

The (theoretical) challenge of precise luminosity measurement

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FCC-ee Physics Workshop (TLEP9)

Scuola Normale Superiore, Pisa

February 3-5, 2015

- Luminosity & theory
- The LEP & flavour factories era
 - Radiative corrections and Monte Carlo generators
 - $\mathcal{O}(0.1)\%$ theoretical accuracy
 - two-loop Bhabha
 - vacuum polarization
- $e^+e^- \rightarrow \gamma\gamma$ vs Bhabha
- How to improve?
- Examples at FCC- ee energies
- Conclusions and outlook

Luminosity & Theory

- Luminosity is a machine (process independent) parameter entering every experimental cross-section

$$\frac{N_{obs}}{\mathcal{L}} = \sigma \rightarrow \mathcal{L} = \frac{N_{obs}^X}{\sigma_{theory}^X}$$

- in order to minimize $\delta\mathcal{L}$, the reference process X has to **have large statistics, be well calculable theoretically, be cleanly detectable** (small systematics)
- the best choice are QED processes, in particular **Bhabha**
 - ★ at small angles at LEP
 - huge statistics, by far dominated by photon t -channel contribution (only QED, no “ Z contamination”)
 - ★ at large angles at flavour factories
 - no need of dedicated detectors, also here dominated by t -channel photon exchange
- to reduce the **theoretical error on σ** all the **relevant radiative corrections (RC)** must be included

$$R_{\text{inv}}^0 = \frac{\Gamma_{\text{inv}}}{\Gamma_{ll}} = \sqrt{\frac{12\pi R_l^0}{\sigma_{\text{had}}^0 m_Z^2}} - R_l^0 - (3 + \delta_\tau)$$

- assuming lepton universality

$$(R_{\text{inv}}^0)_{\text{exp}} = N_\nu \left(\frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{ll}} \right)_{\text{SM}}$$

- from LEP Z -peak measurements

$$N_\nu = 2.9840 \pm 0.0082$$

$$\delta N_\nu \simeq 10.5 \frac{\delta n_{\text{had}}}{n_{\text{had}}} \oplus 3.0 \frac{\delta n_{\text{lept}}}{n_{\text{lept}}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta \mathcal{L}}{\mathcal{L}} = 0.061\% \implies \delta N_\nu = 0.0046$$

ADLO, SLD and LEPEWWG, Phys. Rept. 427 (2006) 257, hep-ex/0509008

- δN_ν severely affected by luminosity uncertainty
(theory dominated at LEP)

- At high energies and luminosities, the *beamstrahlung* effect (beam loss of energy due to beam-beam interaction) has to be accounted for

$$\sigma_{th}(s) = \int dz_1 \int dz_2 \mathcal{D}_{bs}(z_1, z_2) \int d\sigma_{th}(z_1 z_2 s) \Theta(cuts)$$

- to some extent, \mathcal{L} has a continuum spectrum as a function of z_i rather than being a number
- The function $\mathcal{D}_{bs}(z_1, z_2)$ is determined by the machine parameters and specs: it must be known and parameterized with high accuracy
- ★ How well can it be known?

QED Radiative Corrections

- Large part of RC in Bhabha is due to photonic corrections, driven at $\mathcal{O}(\alpha)$ by $\alpha \times$ the collinear log $L = \log \frac{st}{um_e^2} - 1$

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	\dots

- $\star L \simeq \log(s/m_e^2) - 1 \simeq 14$ at flavour factories (large angle)
- $\star L \simeq \log(-t/m_e^2) - 1 \simeq 16$ at LEP1 or FCC- ee -Z (small angle)
- $\star L \simeq \log(-t/m_e^2) - 1 \simeq 19$ at FCC- ee -t

- a $\sim 18\%$ increase of QED RC impact from LEP1/FCC- ee -Z to FCC- ee -t is naively expected

