Hadron Physics from Superconformal Quantum Mechanics in the Light-Front and its Holographic Embedding

Guy F. de Téramond

Universidad de Costa Rica

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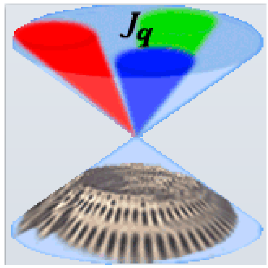
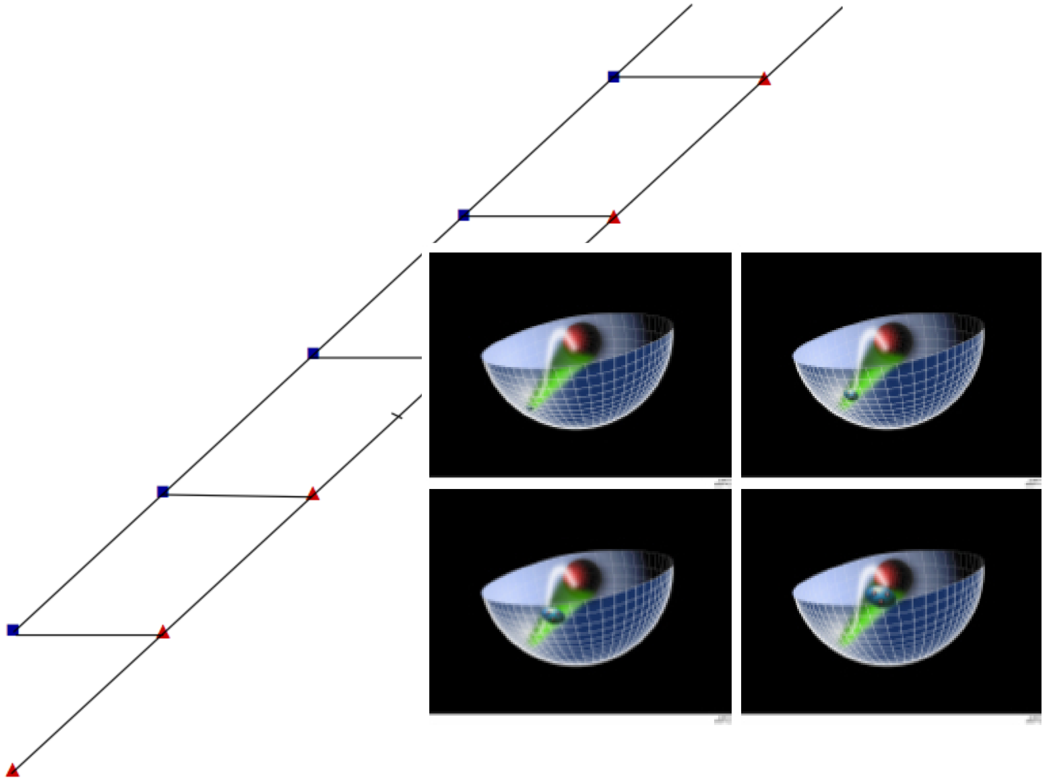




In collaboration with Stan Brodsky and Hans G. Dosch

Quest for a semiclassical approximation to describe bound states in QCD

(Convenient starting point in QCD)

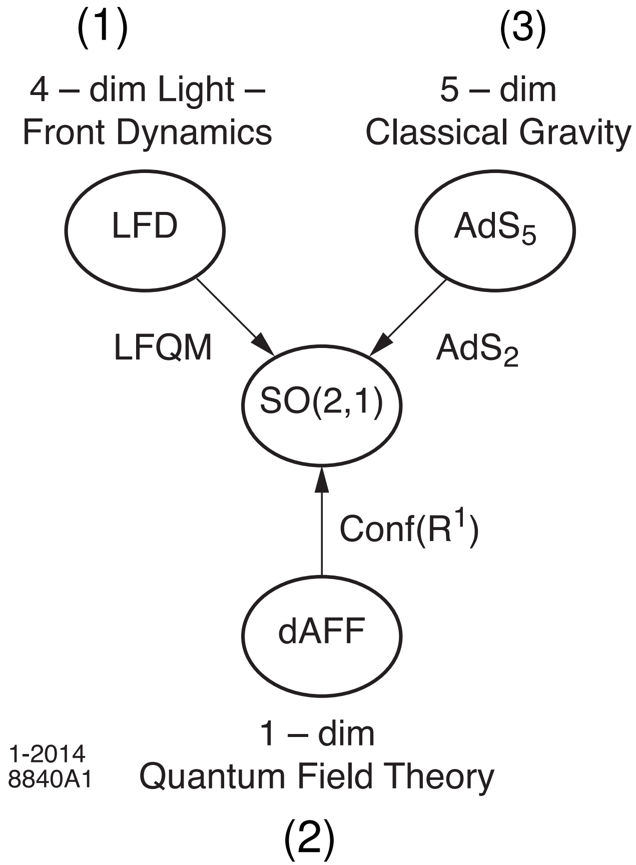
I. Semiclassical approximation to QCD in the light-front: Reduction of QCD LF Hamiltonian leads to a relativistic LF wave equation, where complexities from strong interactions are incorporated in effective potential U

II. Construction of LF potential U: Since the LF semiclassical approach leads to a one-dim QFT, it is natural to extend conformal and superconformal QM to the light front since it gives important insights into the confinement mechanism, the emergence of a mass scale and baryon-meson SUSY

III. Correspondence between equations of motion for arbitrary spin in AdS space and relativistic LF bound-state equations in physical space-time: Embedding of LF wave equations in AdS leads to extension of LF potential U to arbitrary spin from conformal symmetry breaking in the AdS_5 action

(Emerging SUSY from color dynamics $\overline{\mathbf{3}} \to \mathbf{3} \times \mathbf{3}$, No large N_C limit !)

Hadronic triality



Conformal and Superconformal Quantum Mechanics

[de Alfaro, Fubini and Furlan (1976, Fubini and Rabinovici (1984)]

Triple isomorphism *Conf*$(R^1) \sim SO(2,1) \sim \mathrm{AdS}_2$

Outline of this talk

1 Semiclassical approximation to QCD in the light front

2 Conformal quantum mechanics and light-front dynamics

3 Embedding integer-spin wave equations in AdS space

4 Superconformal quantum mechanics and light-front dynamics

5 Embedding half-integer-spin wave equations in AdS space

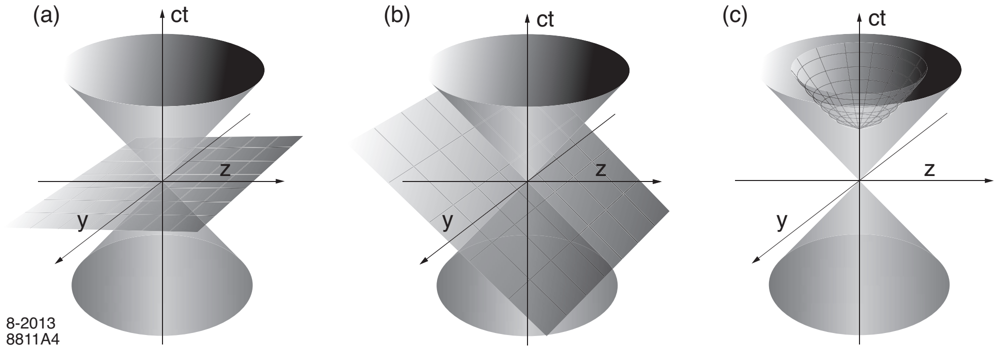
6 Superconformal baryon-meson symmetry and LF holographic QCD

