Precision Observables in the SM: a reexamination

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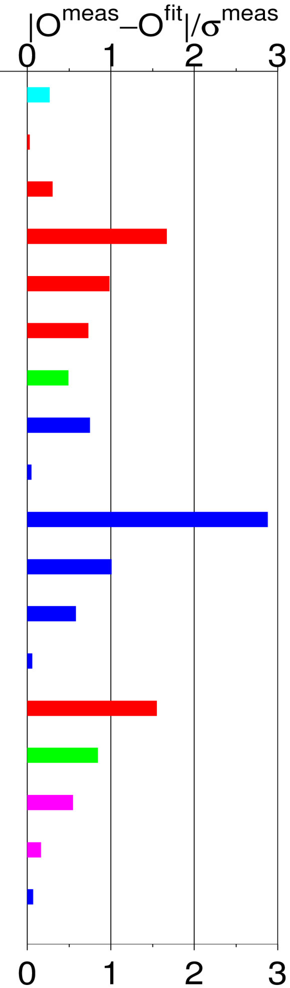
Outline

- The Lep and Tevatron legacy
- Implication of the new information on m_H
- Re-analyzing the prediction of m_W
- Conclusions

The LEP and Tevatron legacy

		Measurement	Fit
	$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
★	m_Z [GeV]	91.1875 ± 0.0021	91.1874
★	Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
★	σ_{had}^{0} [nb]	41.540 ± 0.037	41.478
	R_l	20.767 ± 0.025	20.742
	$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
	$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
	R_b	0.21629 ± 0.00066	0.21579
	R_c	0.1721 ± 0.0030	0.1723
	$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
	$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
	A_b	0.923 ± 0.020	0.935
	A_c	0.670 ± 0.027	0.668
	A_l(SLD)	0.1513 ± 0.0021	0.1481
	$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
★	m_W [GeV]	80.385 ± 0.015	80.377
	Γ_W [GeV]	2.085 ± 0.042	2.092
	m_t [GeV]	173.20 ± 0.90	173.26

