# Open String Field Theory and D-branes:

## *Classical Solutions and Topological Defects*

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1406.3021, JHEP 1410 (2014) 029 *(w/ **Ted Erler**)*
*To Appear, (w/ **Toshiko Kojita, Toru Masuda** and **Martin Schnabl)***

*related works*
1506.03723, JHEP 1508 (2015)149 *(w/ **Martin Schnabl**)*
1402.3546, JHEP 1405 (2014) 004
1207.4785, JHEP 1307 (2013) 033 *(w/ **Matej Kudrna** and **Martin Schnabl**)*
1201.5122, JHEP 1206 (2012) 084 *(w/ **Ted Erler**)*
1201.5119 , JHEP 1204 (2012) 107 *(w/ **Ted Erler**)*

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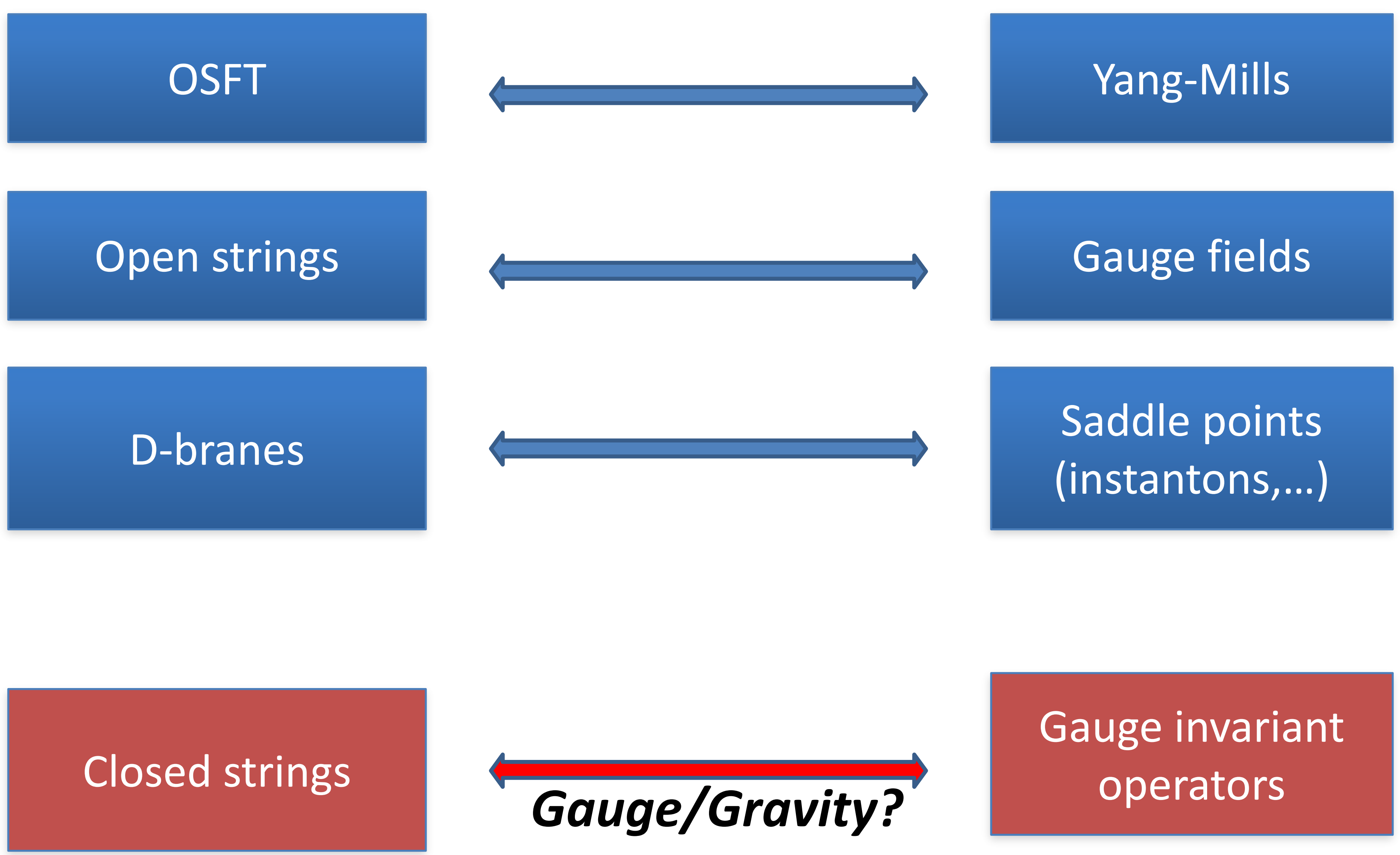
*Physics on the Riviera, Sestri Levante*
*16/09/2015*

