Electroweak precision tests: theory uncertainties

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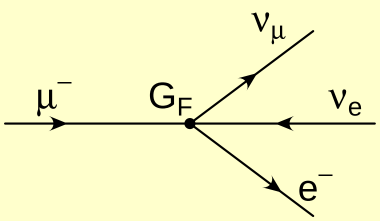
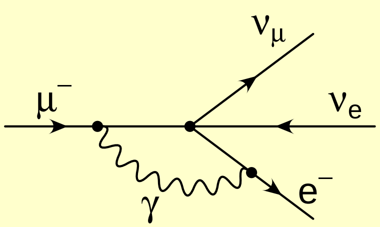
1. **Electroweak precision observables**
2. **Current status of SM loop results**
3. **Future projections**
4. **Parametrization of new physics**

Electroweak precision observables

W mass

μ decay in Fermi Model



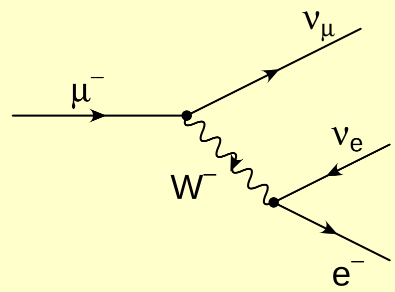


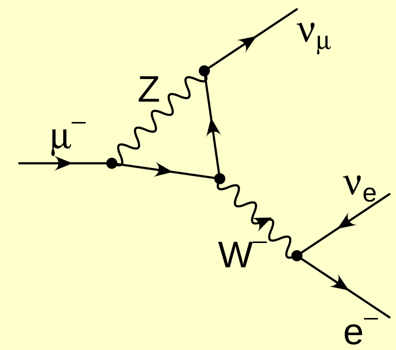
← QED corr. (2-loop)

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1 + \Delta q)$$

Ritbergen, Stuart '98
Pak, Czarnecki '08

μ decay in Standard Model





$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r)$$

electroweak corrections

Z-pole observables

$e^+e^- \to f\bar{f}$ for $\sqrt{s} \sim m_Z$:

$$\sigma = \mathcal{R}_{\text{ini}} \left[12\pi \frac{\Gamma_{ee}\Gamma_{ff}}{(s - m_Z^2)^2 - m_Z^2\Gamma_Z^2} (1 - \mathcal{P}_e\mathcal{A}_e) \; + \; \sigma_{\text{non-res}} \right],$$

$$\Gamma_{ff} = \mathcal{R}_V^f g_{Vf}^2 + \mathcal{R}_A^f g_{Af}^2, \qquad \Gamma_Z = \sum_f \Gamma_{ff},$$

$$\mathcal{A}_f = 2\frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2} = \frac{1 - 4|Q_f| \sin^2\theta_{\text{eff}}^f}{1 - 4|Q_f|\sin^2\theta_{\text{eff}}^f + 8(|Q_f|\sin^2\theta_{\text{eff}}^f)^2}.$$

$$A_{\text{FB}}^f = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \qquad A_{\text{LR}} = \mathcal{A}_e$$

Z-pole observables

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$$A^f_{\rm FB} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \qquad A_{\rm LR} = \mathcal{A}_e$$

QED/QCD corrections on ext. fermions

Chetyrkin, Kataev, Tkachov ’79
Dine, Saphirstein ’79
Celmaster, Gonsalves ’80
Gorishnii, Kataev, Larin ’88,91

Chetyrkin, Kühn ’90
Surguladze, Samuel ’91
Kataev ’92
Chetyrkin ’93

etc...

Z-pole observables

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$$A_{\text{FB}}^f = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \qquad A_{\text{LR}} = \mathcal{A}_e$$

additional initial-state QED corrections ($\mathcal{R}_{\text{ini}}$)

Kuraev, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '89,91
Montagna, Nicrosini, Piccinini '97
etc...

