Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

FROGS
FRont Of pro-Galician Scientists

Holographic Charged Impurities

[1507.02280]

with
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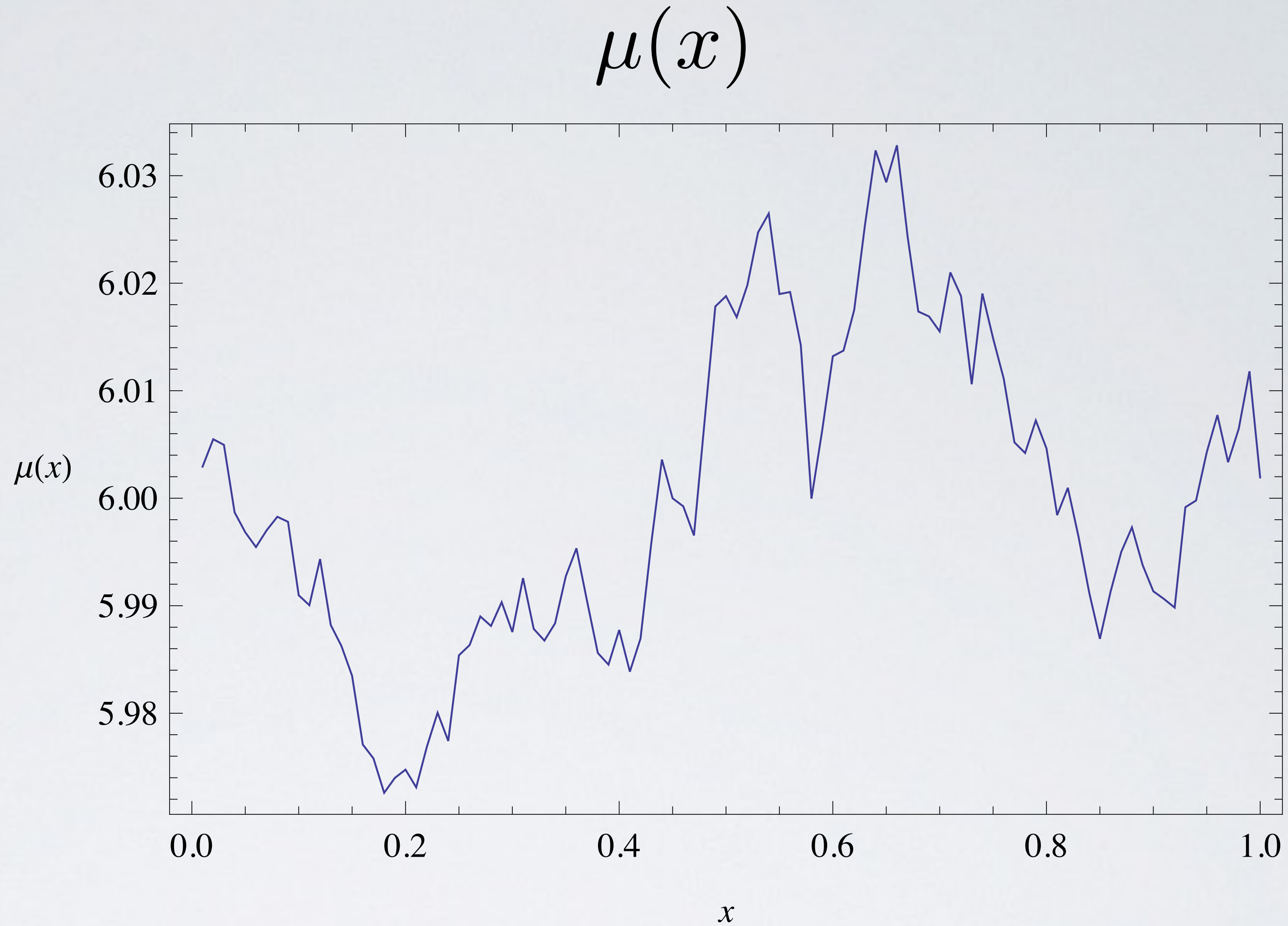
I. Salazar Landea (La Plata, Argentina)
A. Scardicchio (ICTP, Italy)

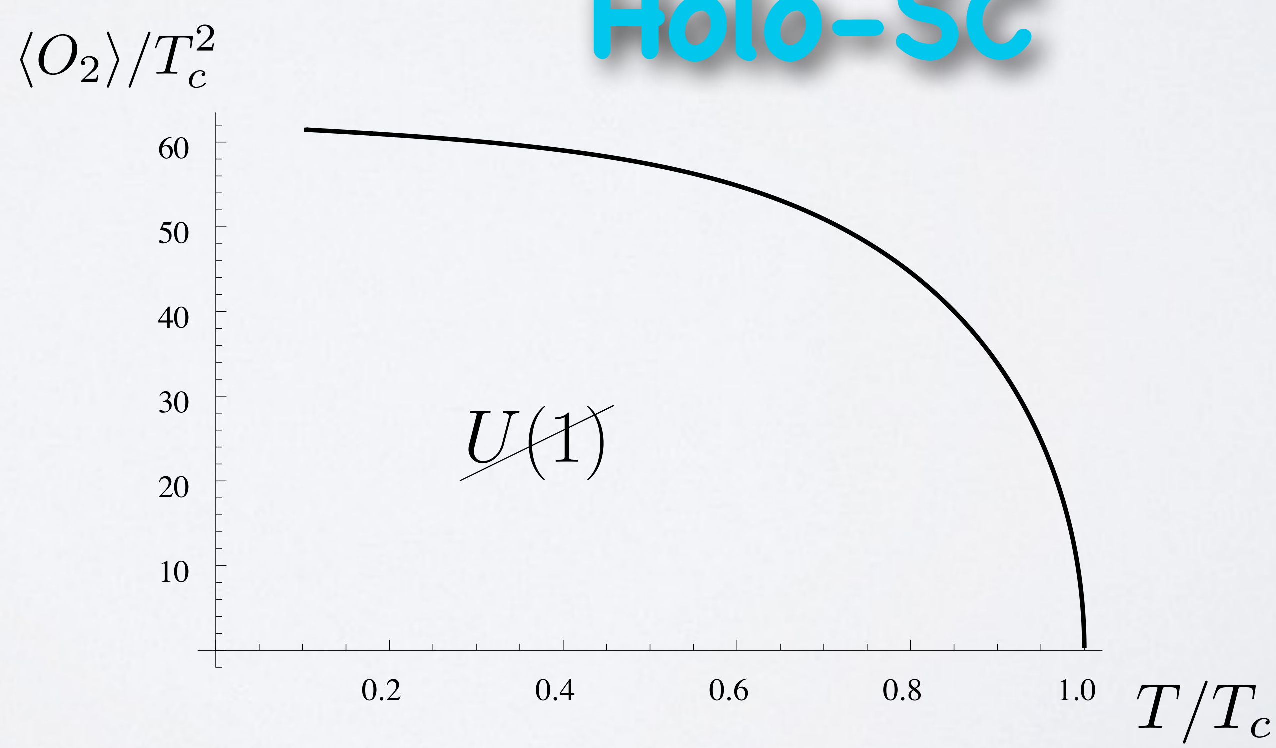
Daniel Areán
Sestri Levante, September 2015

Noise

[charged impurities]