7 Supersymmetry across the light and heavy-light hadronic spectrum

(1) Semiclassical approximation to QCD in the light front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Dirac Forms of Relativistic Dynamics [Dirac (1949)]



- (a) Instant form $x^0 = 0$, (b) Front form $x^0 + x^3 = 0$, (c) Point Form $x^2 = \kappa^2$
- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = M^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2 \phi(\zeta)$$

- Invariant variable in impact space $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$ (N partons: LF cluster decomposition)
- Critical value $L = 0$ corresponds to lowest possible stable solution: ground state of the LF Hamiltonian
- Relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time and comprises all interactions, including those with higher Fock states.

(2) Conformal quantum mechanics and light-front dynamics

[S. J. Brodsky, GdT and H.G. Dosch, PLB **729**, 3 (2014)]

- Incorporate in 1-dim effective QFT the conformal symmetry of 4-dim QCD Lagrangian in the limit of massless quarks: Conformal QM [V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

- Conformal Hamiltonian:

$$H = \tfrac{1}{2}\left(p^2 + \frac{g}{x^2}\right)$$

  g dimensionless: Casimir operator of the representation

- Schrödinger picture: $p = -i\partial_x$

$$H = \tfrac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right)$$

- QM evolution

$$H|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle$$

  H is one of the generators of the conformal group *Conf*(R^1). The two additional generators are:

- Dilatation: $D = -\tfrac{1}{4}\left(px + xp\right)$

- Special conformal transformations: $K = \tfrac{1}{2}x^2$

- H, D and K close the conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

- dAFF construct a new generator G as a superposition of the 3 generators of *Conf*(R^1)

$$G = uH + vD + wK$$

and introduce new time variable τ

$$d\tau = \frac{dt}{u + vt + wt^2}$$

- Find usual quantum mechanical evolution for time τ

$$G|\psi(\tau)\rangle = i\frac{d}{d\tau}|\psi(\tau)\rangle \qquad H|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle$$

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

- Operator G is compact for $4uw - v^2 > 0$, but action remains conformal invariant !

- Emergence of scale: Since the generators of *Conf*$(R^1) \sim SO(2,1)$ have different dimensions a scale appears in the new Hamiltonian G, which according to dAFF may play a fundamental role

Connection to light-front dynamics

- Compare the dAFF Hamiltonian G

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

with the LF Hamiltonian H_{LF}

$$H_{LF} = -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)$$

and identify dAFF variable x with LF invariant variable ζ

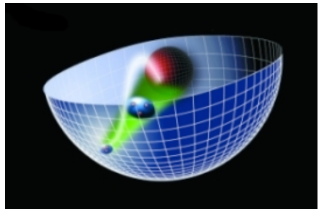
- Choose $u = 2, \;\; v = 0$
- Casimir operator from LF kinematical constraints: $g = L^2 - \frac{1}{4}$
- $w = 2\lambda^2$ fixes the LF potential to harmonic oscillator in the LF plane $\lambda^2\,\zeta^2$

$$U \sim \lambda^2\zeta^2$$

- One can perform a level shift by adding an arbitrary constant to LF potential U: Not true for baryons !

(3) Embedding integer spin wave equations in AdS space

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]



- Integer spin-J in AdS conveniently described by tensor field $\Phi_{N_1 \cdots N_J}$ with effective action

$$S_{\mathit{eff}} = \int d^d x \, dz \, \sqrt{|g|} \, e^{\varphi(z)} \, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{M M'} D_M \Phi^*_{N_1 \ldots N_J} \, D_{M'} \Phi_{N_1' \ldots N_J'} - \mu^2_{\mathit{eff}}(z) \, \Phi^*_{N_1 \ldots N_J} \, \Phi_{N_1' \ldots N_J'} \Big)$$

D_M is the covariant derivative which includes affine connection and dilaton $\varphi(z)$ effectively breaks maximal symmetry of AdS$_{d+1}$

$$ds^2 = \frac{R^2}{z^2} \left(dx_\mu dx^\mu - dz^2 \right)$$

- Effective mass $\mu_{\mathit{eff}}(z)$ is determined by precise mapping to light-front physics

- Non-trivial geometry of pure AdS encodes the kinematics and additional deformations of AdS encode the dynamics, including confinement

- Physical hadron has plane-wave and polarization indices along $3{+}1$ physical coordinates and a profile wavefunction $\Phi(z)$ along holographic variable z

$$\Phi_P(x,z)_{\mu_1 \cdots \mu_J} = e^{iP\cdot x}\Phi(z)_{\mu_1 \cdots \mu_J}, \qquad \Phi_{z\mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Variation of the action gives AdS wave equation for spin-J field $\Phi(z)_{\nu_1 \cdots \nu_J} = \Phi_J(z)\epsilon_{\nu_1 \cdots \nu_J}(P)$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J = M^2\Phi_J}$$

with

$$(\mu\, R)^2 = (\mu_{\mathit{eff}}(z)R)^2 - Jz\,\varphi'(z) + J(d-J+1)$$

and the kinematical constraints to eliminate the lower spin states $J-1$, $J-2, \cdots$

$$\eta^{\mu\nu} P_\mu\, \epsilon_{\nu\nu_2 \cdots \nu_J} = 0, \quad \eta^{\mu\nu}\, \epsilon_{\mu\nu\nu_3 \cdots \nu_J} = 0$$

- Kinematical constrains in the LF imply that μ must be a constant

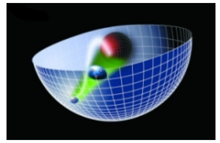
[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

Light-front mapping

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

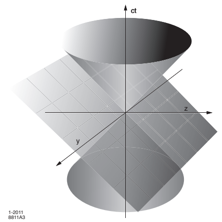
- Upon substitution $\Phi_J(z) \sim z^{(d-1)/2-J} e^{-\varphi(z)/2}\, \phi_J(z)$ and $z \rightarrow \zeta$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z)=M^2\Phi_J(z)$$



we find LFWE $(d=4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta)=M^2\phi_J(\zeta)$$



with

$$U(\zeta)=\tfrac{1}{2}\varphi''(\zeta)+\frac{1}{4}\varphi'(\zeta)^2+\frac{2J-3}{2z}\varphi'(\zeta)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms for the partons
- Effective confining potential $U(\zeta)$ corresponds to the IR modification of AdS space
- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

Meson spectrum

- Dilaton profile in the dual gravity model determined from one-dim QFTh (dAFF)

$$\varphi(z) = \lambda z^2, \qquad \lambda^2 = \tfrac{1}{2} w$$

- Effective potential: $U = \lambda^2 \zeta^2 + 2\lambda(J-1)$

- LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z) = 1$

$$\phi_{n,L}(\zeta) = |\lambda|^{(1+L)/2} \sqrt{\frac{2n!}{(n+L)!}}\, \zeta^{1/2+L} e^{-|\lambda|\zeta^2/2} L_n^L(|\lambda|\zeta^2)$$

- Eigenvalues for $\lambda > 0$

$$\mathcal{M}^2_{n,J,L} = 4\lambda\left(n + \frac{J+L}{2}\right)$$

- $\lambda < 0$ incompatible with LF constituent interpretation

Two relevant points . . .

