T-odd Gluon TMDs inside a Transversely Polarized Hadron

Miguel G. Echevarría

NIKHEF FOM VU VRIJE UNIVERSITEIT AMSTERDAM

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Based on [D. Boer, MGE, P. Mulders, J. Zhou, *in preparation*]

Outline

1. Gluon TMDs

- Definition (factorization theorem!)

Are T-odd gluon TMDs relevant at small-x?

2. T-odd gluon TMDs for a transversely polarized hadron at small-x

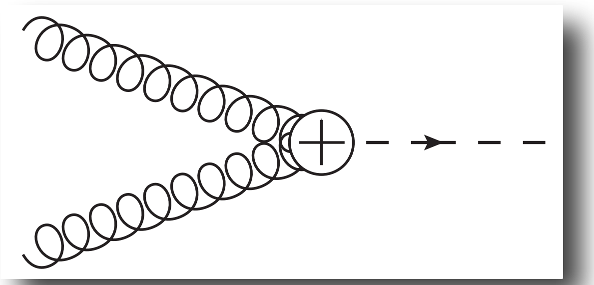
- Introduction
- f-type at NLO: vanish
- d-type at NLO: equal (and given by the Odderon)

3. Conclusions & Outlook

Gluon TMDs: definition (1/2)

- There is no concept of TMDs without factorization! Examples:

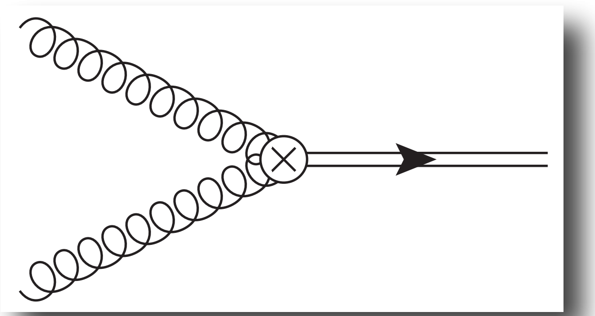
pp —> H(q_T)



$$d\sigma = \sigma_0(\mu)\, C_t^2(m_t^2,\mu)\, H(m_H^2,\mu)\, \frac{m_H^2}{\tau s}\, dy\, \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 y_\perp\, e^{-i q_\perp \cdot y_\perp}$$
$$\times\, 2 J_n^{\mu\nu}(x_A, y_\perp, \mu)\, J_{\bar{n}\,\mu\nu}(x_B, y_\perp, \mu)\, S(y_\perp, \mu)$$

[MGE, Kasemets, Mulders, Pisano 1502.05354]

pp —> η_Q(q_T)



$$\frac{d\sigma}{dy\,dq_T} \propto \Gamma^\dagger_{\mu\alpha}\, \Gamma_{\nu\beta} \int d^2 y_\perp\, e^{-i q_\perp \cdot y_\perp}\, J_n^{\mu\nu}(x_A, y_\perp, \mu)\, J_{\bar{n}}^{\alpha\beta}(x_B, y_\perp, \mu)\, S(y_\perp, \mu)$$

➜ ***Talk of A. Signori***

Collinear and soft matrix elements contain spurious rapidity divergences: They are ill-defined!!!!

Gluon TMDs: definition (2/2)