- The calculations must be implemented into MC event generators
 - any complex event selection criteria can be accounted for
 - realistic simulation of the process
 - A. Arbuzov *et al.*, Phys. Lett. B **383** (1996) 238 [hep-ph/9605239]
 - S. Jadach *et al.*, Physics at LEP2, vol. 2 [hep-ph/9602393]
 - S. Actis *et al.*, Eur. Phys. J. C **66** (2010) 585 [arXiv:0912.0749]
- MCs for LEP
 - ★ **BHLUMI** - Jadach *et al.*
 - ★ **SABSPV** - Cacciari *et al.*
 - ★ **BHAGEN95** - Remiddi *et al.*
 - ★ **NLLBHA** - Arbuzov *et al.*
- MCs for flavour factories
 - ★ **BabaYaga** (**3.5** and **NLO**) - C.M.C.C. *et al.*
 - ★ **BHWIDE** - Jadach *et al.*
 - ★ **MCGPJ** - Arbuzov *et al.*

Monte Carlos building blocks

In order to achieve a $\mathcal{O}(0.1\%)$ accuracy, the most precise event generators implement

- Leading Log QED RC up to all orders (QED PS, YFS, SF,...)
- exact QED NLO corrections (matched with LL)
- relevant weak corrections (at LEP energies)
- photon vacuum polarization corrections (see later)
- part of QED NNLO corrections
- fully exclusive event generation

Besides technical reliability, at present the main sources of theoretical error in MCs are

- ★ partial inclusion of QED NNLO: the error starts at $\mathcal{O}(\alpha^2)$
→ this can be improved in a “short” time
- ★ vacuum polarization: hadronic contribution depends on low energy data
→ upcoming measurements are essential

My personal update of LEP1 theoretical error, Febr. 2003 (red/magenta)

Type of correction/error	Ref.[1]	Ref. [2]	Ref. [3]	My update
Technical precision	—	(0.030%)	(0.030%)	0.030%
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027%	0.027%	0.027%
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	0.015%	0.015%
Vacuum polarization	0.04%	0.04%	0.040%	0.025%
Light pairs	0.03%	0.03%	0.010%	0.010%
Z-exchange	0.015%	0.015%	0.015%	0.015%
Total	0.11%	0.061% (0.068)	0.054% (0.061)	0.53%

[1] A. Arbuzov *et al.* LEP Working Group 1996, Phys. Lett. B **383** (1996) 238

[2] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver
arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262

[3] G. Montagna, M. Moretti, O. Nicosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459**
(1999) 649

S. Jadach

April 21, 2005

- the impressive theoretical accuracy has been achieved with a hard work of many groups and comparing independent calculations/codes

Main conclusion of the Luminosity Section of the WG Report

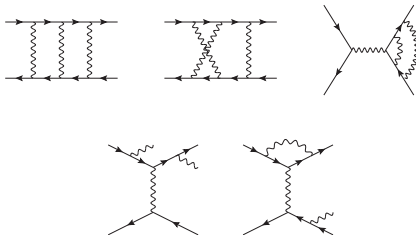
Putting the various sources of uncertainties (for large-angle Bhabha) all together...

Source of error (%)	Φ -factories	$\sqrt{s} = 3.5$ GeV	B -factories
$ \delta_{VP}^{err} $ [Jegerlehner]	0.00	0.01	0.03
$ \delta_{VP}^{err} $ [HMNT]	0.02	0.01	0.02
$ \delta_{SV,\alpha^2}^{err} $	0.02	0.02	0.02
$ \delta_{HH,\alpha^2}^{err} $	0.00	0.00	0.00
$ \delta_{SV,H,\alpha^2}^{err} $ [in progress]	0.05	0.05	0.05
$ \delta_{pairs}^{err} $	0.03	0.016	0.03
$ \delta_{total}^{err} $ linearly	0.12	0.1	0.13
$ \delta_{total}^{err} $ in quadrature	0.07	0.06	0.06

- For the experiments in proximity of the ψ/Υ 's resonances, the accuracy slightly deteriorates
- ★ The present error estimate appears to be rather robust and sufficient for high-precision luminosity measurements. It is comparable with that achieved for small-angle Bhabha luminosity monitoring at LEP/SLC

