University of Trieste
Physics Department

"Collisional description of decoherence"

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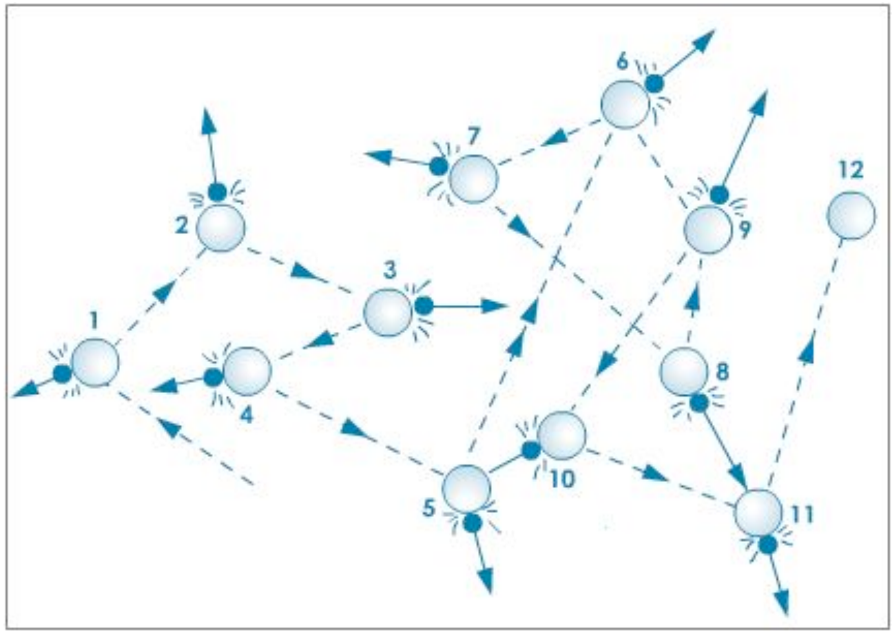
Why collisional Dynamics?

- It is a simple way to describe effects of interaction.

- Works very well in Classical Word:

  Pool game



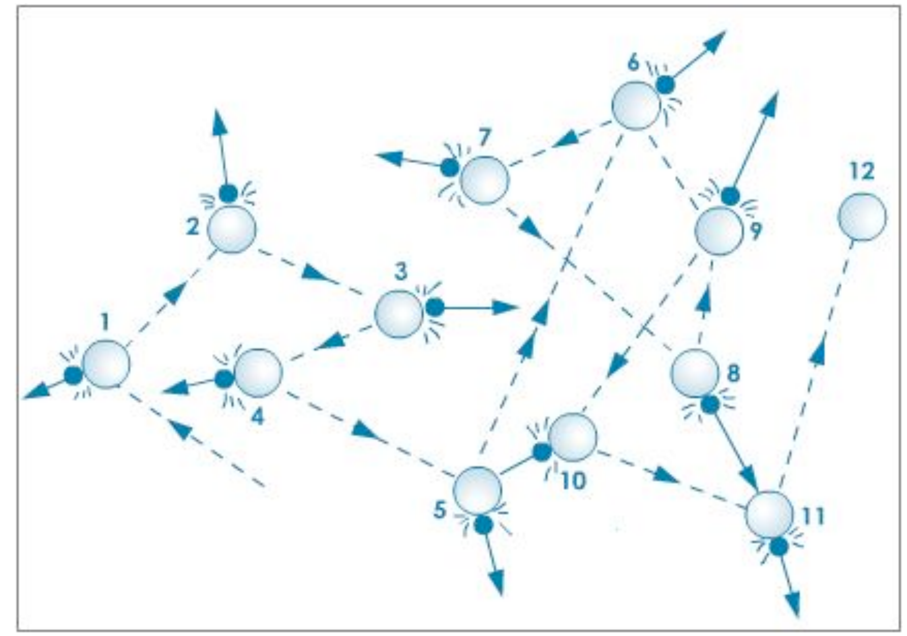
  Boltzmann Equation

$$\frac{\partial \rho(x,v,t)}{\partial t} = -v \cdot \frac{\partial}{\partial x} \rho(x,v,t) + Q(\rho)(x,v,t)$$