- Proper definition it's a bit tricky…

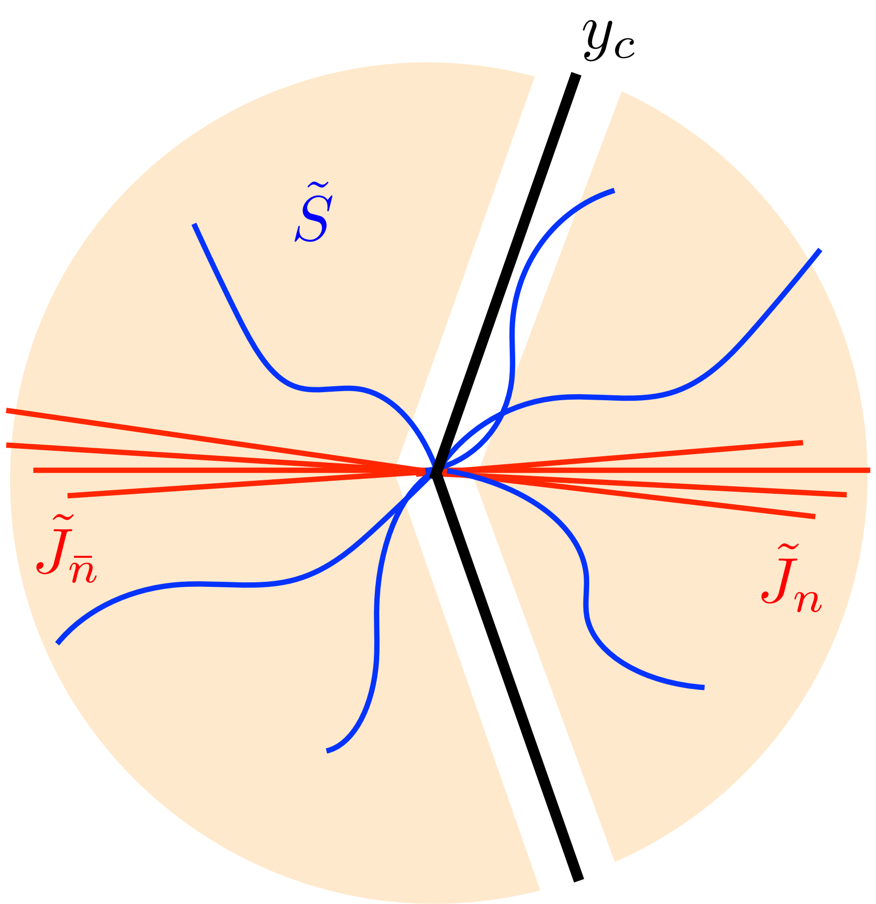
$$k_n \sim Q(1, \lambda^2, \lambda)$$
$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda)$$
$$k_s \sim Q(\lambda, \lambda, \lambda)$$

$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

Different rapidities (mixed under boosts)

$$k_n^2 \sim k_{\bar{n}}^2 \sim k_s^2 \sim Q^2\lambda^2$$

Same invariant mass!



Cancel spurious rapidity divergences

$$\zeta_A = 2(p^+)^2 e^{-2y_c}$$

$$G^{\mu\nu}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu^2) = \tilde{J}_n \sqrt{\tilde{S}}$$
$$G^{\mu\nu}_{g/B}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu^2) = \tilde{J}_{\bar{n}} \sqrt{\tilde{S}}$$

$$\zeta_B = 2(\bar{p}^-)^2 e^{2y_c}$$

[Collins' book '11]
[MGE, Idilbi, Scimemi 1211.1947]
[MGE, Idilbi, Scimemi 1402.0869]

T-odd gluon TMDs for a transv. polarized hadron

- Forgetting about subtleties, gluon TMDs are given by the following correlator:

$$\Gamma^{\mu\nu} = \frac{1}{xP^+}\int \frac{dy^- d^2y_T}{(2\pi)^3} e^{ik\cdot y}\langle P, S_T|2\mathrm{Tr}\left[F^{\mu}_{+T}(0)U F^{\nu}_{+T}(y)U'\right]|P, S_T\rangle|_{y^+=0}$$

U and U' are ***process-dependent*** *gauge links.*

[Mulders, Rodrigues 0009343]

$$\Gamma^{\mu\nu} = \delta_T^{\mu\nu} f_1^g + \left(\frac{2k_T^\mu k_T^\nu}{k_\perp^2} - \delta_T^{\mu\nu}\right)\frac{k_\perp^2}{2M^2} h_1^{\perp g}$$
$$- \delta_T^{\mu\nu}\frac{\epsilon_{T\alpha\beta}k_T^\alpha S_T^\beta}{M} f_{1T}^{\perp g} - i\epsilon_T^{\mu\nu}\frac{k_T\cdot S_T}{M} g_{1T}^g$$
$$- \frac{\tilde{k}_T^{\{\mu}k_T^{\nu\}}}{k_\perp^2}\frac{k_T\cdot S_T}{M} h_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}S_T^{\nu\}} + \tilde{S}_T^{\{\mu}k_T^{\nu\}}}{2M} h_{1T}^g$$

T-odd TMDs are not universal, but can be expressed in terms of a finite set of universal distributions

[Buffing, Mukherjee, Mulders 1306.5897]

- We analyze 2 relevant gauge link structures: simple future/past pointing staple-like

$$\Gamma^{(f)}_{(T-\mathrm{odd})} = \frac{1}{2}\left(\Gamma^{[+,+\dagger]} - \Gamma^{[-,-\dagger]}\right) \qquad \Gamma^{(d)}_{(T-\mathrm{odd})} = \frac{1}{2}\left(\Gamma^{[+,-\dagger]} - \Gamma^{[-,+\dagger]}\right)$$

Are T-odd gluon TMDs relevant at small-x?