NNLO Bhabha calculations

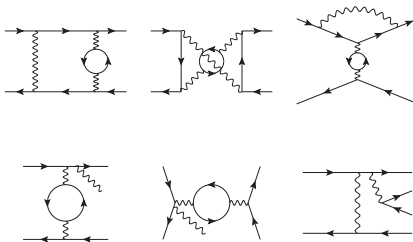
- **Photonic corrections** A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- **Electron loop corrections**

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280

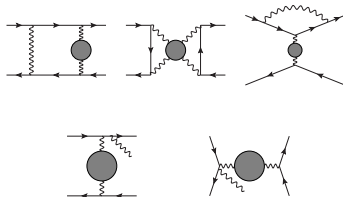
S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26



- Heavy fermion and hadronic loops

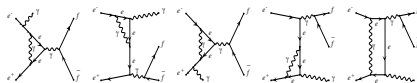
R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601
S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419

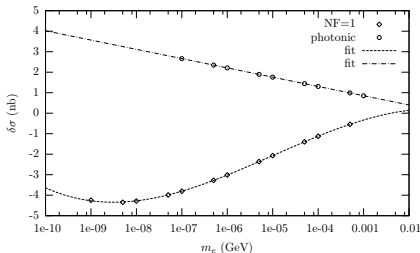
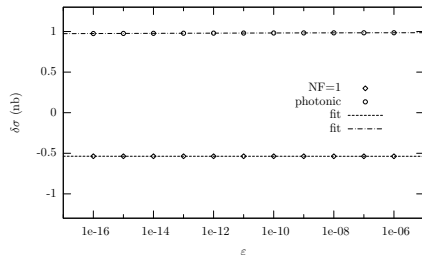


Comparison with (a subset of) NNLO

Comparison of $\sigma_{SV}^{\alpha^2}$ calculation of BabaYaga@NLO with

Using realistic cuts for luminosity @ KLOE

- Penin (photonic): switching off the vacuum polarisation contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut-off (left plot) and of a fictitious electron mass (right plot)



★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{SV}^{\alpha^2}(\text{Penin}) - \sigma_{SV}^{\alpha^2}(\text{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

Vacuum Polarization

- The running of the EM constant α is an important effect
- $\alpha \rightarrow \alpha(q^2) \equiv \frac{\alpha}{1 - \Delta\alpha(q^2)}$ $\Delta\alpha(q^2) = \Delta\alpha_{e,\mu,\tau,\text{top}}(q^2) + \Delta\alpha_{\text{had}}^{(5)}(q^2)$
- $\Delta\alpha_{\text{had}}^{(5)}$ is an **intrinsically non-perturbative** contribution. It can be calculated from $e^+e^- \rightarrow \text{hadrons}$ **data** using dispersion relations

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{q^2\alpha}{3\pi} \left[\oint_{4m_\pi^2}^{E_{\text{cut}}^2} \frac{R_{\text{had}}^{\text{data}}(s)}{s(s-q^2)} ds + \oint_{E_{\text{cut}}^2}^{\infty} \frac{R_{\text{had}}^{\text{pQCD}}(s)}{s(s-q^2)} ds \right]$$

- it is affected by an error, due to low energy data on $\sigma_{\text{had}}(s)$. In turn, it reflects on Bhabha predictions.
New low energy measurements are extremely important.
- at present (F. Jegerlehner '10)

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = 0.027510 \pm 0.000218 \quad [\text{or } \pm 0.000135]$$

$$\alpha^{-1}(M_Z^2) = 128.961 \pm 0.030 \quad [\text{or } \pm 0.018]$$

- Different compilations exist, available in routines by F. Jegerlehner, T. Teubner et al., F. Ignatov, **returning an error**