Free evolution   Collisional Evolution

Model of Collisional Decoherence

- Joosh- Zeh (1985)
- Gallis-Flemming (1990)
- Diosi (1995)
- QLBE by Hornberger-Vacchini (2009)
- QLBE with Finite intercollision time by Diosi (2009)



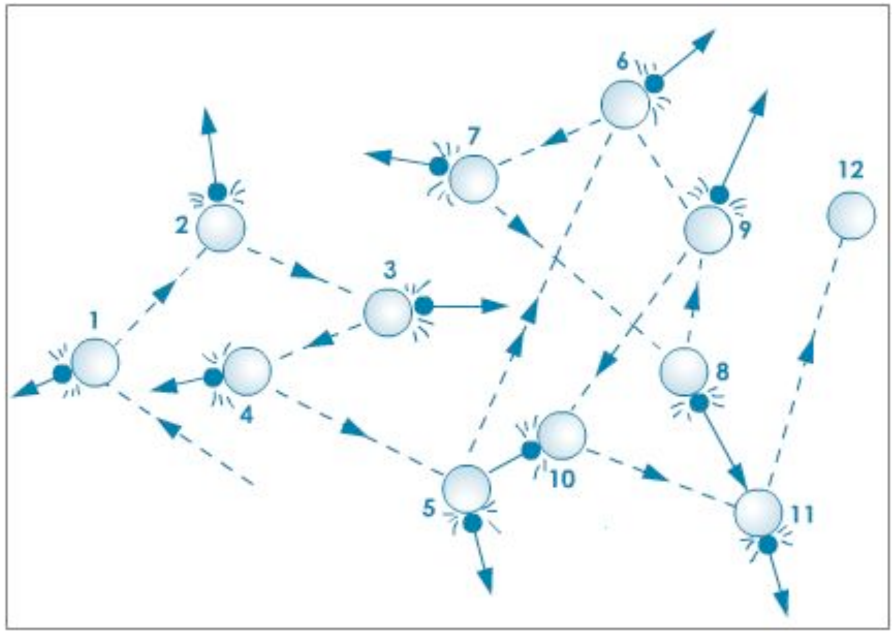
Collision

$$\hat{\rho}_{\mathcal{S}\mathcal{E}} \rightarrow \hat{S}\hat{\rho}_{\mathcal{S}\mathcal{E}}\hat{S}^{\dagger}$$

Model of Collisional Decoherence

...Free evolution...

$$\hat{\rho}_{\mathcal{S}}(t+dt) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}_{\mathcal{S}}]dt$$



Introduction Collisional Dynmamics

$$\hat{\rho}_{\mathcal{S}}(t+dt) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}_{\mathcal{S}}]dt + \mathrm{Tr}_{\mathcal{E}}\left[\hat{S}\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)\hat{S}^{\dagger}\right]$$

Model of Collisional Decoherence

...Free evolution...