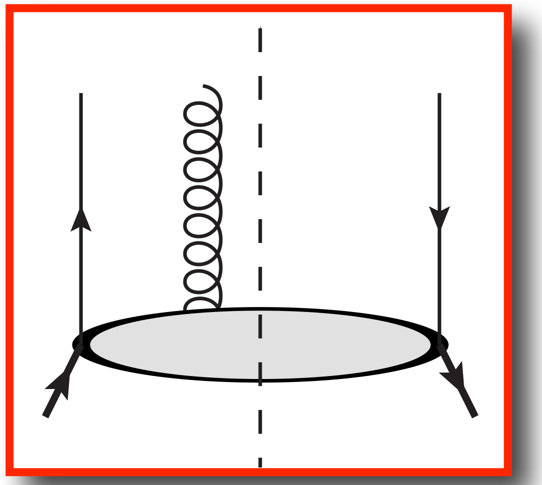
T-odd gluon TMDs for a transv. polarized hadron

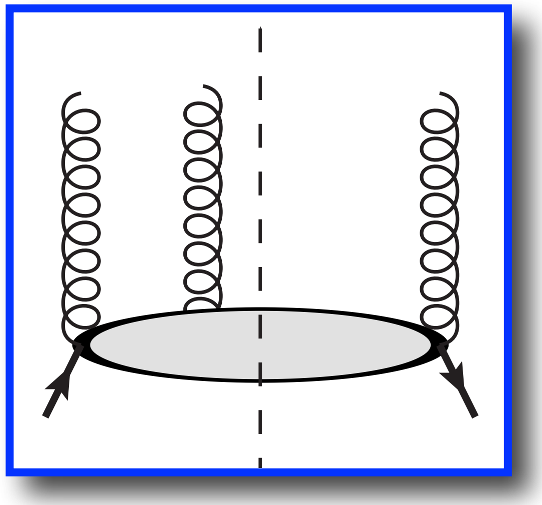
- We will calculate them at large k_T and in the saturation regime (small-x):

$$\tilde{T}_{g/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{g/j}(x, b_T; \zeta, \mu) \otimes t_{j/A}(x; \mu) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

Any gluon TMD

Corresponding integrated function





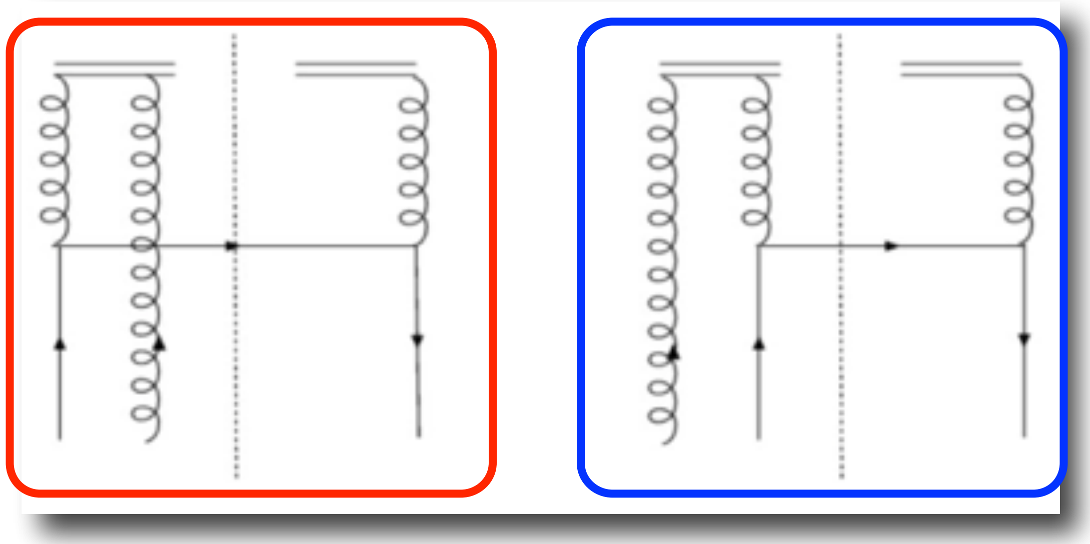
$$T_{F,q}(x,x) = \frac{M}{P^+} \int \frac{dz^- d\eta}{2\pi} e^{ik\cdot z} \frac{(\mathbf{P}\times\mathbf{S}_T)^j}{2M} \langle P,S|\, \bar{\psi}(0)\gamma^+ F^{+j}(\eta z)\psi(z)\,|P,S\rangle\,|_{z^+=|\mathbf{z}_\perp|=\mathbf{0}}$$