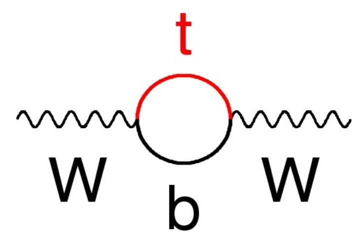


March 2012

precision $\sim 5 - 1 \times 10^{-3}$

★ better than 10^{-3}

Sensitivity to quantum effects



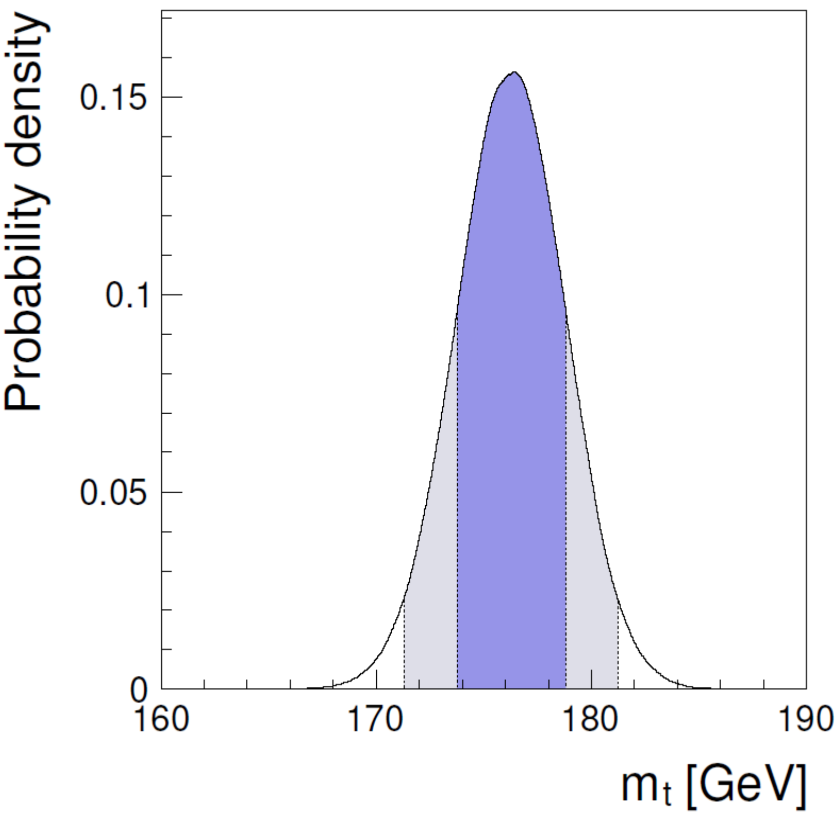
$$\sim \alpha \frac{M_t^2}{m_W^2}$$

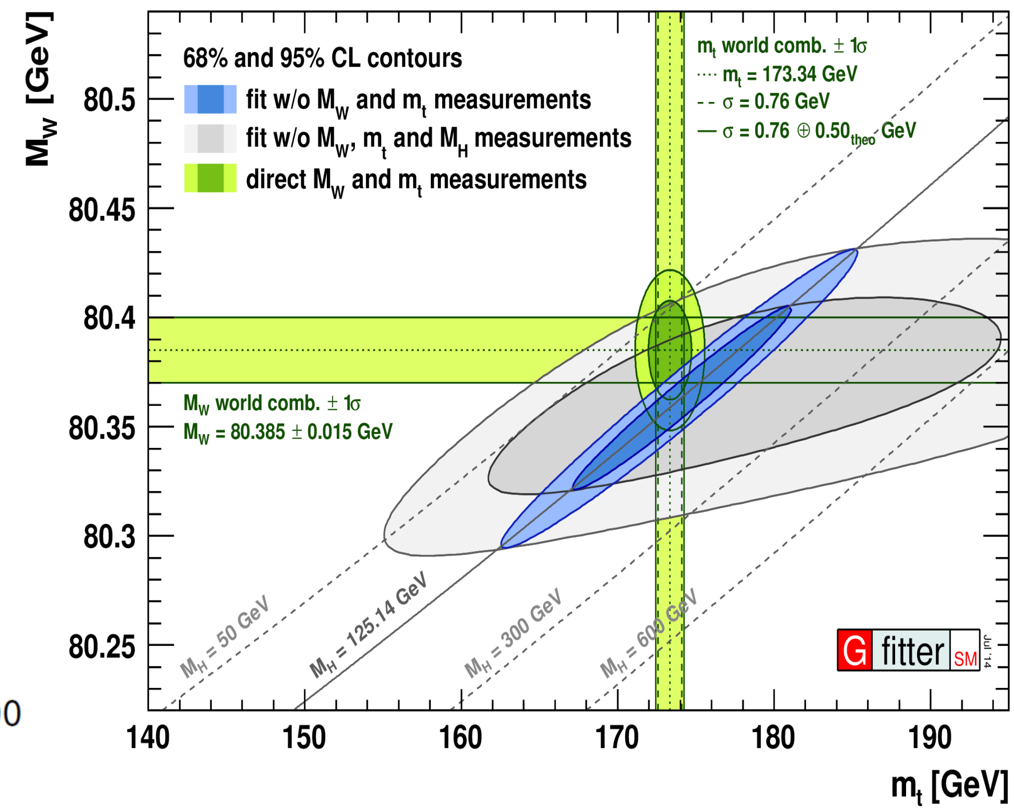
$$\sim \alpha \log \frac{m_H^2}{m_Z^2}$$

$$R_l = \frac{\Gamma_{had}}{\Gamma_l}, \quad R_q = \frac{\Gamma_q}{\Gamma_{had}}$$

