Quantum theory and gravity: which way?

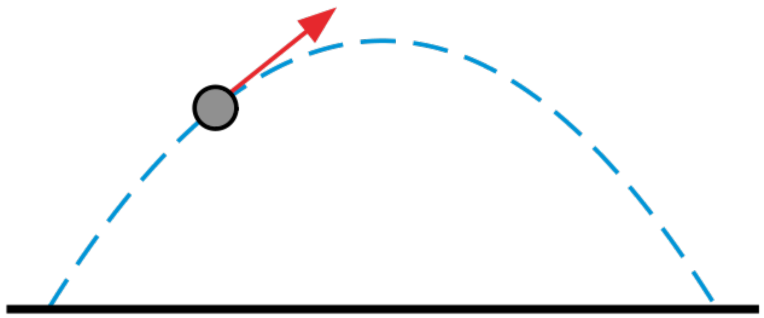
Frascati, 18-19 December 2014

Bell's inequalities

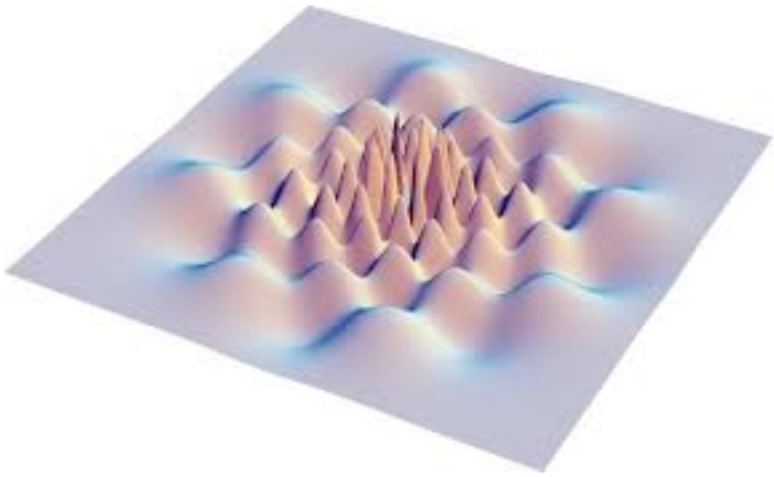
Angelo Bassi – University Trieste & INFN

Quantum theory

Classical mechanics: points, which move in space along trajectories, according to Newton's laws



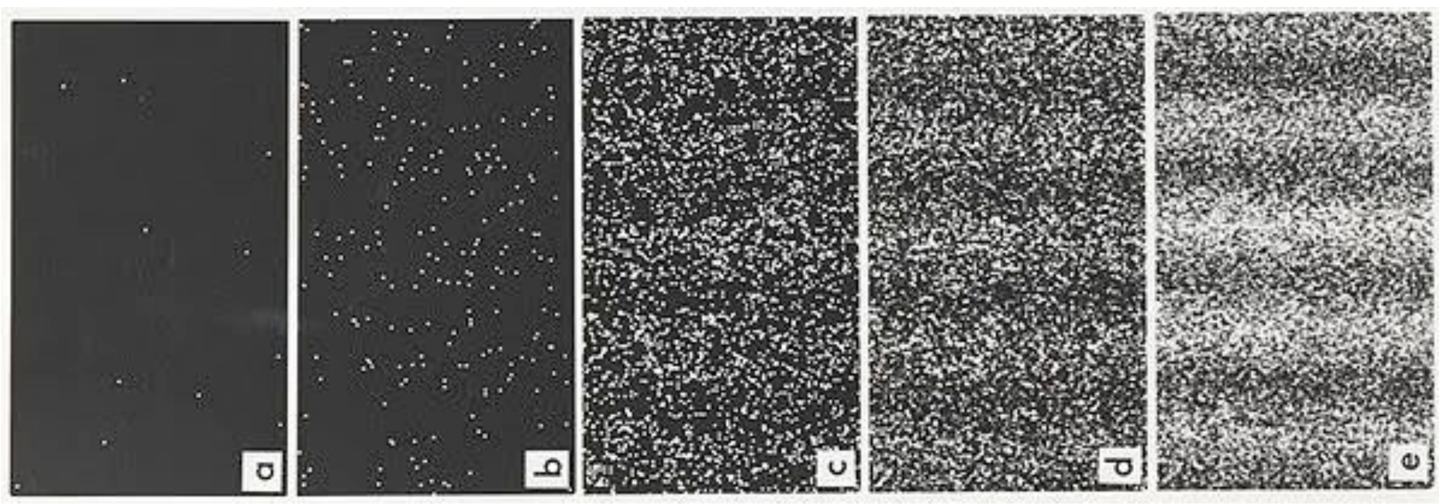
Quantum Mechanics: To every physical system, a wave function is associated, which evolves according to Schrödinger's equation