$$T_G^{\pm}(x,x) = -\frac{2M\delta_T^{lm}}{x(P^+)^2} \int \frac{dz^- d\eta}{2\pi} e^{ik\cdot z} \frac{(\mathbf{P}\times\mathbf{S}_T)^j}{2M} \langle P,S|\, C_{\pm}^{abc} F_a^{+l}(0) F_b^{+j}(\eta z) F_c^{+m}(z)\,|P,S\rangle\,|_{z^+=|\mathbf{z}_\perp|=\mathbf{0}}$$

$C_+^{abc} = if^{abc}$ $\qquad C_-^{abc} = d^{abc}$

Gluon TMDs at small-x: f-type

- For f-type gluon Sivers function we have:



$$f_{1T}^{\perp g(f)}(x, k_\perp^2) = C_1 \int \frac{dz}{z} \sum_q \left\{ T_{F,q}(z,z) \frac{1+(1-\xi)^2}{\xi} - T_{F,q}(z, z-x) \frac{2-\xi}{\xi} \right\}$$

$$\xi = x/z$$

$$C_1 = \frac{N_c}{2} \frac{\alpha_s}{2\pi^2} \frac{M}{k_\perp^4}$$

$x \to 0$

$$f_{1T}^{\perp g/q\,(f)}(x \to 0, k_\perp^2) \approx 0$$

Gluon TMDs at small-x: f-type

- And for the other two TMDs:

$$h_{1T}^{g(f)}(x,k_\perp^2) = C_1 \int \frac{dz}{z} \sum_q \left\{ T_{F,q}(z,z)\frac{2-2\xi}{\xi} - T_{F,q}(z,z-x)\frac{2-\xi}{\xi} \right\}$$

$\xi = x/z$

$x \to 0$

$$h_{1T}^{g/q\,(f)}(x \to 0, k_\perp^2) \approx 0$$

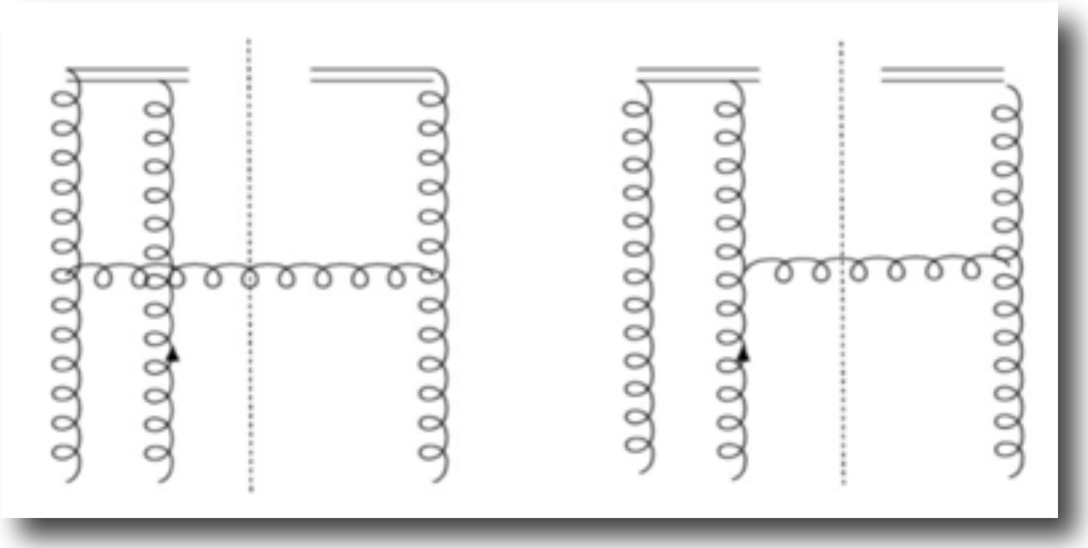
$$h_{1T}^{\perp g(f)}(x,k_\perp^2) = C_1 \int \frac{dz}{z} \frac{4-4\xi}{\xi} \sum_q T_{F,q}(z,z)$$

