Probing deformed commutators with macroscopic harmonic oscillators

Francesco Marin

Phenomenological quantum gravity

- General ‘remark’: one cannot determine a position with an accuracy better than the Planck length **$L_P = \sqrt{hG/c^3} = 1.6\ 10^{-35}$ m**
- Generalized Heisenberg uncertainty relations (GUP)
- Generalized commutators between *p* e *q*
- Modified quantum physics

Detecting signatures of Planck scale-physics in highly-sensitive metrological systems

Outline

- Test of GUP and localization in a large mass oscillator
- Test of modified dynamics in mechanical oscillators

GUP and harmonic oscillator ground state

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \beta_0 \left(\frac{\Delta p}{M_p c}\right)^2\right)$$

$$x = \sqrt{\frac{\hbar}{m\omega_0}}\, X$$

$$p = \sqrt{\hbar m \omega_0}\, P$$

$$H = \frac{\hbar\omega_0}{2}\left(X^2 + P^2\right)$$

$$\Delta X \Delta P \geq \frac{1}{2}\left(1 + \beta \left(\Delta P\right)^2\right)$$

$$\beta = \beta_0 \frac{\hbar m \omega_0}{M_p^2 c^2}$$

GUP and harmonic oscillator ground state

$$H = \frac{\hbar\omega_0}{2}\left(X^2 + P^2\right)$$

$$\Delta X \Delta P \geq \frac{1}{2}\left(1 + \beta\left(\Delta P\right)^2\right)$$

$$\beta = \beta_0 \, \frac{\hbar m \omega_0}{M_p^2 c^2}$$

$$< X > = < P > = 0$$

$$E > \frac{\hbar\omega_0}{2}\left[\left(1 + \frac{\beta^2}{4}\right)(\Delta P)^2 + \frac{1}{4(\Delta P)^2} + \frac{\beta}{2}\right]$$

$$E_{min} = \frac{\hbar\omega_0}{2}\left[\sqrt{1 + \frac{\beta^2}{4}} + \frac{\beta}{2}\right] \simeq \frac{\hbar\omega_0}{2}\beta$$

GUP and cryogenic Weber bars

Higher ground state energy for a quantum oscillator

Test on low-temperature oscillators set limits

$$E_{min} < E_{exp} \quad \Rightarrow \quad \beta < \frac{2E_{exp}}{\hbar\omega_0} \quad \Rightarrow \quad \beta_0 < 2\,\frac{E_{exp}}{\hbar\,\omega_0}\,\frac{M_p}{m}\,\frac{M_p\,c^2}{\hbar\,\omega_0}$$

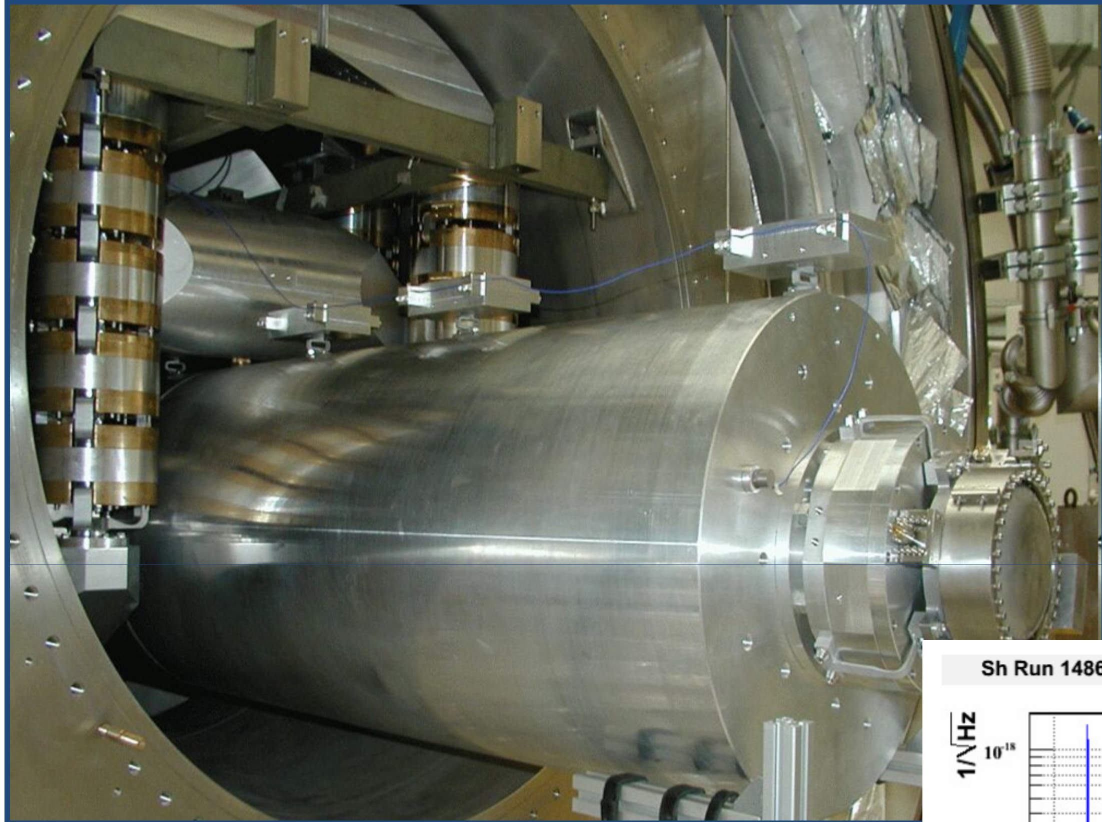
GUP effects expected to scale with the mass *m*

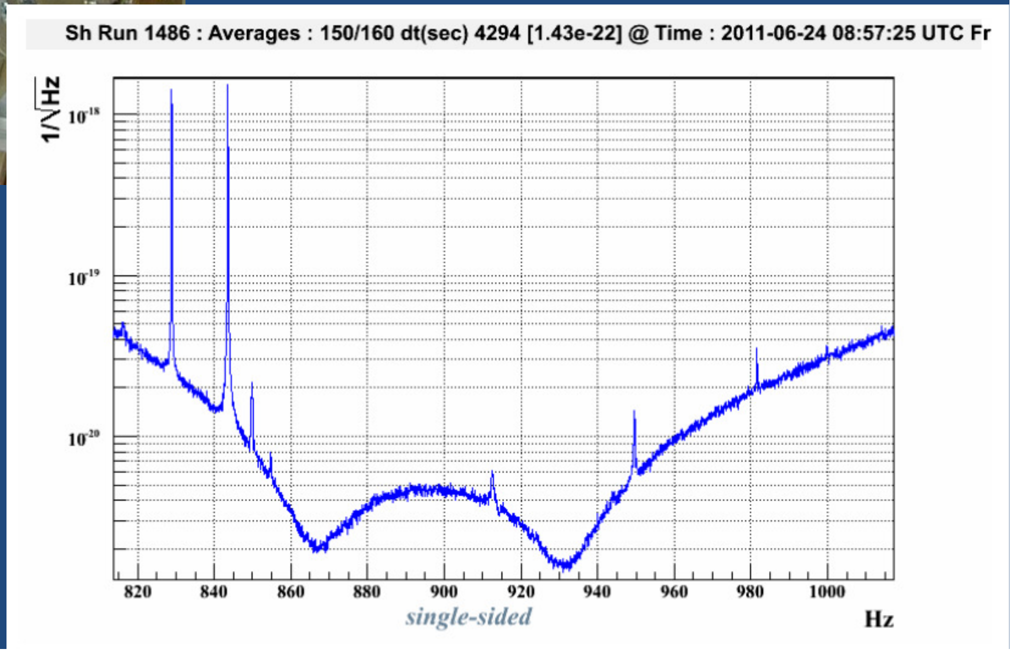
Massive cold oscillators

AURIGA

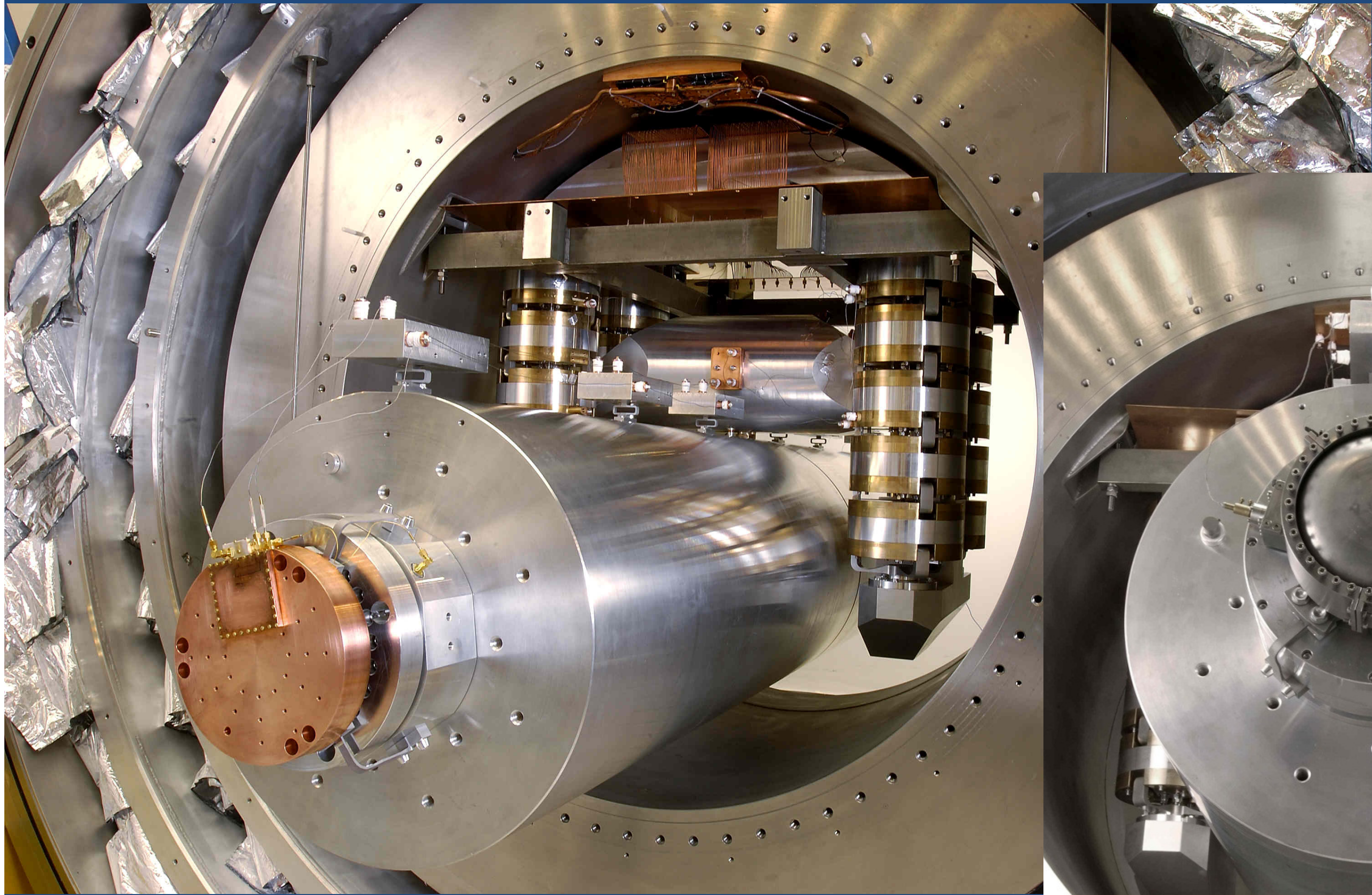
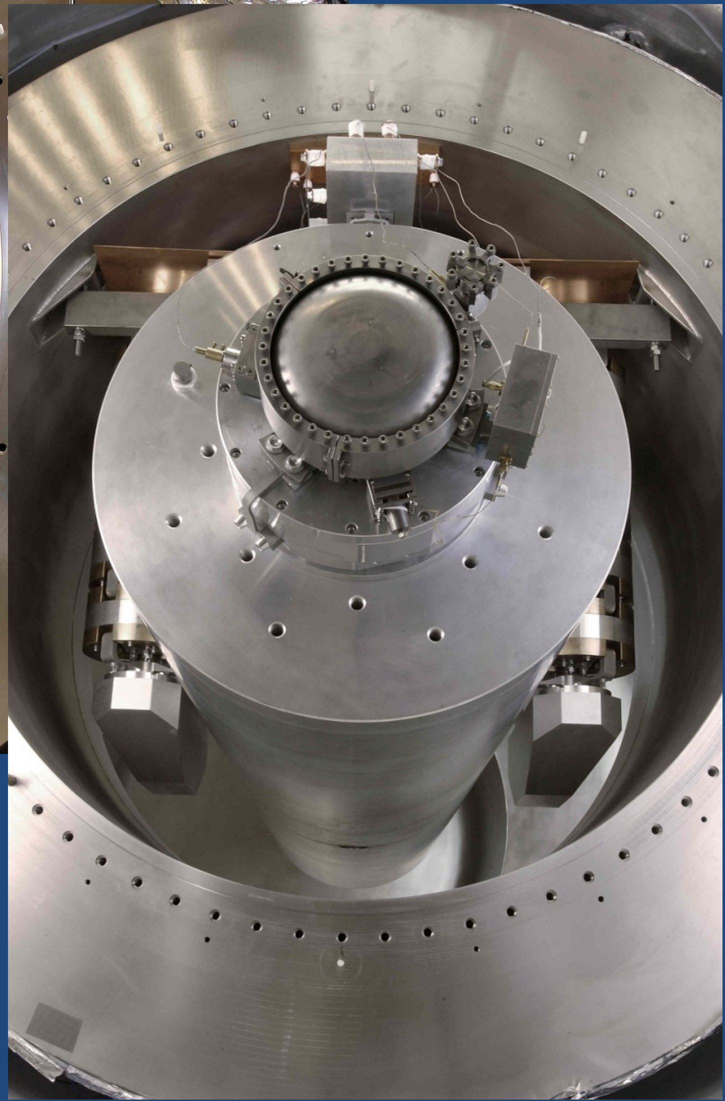
(Sub millikelvin cooling of ton-scale oscillator)