- A linear potential V_{eff} in the *instant form* implies a quadratic potential U_{eff} in the *front form* at large distances $\longrightarrow$ Regge trajectories

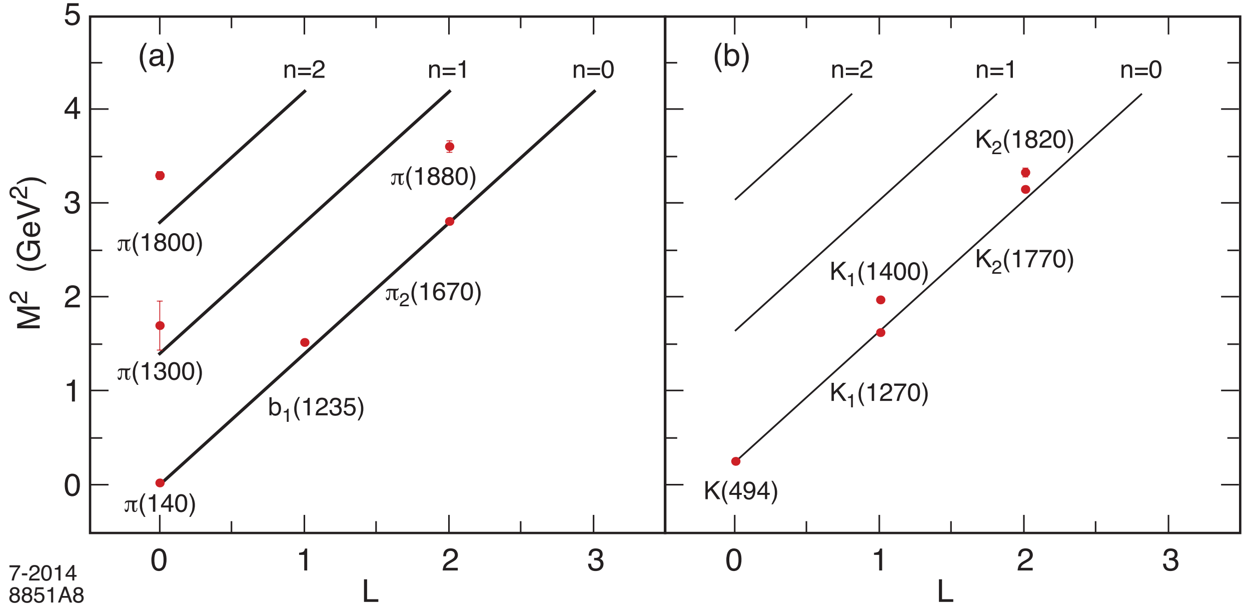
$$U_{\text{eff}} = V_{\text{eff}}^2 + 2\sqrt{p^2 + m_q^2}\, V_{\text{eff}} + 2\, V_{\text{eff}} \sqrt{p^2 + m_{\overline{q}}^2}$$

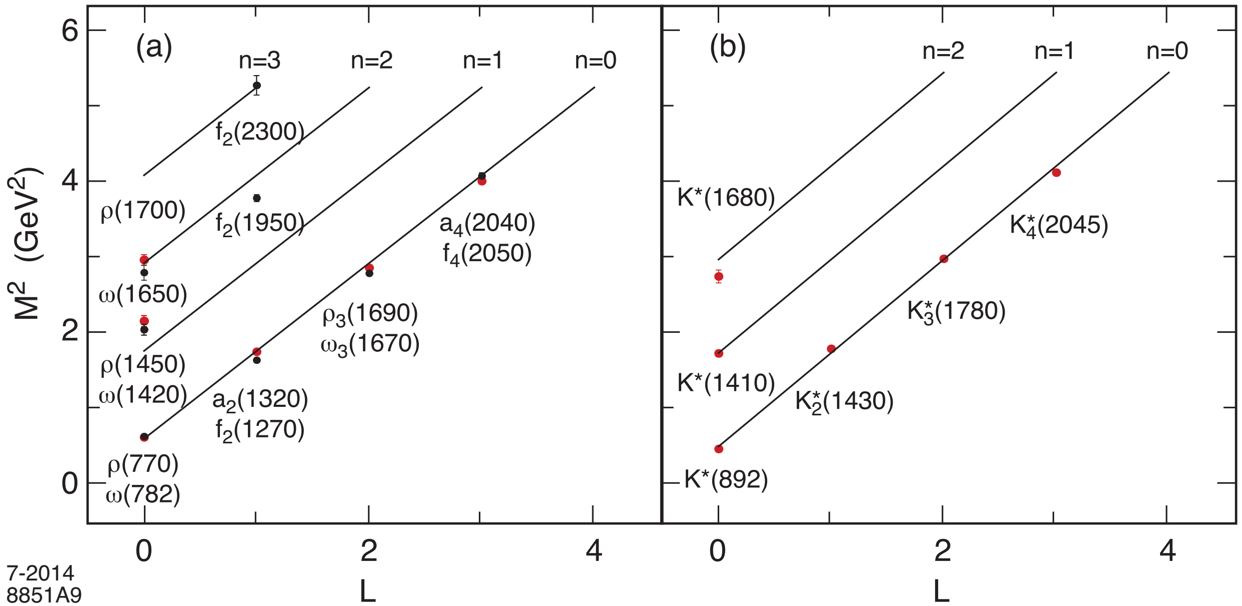
[A. P. Trawiński, S. D. Glazek, S. J. Brodsky, GdT, H. G. Dosch, PRD **90**, 074017 (2014)]

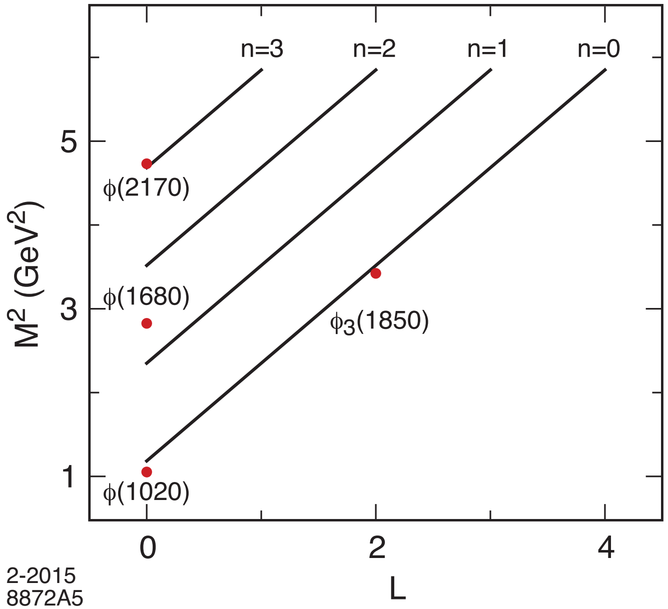
- Results are easily extended to light quarks