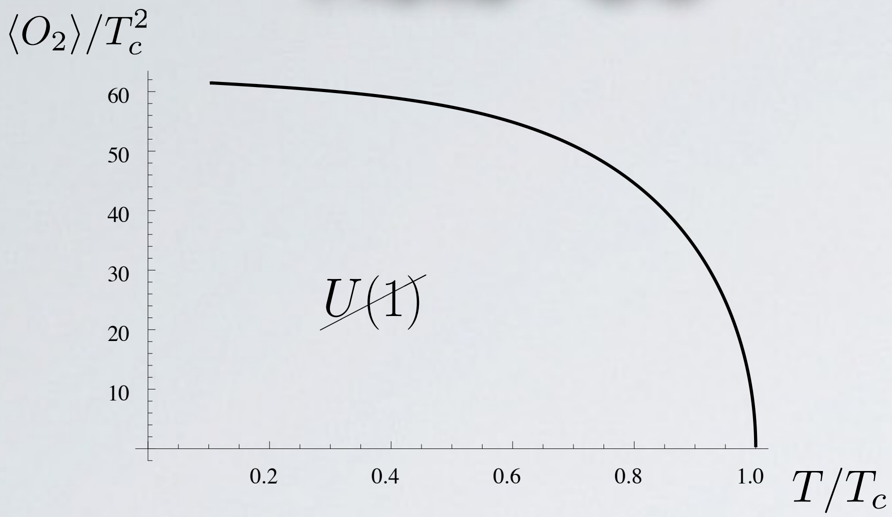


Holo-SC



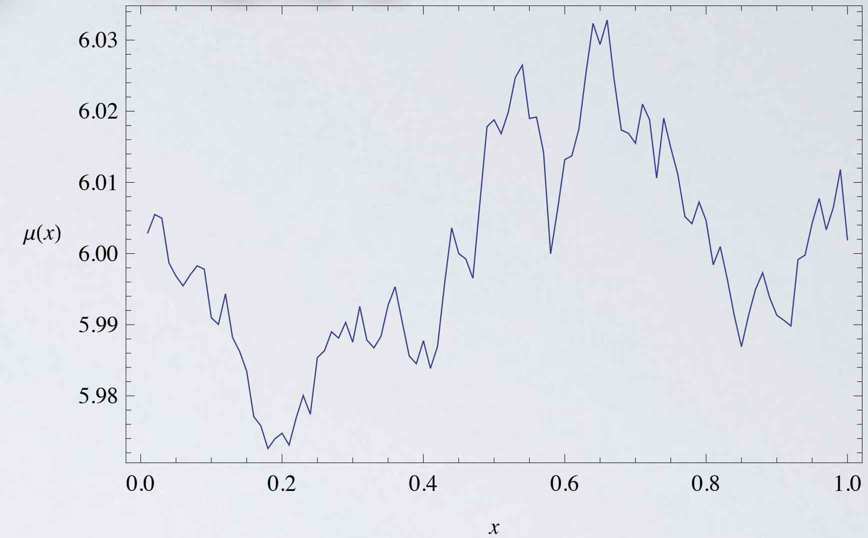
in holography

Holo-SC

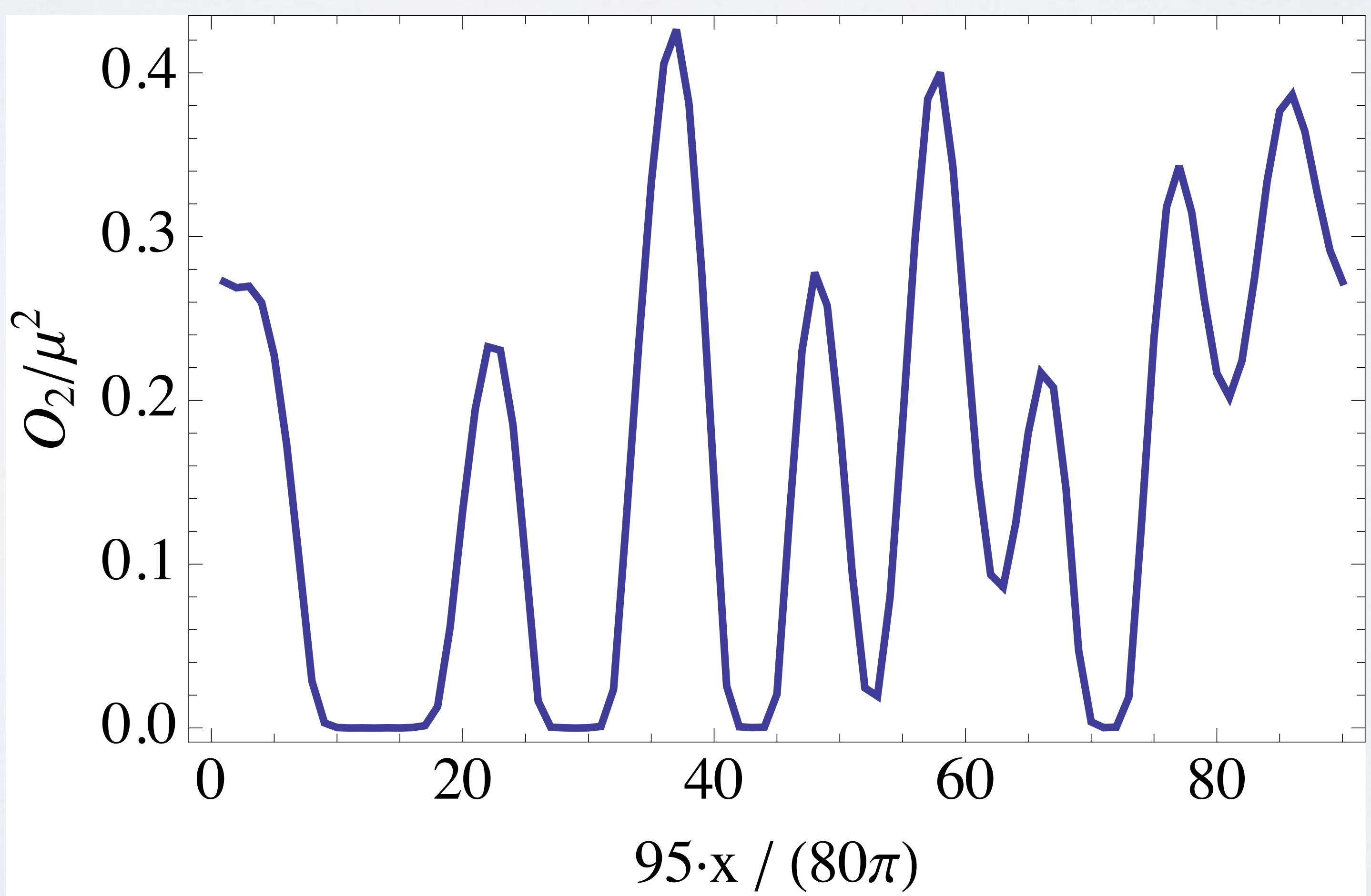


+

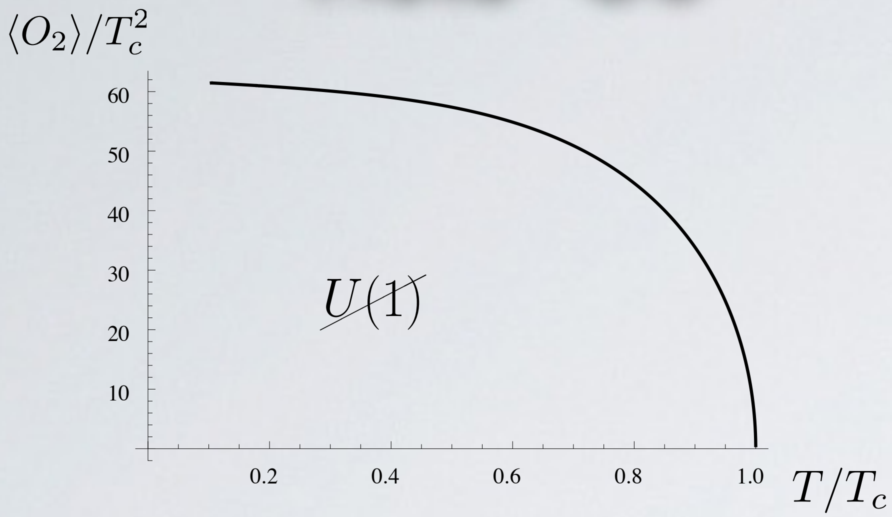
Noise $\mu(x)$



= ⋆ **Islands**

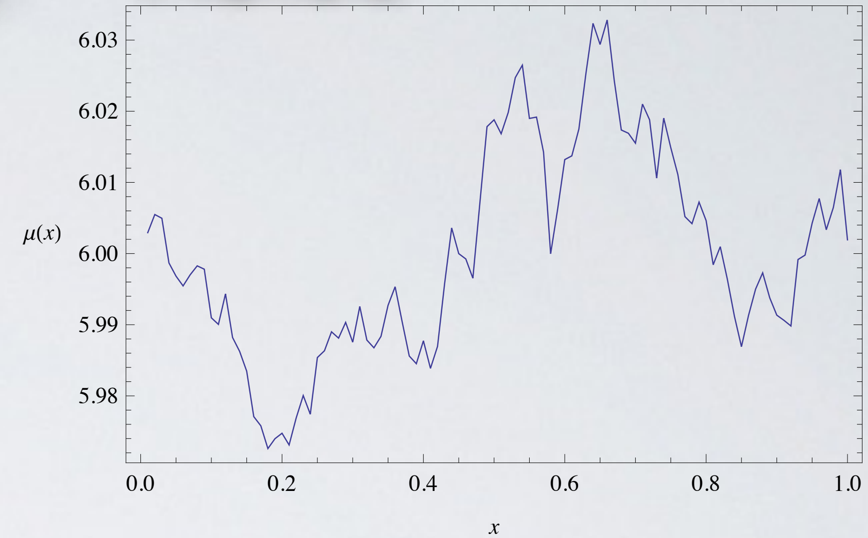


Holo-SC

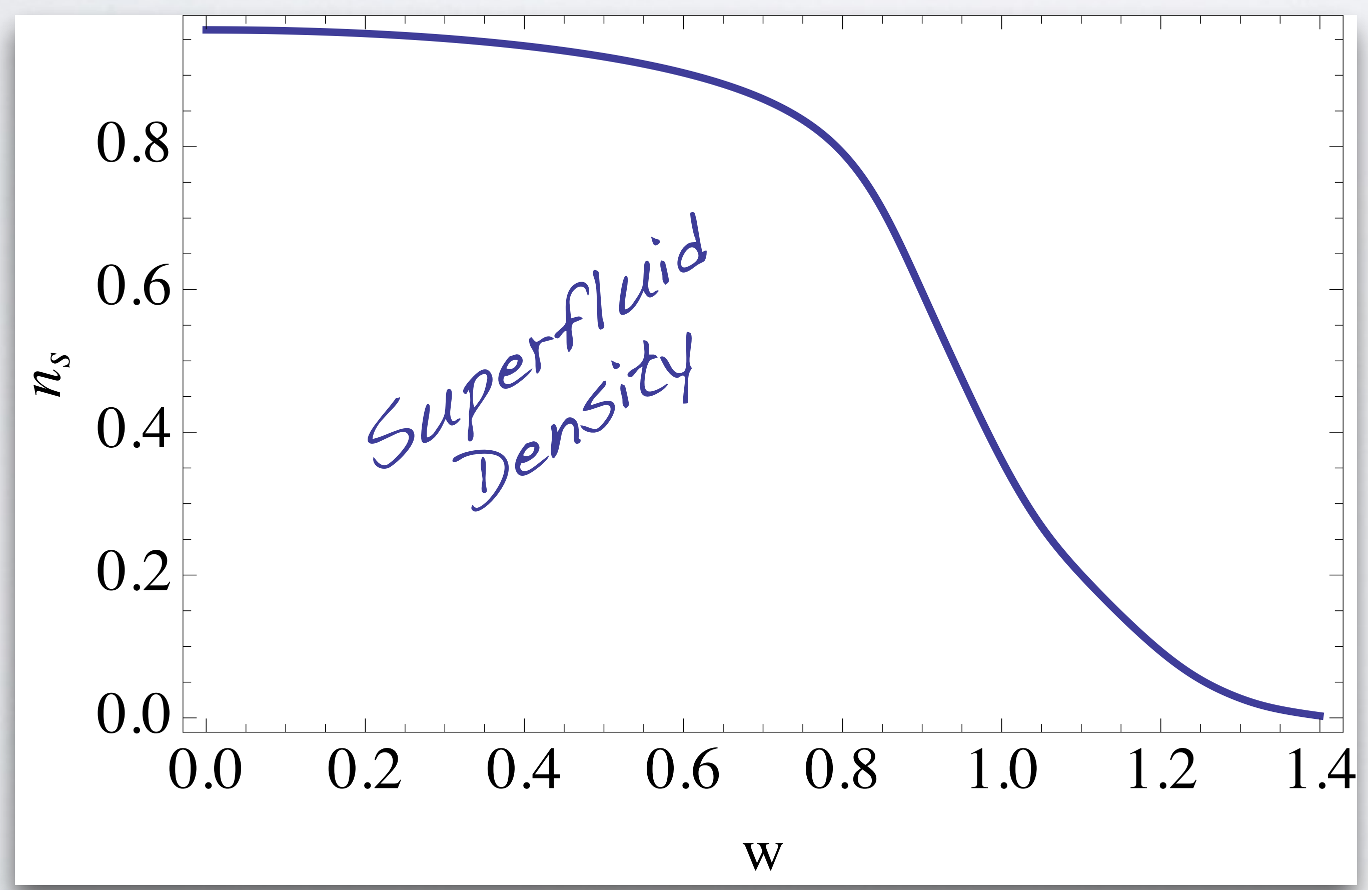


+

Noise $\mu(x)$



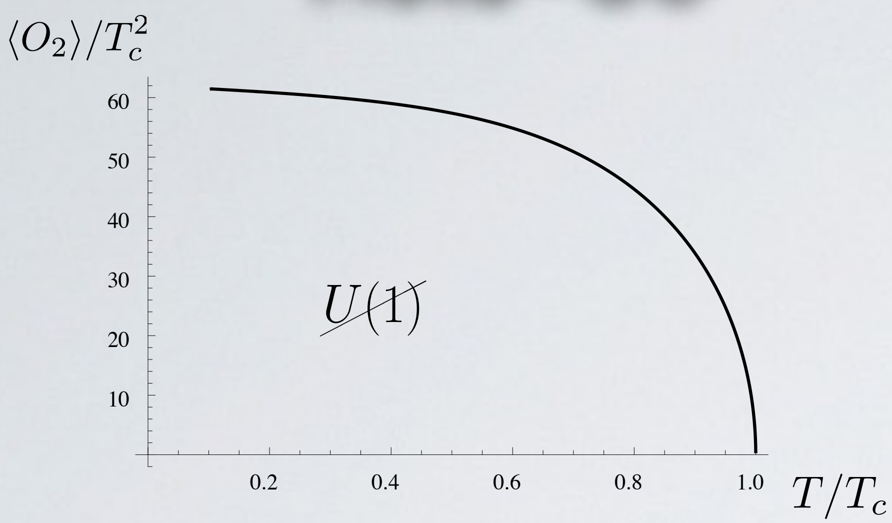
= ⋆ **Disorder can suppress the Superconductivity**



disorder →

Stay tuned:
Novel features in
AC Conductivity !!

