– S-matrix approach to the Z line shape –
A reminiscence.
Prospects?

Tord Riemann, DESY Thanks to: M. Grünewald + S. Riemann

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Outline

S-matrix approach to the Z line shape

- Developed for a model-independent analysis tool of $e^+e^- \to (\gamma, Z) \to f^+f^-$ **around the Z boson resonance**
- In cooperation with M. Grünewald, S. Kirsch, A. Leike, S. Riemann
- Aim: determinations of M_Z and Γ_Z in correlation with the γZ-interference
- Refs.: Leike/Riemann/Rose 1991 [1], Riemann 1992 [2], Kirsch/Riemann 1994 [3]
- First application: L3 1993 [4], also: TOPAZ, VENUS, OPAL, . . .
- Fortran software: stand-alone ZPOLE (Leike/Riemann 1991, unpublished) and SMATASY/ZFITTER (Grünewald/Kirsch/Riemann 1994→2005) [3, 5, 6, 7, 8, 9]

Introduction

The reaction

$$e^+e^- \to (\gamma, Z) \to f^+f^- \; + \; (n\,\gamma) \tag{1}$$

allows to study the Z boson, its mass M_Z, its width Γ_Z, its couplings, and potentially deviations from the Standard Model.

The numerical outcome depends on the model applied.
See experiences with constant and s-dependent Z width

→ Bardin/Leike/Riemann/Sachwitz 1988 [10]

Also: Berends/Burgers/Hollik/v.Neerven 1988 [11]

Unfolding of *Realistic Observables* is needed in order to get *Pseudo Observables*.

→ e.g.:
Borrelli/Consoli/Maiani/Sisto 1990 [12]

Later: Bardin/Passarino 1999 [13], Bardin/Grünewald/Passarino 1999 [14], Passarino 2003 [15], Passarino 2013 [16] and refs. therein.

Introduction

The analysis tool for the Z resonance: ZFITTER

- Complete electroweak radiative corrections
- QED corrections by convolution with some $\sigma_0(s')$ or for initial-final state interferences with some $\sigma_0(s,s')$
- **semi-analytical QED** integrations, using

$$\frac{1}{|s-M_Z^2+iM_Z\Gamma_Z|^2} \sim \frac{i}{\Gamma_Z}\left[\frac{1}{s-M_Z^2+iM_Z\Gamma_Z} - \frac{1}{s-M_Z^2-iM_Z\Gamma_Z}\right] \quad (2)$$

- free choice of $\sigma_0(s')$ by **user interfaces**
- Standard Model interfaces: four weak form factors $\rho, \kappa_e, \kappa_e, \kappa_{ef}$

ZFITTER is **well-tested**, **flexible**, **accurate** and **fast** at the same time.
$\rightarrow$ Interest e.g. at Belle-II for 10^9 events at $\sqrt{s} \sim 10$ GeV.
Compare: Fcc-ee expects 10^{13} Z events

Introduction

Stuart 1991 [17], S-Matrix ansatz for $e^+e^- \longrightarrow Z \longrightarrow f^+f^-$

$$M = \frac{R}{s - s_0} + F(s), \qquad s_0 = M_Z^2 - iM_Z\Gamma_Z \tag{3}$$

Allows to study:

- Mass M_Z and width Γ_Z

  $\rightarrow$ Leike/Riemann/Rose 1991 [1]
- How many independent degrees of freedom are to be introduced?

  $\rightarrow$ Leike/Riemann/Rose 1991 [1]
- Who is correlated with whom?

  $\rightarrow$ Leike/Riemann/Rose 1991 [1] and Kirsch/S.Riemann, L3 [18, 4]
- What about asymmetries and QED corrections?

  $\rightarrow$ Riemann [2]
- But also: How to define mass and width of Z gauge-invariant at higher orders of perturbation theory?

  $\rightarrow$ Denner 2014 [19], Freitas 2014 [20] and Fcc-ee [21], Degrassi FCC-ee [22] and refs. therein

Total cross sections

There are immediate questions, from an experimental point

- What about the photon exchange?
- What about QED corrections, e.g. the $2 \to 3$ part of the cross sections?
- What about asymmetries, besides σ_{tot}?

