LFQ of the Vector Schwinger Model with a Photon Mass Term in the Faddeevian Regularization

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Plan of the Talk:

- ★ Recap of the Basics
- ★ Quantization of the Theory

★ Self References on VSM:

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** The vector Schwinger model(VSM) describes (2D)-dimensional electrodynamics with massless fermions.

** In this vector-like theory the left-handed and right-handed fermions are coupled to the e.m. field with equal couplings.

** VSM is characterized for its exact solvability.
** an Imp. feature is that a fermionic FT in (1+1)-D always has a boson equivalent theory called bosonized FT.
** This fermion-boson equivalence has led to the discovery of many interesting features of the 2D field theories.

** The 2D models are often used as toy models to test several interesting theoretical ideas which are applicable not only for the 2D theories but also to theories in the (3+1)-D, which are more realistic physical theories.

** Quark confinement was proved convincingly only in 2D field theories.

** VSM can also be studied with a mass term for the $U(1)$ gauge field.

Recently, we have studied the IFQ (i.e., quantization on the hyperplanes: $x^0 = t = constant$) as well as the LFQ
(i.e., quantization on the hyperplanes defined by the equal LC time $\tau = x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1) =$ constant) of this model with a mass term for the U(1) gauge field.

**

VSM has been studied widely but mostly without a photon mass term (which was a consequence of demanding the regularization for the VSM to be GI).
** The theory with a mass term for the U(1) gauge field represents a new class of models in the 2D QED with massless fermions
(but with a photon mass term)

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** In this talk, we consider this VSM with PMT in the Faddeevian Regularization (FR).

** This original theory is seen to be GNI. We then construct a GI theory corresponding to this GNI theory using Stueckelberg mechanism and then **recover the physical content of the original GNI theory from the newly constructed GI theory under some special GFC's**. We also study the LFQ of this theory.

** The generalized Schwinger model describing QED in 2D with mass-less fermions is defined by the action:

****

$$
\begin{aligned}
S_1 &= \int \mathcal{L}_1(\psi, \bar{\psi}, A^\mu) d^2x \\
\mathcal{L}_1 &= \Big[i\bar{\psi}\gamma^\mu \partial_\mu \psi + \frac{1}{2} e_R \bar{\psi}\gamma^\mu (1+\gamma^5)\psi A_\mu \\
&\qquad + \frac{1}{2} e_L \bar{\psi}\gamma^\mu (1-\gamma^5)\psi A_\mu - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \Big] \\
g_1 &= \frac{1}{2}(e_L - e_R) \ , \ g_2 = \frac{1}{2}(e_L + e_R) \ , \ \gamma^\mu\gamma^5 = -\epsilon^{\mu\nu}\gamma_\nu \\
\gamma^5 &= \gamma^0\gamma^1 = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \ , \ \epsilon^{\mu\nu} = \epsilon^{-\nu\mu} = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \\
g^{\mu\nu} &:= g_{\mu\nu} = diag(+1, -1) \ , \ F^{\mu\nu} = (\partial^\mu A^\nu - \partial^\nu A^\mu)
\end{aligned}
$$

** This action is equivalent to its bosonized form:

$$S_2 = \int \mathcal{L}_2(\phi, A^\mu) d^2x$$

$$\mathcal{L}_2 = \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + (g_1 g^{\mu\nu} - g_2\epsilon^{\mu\nu})\partial_\mu\phi A_\nu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{M^2}{2\pi}A_\mu A^\mu\right]$$

** Here, the mass term for A_μ arises from the regularization ambiguities associated with the definition of the current.
Chiral SM (CSM) is obtained by setting
$g_1 = g_2 = g$ (i.e., $e_R = 0$); and $M^2 = ag^2$
where a is the regularization parameter in the so-called standard regularization (SR).

VSM is obtained by setting

$$g_1 = 0, \quad g_2 = e \quad (i.e., e_L = e_R = e); \quad M = 0$$

** Here, $e_L \neq e_R$ implies a chiral theory.

** Here, $e_L = e_R$ $(= e)$ implies a vector-like theory.

** e is the coupling constant that couples the mass-less fermion (or equivalently the boson) with the $U(1)$ gauge field A^{μ}.

** For VSM, demanding the regularization to be GI, fixes $a = 0$ i.e., $M = 0$

** But in CSM, no choice for the value of a can make the theory GI and therefore parameter a is left as a free parameter.

*** VSM is defined by the Lagrangian density

$$\mathcal{L}_3 = \left[\bar{\psi}\gamma^\mu(i\partial_\mu + eA_\mu)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

which is equivalent to its bosonized form:

$$\mathcal{L}_4 = \left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - e\epsilon^{\mu\nu}\partial_\mu\phi A_\nu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

where e is the coupling constant that couples the mass-less fermion (or equivalently the boson) with the $U(1)$ gauge field A^μ.
NB: this is a well known GI theory, possessing a set of two ICCs.

** We now modify the theory by including a mass term for the $U(1)$ gauge-field A^{μ}, into the Lag. density, defined by: $(\mathcal{L}_m = \frac{1}{2} ae^2 A_{\mu} A^{\mu})$ where a is the standard regularization (SR) parameter.

** The modified resulting theory then describes the VSM with a PMT defined by the Lagrangian density:

$$\mathcal{L} := \mathcal{L}_4 + \mathcal{L}_m = \left[\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - e \epsilon^{\mu\nu} \partial_{\mu} \phi A_{\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} ae^2 A_{\mu} A^{\mu} \right]$$

** First term in the above action represents a mass-less boson, which is equivalent to a mass-less fermion in 2D, second term represents the vector coupling of this fermion to the electromagnetic (e.m.) field A^{μ}, third term is the kinetic energy term of the e.m. field and the fourth term is the mass term for the e.m. field.

The Lagrangian density $\mathcal{L}$ of the above theory, in the IF of dynamics reads:

$$\mathcal{L} = \Big[\frac{1}{2}(\partial_0\phi\partial_0\phi - \partial_1\phi\partial_1\phi) + e(A_0\partial_1\phi - A_1\partial_0\phi) + \frac{1}{2}(\partial_0 A_1 - \partial_1 A_0)^2 + \frac{1}{2}ae^2(A_0^2 - A_1^2)\Big]$$

** This IF theory is seen to possess one primary and one secondary Gauss-law constraint:

$$\xi_1 = \Pi^0 \approx 0 \quad , \quad \xi_2 = (\partial_1 E + e\partial_1\phi + ae^2 A_0) \approx 0$$

** where π , Π^0 and $E(:= \Pi^1)$, are the momenta conjugate canonically to ϕ , A_0 and A_1.

** The non-vanishing elements of the matrix $N_{\alpha\beta}$ ($:= \{\xi_\alpha, \xi_\beta\}$) of the Poisson brackets of these two constraints ξ_1, ξ_2 among themselves are: $N_{12} = -N_{21} = -ae^2\delta(x^1 - y^1)$.

