Light Front Wave Function (LFWF) for Hadrons with Arbitrary Twist

Alfredo Vega

In collaboration with
I. Schmidt, T. Gutsche and V. Lyubovitskij

Universidad de Valparaíso
CHILE

Light Cone 2015, Frascati, Italy

September 22, 2015

Outline

Introduction

Light Front Wave Functions

Examples

Final Comments and Conclusions

Introduction

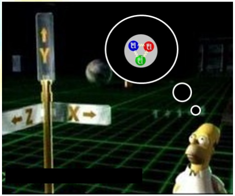
◇ **Brief (and incomplete) list of uses of Bottom - Up models in hadron physics.**

- **DIS** [Polchinski and Strassler; Ballon - Ballona, Boschi and Braga; Braga and A. V; Watanabe and Suzuki; Pire, Roiesnel, Szymanowski, Wallon].
- **GPSs** [A.V, Schmidt, Gutsche and Lyubovitskij; Nishio and Watari].
- **Hadronic spectrum** [Brodsky and de Teramond; A.V and Schmidt; Gutsche, Lyubovitskij, Schmidt and A.V; Forkel, Beyer and Frederico; De Paula and Frederico; Colangelo, De Fazio, Giannuzzi, Jugeau and Nicotri].
- **Transition form factors** [Brodsky, Cao and de Teramond; Gutsche, Lyubovitskij, Schmidt and A.V].
- **Heavy Ion Collisions** [Liu, Rajagopal and Wiedemann; Albacete, Kovchegov and Taliotis].
- **Hadronic wave functions** [Brodsky and de Teramond; Gutsche, Lyubovitskij, Schmidt and A.V].

◇ **To examples in this conference where AdS / QCD ideas are considered, see talks of.**

- Monday.
  - De Teramond.
  - Chakrabarti.
  - Cotogno.
  - Trawinski.
- Tuesday.
  - Lyubovitskij.
  - Vega.
- Friday.
  - Brodsky.

And see posters of Mondan and Bellantuono.



Light Front Wave Functions

◇ **Basic Idea.** [1]

Comparison of Form Factors in light front and in AdS side, offer us a possibility to relate AdS modes that describe hadrons with LFWF.

- In Light Front (for hadrons with two partons),

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int d\zeta\, \zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \frac{|\widetilde{\psi}(x,\zeta)|^2}{(1-x)^2}.$$

- In AdS

$$F(q^2) = \int_0^\infty dz\; \Phi(z) J(q^2,z) \Phi(z),$$

where $\Phi(z)$ correspond to AdS modes that represent hadrons, $J(q^2,z)$ it is dual to electromagnetic current.

[1] S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. **96**, 201601 (2006); Phys. Rev. D **77**, 056007 (2008).

Light Front Wave Functions

Considering a soft wall model with a cuadratic dilaton, Brodsky and de Teramond found [2]

$$\psi(x, \mathbf{b}_\perp) = A\sqrt{x(1-x)}\; e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2},$$

and in momentum space

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi A}{\kappa\sqrt{x(1-x)}} \exp\left(-\frac{\mathbf{k}_\perp^2}{2\kappa_1^2 x(1-x)}\right).$$

[2] S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. **96**, 201601 (2006); Phys. Rev. D **77**, 056007 (2008).

A generalizations of LFWF discused in previous section looks like

$$\psi(x, \mathbf{k}_\perp) = N \frac{4\pi}{\kappa\sqrt{x(1-x)}} g_1(x) \exp\left(-\frac{\mathbf{k}_\perp^2}{2\kappa_1^2 x(1-x)} g_2(x)\right).$$

You can found some examples in

- S. J. Brodsky and G. F. de Teramond, arXiv:0802.0514 [hep-ph].
- A. V, I. Schmidt, T. Branz, T. Gutsche and V. E. Lyubovitskij, PRD 80, 055014 (2009).
- S. J. Brodsky, F. G. Cao and G. F. de Teramond, PRD 84, 075012 (2011).
- J. Forshaw and R. Sandapen, PRL 109, 081601 (2012).
- S. Chabysheva and J. Hiller, Annals of Physics 337 (2013) 143 - 152.
- T. Gutsche, V. Lyubovitskij, I. Schmidt and A. V, PRD 87, 056001 (2013).