We have to describe

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+f^-(\gamma), \tag{4}$$

Ansatz in the complex energy plane, for **four helicity matrix elements**:

$$\mathcal{M}^i(s) = \frac{R^i_\gamma}{s} + \frac{R^i_Z}{s - s_Z} + F^i(s), \quad i = 1, \ldots 4. \tag{5}$$

Beware: Eqn. (5) is mathematically not consistent → Bohm/Sato 2004 [23]

The poles of $\mathcal{M}$ have complex residua R_Z and R_γ, the latter corresponding to the photon, and $F(s)$ is an analytic function without poles:

$$F^i(s) = \sum_{n=0}^{\infty} F^i_n (s - s_0)^n \tag{6}$$

Comment on the photon term (3 Feb. 2015)

$$\begin{aligned}
\frac{R^i_\gamma(s)}{s} &= \frac{\sum_{n=0}^\infty R^i_n (s-s_0)^n}{s} \\
&= \frac{\sum_{n=0}^\infty R^i_n (s-s_0)^n}{s_0-(s_0-s)} \\
&= \sum_{n=0}^\infty R^i_n (s-s_0)^n \, \frac{1}{s_0} \, \frac{1}{1-\frac{s_0-s}{s_0}} \\
&= \sum_{n=0}^\infty R^i_n (s-s_0)^n \, \frac{1}{s_0} \left[1 + \frac{s_0-s}{s_0} + \left(\frac{s_0-s}{s_0}\right)^2 \cdots \right]
\end{aligned} \tag{7}$$

The term $R^i_\gamma(s)/s$ is part of the the background term $B(s)$.

- It is useful to sum up a selected part of self-energy insertions in the propagators in order to derive the Breit-Wigner resonance form,
- It is useful to sum up a selected part of the photonic background of the Z resonance in order to take explicit notice of physically known pieces of the input expressions.

Ansatz for realistic applications

The analysis of the Z line shape will be based here on the cross section

$$\sigma(s) = \sum_{i=1}^{4} \sigma^i(s) = \frac{1}{4} \sum_{i=1}^{4} s |\mathcal{M}^i(s)|^2, \tag{8}$$

where the sum must be performed over four helicity amplitudes with different residua R_Z^i and functions $F^i(s)$. The result is:

$$\sigma_T(s) = \frac{4}{3}\pi\alpha^2 \int \frac{ds'}{s} \left[\frac{r^\gamma}{s} + \frac{sr + (s - M_Z^2)j}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right] \rho_{ini}\left(\frac{s'}{s}\right). \tag{9}$$

The radiation connected with initial-final state interferences can be taken into account by an analogue formula to (9) with a slightly more complicated structure [6, 5]:

$$\sigma_{\rm int}(s) = \int ds' \sigma(s, s') \rho_{\rm int}(s'/s). \tag{10}$$

Some details

The correct ansatz for the S-matrix based cross section is:

$$\sigma(s,s') = \frac{1}{8} s' \sum_i \left[\mathcal{M}_i(s)\mathcal{M}_i^*(s') + \mathcal{M}_i^*(s)\mathcal{M}_i(s') \right]. \tag{11}$$

From Leike/Riemann/Rose:

$$\sigma(s) = \sum_A \sigma_A(s), \qquad A = Z, \gamma, F, \gamma Z, ZF, F\gamma, \tag{12}$$

with the contributions:

$$\begin{aligned}
\sigma_Z(s) &= \frac{s r_Z}{|s-s_Z|^2}, & r_Z &= \tfrac{1}{4}\sum |R_Z^i|^2, \\
\sigma_\gamma(s) &= \frac{r_\gamma}{s}, & r_\gamma &= |R_\gamma|^2, \\
\sigma_F(s) &= s r_F(s), & r_F(s) &= \tfrac{1}{4}\sum |F^i(s)|^2, \\
\sigma_{\gamma Z}(s) &= 2\mathrm{Re}\,\frac{C_\gamma^* C_Z}{s-s_Z}, & C_\gamma &= R_\gamma, \qquad C_Z = \tfrac{1}{4}\sum R_Z^i, \\
\sigma_{ZF}(s) &= 2\mathrm{Re}\,\frac{s C_{ZF}(s)}{s-s_Z}, & C_{ZF}(s) &= \tfrac{1}{4}\sum R_Z^i F^{i*}(s), \\
\sigma_{F\gamma}(s) &= 2\mathrm{Re}\left[C_\gamma^* C_F(s)\right], & C_F(s) &= \tfrac{1}{4}\sum F^i(s).
\end{aligned}$$

