“Black holes, Stokes flows and DC transport at strong coupling”

Talk at “Physics on the Riviera”

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Based on work with J. P. Gauntlett and E. Banks:
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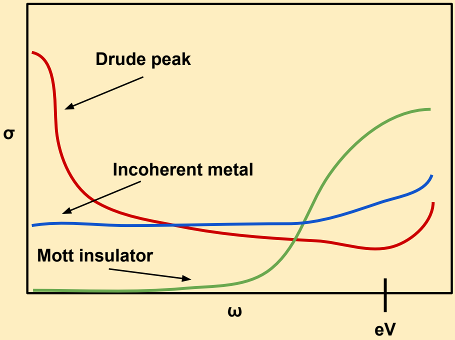
Outline

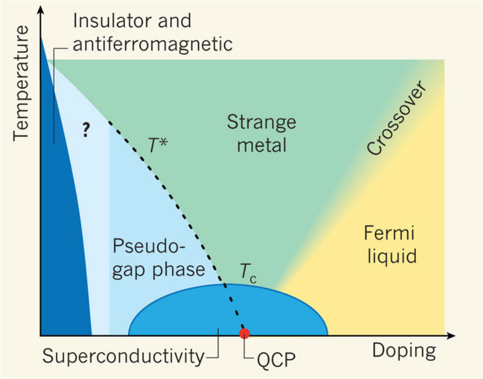
1. Introduction/Motivation
2. The Holographic Lab
   - Incoherent transport
   - Anomalous scaling of Hall angle
   - DC conductivities from BH horizons
3. Summary / Outlook

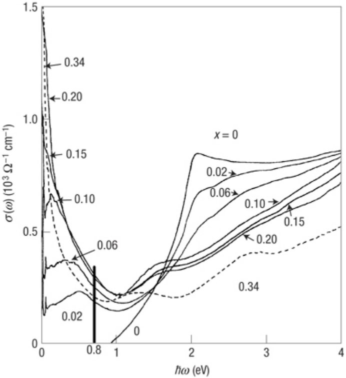
Charge transport in real materials



- Materials with charged d.o.f. can be
  - Coherent metals with a well defined Drude peak
  - Insulators
  - Incoherent conductors of electricity
- Interactions expected to become important in the incoherent phase $\rightarrow$ Possible description in AdS/CFT?

The Cuprates





The Cuprates are real life example of :

- Incoherent transport
- Anomalous scaling of conductivity and Hall angle with T

$$\sigma_{DC}^{B=0} \propto T^{-1}, \quad \theta_H \propto T^{-2}$$

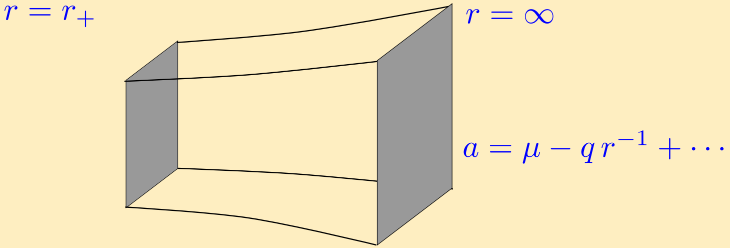
Perfect Holographic Conductor

Do it in $D = 4$ Einstein-Maxwell with AdS asymptotics:

$$\mathcal{L}_{EM} = R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + 12$$

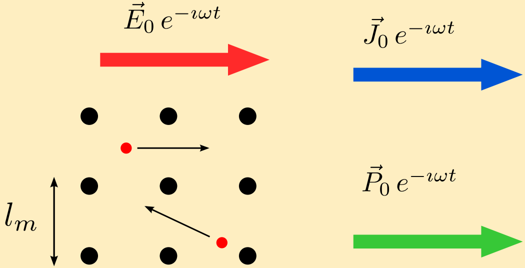
$$ds_4^2 = -U(r)\, dt^2 + U(r)^{-1}\, dr^2 + r^2\, \left(dx_1^2 + dx_2^2\right)$$

$$A = a(r)\, dt$$



Background black hole has temperature T , energy E, pressure P, entropy s and charge q.

