Quark-Hadron Duality: Connecting the Perturbative and Non-Perturbative QCD Regimes

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Quark-Hadron Duality

❖ What is Quark-hadron duality?

Quark-hadron duality = complementarity between quark and hadron descriptions of observables

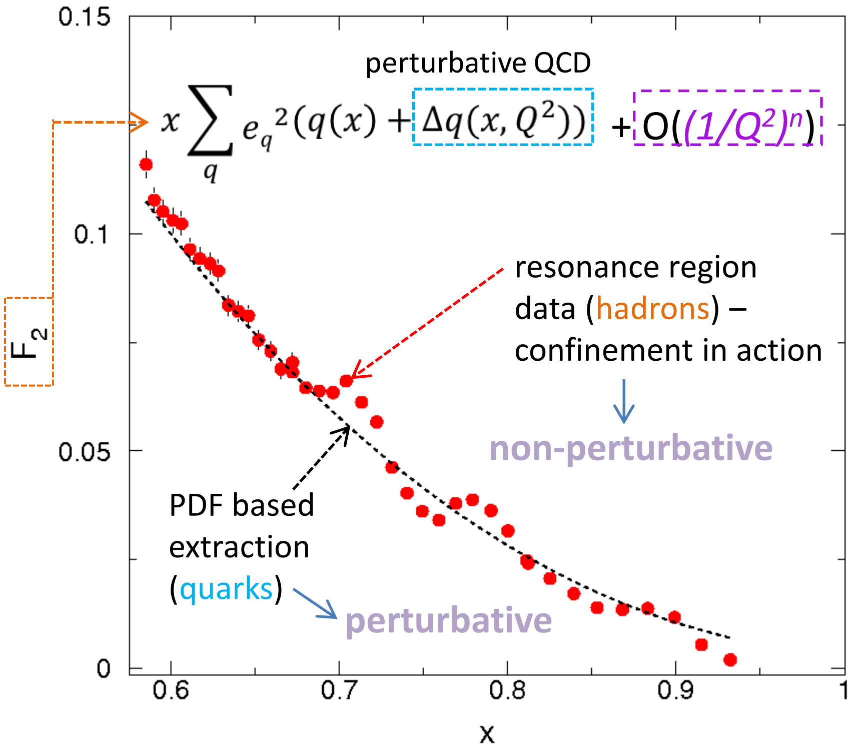
$$\sum_{hadrons} = \sum_{quarks}$$

Description suitable for low-energy regime (confinament)

Description suitable for high-energy regime (asymptotic freedom)

→ We can use either set of **complete** basis states to describe physical phenomena

→ **In practice**, at finite energy we typically have access only to a **limited set of basis states**

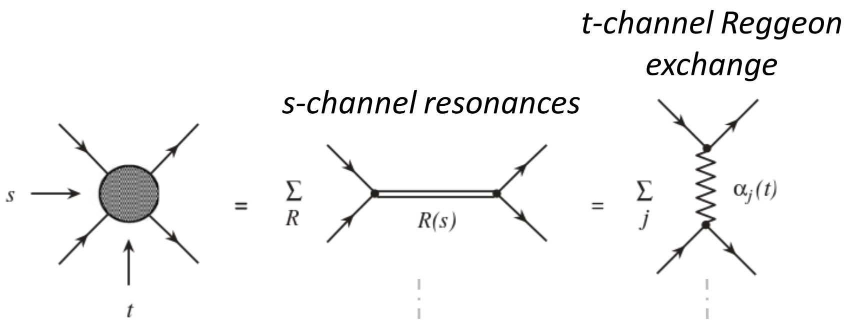


Resonance region data average to PDF based curve: $1/Q^{2n}$ corrections small or cancel on average

→ **Even so**, quark-hadron duality shown to hold globally and locally in **many observables**

Duality: Few Examples

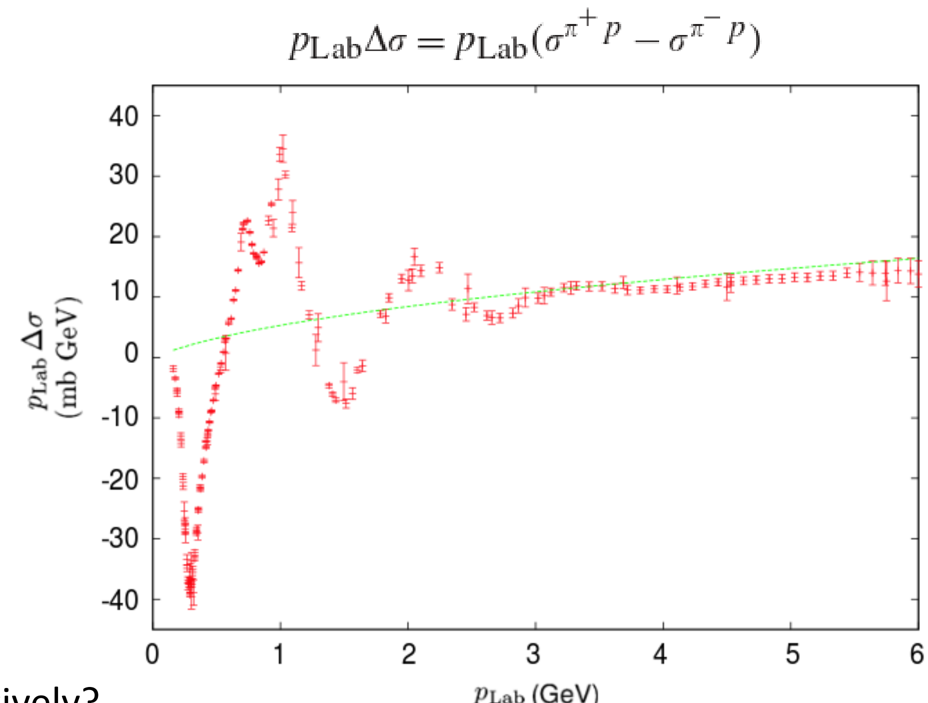
- Duality in pion-hadron scattering: connects description of scattering amplitudes in terms *of s-channel resonances at low energies* and *t-channels Regge poles at high energies*



Scattering amplitude at:

→ low energies (small s): s-channel partial wave sums

→ high energies (large s): t-cannel partial wave sums



Isovector πp cross section as a function of lab momentum compared to the Regge fit to high-energy data

Here the Pomeron-exchange contribution cancels

- How do we connect the two regimes quantitatively?
- What's the best description at intermediate s?

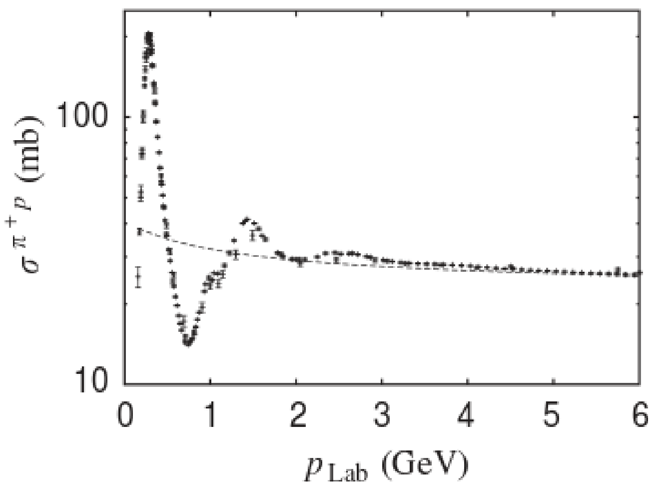
Finite Energy Sum Rule (FESR):

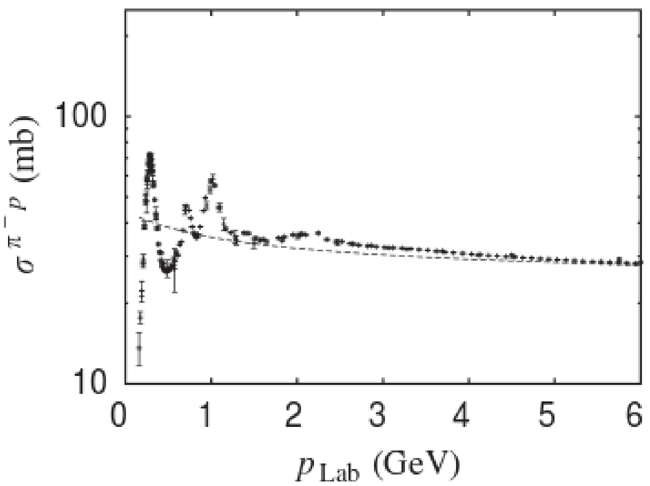
$$\int_0^{\bar{\nu}} \mathrm{d}\nu\, \nu^n \Im m \mathscr{A}(\nu, t) = \int_0^{\bar{\nu}} \mathrm{d}\nu\, \nu^n \Im m \mathscr{A}_{\mathbb{R}}(\nu, t)$$

W. Melnitchouk *et al.*, Physics Reports 406, 127 (2005)

Duality: Few Examples

- Duality in pion-hadron scattering: the low-energy individual cross sections themselves show some degree of duality with the high-energy behavior





- Two-component duality: resonances dual to the non-diffractive Regge pole exchanges and the non-resonant background dual to the Pomeron exchanges

$$\mathscr{A}(s,t) = \sum_{\mathrm{res}} \mathscr{A}_{\mathrm{res}}(s,t) + \mathscr{A}_{\mathrm{bkgd}}(s,t) = \sum_{\mathbb{R}} \mathscr{A}_{\mathbb{R}}(s,t) + \mathscr{A}_{\mathbb{P}}(s,t)$$

→ Since both the non-diffractive and total cross sections satisfy duality the same must be true for the diffractive Pomeron-exchange component

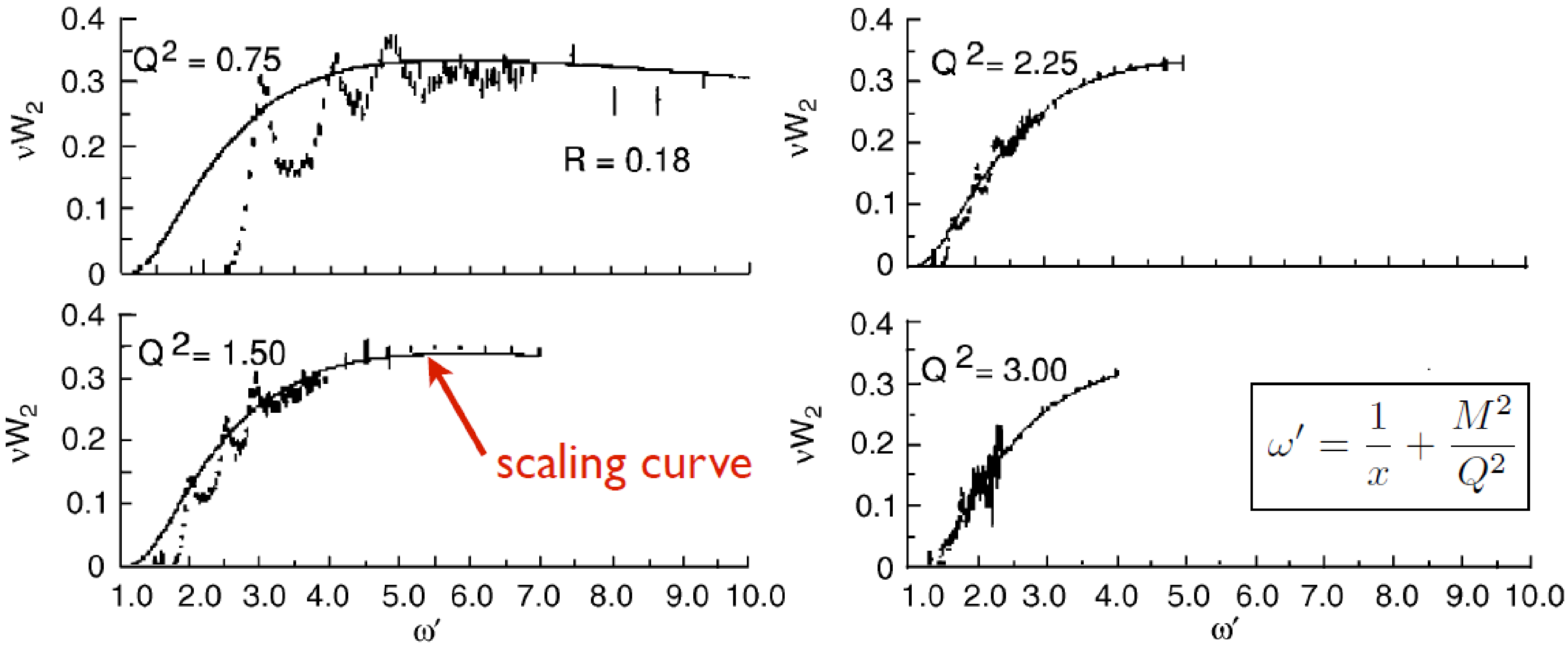
Duality: Few Examples

❖ **Bloom-Gilman Duality** in inclusive **electron-proton** scattering

The resonance region data:

Phys. Rev. Lett. 25, 1140 (1970)

- oscillate around and are on average equivalent to the scaling curve
- "slide" along the deep inelastic curve with increasing Q^2



$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

Finite Energy Sum Rule (FESR): $$\frac{2M}{Q^2}\int_0^{\nu_m} \nu W_2(\nu, Q^2)\,d\nu = \int_1^{\omega'_m} \nu W_2(\omega')\,d\omega'$$

❖ Quantitatively: relative difference 10% for Q^2=1 GeV^2 and <2% beyond Q^2=2 GeV^2

Quark-Hadron Duality in QCD

- Quark-Hadron Duality in inclusive electron-proton scattering: QCD interpretation

De Rujula, Georgi, Politzer, Phys. Lett. B 64, 428 (1976)

Operator Product Expansion

→ Twist (= dimension - spin) expansion of moments of the structure function in QCD

$$\int_0^1 \xi^n F(\xi,Q^2)d\xi = \underbrace{A_n(Q^2)}_{\text{twist-2}} + \underbrace{\sum_{k=1}^{\infty}(nM_0^2/Q^2)^k B_{nk}(Q^2)}_{\text{higher-twist}}$$

Kinematical: target mass effects

Dynamical: quark-quark and quark-gluon correlations that give rise to the resonant enhancements

large Q^2: contribution from the leading-twist => shallow Q^2 dependence of the moments

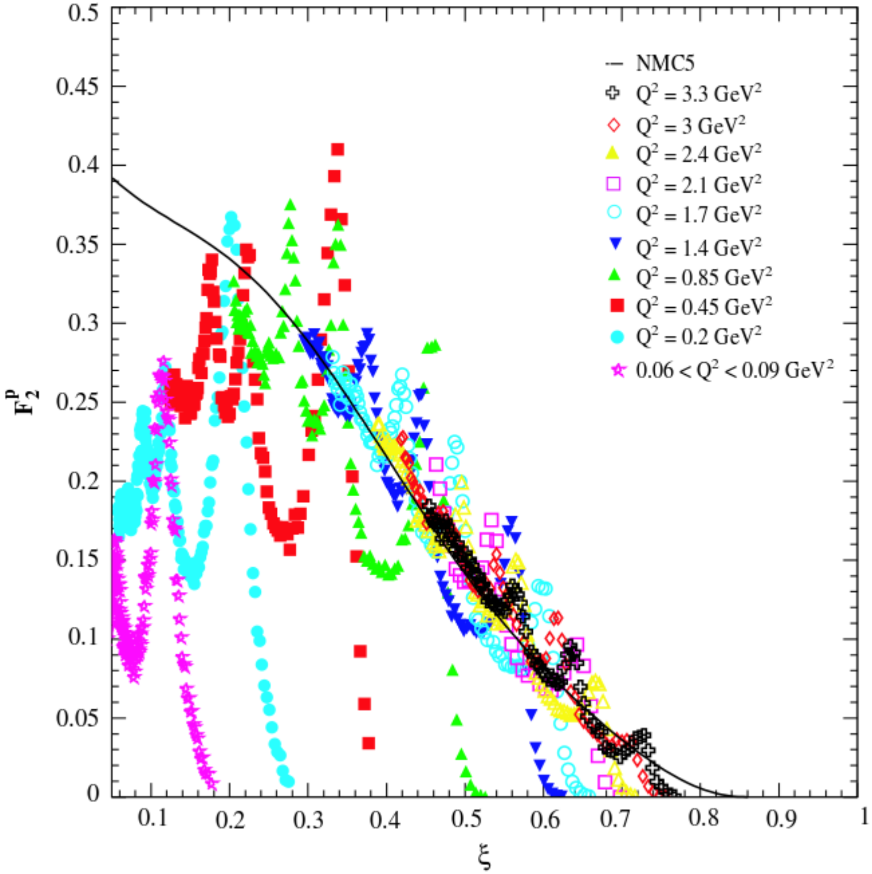
low Q^2: large corrections from the higher-twist terms => very strong Q^2 dependence of the moments

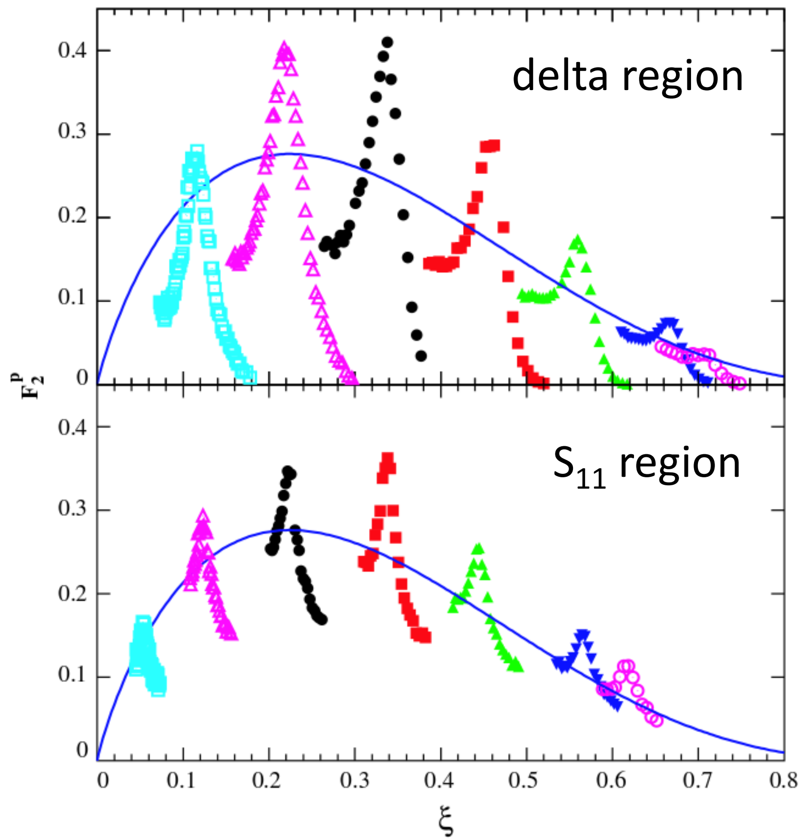
Where Duality holds = higher-twist are either small or cancel on average

The shallow Q^2 dependence of moments at low Q^2 is indeed indicative of duality

Quark-Hadron Duality: Revival

❖ Early (1996) Jefferson Lab experiment re-observes Bloom-Gilman duality





→ ξ variable is used to allow comparison of NMC fit - high (W^2, Q^2) DIS data - to the lower (W^2, Q^2) resonance region data at the same ordinate point

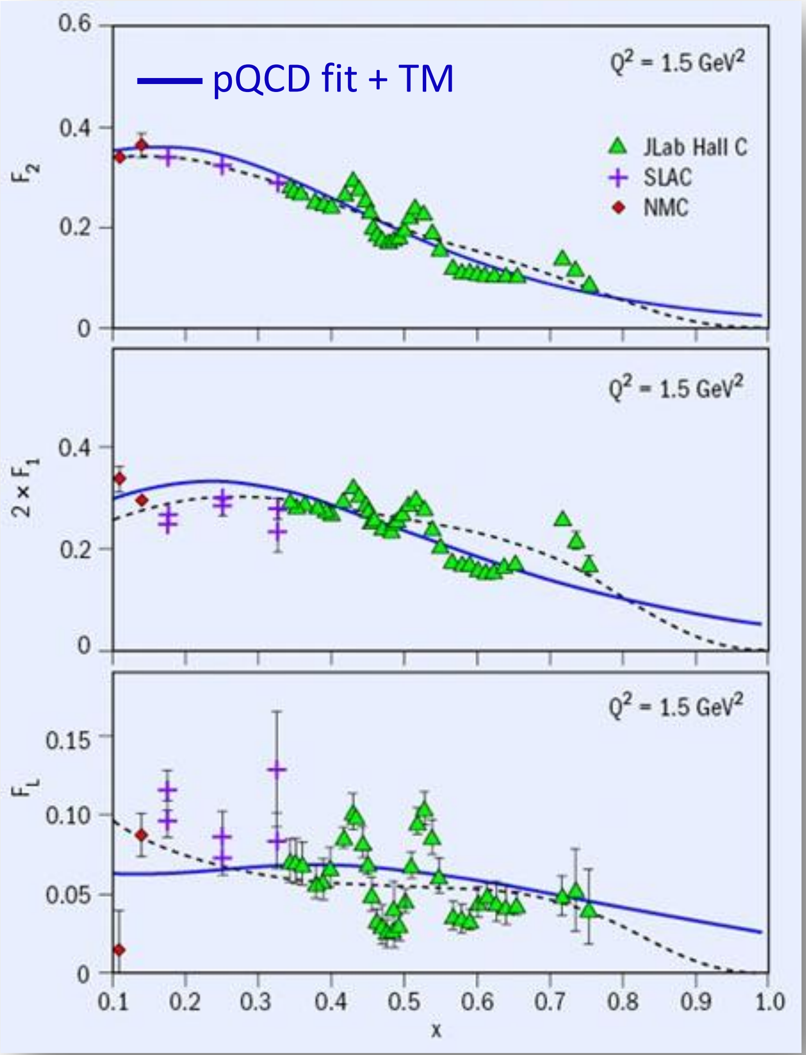
Example: ξ = 0.6 can correspond to $Q^2 = 1.5\ GeV^2$ in the delta region or to a point in DIS with $W^2 = 14\ GeV^2$ and $Q^2 = 20\ GeV^2$

I. Niculescu *et al.*, Phys. Rev. Lett. 85, 1186 (2000)

I. Niculescu *et al.*, Phys. Rev. Lett. 85, 1182 (2000)

Quark-Hadron Duality: Testing with pQCD Curves

- Later Jefferson Lab experiment, E94-110: duality verified in all separated spin-averaged structure functions