[S.J. Brodsky and GdT, arXiv:0802.0514 [hep-ph]]

$$\Delta M^2_{m_q, m_{\overline{q}}} = \frac{\int_0^1 dx\, e^{-\frac{1}{\lambda}\left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)} \left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)}{\int_0^1 dx\, e^{-\frac{1}{\lambda}\left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)}}$$







Orbital and radial excitations for $\sqrt{\lambda} = 0.59$ GeV (pseudoscalar) and 0.54 GeV (vector mesons)

(4) Superconformal quantum mechanics and light-front dynamics

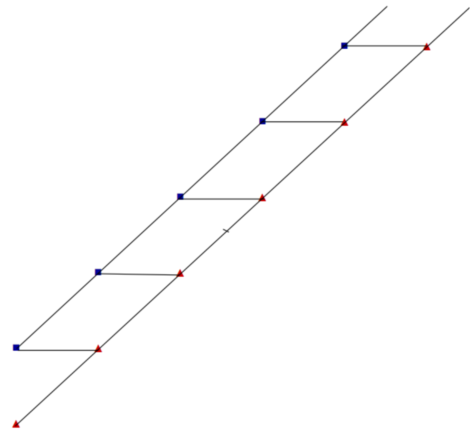
[GdT, H.G. Dosch and S. J. Brodsky, Phys. Rev. D **91**, 045040 (2015)]

- SUSY QM contains two fermionic generators Q and $Q^\dagger$, and a bosonic generator, the Hamiltonian H

  [E. Witten, NPB **188**, 513 (1981)]

- Closure under the graded algebra $sl(1/1)$:

$$\frac{1}{2}\{Q, Q^\dagger\} = H$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[Q, H] = [Q^\dagger, H] = 0$$



Note: Since $[Q^\dagger, H] = 0$ the states $|E\rangle$ and $Q^\dagger|E\rangle$ have identical eigenvalues E

- A simple realization is

$$Q = \chi\,(ip + W)\,, \qquad Q^\dagger = \chi^\dagger\,(-ip + W)$$

where χ and $\chi^\dagger$ are spinor operators with anticommutation relation

$$\{\chi, \chi^\dagger\} = 1$$

- In a 2×2 Pauli-spin matrix representation: $\chi = \frac{1}{2}\,(\sigma_1 + i\sigma_2)\,, \;\; \chi^\dagger = \frac{1}{2}\,(\sigma_1 - i\sigma_2)$

$$[\chi, \chi^\dagger] = \sigma_3$$

- Following Fubini and Rabinovici consider a 1-dim QFT invariant under conformal and supersymmetric transformations [S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]
- Conformal superpotential (f is dimensionless)

$$W(x) = \frac{f}{x}$$

- Thus 1-dim QFT representation of the operators

$$Q \;=\; \chi\left(\frac{d}{dx} + \frac{f}{x}\right), \qquad Q^\dagger = \chi^\dagger\left(-\frac{d}{dx} + \frac{f}{x}\right)$$

- Conformal Hamiltonian $H = \frac{1}{2}\{Q, Q^\dagger\}$ in matrix form

$$H = \frac{1}{2}\begin{pmatrix} -\frac{d^2}{dx^2} + \frac{f(f-1)}{x^2} & 0 \\ 0 & -\frac{d^2}{dx^2} + \frac{f(f+1)}{x^2} \end{pmatrix}$$

- Conformal graded-Lie algebra has in addition to Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to generator of conformal transformations $K \sim \{S, S^\dagger\}$

$$S = \chi\, x, \qquad S^\dagger = \chi^\dagger x$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$\begin{aligned} \tfrac{1}{2}\{Q,Q^\dagger\} &= H, \qquad \tfrac{1}{2}\{S,S^\dagger\} = K \\ \tfrac{1}{2}\{Q,S^\dagger\} &= \frac{f}{2} + \frac{\sigma_3}{4} + iD \\ \tfrac{1}{2}\{Q^\dagger,S\} &= \frac{f}{2} + \frac{\sigma_3}{4} - iD \end{aligned}$$

where the operators

$$\begin{aligned} H &= \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 - \sigma_3 f}{x^2}\right) \\ D &= \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right) \\ K &= \tfrac{1}{2}x^2 \end{aligned}$$

satisfy the conformal algebra

$$[H,D] = iH, \quad [H,K] = 2iD, \quad [K,D] = -iK$$

- Following F&R define a supercharge R, a linear combination of the generators Q and S

$$R = \sqrt{u}\, Q + \sqrt{w}\, S$$

and consider the new generator $G = \frac{1}{2}\{R, R^\dagger\}$ which also closes under the graded algebra $sl(1/1)$

$$\begin{aligned} &\tfrac{1}{2}\{R, R^\dagger\} = G && \tfrac{1}{2}\{Q, Q^\dagger\} = H \\ &\{R, R\} = \{R^\dagger, R^\dagger\} = 0 && \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0 \\ &[R, H] = [R^\dagger, H] = 0 && [Q, H] = [Q^\dagger, H] = 0 \end{aligned}$$

- New QM evolution operator

$$G = uH + wK + \tfrac{1}{2}\sqrt{uw}\,(2f + \sigma_3)$$

is compact for $uw > 0$: Emergence of a scale since Q and S have different units

- Light-front extension of superconformal results follows from

$$x \to \zeta, \quad f \to \nu + \tfrac{1}{2}, \quad \sigma_3 \to \gamma_5, \quad 2G \to H_{LF}$$

- Obtain:

$$H_{LF} = -\frac{d^2}{d\zeta^2} + \frac{\left(\nu + \frac{1}{2}\right)^2}{\zeta^2} - \frac{\nu + \frac{1}{2}}{\zeta^2}\gamma_5 + \lambda^2\zeta^2 + \lambda(2\nu + 1) + \lambda\gamma_5$$

where coefficients u and w are fixed to $u = 2$ and $w = 2\lambda^2$

- Take the ‘square root’ of the LF Hamiltonian $H_{LF} = \{R, R^\dagger\}$

$$H_{LF}\,\psi = D_{LF}^2 \psi = M^2 \psi$$

with the linear Dirac equation

$$(D_{LF} - M)\,\psi = 0$$

- In a 2×2 component representation $\psi_\pm$

$$\begin{aligned} -\frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - \lambda\,\zeta\psi_- &= M\psi_+ \\ \frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - \lambda\,\zeta\psi_+ &= M\psi_- \end{aligned}$$

where the chiral spinors are defined by $\psi_\pm = \frac{1}{2}\,(1 \pm \gamma_5)\,\psi$