Some details

After making denominators real one remains with the following formula for the line shape:

$$\sigma(s) = \frac{R + (s - M_Z^2)I}{|s - s_Z|^2} + \frac{r_\gamma}{s} + r_0 + (s - M_Z^2)r_1 + \dots \tag{13}$$

Besides M_Z, Γ_Z, the real constants R, I, r_0 and r_1 are introduced:

$$\begin{aligned}
R &= M_Z^2 \left[r_Z + 2(\Gamma_Z/M_Z)\left(\Im m C_R + M_Z \Gamma_Z \Re e(C_R') \right) \right], \\
I &= r_Z + 2\Re e C_R, \\
C_R(s) &= C_\gamma^* C_Z + s_Z C_{ZF}(s), \\
r_0 &= M_Z^2 \left[r_F - M_Z \Gamma_Z \Im m(r_F') \right] + \Re e C_r - M_Z \Gamma_Z \Im m C_r', \\
r_1 &= r_F + M_Z^2 \left[\Re e(r_F') - (\Gamma_Z/M_Z) \Im m(r_F') \right] + \Re e C_r', \\
C_r(s) &= C_\gamma^* C_F(s) + C_{ZF}(s).
\end{aligned} \tag{14}$$

The energy-dependent functions C_{ZF}, C_F, r_F, and their (primed) derivatives with respect to s have to be taken at $s = s_Z$. As may be seen, the cross section may be described by only six real parameters as long as one takes into account only the first two terms in the expansion of the functions $F^i(s)$ around $s = s_Z$ and at most terms of the order $(s - M_Z^2)^n, n = 0, 1$ in the cross section parametrization.

Asymmetries

On a Sunday in Summer 1992, I had a discussion with Luciano Maiani in the CERN library.
He had doubt that an analogue to the model-independent ansatz for σ_{tot} might be usefully formulated, especially in view of the QED corrections.

I believed one can do that, and I followed the rule “The proof of the pudding is in the eating” [2]. The result:

$$A(s) = A_0 + A_1 \left(\frac{s}{M_Z^2} - 1 \right) + A_2 \left(\frac{s}{M_Z^2} - 1 \right)^2 + \ldots \tag{15}$$

$$A_{FB} = \frac{\sigma_{FB}}{\sigma_T}, \qquad A_{pol} = \frac{\sigma_{pol}}{\sigma_T}. \tag{16}$$

The coefficients are in QED-Born approximation:

$$A_0 = \frac{R_A}{R_T}, \tag{17}$$

and

$$A_1 = \left[\frac{J_A}{R_A} - \frac{J_T}{R_T} \right] A_0. \tag{18}$$

Some details

$$
\begin{aligned}
R_T &= \kappa^2(a_e^2+v_e^2)(a_f^2+v_f^2) + 2\kappa|Q_eQ_f|v_ev_f\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, \\
R_{FB} &= 3\kappa^2 a_ev_ea_fv_f + \frac{3}{2}\kappa|Q_eQ_f|a_ea_f\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, \\
R_{pol} &= -2\kappa^2(a_e^2+v_e^2)a_\tau v_\tau - 2\kappa|Q_eQ_f|v_ea_\tau\frac{\Gamma_Z}{M_Z}\Im m\frac{\alpha(s)}{\alpha}, && (19) \\
J_A &= 2|Q_eQ_f|\Re e\frac{\alpha(s)}{\alpha}\kappa\kappa_A, \\
\kappa_T &= v_ev_f, && (20) \\
\kappa_{FB} &= \frac{3}{4}a_ea_f, && (21) \\
\kappa_{pol} &= -v_ea_\tau, && (22)
\end{aligned}
$$

$$
\kappa = \frac{G_\mu}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha} = 0.3724\left(\frac{M_Z}{91}\right)^2. \qquad (23)
$$

QED corrections for asymmetries

In the vicinity of the Z boson peak, asymmetries behave relatively smoothly and may be described by a simple, universal formula [2, 3]

$$A(s) = A_0 + C(s)\, A_1 \left(\frac{s}{M_Z^2} - 1 \right) \; + \cdots . \tag{24}$$

The QED corrections are contained in the model-independent factor $C(s)$.