Predictions for m_W, M_t, m_H

The LEP and Tevatorn legacy: top and W

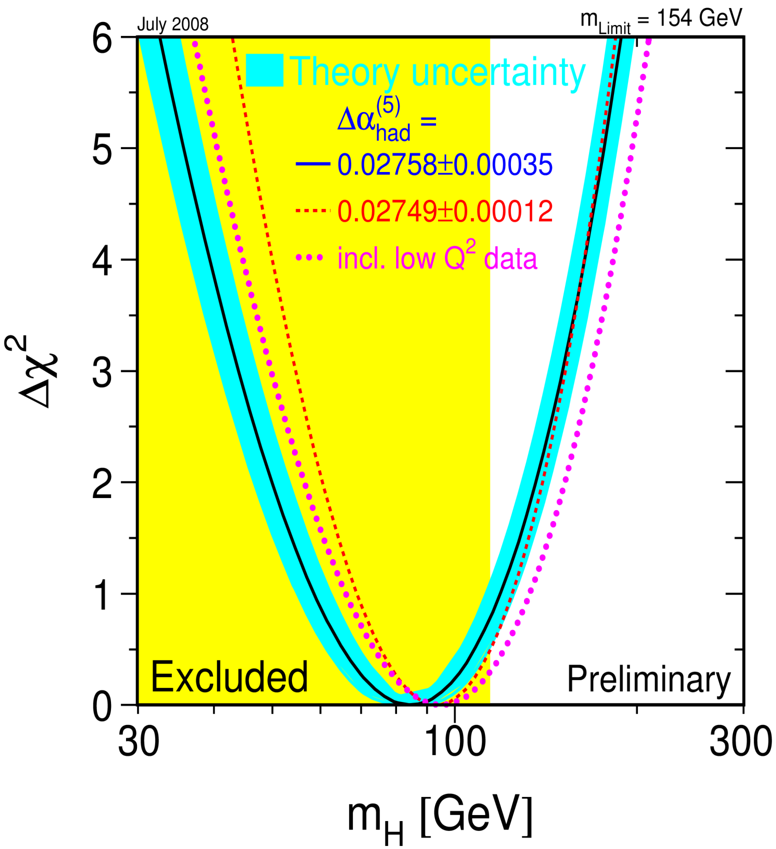


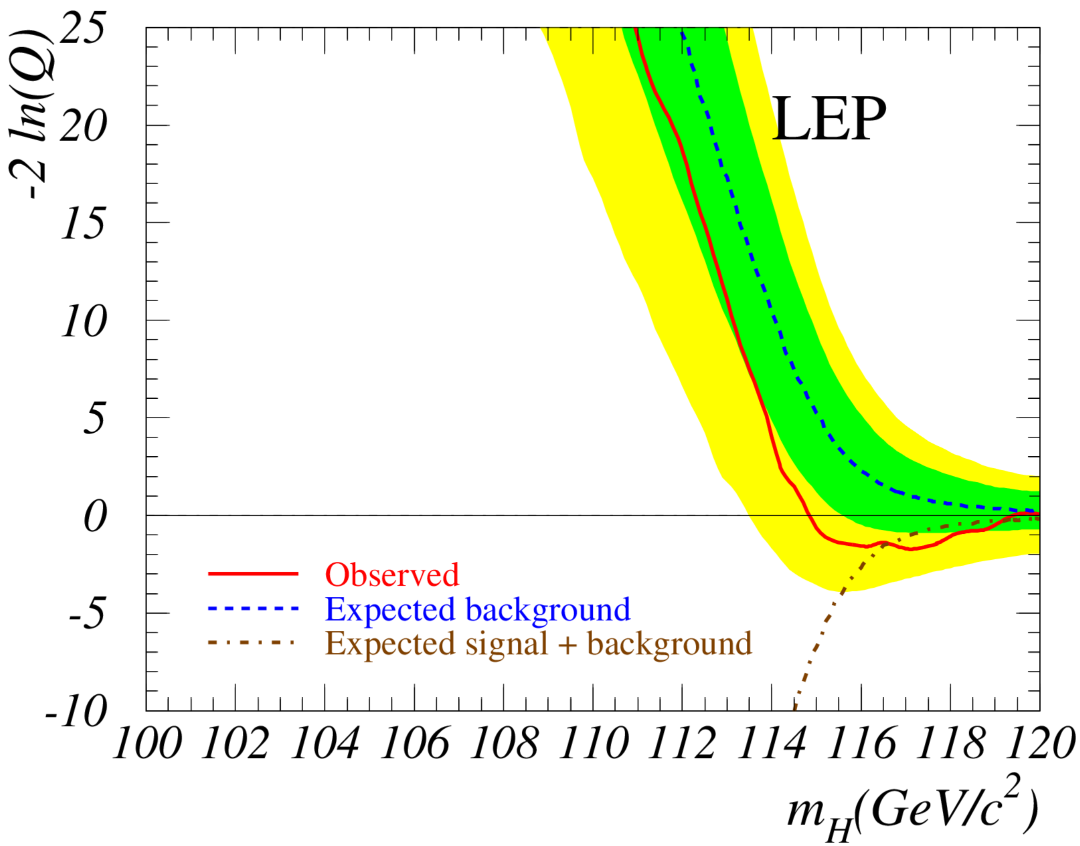


Ciuchini, Franco, Mishima, Silvestrini (13)

Very good agreement between indirect and direct determinations

The LEP and Tevatron legacy: Higgs



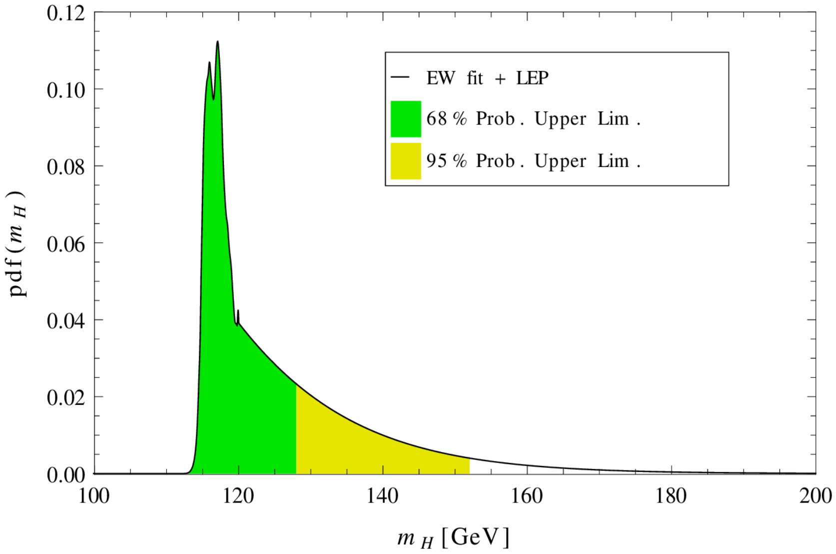


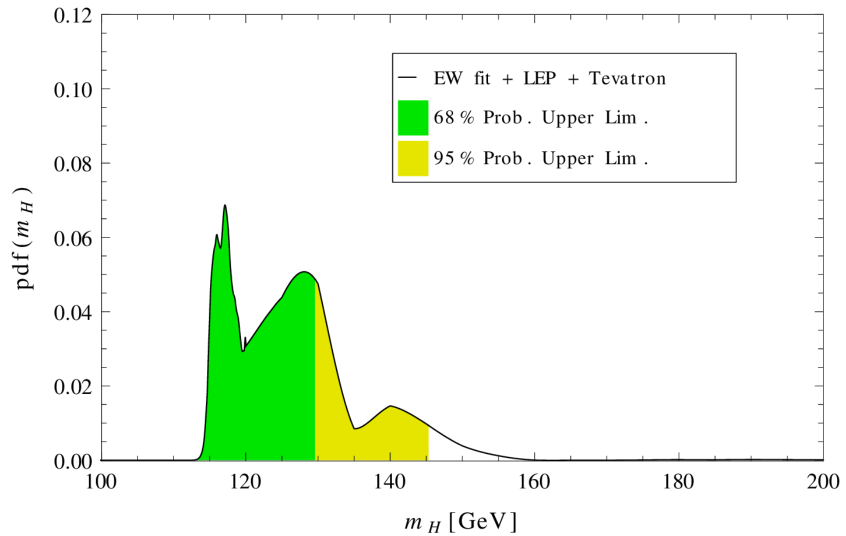
$$Q = \frac{\mathcal{L}(s+b)}{\mathcal{L}(b)}$$

The LEP and Tevatron legacy: Higgs

Combining direct and indirect information: $\text{pdf}(m_h) \propto \frac{Q(m_h)e^{-(\chi^2/2)}}{m_h}$