Classical Drude model



- Average momentum obeys

$$\langle \dot{p} \rangle = qE - \frac{1}{\tau}\langle p \rangle \Rightarrow$$

$$\sigma = \frac{nq^2}{m}\frac{\tau}{1 - \imath\omega\tau}$$

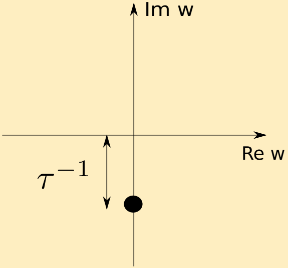
- Without collisions $\tau \to \infty \;\Rightarrow \sigma = \frac{nq^2}{m}\left(\delta(\omega) + \frac{\imath}{\omega}\right)$

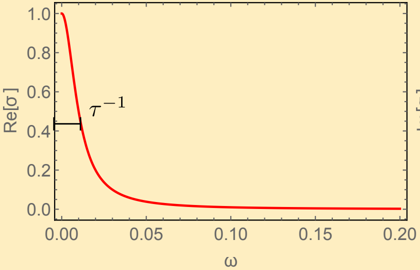
Classical Drude model

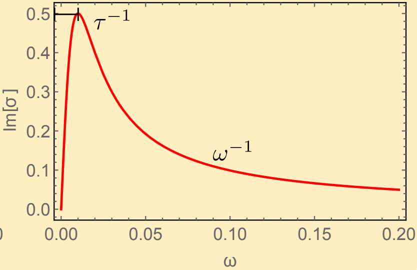
Don't need quasi-particles to have Drude physics.

Coherent metals arise when momentum relaxation is slow with dominant pole on real axis.

[Hartnoll, Hofman]







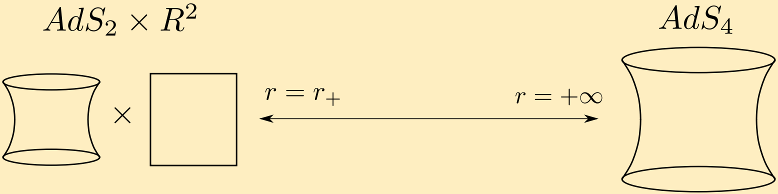
Fourier/Ohm law

- Apart from electric currents we have a thermal current $Q^i = T^{ti} - \mu\, J^i$
- Transport coefficients are packaged in Ohm/Fourier law

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

- With ∇T a temperature gradient

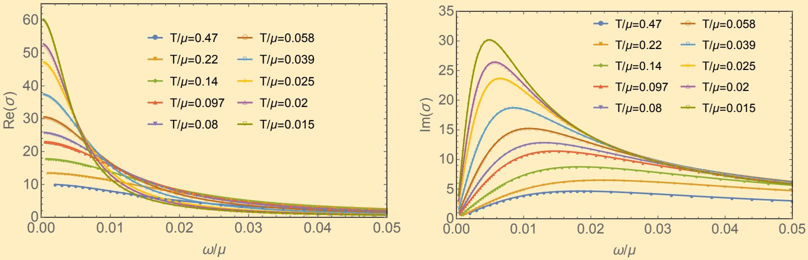
Holographic Lattice



To add momentum dissipation introduce a UV - IR benign lattice:

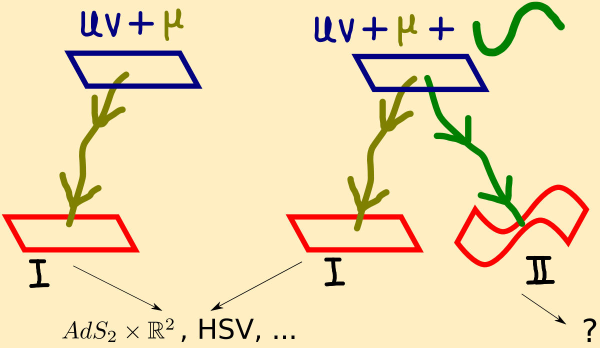
- Keep UV fixed point $\Rightarrow$ relevant deformation $\mathcal{O}(x)$
- Drude physics $\Rightarrow$ $T = 0$ horizon restores translations
- Charge density is a universal relevant operator $\Rightarrow$ Impose
  $A_t = \mu(x) - J^t(x)\, r^{-1} + \cdots$
  [Hartnoll, Hofman][Horowitz, Santos, Tong][AD, Gauntlett]
  $$\mu(x) = \mu_0 + A(x), \quad \langle A \rangle_L = 0$$
- $\mu_0 \Rightarrow$ chemical potential, $A'(x) \Rightarrow$ periodic electric field

Inhomogeneous Lattices



- Drude peaks are there
- Get rid of them!