** This matrix $N_{\alpha\beta}$ is invertible and is therefore non-singular implying that these two constraints together form a *set* of second-class constraints which in turn $\Rightarrow$ that the theory under consideration is gauge-non-invariant (GNI). This as expected, is obviously a consequence of the presence of the mass term for the $U(1)$ vector gauge field A^μ in the Lagrangian density of the theory

** Canonical Hamiltonian density $\mathcal{H}_c^N$ of this IF theory:

$$\mathcal{H}_c^N = \Big[\frac{1}{2}(\pi^2 + E^2 + (\partial_1\phi)^2 + e^2A_1^2) - A_0\partial_1 E + e\pi A_1 - eA_0\partial_1\phi - \frac{a}{2}e^2(A_0^2 - A_1^2)\Big]$$

** Non-vanishing equal-time CR's of the IF theory are:

$$
\begin{aligned}
[\phi(t,x), \pi(t,y)] &= [E(t,x), A_1(t,y)] = (-i)\delta(x-y) \\
[\pi(t,x), A_0(t,y)] &= (-e)[A_0(t,x), A_1(t,y)] = \left[\frac{-i}{ae}\right]\partial_1\delta(x-y)
\end{aligned}
$$

** We have studied the operator solutions of this theory as well as its GI reformulations earlier.

**

** Lagrangian density of this theory in LFQ reads:

$$\mathcal{L} = \Big[\partial_+\phi\partial_-\phi + e(A^+\partial_+\phi - A^-\partial_-\phi) + \frac{1}{2}(\partial_+A^+ - \partial_-A^-)^2 + ae^2A^+A^-\Big]$$

** Canonical momenta obtained from the above action are:

$$\Pi^+ = \frac{\partial\mathcal{L}}{\partial(\partial_+A^-)} = 0$$

$$\Pi^- = \frac{\partial\mathcal{L}}{\partial(\partial_+A^+)} = (\partial_+A^+ - \partial_-A^+)$$

$$\pi = \frac{\partial\mathcal{L}}{\partial(\partial_+\phi)} = (\partial_-\phi + eA^+)$$

** This theory is seen to possess two PC's:

$$\psi_1 = \Pi^+ \approx 0$$
$$\psi_2 = (\pi - \partial_-\phi - eA^+) \approx 0$$

** Canonical Hamiltonian density of the theory is:

$$\mathcal{H}_c = \left[\Pi^+\partial_+A^- + \Pi^-\partial_+A^+ + \pi\partial_+\phi - \mathcal{L}\right]$$
$$= \left[\frac{1}{2}(\Pi^-)^2 + \Pi^-\partial_-A^- + eA^-\partial_-\phi - ae^2A^+A^-\right]$$

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** Total Hamiltonian density of the theory is obtained after including the PC's in the canonical Hamiltonian density of the theory with the help of Lagrange multiplier fields $u(x,t)$ and $v(x,t)$ as:

$$\mathcal{H}_T = \Big[\frac{1}{2}(\Pi^-)^2 + \Pi^-\partial_- A^- + eA^-\partial_-\phi - ae^2 A^+ A^- + u\Pi^+ + v(\pi - \partial_-\phi - eA^+)\Big]$$

** The HE's of motion of the theory preserve the constraints of the theory in the course of time and could be obtained from the total Hamiltonian: $H_T = \int \mathcal{H}_T dx^-$

** Preservation of the PC: ψ_1 in the course of time leads to a SC:

$$\psi_3 = (\partial_- \Pi^- - e\partial_- \phi + ae^2 A^+) \approx 0$$

** Preservation of ψ_2 in the course of time does not lead to any SC.

** Further, the preservation of SC: ψ_3 in the course of time leads to another constraint:

$$\psi_4 = [ae^2 \Pi^- + ae^2 \partial_- A^- - ae^2 \partial_- A^-] = [ae^2 \Pi^-] \approx 0$$

** Preservation of ψ_4 in the course of time does not lead to any further constraint and instead leads only to a condition on the Lagrange multiplier fields.
** The theory is thus seen to possess a set of four constraints: ψ_1 , ψ_2 , ψ_3 and ψ_4 only.

** The matrix $M_{\alpha\beta}$ is clearly a singular matrix and it implies that the *set* of constraints ψ_i (with $i = 1, 2, 3, 4$) represents a *set* of first-class constraints which, in turn, implies that the theory is GI.

** Theory is indeed seen to be invariant under the LVGT's:

$$\begin{aligned} \delta\phi &= -e\beta \ , \quad \delta A^+ = \partial_-\beta \ , \quad \delta A^- = \partial_+\beta \ , \quad \delta u = \partial_+\partial_+\beta \\ \delta v &= -e\partial_+\beta \ , \quad \delta\pi = \delta\Pi^+ = \delta\Pi^- = \delta\Pi_u = \delta\Pi_v = 0 \end{aligned}$$

** Here, Π_u and Π_v , are the momenta conjugate canonically to u and v and $\beta \equiv \beta(\tau, x^-)$ is an arbitrary function of its arguments.

** Divergence of the vector gauge current density of the theory vanishes (giving $\partial_\mu j^\mu = 0$), $\Rightarrow$ that the theory possesses at the classical level, a LVGS.

LFQ of the theory could be studied e.g., under the GFC's:

$\eta_1 = A^+ \approx 0$ and $\eta_2 = A^- \approx 0$

** Theory in the Faddeevian Regularization

** Product of two Fermi fields at the same space-time point is highly singular and leads to regularization ambiguities.
In order to take care of these regularization ambiguities one introduces a regularization parameter which appears in the coefficient of the mass term of the $U(1)$ gauge field A^μ.
** This regularization scheme is often referred to as the standard regularization (SR).

