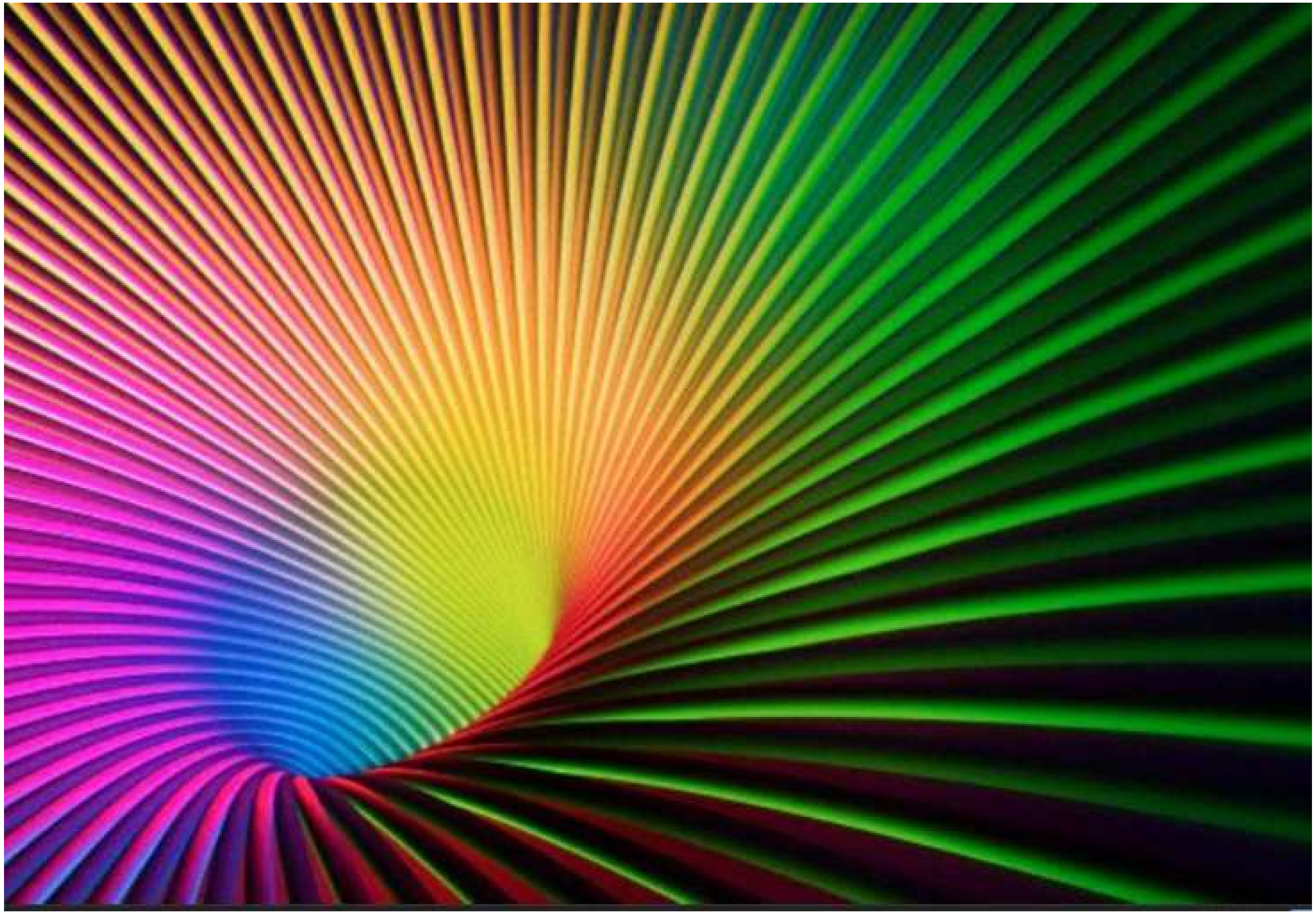
Planck-scale quantum mechanics with deformed relativistic symmetries

Frascati 19.12.2014

Giovanni Amelino-Camelia
Sapienza University of Rome




Giovanni Amelino-Camelia
Group leader


Michele Arzano
Postdoctoral researcher


Giulia Gubitosi
Postdoctoral researcher


Grasiele Batista dos Santos
Postdoctoral researcher


Francisco Nettel Rueda
Postdoctoral researcher


Valerio Astuti
PhD student


Leonardo Barcaroli
PhD student


Giovanni Palmisano
PhD student


Malú Maira Silva
PhD student


Alessandro Moia
PhD student

some previous members


Olaf Dreyer
Postdoctoral researcher


Pierre Martinetti
Postdoctoral researcher


Antonino Marcianò
PhD student
(presently faculty
at Fudan University)


Niccolò Loret
PhD student
(presently postdoc
at Perimeter Institute)


Giacomo Rosati
PhD student
(presently postdoc
at University of Wroclaw)


Stefano Bianco
PhD student
(presently postdoc
at Max Planck Institute)


Flavio Mercati
PhD student
(presently postdoc
at Perimeter Institute)