◇ Background for a generalization to arbitrary twist

- In AdS side, form factors in general looks like

$$F(q^2) = \int_0^\infty dz\; \Phi_\tau(z)\mathcal{V}(q^2, z)\Phi_\tau(z),$$

  Example: Fock expansion in AdS side for Protons [3], Deuteron form factors [4].

- We consider a shape that fulfill the following constraints:
  - At large scales $\mu \to \infty$ and for $x \to 1$, the wave function must reproduce scaling of PDFs as $(1-x)^\tau$.
  - At large Q^2, the form factors scales as $1/(Q^2)^{\tau-1}$.

[3] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D86 (2012) 036007; Phys. Rev. D87 (2013) 016017.

[4] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D91 (2015) 114001.

◇ LFWF with Arbitrary Twist [5]

Recently we have suggested a LFWF at the initial scale μ_0 for hadrons with arbitrary number of constituents that looks like

$$\psi_\tau(x, \mathbf{k}_\perp) = N_\tau \frac{4\pi}{\kappa} \sqrt{log(1/x)} (1-x)^{(\tau-4)/2} Exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2} \frac{log(1/x)}{(1-x)^2} \right]$$

The PDFs $q_\tau(x)$ and GPDs $H_\tau(x, Q^2)$ in terms of the LFWFs at the initial scale can be calculated.

Next we extend our LFWF to an arbitrary scale

$$\psi_\tau(x, \mathbf{k}_\perp, \mu) = N_\tau(\mu) \frac{4\pi}{\kappa} \sqrt{log(1/x)} x^{a_1(\tau,\mu)} (1-x)^{b_1(\tau,\mu)}$$

$$\times (1 + c_1(\tau,\mu)\sqrt{x} + c_2(\tau,\mu)x)^{1/2} Exp\left[-\frac{\mathbf{k}_\perp}{2\kappa^2} \frac{log(1/x)}{x^{a_2(\tau,\mu)}(1-x)^{b_2(\tau,\mu)}} \right],$$

[5] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D89 (2014) 054033.

The PDFs $q_\tau(x)$ and GPDs $H_\tau(x, Q^2)$ in terms of the LFWFs at the initial scale can be calculated.

$$q_\tau(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |\psi_\tau(x, \mathbf{k}_\perp)|^2,$$

$$H_\tau(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi^\dagger_\tau(x, \mathbf{k}^{'}_\perp)\psi_\tau(x, \mathbf{k}_\perp).$$

where $\psi_\tau(x, \mathbf{k}_\perp) = \psi_\tau(x, \mathbf{k}_\perp, \mu_0)$, $\mathbf{k}^{'}_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$ and $Q^2 = \mathbf{q}^2_\perp$.

Note: After evolution of $q_\tau(x)$ and $H_\tau(x, Q^2)$ we can fix parameters in

$$\psi_\tau(x, \mathbf{k}_\perp, \mu) = N_\tau(\mu) \frac{4\pi}{\kappa} \sqrt{log(1/x)} x^{a_1(\tau,\mu)} (1-x)^{b_1(\tau,\mu)}$$

$$\times (1 + c_1(\tau,\mu)\sqrt{x} + c_2(\tau,\mu)x)^{1/2} Exp\left[-\frac{\mathbf{k}_\perp}{2\kappa^2} \frac{log(1/x)}{x^{a_2(\tau,\mu)}(1-x)^{b_2(\tau,\mu)}} \right],$$

Examples

◇ Example 1: Pion Form Factor, GPD and PDF [6].

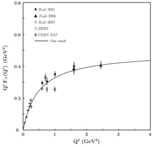


Figure: Pion form factor $Q^2 F_\pi(Q^2)$.

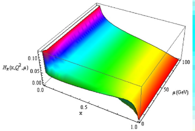


Figure: $H_\pi(x, Q^2, \mu)$ at $Q^2 = 10\ GeV^2$, and $\mu = 1 - 100\ GeV$.