D'Agostini, G.D.1999

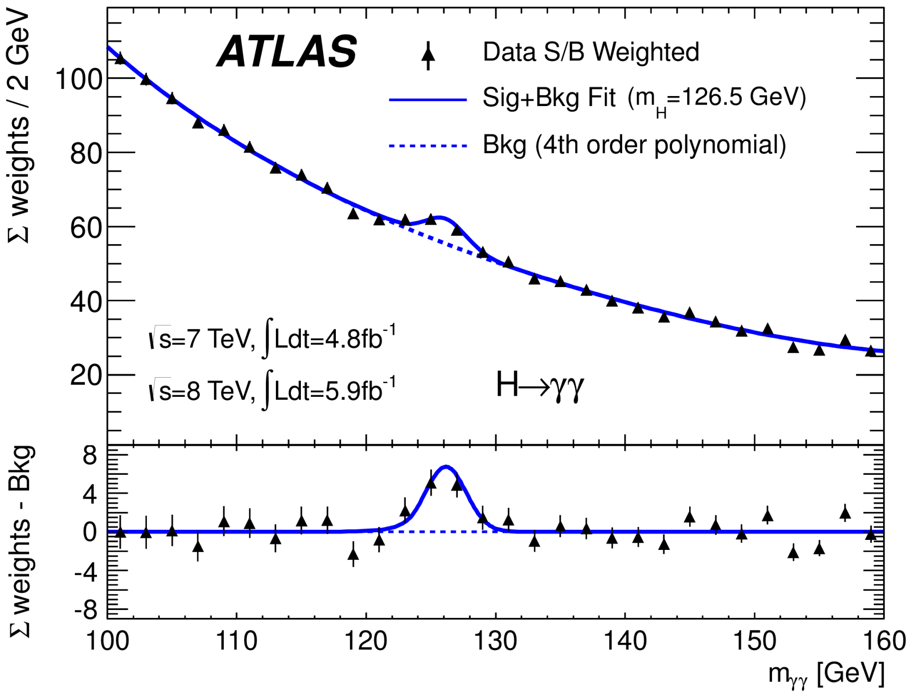


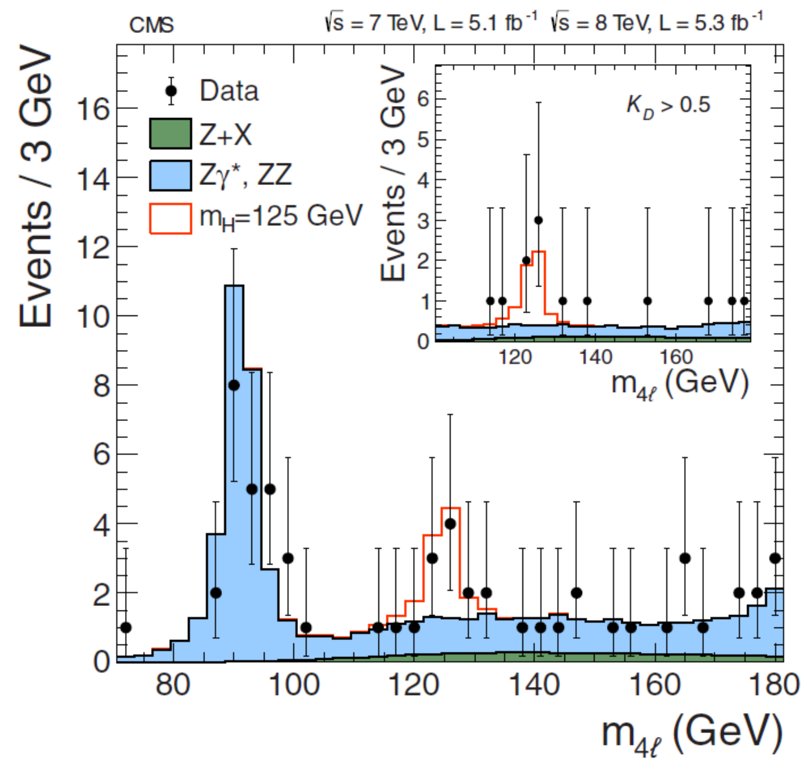


courtesy of S. Di Vita

The consistency of the (minimal) SM at the quantum level predicts a Higgs boson with mass between 110 and 160 GeV

LHC 4th of July 2012 news





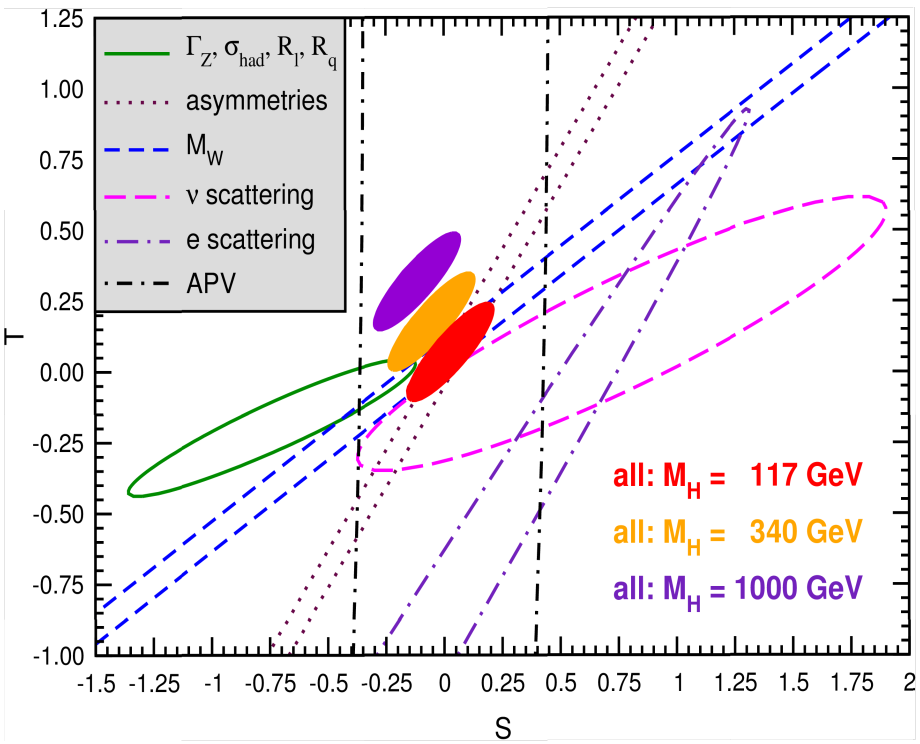
Clear evidence of a new particle
with properties compatible with those of the SM Higgs boson

$$m_H = 125.36 \pm 0.37\,(stat) \pm 0.18\,(syst)\ \text{GeV}\quad (\text{ATLAS})$$
$$m_H = 125.02^{+0.26}_{-0.27}\,(stat)\,^{+0.14}_{-0.15}\,(syst)\ \text{GeV}\quad (\text{CMS})$$
$$\frac{\delta m_H}{m_H} \sim 3\times 10^{-3}$$