Danilo Latini
Laurea student
(presently PhD student
at Università Roma3)


Francesco Brighenti
Laurea student
(presently PhD student
at Università di Bologna)

http://gs51.relativerest.org/people/

not a wild speculation:

the understanding of the quantum-gravity problem might benefit from a better understanding of quantum mechanics…

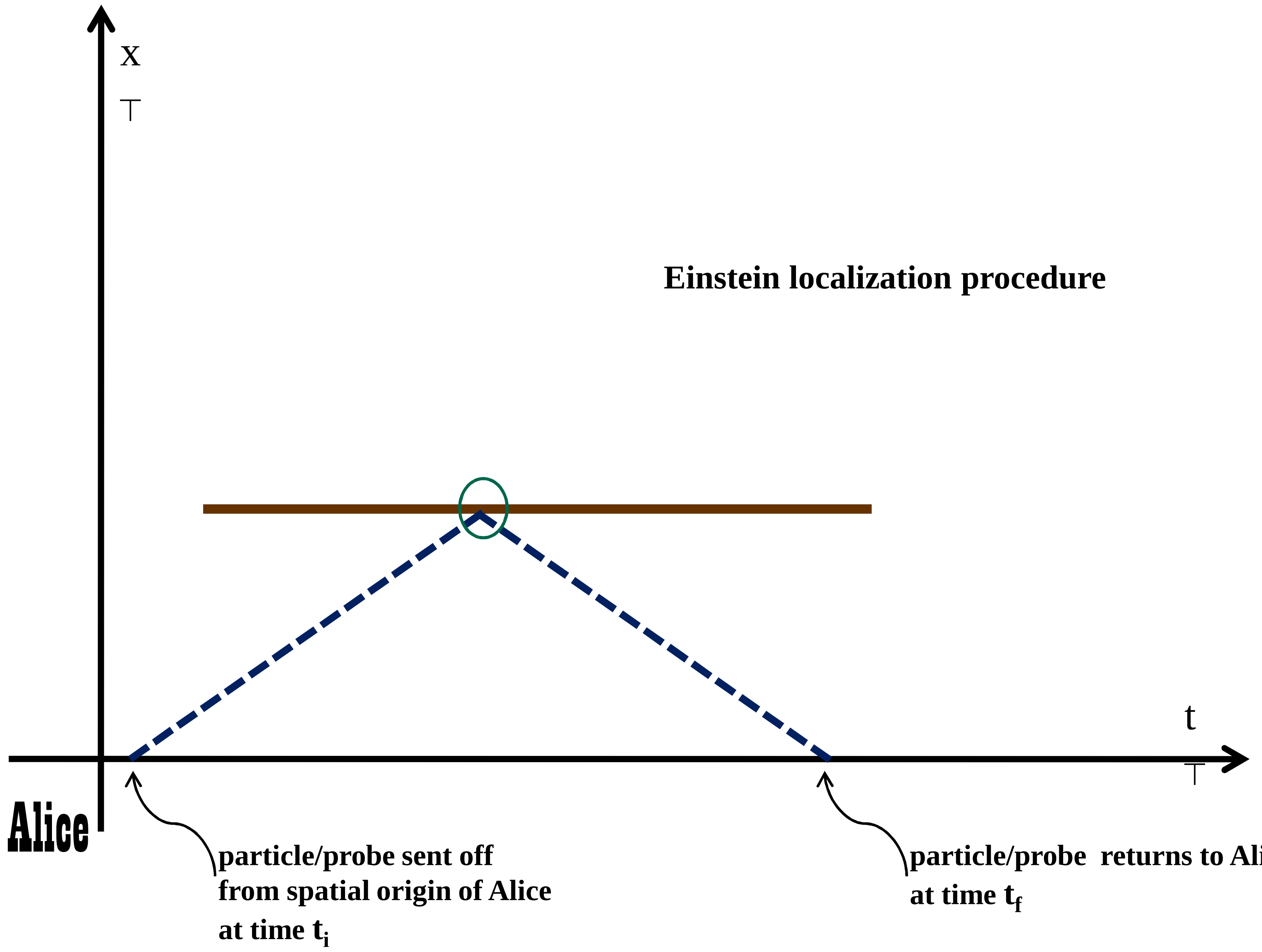
note however that some have argued (perhaps most notably Penrose) that the study of foundational issues in quantum mechanics cannot be disentangled from the study of the quantum gravity problem and the Planck-scale realm

most analyses of the foundations of quantum mechanics choose as arena a Galilean-relativistic framework (colloquially "non-relativistic")

