Identifying the Universal Part of TMDs

Light Cone 2015, Frascati

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Bochum preprint RUB-TPII-02/2015, in preparation

September 24, 2015

Outline

1. Introduction
2. A New Path Layout
3. Retrieving Φ^{SIDIS} and Φ^{DY} from $\widetilde{\Phi}$
4. Conclusion

Universality

Universality of TMDs

- hadron $\sim$ quark correlator $\rightarrow$ parameterise with TMDs
- T-odd TMDs due to Wilson line
- Different paths for different processes $\rightarrow$ not universal!
- Not a real problem, merely a sign difference:

$$f_{\text{T-odd}}^{\text{initial state int.}} = -f_{\text{T-odd}}^{\text{final state int.}}$$

Aim

- Can we create a path layout that combines features of both initial and final state interactions?
- How does it relate to known TMDs?

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Universality

Quark Correlator

$$\Phi_{ij}(P,k,S,n) = \int \frac{\mathrm{d}^\omega z}{(2\pi)^\omega}\ \mathrm{e}^{\mathrm{i}\,k\cdot z}\ \langle PS|\,\overline{\psi}_j(z)\ \mathcal{U}_{(z\,;0)}\ \psi_i(0)\,|P,S\rangle$$

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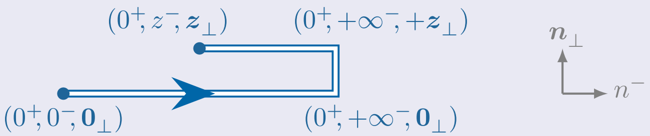
SIDIS:



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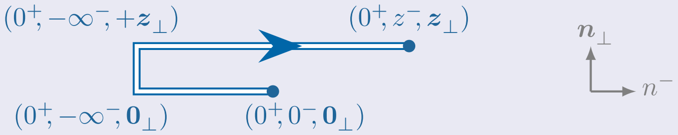
DY:



On Wilson Lines

Definition of Semi-Infinite Line

$$\mathcal{U}_{(+\infty\,;\,a)}[\mathcal{C}] = \mathcal{P}\mathrm{e}^{\mathrm{i}g\int_a^{+\infty}\mathrm{d}z\cdot A(z)} \qquad \mathcal{C}:\, z^\mu = a^\mu + \lambda n^\mu \quad \lambda = 0\ldots\infty$$

Path Inversion

$$\mathcal{U}_{(-\infty\,;\,b)} = \mathcal{U}_{(+\infty\,;\,b)}\big|_{n\,\to -n}$$

Inverted Path in Quark Correlator (in Feynman gauge)

$n \to -n$

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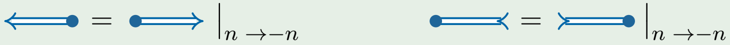

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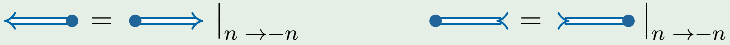

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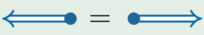
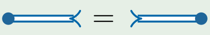


$\Big|_{n\,\to -n}$



$\Big|_{n\,\to -n}$

Inverted Path in Quark Correlator (in Feynman gauge)

$$\Phi^{\text{DY}} = \Phi^{\text{SIDIS}}\Big|_{n\,\to -n}$$

Where Does the Sign Hide?

