

Deuteron Electromagnetic Form Factors in AdS/QCD

Valery Lyubovitskij

Institut für Theoretische Physik, Universität Tübingen
Kepler Center for Astro and Particle Physics, Germany

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN



in collaboration with
Thomas Gutsche
Ivan Schmidt
Alfredo Vega
PRD 91 (2015) 114001

Light Cone 2015, 22 May 2015, Frascati, Italy

Introduction

- AdS/QCD \equiv Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- Motivation: AdS/CFT correspondence 1998 (Maldacena, Polyakov, Witten et al) Extra-Dimensional (ED) theories including gravity are holographically equivalent to gauge theories living on boundary of ED space
- Symmetry arguments: Conformal group acting in boundary theory isomorphic to $SO(4, 2)$ – the isometry group of AdS_5 space
- HQCD models reproduce main features of QCD at low and high energies: chiral symmetry, confinement, power scaling of hadron form factors, Regge trajectories for mass spectrum see papers by Brodsky, Teramond, Lyubovitskij, Gutsche, Schmidt, Vega, ...
- PDFs, GPDs, Structure functions
- LFWFs from matching AdS/QCD and LF QCD

Deuteron in a soft-wall AdS/QCD approach

- We first propose an effective action describing the dynamics of the deuteron in the presence of an external vector field
- Based on this action the deuteron electromagnetic form factors are calculated, displaying the correct $1/Q^{10}$ power scaling for large Q^2 values.
- This finding is consistent with quark counting rules and the earlier observation that this result holds in confining gauge/gravity duals.
- The Q^2 dependence of the deuteron form factors is defined by a single and universal scale parameter κ , which is fixed from data.

Deuteron in a soft-wall AdS/QCD approach

- Effective action in terms of AdS fields $d^M(x, z)$ and $V^M(x, z)$
- $d^M(x, z)$ – dual to Fock component contributing to deuteron with twist $\tau = 6$
- $V^M(x, z)$ – dual to the electromagnetic field

$$\begin{aligned} S &= \int d^4x dz e^{-\varphi(z)} \left[-\frac{1}{4} F_{MN}(x, z) F^{MN}(x, z) - D^M d_N^\dagger(x, z) D_M d^N(x, z) \right. \\ &- ic_2 F^{MN}(x, z) d_M^\dagger(x, z) d_N(x, z) \\ &+ \frac{c_3}{4M_d^2} e^{2A(z)} \partial^M F^{NK}(x, z) \left(-d_M^\dagger(x, z) \overleftrightarrow{D}_K d_N(x, z) + \text{H.c.} \right) \\ &\left. + d_M^\dagger(x, z) \left(\mu^2 + U(z) \right) d^M(x, z) \right] \end{aligned}$$

Deuteron in a soft-wall AdS/QCD approach

- $A(z) = \log(R/z)$, R is the AdS radius

$F^{MN} = \partial^M V^N - \partial^N V^M$ - stress tensor of vector field

$D^M = \partial^M - ieV^M(x, z)$ - covariant derivative

$\mu^2 R^2 = (\Delta - 1)(\Delta - 3)$ - five-dimensional mass

$\varphi(z) = \kappa^2 z^2$ is the background dilaton field

$\Delta = 6 + L$ is the dimension of $d^M(x, z)$

L is the maximal value of orbital angular momentum

$U(z) = U_0 \varphi(z)/R^2$ is the confinement potential

U_0 is constant fixed the deuteron mass.

Use axial gauge for both vector fields $d^z(x, z) = 0$ and $V^z(x, z) = 0$

Deuteron in a soft-wall AdS/QCD approach

- First perform Kaluza-Klein (KK) decomposition for vector AdS field dual to deuteron

$$d^\mu(x, z) = \exp\left[\frac{\varphi(z) - A(z)}{2}\right] \sum_n d_n^\mu(x) \Phi_n(z),$$

$d_n^\mu(x)$ is the tower of the KK fields dual to the deuteron fields with radial quantum number n and twist-dimension $\tau = 6$, and $\Phi_n(z)$ are their bulk profiles.