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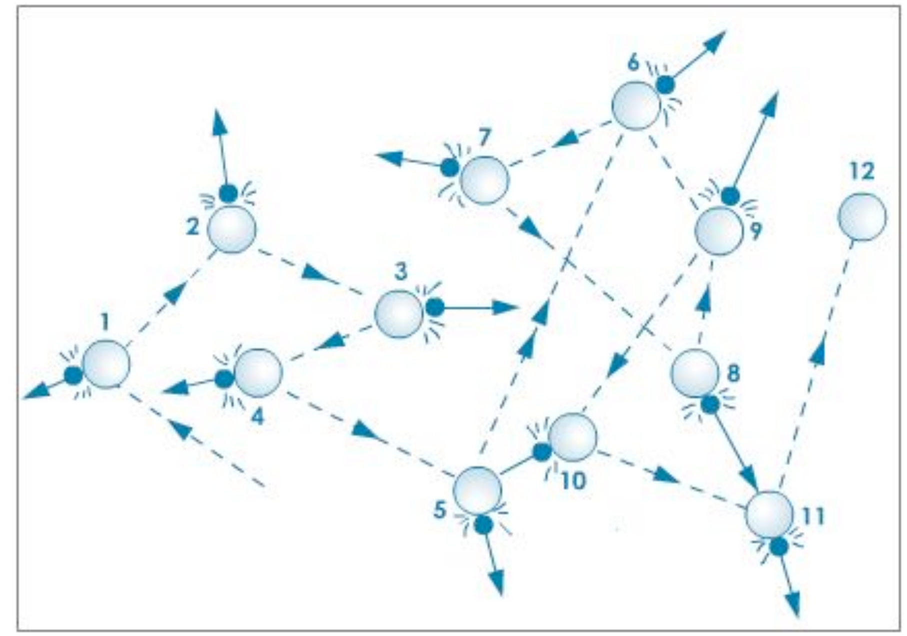