The AURIGA GW detector





- 3m long
- Al5056
- 2200 kg
- 4.5 K

Cooling down to the ground state

$$n_T = \frac{kT}{\hbar\omega_0}$$

(ω_0 = 1 GHz ⟶ T ≈ 50 mK)

Active or passive feedback cooling of one (few) oscillator mode

Cooling down to the ground state

$$n_T = \frac{kT}{\hbar\omega_0}$$

(ω_0 = 1 GHz → T ≈ 50 mK)

Active or passive feedback cooling of one (few) oscillator mode

$$M\left(-\omega^2 + \omega_0^2 - i\frac{\omega\omega_0}{Q}\right)\tilde{x} = \tilde{F} + G\,\tilde{x}$$

$$M\left(-\omega^2 + \omega_{eff}^2 - i\frac{\omega\omega_0}{Q_{eff}}\right)\tilde{x} = \tilde{F}$$

Cold damping

$$\omega_{eff}^2 = \omega_0^2 - \frac{Re\,G}{M} \qquad \frac{1}{Q_{eff}} = \frac{1}{Q} + \frac{Im\,G}{M\omega\omega_0}$$

$$\int S_x^{th} = \frac{kT_{eff}}{M\omega_0^2} \Longrightarrow T_{eff} = T\frac{Q_{eff}}{Q}$$

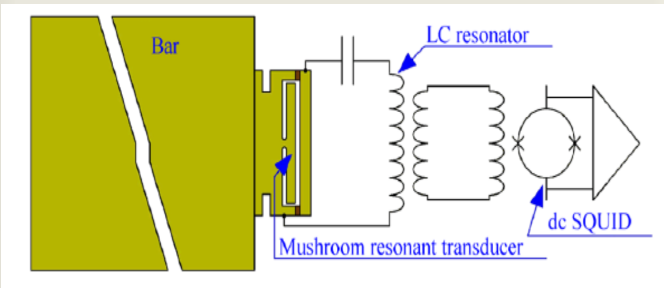
Cooling

Displacement sensitivity improvement ✗ **NO**

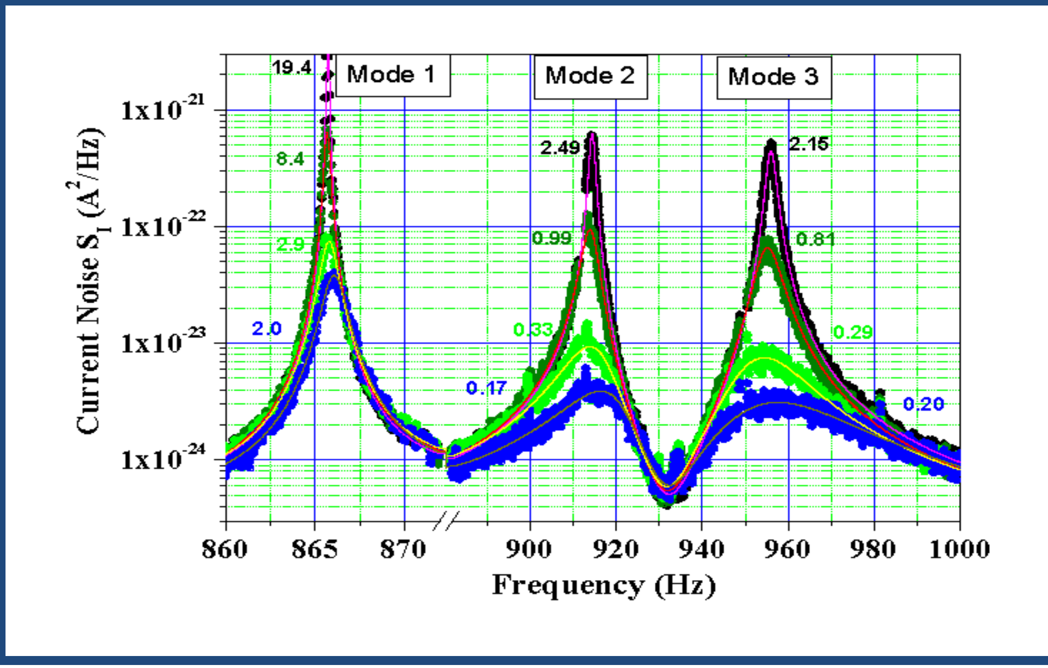
Prepare oscillator in its fundamental state ✓ **YES**

Selected for a Viewpoint in *Physics*
PRL **101,** 033601 (2008) PHYSICAL REVIEW LETTERS week ending 18 JULY 2008

Feedback Cooling of the Normal Modes of a Massive Electromechanical System to Submillikelvin Temperature



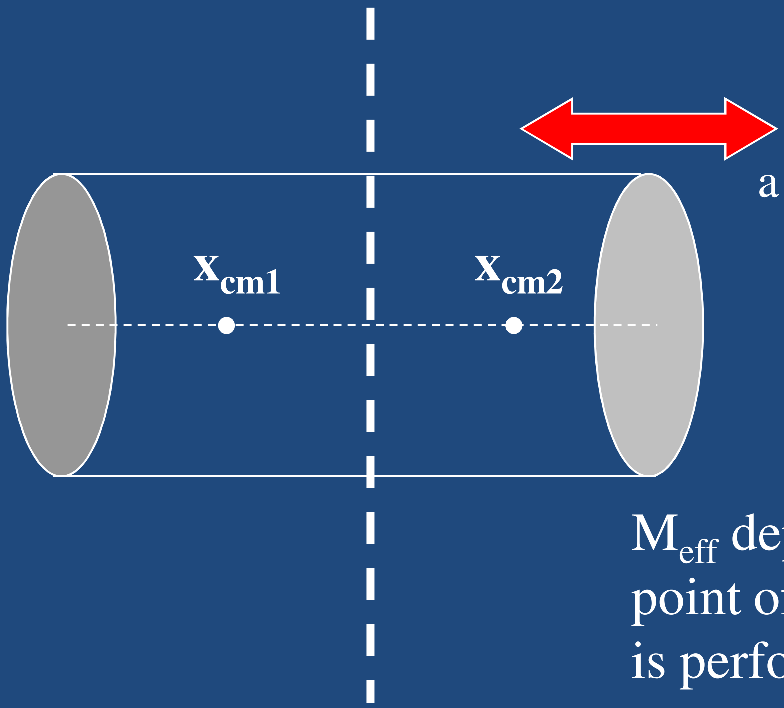
System of three coupled resonators: the bar and the transducer mechanical resonators and the LC electrical resonator



- At increasing feedback gain, the 3 modes of the detector reduce their vibration amplitude.
- The equivalent temperature of the vibration was reduced down to

T_{eff}=0.17 mK

Effective mass vs reduced mass



Readout measures the axial displacement of a bar face corresponding to the first longitudinal mode

$$M_{eff} = M/2$$

M_{eff} depends on the modal shape and interrogation point of the readout (e.g. $M_{eff} \rightarrow \infty$ if the measurement is performed on a node of the vibration mode)

Really moving mass

1) Modal motion implies an oscillation of each half-bar center-of-mass, to which is associated a *reduced mass* *M/2*

2) The energy associated to the oscillation of the couple of c.m.'s, having a reduced mass $M_{red} = M/2$ is about 80% of that of the modal motion

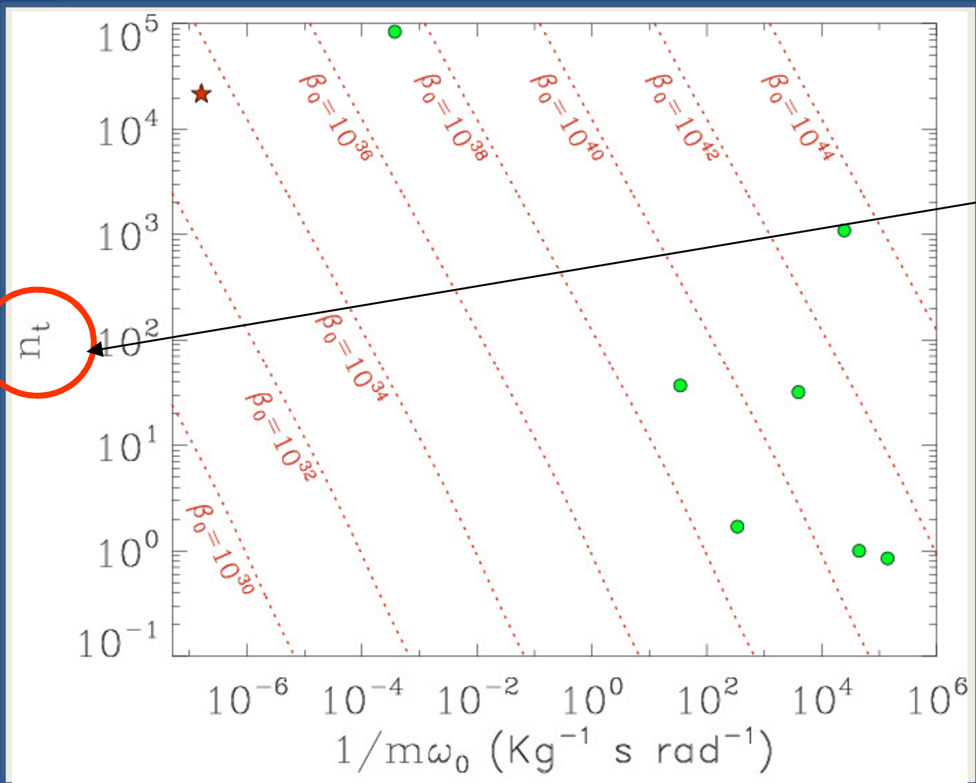
AURIGA minimal energy

$$M_{red} = M/2 = 1.1 \times 10^3 \text{ kg}$$

$$\omega_0 = 900 Hz$$

$$E_{exp} = 1.3 \times 10^{-26} \text{ J}$$

$$\beta < 4.4 \times 10^4 \quad \Rightarrow \quad \beta_0 < 3 \times 10^{33}$$



$$E_{exp} = \hbar\omega_0 (1/2 + n_t)$$

$$n_t = \frac{1}{e^{\frac{\hbar\omega_0}{k_B T_{eff}}} - 1}$$

nature physics LETTERS

PUBLISHED ONLINE: 16 DECEMBER 2012 | DOI: 10.1038/NPHYS2503

Gravitational bar detectors set limits to Planck-scale physics on macroscopic variables