→ Compare resonance region data to pQCD fits with added target mass corrections and use x instead of ξ

Resonances average to pQCD curve down to a surprisingly low Q^2

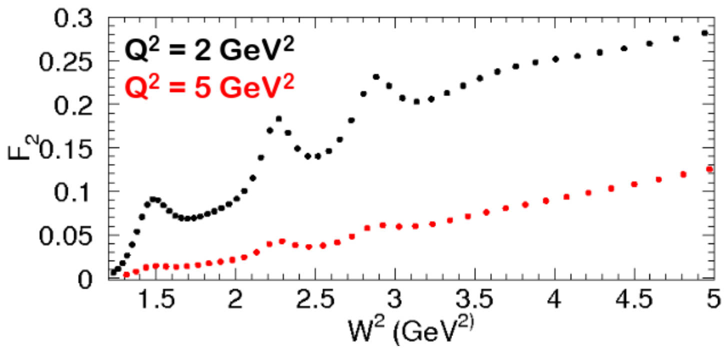
↓

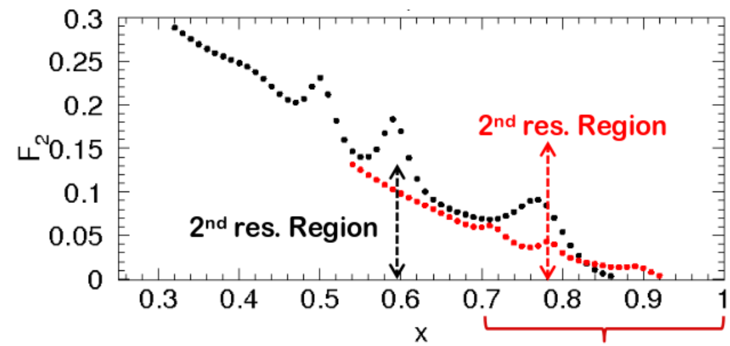
"The successful application of duality to extract known quantities suggests that it should also be possible to use it to extract quantities that are otherwise kinematically inaccessible."
(CERN Courier, 2004)

Y. Liang *et al.*, nucl-ex/0410027

Quark-Hadron Duality: Kinematics

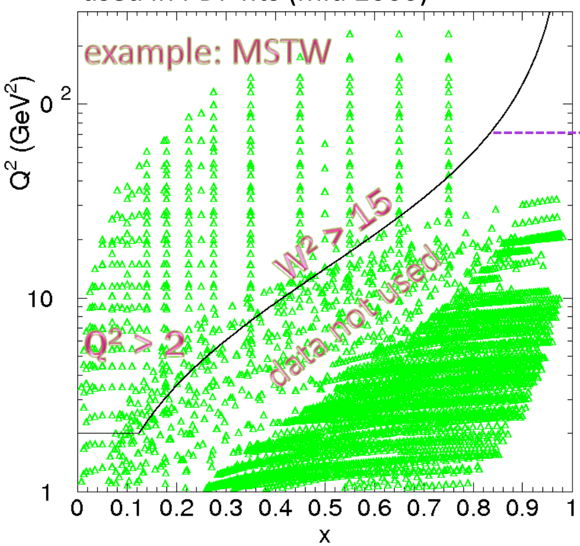
- ❖ Just kinematics: with increasing Q^2 resonances which become less prominent slide towards larger x



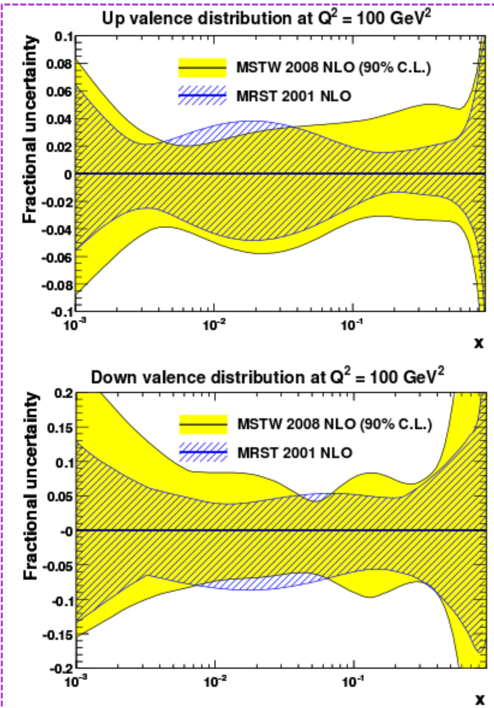


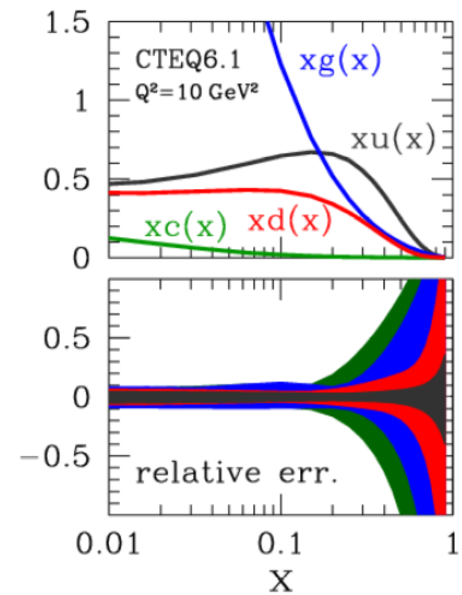
- ❖ "Duality *curve" for verification*: PDF fits ideally well constrained in the x regime where duality needs to be verified

Typical kinematic coverage of data used in PDF fits (mid 2000)



First generation PDF fits





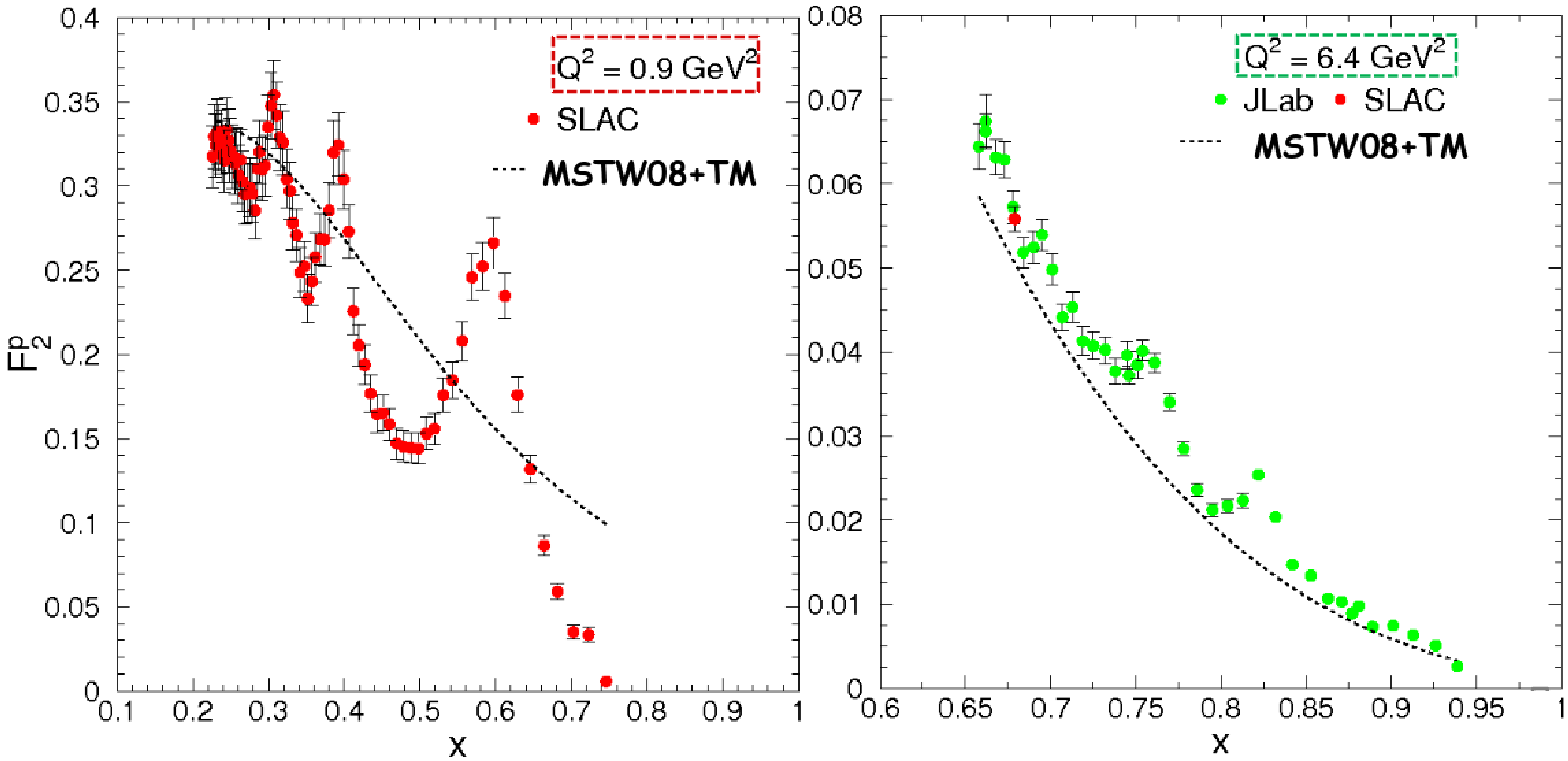
Most PDF extractions not well constrained at large x

Quark-Hadron Duality at Large x

❖ It is not surprising that:

→ though resonances DO average to MSTW08+TM at $Q^2 = 0.9$ GeV^2, x ~ (0.25,0.7)

→ resonances DO NOT average to MSTW08+TM at $Q^2 = 6.4$ GeV^2, x ~ (0.7,0.95)



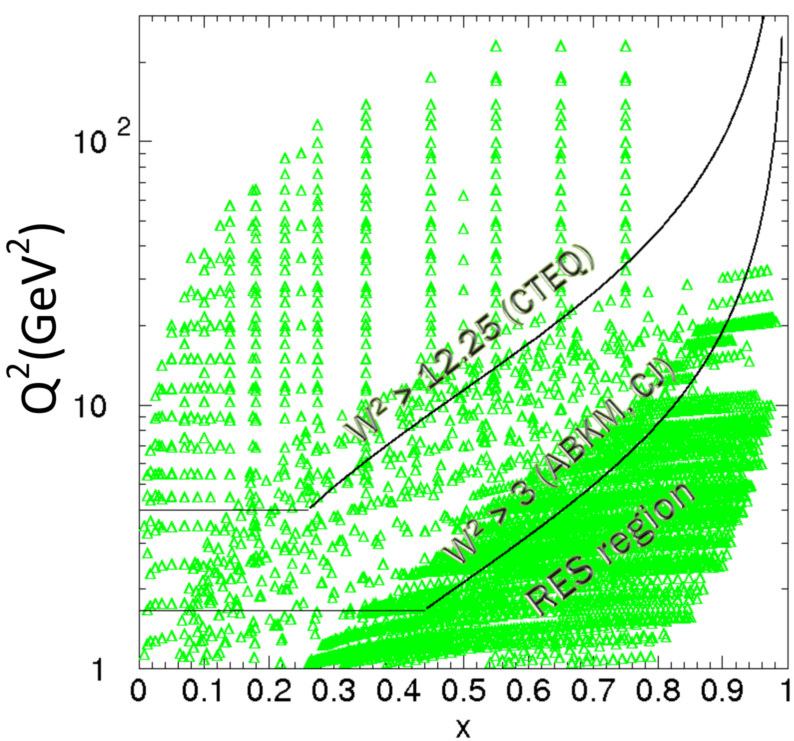
This is not a violation of duality but very likely due to the underestimation of PDFs strength at large x