- Note: In a 4×4 Dirac-matrix representation the spinor operators χ and $\chi^\dagger$ satisfy the relations

$$\{\chi, \chi^\dagger\} = 1 \quad \text{and} \quad [\chi, \chi^\dagger] = \gamma_5$$

(5) Embedding half-integer spin wave equations in AdS space

[J. Polchinski and M. J. Strassler, JHEP **0305**, 012 (2003)]

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

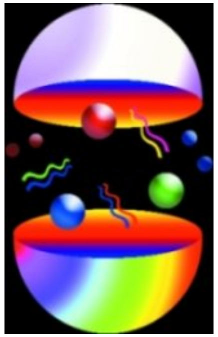


Image credit: N. Evans

- Important similarities between spectra of mesons and baryons: similar slope and spacing of orbital and radial excitations, similar multiplicity
- Holographic embeddings in AdS also explains distinctive features, such as the absence of spin-orbit coupling for baryons
- Half-integer spin-J in AdS described by Rarita-Schwinger (RS) spinor field $\left[\Psi_{N_1 \cdots N_{J-1/2}}\right]_\alpha$ with effective action ($J = T + 1/2$)

$$S_{eff} = \frac{1}{2} \int d^d x \, dz \, \sqrt{|g|} \, g^{N_1 \, N_1'} \cdots g^{N_T \, N_T'} \left[\overline{\Psi}_{N_1 \cdots N_T} \Big(i \, \Gamma^A \, e^M_A \, D_M - \mu - \rho(z) \Big) \Psi_{N_1' \cdots N_T'} + h.c. \right]$$

where covariant derivative D_M includes affine connection and spin connection

- e^A_M is the vielbein and Γ^A tangent space Dirac matrices $\left\{\Gamma^A, \Gamma^B\right\} = \eta^{AB}$

- Dilaton term does not lead to confinement: introduce effective interaction $\rho(z)$ in AdS Dirac equation [Z. Abidin and C. E. Carlson, Phys. Rev. D **79**, 115003 (2009)]

- Baryons described by half-integer spin-J field in AdS

$$\Psi^{\pm}_{\nu_1\cdots\nu_{J-1/2}}(x,z) = e^{iP\cdot x}\, u^{\pm}_{\nu_1\cdots\nu_{J-1/2}}(P)\Psi^{\pm}_J(z)$$

with invariant hadronic mass $P_\mu P^\mu = M^2$ and chiral spinors $u^{\pm} = \frac{1}{2}(1 \pm \gamma_5)u$ with polarization indices along physical coordinates

- Variation of the AdS action leads to Dirac equation $\left(V(z) = \frac{R}{z}\rho(z)\right)$

$$\left[-\frac{d}{dz} - \frac{\mu R}{z} - V(z)\right]\Psi^- = M\Psi^+$$
$$\left[\frac{d}{dz} - \frac{\mu R}{z} - V(z)\right]\Psi^+ = M\Psi^-$$

where

$$V(z) = \frac{R}{z}\rho(z)$$

and the Rarita-Schwinger condition in physical space-time

$$\gamma^\nu \Psi_{\nu\nu_2\ldots\nu_T} = 0$$

- Compare AdS Dirac equation for spin J

$$
\begin{aligned}
-\frac{d}{dz}\Psi_J^- - \frac{\mu R}{z}\Psi_J^- - V(z)\Psi_J^- &= M\Psi_J^+ \\
\frac{d}{dz}\Psi_J^+ - \frac{\mu R}{z}\Psi_J^+ - V(z)\Psi_J^+ &= M\Psi_J^-
\end{aligned}
$$

with LF SUSY Dirac equation

$$
\begin{aligned}
-\frac{d}{d\zeta}\psi_- - \frac{\nu+\frac{1}{2}}{\zeta}\psi_- - \lambda\,\zeta\psi_- &= M\psi_+ \\
\frac{d}{d\zeta}\psi_+ - \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ - \lambda\,\zeta\psi_+ &= M\psi_-
\end{aligned}
$$

- Identify holographic variable z with invariant LF variable ζ and $\Psi_J \to \psi$
- AdS mass is related to parameter ν by $\mu R = \nu + \frac{1}{2}$ and

$$V(\zeta) = \lambda\,\zeta$$

a J-independent potential – No spin-orbit coupling along a given trajectory !

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

Baryon spectrum

- In 2×2 block-matrix form

$$H_{LF} = \begin{pmatrix} -\frac{d^2}{d\zeta^2} - \frac{1-4\nu^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(\nu+1) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} - \frac{1-4(\nu+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda\nu \end{pmatrix}$$

- The light-front eigenvalue equation $H_{LF}|\psi\rangle = M^2|\psi\rangle$ has eigenfunctions

$$\begin{aligned} \psi_+(\zeta) &\sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu}(\lambda\zeta^2) \\ \psi_-(\zeta) &\sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2) \end{aligned}$$