Then we derive the Schrödinger-type equation of motion for the bulk profile

$$\left[-\frac{d^2}{dz^2} + \frac{4(L+4)^2 - 1}{4z^2} + \kappa^4 z^2 + \kappa^2 U_0 \right] \Phi_n(z) = M_{d,n}^2 \Phi_n(z).$$

Deuteron in a soft-wall AdS/QCD approach

- The analytical solutions of this EOM read

$$\Phi_n(z) = \sqrt{\frac{2n!}{(n+L+4)!}} \kappa^{L+5} z^{L+9/2} e^{-\kappa^2 z^2/2} L_n^{L+4}(\kappa^2 z^2),$$
$$M_{d,n}^2 = 4\kappa^2 \left[n + \frac{L+5}{2} + \frac{U_0}{4} \right],$$

where $L_n^m(x)$ are the generalized Laguerre polynomials

- Restricting to the ground state ($n = 0, L = 0$) we get $M_d = 2\kappa \sqrt{\frac{5}{2} + \frac{U_0}{4}}$
- $\kappa = 190$ MeV (fitted from data on electromagnetic deuteron form factors)
- Using central value of deuteron mass $M_d = 1.875613$ GeV we fix $U_0 = 87.4494$

Deuteron in a soft-wall AdS/QCD approach

- We can compare this value for the deuteron scale parameter to the analogous one of κ_N defining the nucleon properties - mass and electromagnetic form factors. In description of nucleon case we fixed the value to $\kappa_N \simeq 380$ MeV, which is 2 times bigger than the deuteron scale parameter κ .
- Difference between the nucleon and deuteron scale parameters can be related to the change of size of the hadronic systems - the deuteron as a two-nucleon bound state is 2 times larger than the nucleon.

Deuteron in a soft-wall AdS/QCD approach

- In case of the vector field dual to the electromagnetic field we perform a Fourier transform with respect to the Minkowski coordinate

$$V_\mu(x, z) = \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} V_\mu(q) V(q, z)$$

where $V(q, z)$ is its bulk profile obeying the following EOM:

$$\partial_z \left(\frac{e^{-\varphi(z)}}{z} \partial_z V(q, z) \right) + q^2 \frac{e^{-\varphi(z)}}{z} V(q, z) = 0 .$$

- Its analytical solution can be written in the form of

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} e^{-\kappa^2 z^2 x/(1-x)} x^a, \quad a = \frac{Q^2}{4\kappa^2}, \quad Q^2 = -q^2 .$$

Deuteron in a soft-wall AdS/QCD approach

- The gauge-invariant matrix element describing the interaction of the deuteron with the external vector field (dual to the electromagnetic field) reads

$$M_{\text{inv}}^{\mu}(p, p') = - \left(G_1(Q^2) \epsilon^*(p') \cdot \epsilon(p) - \frac{G_3(Q^2)}{2M_d^2} \epsilon^*(p') \cdot q \epsilon(p) \cdot q \right) (p + p')^{\mu} \\ - G_2(Q^2) \left(\epsilon^{\mu}(p) \epsilon^*(p') \cdot q - \epsilon^{*\mu}(p') \epsilon(p) \cdot q \right)$$

where $\epsilon(\epsilon^*)$ and $p(p')$ are the polarization and four-momentum of the initial (final) deuteron, and $q = p' - p$ is the momentum transfer.

Deuteron in a soft-wall AdS/QCD approach

- Three EM form factors $G_{1,2,3}$ of the deuteron are related to the charge G_C , quadrupole G_Q and magnetic G_M form factors by
- Expressions for the form factors

$$G_C = G_1 + \frac{2}{3}\tau_d G_Q, \quad G_M = G_2, \quad G_Q = G_1 - G_2 + (1 + \tau_d)G_3, \quad \tau_d = \frac{Q^2}{4M_d^2}.$$

These form factors are normalized at zero recoil as

$$G_C(0) = 1, \quad G_Q(0) = M_d^2 Q_d = 25.83, \quad G_M(0) = \frac{M_d}{M_N} \mu_d = 1.714,$$

- $Q_d = 7.3424 \text{ GeV}^{-2}$ and $\mu_d = 0.8574$ – quadrupole and magnetic moments of the deuteron.