Quark-Hadron Duality Verification

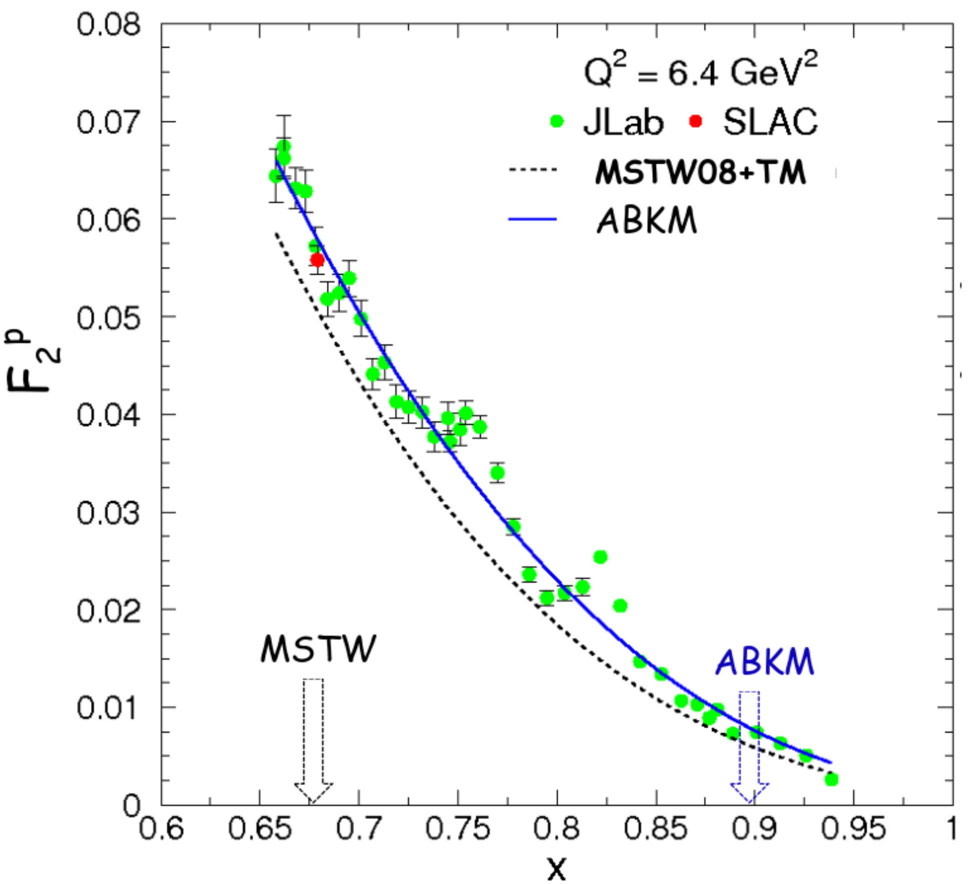
- “Duality *curve” for verification*: PDF fit better constrained at large x

→ Second generation PDF fits: extraction extended to larger x by lowering the W^2 kinematic cuts ABKM, CTEQ-JLab

→ Curve used for duality verification must be from 2nd generation PDF fits



Second generation PDF fits



Quark-Hadron Duality Verification: Local and Global

- Define duality intervals

Region	1^{st}	2^{nd}	3^{rd}	4^{th}	DIS	global
W_{min}	1.3	1.9	2.5	3.1	3.9	1.3
W_{max}	1.9	2.5	3.1	3.9	4.5	4.5

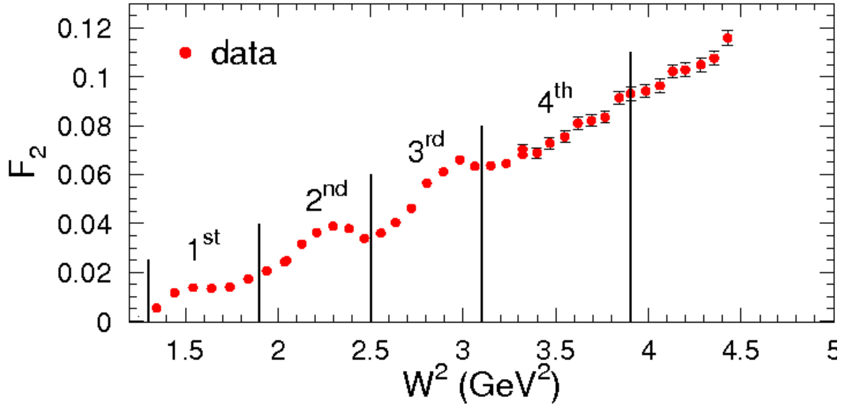
→ any given resonance region will slide towards larger x with increasing Q^2

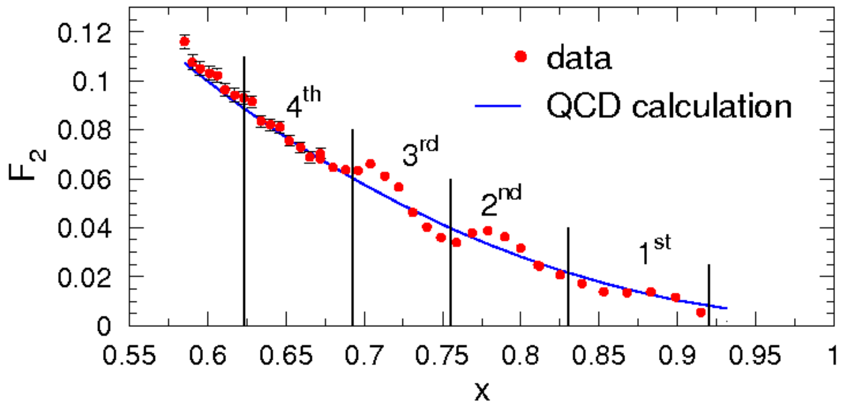
→ There is arbitrariness in defining the local W intervals; typically try to catch peaks and valleys within one interval

How well resonance data average to the scaling curve?

- Calculate ratio:

$$\int_{x_{min}}^{x_{max}} F^{data}(x,Q^2)\,dx \Big/ \int_{x_{min}}^{x_{max}} F^{param.}(x,Q^2)dx$$

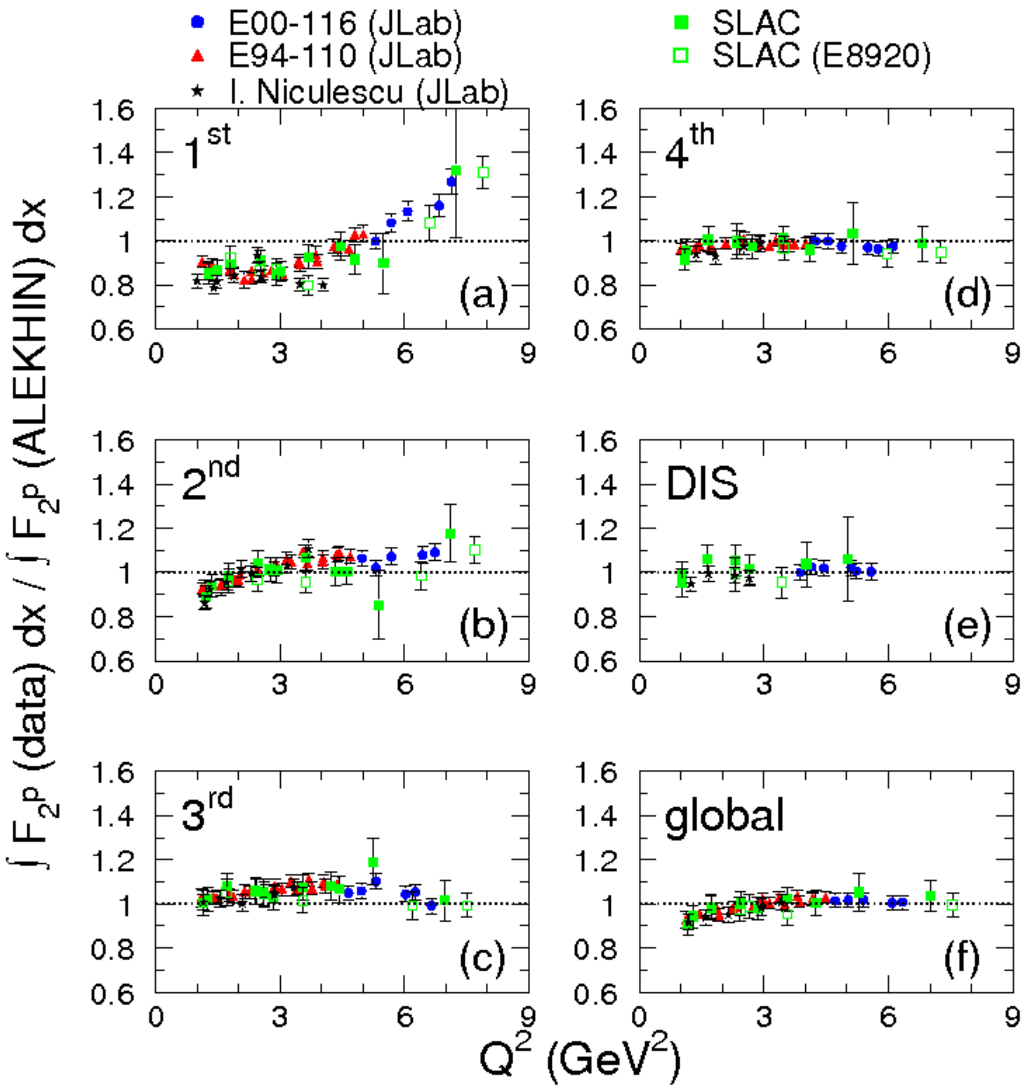




Duality: F_2 Proton Structure Function

Verified!