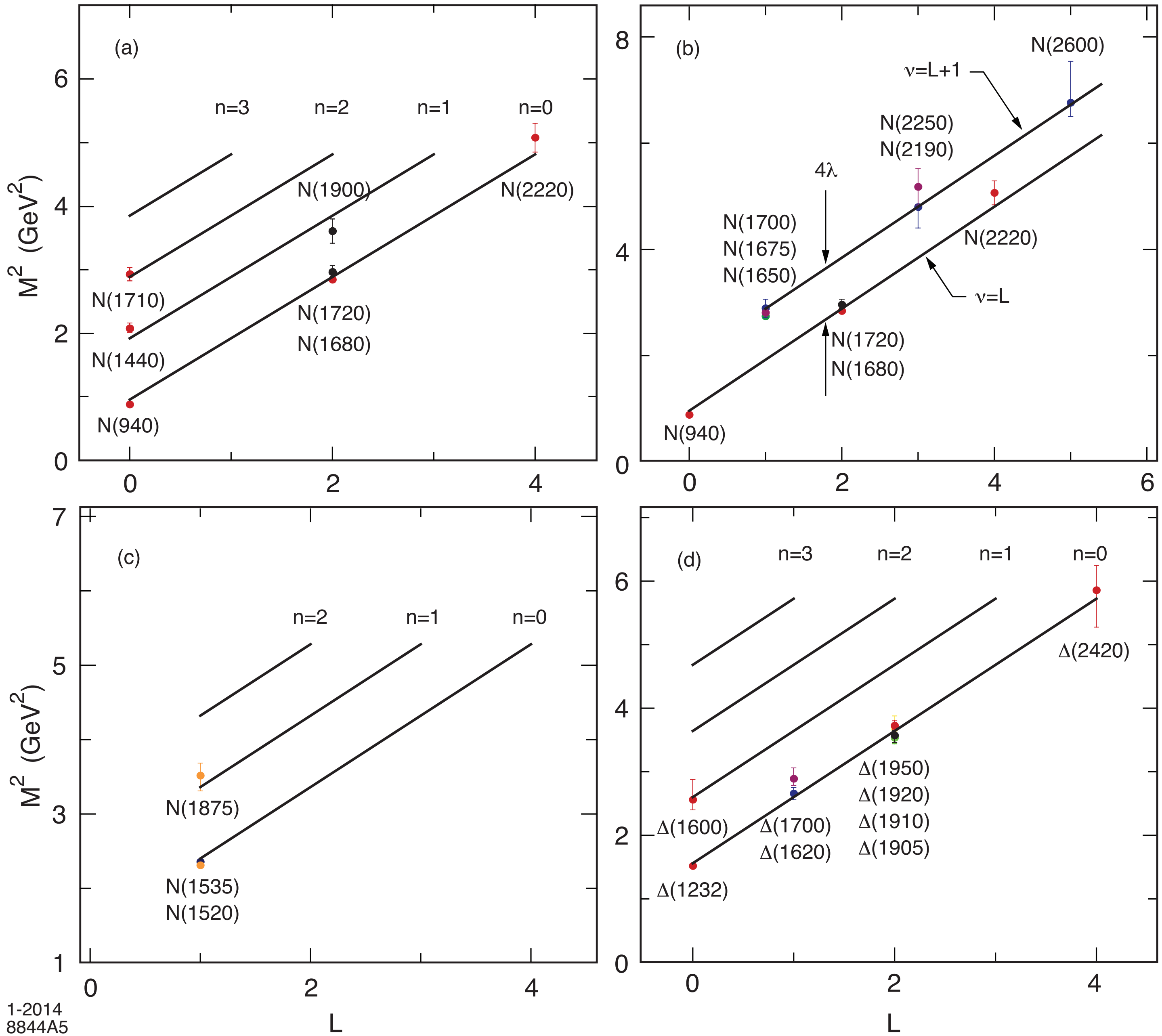
and eigenvalues

$$M^2 = 4\lambda(n + \nu + 1)$$

The assignment

	$S = \frac{1}{2}$	$S = \frac{3}{2}$
P = +	$\nu = L$	$\nu = L + \frac{1}{2}$
P = -	$\nu = L + \frac{1}{2}$	$\nu = L + 1$

describes the full light baryon orbital and radial excitation spectrum



Baryon orbital and radial excitations for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)

(6) Superconformal baryon-meson symmetry and LF holographic QCD

[H.G. Dosch, GdT, and S. J. Brodsky, Phys. Rev. D **91**, 085016 (2015)]

- Previous application: positive and negative chirality components of baryons related by supercharge R

$$R^\dagger |\psi_+\rangle = |\psi_-\rangle$$

with identical eigenvalue M^2 since $[R, G] = [R^\dagger, G] = 0$

- Conventionally supersymmetry relates fermions and bosons

$$R^\dagger |\mathrm{Baryon}\rangle = |\mathrm{Meson}\rangle \quad \text{or} \quad \mathrm{R}\,|\mathrm{Meson}\rangle = |\mathrm{Baryon}\rangle$$

- If $|\phi\rangle_M$ is a meson state with eigenvalue M^2, $G\,|\phi\rangle_M = M^2|\phi\rangle_M$, then there exists also a baryonic state $R\,|\phi\rangle_M = |\phi\rangle_B$ with the same eigenvalue M^2:

$$G\,|\phi\rangle_B = G\,R|\phi\rangle_M = R\,G\,|\phi\rangle_M = M^2|\phi\rangle_B$$

- For a zero eigenvalue M^2 we can have the trivial solution

$$|\phi(M^2 = 0)\rangle_B = 0$$

Special role played by the pion as a unique state of zero energy

Superpartner of the nucleon trajectory

$$|\phi\rangle = \begin{pmatrix} \phi_{\text{Baryon}} \\ \phi_{\text{Meson}} \end{pmatrix}$$

- Compare superconformal equations with LFH nucleon (leading twist) and pion wave equations:

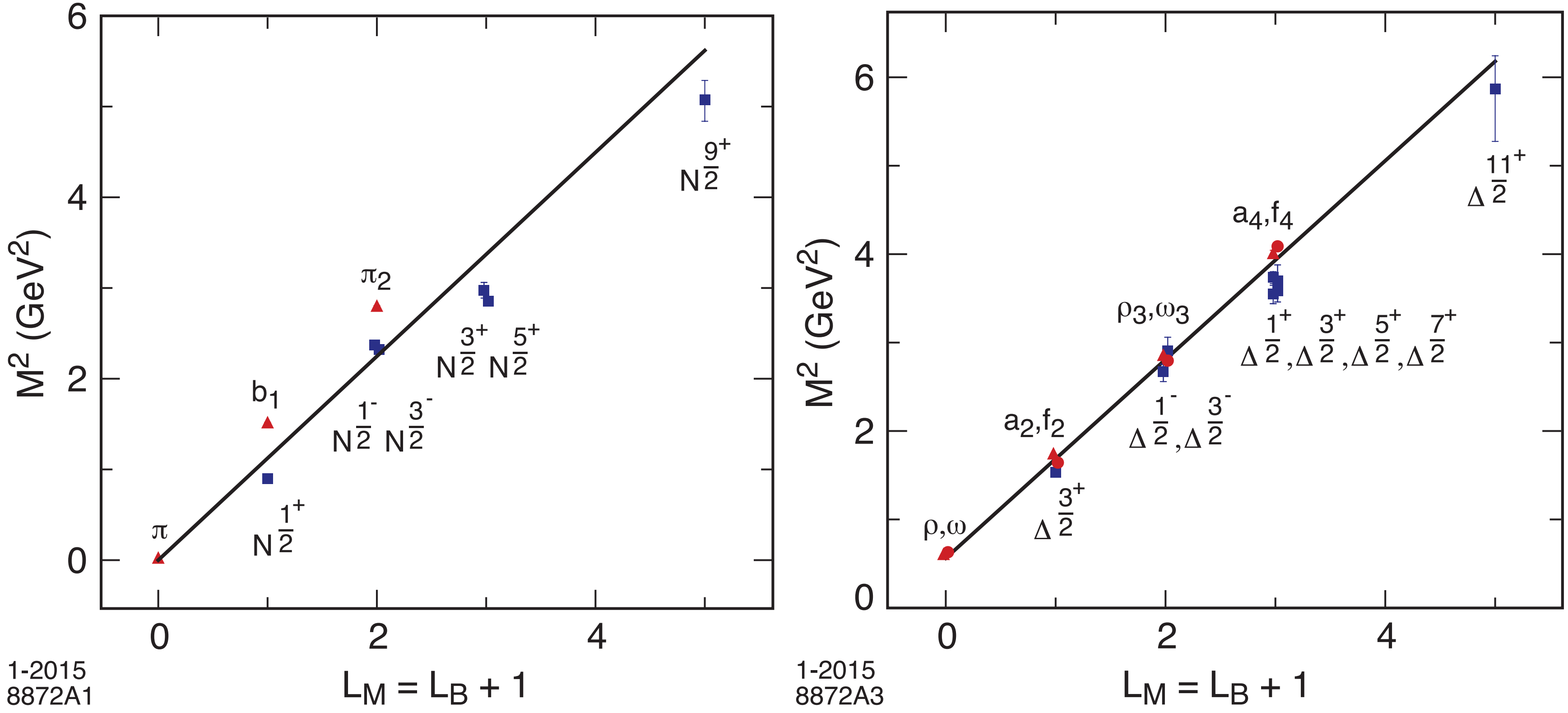
$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f + \lambda + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2}\right) \phi_{\text{Baryon}} = M^2 \phi_{\text{Baryon}}$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2}\right) \psi^+_{L_B} = M^2 \psi^+_{L_B}$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f - \lambda + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2}\right) \phi_{\text{Meson}} = M^2 \phi_{\text{Meson}}$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1) + \frac{4L_M^2 - 1}{4\zeta^2}\right) \psi_{L_M} = M^2 \psi_{L_M}$$