Deuteron in a soft-wall AdS/QCD approach

- In our approach the deuteron form factor $G_1(Q^2) = F(Q^2)$, where $F(Q^2)$ is the twist-6 hadronic form factor, which is given by the overlap of the square of the bulk profile dual to the deuteron wave function (twist-6 hadronic wave function) and the confined electromagnetic current

$$F(Q^2) = \int_0^\infty dz \Phi_0^2(z) V(Q, z) = \frac{\Gamma(6) \Gamma(a+1)}{\Gamma(a+6)}$$

where $a = Q^2/(4\kappa^2)$.

- This formula follows from the result of Brodsky and Teramond – general and universal formula for hadronic FF with twist τ in terms of bulk profile $\phi_\tau(z) = \sqrt{\frac{2}{(\tau-2)!}} \kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2}$ dual to the hadronic wave function with twist τ :

$$F_\tau(Q^2) = \int_0^\infty dz \phi_\tau^2(z) V(Q, z) = \frac{\Gamma(\tau) \Gamma(a+1)}{\Gamma(a+\tau)}$$

Deuteron in a soft-wall AdS/QCD approach

- By analogy we calculate the other two deuteron FF G_2 and G_3 , expressed in terms of the same universal factor $F(Q^2)$

$$G_i(Q^2) = c_i F(Q^2), \quad i = 2, 3.$$

The parameters c_2 and c_3 are defined by normalization of the deuteron form factors as

$$c_2 = G_M(0) = 1.714, \quad c_3 = G_M(0) + G_Q(0) - 1 = 26.544.$$

- $F(Q^2)$ has the correct power-scaling $F(Q^2) \sim 1/(Q^2)^5$ at large $Q^2 \rightarrow \infty$
- It can also be written in the Brodsky-Ji-Lepage form derived within pQCD
- Deuteron FF is factorized in terms of the nucleon FF $F_N(Q^2/4)$ and the so-called “reduced” nuclear form factor $f_d(Q^2)$ (Brodsky-Ji-Lepage):

$$F_d(Q^2) = f_d(Q^2) F_N^2(Q^2/4)$$

Deuteron in a soft-wall AdS/QCD approach

- Our result reads

$$F_d(Q^2) \equiv F(Q^2) = \frac{\Gamma(6) \Gamma(a+1)}{\Gamma(a+6)} = \frac{5!}{(a+1) \dots (a+5)} = f_d(Q^2) F_N^2(Q^2/4)$$

where predictions for $f_d(Q^2)$ and $F_N(Q^2/4)$ are

$$f_d(Q^2) = \frac{30(a+1)(a+2)}{(a+3)(a+4)(a+5)}, \quad F_N(Q^2/4) = \frac{2}{(a+1)(a+2)}$$

where $a = Q^2/(4\kappa^2)$.

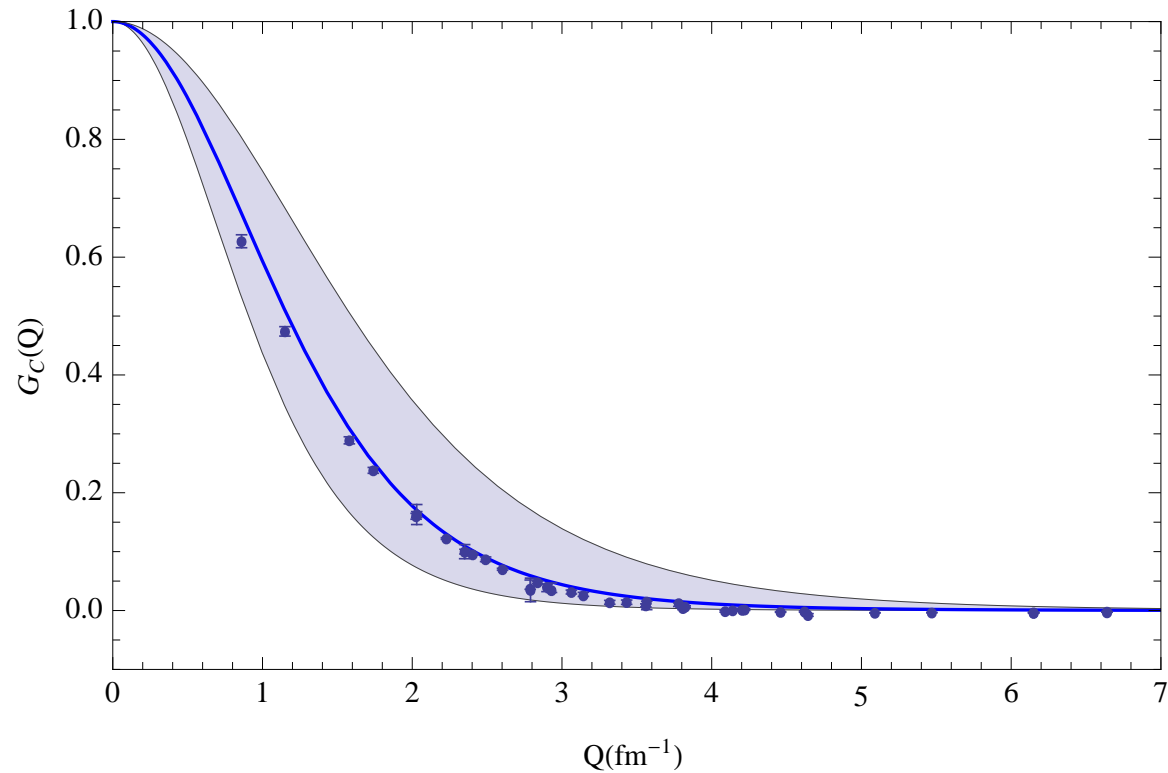
Deuteron in a soft-wall AdS/QCD approach