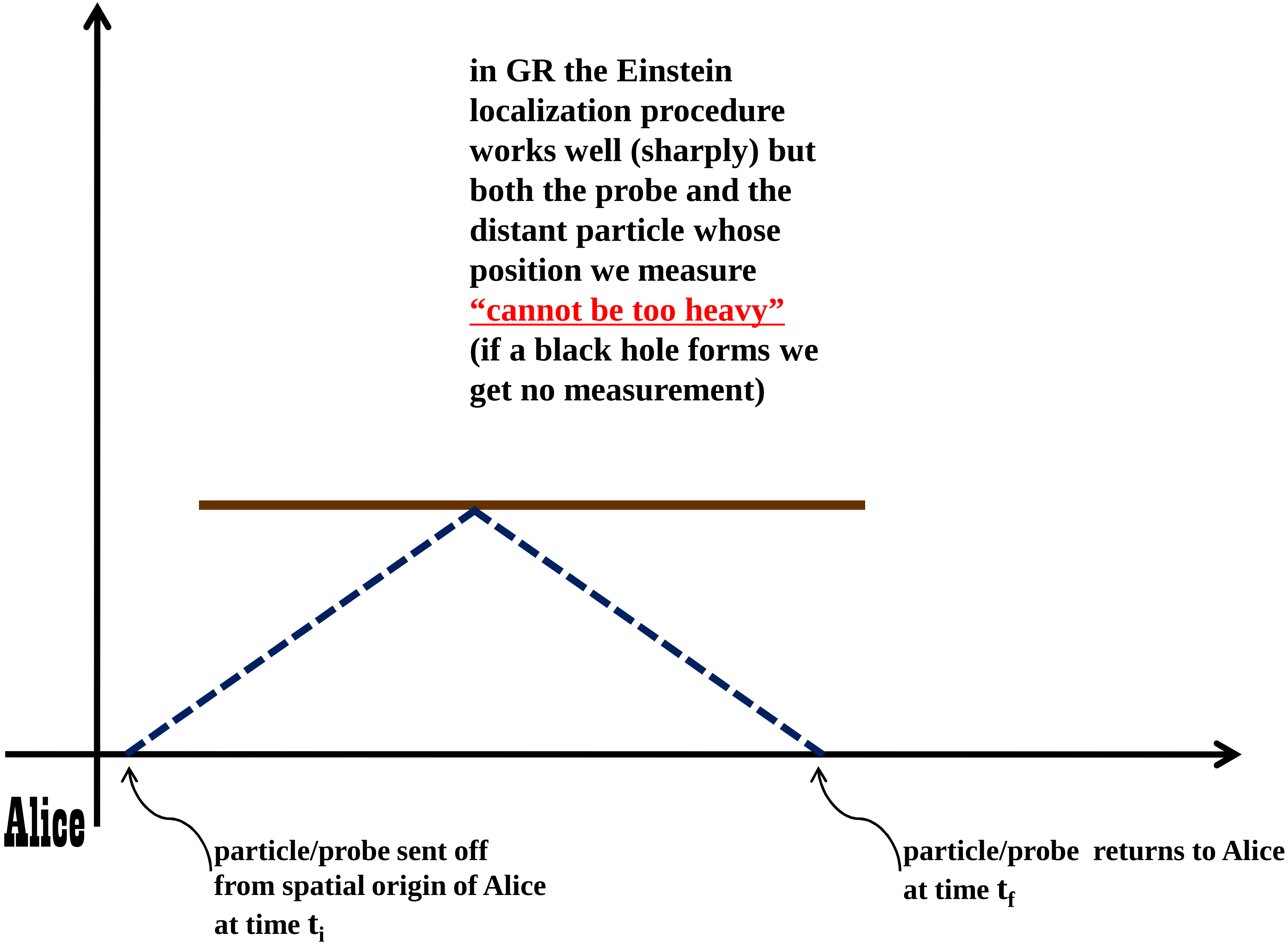
recently increased interest in the foundations of quantum mechanics within special-relativistic frameworks

I focus today on a scenario which has been much studied recently in the quantum-gravity literature which brings about the hypothesis that the relativistic issue and the quantum-gravity issue for the foundations of quantum mechanics might have some overlap

most fascinating example of "conflict" between gravity and quantum mechanics:
localization in GR vs localization in QM