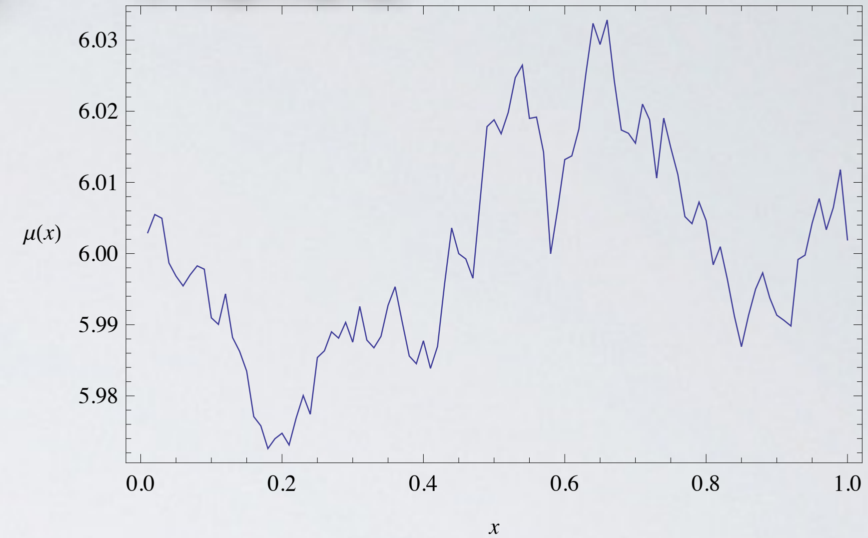
Holo-SC



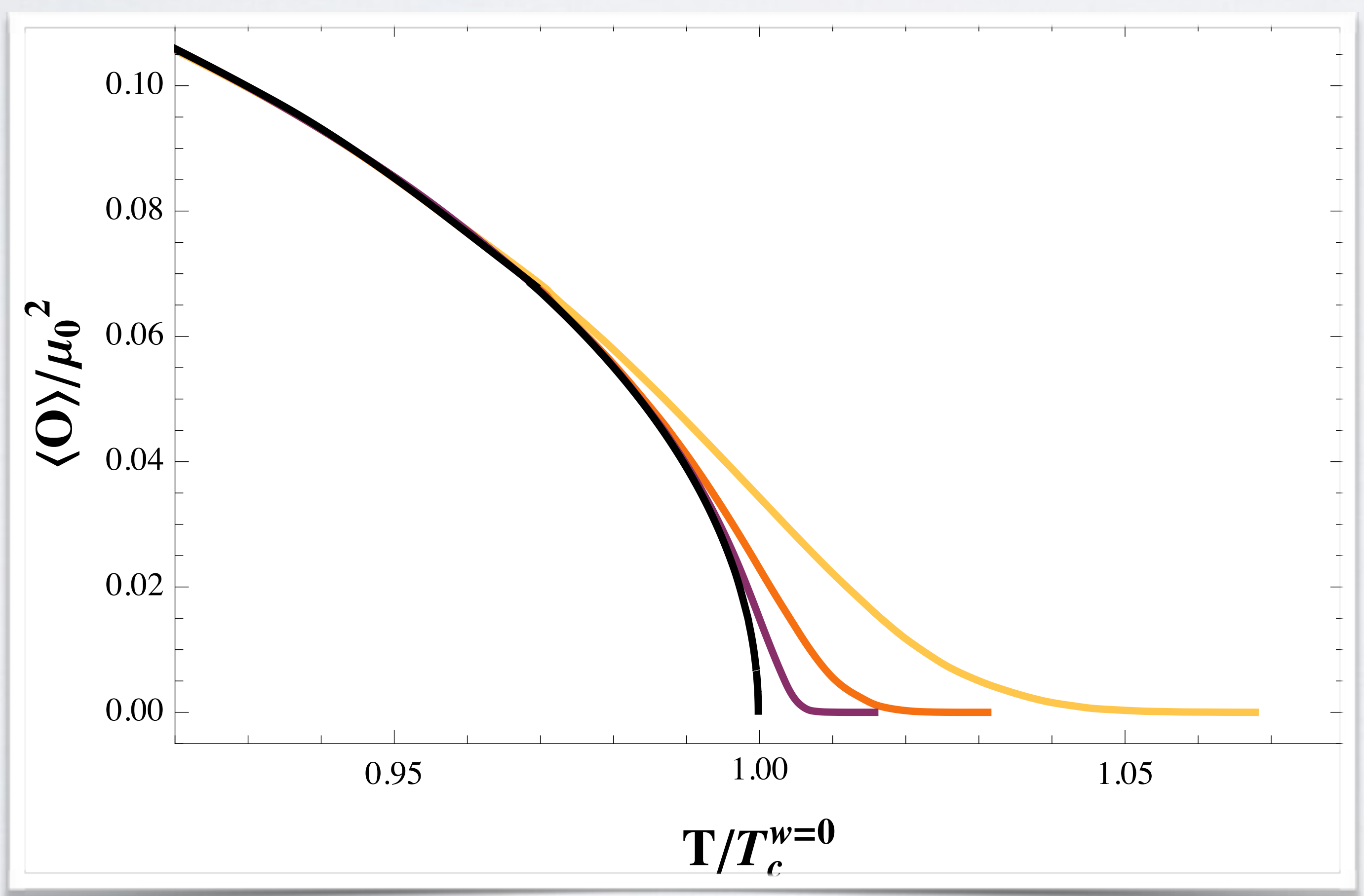
+

Noise

$\mu(x)$



= ★ 'Disordered' SC phase transition (non mean-field)



disorder

- w = 0.016
- w = 0.008
- w = 0.004
- w = 0

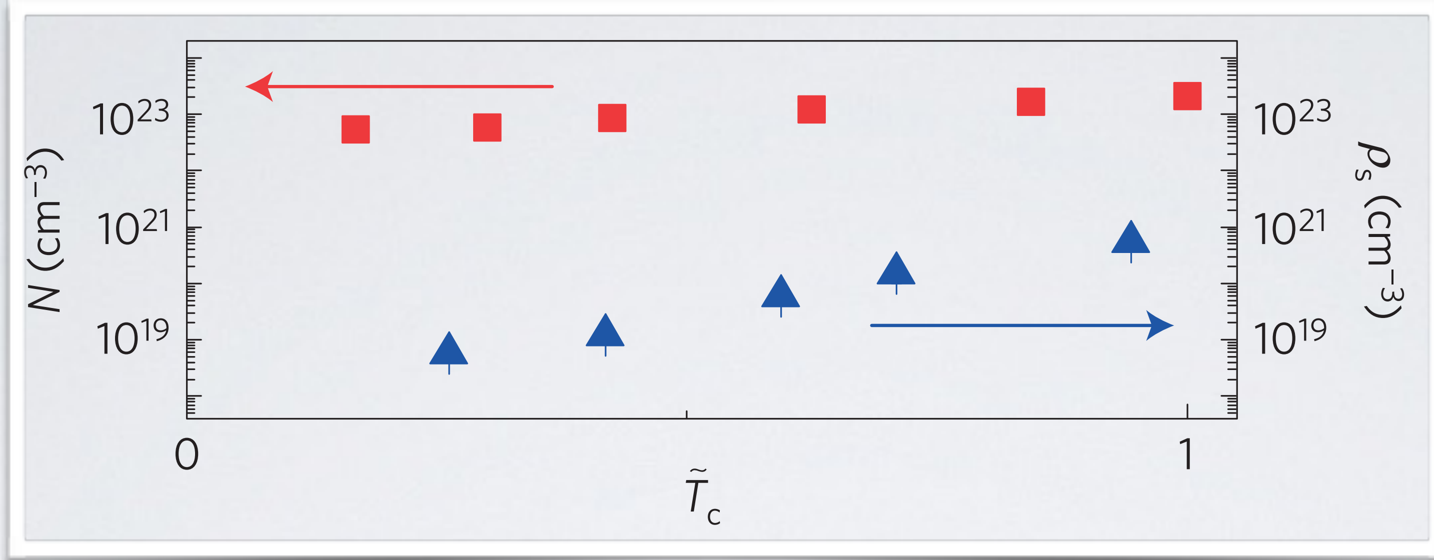
OUTLINE

- **Motivation:** Disorder-induced SIT
- **Setup:** holo SC w/ noise
- **Results:** Islands, phase transition, Conductivity.
- **Future:** Thin Films, backreaction (insulator?), . . .

> SC to insulator disorder-induced phase transition

> Experiment

[Sherman et al, Nature Phys 11, 188–192 (2015)]

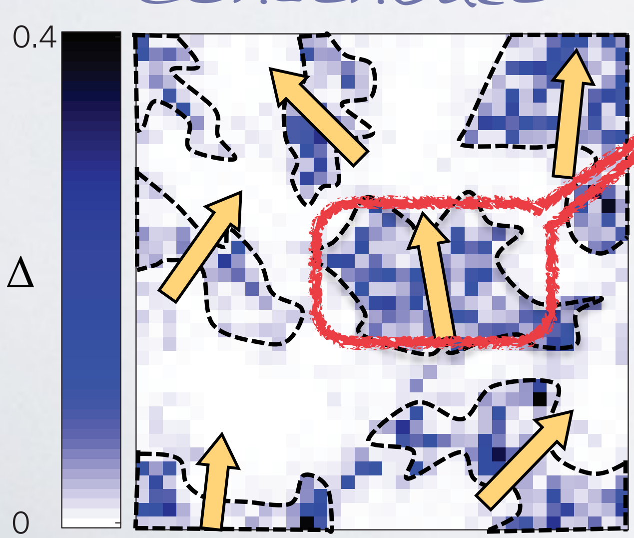


▲ Superfluid Density

disorder

> Theory (quantum Montecarlo)

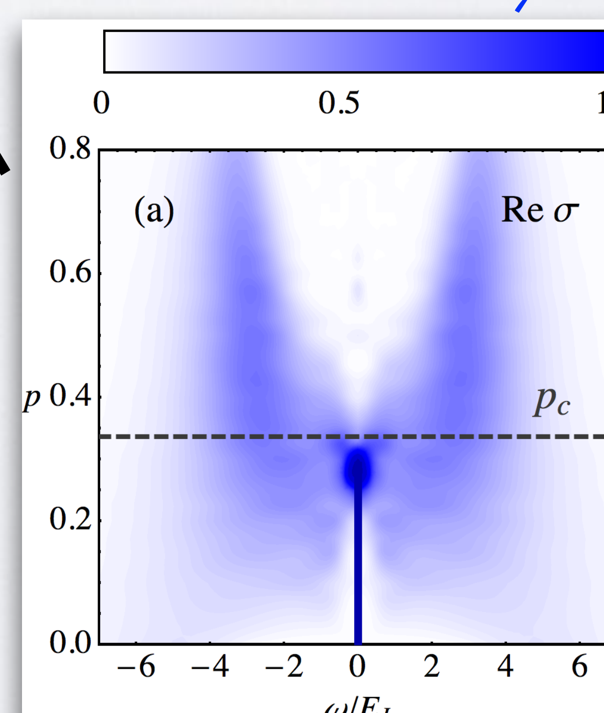
'Condensate'



Islands

disorder

Conductivity



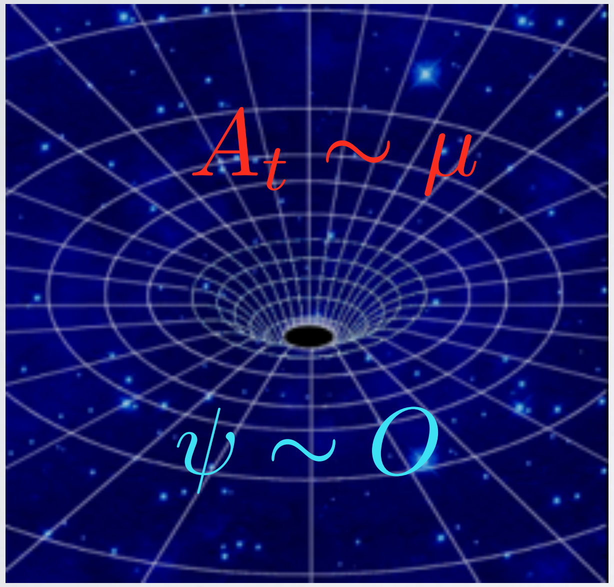
INSULATOR

SUPERCONDUCTOR

[Swanson et al, 1310.1073]

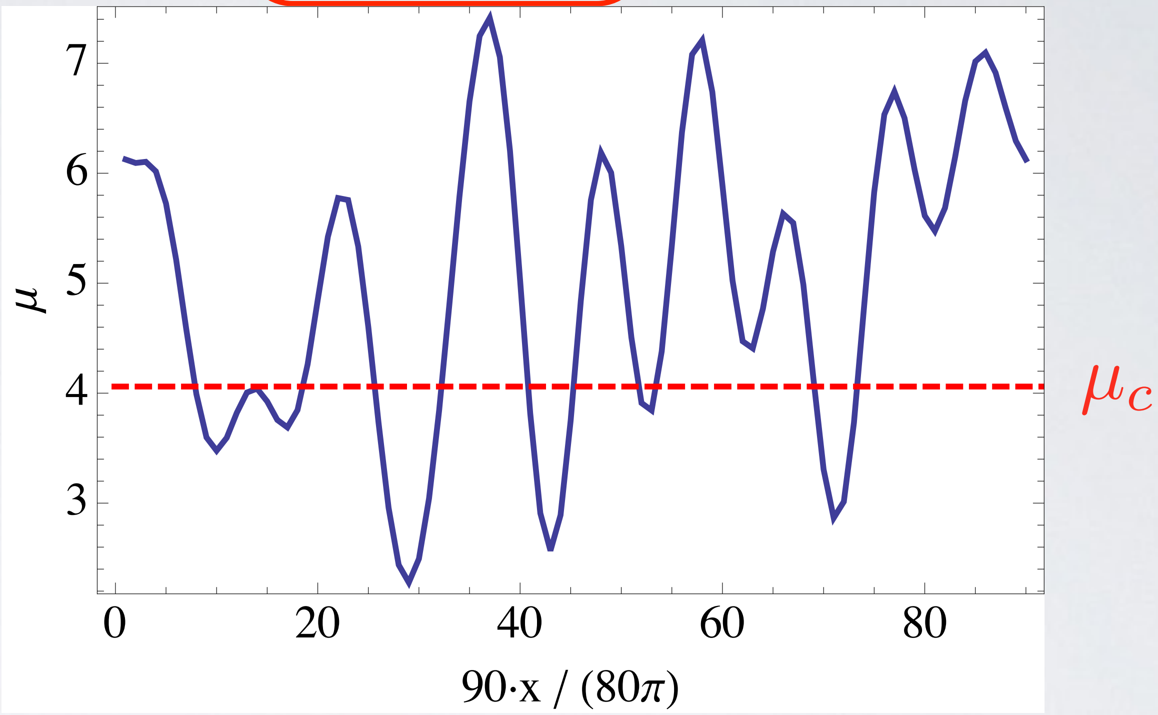
> SETUP: Dirty Holo (s-wave) Superconductor

Holo SC



+

Disordered $\mu(x)$



- **Action (probe limit)** $S = \int d^4x \sqrt{-g}\left(-\frac{1}{4}F_{ab}\,F^{ab} - (D_\mu\Psi)(D^\mu\Psi)^\dagger - m^2\Psi^\dagger\Psi\right)$