** A new regularization called as the Faddeevian regularization (FR) was introduced by Mitra and Mukhopadhyay in the context of CSM, where the mass term of the $U(1)$ gauge field A^μ is expressed in a different manner.
** This FR introduced in the context of CSM, is rather general and could be used in several other models including the VSM with PMT

** We now consider the theory using the FR :

** In FR the mass term of the $U(1)$ gauge field A^μ is defined as:

$$\mathcal{L}_m^{FR} = \frac{1}{2} e^2 \left(A_\mu M^{\mu\nu} A_\nu \right) \quad , \quad M^{\mu\nu} = \begin{pmatrix} +1 & -1 \\ -1 & -3 \end{pmatrix}$$

** In view of this, we define the VSM with PMT in the FR by the Lagrangian density:

$$\begin{aligned} \mathcal{L}^N &:= \mathcal{L}_4 + \mathcal{L}_m^{FR} \\ &= \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - e \epsilon^{\mu\nu} \partial_\mu \phi A_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} e^2 A_\mu M^{\mu\nu} A_\nu \right] \end{aligned}$$

** The last term involving only the e.m. field A^μ has been explicitly derived by using the Pauli-Villars method of regularization and they have obtained the effective action with the above unconventional mass term. This mass term for A^μ arises from the regularization ambiguities associated with the definition of current and contains the fermionic one loop effects.

** This new regularization used by Mitra has been called by him Faddeevian Regularization and this theory is in accordance with the Faddeev's picture of anomalous gauge theories.

** We recall that in a true GI theory the matrix of the PB's of the constraints is a null matrix. Faddeev visualized a situation where anomalies make the PB of the Gauss law constraint with itself non vanishing. If it happens,the constraints become second class and the GI is lost.

** In the VSMPMT with the new FR, the Faddeev's mechanism works, namely, the constraints become second class through an anomaly in the PB of the Gauss Law constraints with itself. Also the mass like term does not have the Lorentz-invariance and therefore the theory lacks manifest Lorentz invariance.

** However Poincare generators of the theory defined on the constraint hypersurface are seen to satisfy the Poincare algebra.

$$
\begin{aligned}
[P_R^0, P_R^1] &= 0 \\
[M_R^{10}, P_R^0] &= -iP_R^1 \\
[M_R^{10}, P_R^1] &= -iP_R^0
\end{aligned}
\tag{1}
$$

Here P_R^0 , P_R^1 and M_R^{10} are opertors corresponding to FE, FM and AM of the theory defined on the constraint hypersurface.

** In vew of this, the theory despite the lack of manifest Lorentz - covariance is seen to be implicitely Lorentz - invariance.

** We now consider the IFQ of this theory with the FR:

** In the IFQ, the Lagrangian density of the theory ($\mathcal{L}^N$) (with $\mu, \nu = 0, 1$) reads:

$$\begin{aligned}\mathcal{L}^N &= \Big[\frac{1}{2}(\partial_0\phi\partial_0\phi - \partial_1\phi\partial_1\phi) + e(A_0\partial_1\phi - A_1\partial_0\phi) \\ &\quad + \frac{1}{2}(\partial_0 A_1 - \partial_1 A_0)^2 + \frac{1}{2}e^2\Big((A_0 - A_1)^2 - 4A_1^2\Big)\Big]\end{aligned}$$

** Canonical momenta obtained from the above Lag. Density are:

$$\begin{aligned}\pi &= \frac{\partial\mathcal{L}^N}{\partial(\partial_0\phi)} = (\partial_0\phi - eA_1) \\ \Pi^0 &= \frac{\partial\mathcal{L}^N}{\partial(\partial_0 A_0)} = 0\ ,\ \Pi^1(:= E) = \frac{\partial\mathcal{L}^N}{\partial(\partial_0 A_1)} = (\partial_0 A_1 - \partial_1 A_0)\end{aligned}$$

** Here $\Pi^0, E(:= \Pi^1)$ and π are the momenta canonically conjugate respectively to A_0, A_1 and ϕ respectively. The theory is thus seen to possess one PC:

$$\xi_1 = \Pi^0 \approx 0$$

** The canonical Hamiltonian density of the theory reads:

$$\mathcal{H}_c^N = \left[\frac{1}{2}\left(\pi^2 + E^2 + (\partial_1\phi)^2 + e^2 A_1^2\right) + E\partial_1 A_0 + e\pi A_1 - eA_0\partial_1\phi - \frac{1}{2}e^2(A_0 - A_1)^2 + 2e^2 A_1^2\right]$$

** After including the primary constraint ξ_1 in the canonical Hamiltonian density $\mathcal{H}_c$ with the help of Lagrange multiplier field $u(x,t)$ which is treated as dynamical, the total Hamiltonian density of the theory $\mathcal{H}_T$ could be written as:

$$\mathcal{H}_T^N = \left[\frac{1}{2}\left(\pi^2 + E^2 + (\partial_1\phi)^2 + e^2 A_1^2\right) + E\partial_1 A_0 + e\pi A_1 - eA_0\partial_1\phi - \frac{1}{2}e^2(A_0 - A_1)^2 + 2e^2A_1^2 + \pi^0 u\right]$$

** Demanding that the primary constraint ξ_1 be preserved for all time leads to a secondary Gauss-law constraint:

$$\xi_2 = [\partial_1 E + e\partial_1\phi + e^2(A_0 - A_1)] \approx 0$$

** Matrix of the PB's among the constraints ξ_1 and ξ_2 is seen to be non-singular implying that the set of constraints ξ_1 and ξ_2 is second-class and that the theory under consideration is GNI, and it could be quantized in the standard manner.