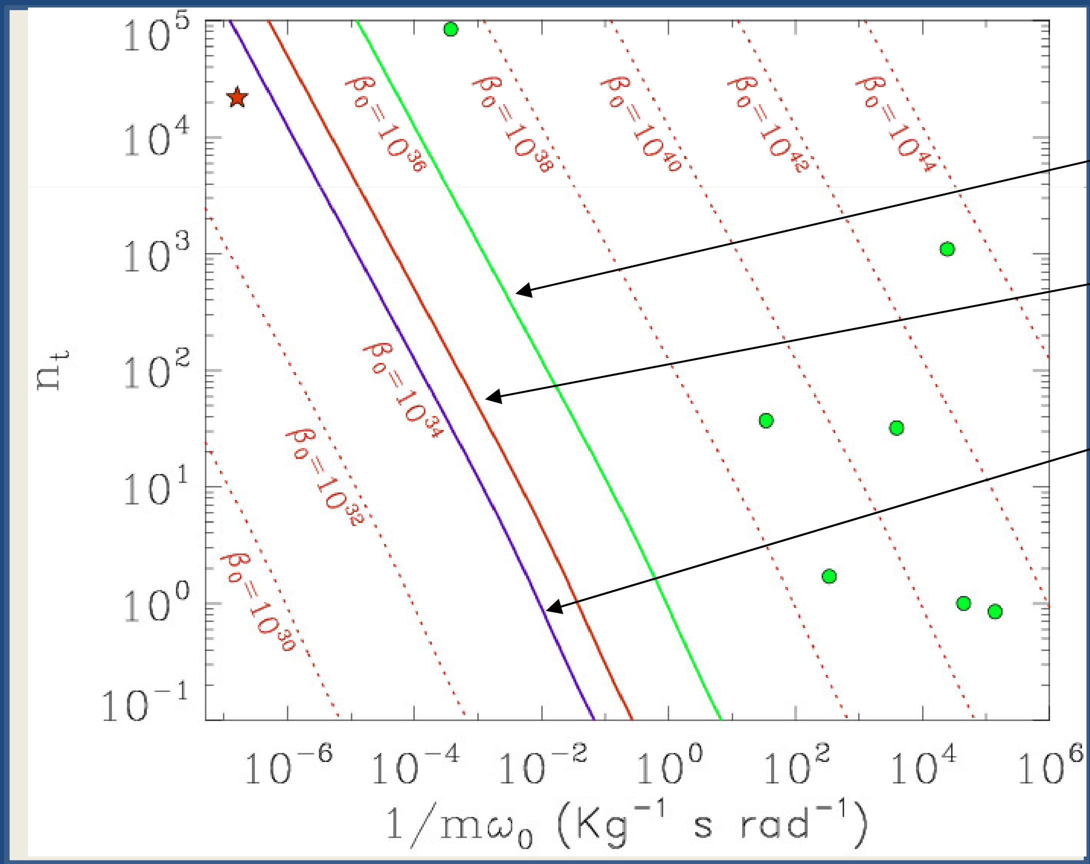
Francesco Marin[1,2,3]*, Francesco Marino[3,4], Michele Bonaldi[5,6], Massimo Cerdonio[7], Livia Conti[7], Paolo Falferi[6,8], Renato Mezzena[6,9], Antonello Ortolan[10], Giovanni A. Prodi[6,9], Luca Taffarello[7], Gabriele Vedovato[7], Andrea Vinante[8,11] and Jean-Pierre Zendri[7]

Modified commutators I

GUP can be associated to a deformed canonical commutator

$$[x,p] = i\hbar\left(1+\beta_0\left(\frac{p}{M_pc}\right)^2\right)$$

Planck scale modifications of the energy spectrum of quantum systems



Lamb shift in hydrogen atoms

1S-2S level energy difference in hydrogen

Lack of observed deviations from theory at the electroweak scale

Modified commutators II

Modifications of commutators are not unique

Experiments could distinguish between the various approaches

$$[x,p] = i\hbar\sqrt{1 + 2\mu_0 \frac{(p/c)^2 + m^2}{M_p^2}}$$

M. Maggiore Phys. Lett. B **319**, 83-86 (1993)

$$[x,p] = i\tilde{\hbar}$$

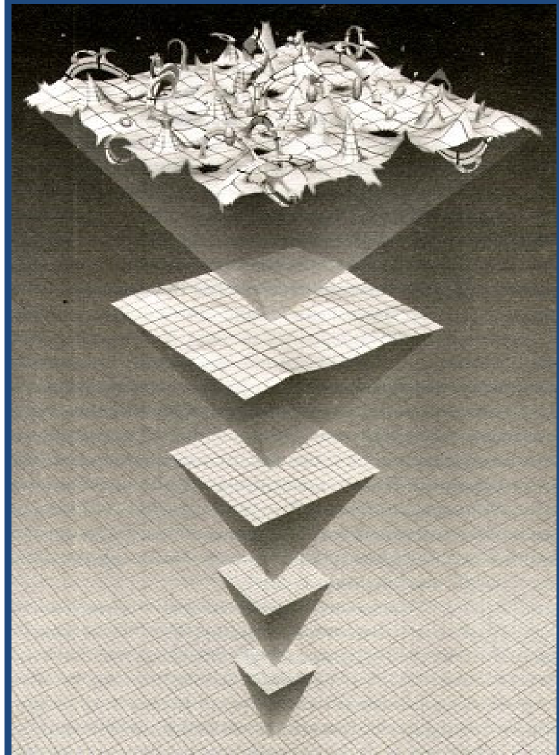
$$\tilde{\hbar} \simeq \hbar\sqrt{1 + 2\mu_0 M^2/M_p^2}$$

$$E_{min} = \frac{1}{2}\tilde{\hbar}\omega_0 \; < E_{exp} \; = 1.3 \times 10^{-26} \text{ J}$$

$\mu_0 < 4 \times 10^{-13}$

F. Marin *et al.*, New J. Phys. **16**, 085012 (2014)

Spacetime granularity (Quantum Foam)



General Relativity → Mass (energy) curves spacetime

Quantum mechanics → Vacuum energy

⇓

Energy of the virtual particles gives space time a "foamy" character at $L \approx L_p$

(Wheeler, 1955)

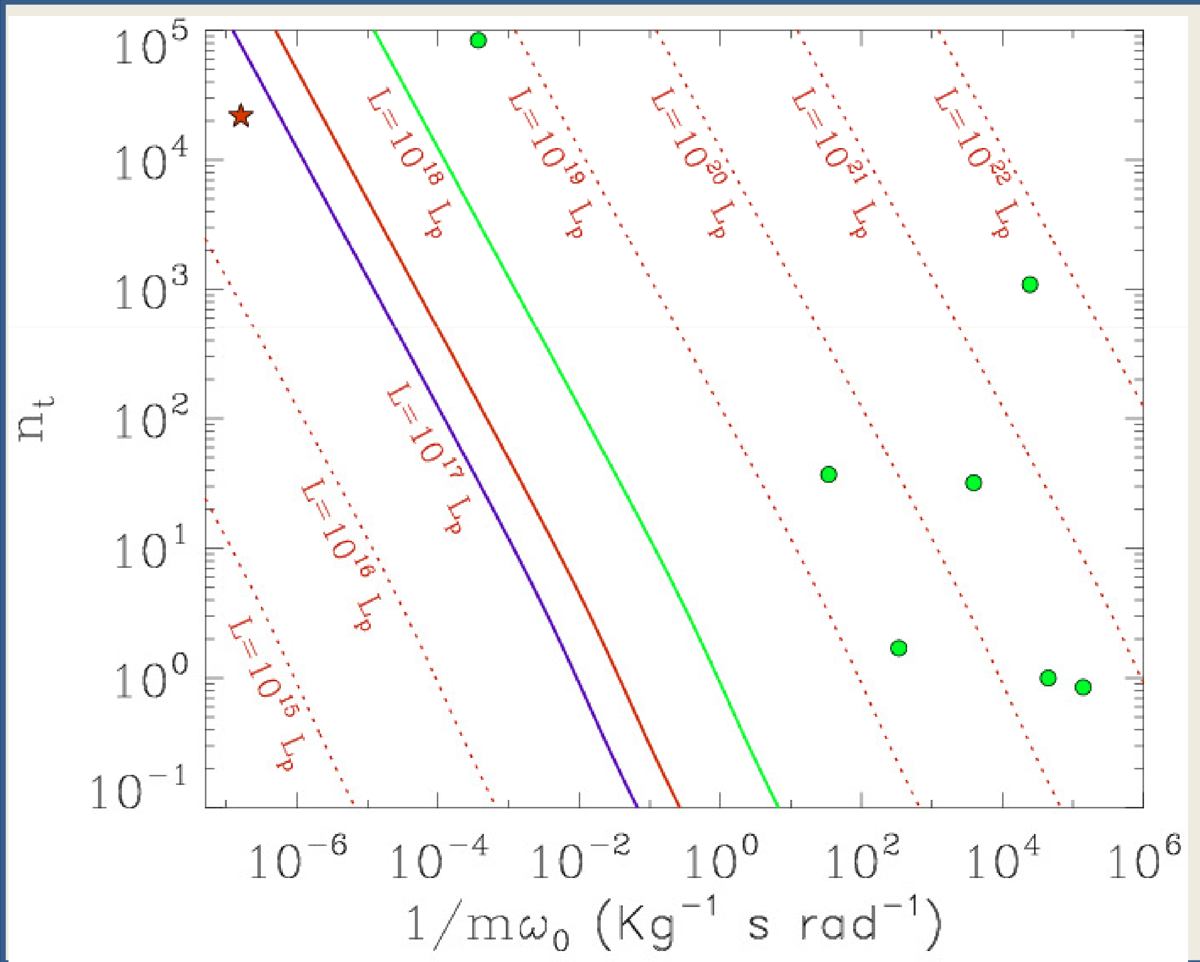
Property of the spacetime geometry and not of physical objects
(Soccer ball problem ?)

Apparatus independent (not based on a specific QG model)

AURIGA: re-interpretation

AURIGA is not the "coolest" oscillator, but is the most motionless

$X_{rms} = (kT/m\omega^2)^{1/2} = (E_{exp}/m\omega^2)^{1/2} \approx 6 \times 10^{-19}$ m $\xrightarrow{T = 0.1K}$ 6×10^{-18} m



$$L = \sqrt{\beta_0} L_p$$

F. Marin *et al.*, New J. Phys. **16**, 085012 (2014)

Outline

- Test of GUP and localization in a large mass oscillator
- Test of modified dynamics in mechanical oscillators

Basic assumptions:

$$\frac{d\hat{O}}{dt} = \frac{1}{i\hbar}[\hat{O}, H]$$

Heisenberg dynamics

$$[x, p] = i\hbar\left(1 + \beta_0\left(\frac{p}{M_p c}\right)^2\right)$$

Deformed commutation relations

$$x = \sqrt{\frac{\hbar}{m\omega_0}}\,X$$

$$p = \sqrt{\hbar m\omega_0}\,P$$

$$\beta = \beta_0\,\frac{\hbar m\omega_0}{M_p^2 c^2}$$

$\Rightarrow$

$$[X, P] = i\left(1 + \beta P^2\right)$$

$$P = \left(1 + \frac{1}{3}\beta\tilde{P}^2\right)\tilde{P}$$

$$H = \frac{\hbar\omega_0}{2}\left(X^2 + \tilde{P}^2\right) + \frac{\hbar\omega_0}{3}\beta\tilde{P}^4$$

See, e.g., C. Quesne and V.M. Tkachuk, Phys. Rev. A **81**, 012106 (2010)

$$\begin{cases} \dot{X} = \omega_0 \tilde{P}\left(1 + \frac{4}{3}\beta\tilde{P}^2\right) \\ \dot{\tilde{P}} = -\omega_0 X \end{cases} \quad \Longrightarrow \quad \ddot{\tilde{P}} + \omega_0^2 \tilde{P} + \frac{4}{3}\beta\omega_0^2 \tilde{P}^3 = 0$$