$x \to 0$

$$h_{1T}^{\perp g(f)}(x \to 0, k_\perp^2) \approx C_1 \frac{4}{x} \int dz \sum_q T_{F,q}(z,z)$$

Gluon TMDs at small-x: f-type

- Now the gluon channel…



$$f_{1T}^{\perp g/g\,(f)}(x \to 0, k_\perp^2) \approx h_{1T}^{g/g\,(f)}(x \to 0, k_\perp^2) \approx 0$$

$$h_{1T}^{\perp g/g\,(f)}(x \to 0, k_\perp^2) \approx C_1 \frac{4}{x} \int dz T_G^{(+)}(z,z)$$

- For h_{1T} we can invoke Burkardt sum rule, adding the quark and gluon channels:

$$h_{1T}^{\perp g(f)}(x \to 0, k_\perp^2) \approx C_1 \frac{4}{x} \int dz \left\{ \sum_q T_{F,q}(z,z) + T_G^{(+)}(z,z) \right\} = 0$$

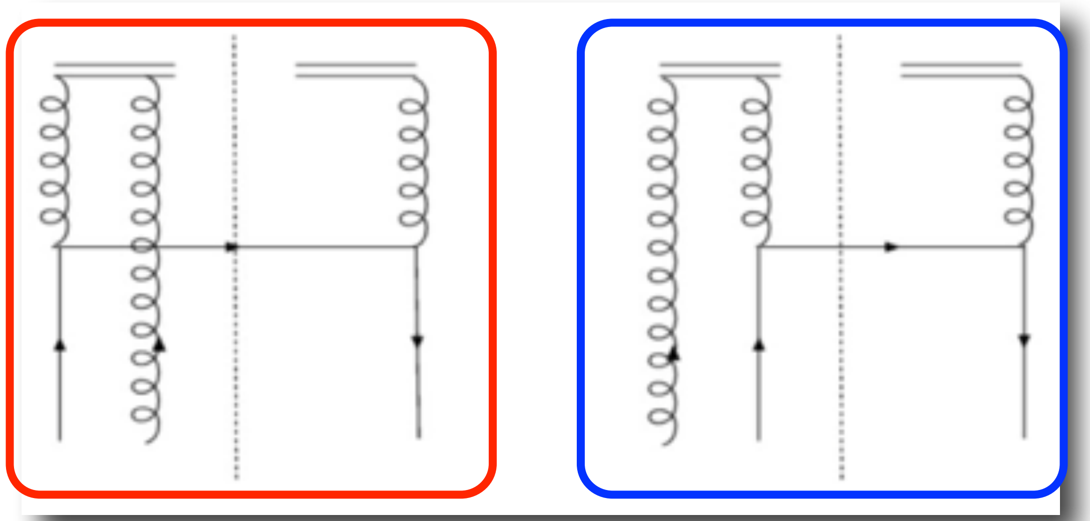
Momentum conservation of the first transverse momentum of Sivers function

[Burkardt 0311013]
[Zhou 1507.02819]

All the 3 T-odd f-type gluon TMDs vanish at small-x

Gluon TMDs at small-x: d-type

- For d-type gluon Sivers function we have:



$$f_{1T}^{\perp g(d)}(x,k_\perp^2) = C_2 \int \frac{dz}{z} \sum_q \left\{ T_{F,q}(z,z)\frac{1+(1-\xi)^2}{\xi} + T_{F,q}(z,z-x)\frac{2-\xi}{\xi} \right\}$$

$$\xi = x/z$$

$$C_2 = \frac{N_c^2-4}{2N_c}\frac{\alpha_s}{2\pi^2}\frac{M}{k_\perp^4}$$

$$x \to 0$$

$$f_{1T}^{\perp g/q\,(d)}(x\to 0, k_\perp^2) \approx C_2 \frac{4}{x} \int dz \sum_q T_{F,q}(z,z)$$

