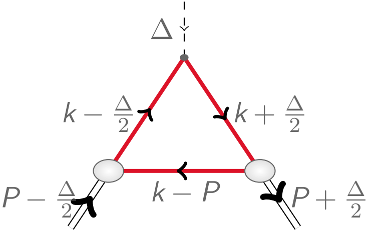


- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.

GPDs in the rainbow ladder approximation.

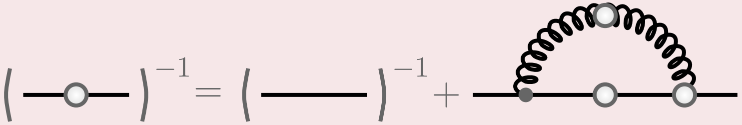
Evaluation of triangle diagrams.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P-\frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.

Dyson - Schwinger equation



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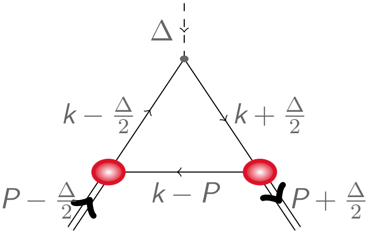

- Compute **Mellin moments** of the pion GPD *H*.
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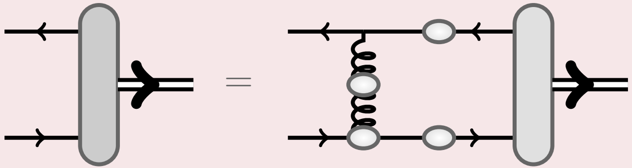
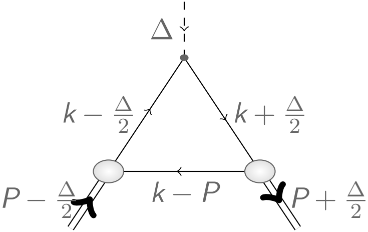
Bethe - Salpeter equation



GPDs in the rainbow ladder approximation.

Evaluation of triangle diagrams.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+ (i \overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

- Most GPD properties **satisfied by construction**.

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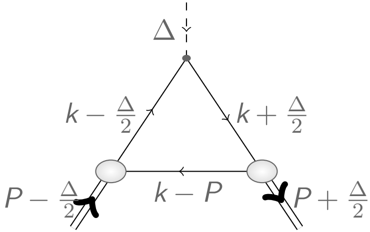
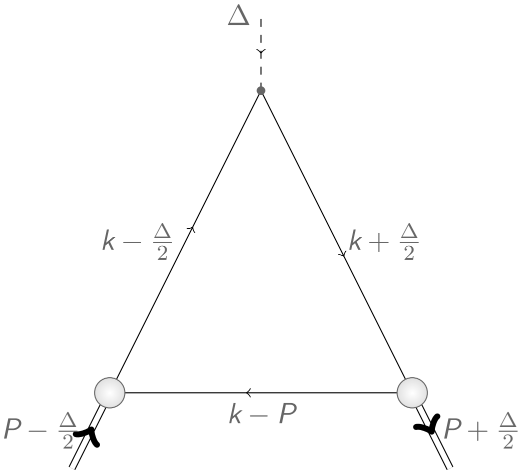
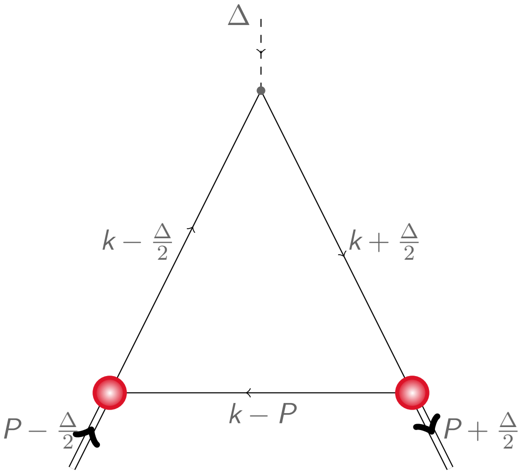


- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

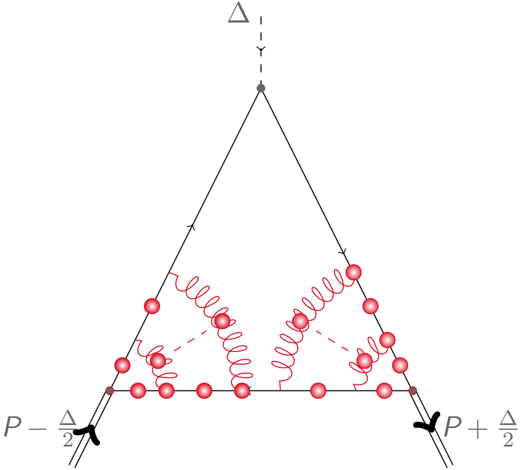
- Most GPD properties **satisfied by construction**.
- Also compute crossed triangle diagram.