❖ Quark-hadron duality: averaged resonance region data vs second generation PDF fits



Alekhin *et al.*: NNLO + HT + TM

→ Ratio within 10% globally

→ For 4th RES region and DIS, ratio very close to 1 for entire Q^2 range analyzed

→ For 2nd and 3rd regions ratio within 5-10 % for entire Q^2 range analyzed

1st : special case

→ models predict stronger violations of duality

→ calculation based on handbag diagram may break at such low W

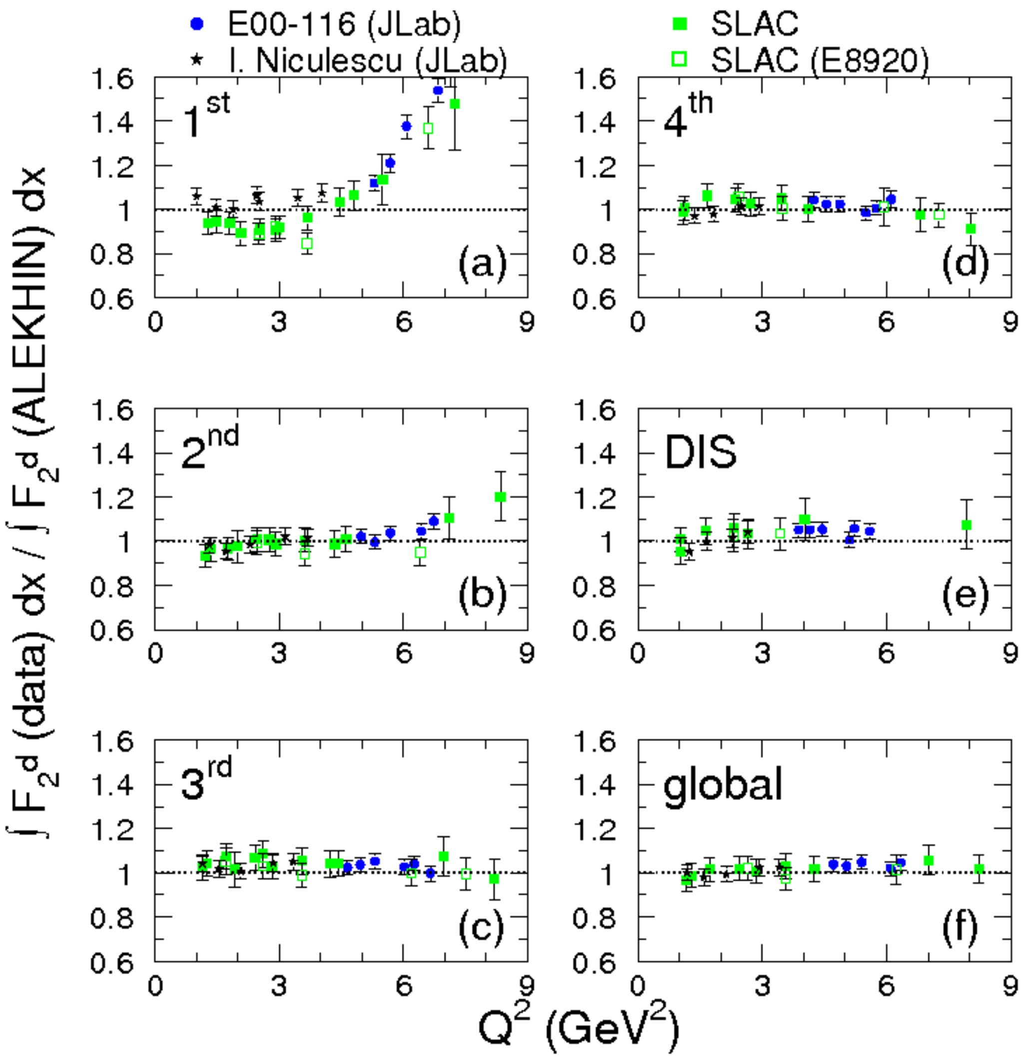
→ at the largest x, QCD fits poorly constrained -> difficult to test duality

S.P. Malace *et al.*, Phys. Rev. C 80 035207 (2009)

Duality: F_2 Deuteron Structure Function

Verified!

❖ Quark-hadron duality: averaged resonance region data vs second generation PDF fits



F_2^d(Alekhin) = F_2^p(Alekhin) * d/p (from empirical fit)

→ Ratio within 5-10% : globally, DIS, 4th, 3rd, 2nd

1st : special case

→ models predict stronger violations of duality

→ calculation based on handbag diagram may break at such low W

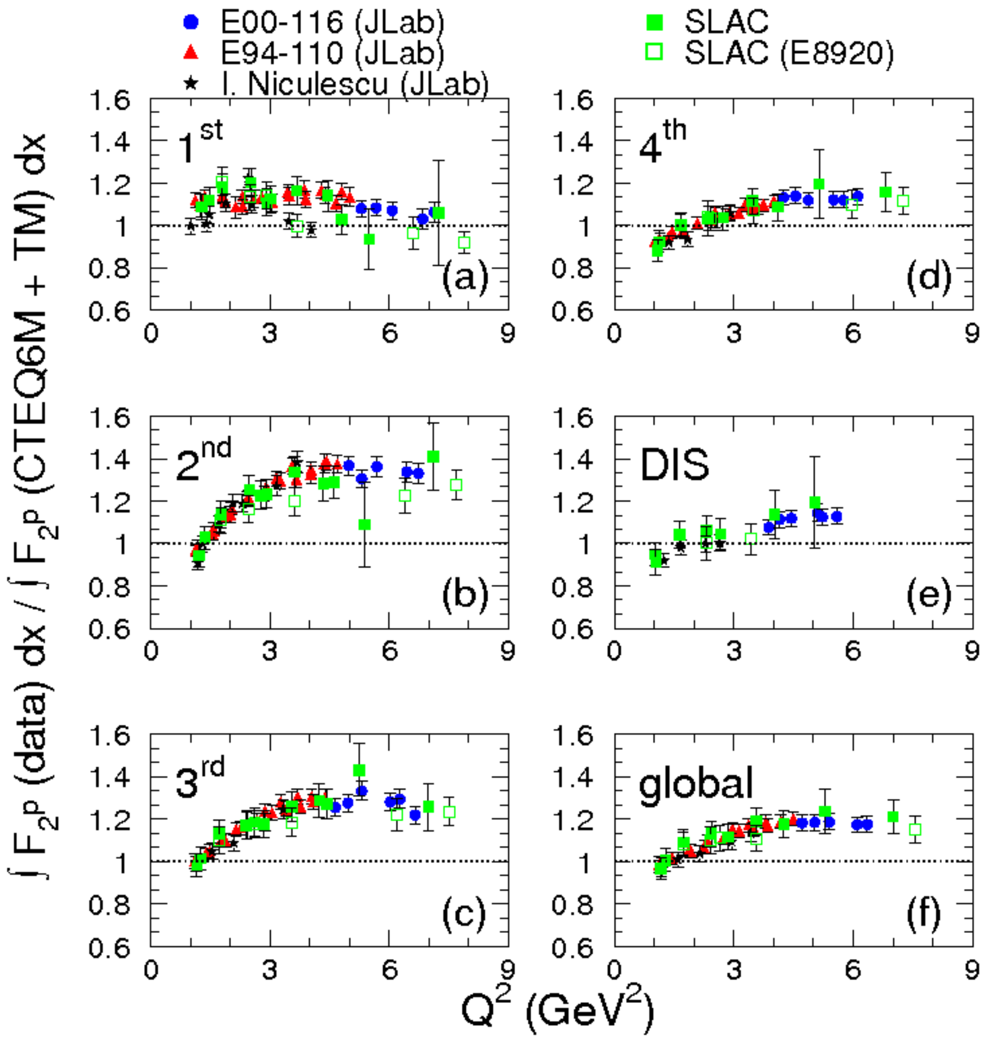
→ at the largest x, QCD fits poorly constrained -> difficult to test duality

→ d/p fit not well constrained at large x

S.P. Malace *et al.*, Phys. Rev. C 80 035207 (2009)

Duality: F_2 Proton Structure Function

❖ Quark-hadron duality: averaged resonance region data vs first generation PDF fits



<u>CTEQ6 + TM</u>

→ Ratio ~ 1 at Q^2 ~ 1.5 GeV^2 then rises with increasing Q^2 and reaches a plateau at ~ 4 GeV^2; above this value Q^2 dependence saturates

→ This behavior displayed when integrating globally and locally except for first resonance region

↓

Not a violation of duality but rather unconstrained PDFs strength at large x

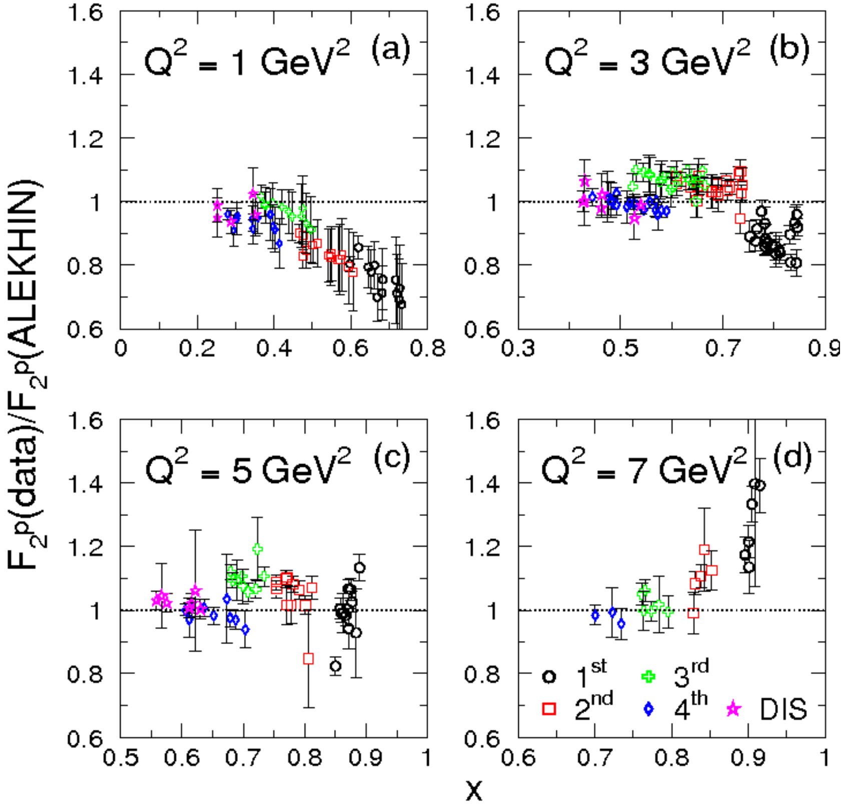
Duality: F_2 Proton Structure Function

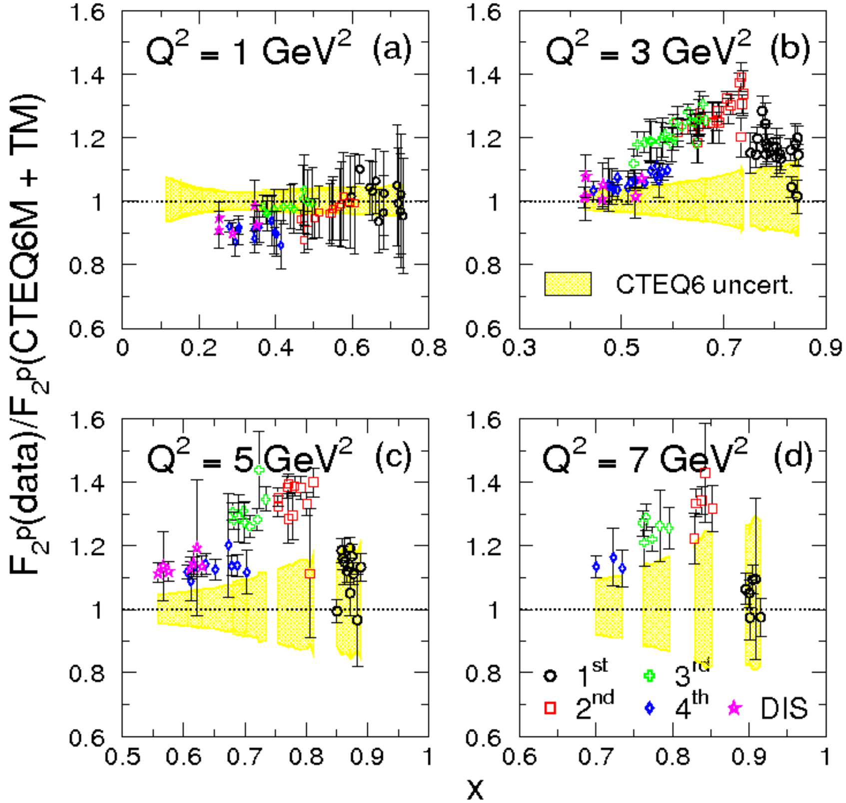
❖ How does it compare in x?