Poincaré solution for $\tilde{P}$, then find X

$$X = X_0\left[\sin(\tilde{\omega}t) + \frac{\beta}{8}X_0^2 \sin(3\,\tilde{\omega}t)\right]$$

3° harmonic

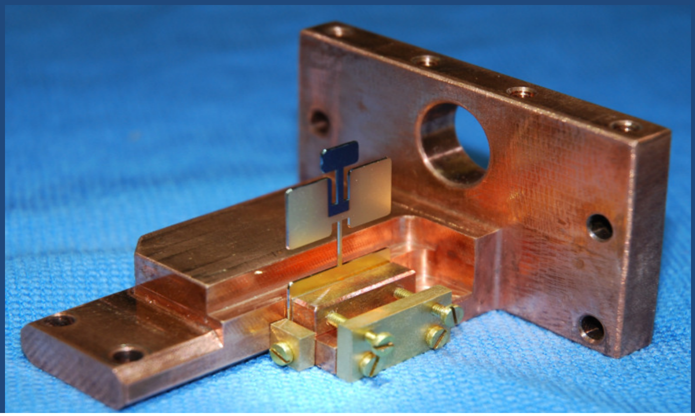
$$\tilde{\omega} = \left(1 + \frac{\beta}{2}X_0^2\right)\omega_0$$

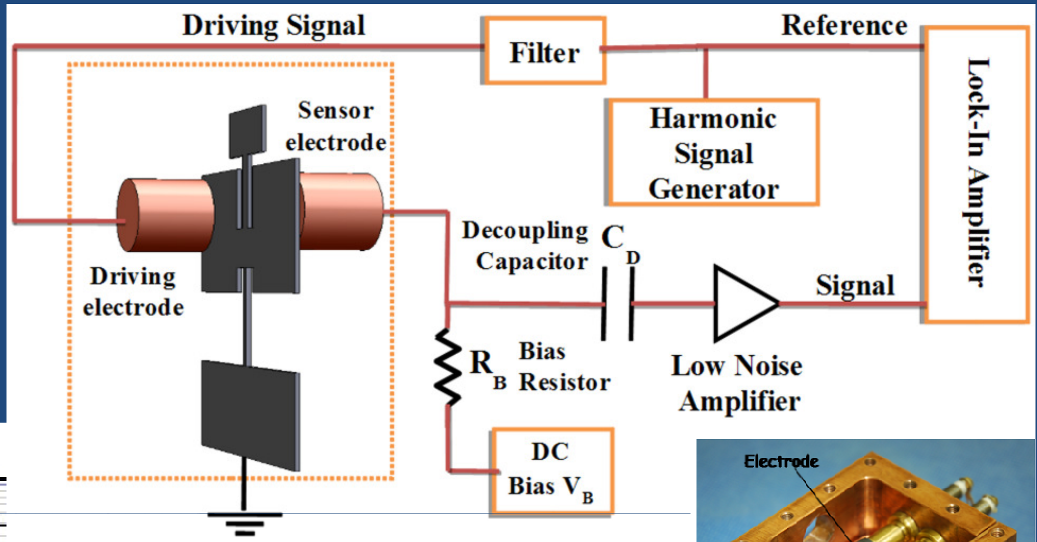
Freq. shift

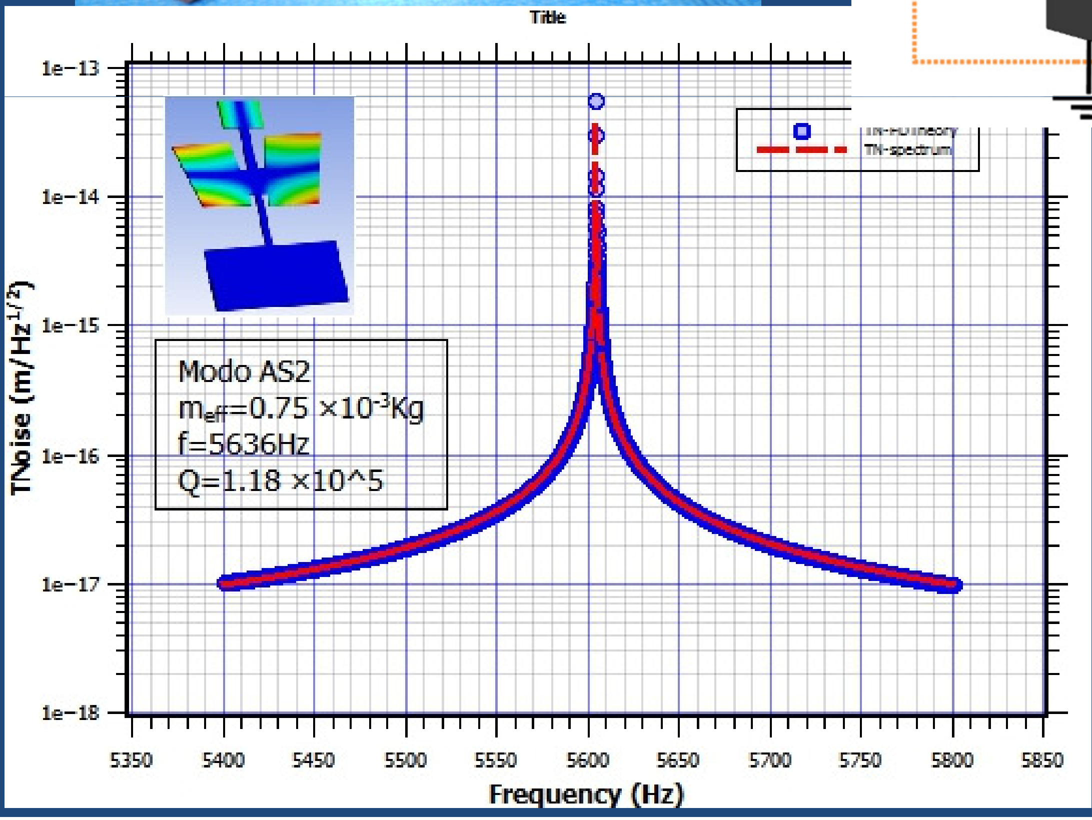
(First order in βX_0^2)

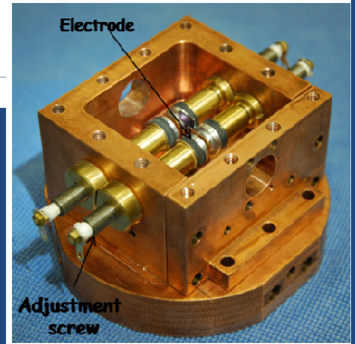
- Test on a wide mass range
- High mechanical quality factor $\rightarrow$ ‘isolated’ oscillators
- Exploit the slow decay to obtain frequency/3° harmonic vs amplitude curves