Z-pole observables

$e^+e^- \to f\bar{f}$ for $\sqrt{s} \sim m_Z$:

$$\sigma = \mathcal{R}_{\text{ini}}\left[12\pi\frac{\Gamma_{ee}\Gamma_{ff}}{(s-m_Z^2)^2 - m_Z^2\Gamma_Z^2}(1-\mathcal{P}_e\mathcal{A}_e) + \sigma_{\text{non-res}}\right],$$

$$\Gamma_{ff} = \mathcal{R}_V^f g_{Vf}^2 + \mathcal{R}_A^f g_{Af}^2, \qquad \Gamma_Z = \sum_f \Gamma_{ff},$$

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$$A_{\text{FB}}^f = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \qquad A_{\text{LR}} = \mathcal{A}_e$$

electroweak corrections

Current status of SM loop results

Known corrections to Δr, $\sin^2\theta^f_{\rm eff}$, g_{Vf}, g_{Af}:

- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\rm eff}$) Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections (g_{Vf}, g_{Af}) Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2)$, $\mathcal{O}(\alpha_{\rm t}^2\alpha_{\rm s})$, $\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^3)$ Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06
  ($\alpha_{\rm t} \equiv \frac{y_{\rm t}^2}{4\pi}$) Boughezal, Czakon '06

Current uncertainties

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_\mathsf{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	α^2_bos, α^3, $\alpha^2\alpha_\mathsf{s}$, $\alpha\alpha_\mathsf{s}^2$
σ^0_had	41540 ± 37 pb	6 pb	α^2_bos, α^3, $\alpha^2\alpha_\mathsf{s}$
$R_b \equiv \Gamma^b_\mathsf{Z}/\Gamma^\mathsf{had}_\mathsf{Z}$	0.21629 ± 0.00066	0.00015	α^2_bos, α^3, $\alpha^2\alpha_\mathsf{s}$
$\sin^2\theta^\ell_\mathsf{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\mathsf{s}$

Methods for theory error estimates:

- Parametric factors, $i.e.$ factors of α, N_c, N_f, ...
- Geometric progression, $e.g.$ $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalization scale dependence (often underestimates error)
- Renormalization scheme dependence (may underestimate error)

Example: Error estimation for Γ_Z

- Goemetric perturbative series $\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z\alpha^2 \sim 0.1 \text{ MeV}$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi}\alpha_s^2 \sim 0.29 \text{ MeV}$$

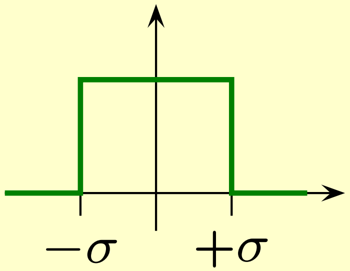
Total: $\delta\Gamma_Z \approx 0.5$ MeV

Combination of theory errors

- Add theory errors from each source **linearly**:

  Idea: each value within error range in equally likely

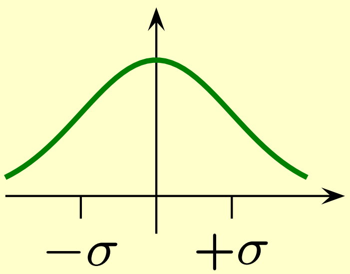
  $\longrightarrow$ Use flat prior in global fits



- Add theory errors from each source **quadratically**:

  Idea: different error sources are uncorrelated

  $\longrightarrow$ Use Gaussian prior in global fits (central limit theorm)



Future projections

	Current exp.	ILC	TLEP	Current perturb.
M_{W} [MeV]	15	3–5	~ 1	4
Γ_Z [MeV]	2.3	~ 1	~ 0.1	0.5
R_b $[10^{-5}]$	66	15	$\lesssim 5$	15
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ $[10^{-5}]$	16	1.3	0.3	4.5

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not ILC/TLEP!