**Open String Field Theory** (OSFT) is a microscopic theory for D-branes, formulated as a ***field theoretic*** description of ***open strings***

## Helpful analogy



# OPEN STRING FIELD THEORY

- Fix a bulk CFT (closed string background)
- Fix a reference $\mathrm{BCFT}_0$ (open string background, D-brane's system)
- The string field is a state in $\mathrm{BCFT}_0$

$$|\psi\rangle = \sum_i t_i\, \psi^i(0)|0\rangle_{SL(2,R)}$$

- There is a non-degenerate inner product (BPZ)

$$\langle\psi,\phi\rangle = \langle\psi(-1)\phi(1)\rangle^{Disk}_{\mathrm{BCFT}_0}$$

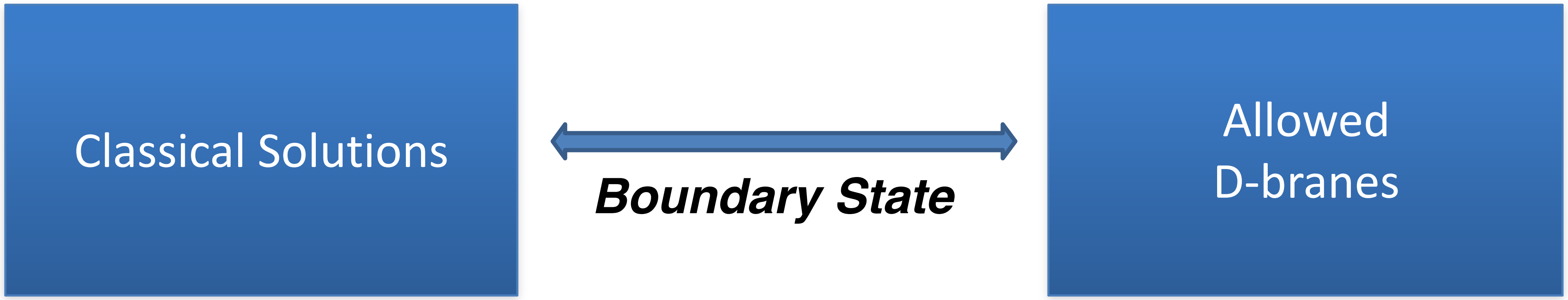
- The bpz-inner product allows to write a target-space action

$$S[\psi] = -\frac{1}{2}\langle\psi, Q\psi\rangle_{\mathrm{BCFT}_0} - \frac{1}{3}\langle\psi,\psi*\psi\rangle_{\mathrm{BCFT}_0} = S_{eff}[t_i]$$

- Witten product *: associative product between states (OPE+conf. map
- ***Equation of motion***

$$Q\Psi + \Psi * \Psi = 0$$

## **OSFT CONJECTURE** (*once known as Sen's Conjectures*)




- Key tool for connecting the two sets is the OSFT construction of the boundary state (**Kiermaier, Okawa, Zwiebach (2008), Kudrna, CM, Schnabl (2012)** )

- The (KMS) boundary state is constructed from gauge invariant quantities starting from a given solution

$$Q\Psi_* + \Psi_*^2 = 0 \quad \Longrightarrow \quad |B_*\rangle = \sum_\alpha n_*^\alpha \, |V_\alpha\rangle\rangle$$

$$n_*^\alpha = \langle V^\alpha | B_* \rangle = \langle V^\alpha(0) \rangle_{\rm disk}^{\rm BCFT_*} = W_{V^\alpha}[\Psi_* - \Psi_{\rm tv}]$$