Comment 1

Why a wave function? Why not keeping using the usual classical formalism of particles which move in space along trajectories, perhaps following new laws?

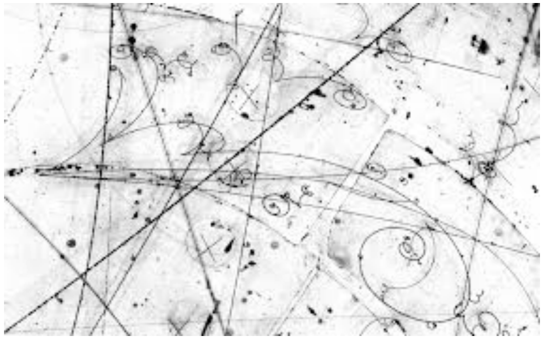
Answer: Dealing only with particle, it is not easy to justify interference phenomena as those seen in interference experiments.

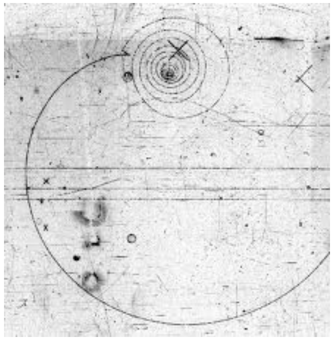


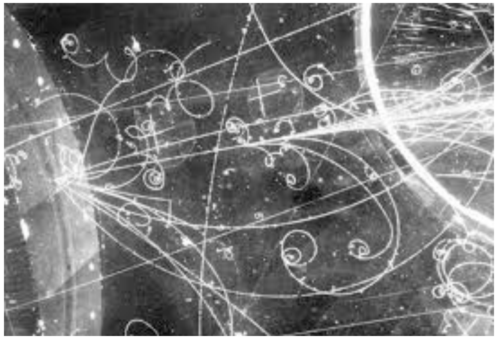
Comment 2

What does the wave function represent? Not the system but (via the square modulus) the probability of finding the system somewhere in space, when its position is measured

Motivation: It never possible to break a system-wave in two parts, as typical of waves. After the measurement, the system is always and entirely localized around a definite position in space







Comment 3

Quantum superposition:

$$\psi(x) = \frac{1}{\sqrt{2}}\left[\psi_{\mathrm{LEFT}}(x) + \psi_{\mathrm{RIGHT}}(x)\right]$$

There is ½ probability to find the particle on the left, and ½ probability to find it on the right, upon a position measurement.