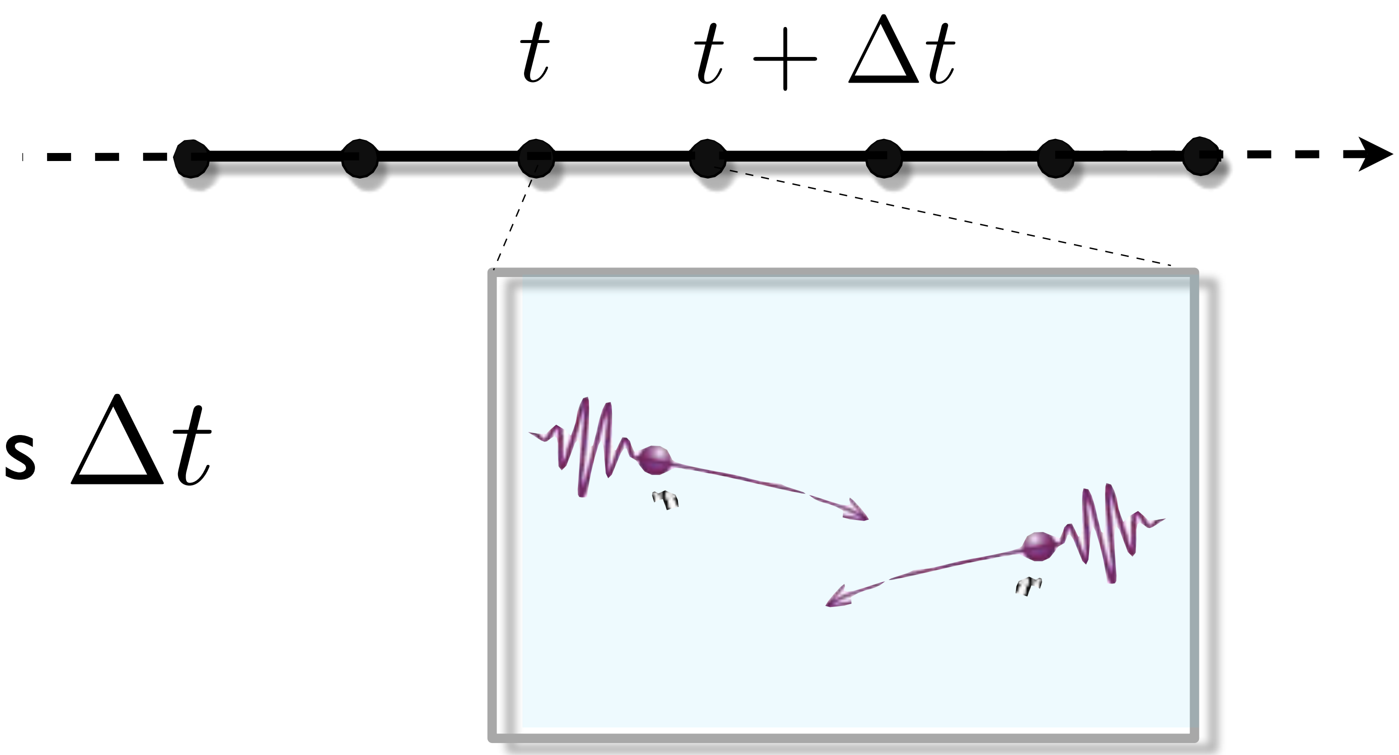
Introduction Collisional Dynmamics

$$\hat{\rho}_{\mathcal{S}}(t+dt) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}_{\mathcal{S}}]dt + \mathrm{Tr}_{\mathcal{E}}\left[\hat{S}\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)\hat{S}^{\dagger}\right]$$

...Quantum mechanics is not so simple!

Microscopic Derivation

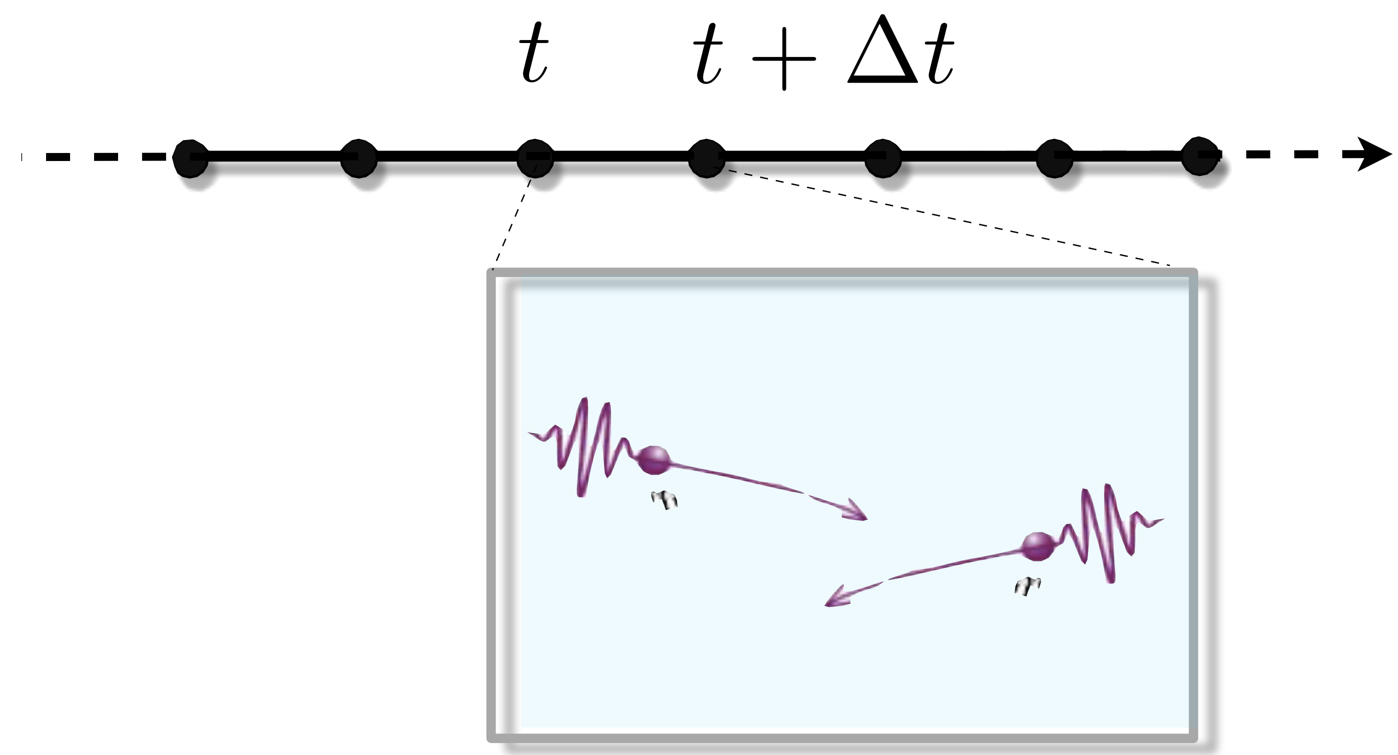
$$\frac{\partial \hat{\rho}_{\mathcal{S}\mathcal{E}}(t)}{\partial t} = \frac{-i}{\hbar}\left[\underbrace{\frac{\hat{P}^2}{2M} + \frac{\hat{p}^2}{2m}}_{\hat{H}_0} + V(\hat{X} - \hat{x}), \hat{\rho}_{\mathcal{S}\mathcal{E}}(t)\right]$$