** LFQ of the theory in Faddeevian Regularization:

** In Front Form, the Lagrangian density of the theory ($\mathcal{L}^N$) (with $\mu, \nu = +, -$) reads:

$$\mathcal{L}^N = \Big[(\partial_+\phi\partial_-\phi) + e(A^+\partial_+\phi) - e(A^-\partial_-\phi) + \frac{1}{2}(\partial_+A^+ - \partial_-A^-)^2 + 2e^2A^+A^- - e^2(A^-)^2\Big]$$

** Canonical momenta obtained from the above Lag. Density are:

$$\Pi = \frac{\partial\mathcal{L}^N}{\partial(\partial_+\phi)} = (\partial_-\phi + eA^-)$$

$$\Pi^+ = \frac{\partial\mathcal{L}^N}{\partial(\partial_+A^-)} = 0 \ , \ \Pi^- = \frac{\partial\mathcal{L}^N}{\partial(\partial_+A^+)} = (\partial_+A^+ - \partial_-A^-)$$

** Here Π, Π^{+} and Π^{-} are the momenta canonically conjugate respectively to ϕ, A^{-} and A^{+} respectively. The theory is thus seen to possess two PC:

$$\begin{aligned} \xi_1 &= \Pi^{+} \approx 0 \\ \psi_1 &= (\Pi - \partial_{-}\phi - eA^{+}) \approx 0 \end{aligned}$$

** The canonical Hamiltonian density of the theory:

$$\mathcal{H}_c^N = \left[\frac{1}{2}(\Pi^{-})^2 + \Pi^{-}(\partial_{-}A^{-}) + eA^{-}(\partial_{-}\phi) - 2e^2A^{+}A^{-} + e^2(A^{-})^2\right]$$

** After including the primary constraint ξ_1 and ψ_1 in the canonical Hamiltonian density $\mathcal{H}_c$ with the help of Lagrange multiplier field $u(x,t)$ and $v(x,t)$ which is treated as dynamical, the total Hamiltonian density of the theory $\mathcal{H}_T$ could be written as:

$$\mathcal{H}_T^N = \left[\frac{1}{2}(\Pi^-)^2 + \Pi^-(\partial_- A^-) + eA^-(\partial_-\phi) - 2e^2 A^+ A^- + e^2 (A^-)^2 + \Pi^+ u + (\Pi - \partial_-\phi - eA^+)\right]$$

** The Hamilton's equations of motion of the theory that preserve the constraints of the theory in the course of time could be obtained from the total Hamiltonian: $H_T = \int \mathcal{H}_T dx^-$.

** Demanding that the PC ξ_1 be preserved for all time leads to a secondary Gauss-law constraint and PC ψ_1 gives a condition on LMF:

$$\xi_2 = [\partial_{-}\Pi^{-} - e\partial_{-}\phi + 2e^2(A^{+}A^{-})] \approx 0 \quad (2)$$

** The matrix of the Poisson brackets among the constraints ξ_1 , ψ_1 and ξ_2 is seen to be non-singular implying that the set of constraints ξ_1 , ψ_1 and ξ_2 is second-class and that the theory under consideration is GNI and it could be quantized in the standard manner using the HF and PIQ.

*** Construction and Quantization of a GI Theory:

** In constructing a GI Model corresponding to $\mathcal{L}^N$, we enlarge the Hilbert space of the theory and introduce a new field θ (called the Stueckelberg Field), through the following redefinition of fields ϕ and A^μ in $\mathcal{L}^N$ through the following Stuckelberg tranformations:

$$\phi \to \Phi = \phi - \theta \quad , \quad A^\mu \to \mathcal{A}^\mu = A^\mu + \partial^\mu \theta$$

Performing the changes in $\mathcal{L}^N$ we obtain:

$$
\begin{aligned}
\mathcal{L}^I &= \mathcal{L}^N + \mathcal{L}^S \\
&= \Big[(\partial_+\phi)(\partial_-\phi) + eA^+(\partial_+\phi) - eA^-(\partial_-\phi) + \frac{1}{2}(\partial_+A^+ - \partial_-A^-)^2 \\
&\quad +2e^2A^+A^- - e^2(A^-)^2 + (e-1)(\partial_+\phi)(\partial_-\theta) \\
&\quad -(1+e)(\partial_-\phi)(\partial_+\theta) + (1-2e^2)(\partial_+\theta)(\partial_-\theta) \\
&\quad -e(1+2e)[A^+(\partial_+\theta) - A^-(\partial_-\theta)] - e^2(\partial_+\theta)^2 - 2e^2A^-(\partial_+\theta)\Big]
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{L}^S &= \Big[(e-1)(\partial_+\phi)(\partial_-\theta) - (1+e)(\partial_-\phi)(\partial_+\theta) + (1-2e^2)(\partial_+\theta)(\partial_-\theta) \\
&\quad -e(1+2e)[A^+(\partial_+\theta) - A^-(\partial_-\theta)] - e^2(\partial_+\theta)^2 - 2e^2A^-(\partial_+\theta)\Big]
\end{aligned}
$$

** Where $\mathcal{L}^S$ is the appropriate Stueckelberg term. We shall see later that the physical contents of the GNI-theory described by $\mathcal{L}^N$ could be recovered under some special GFC's.

** The canonical momenta for the GI theory described by $\mathcal{L}^I$ are:

$$
\begin{aligned}
\Pi &= \frac{\partial \mathcal{L}^I}{\partial(\partial_+\phi)} = \Big[\partial_-\phi + eA^- + (e-1)(\partial_-\theta)\Big] \\
\Pi^+ &= \frac{\partial \mathcal{L}^I}{\partial(\partial_+A^-)} = 0\ ,\ \ \Pi^- = \frac{\partial \mathcal{L}^I}{\partial(\partial_+A^+)} = (\partial_+A^+ - \partial_-A^-) \\
\Pi_\theta &= \frac{\partial \mathcal{L}^I}{\partial(\partial_+\theta)} = -\Big[(1+e)(\partial_-\phi) - (1-2e^2)(\partial_-\theta) \\
&\qquad + e(1+2e)A^+ + 2e^2(\partial_+\theta) + 2e^2A^-\Big]
\end{aligned}
$$

**

Here Π, Π^+, Π^- and Π_θ, are the momenta canonically conjugate respectively to ϕ, A^-, A^+ and θ, respectively.

$\Longrightarrow$ Theory possesses two Primary constraints:

$$\xi_1 = \Pi^+ \approx 0$$

$$\psi_1 = \left[\Pi - \partial\_\phi - eA^+ - (e-1)(\partial\_\theta)\right] \approx 0$$

** The canonical hamiltonian density of the theory:

$$\mathcal{H}_c^N = \Bigg[\frac{1}{2}(\Pi^-)^2 + \Pi^-(\partial\_A^-) + eA^-(\partial\_\phi) - 2e^2A^+A^- + e^2(A^-)^2 - e(1+2e)A^-(\partial_+\theta) - e^2\left(\left(\frac{-1}{2e^2}\right)\{\Pi_\theta + (1+e)\Pi + e^2(\partial\_\theta) + e^2A^+\} - (A^-)\right)^2\Bigg]$$