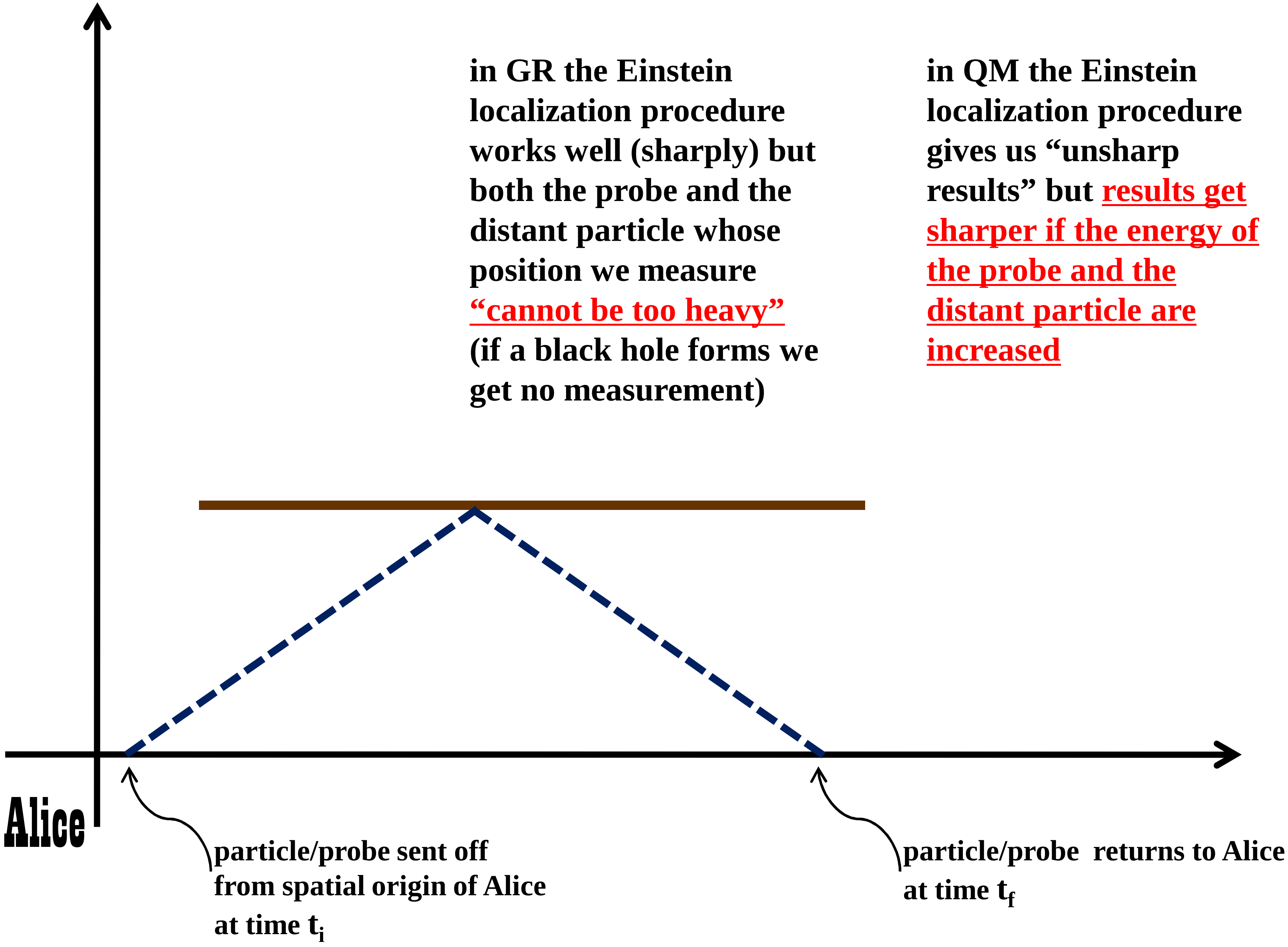
Both theories are formulated in classical spacetime (smooth Reimannian geometry).
In physics this makes sense if points of the spacetime can be identified sharply.
The two theories achieve sharp localization in <u>opposite regimes</u>!!



Both theories are formulated in classical spacetime (smooth Reimannian geometry).
<u>In physics this makes sense if points of the spacetime can be identified sharply.</u>
The two theories achieve sharp localization in <u>opposite regimes</u>!!

in GR the Einstein localization procedure works well (sharply) but both the probe and the distant particle whose position we measure <u>“cannot be too heavy”</u> (if a black hole forms we get no measurement)

in QM the Einstein localization procedure gives us “unsharp results” but <u>results get sharper if the energy of the probe and the distant particle are increased</u>



$E_{QG} \sim E_{Planck} = 1.2 \bullet 10^{19}$GeV= $\left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}}$ **i.e. 10^{-35}meters ("Planck length")**

mainly comes from observing that at the Planck scale

$$\lambda_{compton} \sim \lambda_{schwartzschild}$$

Note that it is only <u>rough order-of-magnitude estimate at best</u>

in particular this estimate <u>assumes that G does not run at all</u>!!!!!!!!!
it most likely does run!!!
and we know the behaviour of gravitation only down to 10^{-6} meters!!!

Planck length as the minimum allowed value for wavelengths:

- suggested by several indirect arguments combining quantum mechanics and GR
- found in some detailed analyses of formalisms in use in the study of the QG problem

But the minimum wavelength is the Planck length for which observer?