SM is constrained

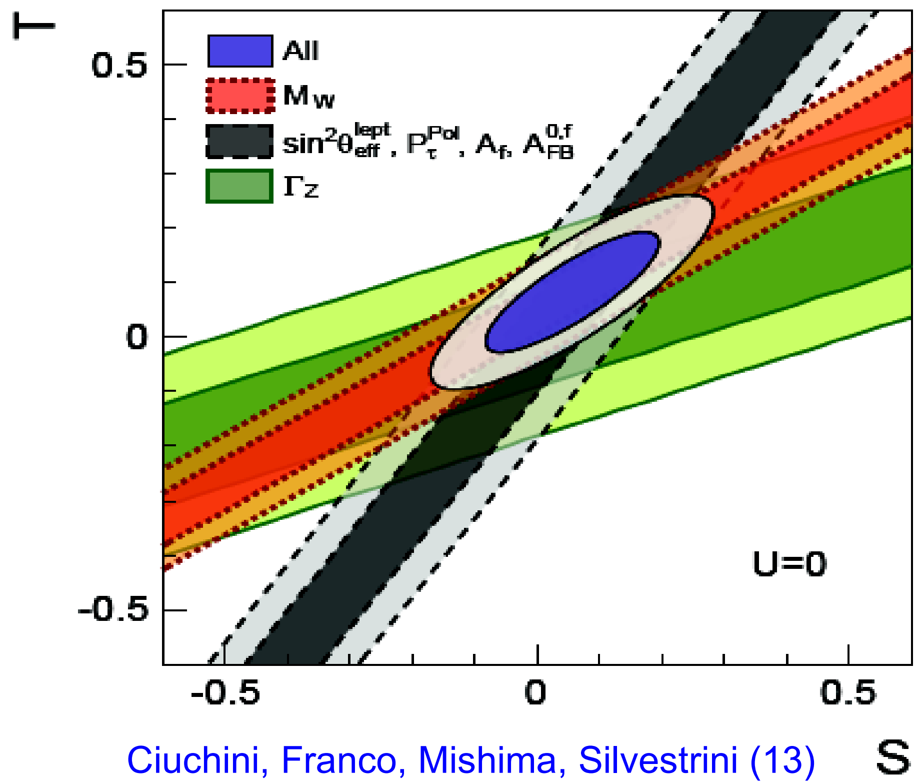
At the time of LEP we could envisage specific type of NP (extra Z, isosplitted (s)fermios, light sleptons etc.) that could allow a heavy Higgs in the EW fit ("conspiracy argument"). With the discovery of the Higgs boson, this is not any more possible

before



P.D.G. (11)

after



Ciuchini, Franco, Mishima, Silvestrini (13)

NP (if there) seems to be of the decoupling type at a high scale: to disentangle it one needs very precise SM predictions

Precision (pseudo)observable at the full two-loop level

- m_W obtained from α, G_μ and m_Z
  Δr known at the 2(+)--loop level
  in the OS scheme

$$M_W^2 = \frac{M_Z^2}{2}\left\{1+\left[1-\frac{4\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}(1+\Delta r)\right]^{1/2}\right\}$$

Freitas, Hollik, Walter, Weiglein (02); Awramik, Czakon (02); Onishchenko, Veretin (02),
Awramik, Czakon, Freitas, Weiglein (04); Chetyrkin, Kuehn, Steinhauser (95);
Faisst et al. (03); Chetyrkin et al. (06); Boughezal, Czakon (06)

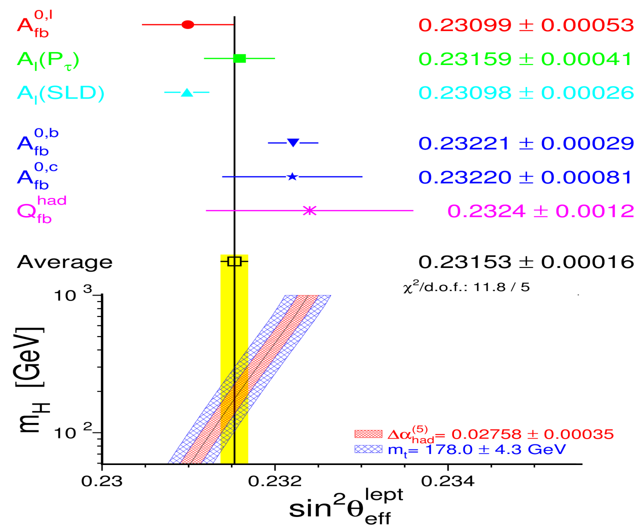
- $\sin^2\theta_{\rm eff}^{\rm lept}$ extracted from asymmetries

  $Z f\bar{f}$ vertex:

$$\rho^f\gamma^\mu\left[\tfrac{1}{4}I_3^f(1-\gamma_5)-Q^f\kappa^f\sin^2\theta_W\right]=\gamma^\mu(g_V^f-g_A^f\gamma_5)$$

$$\Rightarrow \sin^2\theta_{eff}^{\rm lept}=\frac{1}{4}\left[1-{\rm Re}\left(\frac{g_V^l}{g_A^l}\right)\right]={\rm Re}(\kappa^l)\sin^2\theta_W$$

Awramik, Czakon, Freitas, Weiglein (04-06);
Hollik, Meier, Uccirati (05-06)



Estimated Error of the Computation:

$$\begin{aligned}\delta m_W &\sim 4\,{\rm MeV}\\ \delta\sin^2\theta_{\rm eff}^{\rm lept} &\sim 1.5\times10^{-4}\end{aligned}$$

Freitas et al.

Re-analyzing the M_W determination

M_W is obtained from

$$M_W^2 = \frac{M_Z^2}{2}\left\{1+\left[1-\frac{4\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}(1+\Delta r)\right]^{1/2}\right\} \quad (1)$$

with Δr including all the two-loop corrections evaluated in the On-Shell scheme plus some partial three and four-loop contributions.