** After including the primary constraint ξ_1 and ψ_1 in the canonical Hamiltonian density $\mathcal{H}^I_c$ with the help of Lagrange multiplier field u(x,t) and v(x,t) which is treated as dynamical, the total Hamiltonian density of the theory $\mathcal{H}^I_T$ could be written as:

$$\begin{aligned}
\mathcal{H}^I_T \;=\; & \Bigg[\frac{1}{2}(\Pi^-)^2 + \Pi^-(\partial_- A^-) + eA^-(\partial_-\phi) - 2e^2 A^+ A^- \\
& + e^2(A^-)^2 - e(1+2e)A^-(\partial_+\theta) \\
& - e^2\left(\left(\frac{-1}{2e^2}\right)\{\Pi_\theta + (1+e)\Pi + e^2(\partial_-\theta) + e^2 A^+\} - (A^-)\right)^2 \\
& + \Pi^+ u + \Big(\Pi - (\partial_-\phi) - eA^+ - (e-1)(\partial_-\theta)\Big)v
\end{aligned}$$

** Theory is seen to possess one secondary Gauss-law constraint:

$$\xi_2 = \left[\Pi_\theta + (1+2e)\Pi + (\partial_-\Pi^-) + 2e^2 A^+ - 2e(\partial_-\phi) + 2(e+e^2)(\partial_-\theta)\right] \approx 0$$

Matrix of PB's of the constraints ξ_1, ξ_2 and ψ_1 is singular and the constraints form a set of ICC's. The theory described by $\mathcal{L}^I$ is GI.

**Action of the theory is is seen to be invariant under LVGT's:

$$\begin{aligned}
\delta\phi &= -\beta(x^+, x^-)\ ,\ \delta\theta = -\beta(x^+, x^-)\ ,\ \delta A^+ = \partial_-\beta(x^+, x^-) \\
\delta A^- &= \partial_+\beta(x^+, x^-),\ \delta\Pi = \delta\Pi^+ = \delta\Pi^- = \delta\Pi_\theta = 0 \\
\delta\Pi_u &= \delta\Pi_v = 0,\ \delta u = \partial_+\partial_+\beta(x^+, x^-)\ ,\ \delta v = \partial_+\partial_+\beta(x^+, x^-)
\end{aligned}$$

** Gauge parameter $\beta(x^+, x^-)$ is an arbitrary function of its arguments.

** The vector gauge current of the theory (with $\mu = +, -$) is:

$$
\begin{aligned}
J^{\mu} &= \int j^{\mu} dx^{-} \\
j^{+} &= \int dx^{-} \Bigg[(-\beta)\Big(\partial_{-}\phi + eA^{+} + (e-1)(\partial_{-}\theta)\Big) \\
&\quad + (\partial_{-}\beta)(\partial_{+}A^{+} - \partial_{-}A^{-}) \\
&\quad + \beta\Big((1+e)(\partial_{-}\phi) + e(1+2e)A^{+}\Big) \\
&\quad - \beta\Big((1-2e^{2})(\partial_{-}\theta) - 2e^{2}(\partial_{+}\theta) - 2e^{2}A^{-}\Big)\Bigg] \\
j^{-} &= \int dx^{-} \Bigg[(-\beta)\Big(\partial_{+}\phi - eA^{-} - (1+e)(\partial_{+}\theta)\Big) \\
&\quad - \beta\Big((e-1)(\partial_{+}\phi) + (1-2e^{2})(\partial_{+}\theta) + e(1+2e)A^{-}\Big)\Bigg]
\end{aligned}
$$

** Also, $\partial_\mu j^\mu = 0$, implying that the corresponding vector gauge current is conserved

** In quantizing this GI theory using Dirac's procedure, we convert the first-class constraints of the theory into second-class ones by imposing , arbitrarily, some additional constraints on the system in the form of gauge-fixing conditions (GFC's). We choose special GFC's of the theory as

$$\begin{aligned} \zeta_1 &= (-\partial_1 \theta) \approx 0 \\ \zeta_2 &= [\Pi_\theta + (1+e)(\partial_- \phi) + (e+e^2)A^- - 2e^2 A^-] \approx 0 \end{aligned}$$

** Above GFC's reproduce the physical contents of the GNI theory described by $\mathcal{L}^N$ from the GI theory described by $\mathcal{L}^I$.

** Thus the addition of $\mathcal{L}^S$ to $\mathcal{L}^N$ enlarges only the unphysical part of the full Hilbert space of the theory $\mathcal{L}^N$, without modifying the physical contents of the GNI theory $\mathcal{L}^N$.

Finally, following the Dirac quantization procedure, the non-vanishing equal-LC-time commutators of the theory, under the GFC's: ζ_1 and ζ_2 are obtained as:

$$
\begin{aligned}
[\phi(x^+,x^-)\ ,\ \phi(x^+,y^-)] &= i\left(\frac{-1}{4}\right)\epsilon(x^- - y^-) \\
[\phi(x^+,x^-)\ ,\ A^-(x^+,y^-)] &= i\left(\frac{1}{e^2}\right)\delta(x^- - y^-) \\
[A^-(x^+,x^-)\ ,\ A^-(x^+,y^-)] &= i\left(\frac{1+2e^2}{2e^2}\right)\partial_-\delta(x^- - y^-) \\
[A^-(x^+,x^-)\ ,\ A^+(x^+,y^-)] &= i\left(\frac{-1}{2e^2}\right)\partial_-\delta(x^- - y^-) \\
[\phi(x^+,x^-)\ ,\ \Pi(x^+,y^-)] &= i\left(\frac{3}{2}\right)\delta(x^- - y^-) \\
[\phi(x^+,x^-)\ ,\ \Pi^-(x^+,y^-)] &= i\left(\frac{-1}{4}\right)\epsilon(x^- - y^-)
\end{aligned}
$$

$$
\begin{aligned}
[A^-(x^+, x^-)\ ,\ \Pi^-(x^+, y^-)] &= i\left(\frac{4e-1}{2}\right)\delta(x^- - y^-) \\
[\Pi(x^+, x^-)\ ,\ \Pi(x^+, y^-)] &= i\left(\frac{-1}{2}\right)\partial_-\delta(x^- - y^-) \\
[\Pi(x^+, x^-)\ ,\ \Pi^-(x^+, y^-)] &= i\left(\frac{-e}{2}\right)\delta(x^- - y^-) \\
[\Pi^-(x^+, x^-)\ ,\ \Pi^-(x^+, y^-)] &= i\left(\frac{-e^2}{4}\right)\epsilon(x^- - y^-)
\end{aligned}
$$