→ Good description at Q^2= 3,5 GeV^2 except for delta region

→ Q^2= 7 GeV^2 : probing the largest x regime, growing discrepancy

→ Fails to describe the x dependence of data in most regions





❖ Better description of data by ALEKHIN than CTEQ6

Duality: F_2 Neutron Structure Function

- First we extract the neutron F_2^n from proton F_2^p and deuteron F_2^d measurements

  How?

→ Impulse Approximation – virtual photon scatters incoherently from individual nucleons

$$F^D{}_2 = \widetilde{F^p}_2 + \widetilde{F^n}_2 + \underbrace{\boxed{\delta^{off}}}_{\text{off-shell correction}} F^D{}_2 \qquad \widetilde{F^{n,p}}_2 = \int_x^{M_D/M} dy \underbrace{\boxed{f(y,\gamma)}}_{\text{smearing function}} F^{n,p}{}_2\left(\frac{x}{y}\right)$$

→ F_2^n via an additive extraction method, solving equation iteratively

$$f(y,\gamma) = \underbrace{\boxed{\mathrm{N}}}_{\text{normalization of smearing function}}\delta(y-1) + \boxed{\delta f(y,\gamma)} \rightarrow \text{finite width of smearing function}$$

$$\widetilde{F^n}_2(x) = NF^n{}_2(x) + \boxed{\int_x^{M_D/M} dy f(y,\gamma) F^n{}_2\left(\frac{x}{y}\right)} \rightarrow \text{perturbation}$$

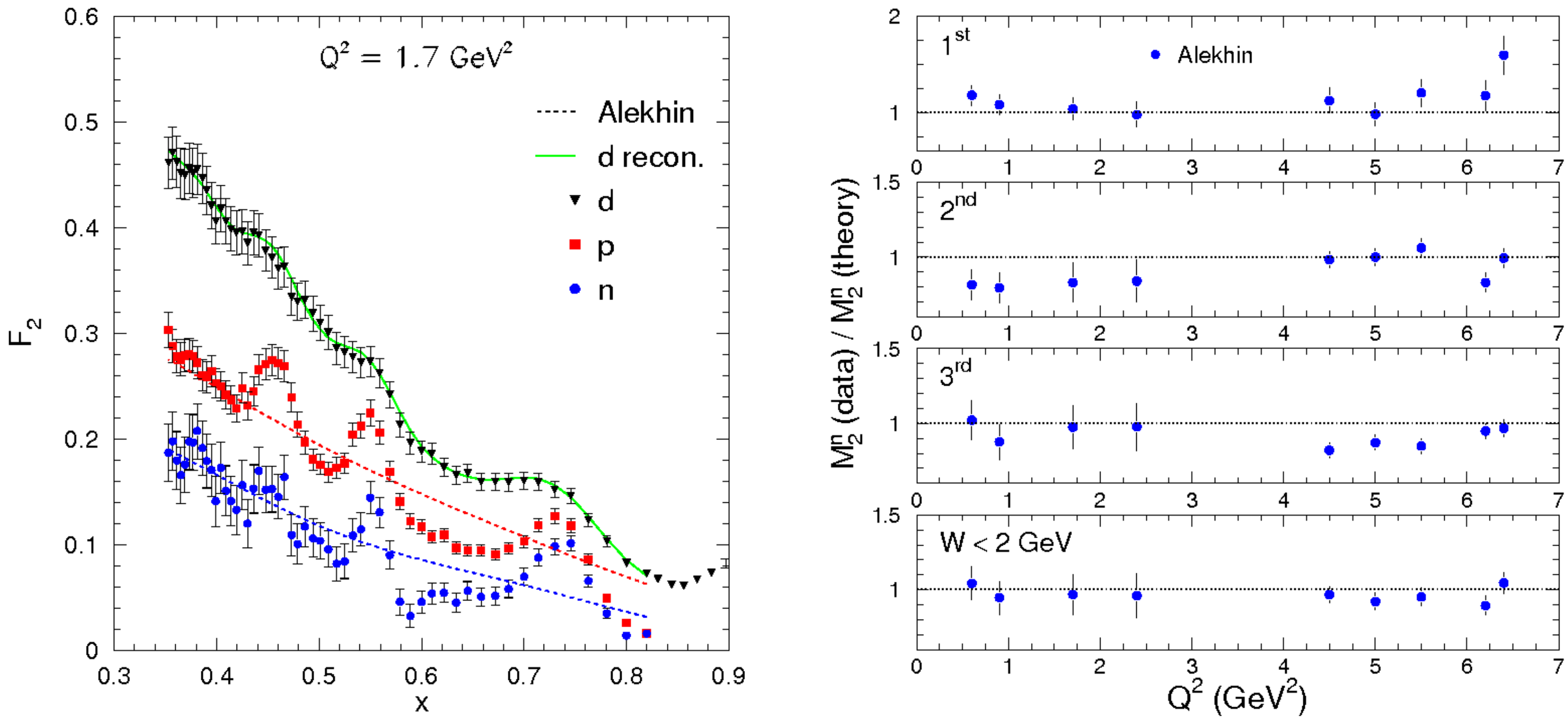
Initial guess for the neutron structure function

$$F^{n(1)}{}_2(x) = F^{n(0)}{}_2(x) + \frac{1}{N}\left[\widetilde{F^n}_2(x) - \int_x^{M_D/M} dy f(y,\gamma) \boxed{F^{n(0)}{}_2}\left(\frac{x}{y}\right)\right]$$

Duality: F_2 Neutron Structure Function

Verified!

- Then we verify quark-hadron duality in the neutron F_2^n using second generation PDF fits (ABKM)



→ Ratio within 10% globally and 15%-20% for 3rd, 2nd resonance regions

→ Our results were later confirmed by the BoNuS experiment at Jefferson Lab

S.P. Malace, Y. Kahn, W. Melnitchouk, C. Keppel, Phys. Rev. Lett. 104, 102001 (2010)

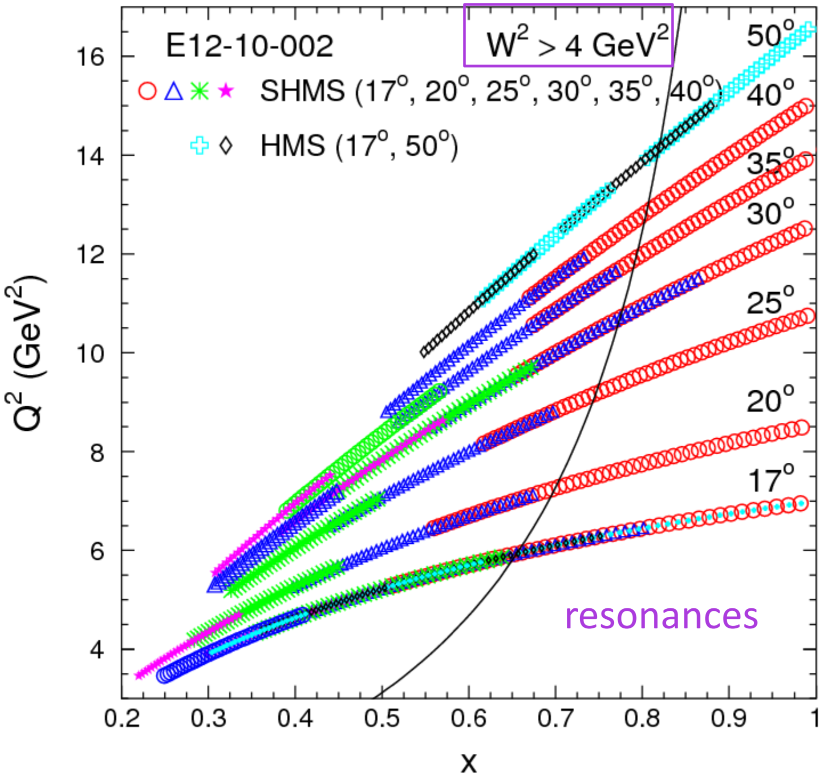
Future: E12-10-002 at JLab, 2016-2017

- **E12-10-002**: new experiment coming up at **Jefferson Lab** to measure cross sections and **F_2 structure functions at large x and low to intermediate Q^2** on proton and deuteron

S.P. Malace – contact and spokesperson
M.E. Christy, C. Keppel, I. Niculescu spokespeople

Measure cross sections in both DIS and resonance regions

DIS data will be included easily in second generation PDF fits: CTEQ-JLAB global fits



Resonance data will be used to study confinement effects and quark-hadron duality

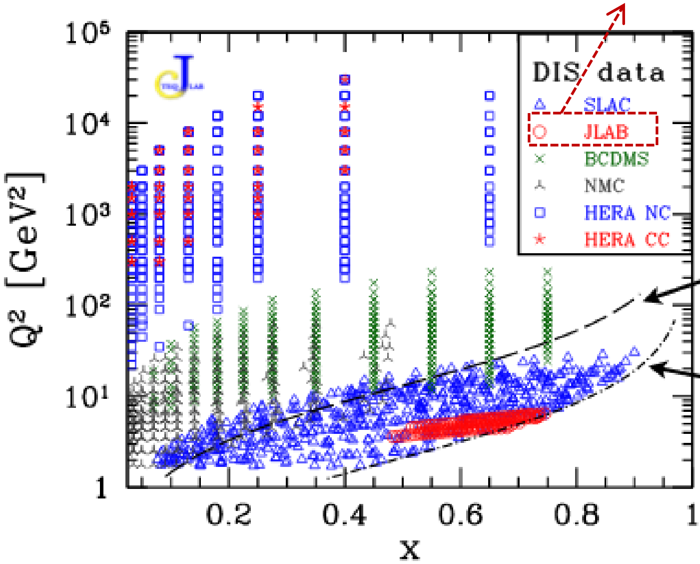
CTEQ-JLab: PDFs at large x

- Next-to-leading order (NLO) analysis of expanded data on proton and deuterium

A. Accardi *et al*., Phys. Rev. D 81 (2010) 034016

→ Improve large-x precision with larger DIS data set on both proton and deuterium: relaxing kinematic cuts to push to larger x leads to a factor of 2 increase in number of DIS data points used for fitting

JLab 6 GeV experiment E00-116: S.P. Malace et al., Phys. Rev. C 80 (2009) 035207



$$W^2 = M^2 + Q^2 \frac{(1-x)}{x}$$

Fits extracted with the "strong" cut will be later called "reference" fits

strong cut:
$Q^2 > 4\ \mathrm{GeV}^2, \quad W^2 > 12.25\ \mathrm{GeV}^2$

weak cut:
$Q^2 > m_c^2\ , \quad W^2 > 3\ \mathrm{GeV}^2$

→ Include all relevant large-x / small-Q^2 theory corrections: use of lower W and Q^2 data requires careful treatment of non-perturbative corrections -- dynamical and kinematic higher-twist (HT)

→ Include nuclear corrections: use of deuterium data requires careful treatment of nuclear corrections -- off-shell effects and sensitivity to the deuteron wave function