GAC, ModPhysLettA (1994)
PhysLettB (1996)

Other results from the 1990s (mainly from spacetime noncommutativity and LoopQG) provided "theoretical evidence" of Planck-scale modifications of the on-shell relation, in turn inviting us to scrutinize the fate of relativistic symmetries at the Planck scale

GAC+Ellis+Nanopoulos+Sarkar, Nature(1998)
Alfaro+Tecotl+Urrutia,PhysRevLett(1999)
Gambini+Pullin, PhysRevD(1999)
Schaefer,PhysRevLett(1999)

a possibility worth exploring: "Planck-scale <u>deformations</u> of Lorentz symmetry"
[jargon: "DSR", for "doubly-special", or "deformed-special", relativity]

GAC, grqc0012051, IntJournModPhysD11,35
hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

<u>change the laws of transformation between observers</u> so that the new properties are observer-independent

- **a law of minimum wavelength can be turned into a DSR law**
- **could be used also for properties other than minimum wavelength, such as deformed on-shellness, deformed uncertainty relations…**

The notion of DSR-relativistic theories is best discussed in analogy with the transition from Galileian Relativity to Special Relativity

analogy with Galilean-SR transition

introduction to DSR case is easier starting from reconsidering the Galilean-SR transition (the SR-DSR transition would be closely analogous)

Galilean Relativity

on-shell/dispersion relation $E = \frac{p^2}{2m} \quad (+m)$

linear composition of momenta $p_\mu^{(1)} \oplus p_\mu^{(2)} = p_\mu^{(1)} + p_\mu^{(2)}$

linear composition of velocities $\vec{V} \oplus \vec{V}_0 = \vec{V} + \vec{V}_0$

from Galilean Relativity to Special Relativity

Maxwell theory was not pointing us toward the demise of relativity!
It was pointing to a "relativistic evolution"

The new law concerning the speed of light is not Galilean invariant but is invariant of a theory, special relativity, no less (and no more) relativistic than Galileo's

Relativistic invariance rescued at the "cost" of replacing Galileian boosts with special-relativistic boosts

of course (since c is invariant of the new theory) the special-relativistic boosts act nonlinearly on velocities (whereas Galilean boosts acted linearly on velocities)

and the special-relativistic law of composition of velocities is nonlinear, noncommutative and nonassociative

$$\mathbf{w} = \mathbf{v} \oplus \mathbf{u} \qquad \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \frac{1}{1+\frac{v_1u_1+v_2u_2+v_3u_3}{c^2}} \left\{ \left[1 + \frac{1}{c^2}\frac{\gamma_{\mathbf{v}}}{1+\gamma_{\mathbf{v}}}(v_1u_1+v_2u_2+v_3u_3) \right] \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \frac{1}{\gamma_{\mathbf{v}}} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right\}$$

much undervalued in the (horrible) textbooks we feed our students: $\dfrac{v+u}{1+(vu/c^2)}$

from Special Relativity to DSR

If there was an observer-independent scale E_P (inverse of length scale ℓ) then, for example, one could have a modified on-shell relation as relativistic law

$$\boxed{m^2 = \Lambda(E, p; E_P) = E^2 - p^2 - \frac{E}{E_P} p^2 + O\left(\frac{E^4}{E_P{}^2}\right)}$$

For suitable choice of $\Lambda(E,p;E_P)$ one can easilyhave a maximum allowed value of momentum, i.e. minimum wavelength ($p_{max}=E_P$ for $\ell=-1/E_P$ in the formula here shown)

$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

I shall later use in particular the fact that this onshellness takes the following form for massless particles

$$p_1 = \frac{1 - e^{-\ell p_0}}{\ell}$$

it turns out that such laws could still be relativistic, part of a relativistic theory where not only c ("speed of massless particles in the infrared limit") but also E_P would be a nontrivial relativistic invariant

action of boosts on momenta must of course be deformed so that

$$\boxed{[N_k, \Lambda(E, p; E_P)] = 0}$$

then it turns out to be necessary to correspondingly deform the law composition of momenta

$$\boxed{p_\mu^{(1)} \oplus p_\mu^{(2)} \neq p_\mu^{(1)} + p_\mu^{(2)}}$$

(and even the simultaneity of coincident events may no longer be observer-independent)

Appreciating these technical and conceptual issues also allowed to shed light on previous results which were thought to be puzzling.
Let us see the case of the kappaMINKOWSKI noncommutative spacetime

$$[x_j, t] = i\lambda x_j \qquad [x_j, x_m] = 0$$

Lukierski+Nowicki+Ruegg+Tolstoy,PLB(1991)
Nowicki+Sorace+Tarlini,PLB(1993)
Majid+Ruegg,PLB (1994)
Lukierski+Ruegg+Zakrzewski, AnnPhys(1995)

evidently not invariant under «classical translations»

$$[x'_0, x'_j] = [x_0 + a_0, x_j + a_j] = [x_0, x_j] = i\lambda x_j \neq i\lambda x'_j$$

but adding commutative numbers to the noncommutative coordinates of kappa-Minkowski is evidently not a sensible thing

Note that a more sensible starting point is to notice that translation transformations of a space are intimately related to the properties of the differential calculus…indeed in kappa-Minkowski it turns out that the properties of translation-transformation parameters ε_μ must be based on the (noncommutative!) differential calculus on kappa-Minkowski

$$[\varepsilon_0, x_\mu] = 0; [\varepsilon_j, x_l] = 0; [\varepsilon_j, x_0] = i\lambda\varepsilon_j$$

Sitarz, PhysLettB349(1995)42 Majid+Oeckl, math.QA/9811054

so that in particular $x_\mu+\varepsilon_\mu$ obeys the kappa-Minkowski commutation relations