RG/Holographic picture



- I Charge dominated RG flows, translations restored in IR $\rightarrow$ Coherent transport
- II Lattice dominated RG flows, translations broken in IR $\rightarrow$ Incoherent transport

[AD, Hartnoll] [AD, Gauntlett]

Q-lattices

Consider a simple model with a global $U(1)$ in addition to the gauged one [AD, Gauntlett]

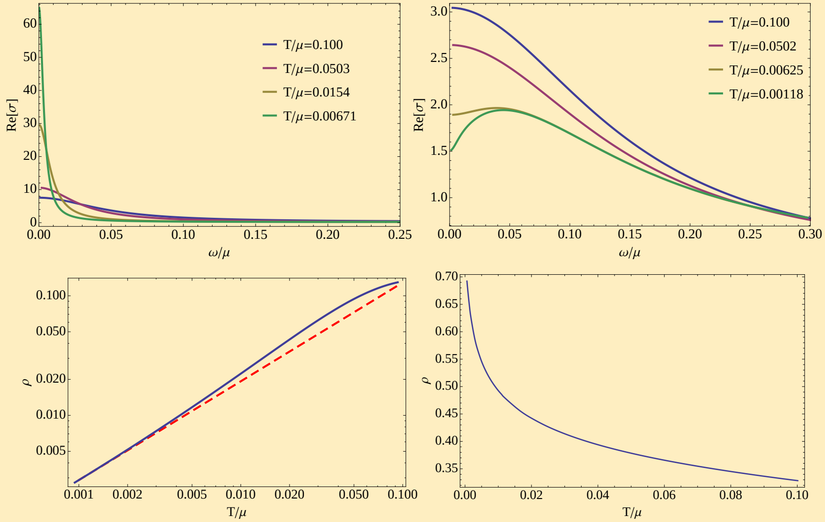
$$S = \int d^4x \sqrt{-g}\, \left[R + 6 - \frac{1}{4}F^2 - |\partial\phi|^2 - m^2\, |\phi|^2\right]$$

along with the ansatz

$$ds^2 = -U(r)\, dt^2 + U(r)^{-1}\, dr^2 + e^{2V_1(r)}\, dx_1^2 + e^{2V_2(r)}\, dx_2^2$$
$$A = a(r)\, dt, \qquad \phi = e^{\imath k x_1}\, \varphi(r)$$

- Leads to ODEs both for background and perturbation
- Two real scalars with $\mathcal{O}_1 \sim \cos(kx)$, $\mathcal{O}_2 \sim \sin(kx)$
- RN still a solution
- Lattice operator relevant for $k < k_c$

Conductivity from Q-lattices



- Can model Metal - Insulator transitions
- Similar story for inhomogeneous lattices
  [Rangamani, Rozali, Smyth]

More general Q-lattices

Consider more general situation where

$$\begin{aligned}\mathcal{L} =& R - \frac{1}{2}\left[(\partial\varphi)^2 + \Phi_1(\varphi)\,(\partial\chi_1)^2 + \Phi_2(\varphi)\,(\partial\chi_2)^2\right]\\ &+ V(\varphi) - \frac{Z(\varphi)}{4}\,F^2\end{aligned}$$

The background is then

$$\begin{aligned}ds_4^2 &= -U(r)\,dt^2 + U(r)^{-1}\,dr^2 + e^{2\,V_1(r)}\,dx_1^2 + e^{2\,V_2(r)}\,dx_2^2\\ A &= a(r)\,dt, \quad \phi = \phi(r)\\ \chi_1 &= k_1\,x_1, \quad \chi_2 = k_2\,x_2\end{aligned}$$

Rich class of new ground states that break translations

- Insulators
- Novel metals

Ohm/Fourier Law

- Perturbative argument to determine horizon at low T
- Analytic argument to express DC transport coefficients in terms of bh horizon data

$$\sigma_{DC} = \left[\frac{Z(\phi)s}{4\pi e^{2V_1}} + \frac{4\pi q^2}{{k_1}^2\Phi_1(\phi)s}\right]_{r=r_+} = \sigma_{ccs} + \sigma_{dis}$$

$$\bar{\kappa}_{DC} = \left[\frac{4\pi sT}{{k_1}^2\Phi_1(\phi)}\right]_{r=r_+}, \quad \alpha_{DC} = \bar{\alpha}_{DC} = \left[\frac{4\pi q}{{k_1}^2\Phi_1(\phi)}\right]_{r=r_+}$$

- Conductivity can be dominated by either σ_{ccs} or σ_{dis}
- Notice that

$$\sigma_{ccs} = \sigma_{Q=0} = \sigma - T\,\alpha\,\bar{\kappa}^{-1}\,\alpha$$

Hall angle [AD, Blake]

- Can include background magnetic field B
- For $B^{1/2} << T << \mu$

$$\theta_H \propto \frac{B}{q}\,\sigma_{dis}^{B=0}$$
$$\sigma_{DC}^{B=0} = \sigma_{ccs}^{B=0} + \sigma_{dis}^{B=0}$$

- It is possible to have

$$\theta_H \propto \frac{B}{q}\,\sigma_{dis}^{B=0}$$
$$\sigma_{DC}^{B=0} \propto \sigma_{ccs}^{B=0}$$

- Not possible to get with weakly coupled fermions.