Fully Unintegrated Quark Correlator

$$
\begin{aligned}
\Phi(P,k,S,n) = \mathbb{1}\, & \Big[M A_1 \;+\; n_\mu\, \varepsilon^{\mu\nu\rho\sigma} \frac{1}{P\cdot n} P_\nu k_\rho S_\sigma B_5 \Big] \\
& + \gamma_\mu \; \Bigg[\bigg(P^\mu A_2 + k^\mu A_3 + \frac{M^2}{P\cdot n} n^\mu B_1 \bigg) + \frac{1}{M} \varepsilon^{\mu\nu\rho\sigma} P_\nu k_\rho S_\sigma A_{12} \\
& \qquad + \frac{1}{P\cdot n} \varepsilon^{\mu\nu\rho\sigma} n_\nu \bigg(P_\rho k_\sigma \bigg(\frac{k\cdot S}{M} B_9 + M \frac{S\cdot n}{P\cdot n} B_{10} \bigg) - M \left(P_\rho B_7 + k_\rho B_8 \right) S_\sigma \bigg) \Bigg] \\
& + i\gamma_5\, M \left[\frac{k\cdot S}{M} A_5 + M \frac{S\cdot n}{P\cdot n} B_6 \right] \\
& + \gamma_\mu \gamma_5 \; \Bigg[M S^\mu A_6 + \frac{k\cdot S}{M} \bigg(P^\mu A_7 + k^\mu A_8 + \frac{M^2}{P\cdot n} n^\mu B_{13} \bigg) \\
& \qquad + M \frac{S\cdot n}{P\cdot n} \bigg(P^\mu B_{11} + k^\mu B_{12} + \frac{M^2}{P\cdot n} n^\mu B_{14} \bigg) + \frac{1}{P\cdot n} \varepsilon^{\mu\nu\rho\sigma} n_\nu P_\rho k_\sigma B_4 \Bigg] \\
& + i\gamma_{\mu\nu} \left[\frac{1}{M} P^\mu k^\nu A_4 + \frac{M}{P\cdot n} \left(P^\mu B_2 + k^\mu B_3 \right) n^\nu \right] \\
& + \gamma_{\mu\nu}\gamma_5 \; \Bigg[\bigg(P^\mu A_9 + k^\mu A_{10} + \frac{M^2}{P\cdot n} n^\mu B_{15} \bigg) S^\nu + \frac{1}{M} P^\mu k^\nu \bigg(\frac{k\cdot S}{M} A_{11} + M \frac{S\cdot n}{P\cdot n} B_{18} \bigg) \\
& \qquad + \frac{k\cdot S}{P\cdot n} \left(P^\mu B_{16} + k^\mu B_{17} \right) n^\nu + \frac{M^2}{P\cdot n} \frac{S\cdot n}{P\cdot n} \left(P^\mu B_{19} + k^\mu B_{20} \right) n^\nu \Bigg]
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\end{aligned}
$$

Where Does the Sign Hide?

Fully Unintegrated Quark Correlator

- Only factors of form $\dfrac{n^\mu}{P\cdot n}$ $\Rightarrow$ no sign change
- Sign change inside T-odd TMDs

Outline

Inspiration

Relation Between Different Layouts

unfortunately

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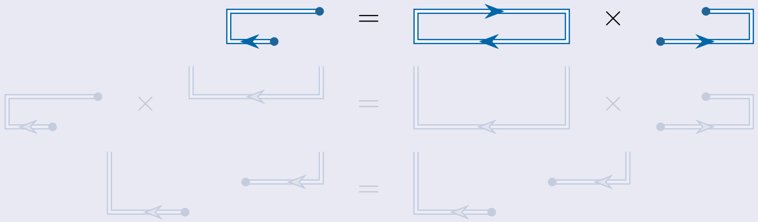
unfortunately

$$\Phi^{\text{DY}} \neq \Phi^{\square}\,\Phi^{\text{SIDIS}}$$

Inspiration

Exploiting the Path Relation



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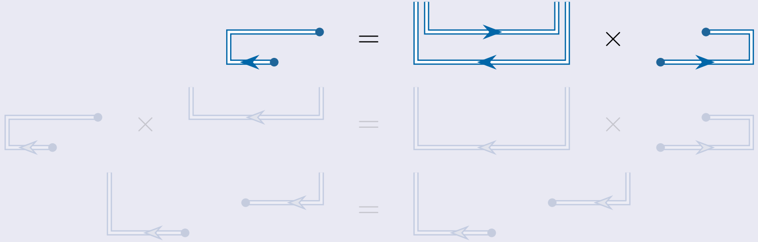
Exploiting the Path Relation



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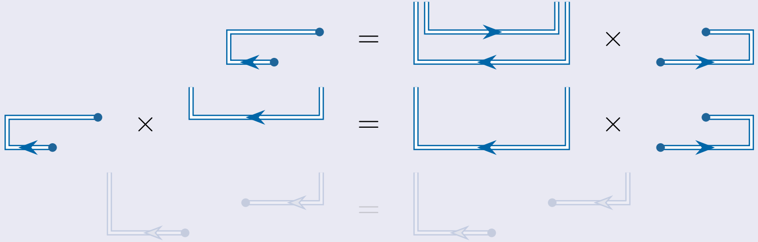