1° oscillator: $m \cong 1$ g

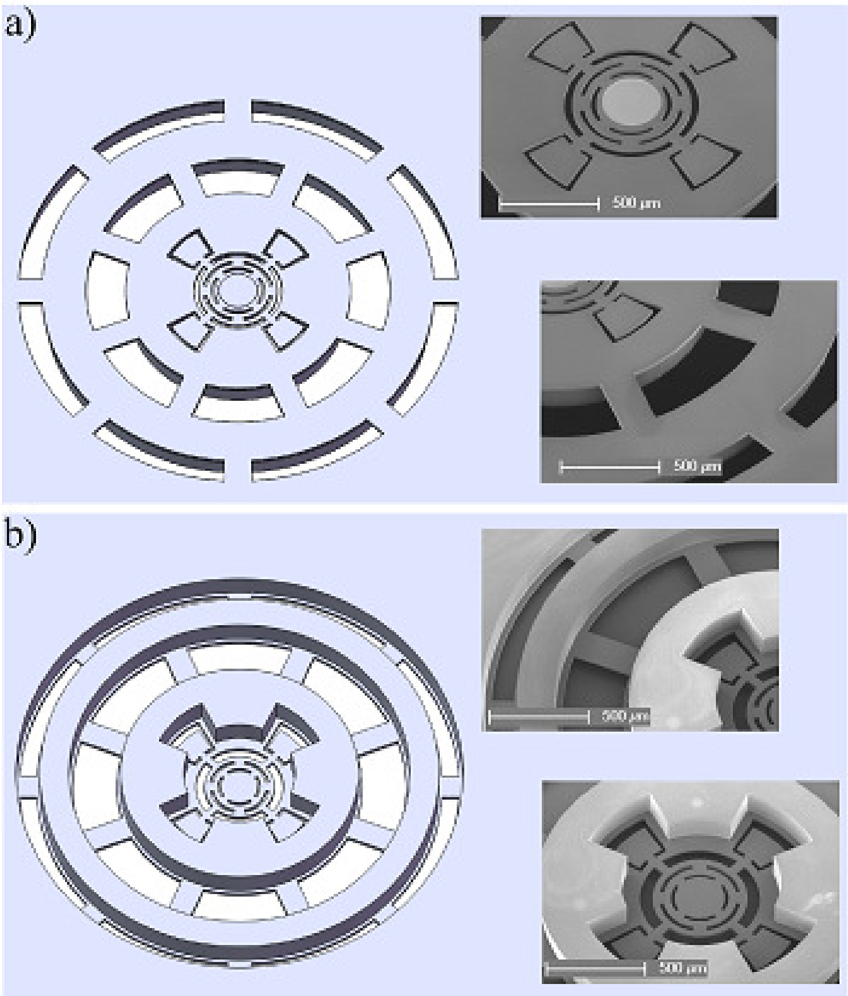








2° oscillator: m ≅ 100 μg

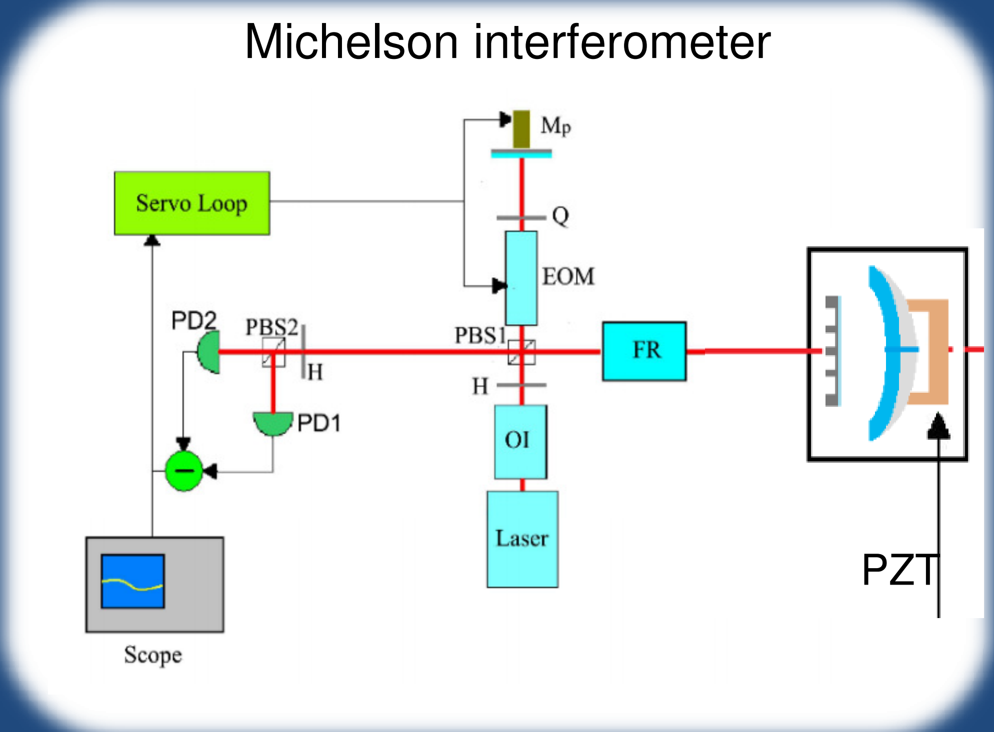


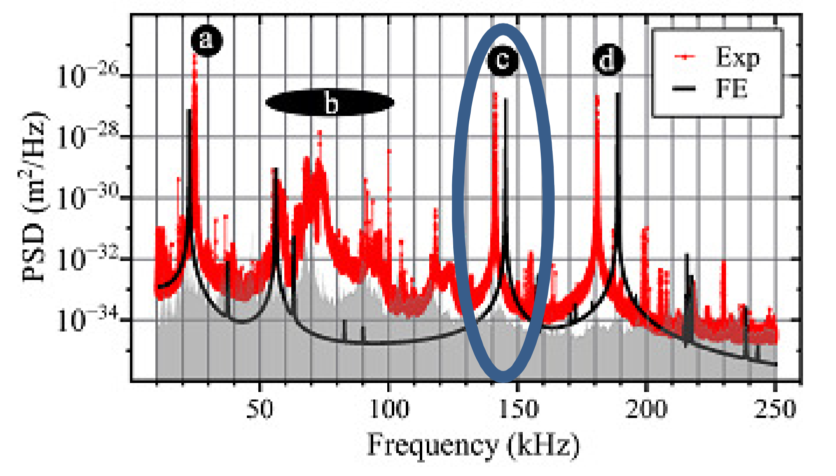
m = 20 μg

f_m = 141 KHz

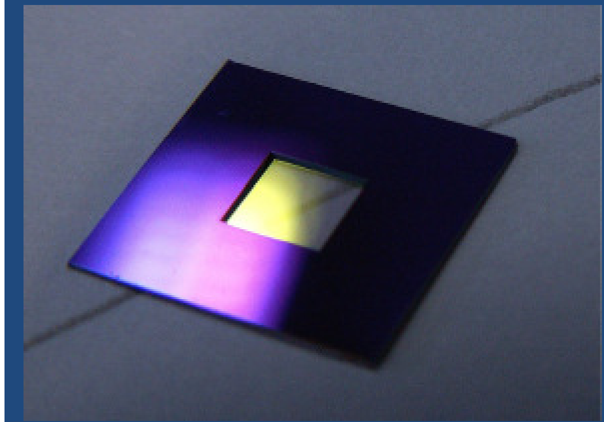
Q = 1.2 x10^6

T = 4.3 K

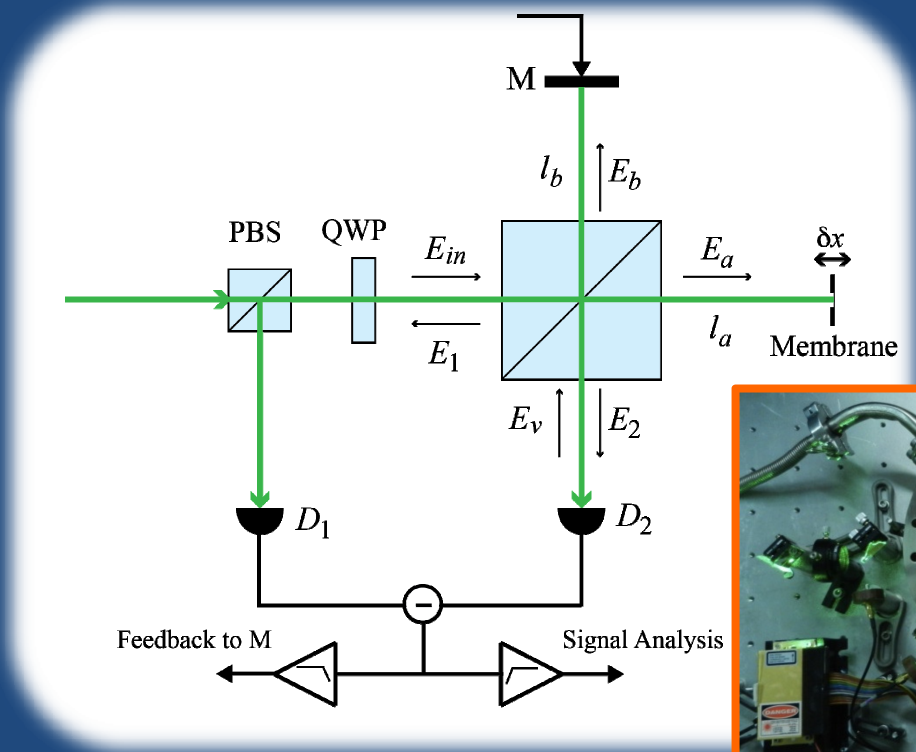


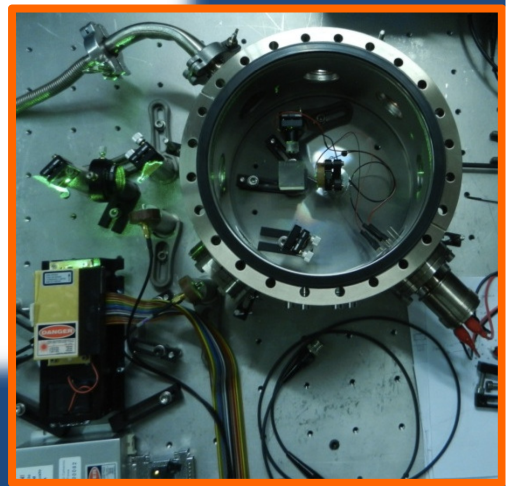


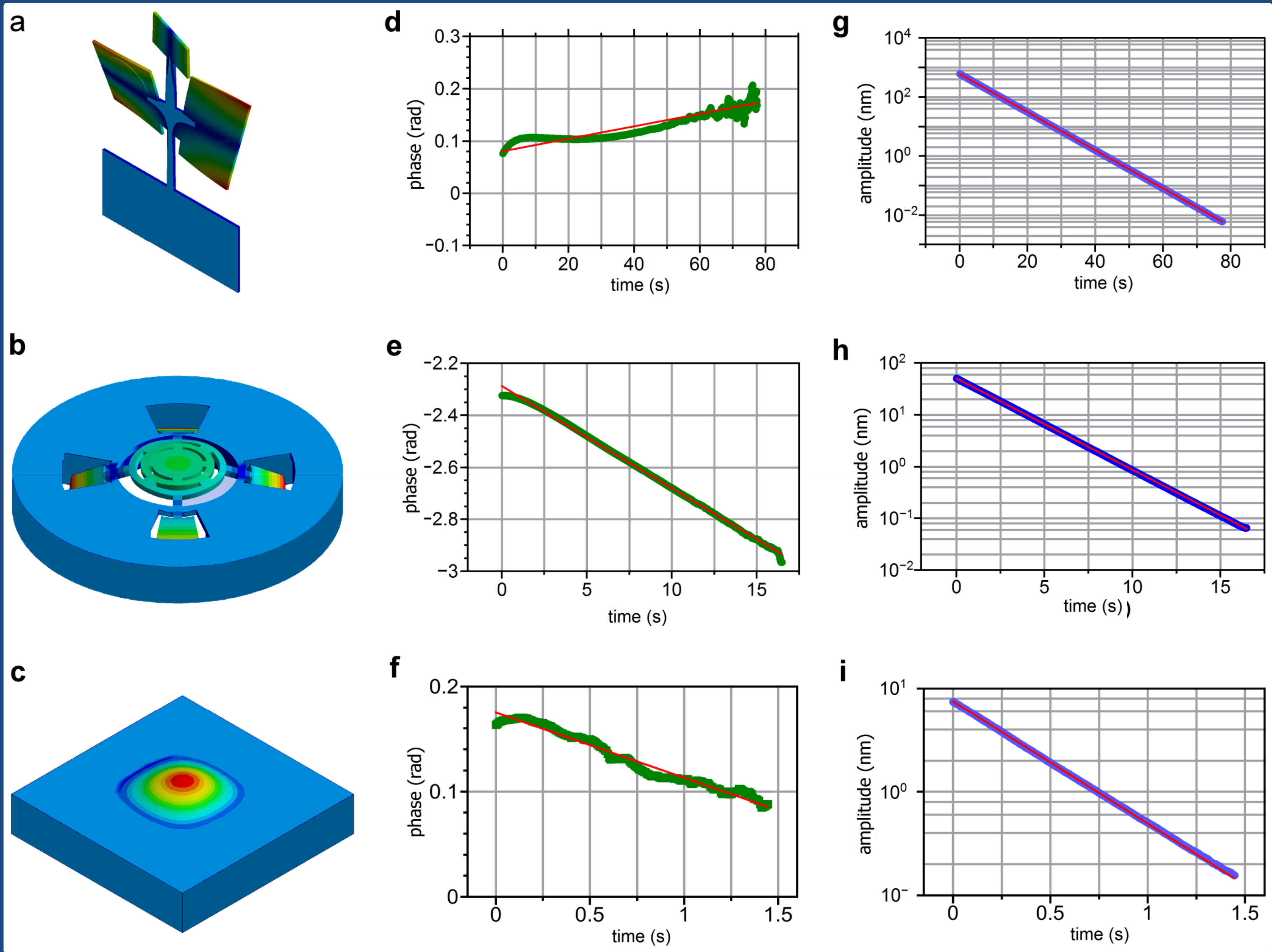
3° oscillator: m ≅ 100 ng

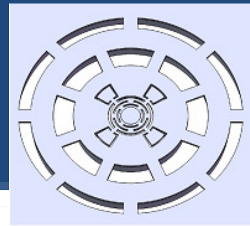


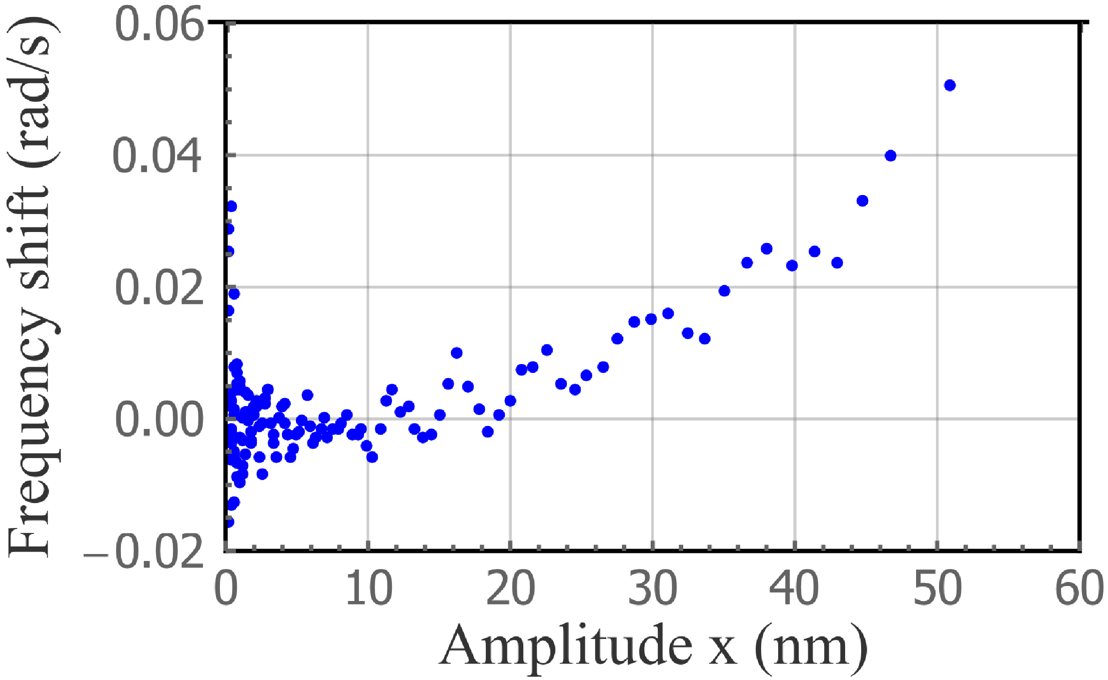
SiN membrane
0.5 x 0.5 mm^2 x 50nm
mass = 135 ng
Q = 23000