- Find: $\lambda = \lambda_M = \lambda_B, \quad f = L_B + \frac{1}{2} = L_M - \frac{1}{2} \quad \Rightarrow \quad \boxed{L_M = L_B + 1}$

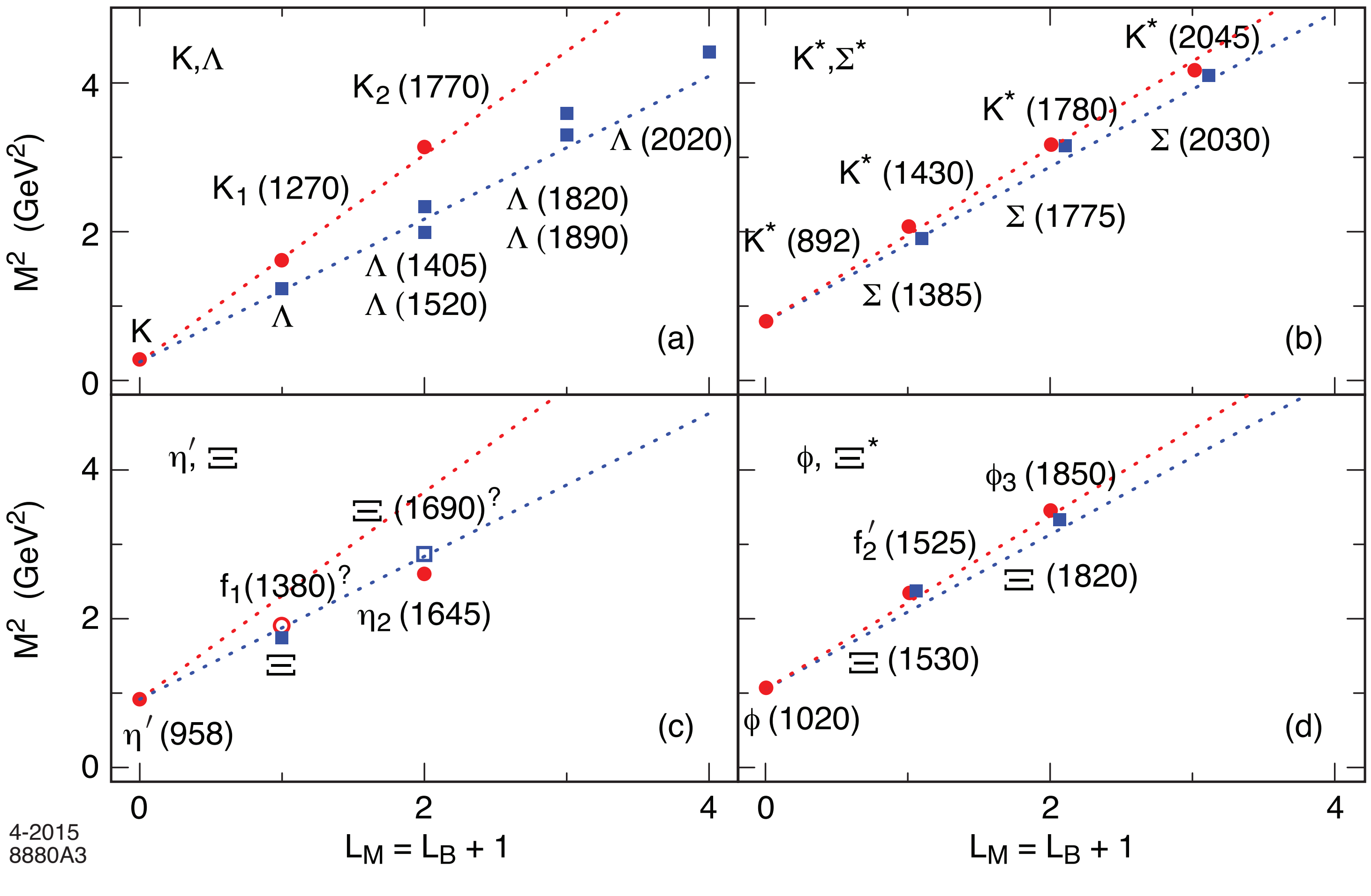


Superconformal meson-nucleon partners: solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV

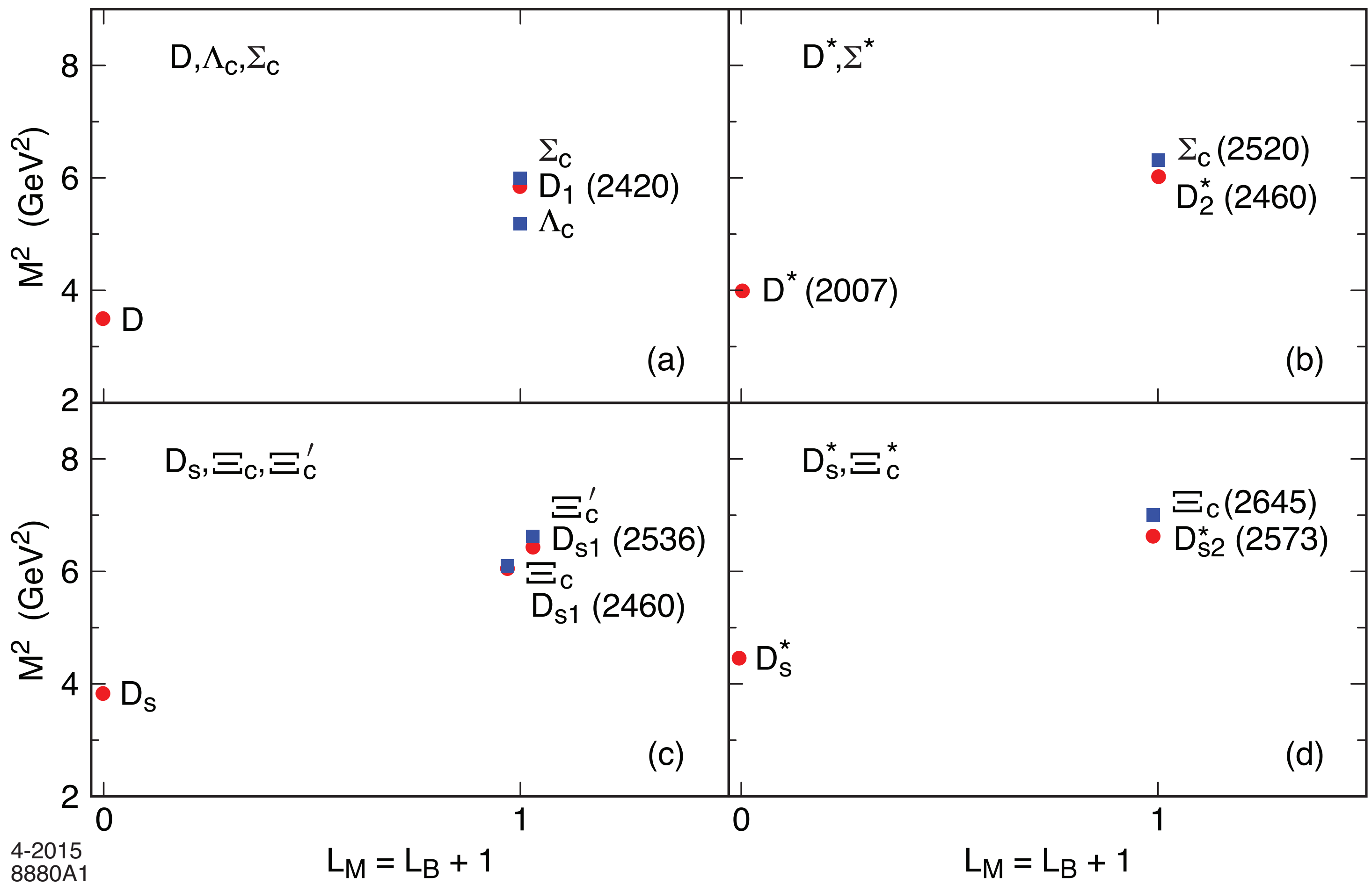
(7) Supersymmetry across the light and heavy-light hadronic spectrum