Theory and parametric uncertainties

	ILC	TLEP	perturb. error with 3-loop[†]	Param. error ILC*	Param. error TLEP**
$M_{\rm W}$ [MeV]	3–5	~ 1	1	2.6	1
Γ_Z [MeV]	~ 1	~ 0.1	$\lesssim 0.2$	0.5	?
R_b $[10^{-5}]$	15	$\lesssim 5$	5–10	< 1	< 1
$\sin^2\theta^{\ell}_{\rm eff}$ $[10^{-5}]$	1.3	0.3	1.5	2	2

[†] **Theory scenario:** $\mathcal{O}(\alpha\alpha_{\rm s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_{\rm s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_{\rm s})$

(N_f^n = at least n closed fermion loops)

Parametric inputs:

*** ILC:** $\delta m_t = 100$ MeV, $\delta\alpha_{\rm s} = 0.001$, $\delta M_{\rm Z} = 2.1$ MeV

****TLEP:** $\delta m_t \lesssim 50$ MeV, $\delta\alpha_{\rm s} =?$, $\delta M_{\rm Z} = 0.1$ MeV

also: $\delta(\Delta\alpha) = 5\times 10^{-5}$

Theory uncertainties in extraction of pseudo-observables

- Subtraction of QED radiation contributions

  → Known to $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^3 L^3)$ for **ISR**, $\mathcal{O}(\alpha^2)$ for **FSR** and $\mathcal{O}(\alpha^2 L^2)$ for **A$_{FB}$**

  $(L = \log \frac{s}{m_e^2})$

  Berends, Burgers, v.Neerven '88
  Kniehl, Krawczyk, Kühn, Stuart '88
  Beenakker, Berends, v.Neerven '89
  Skrzypek '92; Montagna, Nicrosini, Piccinini '97

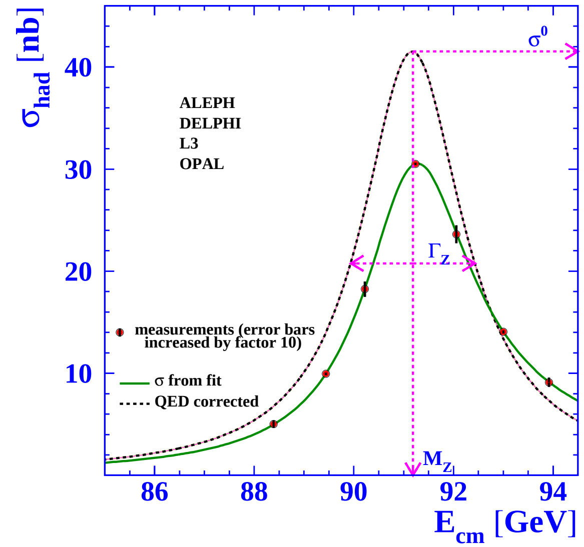
  → $\mathcal{O}(0.1\%)$ uncertainty on σ_Z, A_{FB}
  → Improvement needed for ILC/TLEP

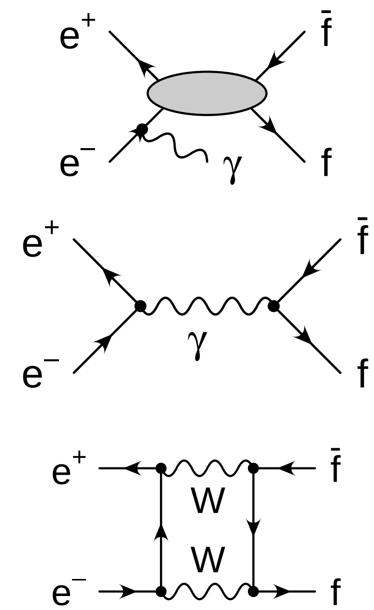
- Subtaction of non-resonant γ-exchange, γ–Z interf., box contributions, Bhabha scattering

  see, e.g., Bardin, Grünewald, Passarino '99

  → $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed)
  → Sensitivity to some NP beyond EWPO