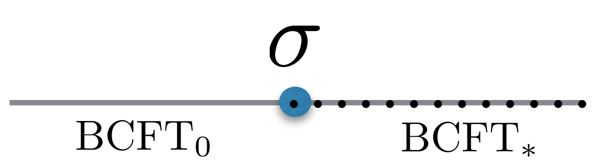
**Tachyon Vacuum**
Sen-Zwiebach 1999
Schnabl 2005

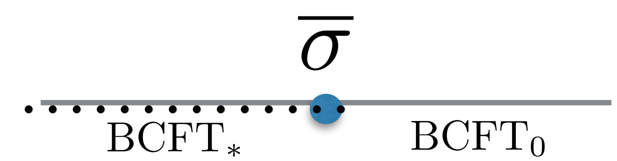
- ***Intriguing possibility of relating BCFT consistency conditions (Cardy-Lewellen, Pradisi-Sagnotti-Stanev) with OSFT equation of motion!***

# SOLUTION FOR ANY BACKGROUND

Erler, CM (2014)

- A change in boundary conditions is encoded in a bcc operator (*Cardy, '86-'89*)



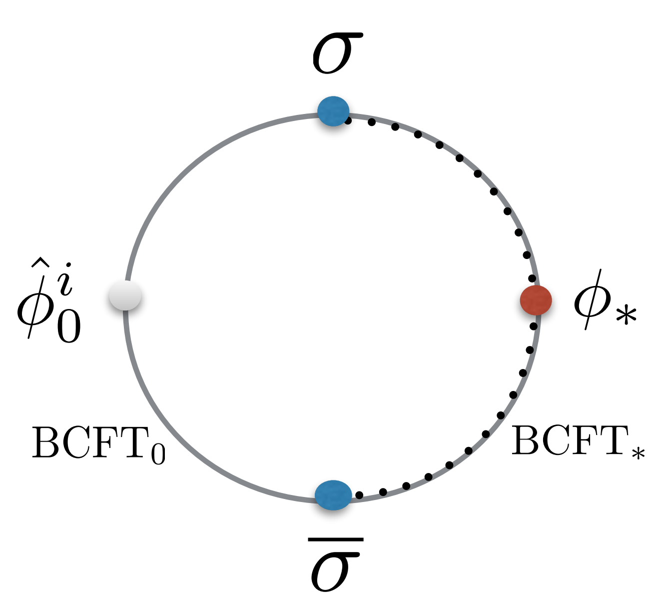




- OSFT: describe the dof of a target **BCFT*** using the dof of a reference **$\mathbf{BCFT_0}$**

$$\phi_*(0)|0\rangle_* \to \sigma(1)\phi_*(0)\overline{\sigma}(-1)|0\rangle_0 = \sum_i t_i \phi_0^i(0)|0\rangle_0$$