Mezrag *et al.*, arXiv:1406.7425 [hep-ph] and Phys. Lett. **B741**, 190 (2015)


Δ
$k - \frac{\Delta}{2}$
$k + \frac{\Delta}{2}$
$k - P$
$P - \frac{\Delta}{2}$
$P + \frac{\Delta}{2}$



- Bethe-Salpeter vertex.



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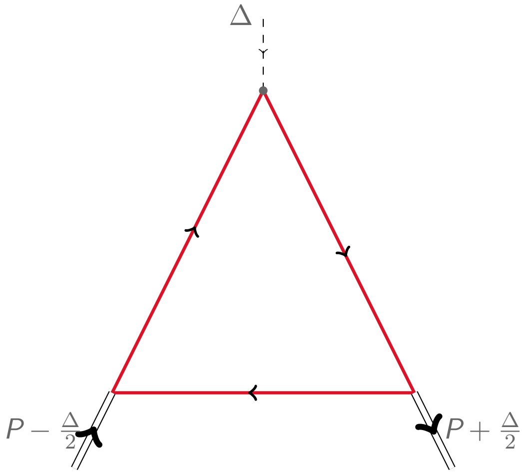
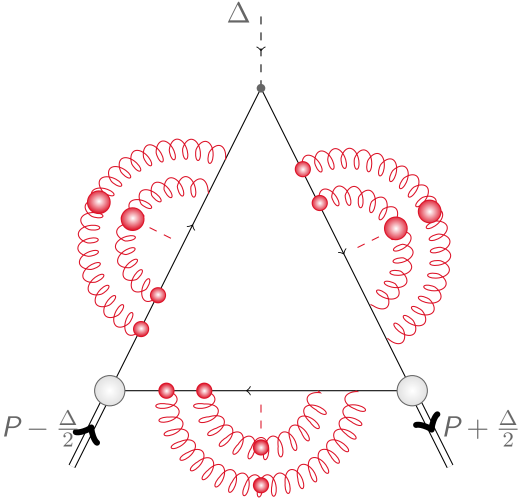


- Bethe-Salpeter vertex.
- Dressed quark propagator.



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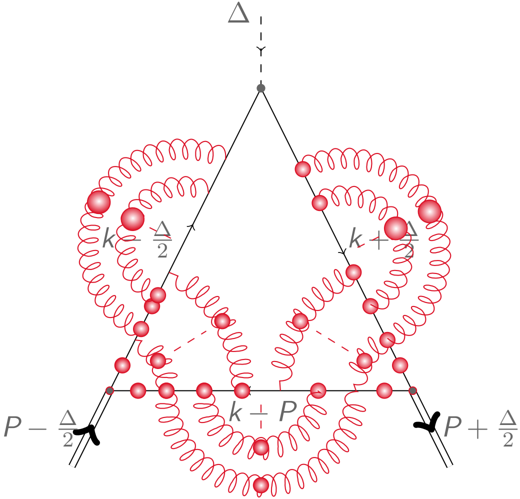
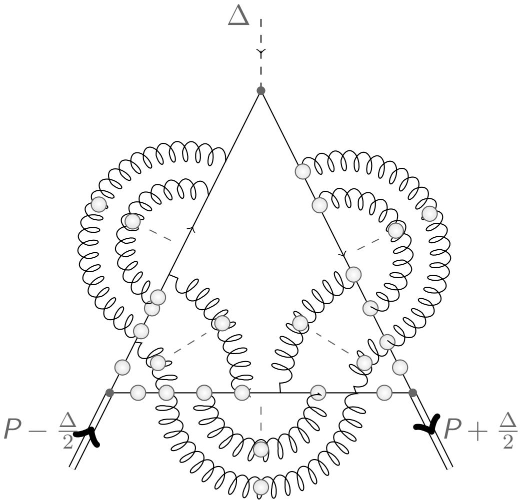


- Bethe-Salpeter vertex.
- Dressed quark propagator.
- Much more than tree level perturbative diagram!