δM_W estimated by the size of the last known terms

few points:

- M_Z and M_W in (1) are not directly identified with the experimentally determined quantities;
- No resummation of lower-order contributions is performed in the OS scheme using (1);
- No estimate of δM_W via the comparison of results obtained in different renormalization scheme

Address these issues via an $\overline{\text{MS}}$ computation

Theory vs. Experimental masses

W and Z bosons are unstable particles. Different possible definitions of the masses

zero of the real part of the propagator

$$\Pi_{\mu\nu}(q^2) = \frac{-ig_{\mu\nu}}{q^2 - m_0^2 - A(q^2)} + \cdots \Rightarrow \begin{cases} m^2 = m_0^2 + \mathrm{Re}\, A(m^2) \\ s_p = m_0^2 + A(s_p) \end{cases} \quad s_p = M^2 - iM\Gamma$$

Real part of the complex pole

$$m^2 = M^2 - \mathrm{Im} A'(M^2) M\Gamma + \mathcal{O}(\alpha^3) = M^2 + \Gamma^2 + \mathcal{O}(\alpha^3)$$

Energy-dependent width

$$\sigma(s) \sim \frac{1}{(s-m^2)^2 + s^2 \frac{\Gamma^2}{m^2}}$$

Avoid pathological gauges

Constant width

$$\sigma(s) \sim \frac{1}{(s-M^2)^2 + M^2\Gamma^2}$$

Experimentally gauge boson masses are determined using a Breit-Wigner function with an energy-dependent width

$$\begin{aligned} m &= m_{\rm exp} \\ M &= K\, m_{\rm exp} \\ \Gamma &= K\, \Gamma_{\rm exp} \\ K &= \frac{1}{\left(1 + \frac{\Gamma_{\rm exp}^2}{m_{\rm exp}^2}\right)^{1/2}} \end{aligned}$$

M is the mass used in the OS calculation

m is the mass we use in the $\overline{\rm MS}$ calculation

$M_W - m_W \approx 27$ MeV

$\overline{\rm MS}$ formulation of SM radiative corrections

In the OS scheme large corrections are connect to OS Weinberg angle countertems

~3

$$\delta \sin^2\theta_W \sim \frac{\cos^2\theta_W}{\sin^2\theta_W}\alpha\frac{M_t^2}{m_W^2}$$

Absorb these contributions in a $\overline{\rm MS}$ definition of the angle $\sin^2\hat{\theta}_W(\mu)$

$$\frac{m_W^2}{m_Z^2\,\cos^2\hat{\theta}_W(\mu)} \equiv \hat{\rho} = 1 + \mathcal{O}(\alpha)$$

Basic idea: distinguish between couplings (g, g', $\overline{\rm MS}$) and masses (g v, m)

Radiative parameters

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\hat{\alpha}(m_Z)}{2m_W^2\sin^2\hat{\theta}_W(m_Z^2)}\left[1+\Delta\hat{r}_W\right], \qquad \hat{\alpha}(m_Z) = \frac{\alpha}{1-\Delta\hat{\alpha}(m_Z)}, \qquad \hat{\rho} = \frac{1}{1-\mathrm{Re}\left[\frac{A_{WW}(m_W^2)}{m_W^2} - \hat{c}^2\frac{A_{ZZ}(m_Z^2)}{m_W^2}\right]_{\overline{MS}}}$$

resummed

$$\sin^2\hat{\theta}_W(m_Z^2) = \frac{1}{2}\left\{1-\left[1-\frac{4\pi\hat{\alpha}(m_Z)}{\sqrt{2}G_\mu m_Z^2\hat{\rho}}(1+\Delta\hat{r}_W)\right]^{1/2}\right\}, \quad m_W^2 = \frac{\hat{\rho}\,m_Z^2}{2}\left\{1+\left[1-\frac{4\pi\hat{\alpha}(m_Z)}{\sqrt{2}G_\mu m_Z^2\hat{\rho}}(1+\Delta\hat{r}_W)\right]^{1/2}\right\}$$

$\overline{\text{MS}}$ electromagnetic coupling

$$e^2 = \frac{e_0^2}{1 - e_0^2\,\Pi_{\gamma\gamma}(0)} \Longrightarrow \hat{\alpha}(\mu) = \frac{\alpha}{1-\Delta\hat{\alpha}(\mu)}, \qquad \Delta\hat{\alpha}(m_z) \sim \mathcal{O}\left(\alpha \log\frac{m_f}{m_z}\right) \sim 6.5\times 10^{-2}$$

BFM gauge, resummation valid
also for the bosonic contribution

$$\Delta\hat{\alpha}(\mu) = -4\,\pi\,\alpha \left[\underbrace{\Pi^{(\mathrm{b})}_{\gamma\gamma}(0) + \Pi^{(\mathrm{l})}_{\gamma\gamma}(0)}_{\mathcal{O}(\alpha)} + \underbrace{\Pi^{(\mathrm{p})}_{\gamma\gamma}(0)}_{\mathcal{O}(\alpha\alpha_s)} + \underbrace{\left(\Pi^{(5)}_{\gamma\gamma}(0) - \mathrm{Re}\,\Pi^{(5)}_{\gamma\gamma}(m_z^2)\right)}_{\text{data}\quad \Delta\alpha^{(5)}_{\mathrm{had}}(m_z^2)} + \underbrace{\mathrm{Re}\,\Pi^{(5)}_{\gamma\gamma}(m_z^2)}_{\mathcal{O}(\alpha_s^2)}\right]_{\overline{MS}}$$

Recent evaluations of $\Delta\alpha^{(5)}_{\mathrm{had}}(m_z^2)$

$$\Delta\alpha^{(5)}_{\mathrm{had}}(m_z^2) = (275.7 \pm 1.0)\times 10^{-4}$$ Davier et al. (11)