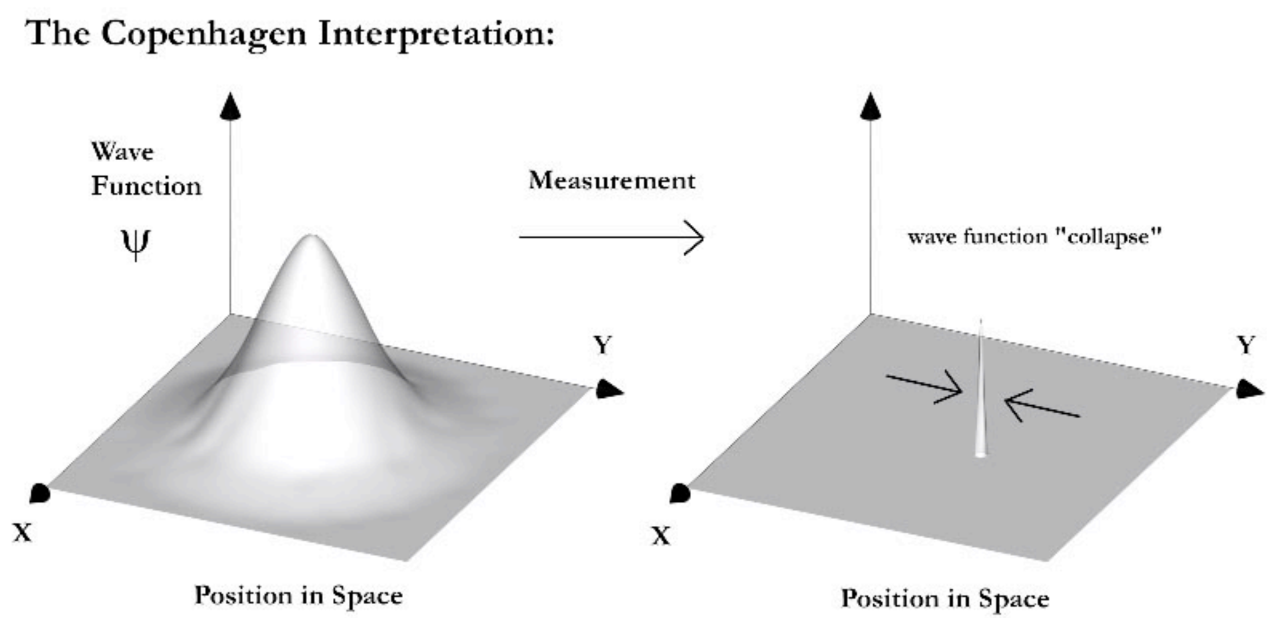
We cannot say anything about the position, prior to the measurement. More than this: there is not fact about the position of the particle, before the measurement. It is as if we ask whether the particle is married (cit. David Albert)

If we think that the particle is somewhere, even if we do not know where till we measure it, then we are assuming that the **theory is incomplete**

Comment 4

What happens at the end of a meaurement process? The wave function collapses around the place where it is found

Reason: If the wave function did not collapse and we repeat the measurement again, immediately after the first one, we would not obtain the same result with probability 1



Quantum Mechanics

1. Every physical system is described by a wave function $\psi(x)$

2. The wave function evolves according to Schrödinger's equation

3. $|\psi(x)|^2$ represents the probability (density) of finding the particle around x, upon a measurement of its position.

4. At the end of a measurement process, the wave function collapses around the point, where the system has been found.

The measurement problem

Let us consider the following two postulates

2. The wave function evolves according to Schrödinger's equation

4. At the end of a measurement process, the wave function collapses around the point, where the system has been found.

They imply two opposite kind of evolutions, they are mutually incompatible.

Problem: when does the collapse occur?

Only the Schrödinger equation?

The entire universe is quantum. Everything evolves linearly. Then there exist macroscopic quantum superpositions. What do they mean?

Entanglement

$$\psi(x,y) = \frac{1}{\sqrt{2}}\left[\psi^{(1)}_{\mathrm{LEFT}}(x)\psi^{(2)}_{\mathrm{RIGHT}}(y) + \psi^{(1)}_{\mathrm{RIGHT}}(x)\psi^{(2)}_{\mathrm{LEFT}}(y)\right]$$

1. With probability ½ particle 1 is found on the left, and with probability ½ if found on the right, upon a position measurement. Same thing for particle 2.

2. Suppose we measure the position of particle 1 and we find it on the left. Then, because of the collapse of the wave function

$$\psi_{\mathrm{COLL}}(x,y) = \psi^{(1)}_{\mathrm{LEFT}}(x)\psi^{(2)}_{\mathrm{RIGHT}}(y)$$

If we measure the position of particle 2, then we will certainly find it on the right.