[6] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D89 (2014) 054033.

◇ Example 1: Pion Form Factor, GPD and PDF [7].

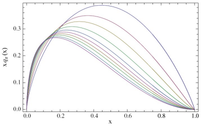


Figure: Hard evolution of the pion PDF for $\mu = 1,2,4,10,25,50,100,200,500$ and $1000\ GeV$. An increase of the scale leads to lowering of the maximum of the curves.

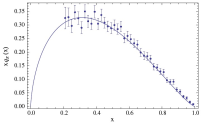


Figure: Comparison of the evolved pion PDF at the scale $\mu = 4\ GeV$ in our approach to the analysis of the E615 experiment.

[7] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D89 (2014) 054033.

◇ Example 2: Nucleon Properties in a Light-Front Quark - Diquark Model [8].

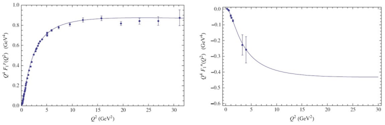


Figure: Dirac Proton and Neutron form factors multiplied by Q^4.

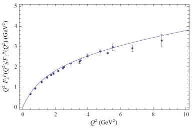


Figure: Ratio $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$.

[8] Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt y A. V, Phys. Rev. D89 (2014) 054033.

◇ Example 3: $s - \bar{s}$ Asymmetry in a Light Front Model (In Progress).

Considering a model proposed by Brodsky and Ma (1996).

$$s(x) = \int_x^1 \frac{dy}{y} f_{\Lambda/K^+\Lambda}(y) q_{s/\Lambda}\left(\frac{x}{y}\right),$$

$$\bar{s}(x) = \int_x^1 \frac{dy}{y} f_{K^+/K^+\Lambda}(y) q_{\bar{s}/K^+}\left(\frac{x}{y}\right).$$

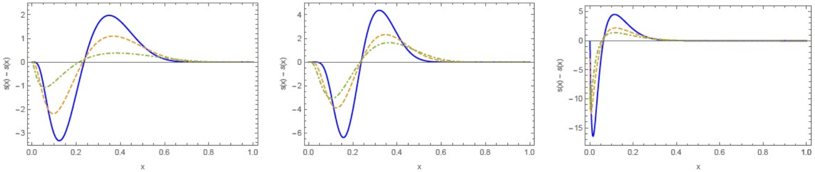


Figure: $s(x) - \bar{s}(x)$ calculated with three different LFWFs (Gaussian, Holographic and Holographic with arbitrary twist).

◇ Example 3: $s-\bar{s}$ Asymmetry in Light Front Model (In Progress).

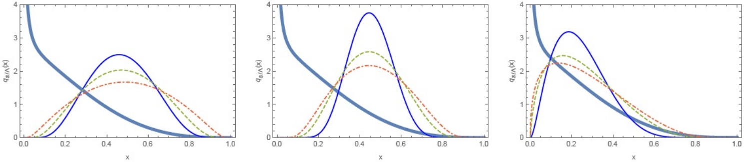


Figure: Densities of quarks s in Λ calculated with three LFWFs.

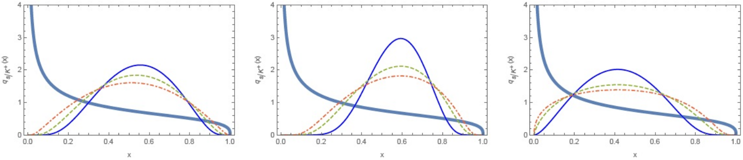


Figure: Densities of quarks $\bar{s}$ in K^+ calculated with three LFWFs.

Final Comments and Conclusions

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- We presented a light-front quark model (or wave function) consistent with model independent scaling laws.
- The LFWF is explicitly dependent on the scale and consider different number of constituent in hadrons.
- The LFWF proposal is an interesting alternative to other commonly used wave functions.
- In our opinion the examples considered show that this function is versatile, and it can be used in different models.

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That's all Folks!