- Discretize time in intervals Δt
- $\Delta t \ll \tau_{free}$
- Complete collision process in the time interval

Microscopic Derivation

$$\frac{\partial \hat{\rho}_{\mathcal{SE}}(t)}{\partial t} = \frac{-i}{\hbar}\Big[\underbrace{\frac{\hat{P}^2}{2M} + \frac{\hat{p}^2}{2m}}_{\hat{H}_0} + V(\hat{X} - \hat{x}), \hat{\rho}_{\mathcal{SE}}(t)\Big]$$



Discrete Evolution in interaction picture

$$\hat{\rho}^{I'}_{\mathcal{SE}}(t + \Delta t) = \hat{S}(\Delta t)\hat{\rho}^{I'}_{\mathcal{SE}}(t)\hat{S}^{\dagger}(\Delta t)$$

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incomplete Scattering operator

$$\hat{S}(\Delta t)$$

complete Scattering operator

$$\lim_{\Delta t \to \infty} \hat{S}(\Delta t) = \hat{S}$$

Microscopic Derivation

Moving to Schrödinger Picture...

$$\hat{S}(\Delta t) = 1 + i\hat{T}(\Delta t)$$

$$\frac{\Delta\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)}{\Delta t} = \frac{e^{-\frac{i}{\hbar}\hat{H}_0\Delta t}\,\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)\,e^{\frac{i}{\hbar}\hat{H}_0\Delta t} - \hat{\rho}_{\mathcal{S}\mathcal{E}}}{\Delta t}$$

$$- ie^{-\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\left[\frac{\hat{T}(\Delta t) + \hat{T}^\dagger(\Delta t)}{2\Delta t},\, e^{-\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\,\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)\,e^{\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\right]e^{\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}$$

$$+ e^{-\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\left\{\frac{\hat{T}^\dagger(\Delta t)\hat{T}(\Delta t)}{2\Delta t},\, e^{-\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\,\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)\,e^{\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\right\}e^{\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}$$

$$- \frac{1}{\Delta t}e^{-\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\hat{T}(\Delta t)e^{-\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\,\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)e^{\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\hat{T}^\dagger(\Delta t)e^{\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}$$

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$$- i e^{-\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}} \left[\frac{\hat{T}(\Delta t) + \hat{T}^{\dagger}(\Delta t)}{2\Delta t},\, e^{-\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}}\,\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)\,e^{\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}} \right] e^{\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}}$$

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$$- \frac{1}{\Delta t} e^{-\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}}\hat{T}(\Delta t) e^{-\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}}\,\hat{\rho}_{\mathcal{S}\mathcal{E}}(t) e^{\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}}\hat{T}^{\dagger}(\Delta t) e^{\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}}$$

Microscopic Derivation

$$\Delta t \ll \tau_{free}$$

first term..

$$\frac{e^{-\frac{i}{\hbar}\hat{H}_0 \Delta t}\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)\, e^{\frac{i}{\hbar}\hat{H}_0 \Delta t} - \hat{\rho}_{\mathcal{S}\mathcal{E}}}{\Delta t}$$

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in momentum basis

$$\rho_{\mathcal{S}\mathcal{E}}(p, p', t) e^{-\frac{i}{\hbar}[E_0(p) - E_0(p')]\Delta t}$$

$$\tau_{free} \equiv \frac{\hbar}{\max_{\rho} |E_0(p) - E_0(p')|}$$

Microscopic Derivation

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first term..