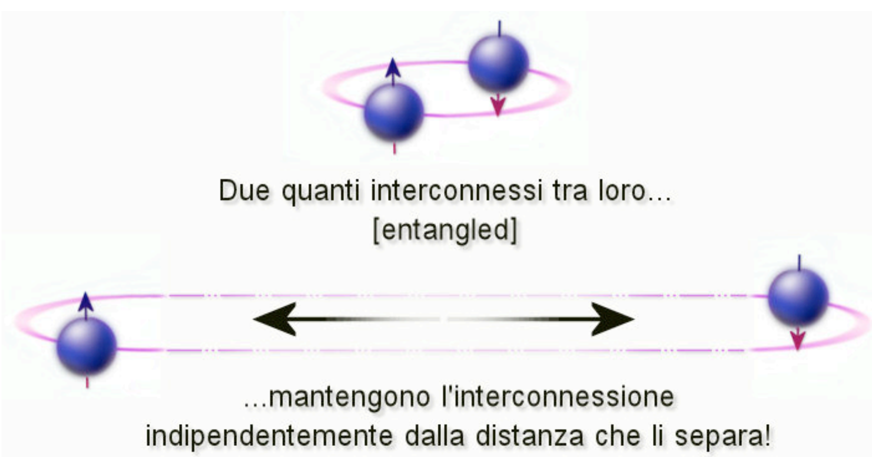
There is perfect correlation in position for the two particles

Entanglement

$$\psi(x,y) = \frac{1}{\sqrt{2}}\left[\psi^{(1)}_{\text{LEFT}}(x)\psi^{(2)}_{\text{RIGHT}}(y) + \psi^{(1)}_{\text{RIGHT}}(x)\psi^{(2)}_{\text{LEFT}}(y)\right]$$

Before the measurement, none of the two particles had a definite position in space.

If the position of one of the two particles is measured, then also the position of the other particles is instantly determined, no matter how far they are.



Entanglement: quantum correlations at a distance. Correlations, not interactions! Quantum correlations are independent of distance! They are nonlocal

E$_{\text{instein}}$ P$_{\text{odolski}}$ R$_{\text{osen}}$ - 1935

1. In an entangled two-particle state, the measurement of the position of one of the two particles determines also the position of the other particle, which can be **arbitrarily far away.**

2. **Special relativity** tells that it is not possible to instantly change the state of a distant system. Interactions and signals propagate at most at the speed of light.

3. Then the second particle must have a **definite position** also prior to the measurement of the first particle.

4. Quantum mechanics is **incomplete.**

Bell's inequality - 1964

John S. Bell was impressed by Bohm's theory. What seemed to be possible was actually done: quantum particles do move in space along trajectories

Was seemed to be impossible, was actually achieved.

But Bohm's theory is clearly nonlocal.

Bell tried to reformulate the theory in local terms, without success

Eventually he asked himself: is it perhaps impossible to make the theory local and still compatible quantum theory? Is quantum theory itself nonlocal?

Interlude for a music-hall
(Mermin & Squires)