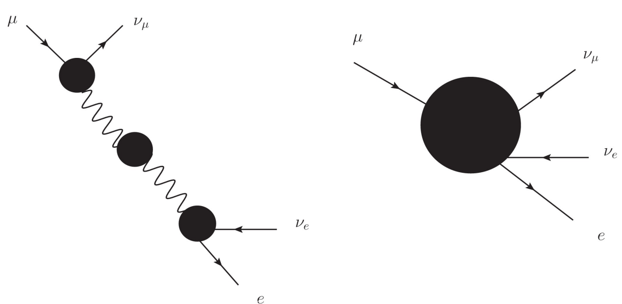
$$\Delta\alpha^{(5)}_{\mathrm{had}}(m_z^2) = (275.0 \pm 3.3)\times 10^{-4}$$ Burkhardt, Pietrzyk (11)

$$\hat{\alpha}(M_t) = (127.73)^{-1} \pm 0.0000003$$

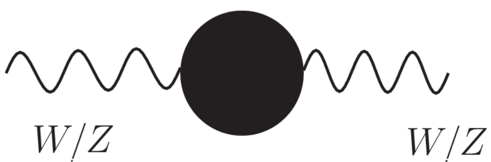
$\Delta\hat{r}_W\,,\ \hat{\rho}$

$$\Delta\hat{r}_W = \frac{\mathrm{Re}A^{(1+2)}_{WW}(m^2_W)}{m^2_W} - \frac{A^{(1+2)}_{WW}(0)}{m^2_W} + E^{(1+2)} + \delta^\epsilon\Delta\hat{r}^{(1)}_W + A^{(1)}_{WW}(0)B^{(1)}_W + \left(\frac{\mathrm{Re}A^{(1)}_{WW}(m^2_W)}{m^2_W} - \frac{A^{(1)}_{WW}(0)}{m^2_W}\right)\left(\frac{\mathrm{Re}A^{(1)}_{WW}(m^2_W)}{m^2_W} - \frac{A^{(1)}_{WW}(0)}{m^2_W} + E^{(1)}\right)\Bigg|_{\overline{MS}}\,.$$

$$\hat{\rho} = \frac{1}{1 - \mathrm{Re}\left[\frac{A_{WW}(m^2_W)}{m^2_W} - \hat{c}^2\frac{A_{ZZ}(m^2_Z)}{m^2_W}\right]_{\overline{MS}}}$$



E contribution (vertices+ box)
analytical, vacuum integrals



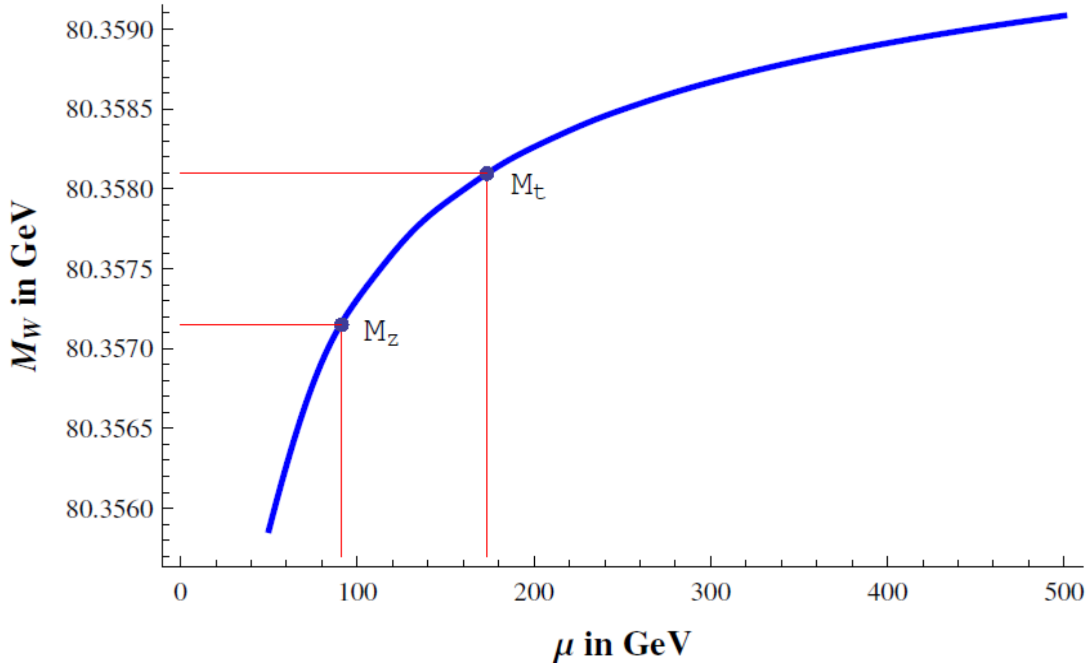
Self-energy contribution
Zero external momentum, analytical
Non vanishing external momentum
Numerical, Martin's loop functions

Martin (02,03)

Computation done in the R_ξ=1 gauge and cross-checked in the ξ=1 BFM gauge

m_w prediction in the $\overline{MS}$ framework

Check of the μ dependence



$100 \leq \mu \leq 200$ GeV, $\delta m_w \sim 1$ MeV

$50 \leq \mu \leq 500$ GeV, $\delta m_w \sim 3$ MeV

$m_w = m_w(M_t, \Delta\alpha^{(5)}_{\rm had}(m_z^2), m_h, \alpha_s, \ldots)$

$$m_W = 80.357 + 0.52749\left[\left(\frac{M_t}{173.34}\right)^2 - 1\right] - 0.50530\left[\frac{\Delta\alpha^{(5)}_{\rm had}(m_Z^2)}{0.02750} - 1\right] + \ldots \quad \text{GeV}$$

$$\left.\begin{aligned} m_W &= 80.357 \pm 0.009_{\rm par} \pm 0.003_{\rm th} \quad \text{GeV} \\ m_W^{\rm exp} &= 80.385 \pm 0.015 \quad \text{GeV} \end{aligned}\right\}$$