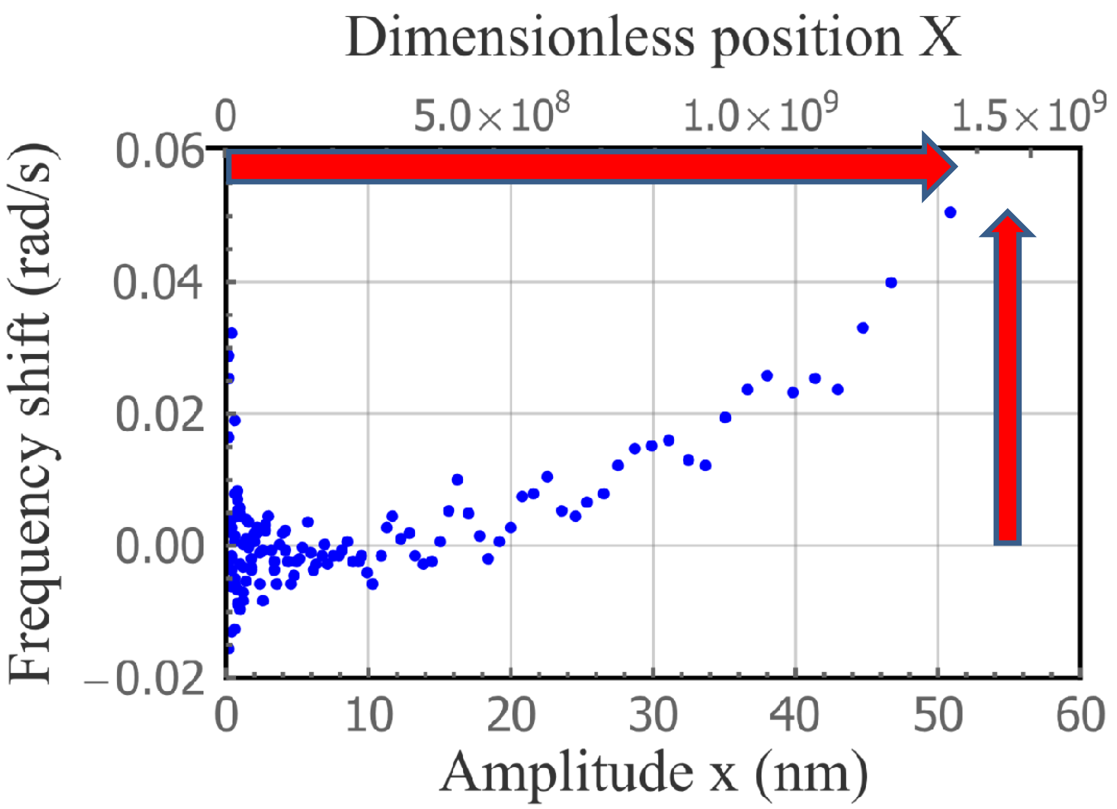


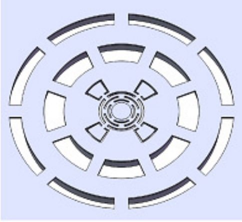



a
b
c
d
e
f
g
h
i
phase (rad)
amplitude (nm)
time (s)



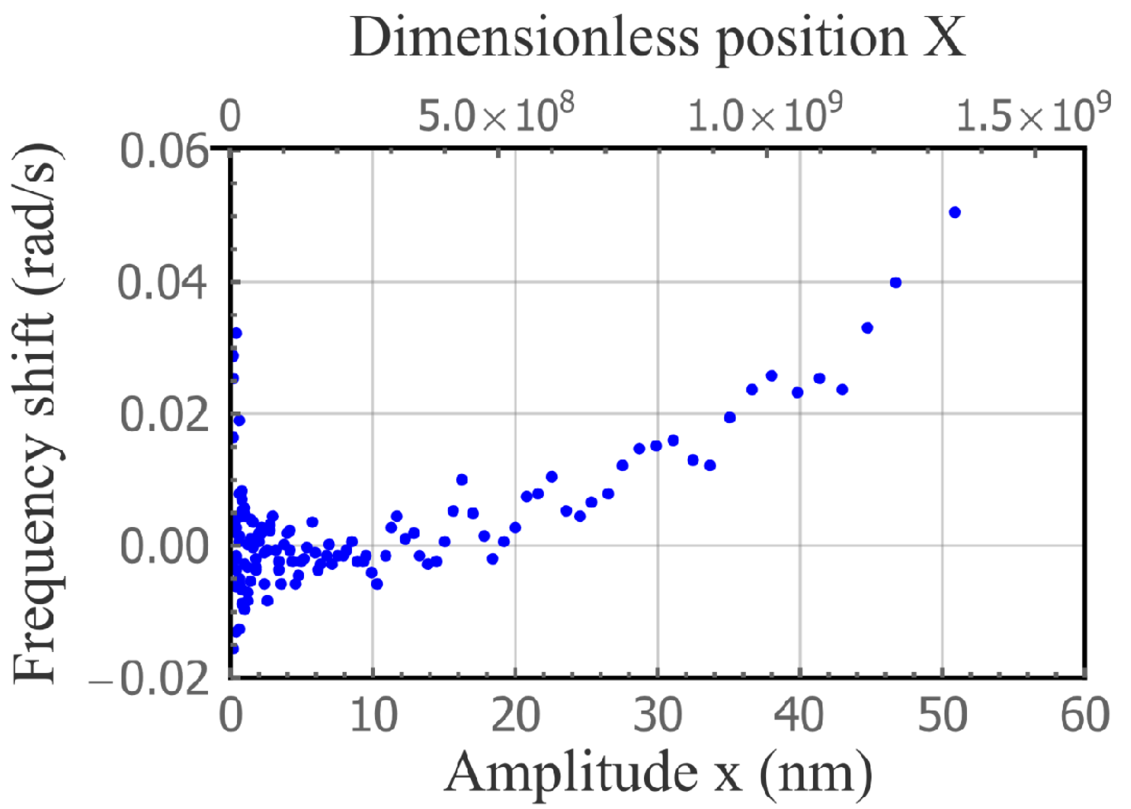

Frequency shift (rad/s)
0.06
0.04
0.02
0.00
−0.02
0
10
20
30
40
50
60
Amplitude x (nm)


Dimensionless position X
0
5.0×10^8
1.0×10^9
1.5×10^9
Frequency shift (rad/s)
0.06
0.04
0.02
0.00
−0.02
0
10
20
30
40
50
60
Amplitude x (nm)

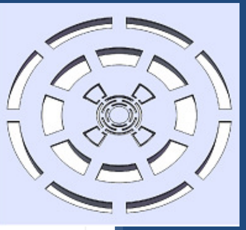


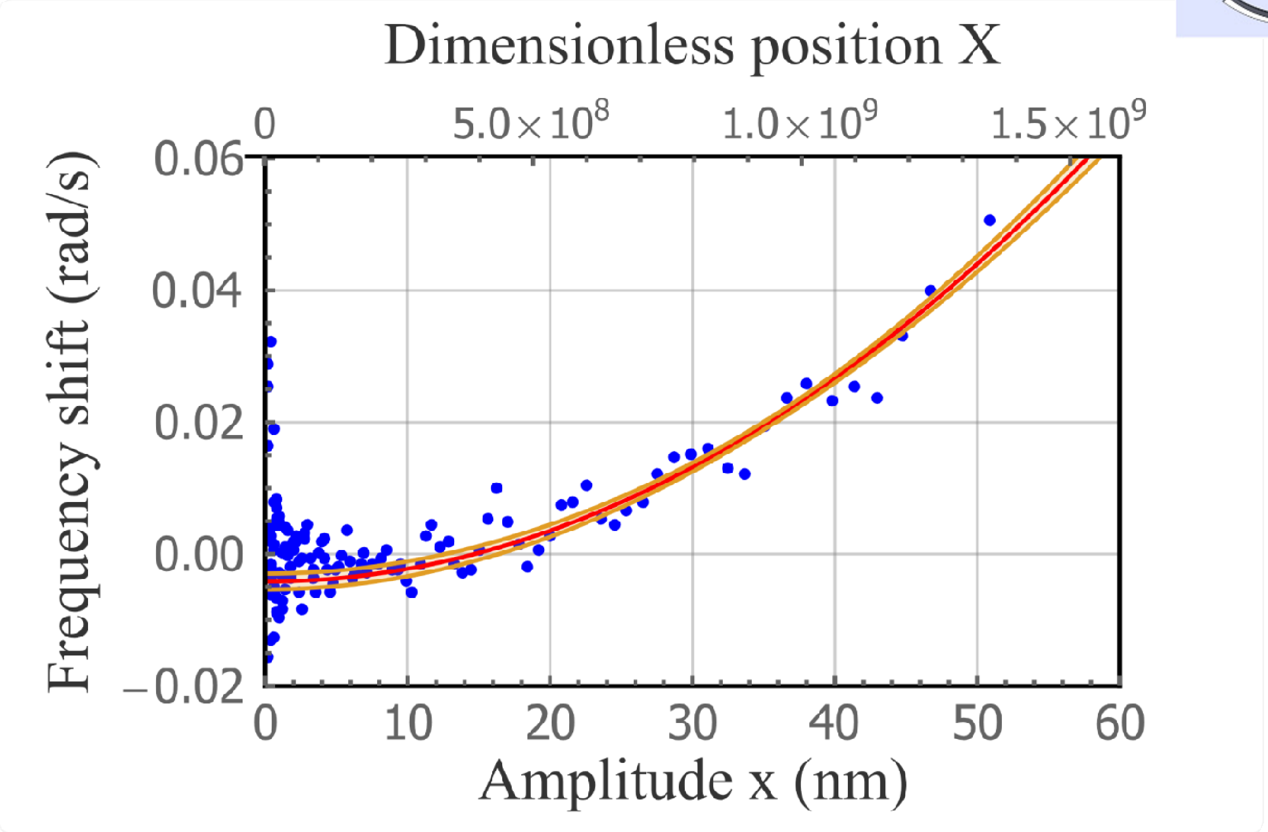
'Model-independent' limits

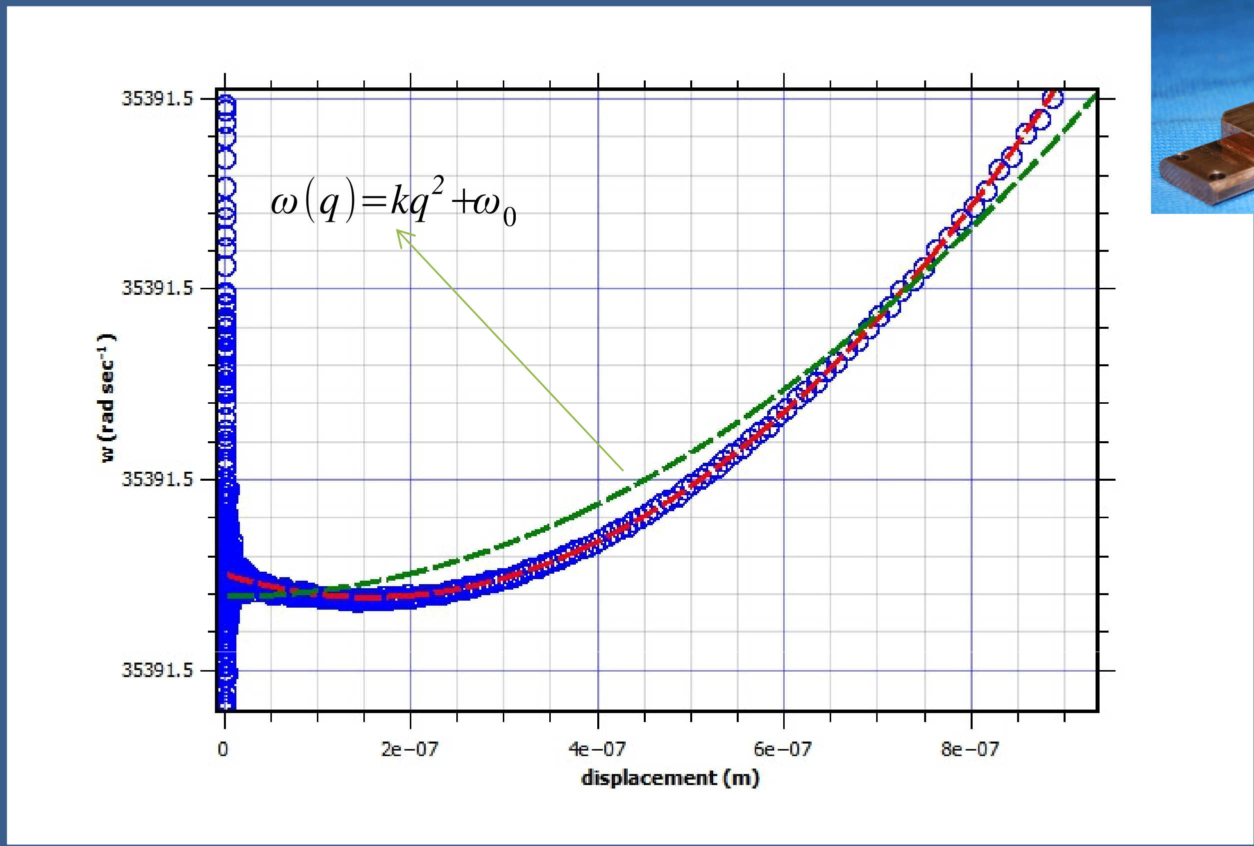
Mass (kg)	Frequency (Hz)	Max. ampl. (nm)	Max. Q_0	Max. $\Delta\omega/\omega_0$
3.3×10^{-5}	5.64×10^{3}	600	6×10^{10}	4×10^{-7}
2×10^{-8}	1.42×10^{5}	55	7×10^{8}	6×10^{-8}
2×10^{-11}	7.47×10^{5}	7.5	7×10^{6}	4×10^{-8}