QED corrections for asymmetries

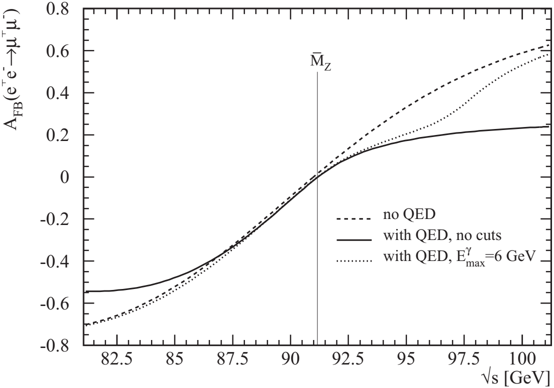

Figure 1 : The forward-backward asymmetry for the process $e^+e^- \to \mu^+\mu^-$ near the Z boson peak. From Kirsch/Riemann 1994 [3], license Number: 3557090997554.

Applications

In Leike/S.Riemann/Riemann 1992 [24] correlations are discussed.

For the Z peak position s_{peak}, one may derive the relation:

$$\Delta\sqrt{s_{peak}} = \Delta M_Z + \frac{1}{4}\frac{\Gamma_Z^2}{M_Z}\,\Delta\left(\frac{J_T}{R_T}\right) + \ldots \tag{25}$$

between an uncertainty in M_Z and an uncertainty in the γZ interference.
The latter influences A_1.

Similarly, for a **hypothetical heavy gauge boson** Z', the effects from its virtual exchange transform after a partial fraction decomposition into simple shifts of the γZ interferences [24

$$\Delta\left(\frac{J_T}{R_T}\right) = -2\frac{g'^2}{g^2}\frac{M_{Z'}^2}{M_{Z'}^2 - M_Z^2}\frac{(a_e a_e' + v_e v_e')(a_f a_f' + v_f v_f')}{(a_e^2 + v_e^2)(a_f^2 + v_f^2)}, \tag{26}$$

Correlations

From Phys. Rept. 2006, section 2 [25]:

The extra free parameter j_{had}^{tot} is strongly anti-correlated with m_Z, resulting in errors on m_Z enlarged by a factor of almost three, as is observed in the existing S-matrix analyses of LEP-I data [77].

The dependence of m_Z on j_{had}^{tot} is given by:

$$j_{had}^{tot} = \frac{G m_Z^2}{\sqrt{2}\pi\alpha(m_Z^2)} Q_e g_{V_e} \cdot 3 \sum_{q\neq t} Q_q g_{V_q} \quad (27)$$

$$\frac{\partial m_Z}{\partial j_{had}^{tot}} = -1.6\mathrm{MeV}/0.1 \quad (28)$$

. . .

Improved experimental constraints on the hadronic interference term are obtained by including measurements of the hadronic total cross-section at centre-of-mass energies further away from the Z pole than just the off-peak energies at LEP-I. Including the measurements of the **TRISTAN** *collaborations at KEK,* **TOPAZ** *[78] and* **VENUS** *[79], at $q(s) = 58$ GeV,* **the error on j_{had}^{tot} is about** ± 0.1, *while its central value is in good agreement with the SM expectation.*

Correlations

From Phys. Rept. 2006, section 2 [25], continued:

Measurements at centre-of-mass energies above the Z resonance at **LEP-II** *[80-83] also provide constraints on j_{had}^{tot}, and in addition test modifications to the interference terms arising from the possible existence of a heavy Z' boson."*

[77] = L3, OPAL 1993 . . . 2003 [26, 27, 28] (see also K. Sachs, L3 [29, 30])
[78] = TOPAZ 1994 [31]
[79] = VENUS 1999 [32]
[80] => correct ref: ALEPH 1996 [33]
[81] = DELPHI 1999 [34]
[82] = L3 1993 . . . 2000 [4, 35, 30]
[83] = OPAL 1997 [36]

Correlations

From Phys. Rept. 2013, App. A [37]:

"In the LEP-I combination the measured values of **the Z boson mass**

$m_Z = 91.1929 \pm 0.0059$ **GeV**

agrees well with the results of the **standard nine parameter fit,**

$[m_Z =]\ 91.1876 \pm 0.0021$ **GeV**,

albeit with a significantly larger error, resulting from the correlation with the large uncertaint
on j_{had}^{tot}.
This uncertainty is the dominant source of uncertainty on mZ in the S-Matrix fits.
The measured value of

$j_{had}^{tot} = -0.10 \pm 0.33$

also agrees with the prediction of the SM,

$[j_{had}^{tot} =]\ 0.2201^{+0.0032}_{-0.0137}$."

Correlations

We miss an analysis of the LEP-2 data in terms of j_{had}^{tot}, but see Holt 2001 [38] and Sachs 2003 [29].

Including more measurements from LEP*II* solves this problem, reducing the correlation. The final result of $M_Z = 91\ 186.9 \pm 2.3$ MeV [8] is in very good agreement with the result of the standard lineshape fit $M_Z = 91\ 187.6 \pm 2.1$ MeV [9] with only slightly increased error.

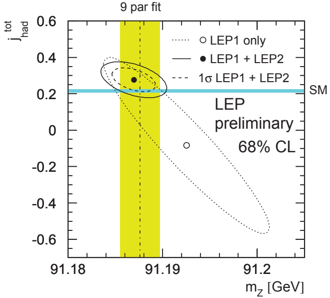

Figure 2: Correlation between the mass of the Z and $j_{\rm had}^{\rm tot}$. Results are shown for LEP*I* data only and for a combined fit to LEP*I* and LEP*II* data. The yellow band indicates the 1 σ error from the 9 parameter fit.

Figure 2 : K. Sachs, "Standard model at LEP II", talk held at Moriond 2001, fig. 2 [29]

Correlations

LEP experiments use cross-section and forward-backward asymmetry results from $\sqrt{s} \sim M_Z$ and LEP II. OPAL and L3 have reported preliminary results which are given in Table 1, and are compared to the value obtained by VENUS [9] using data at $\sqrt{s} \sim 60$ GeV and preliminary LEP I S-Matrix results. The results are consistent with each other, and with the $\mathcal{SM}$ prediction $j_{had}^{tot} = 0.22$.

Expt	Data	j_{had}^{tot}
L3:	LEP I + LEP II	0.30 ± 0.10
OPAL:	LEP I + LEP II	0.21 ± 0.12
VENUS:	VENUS + LEP I	0.20 ± 0.08

Table 1: Measurements of j_{had}^{tot}

Figure 3 : P. Holt, “Fermion pair production above the Z^0 resonance”, talk held at HEP 2001, table 1 [38]

Fortran programs: ZPOLE and SMATASY/ZFITTER

An older version of the Fortran test package **ZPOLE** (Leike/Riemann, v.0.5, July 1991) is available on request.
It was used for the numerics of [1]

Older ZFITTER versions had an **S-Matrix interface ZUSMAT**.
ZUSMAT was used for analysing the total cross sections, but could not treat asymmetries.

Both codes got later replaced by **SMATASY**, in order to have the full functionality of the **ZFITTER** [7, 8, 9] radiative corrections.

The actual Fortran program for the S-matrix Z line shape approach

M. Grünewald, S. Kirsch, T. Riemann 1994 [3]
SMATASY v.6.42.01 = SMATA642 (2 June 2005)
available at `https://gruenew.web.cern.ch/gruenew/smatasy.html`

Summary

- The S-matrix approach is absolutely independent of the Standard Model approach.
- The degrees of freedom are, at minimum:
  M_Z
  Γ_Z
  R – the residue of the Z resonance, *per scattering channel*
  J – the value of the γZ interference, *per scattering channel*
- So we have at least four degrees of freedom.
  This deserves at least five data points as a function of s.
- Asymmetries may be described as well as σ_{tot}.
- For a exact numerical analysis of data, an accurate description of QED corrections is mandatory.
  This has been realised by combining SMATASY with ZFITTER.
- With so much more statistics at the Fcc-ee compared to LEP-1 and LEP-2:

  The S-matrix approach might gain at the Fcc-ee even more interest as an alternative to the Standard Model approach.

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