DC conductivities from BH horizons

- Poweful
- Reveal interesting physics
- Have been found for more examples of homogeneous lattices e.g. Bianchi VII_0
  [Iqbal, Liu] [Blake, Tong] [Andrade, Withers] [Gouteraux] [AD, Gauntlett] [AD, Gouteraux, Kiritsis] [AD, Gauntlett, Pantelidou]
- Where do they come from?

DC conductivities from BH horizons

- Back to Einstein-Maxwell
- Consider electrically charged, static black branes

$$ds^2 = -UG\,dt^2 + \frac{F}{U}\,dr^2 + ds^2(\Sigma_d)\,, \quad A = a_t\,dt$$

$$ds^2(\Sigma_d) = g_{ij}(r,x)dx^i dx^j$$

- Asymptotically, $r \to \infty$

$$U \to r^2, \qquad F \to 1, \qquad G \to \bar{G}(x),$$

$$g_{ij}(r,x) \to r^2 \bar{g}_{ij}(x), \qquad a_t(r,x) \to \mu(x)\,.$$

DC conductivities from BH horizons

For the perturbation write

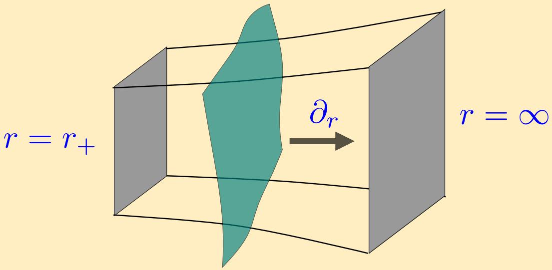
$$\delta(ds^2) = \delta g_{\mu\nu}(r,x)dx^\mu dx^\nu - 2tGU\zeta_i dt dx^i\,,$$
$$\delta A = \delta a_\mu(r,x)dx^\mu - tE_i dx^i + ta_t\zeta_i dx^i$$

- $E(x^i)$ and $\zeta(x^i)$ are closed forms
- ζ is boundary temperature gradient
- E is boundary electric field
- Count functions:
  - $g_{\mu\nu} \to \frac{1}{2}(d+2)(d+3) - (d+2)$ functions
  - $A_\mu \to (d+2) - 1$ functions

Radial Hamiltonian

- Imagine radial foliation by hypersurfaces e.g. normal to ∂_r
- Radial evolution Hamiltonian is sum of constraints

$$H_{\partial_r} = N\,\mathcal{H} + N_\mu\,\mathcal{H}^\mu + D\,\mathcal{G} + \text{b.t.}$$



- At infinity they yield Ward identities

$$\nabla_\mu \langle T^{\mu\nu}\rangle = F^{\mu\nu}\,\langle J_\nu\rangle\,, \qquad \nabla_\mu \langle J^\mu\rangle = 0, \qquad \langle T^\mu{}_\mu\rangle = \text{anom}$$

- Meaningful but not closed system without hydro

DC conductivities from BH horizons

Examine constraints close to the horizon

- Define

$$v_i \equiv \delta g_{it}^{(0)}, \qquad w \equiv \delta a_t^{(0)}\,,$$

$$p \equiv 4\pi T \frac{\delta g_{rt}^{(0)}}{G^{(0)}} + \delta g_{it}^{(0)} g_{(0)}^{ij} \nabla_j \,\ln G^{(0)}$$

- To find

$$\mathcal{H}^t \Rightarrow \quad \nabla_i v^i = 0$$

$$\mathcal{H}^j \Rightarrow \quad 2\,\nabla^i \nabla_{(i} v_{j)} - a_t^{(0)} \nabla_j w - \nabla_j\, p = 4\pi T\, \zeta_j + a_t^{(0)} E_j$$

$$\mathcal{G} \Rightarrow \quad \nabla^2 w - v^i\, \nabla_i a_t^{(0)} = -\nabla_i E^i$$

- Solve for a Stokes flow on the curved black hole horizon
- Nowhere made hydro assumptions!