Gluon TMDs at small-x: d-type

- For the other two TMDs we have:

$$h_{1T}^{g(d)}(x, k_\perp^2) = C_2 \int \frac{dz}{z} \sum_q \left\{ T_{F,q}(z,z) \frac{2-2\xi}{\xi} + T_{F,q}(z, z-x) \frac{2-\xi}{\xi} \right\}$$

$$\xi = x/z$$

$x \to 0$

$$h_{1T}^{g/q\,(d)}(x \to 0, k_\perp^2) \approx C_2 \frac{4}{x} \int dz \sum_q T_{F,q}(z,z)$$

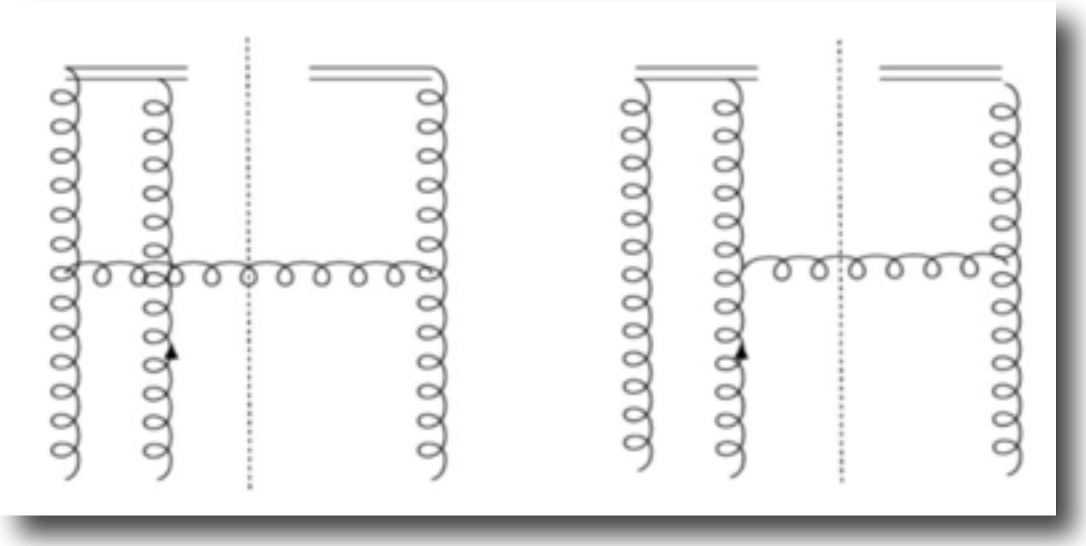
$$h_{1T}^{\perp g(d)}(x, k_\perp^2) = C_2 \int \frac{dz}{z} \frac{4-4\xi}{\xi} \sum_q T_{F,q}(z,z)$$

$x \to 0$

$$h_{1T}^{\perp g/q\,(d)}(x \to 0, k_\perp^2) \approx C_2 \frac{4}{x} \int dz \sum_q T_{F,q}(z,z)$$

Gluon TMDs at small-x: d-type

- Now the gluon channel…



$$f_{1T}^{\perp g(d)}(x,k_\perp^2) \approx h_{1T}^{g(d)}(x,k_\perp^2) \approx h_{1T}^{\perp g(d)}(x,k_\perp^2)$$
$$\approx C_2 \frac{4}{x} \int dz\, T_G^{(-)}(z,z)$$

All the 3 T-odd d-type gluon TMDs are enhanced and equal at small-x

Gluon TMDs at small-x: summary

$f_{1T}^{\perp g\,(f,d)}$ $h_{1T}^{g\,(f,d)}$ $h_{1T}^{\perp g\,(f,d)}$

$$f_{1T}^{\perp g\,(f)}(x \to 0, k_\perp^2) \approx h_{1T}^{g\,(f)}(x \to 0, k_\perp^2) \approx h_{1T}^{\perp g\,(f)}(x \to 0, k_\perp^2) \approx 0$$

All the 3 T-odd f-type gluon TMDs vanish at small-x

$$C_2 = \frac{N_c^2 - 4}{2N_c}\frac{\alpha_s}{2\pi^2}\frac{M}{k_\perp^4}$$

$$f_{1T}^{\perp g\,(d)}(x \to 0, k_\perp^2) \approx h_{1T}^{g\,(d)}(x \to 0, k_\perp^2) \approx h_{1T}^{\perp g\,(d)}(x \to 0, k_\perp^2) \approx C_2 \frac{4}{x} \int dz \left\{ \sum_q T_{F,q}(z,z) + T_G^{(-)}(z,z) \right\}$$

All the 3 T-odd d-type gluon TMDs are enhanced and equal at small-x

Gluon TMDs and the Odderon (1/2)

- In pQCD, the Odderon is a color-singlet exchange and can be formed by 3 gluons in a symmetric color state. It has negative C-parity and therefore dominates the differences between particle-particle and particle-antiparticle scatterings at high energy.