$$
\begin{aligned}
[A^-(x^+, x^-)\ ,\ \theta(x^+, y^-)] &= i\left(\frac{-1}{2e^2}\right)\delta(x^- - y^-) \\
[\phi(x^+, x^-)\ ,\ \Pi_\theta(x^+, y^-)] &= i\left(\frac{e-3}{2}\right)\delta(x^- - y^-)
\end{aligned}
$$

$$
\begin{aligned}
[A^-(x^+,x^-)\ ,\ \Pi_\theta(x^+,y^-)] &= i\left(\frac{6e^2-e+1}{2e^2}\right)\partial_-\delta(x^- - y^-) \\
[A^+(x^+,x^-)\ ,\ \Pi_\theta(x^+,y^-)] &= i\ \partial_-\delta(x^- - y^-) \\
[\theta(x^+,x^-)\ ,\ \Pi_\theta(x^+,y^-)] &= i\ 2\delta(x^- - y^-) \\
[\Pi(x^+,x^-)\ ,\ \Pi_\theta(x^+,y^-)] &= i\left(\frac{1-e}{2}\right)\partial_-\delta(x^- - y^-) \\
[\Pi^-(x^+,x^-)\ ,\ \Pi_\theta(x^+,y^-)] &= i\left[\left(\frac{e(1-e)}{2} - 4e^2\right)\right]\delta(x^- - y^-) \\
[\Pi^-(x^+,x^-)\ ,\ \Pi_\theta(x^+,y^-)] &= i\left[\left(\frac{e(1-e)}{2} - 4e^2\right)\right]\delta(x^- - y^-) \\
[\Pi_\theta(x^+,x^-)\ ,\ \Pi_\theta(x^+,y^-)] &= (-i)\left[\frac{(1-e)^2}{2} - 8e^2\right]\partial_-\delta(x^- - y^-)
\end{aligned}
$$

** It is now possible to study the path integral and BRST quantization of this theory under the above GFC's or under some other suitable set of GFC's.

** In the path integral formulation, the transition to quantum theory is made again by writing the vacuum to vacuum transition amplitude for the theory, called the generating functional $Z[J_k]$ of the theory in the presence of the external sources: J_k as:

$$\begin{aligned} Z[J_k] \;&=\; \int [d\mu] \exp \Bigg[i \int d^2x^- \Bigg(J_k \Phi^k + \Pi(\partial_+ \phi) + \Pi^+(\partial_+ A^-) \\ &\quad + \Pi^-(\partial_+ A^+) + \Pi_\theta(\partial_+ \theta) + \Pi_u(\partial_+ u) + \Pi_v(\partial_+ v) - \mathcal{H}_T \Bigg) \Bigg] \end{aligned}$$

** where the phase space variables of the theory are: $\Phi^k \equiv (\phi, A^-, A^+, \theta, u, v)$ with the corresponding respective canonical conjugate momenta: $\Pi_k \equiv (\Pi, \Pi^+, \Pi^-, \Pi_\theta, \Pi_u, \Pi_v)$. The functional measure $[d\mu]$ of the generating functional $Z[J_k]$ under the above LC gauges is obtained as:

$$
\begin{aligned}
[d\mu] \; = \; & 2\sqrt{2}e^2\delta(x^- - y^-)\delta'(x^- - y^-)[\delta'(x^- - y^-)]^{\frac{1}{2}} \\
& [d\phi][dA^-][dA^+][d\theta][du][dv][d\Pi] \\
& [d\Pi^+][d\Pi^-][d\Pi_\theta][d\Pi_u][d\Pi_v]\delta[\Pi^+ \approx 0] \\
& \delta[(\Pi - \partial\_\phi - eA^+ - (e-1)\partial\_\theta) \approx 0] \\
& \delta[(\Pi_\theta + (1+2e)\Pi + \partial\_\Pi^- + 2e^2A^+ - 2e\partial\_\phi \\
& \qquad + 2(e+e^2)\partial\_\theta) \approx 0]\delta[(-\partial\_\theta) \approx 0] \\
& \delta[(\partial\_\theta + (1+e)\partial\_\phi + (e+e^2)A^+ - 2e^2A^-) \approx 0]
\end{aligned}
$$

This completes the Hamiltonian and PIQ of the GI theory.

★ BRST Formulation of the Gauge-Invariant Theory

** In BRSTQ, we rewrite the theory as a quantum system which possesses the generalized gauge invariance called BRST symmetry.
** For this, we enlarge the Hilbert space of GI theory and replace the notion of GT (which shifts operators by c-number functions), by a BRST transformation, which mixes operators with Bose and Fermi statistics.
** We then introduce new anti-commuting variables c and $\bar{c}$ (Grassman numbers at the classical level and operators in the quantized theory) and a commuting variable b such that:

$$\begin{aligned} \hat{\delta}\phi &= -c, \ \hat{\delta}\theta = -c, \quad \hat{\delta}A^{+} = (\partial_{-}c), \ \hat{\delta}A^{-} = (\partial_{+}c) \\ \hat{\delta}\Pi &= 0, \hat{\delta}\Pi_{\theta} = 0, \hat{\delta}\Pi^{-} = 0, \hat{\delta}\Pi^{+} = 0, \hat{\delta}\Pi_{u} = 0, \hat{\delta}\Pi_{v} = 0 \\ \hat{\delta}u &= \partial_{+}\partial_{+}c, \quad \hat{\delta}v = -(\partial_{+}c), \quad \hat{\delta}c = 0, \quad \hat{\delta}\bar{c} = b, \quad \hat{\delta}b = 0 \end{aligned} \tag{3}$$

**with the property $\hat{\delta}^2 = 0$.

**We now define a BRST-invariant function of the dynamical variables to be a function f(Π, Π^+, Π^-, Π_θ, Π_u, Π_v, p_b, Π_c, $\Pi_{\bar{c}}$, ϕ, A^-, A^+,θ, u, v, b, c, $\bar{c}$) such that $\hat{\delta}\mathrm{f} = 0$.

** Performing gauge-fixing in the BRST formalism implies adding to the first-order Lagrangian density $\mathcal{L}_{I0}$, a trivial BRST-invariant function.