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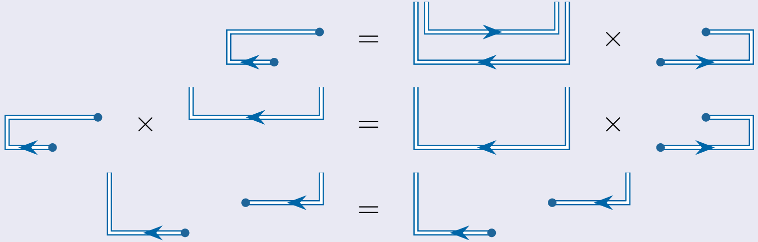
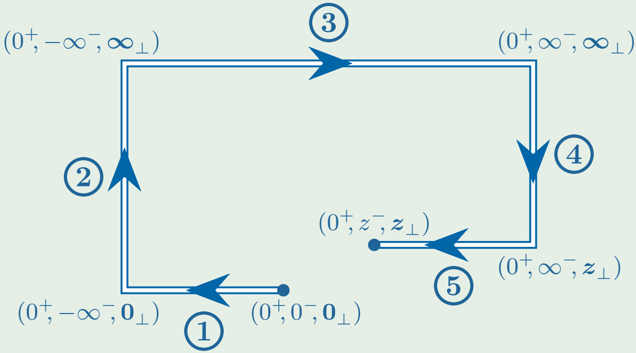


New Layout

Proposed Layout



New Layout

Properties

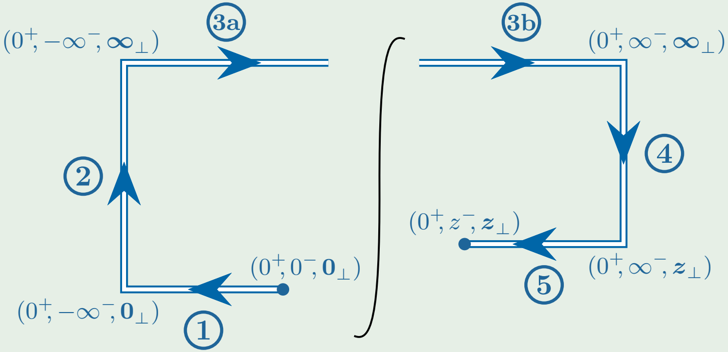
- Combines features from both SIDIS and DY
- More specifically, $\widetilde{\mathcal{U}} \sim \mathcal{U}^{\dagger}_{\text{SIDIS}}\,\mathcal{U}_{\text{DY}}$
- But extra line at $\infty_{\perp}$: $\mathcal{U}_3 \qquad \Rightarrow \qquad \widetilde{\mathcal{U}} = \mathcal{U}^{\dagger}_{\text{SIDIS}}\,\widetilde{\mathcal{U}}\,\mathcal{U}_{\text{DY}}$
- Ensures gauge invariance, but also ensures that

$$\int \frac{d^2\boldsymbol{k}_{\perp}}{(2\pi)^2}\,\widetilde{\mathcal{U}} = \mathcal{U}_{\text{PDF}}$$

- Not necessarily physical

New Layout

Proposed Layout



Outline

Relating New Structure to SIDIS and DY

Correction Factors

Define:

$$\Phi^{\text{SIDIS}} = C^{\text{SIDIS}} \otimes \widetilde{\Phi}$$
$$\Phi^{\text{DY}} = C^{\text{DY}} \otimes \widetilde{\Phi}$$

then

$$C_0 = \delta(1-x)\ \delta^{(2)}(\boldsymbol{k}_\perp)$$

because

$$\Phi_0^{\text{SIDIS}} = \Phi_0^{\text{DY}} = \widetilde{\Phi}_0$$

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Parameters

Parameters of the Layout

Feynman gauge:

- Only n^- as defining parameter
- Assign one n^- to each side of the cut

$$\Rightarrow \quad n_1, n_5$$

Light-cone gauge:

- Only r^- as defining parameter
- Assign one r^- to each side of the cut

$$\Rightarrow \quad r_2, r_4$$

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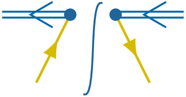
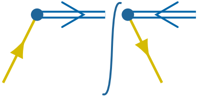
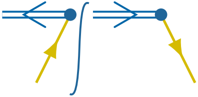
Comparison

	$\widetilde{\Phi}$	Φ^{SIDIS}	Φ^{DY}
Feynman	 $n_1 = -n^-$ $n_5 = -n^-$	 $n_1 = +n^-$ $n_5 = -n^-$	 $n_1 = -n^-$ $n_5 = +n^-$
Light-cone	$r_2 = -\infty^-$ $r_4 = +\infty^-$	$r_2 = +\infty^-$ $r_4 = +\infty^-$	$r_2 = -\infty^-$ $r_4 = -\infty^-$

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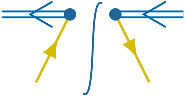
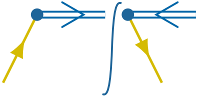
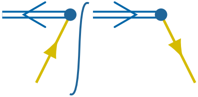
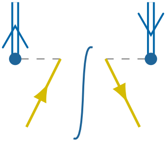
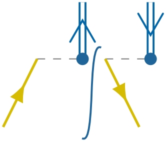
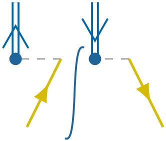
	$\widetilde{\Phi}$	Φ^{SIDIS}	Φ^{DY}
Feynman	 $n_1 = -n^-$ $n_5 = -n^-$	 $n_1 = +n^-$ $n_5 = -n^-$	 $n_1 = -n^-$ $n_5 = +n^-$
Light-cone	 $r_2 = -\infty^-$ $r_4 = +\infty^-$	 $r_2 = +\infty^-$ $r_4 = +\infty^-$	 $r_2 = -\infty^-$ $r_4 = -\infty^-$

Comparison

Calculating the Correction Factors

$$C_1^{\text{SIDIS}} \sim \Phi_1\big|_{n_1 \to -n_1} - \Phi_1$$
$$C_1^{\text{DY}} \sim \Phi_1\big|_{n_5 \to -n_5} - \Phi_1$$

Preliminary Results in Feynman Gauge

Generic Structure of Correlator

$$\Phi = \Phi_0 + \alpha_s \left(A + \frac{n_1 + n_5}{2} B + \frac{n_1 - n_5}{2} C \right) + \mathcal{O}(\alpha_s^2)$$

Results

$$C_1^{\text{SIDIS}} \sim -\alpha_s \, n^- \, (B + C)$$
$$C_1^{\text{DY}} \sim -\alpha_s \, n^- \, (B - C)$$

Preliminary Results in Feynman Gauge

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$\widetilde{\Phi}$

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Φ^{SIDIS}, Φ^{DY}

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T-even

Results

$$f_{\text{T-even}} = f_0 + \alpha_s A + \mathcal{O}(\alpha_s^2)$$

$$f_{\text{T-odd}} = \alpha_s \frac{n_1 - n_5}{2} C + \mathcal{O}(\alpha_s^2)$$

Preliminary Results in Feynman Gauge

Generic Structure of Correlator

$$\Phi = \Phi_0 + \alpha_s \left(A + \underbrace{\frac{n_1 + n_5}{2} B + \frac{n_1 - n_5}{2} C}_{\text{T-odd}} \right) + \mathcal{O}(\alpha_s^2)$$

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Preliminary Results in Feynman Gauge

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$$f_{\text{T-odd}} = \alpha_s \frac{n_1 - n_5}{2} C + \mathcal{O}(\alpha_s^2)$$

Preliminary Results in Feynman Gauge

Universality

Only virtual diagrams contribute to B

$\Rightarrow$ $\widetilde{\Phi}$ is universal everywhere except in its endpoints

Outline

Conclusions

- Using double parameters is an easy way to generalise calculations
- The proposed layout has some universal features

Outlook

- Extend calculation to light-cone gauge
- Investigate universality of $\widetilde{\Phi}$ deeper

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Thank you for your attention!