Hydro temptation

- Meaningful quantities are

$$Q = \mathrm{vol}_d^{-1} \int \sqrt{g_{(0)}}\, a_t^{(0)}, \quad S = \mathrm{vol}_d^{-1} \int 4\pi \sqrt{g_{(0)}}$$

- Very tempting to think of it as

$$\nabla_i v^i = 0$$

$$2\eta\, \nabla^i \nabla_{(i} v_{j)} - \rho\, \nabla_j \delta\mu - s\, \nabla_j\, \delta T = T\, s\, \zeta_j + \rho\, E_j$$

$$\nabla^2 \delta\mu - v^i\, \nabla_i \rho = -\nabla_i E^i$$

- Looks like first order hydro
- But, is there a fluid?

DC conductivities from BH horizons

Electric Current

Define

$$J^i = \sqrt{-g} F^{ir}$$

- At $r \to \infty$ gives field theory current densities
- Anywhere in the bulk

$$\partial_i J^i = 0\,, \qquad \partial_r J^i = \partial_j \left(\sqrt{-g} F^{ji}\right)$$

DC conductivities from BH horizons

Heat Current

Let $k = \partial_t$ and define

$$G^{\mu\nu} = -2\,\nabla^{[\mu}k^{\nu]} - k^{[\mu}F^{\nu]\sigma}A_\sigma - \frac{1}{2}\,(\phi - \theta)\,F^{\mu\nu}$$

Let

$$Q^i = \sqrt{-g}G^{ir}$$

- At $r \to \infty$ gives field theory heat current densities

$$Q^i = -\left\langle \delta T^i{}_t \right\rangle - \mu\,\left\langle \delta J^i \right\rangle$$

- Anywhere in the bulk

$$\partial_i Q^i = 0\,, \qquad \partial_r Q^i = -\partial_j\left(2\sqrt{-g}G^{ji}\right)$$

DC conductivities from BH horizons

- Solutions for v^i, w and p are uniquely fixed by sources E and ζ
- Then

$$J^i|_{r=rh} = \frac{s}{4\pi}\left(\partial^i w + E^i\right) + \rho\, v^i$$

$$Q^i|_{r=rh} = T s\, v^i, \quad s = 4\pi\sqrt{g_{(0)}}, \quad \rho = \sqrt{g_{(0)}}\, a_t^{(0)}$$

- Exploit

$$\partial_r J^i = \partial_j\left(\sqrt{-g}F^{ji}\right), \quad \partial_r Q^i = -\partial_j\left(2\sqrt{-g}G^{ji}\right)$$

- To find field theory currents $\bar{J}^i$ and $\bar{Q}^i$ in e.g. $d = 2$

$$\bar{J}^1 = \int dx^2\, J^1, \quad \bar{J}^2 = \int dx^1\, J^2$$

- Conductivities determined solely by BH horizon data!

DC conductivities from BH horizons

Examples

- Can recover earlier results for e.g. Q-lattices and 1-dim lattices
- Perturbative, periodic lattices about AdS-RN black brane
  Let λ be the expansion parameter
  The black hole horizon is a small expansion about flat space

$$
\begin{aligned}
g_{(0)ij} &= g\,\delta_{ij} + \lambda\, h^{(1)}_{ij} + \lambda^2\, h^{(2)}_{ij} + \cdots \\
a^{(0)}_t &= a + \lambda\, a_{(1)} + \lambda^2\, a_{(2)} + \cdots \\
G^{(0)} &= f_{(0)} + \lambda\, f_{(1)} + \cdots
\end{aligned}
$$

Solve Navier-Stokes perturbatively in λ

DC conductivities from BH horizons

At leading order in λ we find

$$\alpha_{ij} = \bar{\alpha}_{ij} = \frac{L_{ij}^{-1}}{\lambda^2}\, 4\pi\rho + \ldots, \quad \bar{\kappa}_{ij} = \frac{L_{ij}^{-1}}{\lambda^2}\, 4\pi\, sT + \ldots$$

$$\sigma_{ij} = \frac{L_{ij}^{-1}}{\lambda^2}\, \frac{4\pi\rho^2}{s} + \ldots$$

Where $L_{ij} = \int_H\, l_{ij}\left(h_{kl}^{(1)}, a^{(1)}\right)$

Consistent with memory matrix formalism
[Barkeshli,Hartnol,Mahajan]

Other applications?

- Conductivity bounds: e.g. [Lucas, Sachdev, Schalm]
- Random geometries: e.g. [Hartnoll, Santos, Ramirez]

Summary / Outlook

- Holography is a tool to study transport in strongly coupled systems
- No assumption of quasiparticles
- Understand better the physics of the new ground states
- Fluid/gravity can be used to obtain DC thermoelectric conductivities
- Connection with fluid/gravity beyond DC?