~1.5 σ away, $M_t \uparrow$, $\Delta\alpha^{(5)}_{\rm had}(m_Z^2) \downarrow$

Parametric uncertainties

The most important (by far) are due to the hadronic contributions and to the top mass, contributing both at the same level (≈½ 0.009), but

$$\frac{\delta M_t}{M_t} \sim 0.6 \times 10^{-3}, \quad \frac{\delta \Delta\alpha^{(5)}_{\rm had}(m_Z^2)}{\Delta\alpha^{(5)}_{\rm had}(m_Z^2)} \sim 1.2 \times 10^{-3}$$

$$\Delta\alpha^{(5)}_{\rm had}(m_Z^2) \equiv Re\,\Pi^h_{\gamma\gamma}(m_Z^2) - \Pi^h_{\gamma\gamma}(0) = -\frac{\alpha m_Z^2}{3\pi} Re \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s - m_Z^2 - i\epsilon)}$$

$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to hadrons)}{\sigma_{tot}(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

We use $\Delta\alpha^{(5)}_{\rm had}(m_Z^2) = (275.0 \pm 3.3) \times 10^{-4}$ that employ only experimental data in R below 12 GeV

Using $\Delta\alpha^{(5)}_{\rm had}(m_Z^2) = (275.7 \pm 1.0) \times 10^{-4}$ (PQCD down to 1.8 GeV) 0.009 → 0.007

Future improvements in the 2-5 GeV energy region (VEPP-4, BESIII)

Parametric uncertainties: M_t

Predictions of M_w are given in terms of the pole top mass identified with the Tevatron-LHC number. Is that number really the "pole" (what is?) mass?

Monte Carlo are used to reconstruct the top pole mass form its decays products. Modeling of the event that contain jets, missing energy and initial state radiation is required. Extraction of the top mass in hadron collisions with a precision below $O(\Gamma_{top})$ ~ 1 GeV is extremely challenging

$$M_t = M_t^{MC} + \Delta, \qquad \delta M_t^{MC} = \pm 0.76 \text{ GeV}, \qquad \Delta = ?$$

- M_t^{MC} is interpreted as M_t within the intrinsic ambiguity in the definition of M_t

  $\Delta \sim O(\Lambda_{QCD})$ ~250-500 MeV

  Mangano (13),

- M_t^{MC} ⟶ $M_t^{SD}(\Gamma_t)$ ⟶ M_t (M_t^{SD} better defined theoretically)

  ~1 GeV ~ 0.5 GeV

  Moch, (14)

Alternative: $M_t^{\overline{MS}}$ is free of renormalon ambiguity. It can be extracted from total production cross section $\sigma(t\bar{t} + X)$

$$M_t^{\overline{MS}}(M_t) = 162.3 \pm 2.3\,\text{GeV} \to M_t = 171.2 \pm 2.4\,\text{GeV}$$

Moch, (14)

quite low value

δm_w ~18 MeV

Caution

Fermion masses are parameters of the QCD Lagrangian, not of the EW one.
The Yukawa (and gauge) couplings are the parameters of the EW Lagrangian.
The vacuum is not a parameter of the EW Lagrangian.

$\overline{MS}$ masses are gauge invariant objects in QCD, not in EW, Yukawas are
A $\overline{MS}$ mass in the EW theory has not a unique definition (RGE is not unique).
It depends upon the definition of the vacuum:

- Minimum of the tree-level potential
  $\rightarrow$ $M_t^{\overline{MS}}$ g.i. but large EW corrections in the relation pole-$\overline{MS}$ mass ($\sim M_t^4$)
  But direct extraction of $M_t^{\overline{MS}}$ requires EW correction Jegerlehner, Kalmykov, Kniehl, (12)

- Minimum of the radiatively corrected potential
  $\rightarrow$ $M_t^{\overline{MS}}$ not g.i. (problem? $\overline{MS}$ mass is not a physical quantity)
  no large EW corrections in the relation pole-$\overline{MS}$ mass

Theoretical uncertainties

OS and $\overline{\text{MS}}$ calculations equivalent at the 2(+)--loop level but $\overline{\text{MS}}$ one includes resummation of lower-order contributions.
Numerical difference between the two-results can be taken as an estimate of the size of the unknown higher-order contributions.

Rule of thumb: $\delta m_w \sim -15\,\delta(\Delta r)$ → $1 \times 10^{-4}\,(\Delta r) \to -1.5\,\text{MeV}\,(m_W)$

Large one-loop terms: $\Delta\alpha \sim 6.5 \times 10^{-2}$ $\quad X \equiv \frac{\cos^2\theta_W}{\sin^2\theta_W}\text{Re}\left[\frac{A_{WW}(m_W^2)}{m_W^2} - \frac{A_{ZZ}(m_Z^2)}{m_Z^2}\right] \sim -3.3 \times 10^{-2}$

Three-loop contribution:

	$(\Delta\alpha)^3$	$(\Delta\alpha)^2 X$	$\Delta\alpha X^2$	$\Delta\alpha X^3$	$\Delta\alpha X$	X^2
$\text{O}(\alpha^3) \times 10^4$	~ 2.7	~ -1.4	~ 0.7	~ -0.4		
$\text{O}(\alpha^2\alpha_s) \times 10^4$					~ 2.4	~ -1.2

Three-loop effects are several MeV

Difference between the OS and MS calculation $\delta m_w \sim 6$ MeV

Conclusions

- The SM works fine
- The present level of precision in the theoretically determination of m_W is comparable to the present experimental error
- A solid prediction of m_W with a precision below 10 MeV requires a better understanding of the top pole mass (and an improvement in $\Delta\alpha^{(5)}_{\rm had}(m_z^2)$)
- A prediction of m_W with a precision 1 MeV seems not feasible in the near future