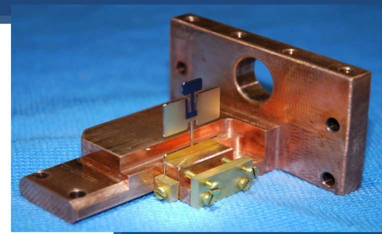

Dimensionless position X
0
5.0×10^8
1.0×10^9
1.5×10^9
0.06
0.04
0.02
0.00
−0.02
Frequency shift (rad/s)
0
10
20
30
40
50
60
Amplitude x (nm)

$$\tilde{\omega} = \left(1 + \frac{\beta}{2} X_0^2\right)\omega_0$$



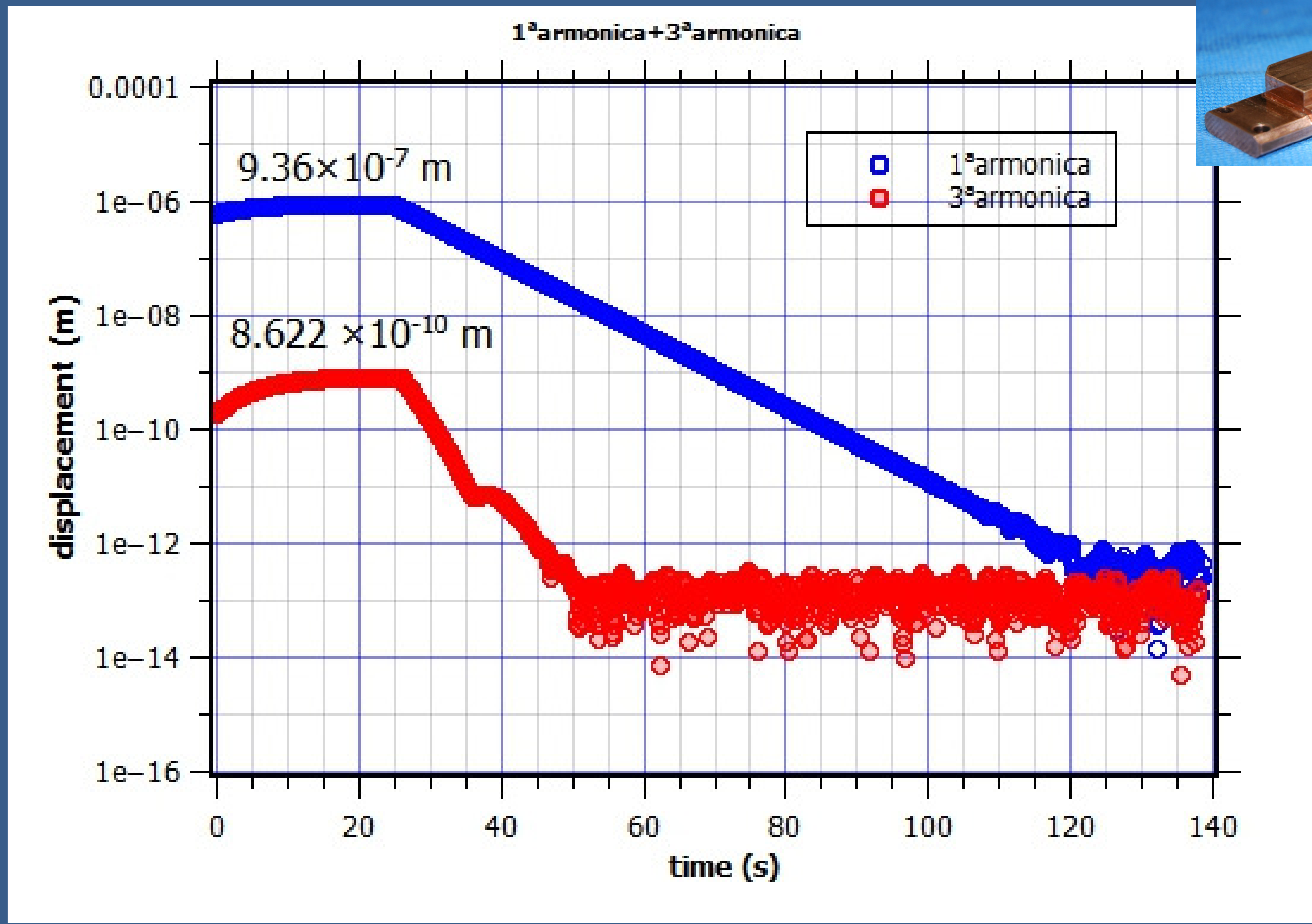


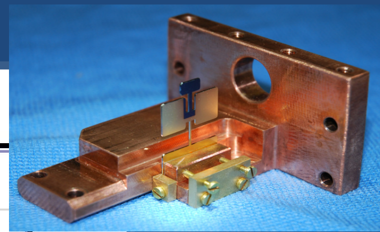




NB: stiffening in clamped-free microcantilevers has been observed in AFM tips.
Hardening geometric nonlinearity

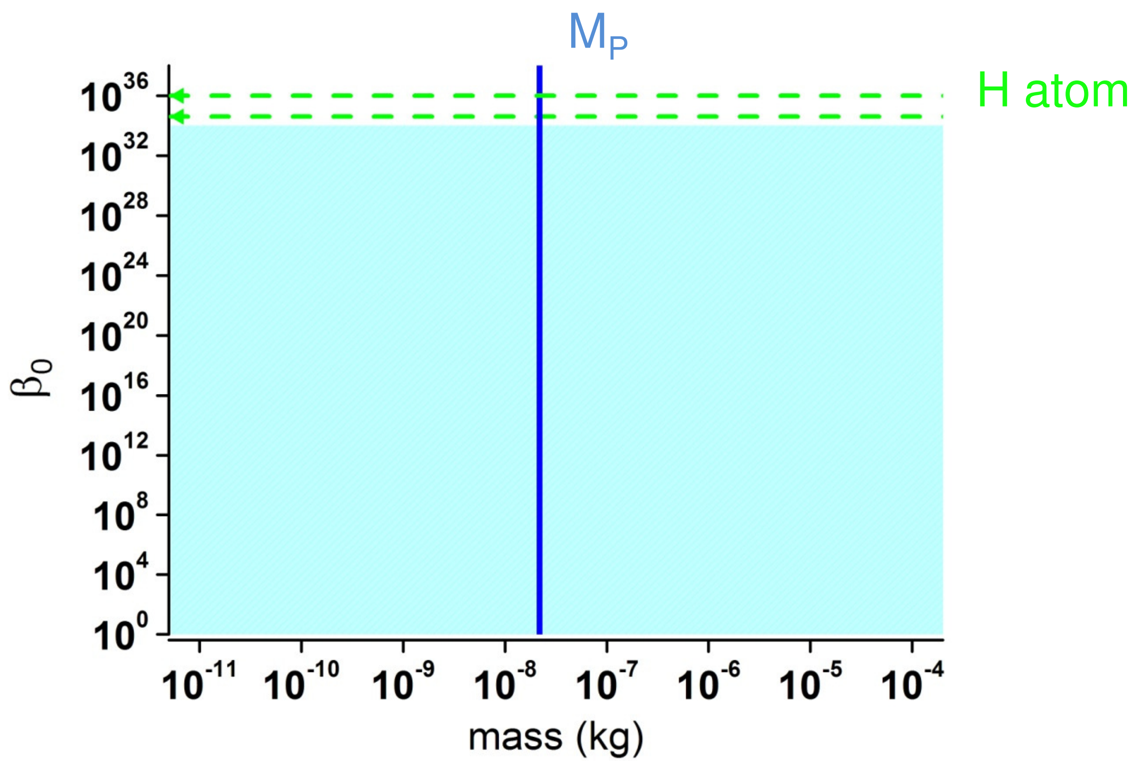
$$X = X_0\left[\sin(\tilde{\omega}t) + \frac{\beta}{8}X_0^2 \sin(3\,\tilde{\omega}t)\right]$$



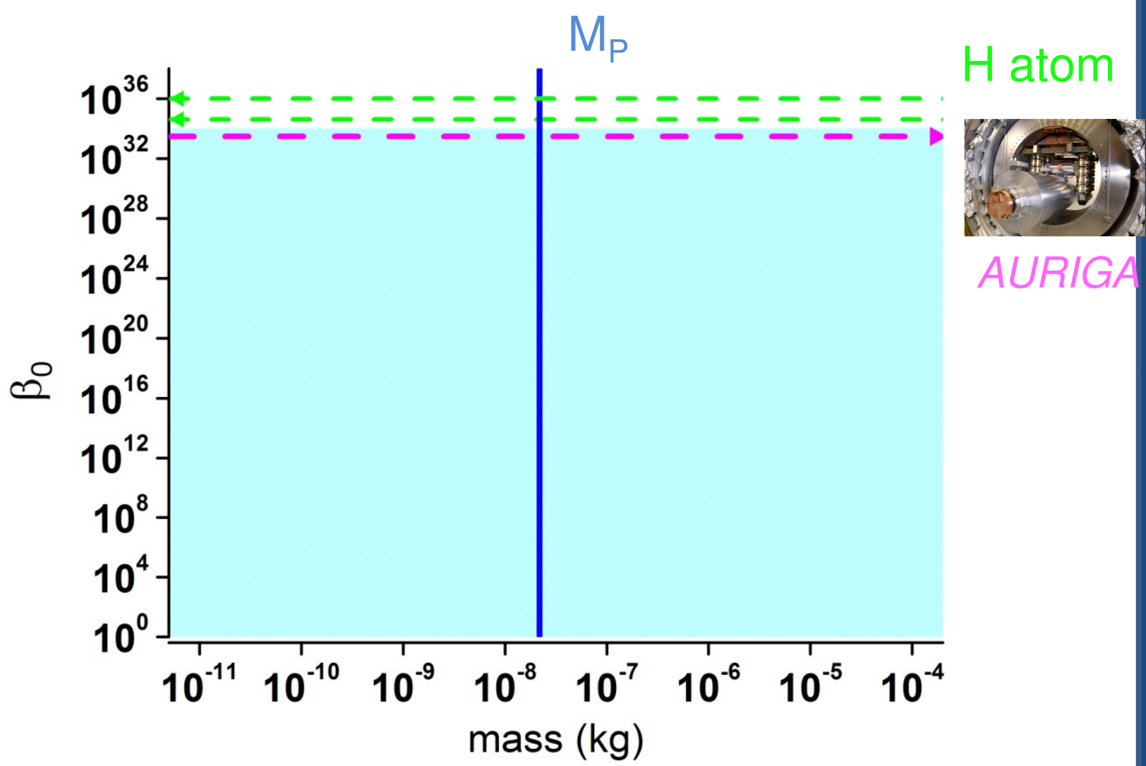


Mass (kg)	Frequency (Hz)	β	β_0	indicator
3.3×10^{-5}	5.64×10^{3}	7×10^{-29}	3×10^{7}	$\Delta\omega$
"	"	7×10^{-25}	2×10^{11}	3rd harmonic
7.7×10^{-8}	1.29×10^{5}	8×10^{-24}	5×10^{13}	$\Delta\omega$
"	"	2×10^{-19}	2×10^{18}	3rd harmonic
2×10^{-8}	1.42×10^{5}	3×10^{-25}	6×10^{12}	$\Delta\omega$
2×10^{-11}	7.47×10^{5}	4×10^{-21}	2×10^{19}	$\Delta\omega$
"	"	2×10^{-14}	1×10^{26}	3rd harmonic