- **Geometry: Sch-AdS BH** $ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2 + dy^2\right)\,, \quad f(z) = 1 - z^3$

- **Field content**

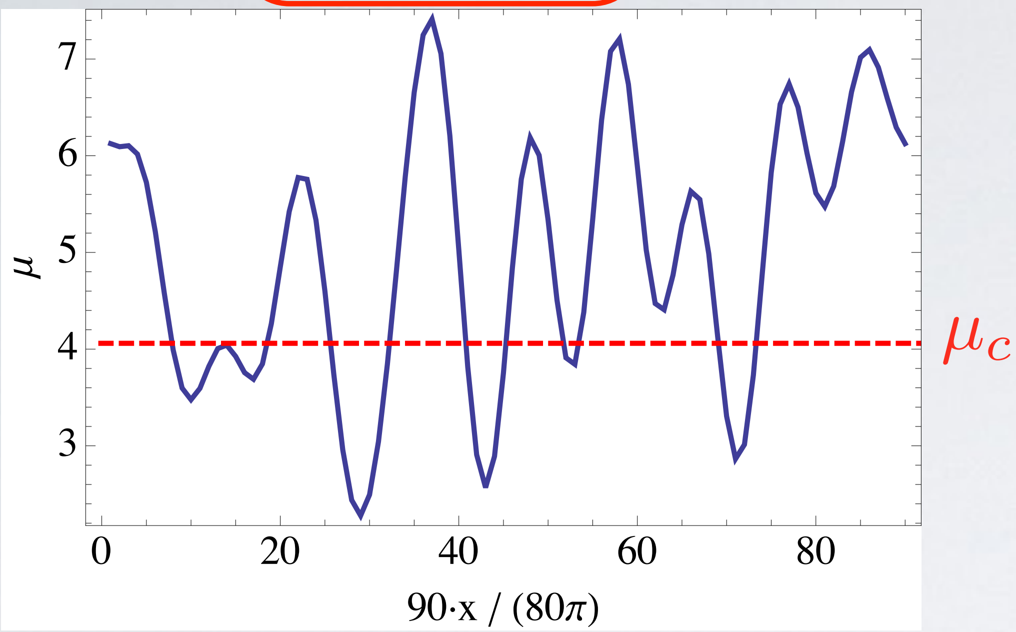
$$\Psi(x,z) = \psi(x,z)\,, \quad \psi(x,z) \in \mathbb{R} \qquad \sim \langle O(x)\rangle$$

$$A = \phi(x,z)\,dt \qquad \sim \quad \mu(x)$$

[Hartnoll et al'08]

> SETUP: Dirty Holo (s-wave) Superconductor

Disordered $\mu(x)$



Flat Noise

$$\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)$$

- w **Noise strength**
- $k_0 \sim$ **1/(System Size). [IR Scale]**
- $k_* \sim$ **1/Correlation length [UV Scale]**

EoMs ➡ **2 Coupled PDEs** **with...**

- **UV (z=0) Boundary Conditions**

$$\phi(x,z) = \mu(x) - \rho(x)\,z + \ldots$$

$$\psi(x,z) = \psi^{(1)}(x)\,z + \langle O(x)\rangle\, z^2 + \ldots$$

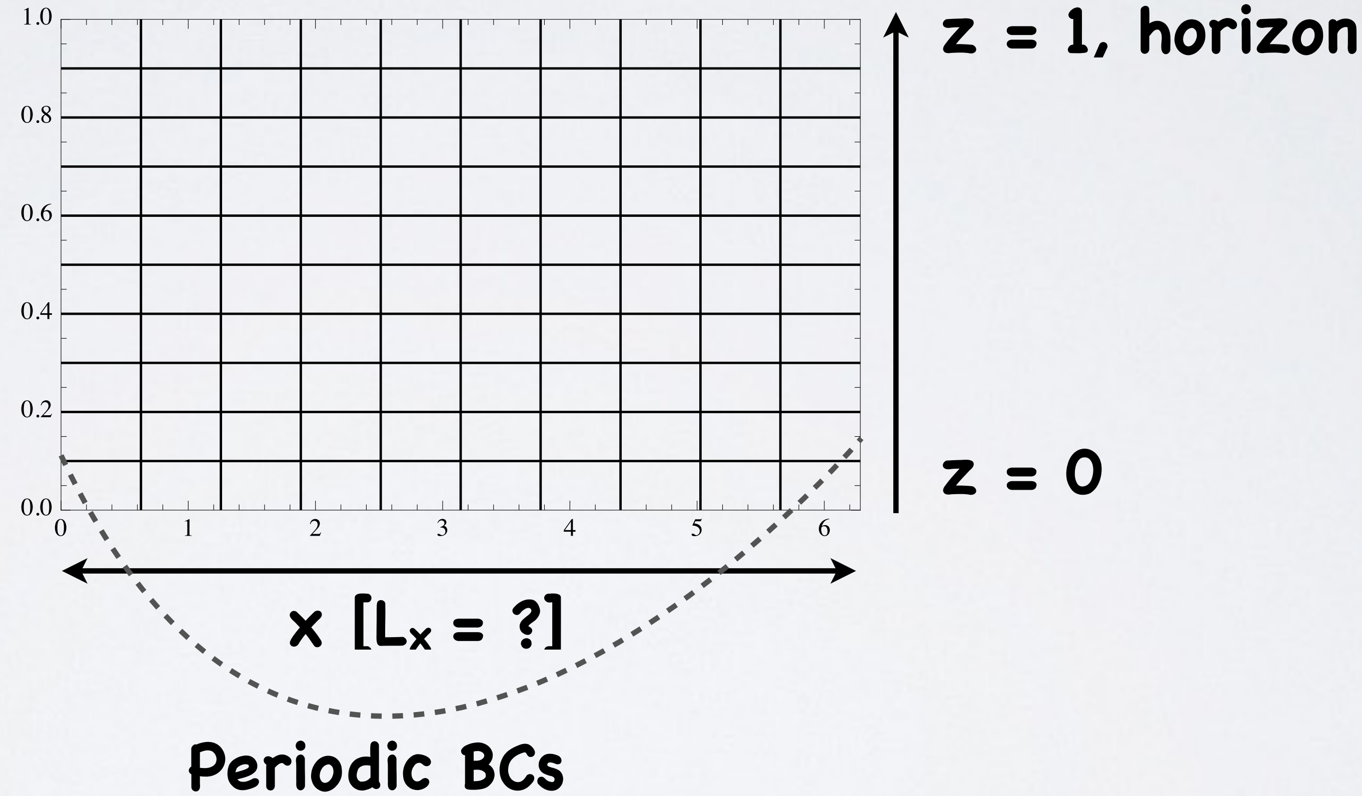
> SETUP: Solving the background ....

Flat Noise

$$\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)$$

- w Noise strength
- $k_0 \sim$ 1/(System Size). [IR Scale]
- $k_* \sim$ 1/Correlation length [UV Scale]

- System on a grid



- With (ideally): $L_x \gg 1 \, [\rightarrow \; k_0 \ll 1] \, , \; k_* \gg 1$

'Uncorrelated Noise'

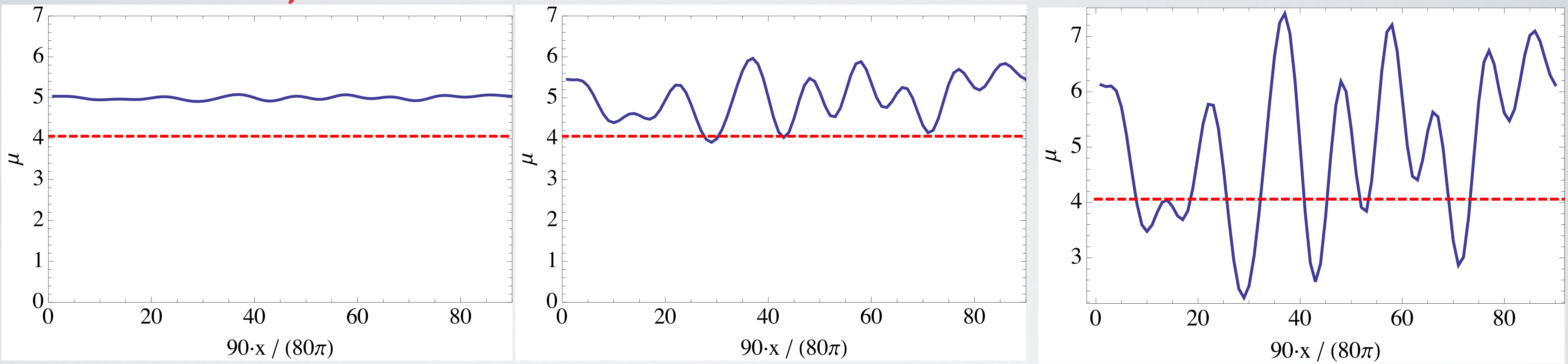
* [in units of temperature]

> Results: The Inhomogeneous Condensate

$\mu = 5 \rightarrow T \sim 0.8\, T_c$

$L_x = 80\pi$, 9 modes

Chemical potential

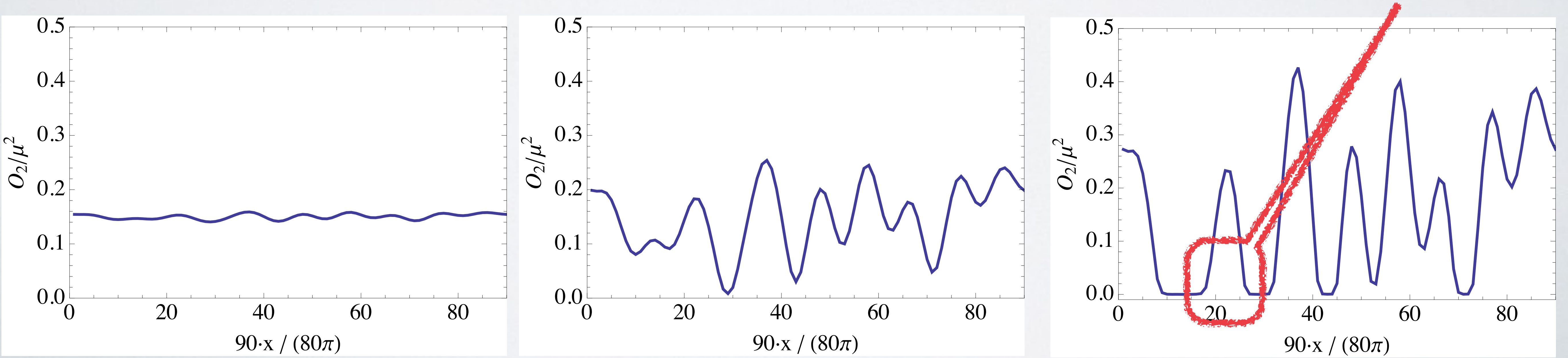


$w = 0.004$ $w = 0.048$ $w = 0.12$

disorder →

Condensate

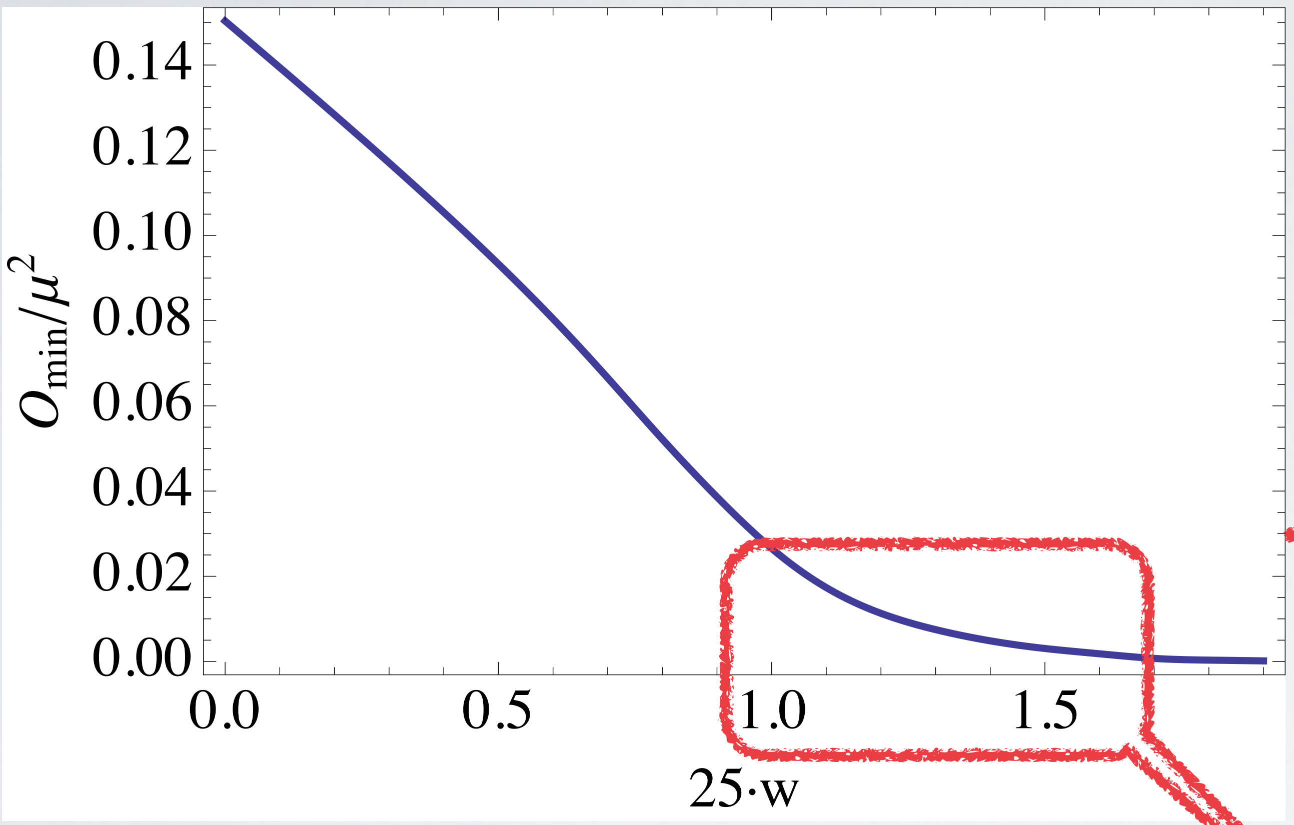
Islands?