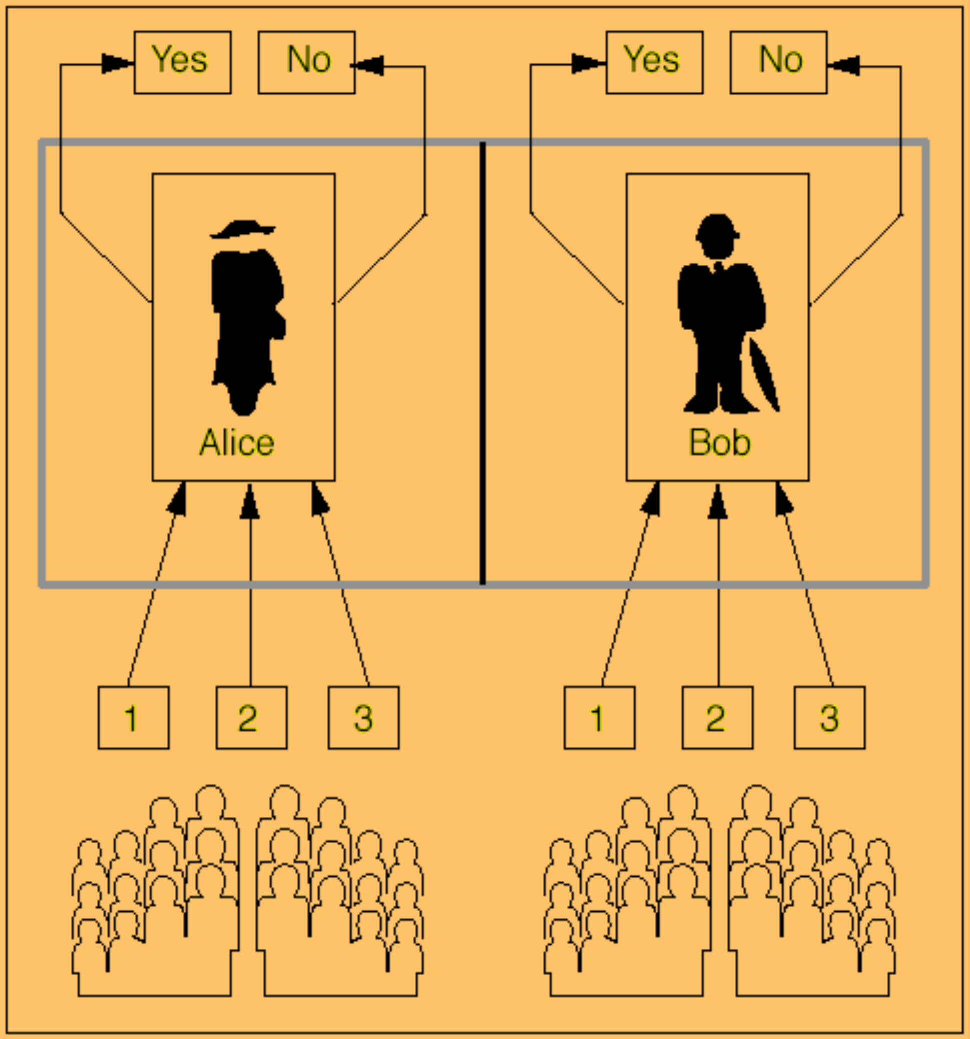
Characters: **Alice** and **Bob**.

Two groups of people. One gives Alice a card, the other gives Bob a card.

On each card there is a number: **1, 2 o 3**.