- Bethe-Salpeter vertex.
- Dressed quark propagator.
- Much more than tree level perturbative diagram!
- Enable description of **non perturbative phenomena**.

- **Polynomiality** from Poincaré covariance.

Symmetry-preserving truncation.

Most of the GPD properties are obtained *a priori*.

- **Polynomiality** from Poincaré covariance.
- **Soft pion theorem** from **symmetry-preserving** truncation of Bethe-Salpeter and gap equations.

Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)

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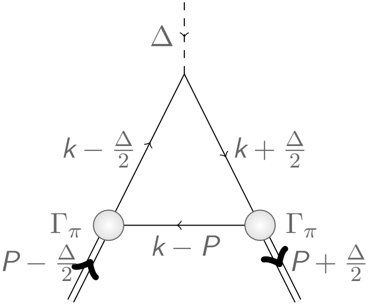
Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)

- Mellin moments.



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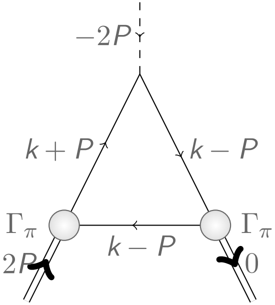
Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)



- Mellin moments.
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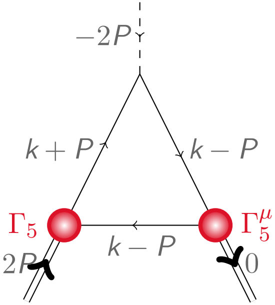

- Mellin moments.
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- Axial and axial vector vertices Γ_5, Γ_5^μ in chiral limit.

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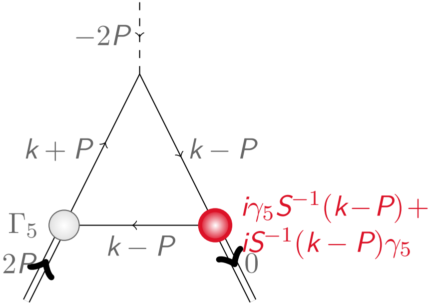
Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)



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- Axial-vector Ward identity.

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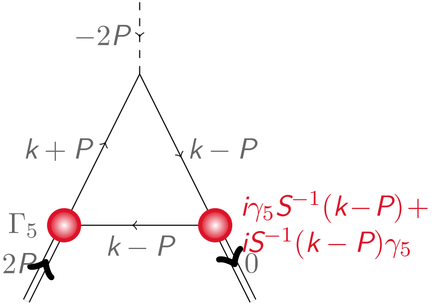
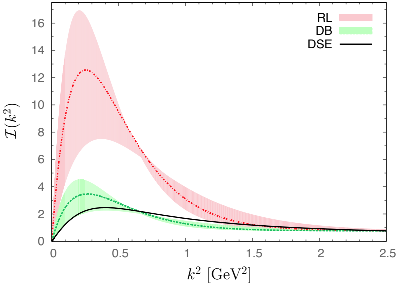
Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)



- Mellin moments.
- Soft pion kinematics.
- Axial and axial vector vertices Γ_5, Γ_5^μ in chiral limit.
- Axial-vector Ward identity.
- Recover pion DA Mellin moments.

- Gap equation kernel depends on **interaction strength** function $\mathcal{I}(k^2)$.
- Current model of $\mathcal{I}(k^2)$ yields ground and excited-state hadron masses with a **10-15 % accuracy** compared to experimental data.

Roberts *et al.*, Few Body Syst. **51**, 1 (2011)



- Good agreement with **independent evaluation** from lattice data + Dyson-Schwinger equations.

Binosi *et al.*, Phys. Lett. **B742**, 183 (2015)

- Numerical resolution of gap and Bethe-Salpeter equations in Euclidean space.
- Analytic continuation to Minkowskian space required.
- **Ill-posed** problem in the sense of Hadamard.
- Parameterize solutions and fit to numerical solution:

Gap Complex-conjugate pole representation:

$$S(k) = \sum_{i=0}^{N} \left[\frac{z_i}{i\not{k} + m_i} + \frac{z_i^*}{i\not{k} + m_i^*} \right]$$

Bethe-Salpeter Nakanishi representation of amplitude $\mathcal{F}_\pi$:

$$\mathcal{F}_\pi(q^2, q \cdot P) = \int_{-1}^{+1} \mathrm{d}\alpha \int_0^\infty \mathrm{d}\lambda \frac{\rho(\alpha, \lambda)}{(q^2 + \alpha q \cdot P + \lambda^2)^n}$$

Algebraic model.