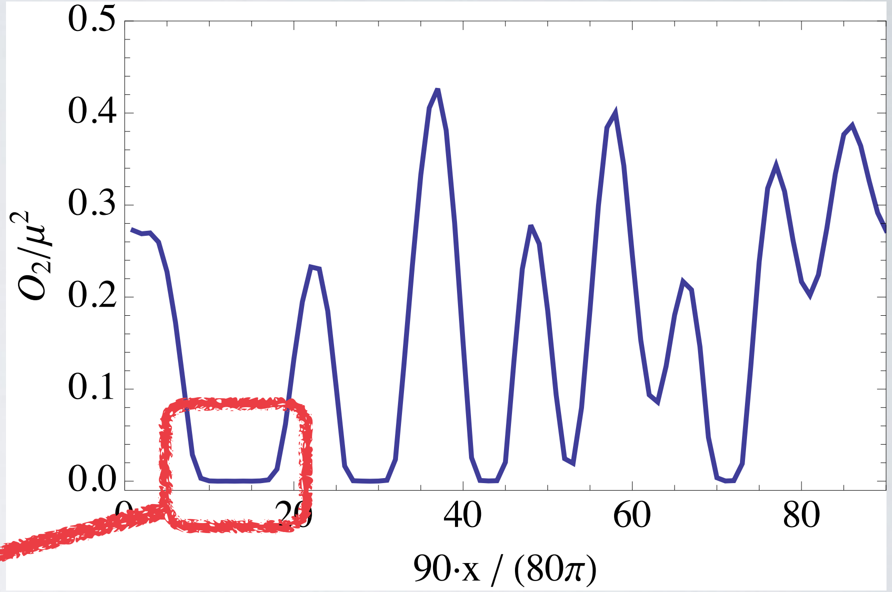


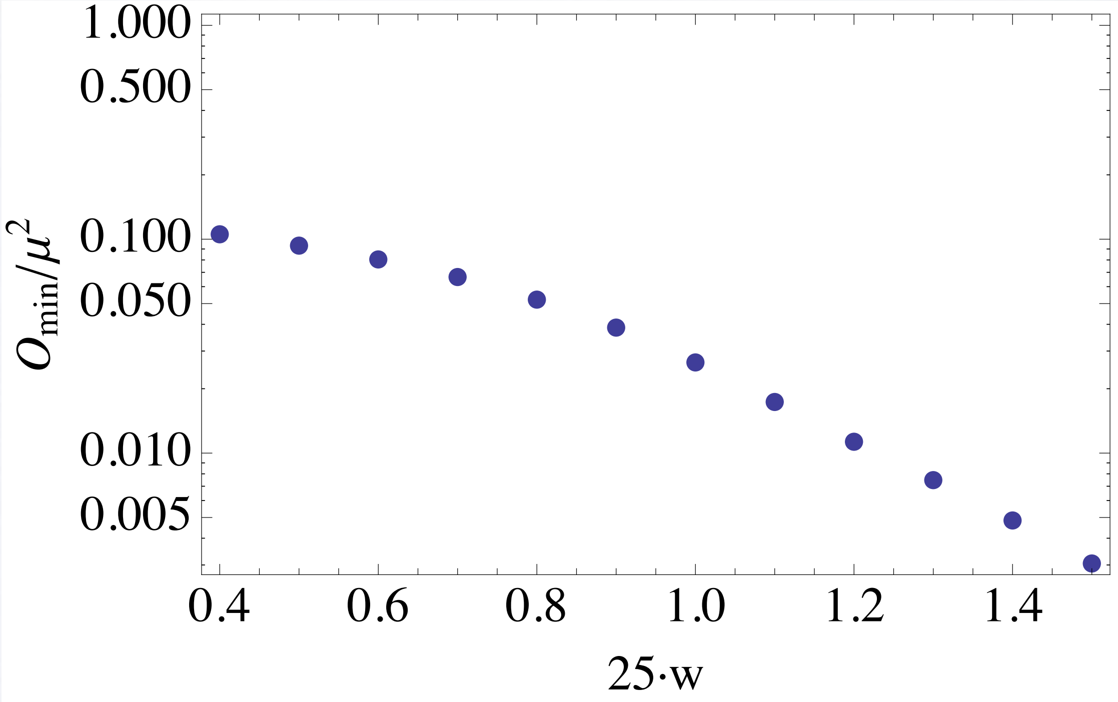
> Results: ISLANDS?

$\mu = 5 \to T \sim 0.8\,T_c$
$L_x = 80\pi\ ,\ 9$ modes

> Let's plot the minimum of the condensate







The condensate dies away exp-like

➡ *no critical noise?*

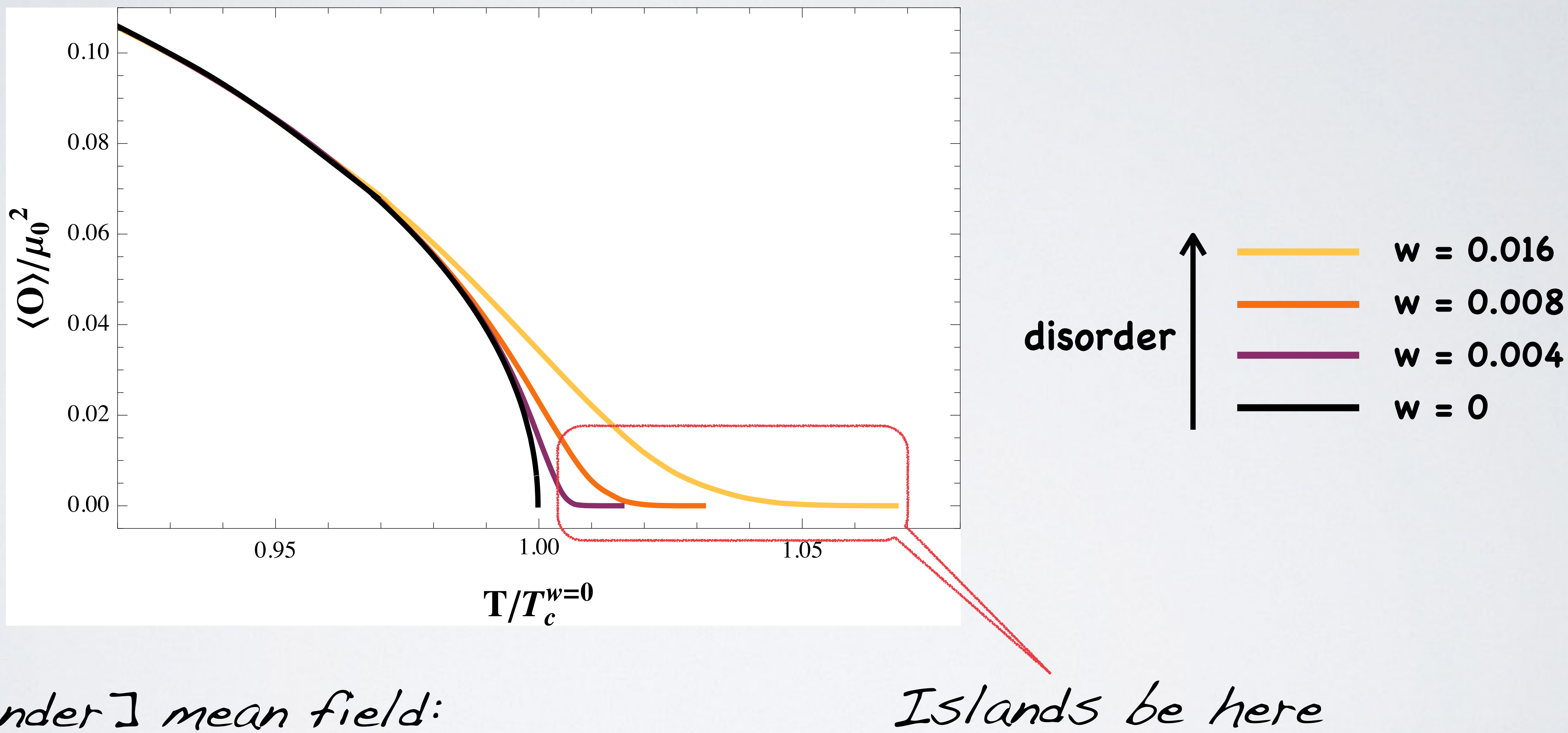
↕

Finite Size + Spec. Renorm. (kills higher modes)

➡ *Condensate leaks!*

> Results: Phase transition @ finite disorder

> Let's plot the average of the condensate vs Temperature



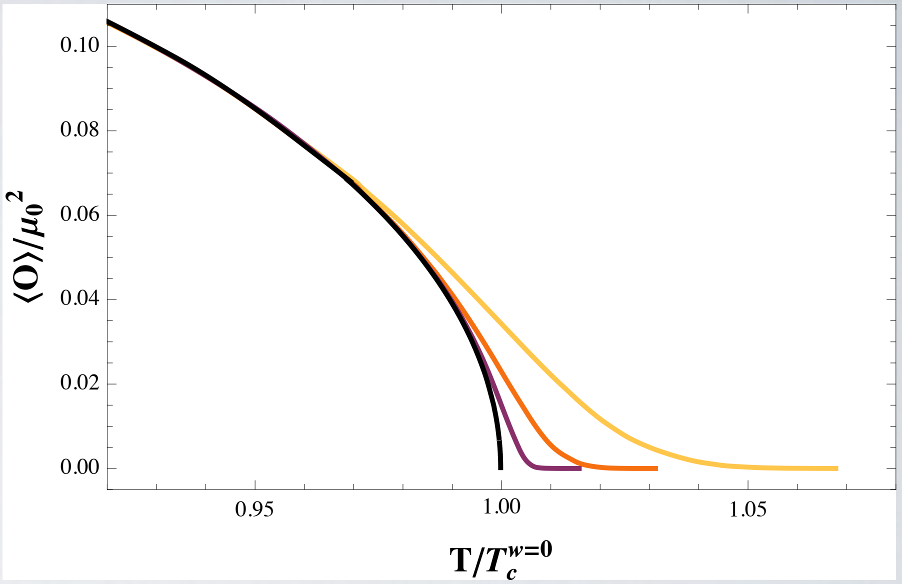
[reminder] mean field:

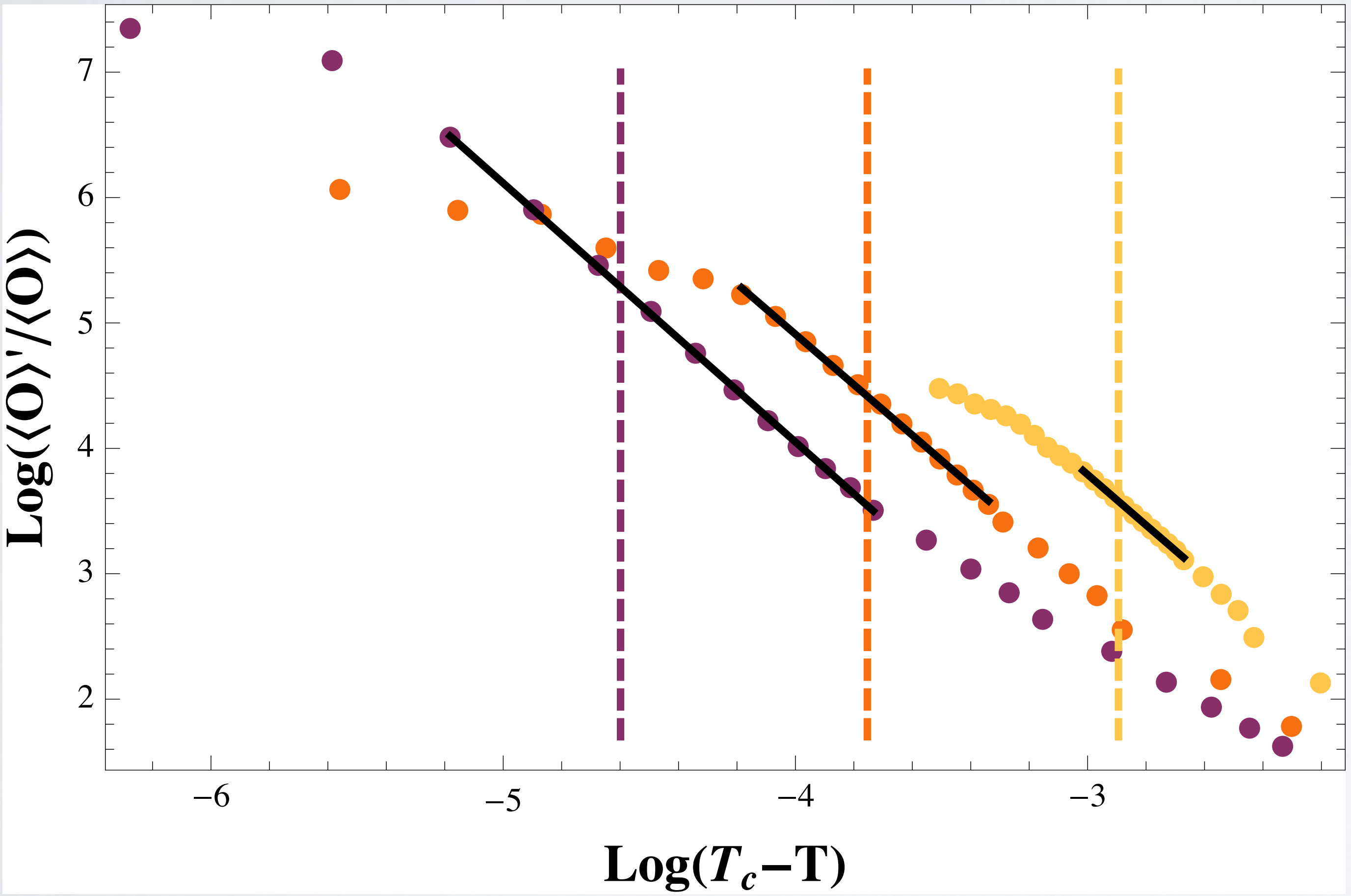
w = 0 → $\langle \mathcal{O} \rangle \sim \sqrt{T - T_c}$

[See Griffiths' phases... T. Vojta, PRL'03]

> Results: Phase transition @ finite disorder

> Average of the condensate vs Temperature…





$$\langle \mathcal{O} \rangle \sim \exp\left(-A|T - T_c|^{-\nu}\right)$$

with

$$\nu = 1.03 \pm 0.02$$

[See Griffiths' phases… T. Vojta, PRL'03]

> Computing the conductivity [⇨ Superfluid density]

$$\sigma_x(\omega) = \frac{\langle j_x(x,\omega)\rangle}{E_x(\omega)}$$

→

Study perturbations

$$\delta A_x = a_x(z,x)\, e^{-i\omega\, t} \sim j_x(x,\omega)$$

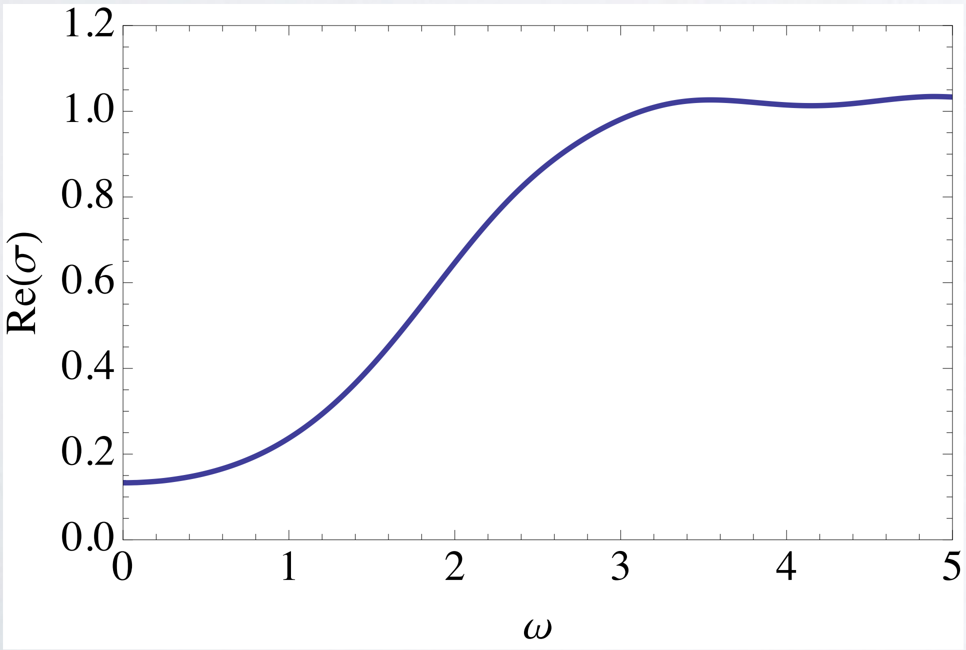
which couple to

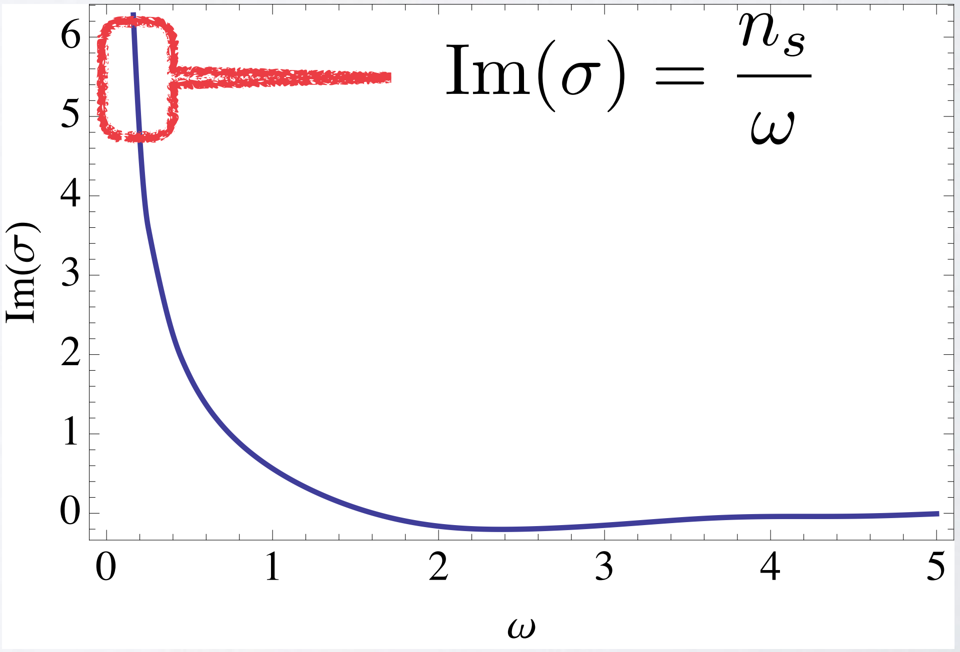
$$(\delta A_t\,,\,\delta\Psi) = (a_t(z,x)\,,\,\psi_r(z,x)\,,\,\psi_i(z,x))e^{-i\omega\, t}$$