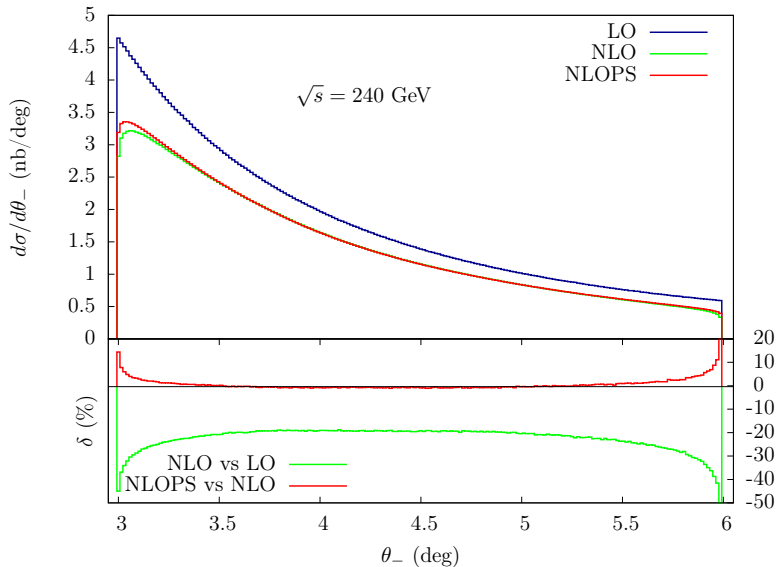
BabaYaga for Bhabha at FCC- ee , examples

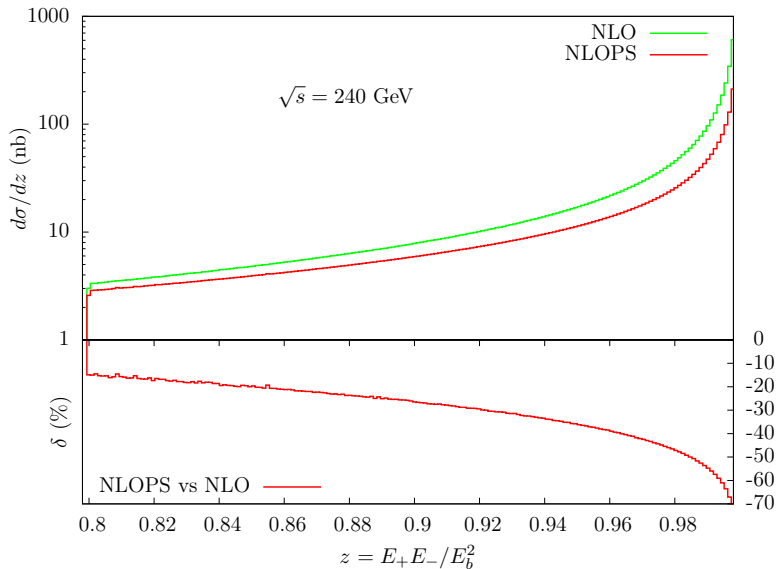
- example with **BabaYaga** run at FCC- ee , in the small angle regime
- e.g. $3^\circ < \theta_- < 6^\circ$, $174^\circ < \theta_+ < 177^\circ$, $E_- E_+ / E_b^2 > z_{min} = 0.8$

	FCC- ee -Z 91 GeV	FCC- ee -W 160 GeV	FCC- ee -H 240 GeV	FCC- ee -t 350 GeV
σ_0^{VP} (nb)	36.0030	11.8062	5.28998	2.50709
+ Z	+0.064%	-0.062%	-0.044%	-0.030%
+ QED NLO	-17.41%	-18.73%	-19.57%	-20.35%
+ QED h.o.	+0.54%	+0.66%	+0.71%	+0.77%
VP	+5.17%	+6.27%	+7.14%	+7.99%
$\delta\Delta\alpha_h$ ¹	$\pm 0.021\%$	$\pm 0.027\%$	$\pm 0.030\%$	$\pm 0.032\%$

- ★ Once full NNLO is merged into MCs, **VP error will become the dominant one if the accuracy has to be pushed up to $\mathcal{O}(0.01\%)$**