Intermediate step before using numerical solutions of Dyson-Schwinger and Bethe-Salpeter equations.

- Expressions for vertices and propagators:

$$S(p) = \left[-i\gamma \cdot p + M\right]\Delta_M(p^2)$$

$$\Delta_M(s) = \frac{1}{s + M^2}$$

$$\Gamma_\pi(k, p) = i\gamma_5 \frac{M}{f_\pi} M^{2\nu} \int_{-1}^{+1} \mathrm{d}z\, \rho_\nu(z)\ \left[\Delta_M(k_{+z}^2)\right]^\nu$$

$$\rho_\nu(z) = R_\nu (1 - z^2)^\nu$$

with R_ν a normalization factor and $k_{+z} = k - p(1 - z)/2$.

Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:

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- Only two parameters:
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with R_ν a normalization factor and $k_{+z} = k - p(1-z)/2$.

Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:
  - Dimensionful parameter M.
  - Dimensionless parameter ν. **Fixed to 1** to recover asymptotic pion DA.

Results: Theoretical Constraints and Phenomenology

Verification of theoretical constraints.

Analytic expressions for the GPD H.

- **Analytic expression** in the DGLAP region.

$$H^u_{x\geq\xi}(x,\xi,0) = \frac{48}{5}\Bigg\{ \frac{3\left(-2(x-1)^4\left(2x^2-5\xi^2+3\right)\log(1-x)\right)}{20\left(\xi^2-1\right)^3}$$

$$\frac{3\left(+4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\tanh^{-1}\left(\frac{(x-1)\ldots}{x-\xi^2\ldots}\right.\right.}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(x^3(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^4\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(-5x(x(x(x+2)+36)+18)\xi^2-15\xi^6\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(2(x-1)\left((23x+58)\xi^4+(x(x(x+67)+112)+6)\xi^2+x(x((5-2x)x+15)+3\ldots\right.\right.}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(\left(15(2x(x+5)+5)\xi^4+10x(3x(x+5)+11)\xi^2\right)\log\left(1-\xi^2\right)\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(2x(5x(x+2)-6)+15\xi^6-5\xi^2+3\right)\log\left(1-\xi^2\right)}{20\left(\xi^2-1\right)^3}\Bigg\}$$

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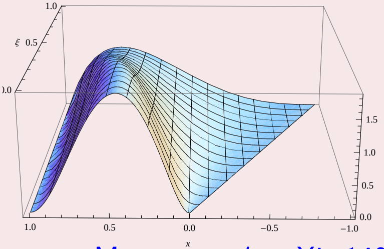
Valence $H^u(x, \xi, t)$ as a function of x and ξ at vanishing t.

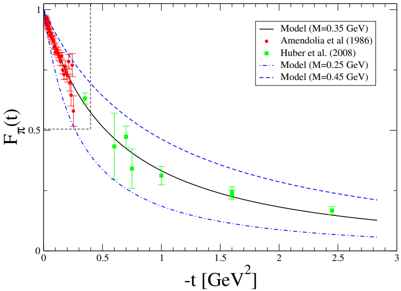


Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Pion form factor obtained from isovector GPD:

$$\int_{-1}^{+1} \mathrm{d}x \, H^{I=1}(x, \xi, t) = 2F_\pi(t)$$

- Single dimensionful parameter $M \simeq 350$ MeV.



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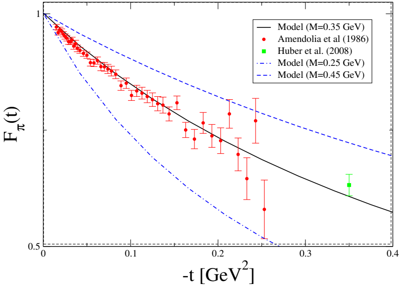

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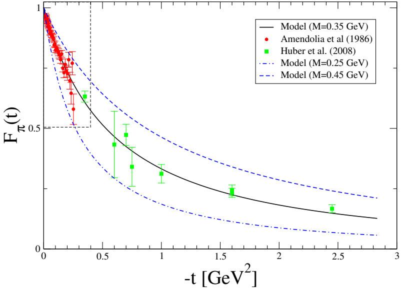
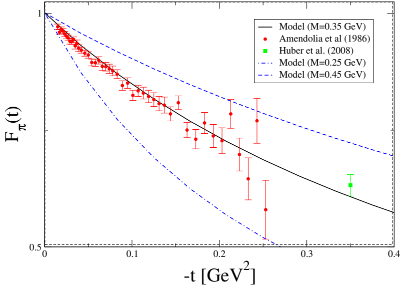