- Numerical results for deuteron FF
- Shaded band corresponds to values of κ in range of $150 \text{ MeV} < \kappa < 250 \text{ MeV}$.
- Increase of the parameter κ leads to an enhancement of the form factors.
- The best description of the data on the deuteron form factors is obtained for $\kappa = 190 \text{ MeV}$ and is shown by the solid line.
- With $\kappa = 190 \text{ MeV}$ our result for the deuteron charge radius

$$r_C = \sqrt{-6G'_C(0)} = \sqrt{\frac{137}{40\kappa^2} - Q_d} = 1.846 \text{ fm close to data}$$

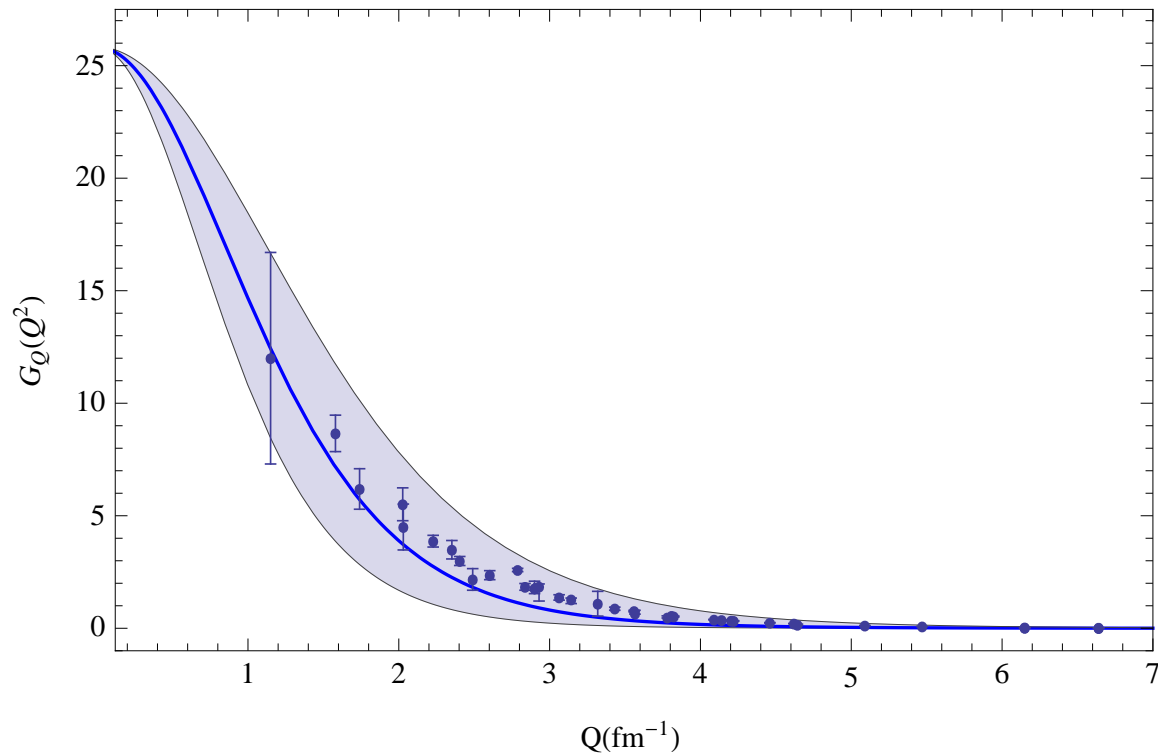
$$r_C = 2.130 \pm 0.010 \text{ fm.}$$