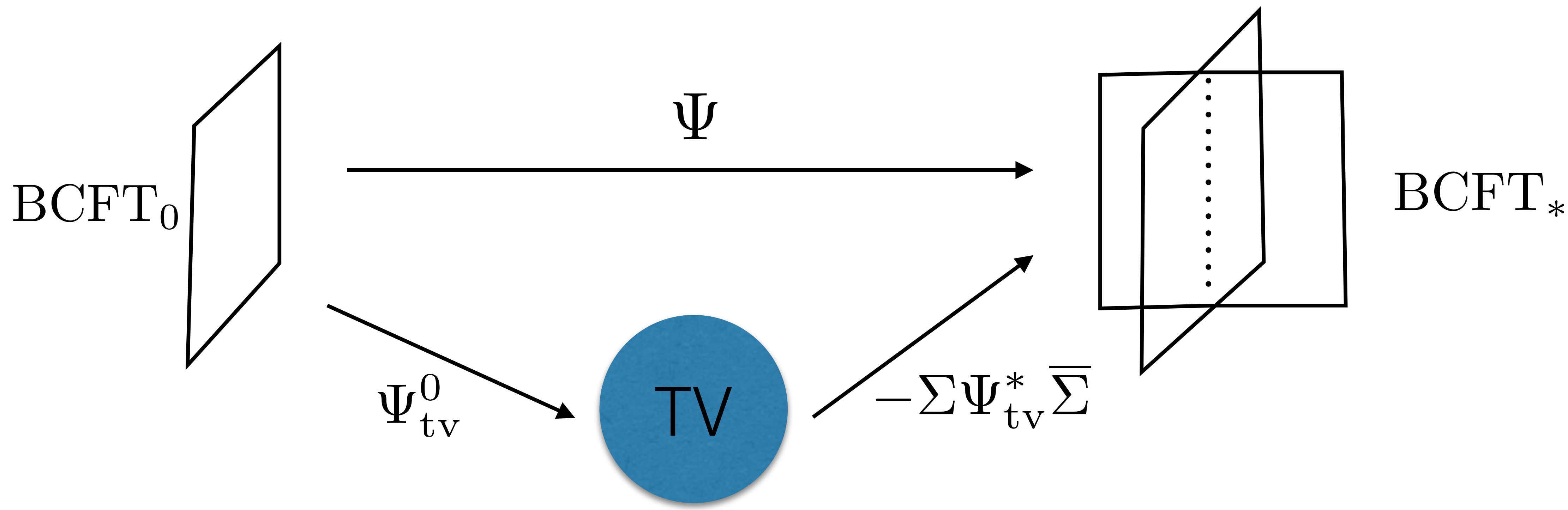
*Idea back from Vacuum SFT (Rastelli, Sen, Zwiebach,2000)*

$t_i =$




$$\langle \hat{\phi}_0^i | \phi_0^j \rangle = \delta^{ij}$$

- Connect two generic backgrounds by passing through the ***tachyon vacuum*** (***simplest universal solution: no D-branes***)




$$\Psi = \Psi^0_{\mathrm{tv}} - \Sigma\Psi^*_{\mathrm{tv}}\overline{\Sigma}$$

$$\Sigma \in \mathcal{H}_{0*}$$

$$\overline{\Sigma} \in \mathcal{H}_{*0}$$

$$\overline{\Sigma}\Sigma = 1$$

$$Q\Psi + \Psi^2 = 0$$

$$Q_{\mathrm{tv}}\overline{\Sigma} = Q_{\mathrm{tv}}\Sigma = 0$$

- **The Sigma's can be constructed due to the trivial cohomology at the Tachyon Vacuum, using bcc's**

$$Q_{\rm tv}A \equiv QA + [\Psi_{\rm tv}, A] = 1$$

*No open strings at TV*

$$Q_{\rm tv}\overline{\Sigma} = Q_{\rm tv}\Sigma = 0 \longrightarrow \begin{array}{l} \Sigma = Q_{\rm tv}(A\sigma) \\ \overline{\Sigma} = Q_{\rm tv}(A\overline{\sigma}) \end{array}$$

$$\overline{\Sigma}\Sigma = Q_{\rm tv}\left(A\overline{\sigma}\sigma\right) = 1$$

**IF** $\overline{\sigma}\sigma = 1$

- **Convenient universal choice**

$$\sigma = e^{i\sqrt{h}X^0}\sigma_*^{(c=25)}$$

$$\overline{\sigma} = e^{-i\sqrt{h}X^0}\overline{\sigma}_*^{(c=25)}$$

*Explicitly possible for*
*time independent backgrounds!*
*(this adds a pure gauge time-like Wilson line)*
***other possible constructions??***

# Background independence

- Action for fluctuations

$$S[\Psi+\phi]=\frac{1}{2\pi^2}(g_0-g_*)-\frac{1}{2}\mathrm{Tr}[\phi\, Q_\Psi\phi]-\frac{1}{3}\mathrm{Tr}[\phi^3]$$

- Consider the peculiar **BCFT$_0$** states

$$\phi=\Sigma\phi_*\overline{\Sigma},\qquad \phi_*\in\mathrm{Fock}_{\mathrm{BCFT}_*}$$

- Remarkably

$$Q_\Psi\left(\Sigma\phi_*\overline{\Sigma}\right)=\Sigma(Q\phi_*)\overline{\Sigma}$$

- And we get the theory DIRECTLY formulated in **BCFT*!**

$$S[\Psi+\phi]=\frac{1}{2\pi^2}(g_0-g_*)-\frac{1}{2}\mathrm{Tr}[\phi_*Q\phi_*]-\frac{1}{3}\mathrm{Tr}[\phi_*^3]$$

- **BCFT*** can be composite (multibranes) and this construction gives rise to **Chan-Paton's factors**, out of a **single** D-brane!