Alice and Bob write **"yes"** or **"no"** on each card they receive

Alice and Bob **cannot communicate** with each other



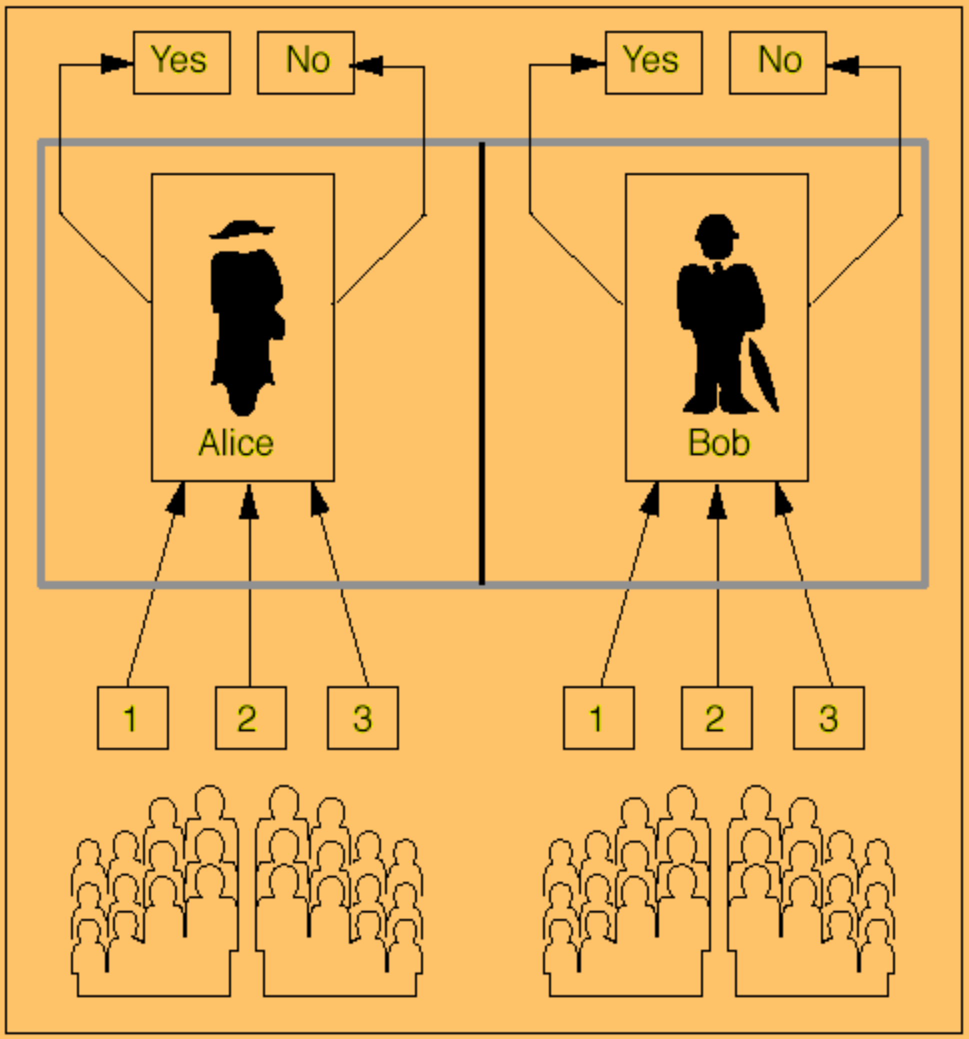
Outcome

The game is repeated over and over.

The sequence of yes and no is **fully random**.

On the average, there is the **same number** of yes and no.

BUT: each time Alice and Bob receive a card with the **same number**, they always give the **same answer**: either yes or no



Conclusion (so far…)

1. Alice and Bob cannot communicate with each other
2. In certain specific cases, they give the very same answer

Alice and Bob are **telepathic**!

Einstein - 1935

Wait: there is a simple explanation: Alice and Bob agreed beforehand on the answers to give

An example:

First run:

1	2	3
Yes	No	No

Second run:

1	2	3
No	No	Yes

and so on changing all times the answers in such a way to simulate the randomness.

Quantum mechanics: quantum particles have definite positions even before measurements.

Bell - 1964

However someone in the public took notice of all measurements – even when Alice and Bob were given cards with different numbers, and analyzed the statistics. The conclusion is:

Alice and Bob are really telepathic!

Motivation

First run:

1	2	3
Yes	No	No

Second run:

1	2	3
No	No	Yes

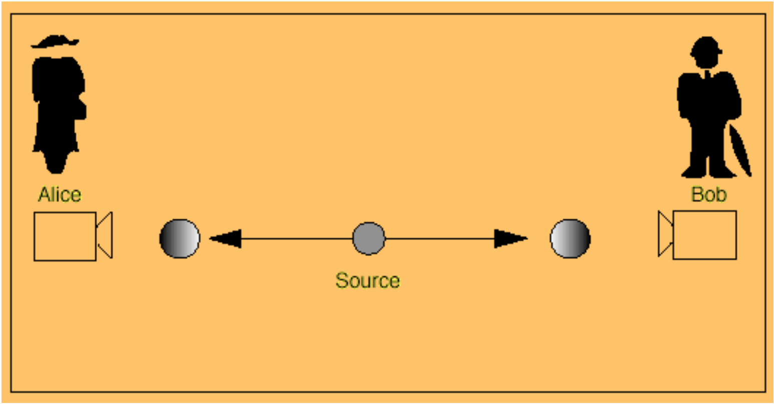
1/1	1/2	1/3	2/1	2/2	2/3	3/1	3/2	3/3
Y/Y	Y/N	Y/N	N/Y	N/N	N/N	N/Y	N/N	N/N
A	D	D	D	A	A	D	A	A