Deuteron in a soft-wall AdS/QCD approach



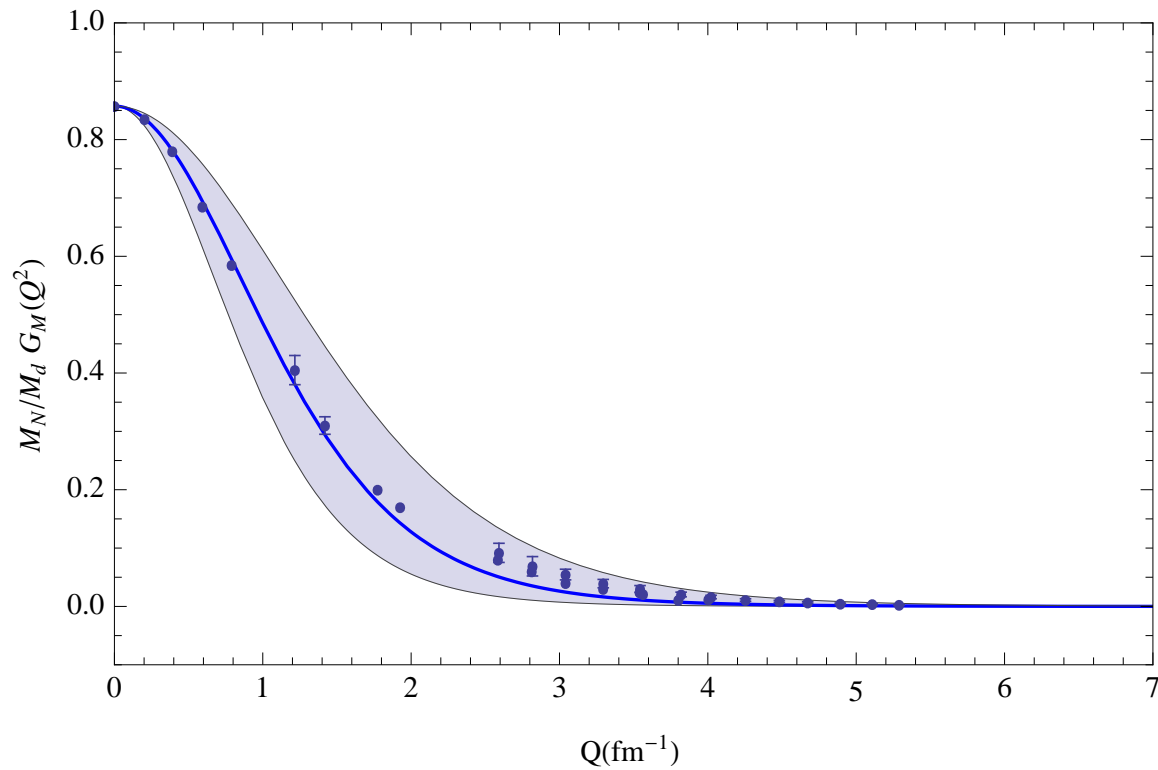
Charge deuteron form factor

Deuteron in a soft-wall AdS/QCD approach



Quadrupole deuteron form factor

Deuteron in a soft-wall AdS/QCD approach



Magnetic deuteron form factor

Summary

- AdS/QCD \equiv Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies
- Soft-wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Mesons, baryons, exotics from unified point view and including high Fock states
- Thanks to Stan Brodsky, Guy de Téramond, Werner Vogelsang for useful discussions and comments