# So we are now in a new phase for OSFT

- *Tantalizing conjecture*

**OSFT EOM implies BCFT constraints (bootstrap)**

*This is why EM works, essentially!*

- *Can we find the most generic solution to OSFT?*
- *All known D-branes give rise to solutions, can we reverse the argument to DISCOVER new D-branes?*
- *Long-standing problem in CFT!*

***…As a first step in this challenge let's see how to generate new solutions from known ones:***

***Topological Defects in OSFT***

# Open string defect operators in OSFT

(Kojita, Masuda, CM, Schnabl, 2015)

$$\mathcal{D} : H_{\text{open}} \to H'_{\text{open}}$$

$$[\mathcal{D}, Q] = 0$$

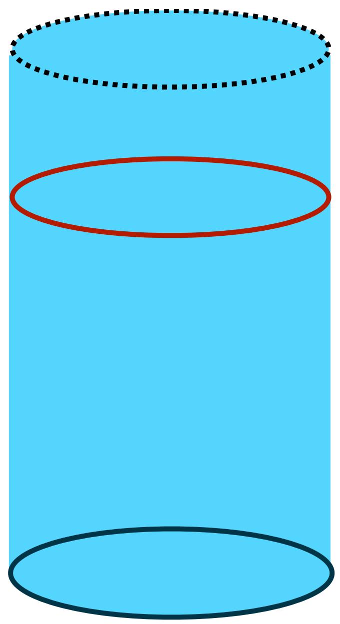
$$\mathcal{D}(\Psi_1 * \Psi_2) = (\mathcal{D}\Psi_1) * (\mathcal{D}\Psi_2)$$

- They map solutions to solutions

$$Q\Psi + \Psi * \Psi = 0 \qquad \to \qquad Q(\mathcal{D}\Psi) + (\mathcal{D}\Psi) * (\mathcal{D}\Psi) = 0$$

- ***Generalization of symmetries*** (which are ***group-like defects***)

- An operator $\mathcal{D}$ can be explicitly constructed starting from a ***closed*** topological defect line $D_{\rm cl}$ .




$$[D_{\rm cl}, T(z)] = [D_{\rm cl}, \bar{T}(\bar{z})] = 0$$

$$D^d_{\rm cl} = \sum_{i,\bar{i} \in H_{\rm cl}} D^d_{i\bar{i}} \, P_{i\bar{i}}$$ *Petkova-Zuber (2000)*

- In (diagonal) RCFT: as many fundamental defects as irreps. The fusion rules govern their composition and the action on boundary states

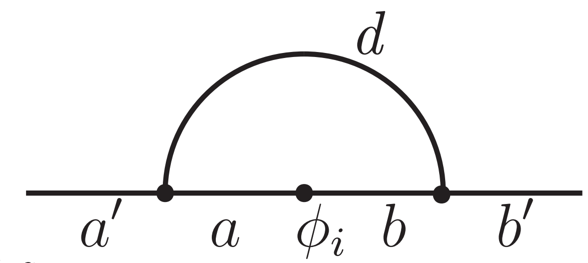
$$D^d_i = \frac{S_{di}}{S_{1i}}$$

$$[\phi_a][\phi_b] = \sum_i N^i_{ab} [\phi_i] \qquad D^d D^c = \sum_i N^i_{cd} \, D^i$$

$$D^c ||a\rangle\rangle = \sum_i N^i_{ca} \, ||i\rangle\rangle$$ *Graham-Watts (2003)*

- In the open string sector we must have

$$\mathcal{D}^d\,\psi_i^{(ab)} = \sum_{\substack{a' \in d\times a \\ b' \in d\times b}} X^{dab}_{ia'b'}\,\psi_i^{(a'b')} = \sum_{\substack{a' \in d\times a \\ b' \in d\times b}}$$