Making a very long story short: these noncommutative properties of the translation-transformation parameters can be faithfully reflected on properties of translation generators, even by keeping a classical action of the generators on suitably ordered functions of the coordinates

Translation generators in kappa-Minkowski:

$$P_\mu \left(e^{ikx} e^{ik_0 t}\right) = k_\mu \left(e^{ikx} e^{ik_0 t}\right)$$

classical action

$$[x_j, t] = i\lambda x_j$$
$$[x_j, x_m] = 0$$

$$\Rightarrow \quad e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} = e^{i(k+e^{\lambda k_0} K)x} e^{i(k_0+K_0)t}$$

!!

then "non-primitive coproduct"

$$P_\mu \left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t}\right) = P_\mu \left(e^{i(k+e^{\lambda k_0} K)x} e^{i(k_0+K_0)t}\right)$$
$$= \left(k_\mu + e^{-\lambda k_0} K_\mu\right)\left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t}\right)$$
$$= \left[P_\mu \left(e^{ikx} e^{ik_0 t}\right)\right]\left(e^{iKx} e^{iK_0 t}\right) + \left[e^{-\lambda P_0}\left(e^{ikx} e^{ik_0 t}\right)\right] P_\mu \left(e^{iKx} e^{iK_0 t}\right)$$

!!

Generalization of Noether theorem applicable to this sort of Hopf-algebra symmetries of field theories in noncommutative spacetime has been achieved

PLB671(2009)298, PRD78(2008) 025005 ,MPLA22(2007)1779
(Agostini+Arzano+Gubitosi+Marciano+Martinetti+Mercati+GAC)

relativistic kinematics in kappa-Minkowski (based on nearly two decades of results)

GAC,arXiv:1111.5081,PhysRevD(2012)

on-shell/dispersion relation $$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

[notice that this, for $\boldsymbol{\ell} = -1/E_P$, sets maximum momentum E_P]

modified law of composition of momenta

$$(p \oplus_\ell p')_1 = p_1 + e^{\ell p_0} p'_1$$

$$(p \oplus_\ell p')_0 = p_0 + p'_0$$

$$\boxed{\ell \equiv \lambda \approx \frac{1}{E_P}}$$

modified boost action

$$[N, p_0] = p_1$$

$$[N, p_1] = \frac{e^{2\ell p_0} - 1}{2\ell} + \frac{\ell}{2} p_1^2$$

ensures observer-independence of on-shell relation

$$[N, \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2] = 0$$

It was recently realized that this sort of theoretical frameworks a la kappa-Minkowski (with DSR-deformed relativistic laws) may be connected to an old idea advocated by Max Born

one of the first papers on the quantum gravity problem **was a paper by Max Born [***Proc.R.Soc.Lond.*A165,29(**1938**)**] centered on the dual role within quantum mechanics between momenta and spacetime coordinates (Born reciprocity)**

$$p_\mu \leftrightarrow x^\mu$$

Born argued that it might be impossible to unify gravity and quantum theory unless we make room for curvature of momentum space

this idea of curvature of momentum space had no influence on quantum-gravity research for several decades, until very recently

We now understand that momentum space for certain models based on spacetime noncommutativity is curved

For example there is a connection between the onshelleness of massless particles in the kappa-minkowski quantum spacetime and the form of worldlines of massless particles in classical deSitter spacetime (crossing the origin of the reference frame)

$$p_1 = \frac{1 - e^{-\ell p_0}}{\ell}$$

KAPPAMINKOWSKI SPACETIME

$$x^1 = \frac{1 - e^{-H x^0}}{H}$$

DESITTER SPACETIME

The kappa-Minkowski quantum spacetime has curved momentum space (lateshift)

GAC+Matassa+Mercati+Rosati, PhysicalReviewLetters106,071301 (2011)
GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)
Carmona+Cortes+Mercati, PhysRevD84,084010 (2011)
GAC, PhysicalReviewLetters111,101301 (2013)

there is "preliminary theoretical evidence" that the momentum space of Loop Quantum Gravity is also curved

and perhaps most importantly we learned that the only quantum gravity we actually can solve, which is 3D quantum gravity, definitely has curved momentum space

in 3D quantum gravity

see, e.g., Freidel+Livine,
PhysRevLett96,221301(2006)

consider a matter field ϕ coupled to gravity,

$$Z = \int Dg \int D\phi \, e^{iS[\phi,g]+iS_{GR}[g]}, \qquad (1)$$

where g is the space-time metric, $S_{GR}[g]$ the Einstein gravity action and $S[\phi, g]$ the action defining the dynamics of ϕ in the metric g.

integrate out the quantum gravity fluctuations and derive an *effective action* for ϕ taking into account the quantum gravity correction:

$$Z = \int D\phi e^{iS_{eff}[\phi]}.$$

the effective action obtained through this constructive procedure gives matter fields in a noncommutative spacetime (similar to, but not exactly given by, kappa-Minkowski) and with curved momentum space, as signalled in particular by the deformed on-shellness

(anti-deSitter momentum space)

$$\cos(E) - e^{\ell E} \frac{\sin(E)}{E} P^2 = \cos(m)$$

The notion of geometry of momentum space which is getting now used primarily connects the metric of momentum space to the on-shell relation (on-shell relation obtained as the geodesic distance of a momentum-space point from the origin) and connects the affine connection of momentum space to the law of composition of momenta (by describing parallel transport in terms of the law of composition)

This could have been just a futile "geometric interpretation" but it is proving useful

It establishes valuable similarities between different theories.