⇨ **4 Coupled linear PDEs**

> Reminder: Homogeneous case

$$\mu = 5 \rightarrow T \sim 0.8\, T_c$$

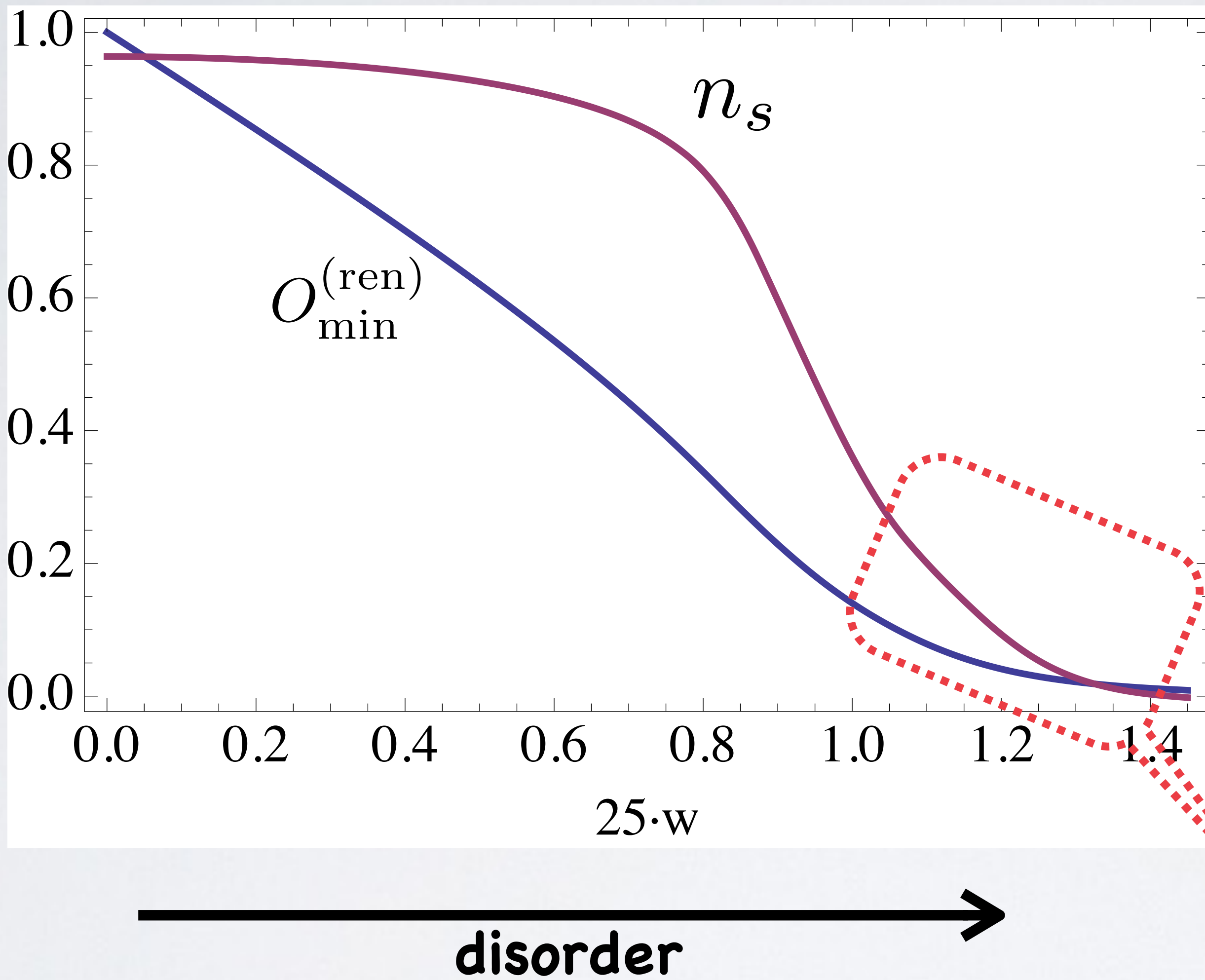


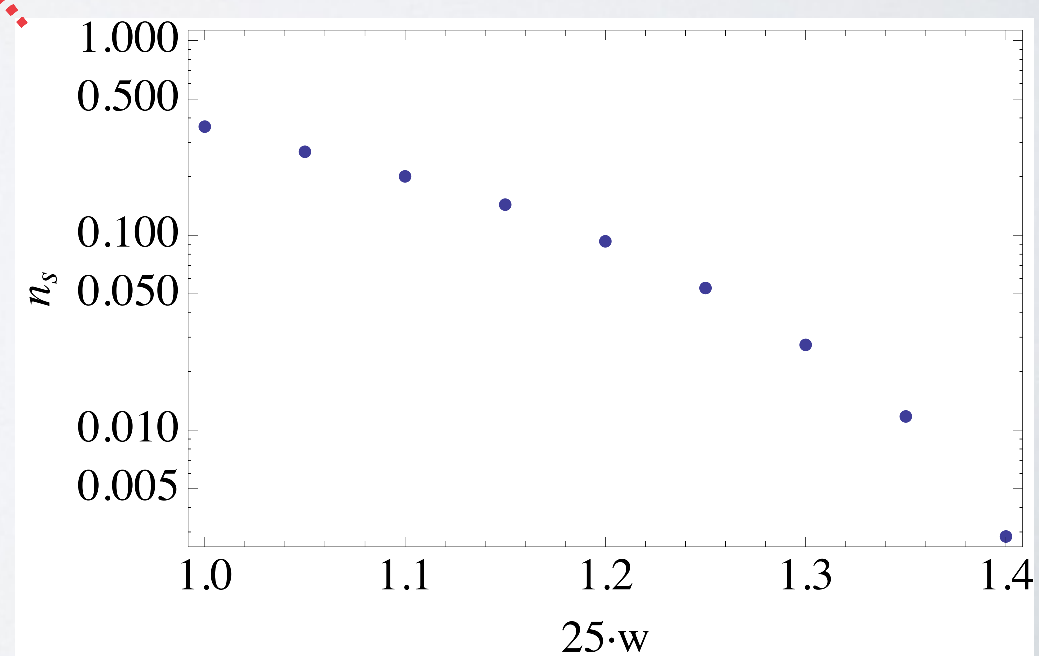


> Noisy conductivity

$$\mu = 5 \to T \sim 0.8\, T_c$$
$$L_x = 80\pi\ ,\ 9 \text{ modes}$$

> The superfluid density



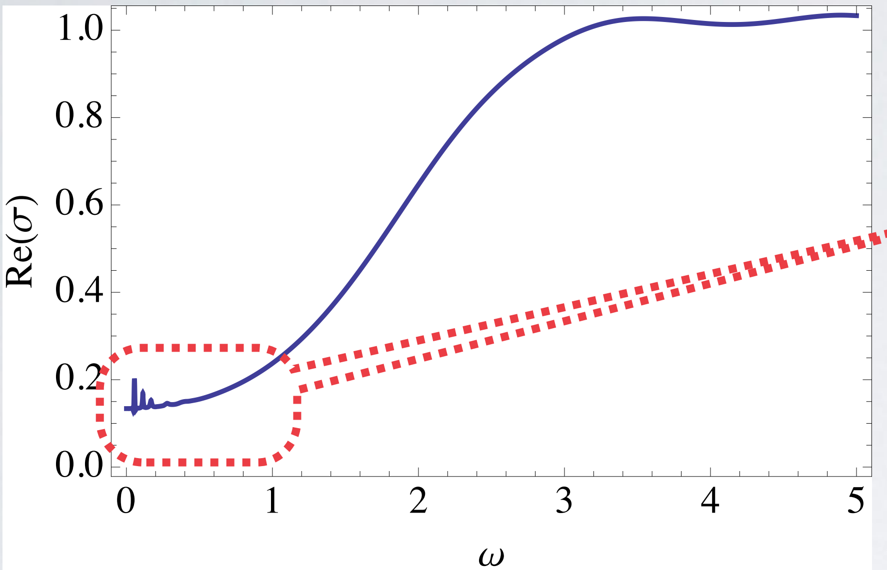


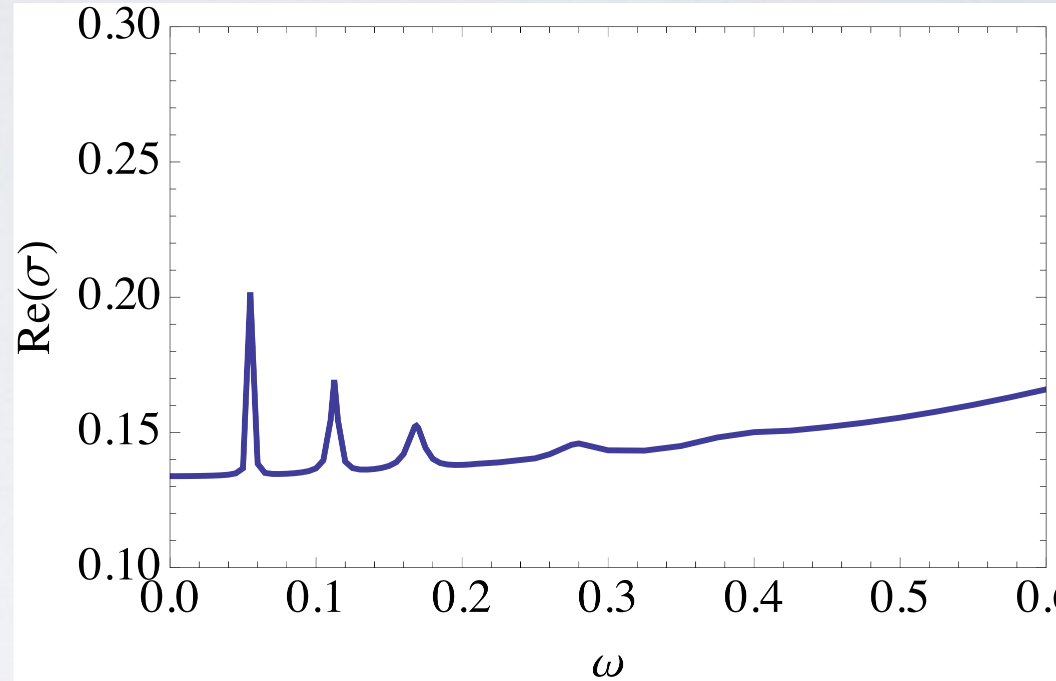
➡ Superfluid density follows O_{min} and dies away (exp-like)

> Noisy conductivity

$\mu = 5 \to T \sim 0.8\, T_c$
$L_x = 20\pi\ ,\ 9$ modes

> The AC Conductivity [averaged over x]

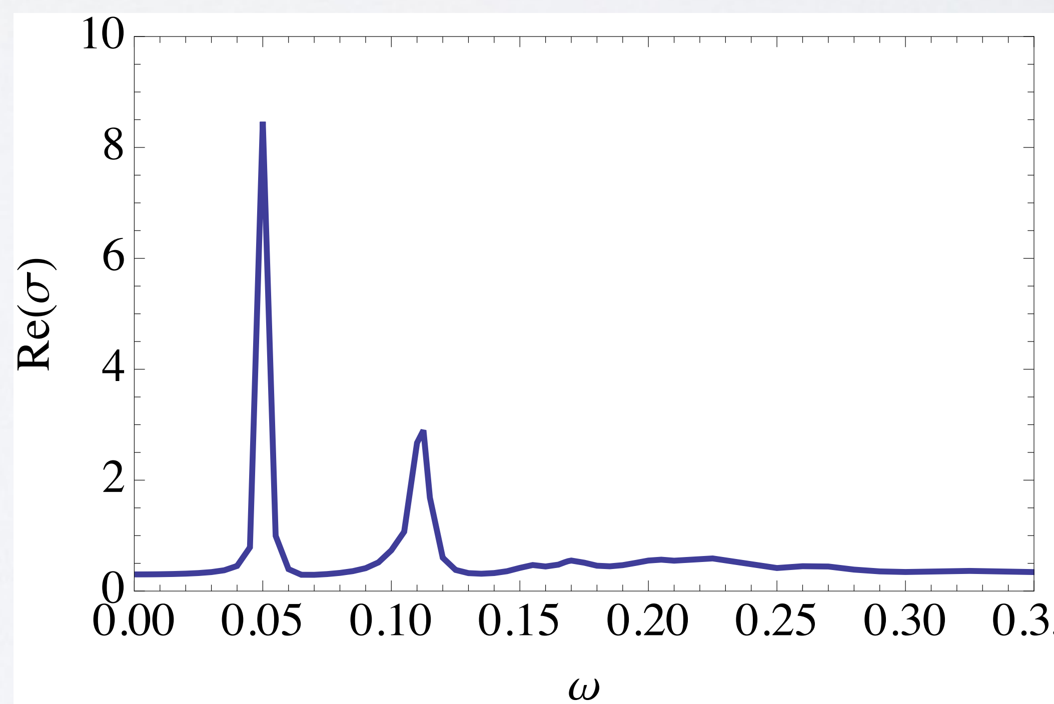




$w = 0.004$

disorder

$w = 0.04$



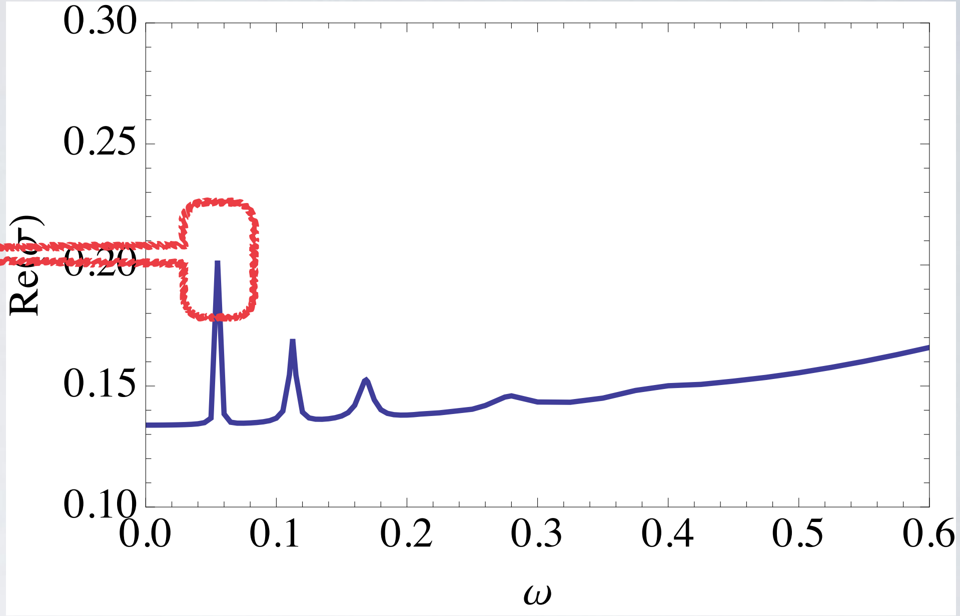
- Resonances appear
- 'Steal' spectral weight from the SC delta

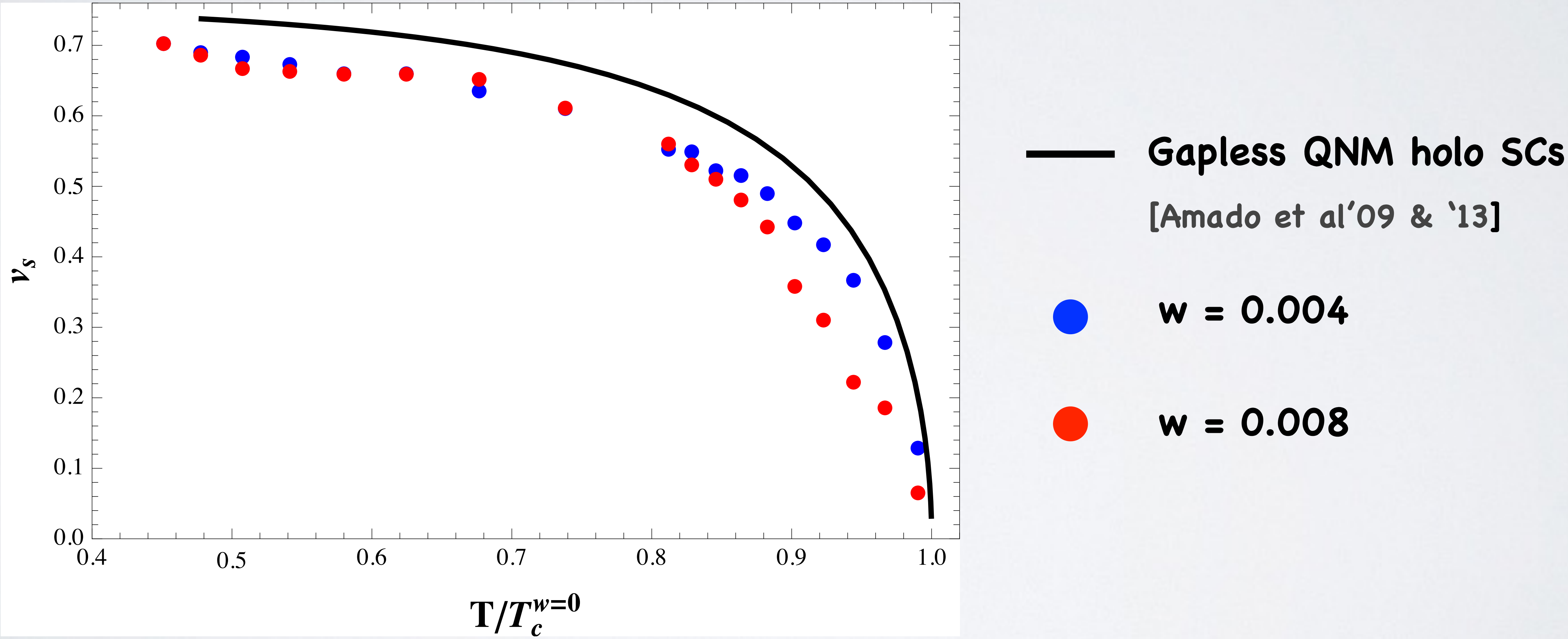
[See also Donos&Gauntlett, 1409.6875]

> Noisy AC conductivity

Let's follow the peaks...