- Let us consider the d-type T-odd correlator and manipulate it:

$$\Gamma^{(d)}_{(T-\text{odd})} = \frac{1}{2}\left(\Gamma^{[+,-\dagger]} - \Gamma^{[-,+\dagger]}\right)$$

$$\Gamma^{\mu\nu}_{\text{T}-\text{odd}} = \frac{1}{xP^+}\int \frac{dy_1^- d^2y_{1T} dy_2^- d^2y_{2T}}{(2\pi)^3} e^{ik\cdot y} \times \Big\{ \langle P, S_T| \text{Tr}\left[F^\mu_{+T}(y_1)U^{[+]\dagger}F^\nu_{+T}(y_2)U^{[-]} - F^\mu_{+T}(y_1)U^{[-]}F^\nu_{+T}(y_2)U^{[+]\dagger}\right]|P, S_T\rangle\Big\}$$

$y = y_1 - y_2$

Approximation:

$e^{ik^+(y_1^- - y_2^-)} \to 1$

$$\Gamma^{\mu\nu}_{\text{T}-\text{odd}} = \frac{k_T^\mu k_T^\nu}{g^2 xP^+}\int \frac{d^2y_{1T} d^2y_{2T}}{(2\pi)^3} e^{ik_T\cdot y_T} \times \langle P, S_T|\text{Tr}\left[U^{[\Box]}(y_T) - U^{[\Box]\dagger}(y_T)\right]|P, S_T\rangle$$

MV-model

[Zhou 1308.5912]

$$\Gamma^{\mu\nu}_{\text{T}-\text{odd}} = \frac{k_T^\mu k_T^\nu N_c}{2\pi^2 \alpha_s x}\frac{\epsilon_T^{\alpha\beta} S_{T\alpha} k_{T\beta}}{M} O^\perp_{1T}(x, k^2_\perp)$$

Gluon TMDs and the Odderon (2/2)

$$\Gamma^{\mu\nu}_{\mathrm{T-odd}} = \frac{k_T^\mu k_T^\nu N_c}{2\pi^2\alpha_s x}\frac{\epsilon_T^{\alpha\beta}S_{T\alpha}k_{T\beta}}{M}O_{1T}^\perp(x,k_\perp^2)$$

- Thus we have the equality:

$$\frac{k_T^\mu k_T^\nu N_c}{2\pi^2\alpha_s x}\frac{\epsilon_T^{\alpha\beta}S_{T\alpha}k_{T\beta}}{M}O_{1T,x}^\perp(k_\perp^2) = -\delta_T^{\mu\nu}\frac{\epsilon_{T\alpha\beta}k_T^\alpha S_T^\beta}{M}f_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}k_T^{\nu\}}}{k_\perp^2}\frac{k_T\cdot S_T}{M}h_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}S_T^{\nu\}}+\tilde{S}_T^{\{\mu}k_T^{\nu\}}}{2M}h_{1T}^g$$

Given that the d-type T-odd TMDs are equal at small-x

$$x f_{1T}^{\perp g}(x,k_\perp^2) = x h_{1T}^g(x,k_\perp^2) = x h_{1T}^{\perp g}(x,k_\perp^2) = \frac{k_\perp^2 N_c}{4\pi^2\alpha_s}O_{1T}^\perp(x,k_\perp^2)$$

All the 3 T-odd d-type gluon TMDs are given by the spin-dependent Odderon

Conclusions & Outlook

★ (Un)polarized gluon TMDPDFs are defined such that they are free from spurious rapidity divergences

★ f-type T-odd gluon TMDs inside a transversely polarized hadron are suppressed at small-x, for which momentum conservation was used.

★ d-type T-odd gluon TMDs inside a transversely polarized hadron are enhanced at small-x. Moreover they are equal, and can be given in terms of the spin-dependent Odderon.

✤ Next steps: phenomenology

✤ We need experimental measurements (RHIC, EIC, AFTER@LHC,…)

Thanks!