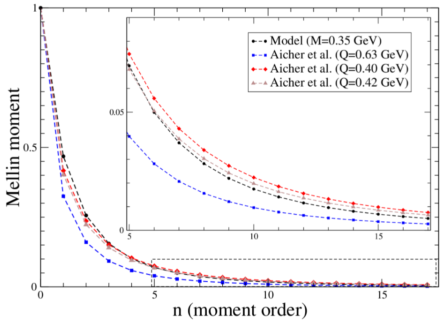


Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Pion PDF obtained from forward limit of GPD:

$$q(x) = H^q(x, 0, 0)$$

- Use LO DGLAP equation and compare to PDF extraction.
  Aicher *et al.*, Phys. Rev. Lett. **105**, 252003 (2010)



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Find model initial scale $\mu \simeq 400$ MeV.

Extension: Implementing Positivity and Polynomiality

- Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

$$|H; P, \lambda\rangle = \sum_{N,\beta} \int [\mathrm{d}x \mathrm{d}\mathbf{k}_\perp]_N \psi_N^{(\beta,\lambda)}(x_1, \mathbf{k}_{\perp 1}, \ldots, x_N, \mathbf{k}_{\perp N}) \, |\beta, k_1, \ldots, k_N\rangle$$

- Derive an expression for the pion GPD in the DGLAP region $\xi \leq x \leq 1$:

$$H^q(x, \xi, t) \propto \sum_{\beta,j} \int [\mathrm{d}\bar{x} \mathrm{d}\bar{\mathbf{k}}_\perp]_N \delta_{j,q} \delta(x - \bar{x}_j) \psi_N^{(\beta,\lambda)*}(\hat{x}', \hat{\mathbf{k}}'_\perp) \psi_N^{(\beta,\lambda)}(\tilde{x}, \tilde{\mathbf{k}}_\perp)$$

  with $\tilde{x}, \tilde{\mathbf{k}}_\perp$ (resp. $\hat{x}', \hat{\mathbf{k}}'_\perp$) generically denoting incoming (resp. outgoing) parton kinematics.

  Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

- Similar expression in the ERBL region $-\xi \leq x \leq \xi$, but with overlap of N- and $(N+2)$-body LFWF.

Pion Parton Distribution Function.

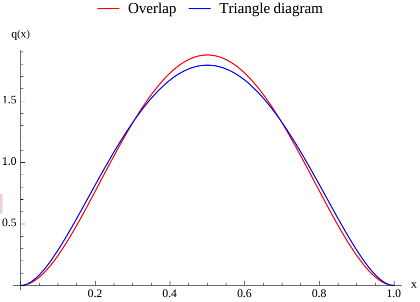
Algebraic model in the overlap representation.

- Evaluate LFWF in algebraic model:

$$\psi(x, \mathbf{k}_\perp) \propto \frac{x(1-x)}{[(\mathbf{k}_\perp - x\mathbf{P}_\perp)^2 + M^2]^2}$$

- Expression for the GPD at $t = 0$:

$$H(x, \xi, 0) \propto \frac{(1-x)^2(x^2-\xi^2)}{(1-\xi^2)^2}$$



- Manifest 2-body symmetry.
- Expression for the PDF:

$$q(x) = 30x^2(1-x)^2$$

- Off-forward case: *in progress*.

Conclusions

- Computation of GPDs, DDs, PDFs, LFWFs and form factors in the **nonperturbative framework** of Dyson-Schwinger and Bethe-Salpeter equations.
- **Explicit check** of several theoretical constraints, including polynomiality, support property and soft pion theorem.
- Simple algebraic model exhibits **most features of the numerical solutions** of the Dyson-Schwinger and Bethe-Salpeter equations.
- **Very good agreement** with existing pion form factor and PDF data.
- In progress: *a priori* implementation of polynomiality and positivity.

Commissariat à l'énergie atomique et aux énergies alternatives
Centre de Saclay | 91191 Gif-sur-Yvette Cedex
T. +33(0)1 69 08 73 88 | F. +33(0)1 69 08 75 84

DSM
Irfu
SPhN