1/1	1/2	1/3	2/1	2/2	2/3	3/1	3/2	3/3
N/N	N/N	N/Y	N/N	N/N	N/Y	Y/N	Y/N	Y/Y
A	A	D	A	A	D	D	D	A

If Alice and Bob agreed beforehand, then there would be **5 possibilities of agreement (A)** against **4 of disagreement (D).** However, in the experiment A and D occur with the same probability! Therefore Alice and Bob did not agree beforehand on the answer to give

"5 versus4" instead of "5 versus 5" is an example of Bell inequality

How alice and Bob did it



N. 1: Vertical polarization measurement

N. 2: Polarization measurement at 60°

N. 2: Polarization measurement at 120°

Pairs of entangled photons in singlet state are created and sent one to Alice and one to Bob.

Alice and Bon make spin measurement. Based on the outcome they write the number on the card

Alice and Bob measure along appropriate directions. If the photon passes the test, then the write “yes”, otherwise they write “no”

Measurements are performed “instantaneously”, without possibility of communication

Predizioni della teoria

Quantum theory tells us that

1. Outcomes of measurements are randomly distributed, with an equal probability of yes and no.

2. If Alice and Bon measure along the same direction, they obtain the same outcome.

3. If they measure along different directions, the probability of agreement is ¼.

Predizioni della teoria

Since 3/9 of times Alice and Bob receive the same number, and 6/9 times of the times they receive different numbers, then the probability of agreement is:

$$(3/9) \times 1 + (6/9) \times (1/4) = 1/2$$

Alice and Bob will give the same answer 50% of times

Quantum mechanics is nonlocal

The experiment has been performed and agrees with quantum theory: **nature is nonlocal**

Bell's theorem

Consider an entangled pair of ½ spin particles

$$p_\lambda(\alpha, \beta | a, b)$$

λ = complete specification of the state of the system

For example:

λ = (p,q) in classical mechnics

λ = ψ in quantum mechanics

λ = (q,ψ) in Bohmian mechanics

α, β = outcome of spin measurements

a, b = direction of spin measurements

Bell's theorem

Bell locality

$$\boxed{p_\lambda(\alpha,\beta|a,b) = p_\lambda(\alpha,|a)p_\lambda(\beta|b)}$$

Probabilities factorize

Lemma

$$q, q', r, r' \in [-1,+1] \quad \Longrightarrow \quad |qr + qr' + q'r - q'r'| \leq 2$$

Bell's theorem

$$E_\lambda(a,b) = \sum_{\alpha,\beta} \alpha\beta \, p_\lambda(\alpha,\beta|a,b)$$

Sum of agreements minus sum of disagreements

Then

$$|E_\lambda(a,b) + E_\lambda(a,b') + E_\lambda(a',b) - E_\lambda(a',b')| \leq 2$$

Bell's theorem

λ = (μ,ν)

μ = <u>controllable</u> degrees of freedom (μ = ψ in QM)

ν = <u>uncontrollable</u> degrees of freedom. ρ(ν) = distribution of ν

$$p_\mu(\alpha,\beta|a,b) = \int d\nu\rho(\nu)p_{(\mu,\nu)}(\alpha,\beta|a,b) \quad \Rightarrow \quad E_\mu(a,b) = \int d\nu\rho(\nu)E_{(\mu\nu)}(a,b)$$

Then

$$\boxed{|E_\mu(a,b) + E_\mu(a,b') + E_\mu(a',b) - E_\mu(a',b')| \leq 2}$$

Comments

- "Hidden variables" (sometimes referred to as "realism") play no role in the derivation. It is only about locality
- Quantum theory is nonlocal (Bell's theorem). Nature is nonlocal (Aspect's experiments)
- It is not the nonlocality of classical mechanics: it cannot be controlled (non superluminal signalling, no 1/r decay), nevertheless it is nonlocality
- It challenges our relativistic understanding of the world