- Determine $X$ coeff. imposing the star algebra homomorphism

$$\mathcal{D}^d\left(\phi_i^{(ab)}(x)\phi_j^{(bc)}(y)\right) = \left(\mathcal{D}^d\phi_i^{(ab)}(x)\right)\left(\mathcal{D}^d\phi_j^{(bc)}(y)\right)$$

$$X^{dac}_{ka'c'}C^{(abc)k}_{ij} = \sum_{b'\in d\times b} C^{(a'b'c')k}_{ij} X^{dab}_{ka'b'} X^{dbc}_{kb'c'},$$

- In explicit case of diag. minimal models we find (pentagon identity)

$$X^{dab}_{ia'b'} = F_{di}\begin{bmatrix} a & b \\ a' & b' \end{bmatrix} \frac{\sqrt{F_{1a'}\begin{bmatrix} a & d \\ a & d \end{bmatrix} F_{1b'}\begin{bmatrix} b & d \\ b & d \end{bmatrix}}}{F_{1i}\begin{bmatrix} a & b \\ a & b \end{bmatrix}}$$

*Generalizes Graham and Watts (2003)*

- The composition (fusion) is trickier than in the bulk case. Naively we would expect

$$\mathcal{D}^d \mathcal{D}^c \psi = \bigoplus_e {N_{dc}}^e \mathcal{D}_e \psi \qquad (?)$$

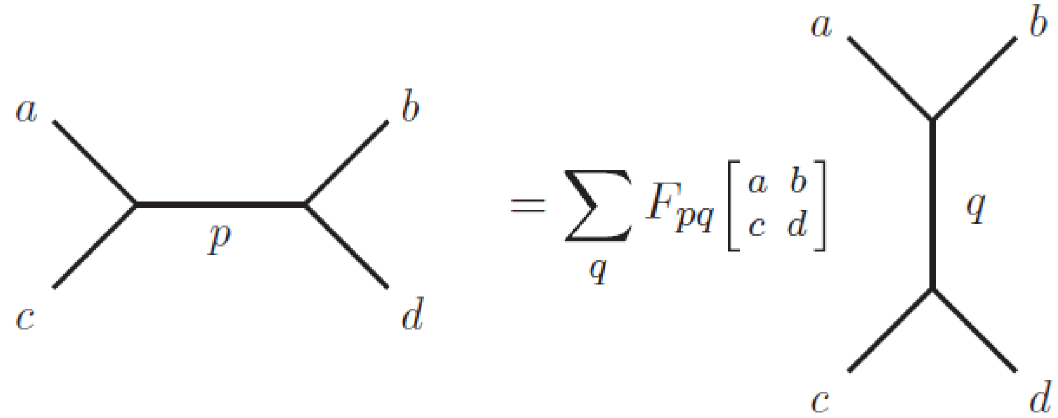
- However explicit computation reveals a similarity transformation !

$$\mathcal{D}^d \mathcal{D}^c \psi = U_{dc} \left( \bigoplus_e {N_{dc}}^e \mathcal{D}_e \psi \right) U_{dc}^{-1}$$

$$(U_{dc})^T = (U_{dc})^{-1}$$

$$(U_{dc})^{\{aa'a''\}[e;a,a'']} = \sqrt{\frac{F_{1a''}\begin{bmatrix} d & a' \\ d & a' \end{bmatrix} F_{1a'}\begin{bmatrix} c & a \\ c & a \end{bmatrix}}{F_{1a''}\begin{bmatrix} e & a \\ e & a \end{bmatrix} F_{1e}\begin{bmatrix} d & c \\ d & c \end{bmatrix}}}\, F_{a'e}\begin{bmatrix} d & c \\ a'' & a \end{bmatrix}$$