CTEQ-JLab: PDFs at large x

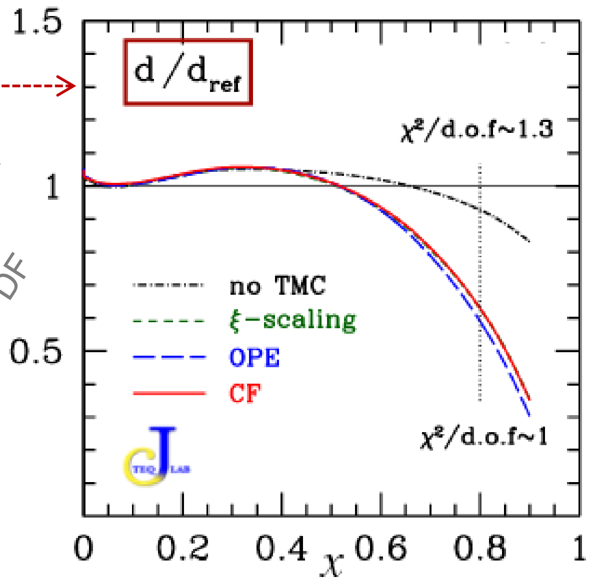
- Next-to-leading order (NLO) analysis of expanded data on proton and deuterium

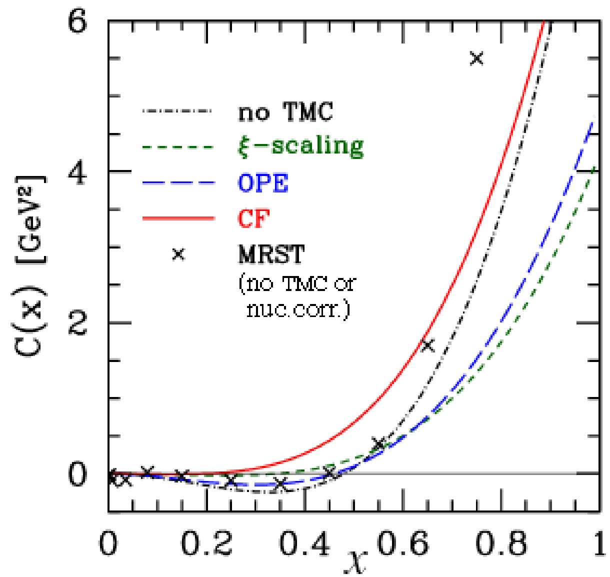
A. Accardi *et al.*, Phys. Rev. D 81 (2010) 034016

→ Non-perturbative 1/Q² corrections: dynamical and kinematic higher-twist

$$F_2(x,Q^2) = F_2^{\mathrm{LT}}(x,Q^2)\left(1+\frac{C(x)}{Q^2}\right) \qquad C(x) \text{ polynomial}$$

TMCs and dynamical HTs compensate each other in the fitting procedure leading to stable PDF





1) Almost identical results for the d-quark distribution when different prescriptions of TMCs are used in conjunction with the dynamical HT → that's great! We don't want the PDF extraction to be affected by our imperfect knowledge of TMCs

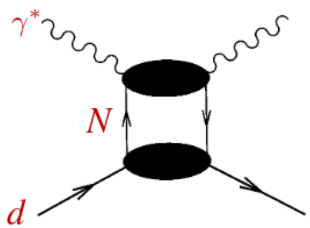
2) The dynamical HT extraction depends on the TMC prescription used

CTEQ-JLab: PDFs at large x

- Next-to-leading order (NLO) analysis of expanded data on proton and deuterium

A. Accardi *et al*., Phys. Rev. D 81 (2010) 034016

→ Nuclear corrections: wave function & off-shell dependence



$N=p+n$

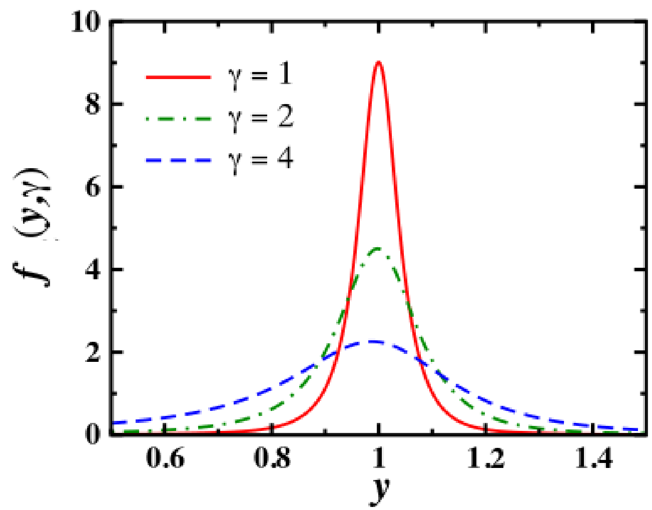
$$F_2^d(x,Q^2) = \int_x dy\ f(y,\gamma)\ F_2^N(x/y,Q^2)\ +\ \delta^{(\mathrm{off})} F_2^d$$

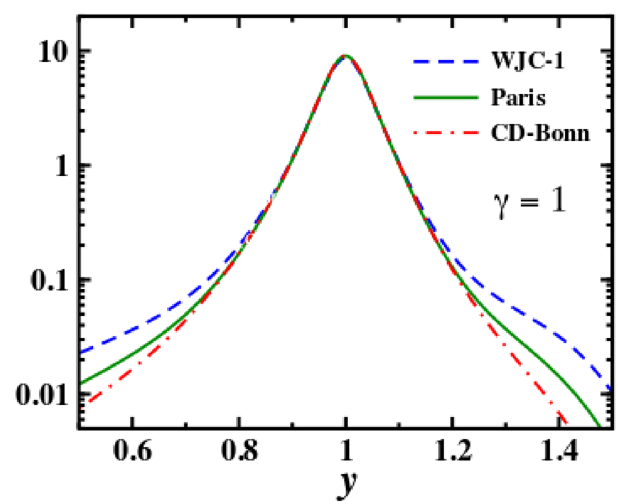
nucleon momentum distribution in d ("smearing function")

off-shell correction

$$f(y,\gamma) = \int \frac{d^3p}{(2\pi)^3}\ |\psi_d(p)|^2\ \delta\Big(y - 1 - \frac{\varepsilon + \gamma p_z}{M}\Big) \times \frac{1}{\gamma^2}\Big[1 + \frac{\gamma^2-1}{y^2}\Big(1 + \frac{2\varepsilon}{M} + \frac{\vec{p}^{\,2}}{2M^2}(1 - 3\hat{p}_z^2)\Big)\Big]$$

$$\gamma = |\mathbf{q}|/q_0 = \sqrt{1 + 4M^2x^2/Q^2}$$





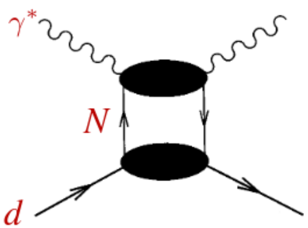
→ more smearing for larger x and lower Q^2

→ greater wave function dependence at large y (and x)

CTEQ-JLab: PDFs at large x

- Next-to-leading order (NLO) analysis of expanded data on proton and deuterium

→ Nuclear corrections: wave function & off-shell dependence



$$F_2^d(x,Q^2) = \int_x dy\; f(y,\gamma)\; F_2^N(x/y,Q^2) \;+\; \delta^{(\text{off})} F_2^d$$

$N{=}p{+}n$

nucleon momentum distribution in d ("smearing function")

off-shell correction

Expand the distribution in the vicinity of mass shell in series of p^2-M^2

$$\widetilde{q}(x,p^2) = q(x)\left[1 + \frac{(p^2-M^2)}{M^2}\,\delta q(x)\right]$$

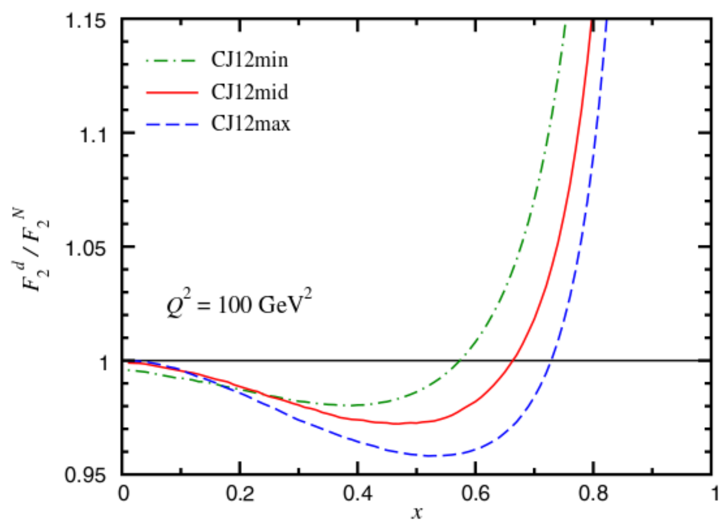
$$\delta q(x) = \left.\frac{\partial \log \widetilde{q}}{\partial \log p^2}\right|_{p^2=M^2}$$

Spectral representation of a quark *q* in an off-shell nucleon with invariant mass *p²*

$$\widetilde{q}(x,p^2) = \int d\hat{p}^2\, \Phi_q(\hat{p}^2, \Lambda(p^2))$$

applied to q_v, $\bar{q}$ & g

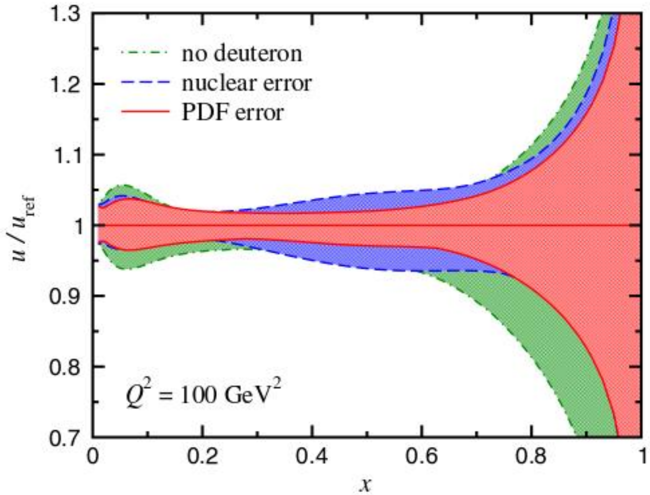
momentum distribution of quarks with virtuality $\hat{p}^2$ in bound nucleon

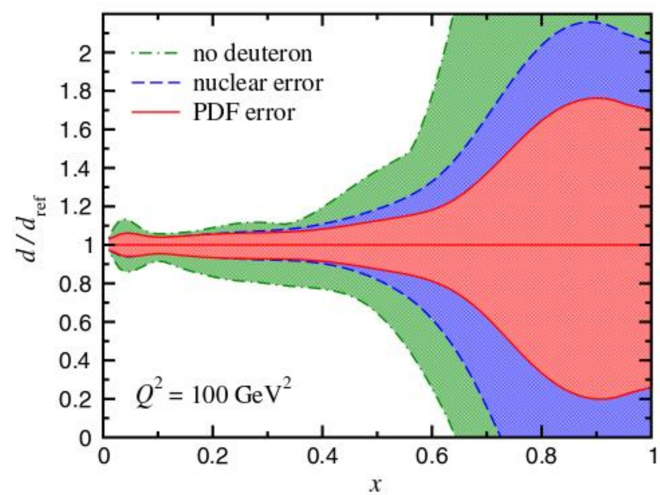


Off-shell rescaling parameter varied in fit to minimize chi²

CTEQ-JLab: PDFs at large x

- Next-to-leading order (NLO) analysis of expanded data on proton and deuterium





Yes!