LEP EWWG '05





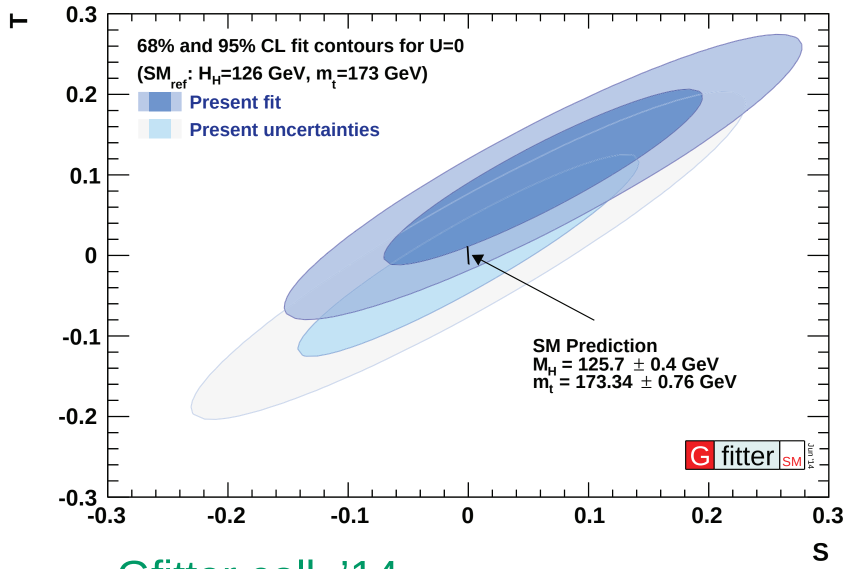
Parametrization of new physics

Oblique parameters:

$$\alpha T = \frac{\Sigma_{WW}(0)}{M_W} - \frac{\Sigma_{ZZ}(0)}{M_Z}$$

$$\frac{\alpha}{4s^2c^2} S = \frac{\Sigma_{ZZ}(M_Z^2) - \Sigma_{ZZ}(0)}{M_Z} + \frac{s^2 - c^2}{sc} \frac{\Sigma_{Z\gamma}(M_Z^2)}{M_Z} - \frac{\Sigma_{\gamma\gamma}(M_Z^2)}{M_Z}$$

$$\frac{\alpha}{4s^2}(S+U) = \frac{\Sigma_{WW}(M_W^2) - \Sigma_{WW}(0)}{M_W} - \frac{c}{s}\frac{\Sigma_{Z\gamma}(M_Z^2)}{M_Z} - \frac{\Sigma_{\gamma\gamma}(M_Z^2)}{M_Z}$$



Gfitter coll. '14

$\longrightarrow$ Not adequate for new physics that affects flavor ($Z \to \ell\ell$, $Z \to bb$, ...)

Parametrization of new physics

More general setup: Use pseudo-observables

M_W, Γ_Z, σ^0_{had}, R_b, R_ℓ, A_ℓ, A_b, A_{lq} $(\ell = e, \mu, \tau)$ $\longrightarrow$ 12 quantities

Effective field theory: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3})$ $(\Lambda \gg M_Z)$

$$\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \, \Phi^\dagger (D^\mu \Phi) \qquad \alpha \Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$$

$$\mathcal{O}_{BW} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi \qquad \alpha \Delta S = -e^2 v^2 \frac{c_{BW}}{\Lambda^2}$$

$$O^{(3)e}_{LL} = (\bar{L}^e_L \sigma^a \gamma_\mu L^e_L)(\bar{L}^e_L \sigma^a \gamma^\mu L^e_L) \qquad \Delta G_F = -\sqrt{2}\frac{c^{(3)e}_{LL}}{\Lambda^2}$$

$$O^f_R = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_R \gamma^\mu f_R) \qquad f = e, \mu\, \tau, b, lq$$

$$O^F_L = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_L \gamma^\mu F_L) \qquad F = \binom{\nu_e}{e}, \binom{\nu_\mu}{\mu}, \binom{\nu_\tau}{\tau}, \binom{u, c}{d, s}, \binom{t}{b}$$