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in momentum basis

$$\rho_{\mathcal{S}\mathcal{E}}(p, p', t) e^{-\frac{i}{\hbar}[E_0(p) - E_0(p')]\Delta t}$$

Gives the following condition for the state

$$\Delta t \ll \tau_{free} \longrightarrow \frac{\hbar}{\max_\rho |E_0(p) - E_0(p')|} \gg \Delta t$$

Microscopic Derivation

Complete Collision $(\Delta t \to \infty)$

$$\lim_{\Delta t \to \infty} \left[\frac{\hbar}{\max_\rho |E_0(p) - E_0(p')|} \gg \Delta t \right]$$

Microscopic Derivation

Complete Collision $(\Delta t \to \infty)$

$$\lim_{\Delta t \to \infty} \left[\frac{\hbar}{\max_{\rho} |E_0(p) - E_0(p')|} \gg \Delta t \right] \longrightarrow \frac{i}{\hbar}[\hat{H}_0, \hat{\rho}_{\mathcal{S}\mathcal{E}}] = 0$$

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The state should be in an energy eigenstate !!!

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4-th term

$$-\frac{1}{\Delta t} e^{-\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\hat{T}(\Delta t)e^{-\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\hat{\rho}_{\mathcal{S}\mathcal{E}}(t)e^{\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}\hat{T}^{\dagger}(\Delta t)e^{\frac{i}{\hbar}\hat{H}_0\frac{\Delta t}{2}}$$

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$$-\frac{1}{\Delta t} e^{-\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}} \hat{T}(\Delta t) e^{-\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}} \hat{\rho}_{\mathcal{S}\mathcal{E}}(t) e^{\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}} \hat{T}^\dagger(\Delta t) e^{\frac{i}{\hbar}\hat{H}_0 \frac{\Delta t}{2}}$$

$$\downarrow \qquad [\hat{T}, \hat{H}] = 0$$

$$\frac{1}{\Delta t} \hat{T} \hat{\rho}_{\mathcal{S}\mathcal{E}} \hat{T}^\dagger$$

System Reduced Dynamics:

Tracing out the degrees of freedom of the environment...

$$\frac{\partial \hat{\rho}_{\mathcal{S}}(t)}{\partial t} \simeq -\frac{i}{\hbar}\left[\frac{\hat{P}^2}{2M}, \hat{\rho}_{\mathcal{S}}(t)\right] + \frac{1}{\Delta t}\mathrm{Tr}_{\mathcal{E}}\left\{\frac{i}{2}[\hat{T} - \hat{T}^\dagger, \hat{\rho}_{\mathcal{SE}}(t)]\right\}$$

$$+ \frac{1}{\Delta t}\mathrm{Tr}_{\mathcal{E}}\left\{\hat{T}\hat{\rho}_{\mathcal{SE}}(t)\hat{T}^\dagger - \frac{1}{2}\{\hat{T}^\dagger\hat{T}, \hat{\rho}_{\mathcal{SE}}(t)\}\right\}$$

with $\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}_{\mathcal{SE}}] = 0$

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$$+\frac{1}{\Delta t}\mathrm{Tr}_{\mathcal{E}}\left\{\hat{T}\hat{\rho}_{\mathcal{SE}}(t)\hat{T}^\dagger - \frac{1}{2}\{\hat{T}^\dagger\hat{T}, \hat{\rho}_{\mathcal{SE}}(t)\}\right\}$$

with $\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}_{\mathcal{SE}}] = 0$

$$\hat{T}\hat{\rho}_{\mathcal{SE}}\hat{T}^\dagger \propto \frac{\Delta T}{2\pi\hbar}\delta(\Delta E)$$

Conclusions

- Complete Collisional Effective Dynamics are allowed only for free energy eigenstates

- We are still far from having a good dynamical description of a quantum particle in a gas!

- Necessity to find a different approaches to treat the problem.