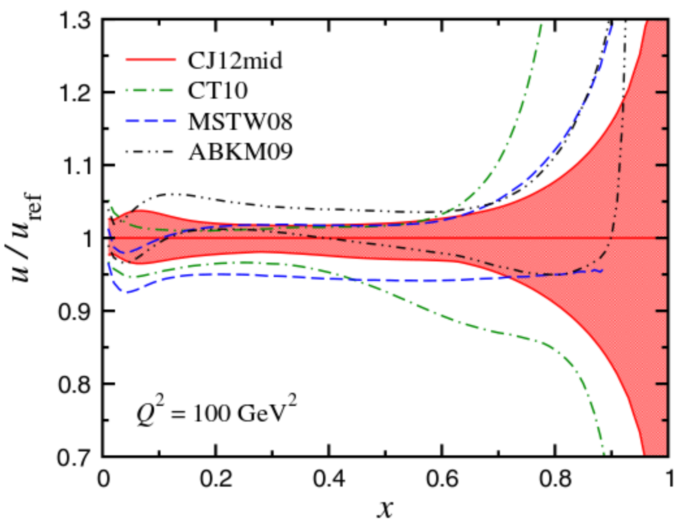
<u>Global fits with/without deuteron data show</u>:

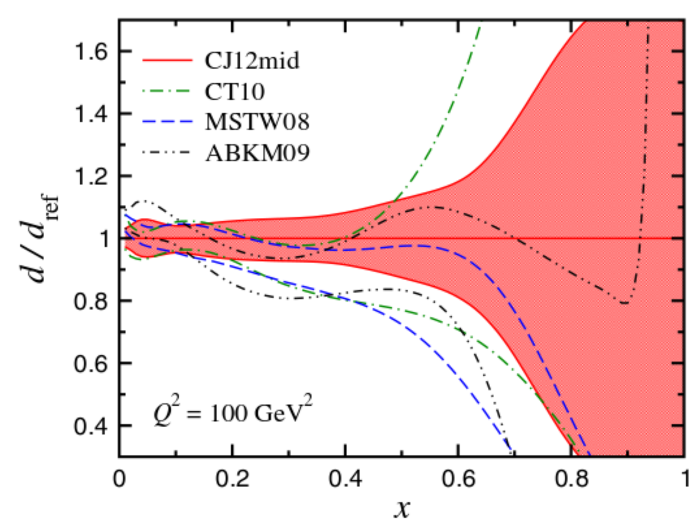
→ modest increase in the **u** error band for x > 0.7

→ significant increase in error band for **d**

→ smaller error band for **d** if deuterium hence nuclear corrections included

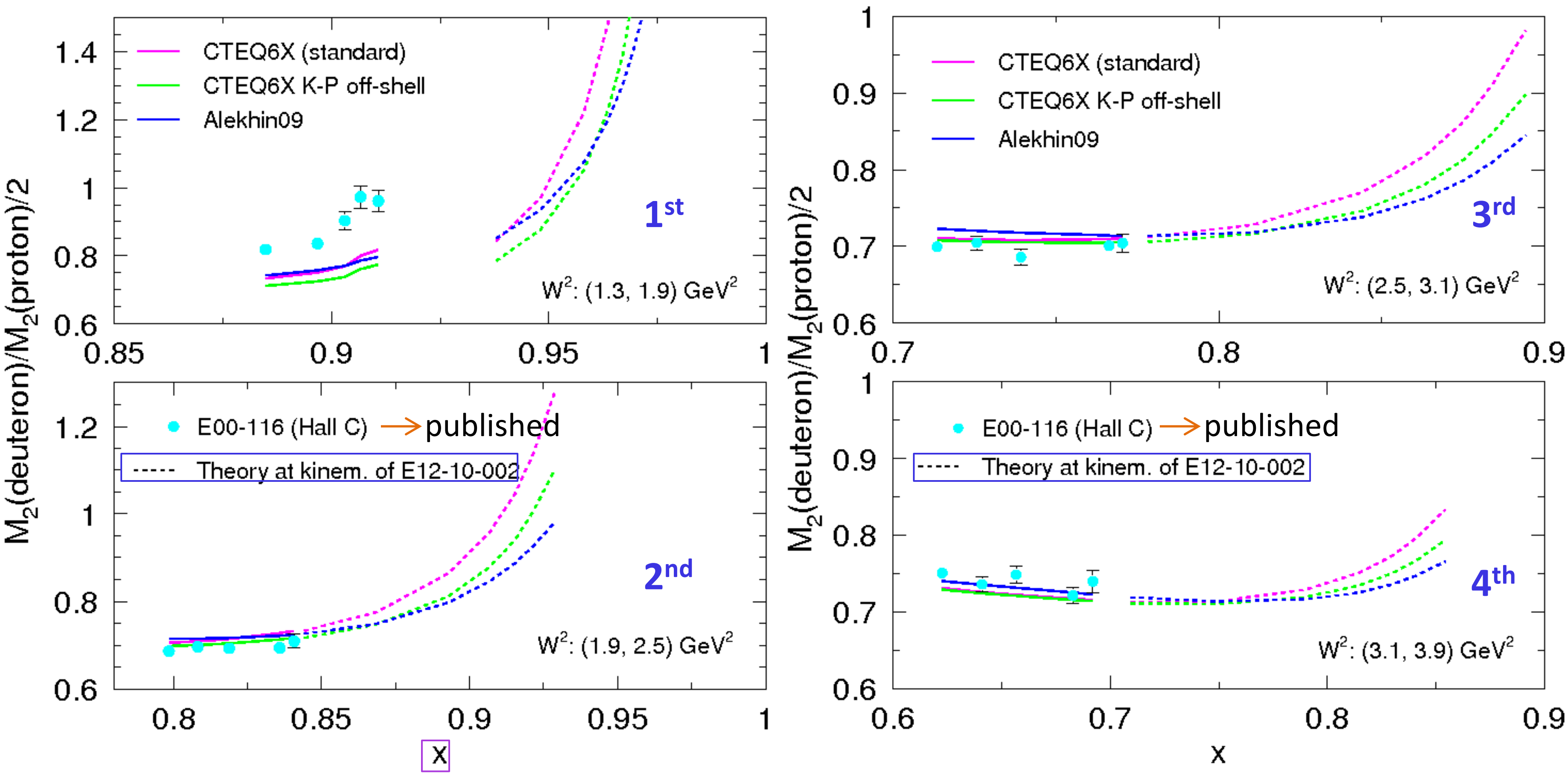
Inclusion of more data at large x leads to better constrained PDFs for CJ





Future: E12-10-002 at JLab, 2016-2017

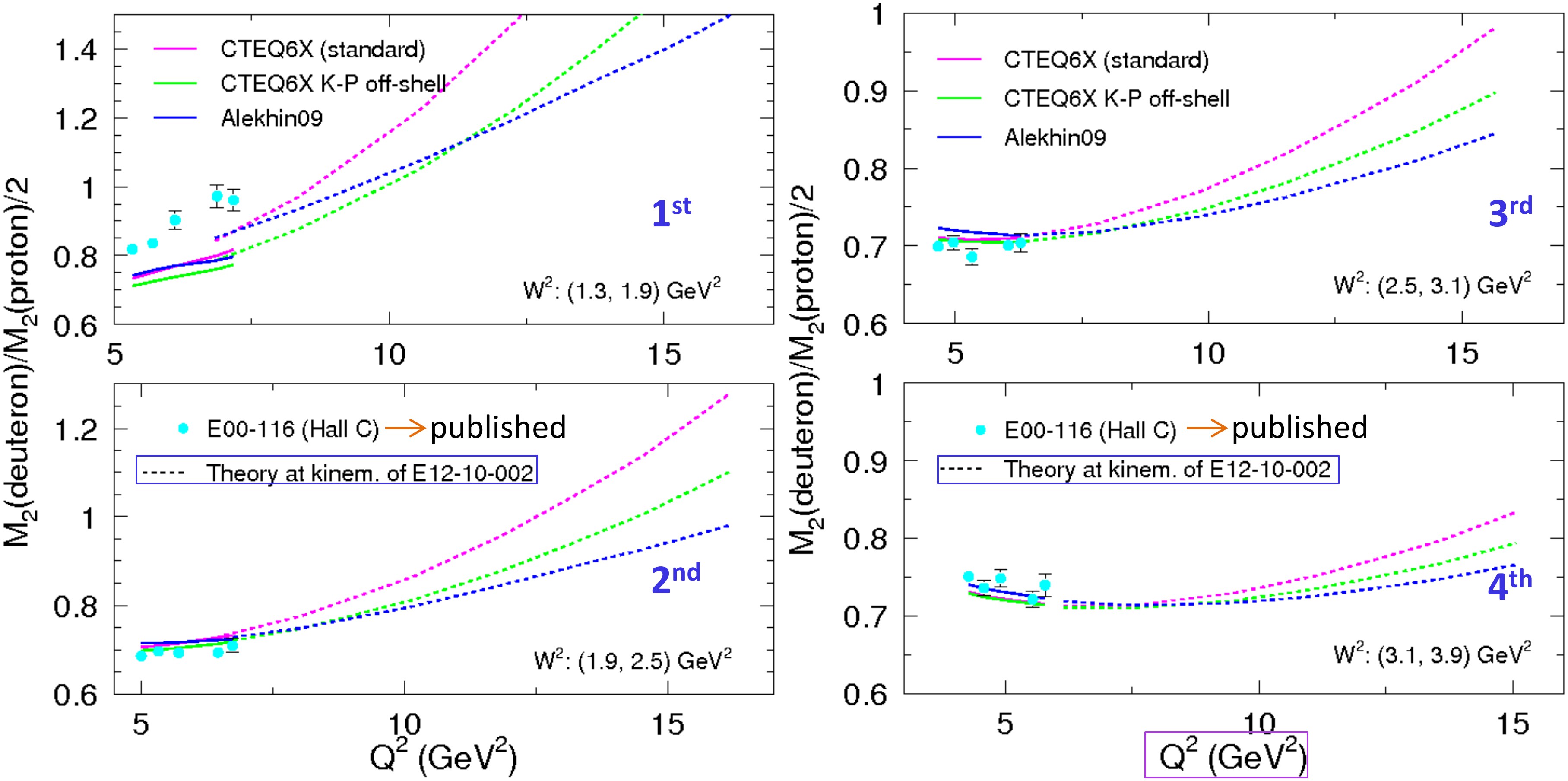
Resonance Region coverage



❖ E12-10-002: greatly extends the **x coverage** per resonance region

Future: E12-10-002 at JLab, 2016-2017

Resonance Region coverage



❖ E12-10-002: greatly extends the **Q^2 coverage** per resonance region

Summary

→ I focused on studies of quark-hadron duality in the proton, deuteron and neutron F_2 structure functions

The procedure to verify how well do resonance region data average to "scaling curves" is rather simple:

→ We define local and global resonance regions using W as parameter
→ We generate the "duality curve" at the exact same kinematics as the data
→ We apply the same integration procedure to data and generated "duality curve"
→ The ratio of integrals from data and duality curves will then ONLY be a measure of how well the data average to the curve

→ Second generation PDF fits better constrained at large x are ideal for these studies

→ Quark-hadron duality has been verified and holds at ~10-15% level or better globally and locally except for the delta region up to Q^2 of 7-8 GeV^2

→ New experiment coming up at Jefferson Lab: will contribute data at large x to extractions of second generation PDFs (CTEQ-JLab) and will extend quark-hadron duality verification and studies of non-perturbative effects in the resonance region to Q^2 of 16 GeV^2