We could thus write e.g.:
$\mathcal{L}_{BRST}$

$$
\begin{aligned}
= \quad & \Big[\Pi^{-}(\partial_{+}A^{+}) + \Pi_{\theta}(\partial_{+}\theta) + \Pi_{u}(\partial_{+}u) + \Pi_{v}(\partial_{+}v)\Big] \\
& -\frac{1}{2}(\Pi^{-})^{2} - \Pi^{-}(\partial_{-}A^{-}) - eA^{-}(\partial_{-}\phi) + 2e^{2}A^{+}A^{-} \\
& -e^{2}(A^{-})^{2} + e(1+2e)A^{-}(\partial_{-}\theta) \\
& +e^{2}\left[-\left(\frac{1}{2e^{2}}\right)\left(\Pi_{\theta} + (1+e)\Pi + e^{2}(\partial_{-}\theta) + e^{2}A^{+}\right) - (A^{-})\right]^{2} \\
& +\Big[(\partial_{-}\phi) + eA^{+} + (e-1)(\partial_{-}\theta)\Big](\partial_{+}\phi) \\
& +\hat{\delta}\left[\bar{c}(\partial_{+}A_{-} - \phi - \frac{1}{2}b)\right]
\end{aligned}
$$

** The last term is the extra BRST-invariant gauge-fixing term

After one integration by parts, the above equation could now be written as:

$$
\begin{aligned}
\mathcal{L}_{BRST} \\
= \; & \Big[\Pi^{-}(\partial_{+}A^{+}) + \Pi_{\theta}(\partial_{+}\theta) + \Pi_{u}(\partial_{+}u) + \Pi_{v}(\partial_{+}v)\Big] \\
& -\frac{1}{2}(\Pi^{-})^{2} - \Pi^{-}(\partial_{-}A^{-}) - eA^{-}(\partial_{-}\phi) + 2e^{2}A^{+}A^{-} \\
& -e^{2}(A^{-})^{2} + e(1+2e)A^{-}(\partial_{-}\theta) \\
& +e^{2}\left[-\left(\frac{1}{2e^{2}}\right)\left(\Pi_{\theta} + (1+e)\Pi + e^{2}(\partial_{-}\theta) + e^{2}A^{+}\right) - (A^{-})\right]^{2} \\
& +\Big[(\partial_{-}\phi) + eA^{+} + (e-1)(\partial_{-}\theta)\Big](\partial_{+}\phi) \\
& +\left[-\frac{1}{2}b^{2} + b(\partial_{+}A^{-} - \phi) + (\partial_{+}\bar{c})(\partial_{+}c) - \bar{c}c\right]
\end{aligned}
$$

** Proceeding classically, the ELE for b reads:

$$b = [\partial_+ A^+ - \phi] \tag{4}$$

** the requirement $\hat{\delta} b = 0$ then implies

$$\hat{\delta} b = [\hat{\delta}(\partial_+ A^+) - \hat{\delta}\phi] = 0 \tag{4}$$

** which in turn implies:

$$c\partial_+\partial_+ c = (-c)$$

** The above equation is also an ELE obtained by the variation of $\mathcal{L}_{BRST}$ with respect to $\bar{c}$. In introducing momenta one has to be careful in defining those for the fermionic variables. We thus define the bosonic momenta in the usual manner so that:

$$\Pi^+ := \frac{\partial}{\partial(\partial_+ A^-)} \mathcal{L}_{BRST} = +b \tag{4}$$

** for the fermionic momenta with directional derivatives we set

$$\Pi_c := \mathcal{L}_{BRST} \frac{\overleftarrow{\partial}}{\partial(\partial_+ c)} = \partial_+ \bar{c} \quad ; \quad \Pi_{\bar{c}} := \frac{\overrightarrow{\partial}}{\partial(\partial_+ \bar{c})} BRST = \partial_+ c$$

** implying that the variable canonically conjugate to c is $(\partial_+ \bar{c})$ and the variable conjugate to $\bar{c}$ is $(\partial_+ c)$. For writing the quantum Hamiltonian density from the Lagrangian density in the usual manner we remember that the former has to be Hermitian so that:

$\mathcal{H}_{BRST}$

$$= \Bigg[\frac{1}{2}(\Pi^-)^2 + \Pi^-(\partial_- A^-) + eA^-(\partial_-\phi) - 2e^2 A^+ A^-$$
$$+ e^2(A^-)^2 - e(1+2e)A^-(\partial_-\theta)$$
$$- e^2\left[-\left(\frac{1}{2e^2}\right)\left(\Pi_\theta + (1+e)\Pi + e^2(\partial_-\theta) + e^2 A^+\right) - (A^-)\right]^2$$
$$- \Pi^+(\partial_+ A^- - \phi) + \frac{(\Pi^+)^2}{2}\Bigg] + (\Pi_c \Pi_{\bar{c}} + \bar{c}c)$$

** We can check the consistency of our definitions of the the Fermionic momenta by looking at the Hamiltons equations for the Fermionic variables:

$$\partial_+ c = \frac{\overrightarrow{\partial}}{\partial \Pi_c} \mathcal{H}_{BRST} \quad ; \quad \partial_+ \bar{c} = \mathcal{H}_{BRST} \frac{\overleftarrow{\partial}}{\partial \Pi_{\bar{c}}}$$

**Thus we see that

$$\partial_+ c = \frac{\overrightarrow{\partial}}{\partial \Pi_c} \mathcal{H}_{BRST} = \Pi_{\bar{c}} \quad ; \quad \partial_+ \bar{c} = \mathcal{H}_{BRST} \frac{\overleftarrow{\partial}}{\partial \Pi_{\bar{c}}} = \Pi_c$$

is in agreement with our definitions of the Fermionic momenta. For the operators $c, \bar{c}, \partial_+ c$ and $\partial_+ \bar{c}$, one needs to satisfy the anti-commutation relations of $\partial_+ c$ with $\bar{c}$ or of $\partial_+ \bar{c}$ with c, but not of c, with $\bar{c}$.