[H.G. Dosch, GdT, S. J. Brodsky, arXiv:1504.05112 (to appear in PRD)]

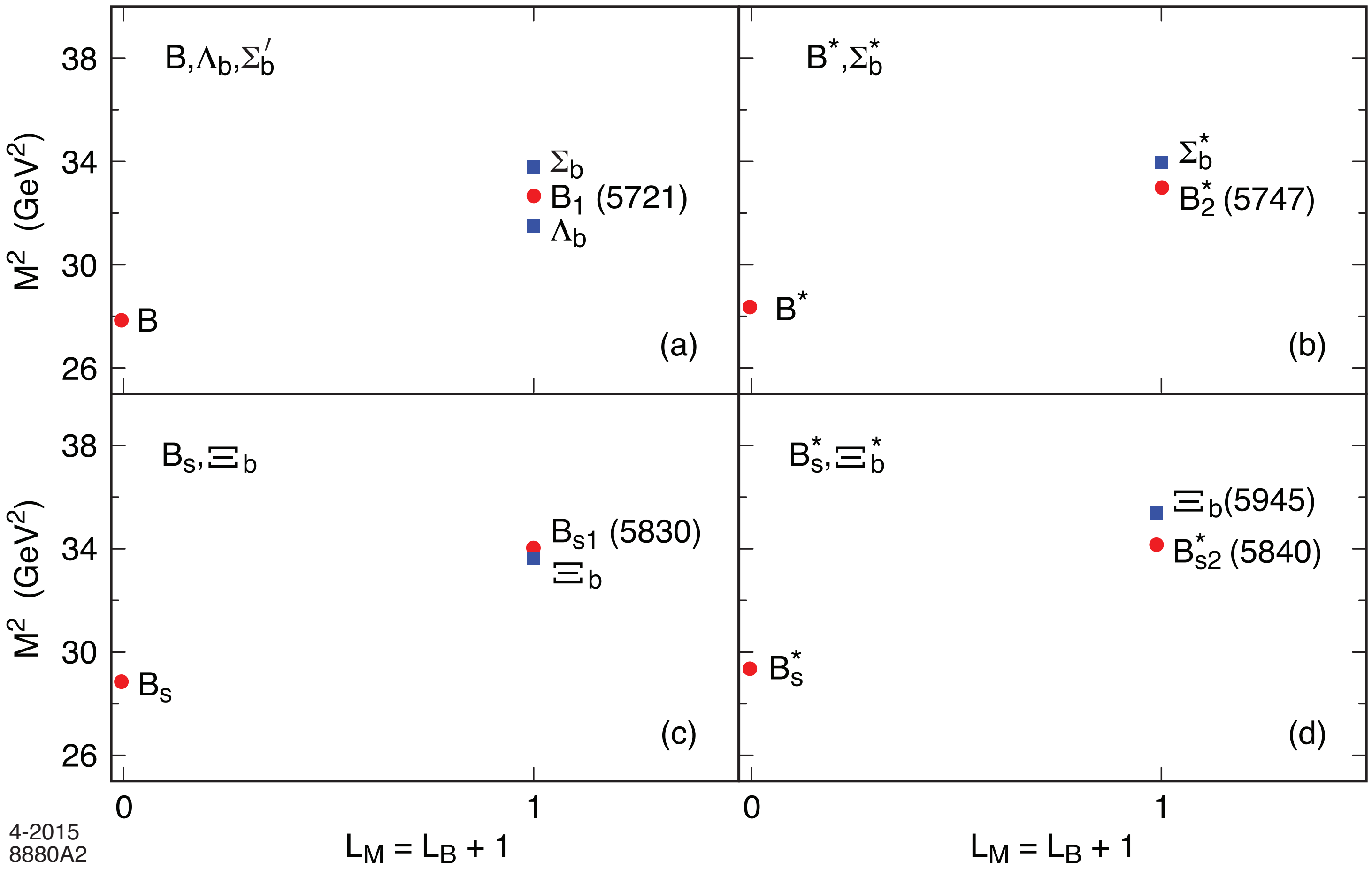
- Introduction of quark masses breaks conformal symmetry without violating supersymmetry



Supersymmetric relations between mesons and baryons with strangeness



Supersymmetric relations between mesons and baryons with charm

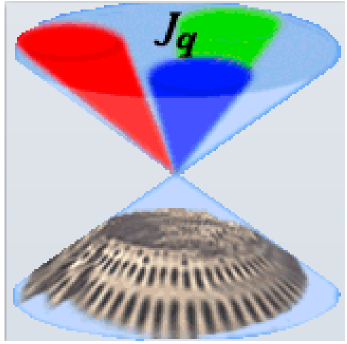


Supersymmetric relations between mesons and baryons with beauty

- Extension to double-heavy hadrons: mesons containing two heavy quarks and their supersymmetric baryon partners containing two heavy and one light quark

Double-heavy meson	Corresponding baryon
$h_c(1P)(3525)$	$\Xi_{ccq},\ \frac{1}{2}^+$
$\chi_{c2}(1P)(3556)$	$\Xi^*_{ccq},\ \frac{3}{2}^+$
$h_b(1P)(9899)$	$\Xi_{bbq},\ \frac{1}{2}^+$
$\chi_{b2}(1P)(9912)$	$\Xi^*_{bbq},\ \frac{3}{2}^+$

- Predicted values higher than the masses of the SELEX double-charm ccu and ccd candidates, but below the predictions from quark models and lattice computations



Thanks !

For a review: S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rept. **584**, 1 (2015)