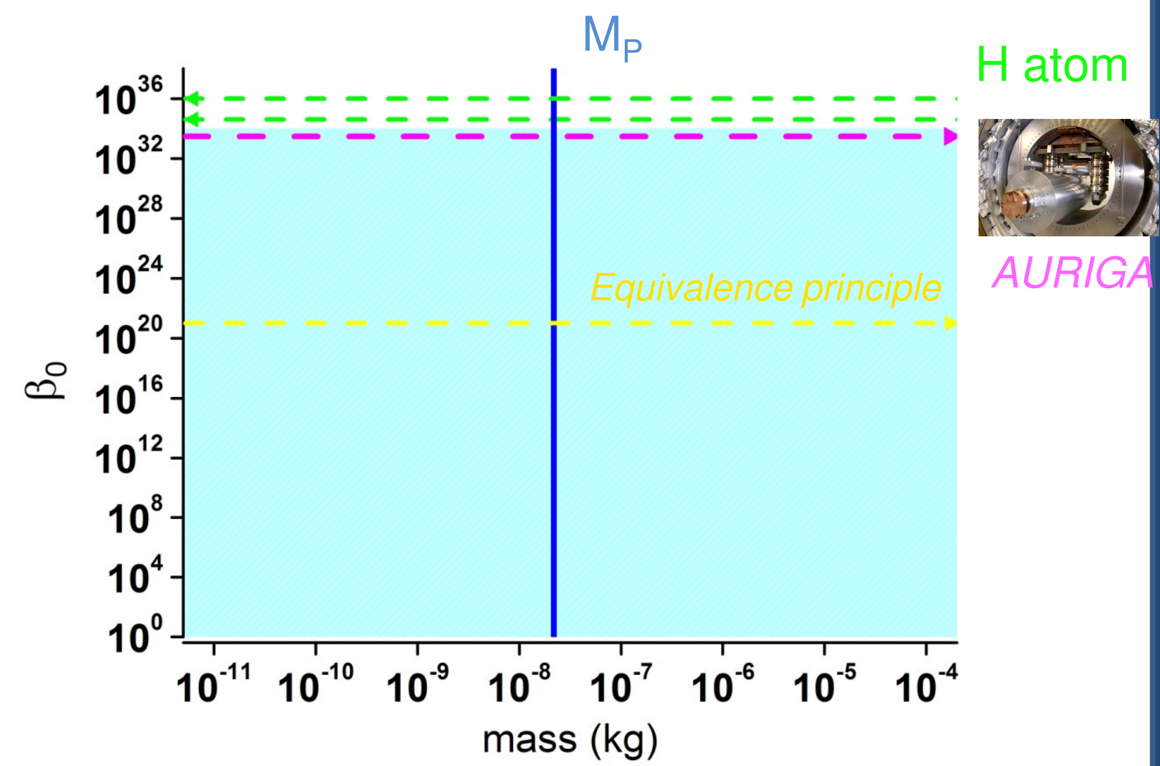
$$\beta = \beta_0 \frac{\hbar m \omega_0}{M_p^2 c^2}$$



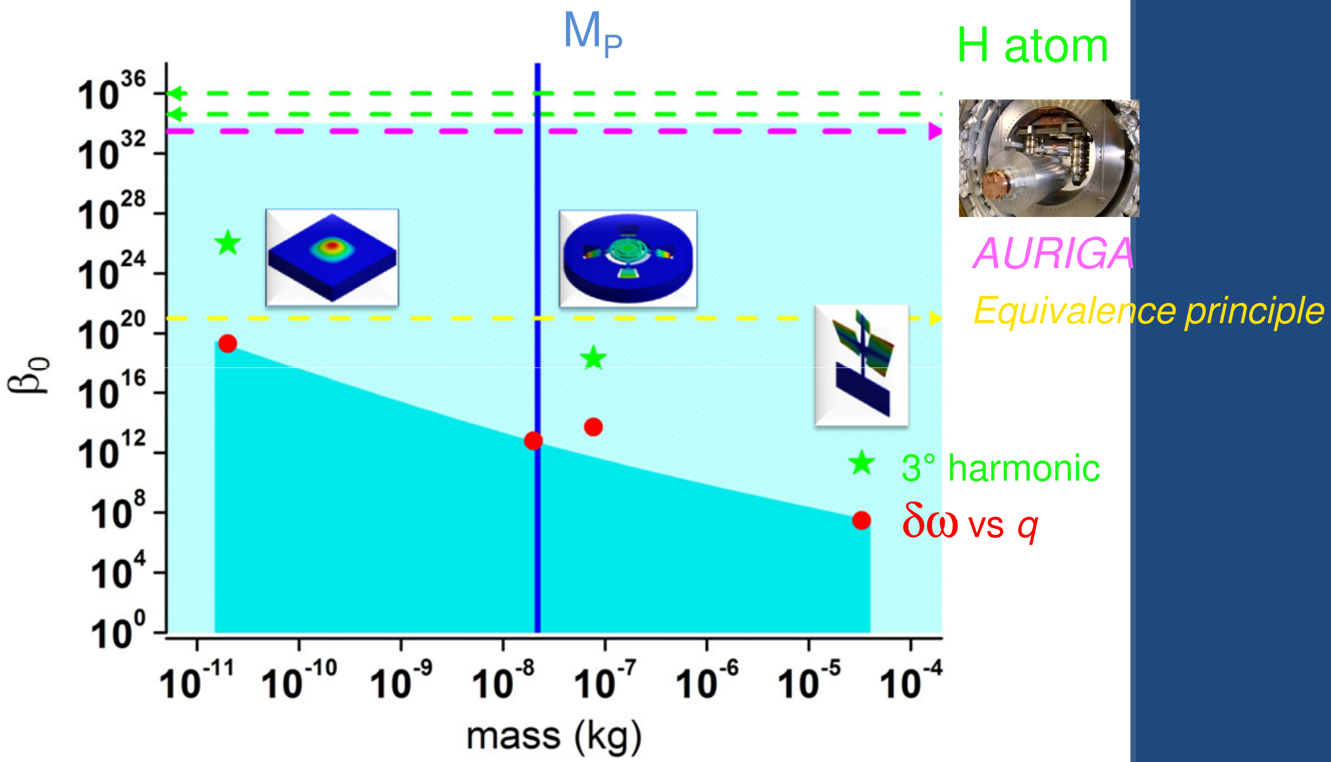
M_P
H atom
10^36
10^32
10^28
10^24
10^20
10^16
10^12
10^8
10^4
10^0
β_0
10^-11
10^-10
10^-9
10^-8
10^-7
10^-6
10^-5
10^-4
mass (kg)



M_P
H atom
AURIGA
β_0
10^36
10^32
10^28
10^24
10^20
10^16
10^12
10^8
10^4
10^0
10^-11
10^-10
10^-9
10^-8
10^-7
10^-6
10^-5
10^-4
mass (kg)



M_P
H atom
AURIGA
Equivalence principle
β_0
10^36
10^32
10^28
10^24
10^20
10^16
10^12
10^8
10^4
10^0
10^-11
10^-10
10^-9
10^-8
10^-7
10^-6
10^-5
10^-4
mass (kg)



M. Bavaj. et al., arXiv: 1411.6410

Perspectives

The link between these models and an underlying fundamental theory of quantum gravity is unclear

The description of macroscopic objects in the framework of quantum gravity models is still lacking

A macroscopic mechanical oscillator really behaves as quantum oscillator (recent experimental verification in cooled micro-oscillators)

Macroscopic oscillators in their quantum ground state

LETTER

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Laser cooling of a nanomechanical oscillator into its quantum ground state

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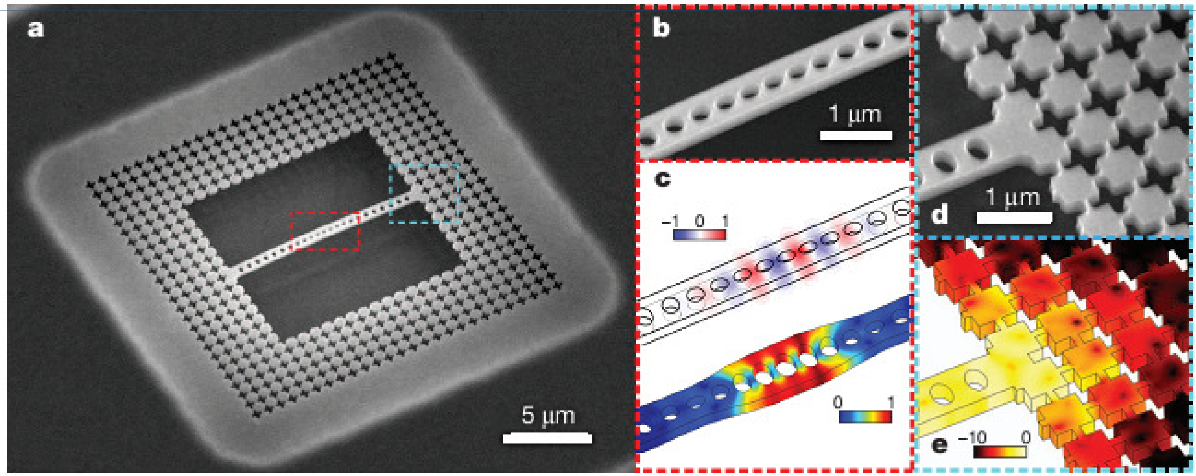

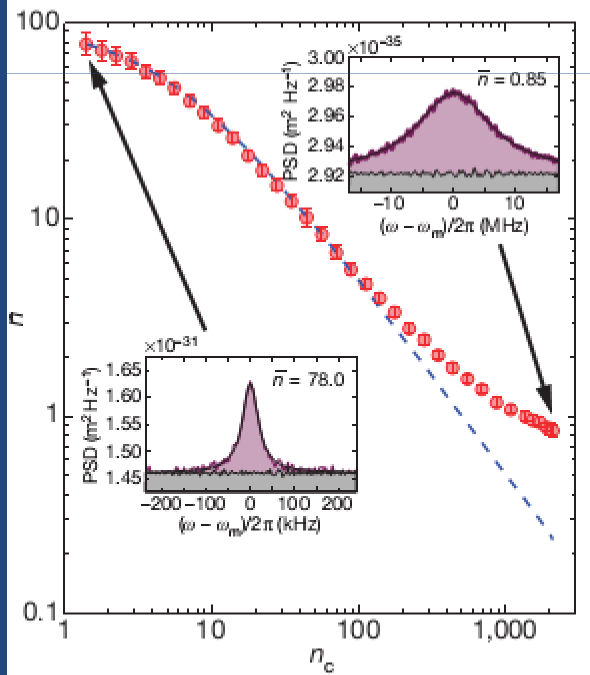

Perspectives

The link between these models and an underlying fundamental theory of quantum gravity is unclear

The description of macroscopic objects in the framework of quantum gravity models is still lacking

A macroscopic mechanical oscillator really behaves as quantum oscillator (recent experimental verification in cooled micro-oscillators)

Quantum gravity effects could be linked to 'really quantum' properties, i.e., to quantum coherence

⇒ Test on oscillators cooled down to the ground state

AURIGA

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HUMOR

Heisenberg Uncertainty Measured with Opto-mechanical Resonators

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