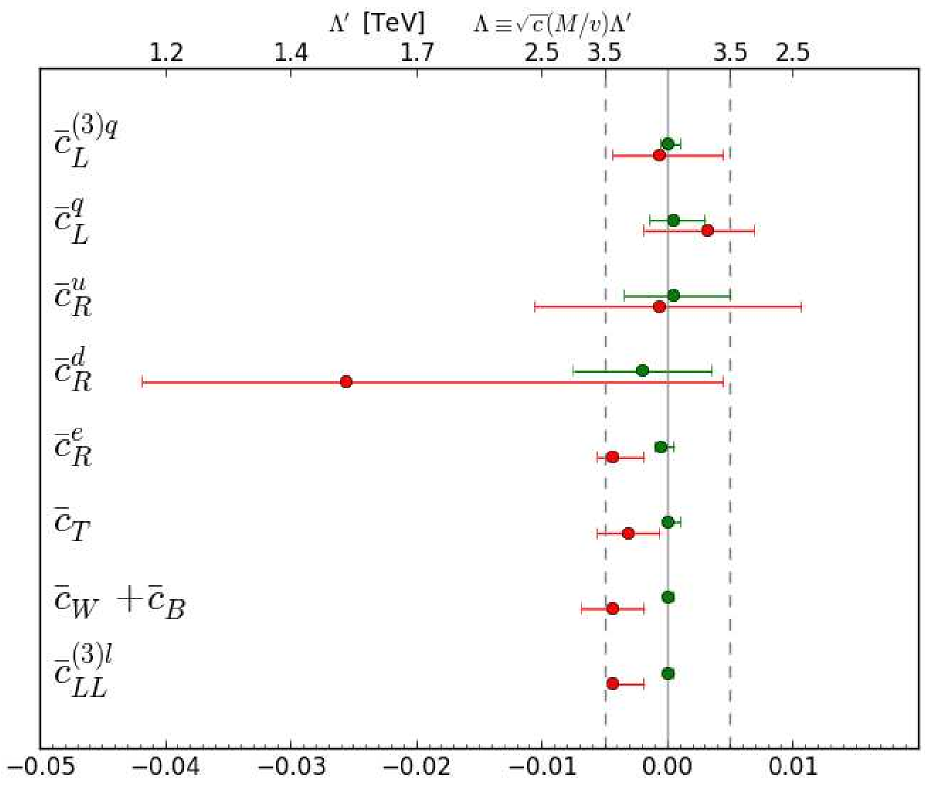
$$O^{(3)F}_L = i(\Phi^\dagger \overleftrightarrow{D}^a_\mu \Phi)(\bar{F}_L \sigma_a \gamma^\mu F_L)$$

More operators than EWPOs

$\longrightarrow$ Some can be constrained by $W \to \ell\nu$, had., $e^+e^- \to W^+W^-$

Current constraints on some dim-6 operators

Assuming flavor universality:



Significant correlation/ degeneracy between different operators

Pomaral, Riva '13

Ellis, Sanz, You '14

Conclusions

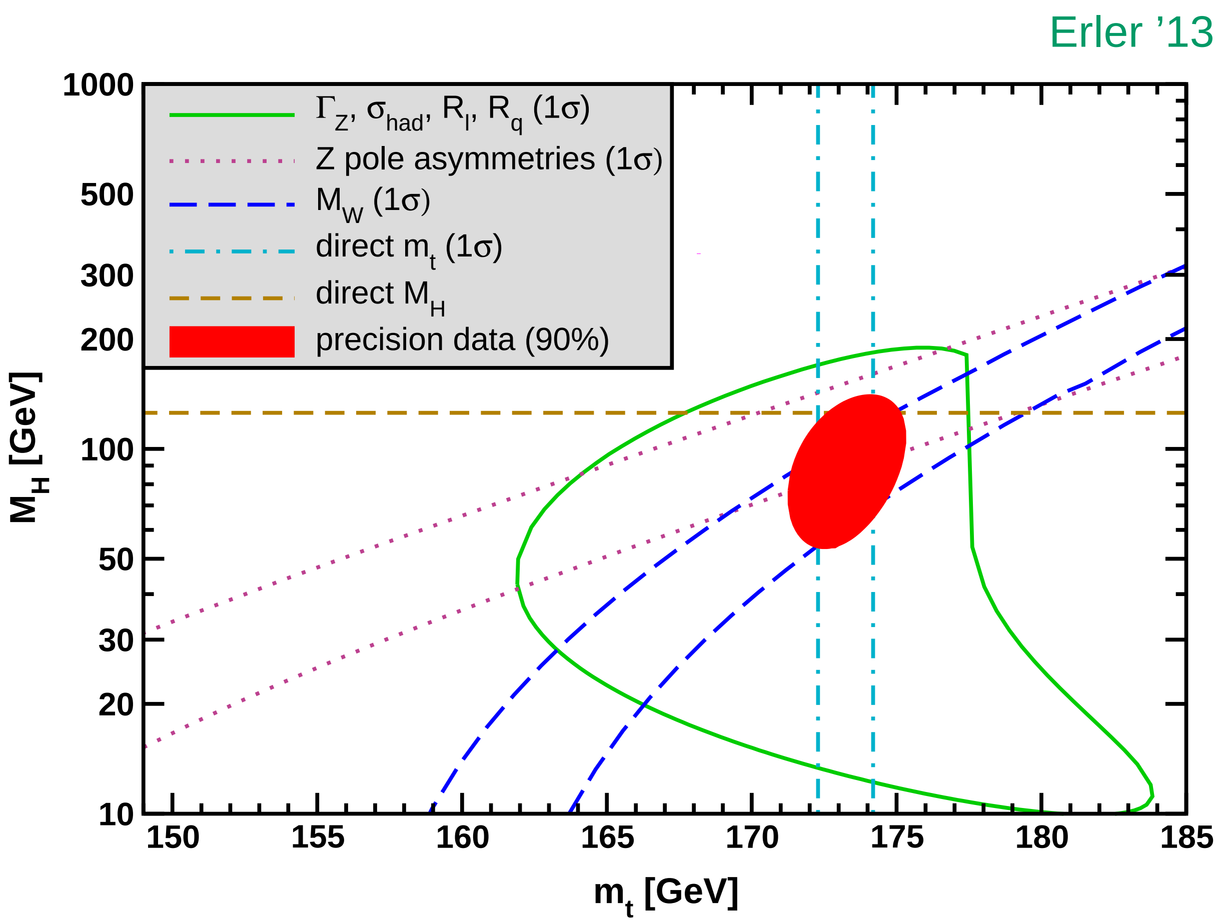
- Experimental precision from LEP/SLC demands SM prediction with **2-loop corrections** and **partial 3-loop corrections**
- **LHC** will provide independent results for $\sin^2\theta_{\rm eff}$ and $M_{\rm W}$, but overall precision not improved
- **ILC/TLEP** with $\sqrt{s} \sim M_{\rm Z}$ will reduce exp. error of some EWPO by $\mathcal{O}(10)$
  $\longrightarrow$ 3-loop (and maybe some 4-loop) corrections needed!
- Procedure for combination of experimental, perturbative and parametric uncertainties needs consensus
- At current and future level of precision, new physics should not be parametrized just by S/T/U parameters
  $\longrightarrow$ For TeV-scale physics: use effective operators (which is the best basis?)
  $\longrightarrow$ For new physics at weak scale and below: directly compute pseudo-obs.

Backup slides

Electroweak precision fit

Standard Model after Higgs discovery:

- Very good agreement over large number of observables
- Sensitivity to TeV-scale new physics



Direct measurements:

$M_{\rm H} = 125.6 \pm 0.4$ GeV

$m_{\rm t} = 173.24 \pm 0.95$ GeV

Indirect prediction:

$M_{\rm H} = 123.7 \pm 2.3$ GeV (with LHC BRs)

$M_{\rm H} = 89^{+22}_{-18}$ GeV (w/o LHC data)

$m_{\rm t} = 177.0 \pm 2.1$ GeV

Z-pole observables

After deconvolution of initial-state QED radiation and subtraction of γ-exchange:

$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)S'$$

$$s_0 \equiv M_Z^2 - iM_Z\Gamma_Z$$

$s_0,\ R,\ S,\ S'$ are gauge-invariant

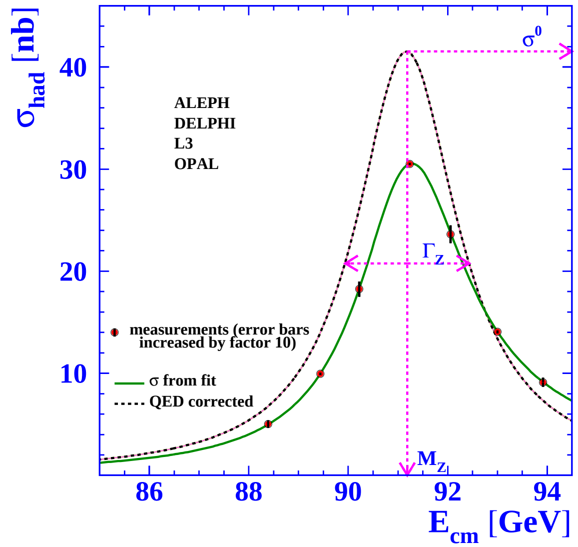
Willenbrock, Valencia '91; Sirlin '91; Stuart '91

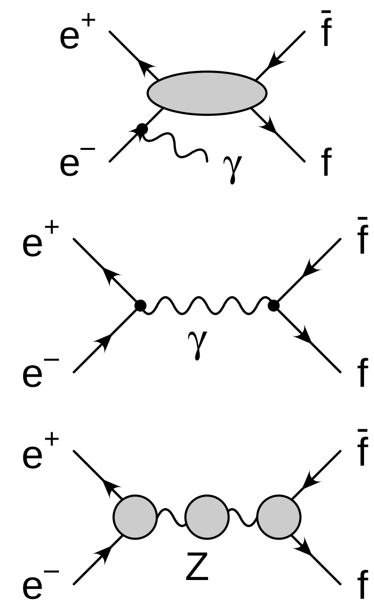
Gambino, Grassi '00

$$\sigma \sim \frac{1}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \text{non.res.}$$

$M_Z = M_Z^{\text{exp}} - 34$ MeV
$\Gamma_Z = \Gamma_Z^{\text{exp}} - 0.9$ MeV

LEP EWWG '05





Electroweak precision tests: new physics

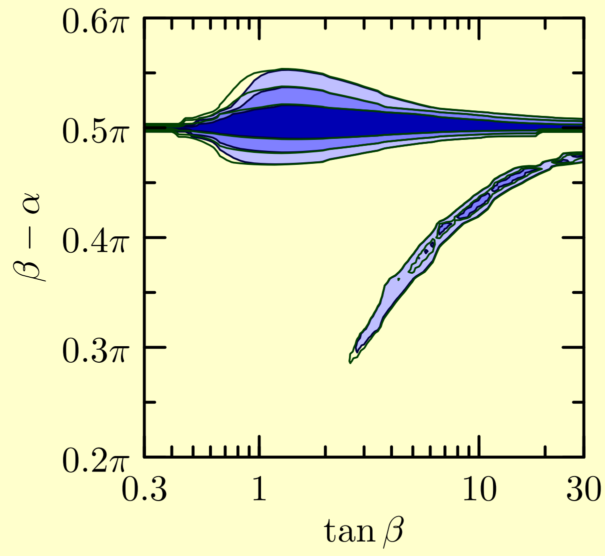
Constraints on Higgs physics:

- Indirect determination of M_{H}

- Couplings: need to go beyond SM, *e.g.* THDM:

$$\left|\frac{g_{hVV}^{\text{THDM}}}{g_{hVV}^{\text{SM}}}\right| = \sin(\beta-\alpha),$$

$$\left|\frac{g_{hff}^{\text{THDM}}}{g_{hff}^{\text{SM}}}\right| = \frac{\cos\alpha}{\sin\alpha} \text{ or } \frac{\sin\alpha}{\cos\alpha}$$



Eberhardt, Nierste, Wiebusch '13

- Couplings: effective operators

$$\mathcal{O}_W = (D^\mu\Phi)^\dagger W_{\mu\nu}(D^\nu\Phi)$$
$$\mathcal{O}_{WW} = \Phi^\dagger W^{\mu\nu}W_{\mu\nu}\Phi$$

etc.

Mebane, Greiner, Zhang, Willenbrock '13
Chen, Dawson, Zhang '13