¹ This uncertainty is by far dominated by the error on $\alpha(t)$





$$e^+e^- \rightarrow \gamma\gamma$$

- $e^+e^- \rightarrow \gamma\gamma$ could be used to **cross-check independently \mathcal{L} measurements**
 - ★ At present, its theoretical accuracy is similar to Bhabha (NLO + h.o.)
 - ★ **Advantages:** no Z exchange diagrams (at LO), no photon VP corrections (up to NNLO)
 - ★ **Disadvantages:** lower x-section, less clean signal (?)
 - ★ Efficiency in detecting $\gamma\gamma$ events?
- e.g. at least 2 γ 's with $10^\circ < \theta_\gamma < 170^\circ$, $E_1 E_2 / E_b^2 > z_{min} = 0.8$

	FCC-ee-Z	FCC-ee-W	FCC-ee-H	FCC-ee-t
σ_0 (pb)	60.962	19.785	8.793	4.135
+ QED NLO	-8.61%	-9.06%	-9.40%	-9.71%
+ QED h.o.	-0.37%	-0.41%	-0.42%	-0.44%

- ★ **Could it be used as normalization for Bhabha related measurements (e.g. $\alpha(-t)$)?**

see e.g. A. B. Arbuzov et al., Eur. Phys. J. C **34** (2004) 267

Conclusions

- On the theory side, precise luminosity determination at FCC- ee can greatly benefit from past experience, LEP and flavour factories
- Tools to reach a theoretical accuracy at less than 0.1%, with small angle Bhabhas, are already on the market
- There is room for improvements, to reach $\mathcal{O}(0.01\%)$ accuracy
 - new data at low energies for hadronic contribution to VP
 - systematically include QED NNLO into event generators
 - include in MCs exact $\mathcal{O}(\alpha)$ weak corrections to full Bhabha
- QED exponentiation still needed to achieve high accuracy
- Can $e^+e^- \rightarrow \gamma\gamma$ be a cross-reference process?
- To which accuracy will beamstrahlung be under control?

BACKUP SLIDES

Theoretical accuracy, comparisons with NNLO

- the theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) (though large NNLO RC already included by expon. and by $\mathcal{O}(\alpha)$ LL \times finite-NLO)
- ★ The NNLO QED RC to Bhabha have been calculated but not available into EGs yet
- e.g., **BabaYaga** formulae can be truncated at $\mathcal{O}(\alpha^2)$ to be consistently compared with all the classes of NNLO corrections

$$\sigma^{\alpha^2} = \sigma_{\text{SV}}^{\alpha^2} + \sigma_{\text{SV,H}}^{\alpha^2} + \sigma_{\text{HH}}^{\alpha^2}$$

- $\sigma_{\text{SV}}^{\alpha^2}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2)$ \rightarrow compared with the corresponding available NNLO QED calculation
- $\sigma_{\text{SV,H}}^{\alpha^2}$: one-loop soft+virtual corrections to single hard bremsstrahlung \rightarrow **presently** estimated relying on existing (partial) results
- $\sigma_{\text{HH}}^{\alpha^2}$: double hard bremsstrahlung \rightarrow compared with the exact $e^+e^- \rightarrow e^+e^-\gamma\gamma$ cross section, to register **really negligible differences (at the 1×10^{-5} level)**

Resummation beyond α^2

- ★ with a complete 2-loop generator at hand, (leading-log) resummation beyond α^2 can be neglected?

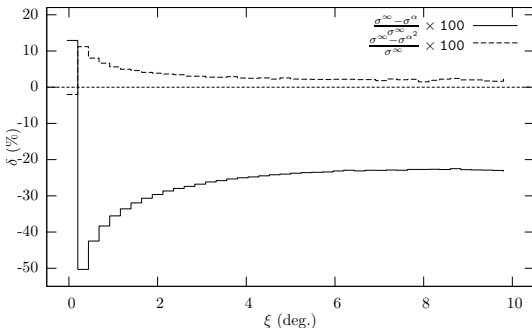


Figure: Impact of α^2 (solid line) and resummation of higher order ($\geq \alpha^3$) (dotted) corrections on the acollinearity distribution at DAΦNE

- ★ resummation beyond α^2 still important