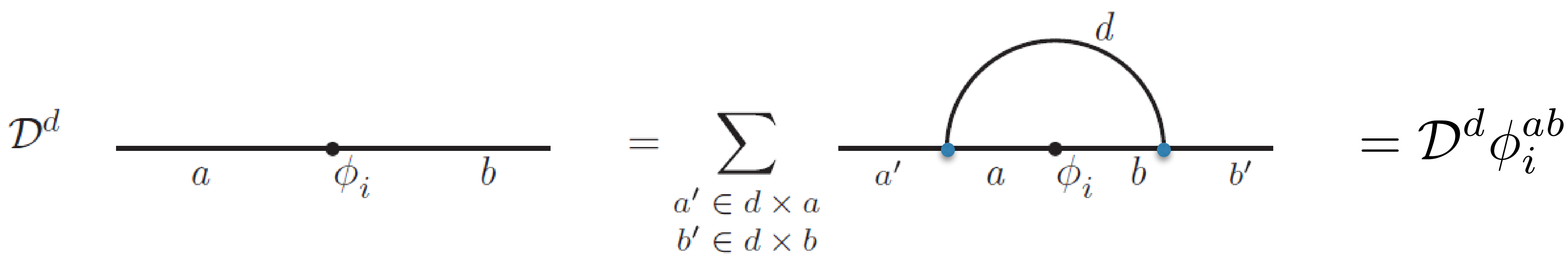
- This becomes transparent using defect-network manipulations



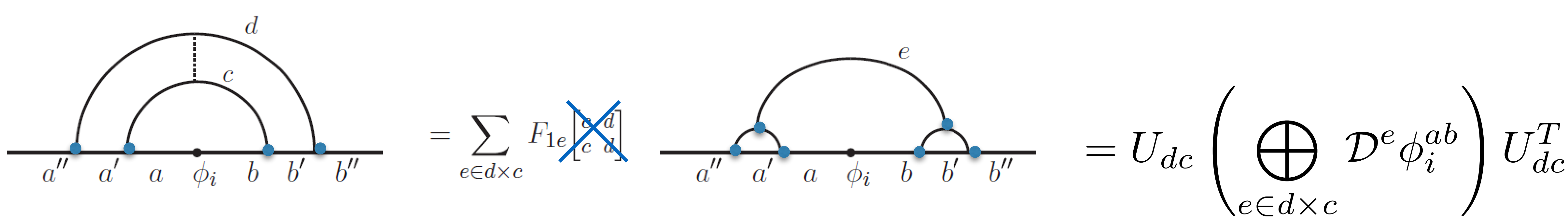

*Frolich, Fuchs, Runkel , Schwieger (2006)*

***[These are defect networks, not (a priori) conformal blocks!]***

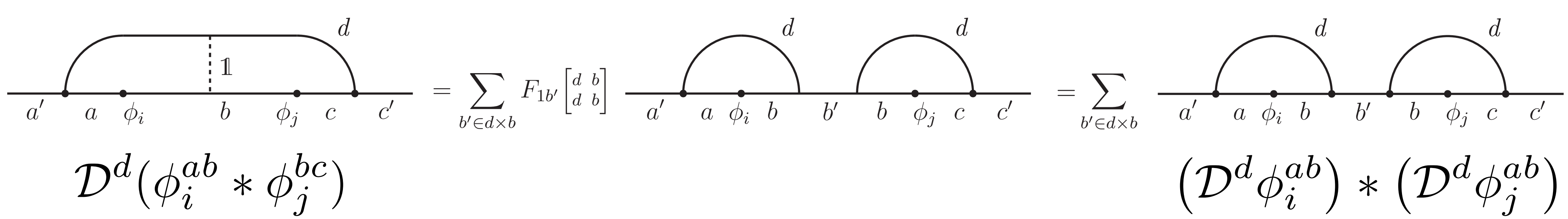
- Defect action




- Defect fusion




- Important to keep track of ***defect junctions*** (from which the square roots of F originates), example:



# OSFT observables and defects

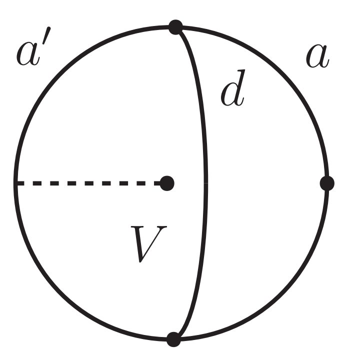
- Change in the (off-shell) action

$$S_{\mathrm{OSFT}}(\mathcal{D}^d\Psi) = \frac{g_d}{g_1} S_{\mathrm{OSFT}}(\Psi) \qquad g_a \equiv \langle\langle a \| 0 \rangle_{SL(2,C)} = \langle 1 \rangle^{\mathrm{BCFT}_a}_{\mathrm{disk}}$$

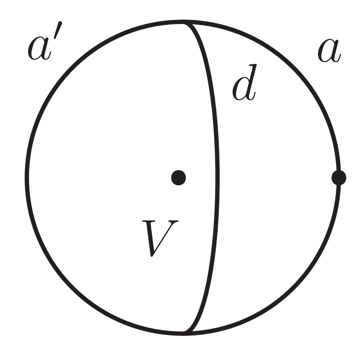
- Change in the gauge-invariant coupling to closed strings (Ellwood invariant)

$$\mathrm{Tr}_V[\mathcal{D}^d\Psi] = \mathrm{Tr}_{D^d_{\mathrm{cl}}V}[\Psi]$$

$\mathrm{Tr}_V\left[\mathcal{D}^d\Psi\right] = \sum_a \sum_{a' \in d\times a}$

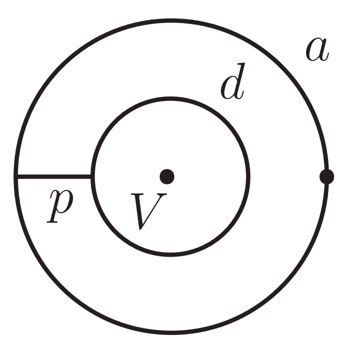



$f^{(1)} \circ \Psi^{aa} = \sum_a \sum_{a' \in d\times a} F_{1a'}\begin{bmatrix} d & a \\ d & a \end{bmatrix}$

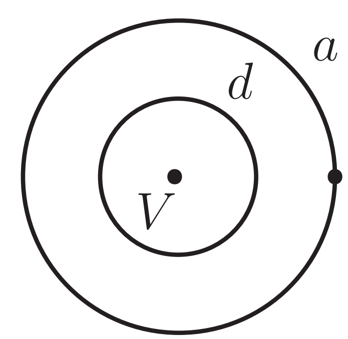



$f^{(1)} \circ \Psi^{aa}$

$= \sum_{a,p} \sum_{a' \in d\times a} F_{1a'}\begin{bmatrix} d & a \\ d & a \end{bmatrix} F_{a'p}\begin{bmatrix} a & a \\ d & d \end{bmatrix}$




$f^{(1)} \circ \Psi^{aa} = \sum_a$




$f^{(1)} \circ \Psi^{aa} = \mathrm{Tr}_{D^d V}[\Psi]$

# OSFT boundary state and defects

- Given a solution $\Psi_{X\to Y}$, we can compute its boundary state

$$|B(\Psi_{X\to Y})\rangle_{\rm OSFT} = \|Y\rangle\!\rangle_{\rm BCFT}$$

*Kiermaier, Okawa, Zwiebach (2008)*
*Kudrna, CM, Schnabl (2012)*

- Previous slide computation has the important consequence that

$$|B(\mathcal{D}^d\Psi_{X\to Y})\rangle_{\rm OSFT} = D^d_{\rm cl}\|Y\rangle\!\rangle_{\rm BCFT}$$

*Kojita, Masuda, CM, Schnabl (2015)*

$$\mathcal{D}\Psi_{X\to Y} = \Psi_{DX\to DY}$$

# CONCLUSIONS

- OSFT as a dynamical “field” theory for BCFT.
- All known (time ind.) BCFT’s remarkably give exact analytic solutions of OSFT. The string field is indeed “big enough”!
- OSFT is (open) string background independent: background shift + field redefinition …*some subtlety still to address here, (1to1?)*
- Topological defects give rise to new operators in the open string algebra which map solutions to solutions.
- As solution generating operators, they must play an important role in the (so far mysterious) classification of OSFT solutions.
- *Superstrings??*

Thank you.