In particular theories with curved momentum spaces can still be relativistic, but this requires that momentum space is <u>maximally symmetric</u> (dS/anti-dS cases discussed above)

GAC,arXiv:11105081, PhysRevD85,084034

and the relativistic symmetries are a "deformation" of ordinary special-relativistic symmetries, examples of the above-mentioned <u>DSR-relativistic theories</u>

GAC, grqc0012051, IntJournModPhysD11,35
GAC, hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
Magueijo+Smolin,grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)

mass of a particle with four-momentum p_μ is determined by the metric geodesic distance on momentum space from p_μ to the origin of momentum space

$$m^2 = d_\ell^2\,(p,0) = \int dt \sqrt{g^{\mu\nu}(\gamma^{[A;p]}(t))\dot{\gamma}_\mu^{[A;p]}(t)\dot{\gamma}_\nu^{[A;p]}(t)}$$

where $\gamma^{[A;p]}{}_\mu$ is the metric geodesic connecting the point p_μ to the origin of momentum space

$$\frac{d^2\gamma_\lambda^{[A]}(t)}{dt^2} + A^{\mu\nu}{}_\lambda \frac{d\gamma_\mu^{[A]}(t)}{dt}\frac{d\gamma_\nu^{[A]}(t)}{dt} = 0$$

with $A^{\mu\nu}{}_\lambda$ the Levi-Civita connection

the affine connection on momentum space determines the law of composition of momenta, and it might not be the Levi-Civita connection of the metric on momentum space (it is not in 3D quantum gravity and in all cases based on noncommutative geometry, where momentum space is a group manifold)

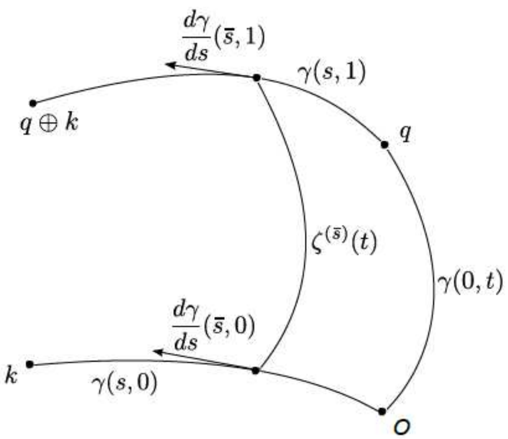


Figure 1. We determine the law of composition of momenta from the affine connection by associating to the points q and k of momentum space the connection geodesics $\gamma^{(q)}$ and $\gamma^{(k)}$ which connect them to the origin of momentum space. We then introduce a third curve $\bar{\gamma}(s)$, which we call the parallel transport of $\gamma^{(k)}(s)$ along $\gamma^{(q)}(t)$, such that for any given value $\bar{s}$ of the parameter s one has that the tangent vector $\frac{d}{ds}\bar{\gamma}(\bar{s})$ is the parallel transport of the tangent vector $\frac{d}{ds}\gamma^{(k)}(\bar{s})$ along the geodesic connecting $\gamma^{(k)}(\bar{s})$ to $\bar{\gamma}(\bar{s})$. Then the composition law is defined as the extremal point of $\bar{\gamma}$, that is $q\oplus_\ell k = \bar{\gamma}(1)$.

…and is proving valuable for phenomenology.

Much studied opportunity for phenomenology comes from fact that several pictures of quantum spacetime predict that the speed of photons is energy dependent.

Calculation of the energy dependence in a given model used to be lengthy and cumbersome. We now understand those results as dual redshift on Planck-scale-curved momentum spaces:

In particular,
ordinary redshift in deSitter spacetime implies in particular that massless particles emitted with same energy but at different times from a distant source reach the detector with different energy

dual redshift in deSitter momentum space implies that massless particles emitted simultaneously but with different energies from a distant source reach the detector at different times

GAC+Barcaroli+Gubitosi+Loret, Classical&QuantumGravity30,235002 (2013)
GAC+Matassa+Mercati+Rosati, PhysicalReviewLetters106,071301 (2011)

dual redshift on Planck-scale-curved momentum spaces produces time-of-arrival effects which at leading order are of the form (n$\in${1,2})

$$\Delta T = \left(\frac{E}{E_P} \right)^n T$$

and could be described in terms of an energy-dependent "physical velocity" of ultrarelativistic particles

$$\mathrm{v} = c + s_{\pm} \left(\frac{E}{E_P} \right)^n c$$

<u>these are very small effects but (at least for the case n=1) they could cumulate to an observably large ΔT if the distances travelled T are cosmological and the energies E are reasonably high (GeV and higher)!!!</u>