** In general, c and $\bar{c}$ are independent canonical variables and one assumes that:

$$\{\Pi_c, \Pi_{\bar{c}}\} = \{\bar{c}, c\} = 0 \; ; \; \partial_+\{\bar{c}, c\} = 0 \; ; \; \{\partial_+\bar{c}, c\} = (-1)\{\partial_+ c, \bar{c}\}$$

** where { , } means an anti commutator. We thus see that the anti-commulators in the above equation are non-trivial and need to be fixed. In order to fix these, we demand that c satisfy the Heisenberg equation

$$[c, \mathcal{H}_{BRST}] = i\partial_+ c$$

and using the property $c^2 = \overline{c}^2 = 0$ one obtains

$$[c, \mathcal{H}_{BRST}] = \{\partial_+\bar{c}, c\}\partial_+ c$$

The last three equations then imply:

$$\{\partial_+\bar{c}, c\} = (-1)\{\partial_+ c, \bar{c}\} = i$$

** Here the minus sign in the above equation is nontrivial and implies the existence of states with negative norm in the space of state vectors of the theory.
** The BRST charge operator Q is the generator of the BRST transformations. It is nilpotent and satisfies $Q^2 = 0$. It mixes operators which satisfy Bose and fermi statistics.

**In view of this, the BRST charge operator of the present theory could be written as:

$$
\begin{aligned}
Q \; = \; & \int dx^- \Big[ic[\Pi_\theta + (1+2e)\Pi + (\partial_-\Pi^-) + 2e^2 A^+ \\
& -2e(\partial_-\phi) + 2(e+e^2)(\partial_-\theta)] \\
& -i(\partial_+ c)[\Pi + \Pi^+ - (\partial_-\phi) - eA^+ - (e-1)(\partial_-\theta)] \Big]
\end{aligned}
$$

** This equation implies that the set of states satisfying the conditions:

$$\Pi^+|\psi\rangle = 0$$
$$[\Pi - \partial_-\phi - eA^+ - (e-1)(\partial_-\theta]|\psi\rangle = 0$$
$$\Big[\Pi_\theta + (1+2e)\Pi + (\partial_-\Pi^-) + 2e^2A^+ - 2e(\partial_-\phi) + 2(e+e^2)(\partial_-\theta)\Big]|\psi\rangle = 0$$

belong to the dynamically stable subspace of states $|\psi\rangle$ satisfying $Q|\psi\rangle = 0$, i.e., it belongs to the set of BRST-invariant states.

** The Hamiltonian is also invariant under the anti-BRST transformation given by:

$$\begin{aligned}
\hat{\bar{\delta}}\phi &= \bar{c} \quad , \quad \hat{\bar{\delta}}\theta = \bar{c} \quad , \quad \hat{\bar{\delta}}A^+ = -(\partial_-\bar{c}) \quad , \quad \hat{\bar{\delta}}A^- = -(\partial_+\bar{c}) \\
\hat{\bar{\delta}}\Pi &= 0 \quad , \quad \hat{\bar{\delta}}\Pi_\theta = 0 \quad , \quad \hat{\bar{\delta}}\Pi^+ = 0 \quad , \quad \hat{\bar{\delta}}\Pi^- = 0 \\
\hat{\bar{\delta}}u &= -(\partial_+\partial_+\bar{c}) \quad , \quad \hat{\bar{\delta}}v = -(\partial_+\bar{c}) \quad , \quad \hat{\bar{\delta}}\Pi_u = 0 \quad , \quad \hat{\bar{\delta}}\Pi_v = 0 \\
\hat{\bar{\delta}}\bar{c} &= 0, \quad \hat{\bar{\delta}}c = -b, \quad \hat{\bar{\delta}}b = 0
\end{aligned}$$

** with generator or anti-BRST charge:

$$\begin{aligned}
\bar{Q} &= \int dx^- \Big[-i\bar{c}[\Pi_\theta + (1+2e)\Pi + (\partial_-\Pi^-) + 2e^2A^+ \\
&\quad -2e(\partial_-\phi) + 2(e+e^2)(\partial_-\theta)] \\
&\quad +i(\partial_+\bar{c})[\Pi + \Pi^+ - (\partial_-\phi) - eA^+ - (e-1)(\partial_-\theta)] \Big]
\end{aligned}$$

We also have

$$\partial_+ Q = [Q, H_{BRST}] = 0 \quad , \quad \partial_+ \bar{Q} = [\bar{Q}, H_{BRST}] = 0$$

with $H_{BRST} := \int dx \mathcal{H}_{BRST}$

** and we further impose the dual condition that both Q and $\bar{Q}$ annihilate physical states, implying that:

$$Q|\psi\rangle = 0 \ \text{ and } \ , \quad \bar{Q}|\psi\rangle = 0 \tag{5}$$

** The states for which the constraints of the theory hold, satisfy both of above conditions and are in fact, the only states satisfying both of these conditions.

** Thus the only states satisfying $Q|\psi\rangle = 0$ and $\bar{Q}|\psi\rangle = 0$ are those that satisfy the constraints of the theory.

** Now because $Q|\psi\rangle = 0$, the set of states annihilated by Q contains not only the set of states for which the constraints of the theory hold but also additional states for which the constraints of the theory do not hold in particular.
** This situation is, however, easily avoided by additionally imposing on the theory, the dual condition: $Q|\psi\rangle = 0$ and $\bar{Q}|\psi\rangle = 0$.

** Thus by imposing both of these conditions on the theory simultaneously, one finds that the states for which the constraints of the theory hold satisfy both of these conditions and, in fact, these are the only states satisfying both of these conditions because in view of the conditions on the fermionic variables c and $\bar{c}$ one cannot have simultaneously c, $\partial_0 c$ and $\bar{c}$, $\partial_0 \bar{c}$, applied to $|\psi\rangle$ to give zero.

** Thus the only states satisfying $Q|\psi\rangle = 0$ and $\bar{Q}|\psi\rangle = 0$ are those that satisfy the constraints of the theory and they belong to the set of BRST-invariant as well as to the set of anti-BRST-invariant states.

** In conclusions

In the BRST formalism we embed a GI th. (L^I) into a BRST inv. system and the new BRST-Symmetry is maintained (even under GF) and hence projecting any state onto the sector of BRST and the anti-BRST invariant states yields a theory isomorphic to the original GI theory.

Conclusions

★ To summarize, we have considered in this work, the VSM with a photon mass term in the FR. Then we have studied its LFQ using Hamiltonian and path integral formulations, on the hyperplanes: $x^+ = t =$constant.

★ This model is seen to be GNI and anomalous, in contrast to the usual model without a photon mass term, which describes a GI theory.

★ Further, we have explicitly constructed a corresponding GI theory by calculating an appropriate Stueckelberg term for the theory and we have explicitly demonstrated that this new theory is GI even though we have photon mass term in the theory.

★ This model represents a new class of models in the 2D QED with mass-less fermions but with a photon mass term.

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