GRBs are ideally suited for testing this:
cosmological distances (established in 1997)
photons (and neutrinos) emitted nearly simultaneously
with rather high energies (GeV…..TeV…100 TeV…)

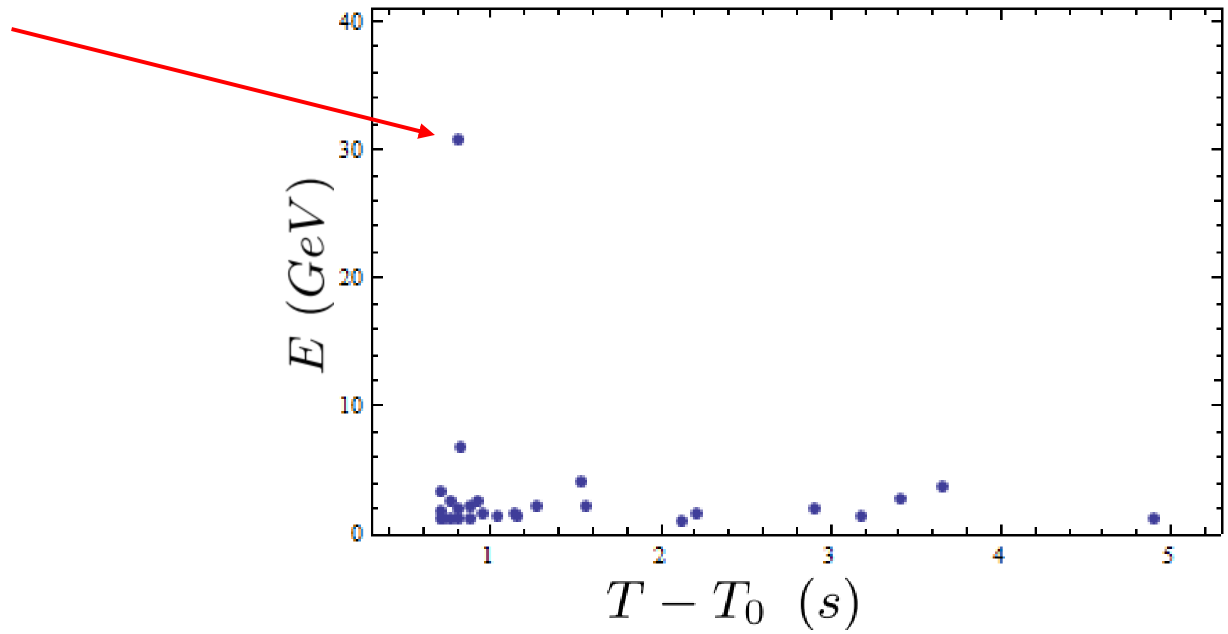
GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)
GAC, NaturePhysics10,254(2014)

focus on n=1 case (sensitivity to the n=2 case still far beyond our reach presently but potentially within reach of future neutrino astrophysics)

first came GRB080916C data providing a limit of $M_{QG}>10^{-1}M_{planck}$ for hard spectral lags and $M_{QG}>10^{-2}M_{planck}$ for soft spectral lags

analogous studies of blazars lead to comparable limits

then came GRB090510 (magnificent short burst) allowing to establish a limit at M_{planck} level on both signs of dispersion (soft and hard spectral lags)



a test with accuracy of about one part in 10^{20}!!!

this Planck-scale limit is illustrative of how we have learned over this past decade that there are ways for achieving in some cases sensitivity to Planck-scale-suppressed effects,
<u>something that was thought to be impossible up to the mid 1990s</u>

Quantum-Gravity Phenomenology exists!!!

a collection of other plausible quantum-gravity effects and of some associated data analyses where <u>Planck-scale sensitivity</u> was achieved (or is within reach) can be found in my "living review"

GAC, LivingRev.Relativity16,5(2013)

http://www.livingreviews.org/lrr-2013-5

part of this quantum-gravity phenomenology is low-energy phenomenology!!!!

First paper making this point was mentioned yesterday by Tino

GAC+Laemmerzahl+Mercati+Tino, PRL(2009)

remarkably in cold-atom interferometry a term of the form $\frac{mp}{2M_P}$

(loopQG) could be appreciated as a correction of one part in 10^9 to $\frac{p^2}{2m}$

other low-energy opportunities come from "infrared-ultraviolet mixing":
in several candidates for the formalization of quantum spacetime the renormalization group work differently…
Wilson decoupling works as usual only down to some characteristic low-energy scale M_*^2/M_{pl}

Also think of Hawking-Bekenstein black-hole entropy, scaling with area (rather than with volume as one would naivey expect)….
Black-hole entropy truly is a macroscopic problem!!!

working on quantum gravity one cannot avoid getting the feeling that Nature might have hidden very well some of its most fascinating secrets

still we have no other option but to keep looking

and maybe we are wrong and the secrets are not so well hidden