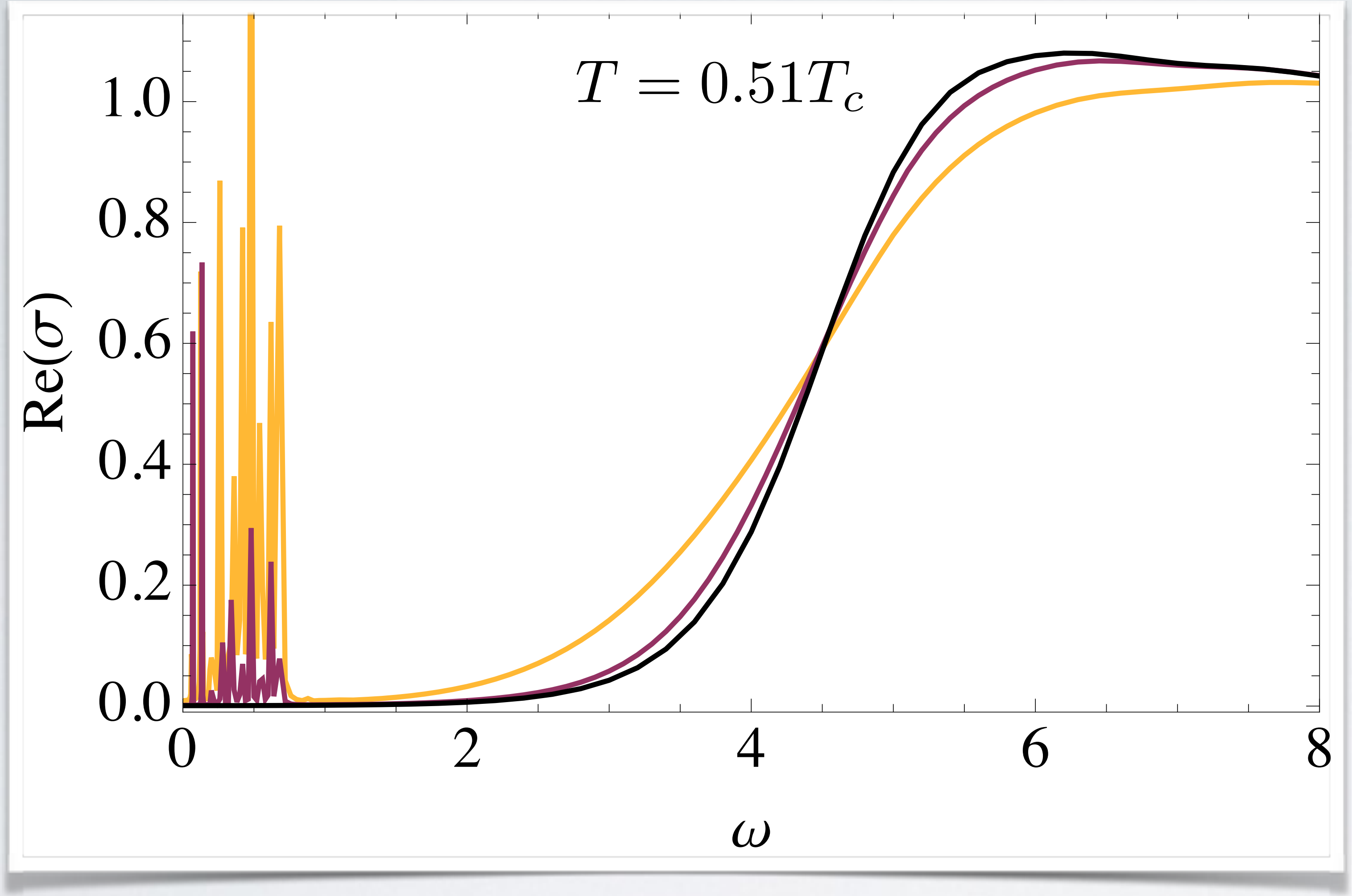
& $\omega = v_s(T, w)\, k_0$



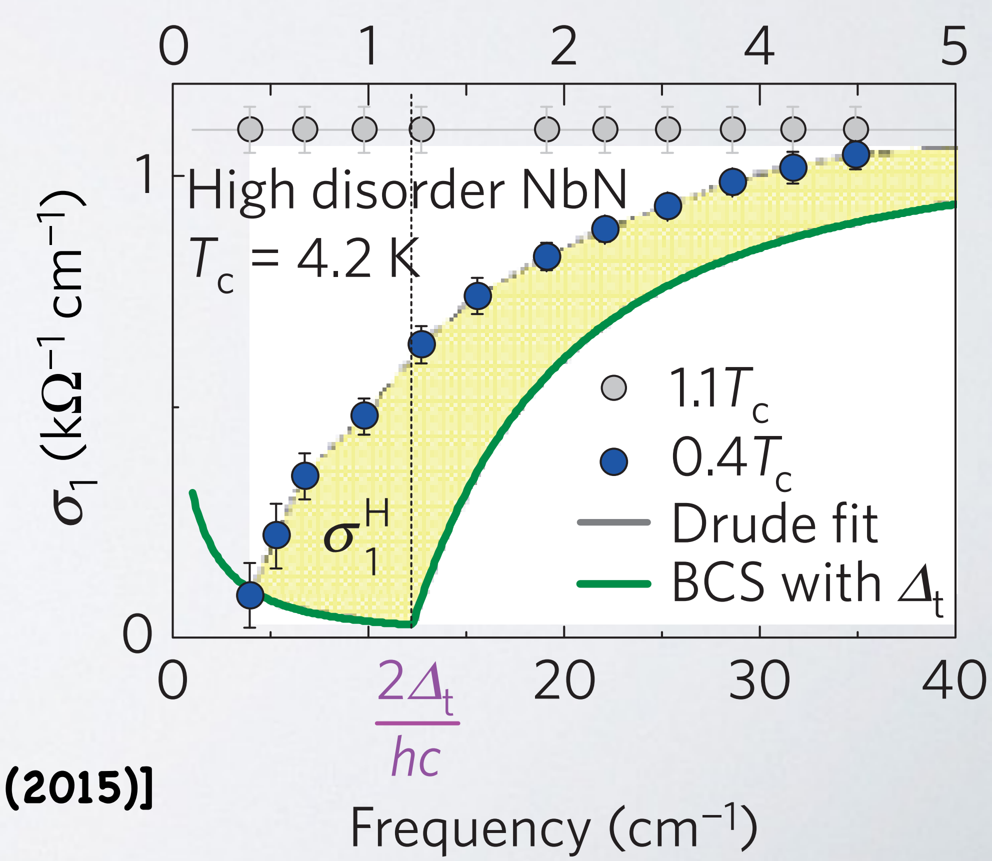


disorder 'excites' the Goldstone

> The AC Conductivity. Large disorder [Higgs mode?]

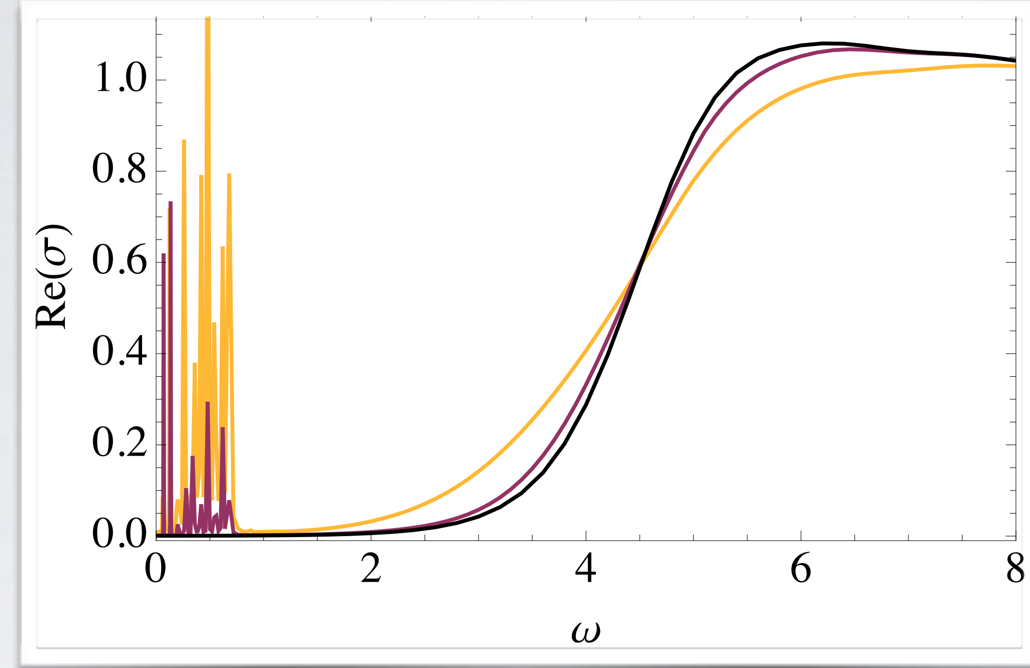


Disorder suppresses the gap

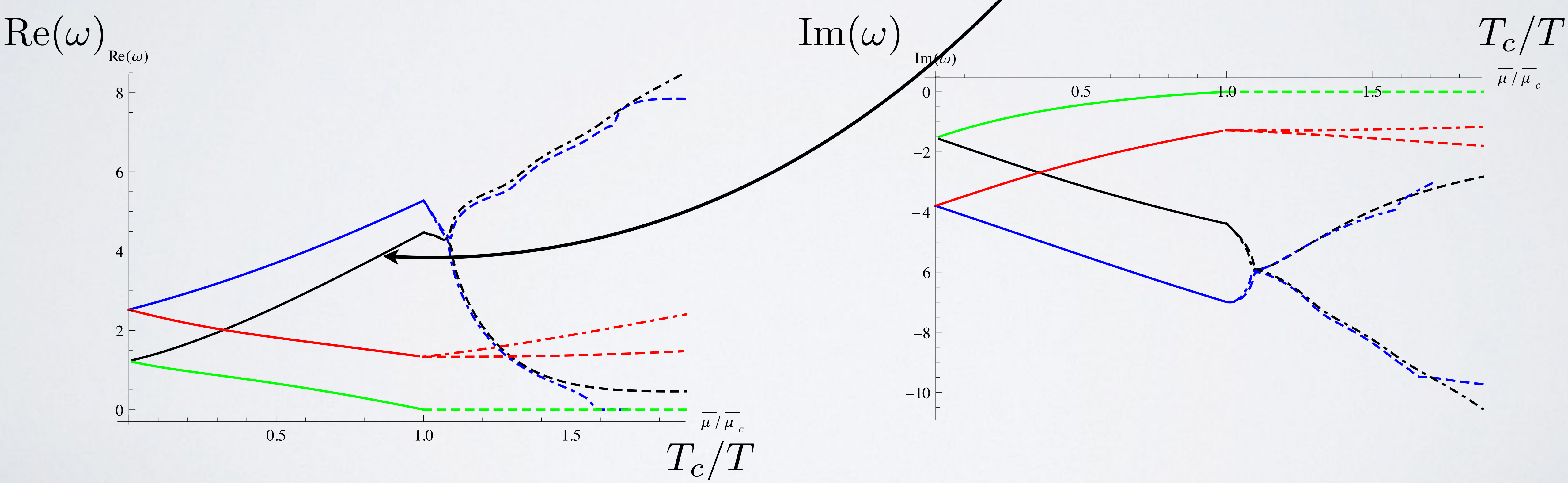


[Sherman et al, Nature Phys 11, 188–192 (2015)]

> The AC Conductivity. [Higgs mode...]



The Goldstone QNM has a massive partner...



> Outlook &To Do

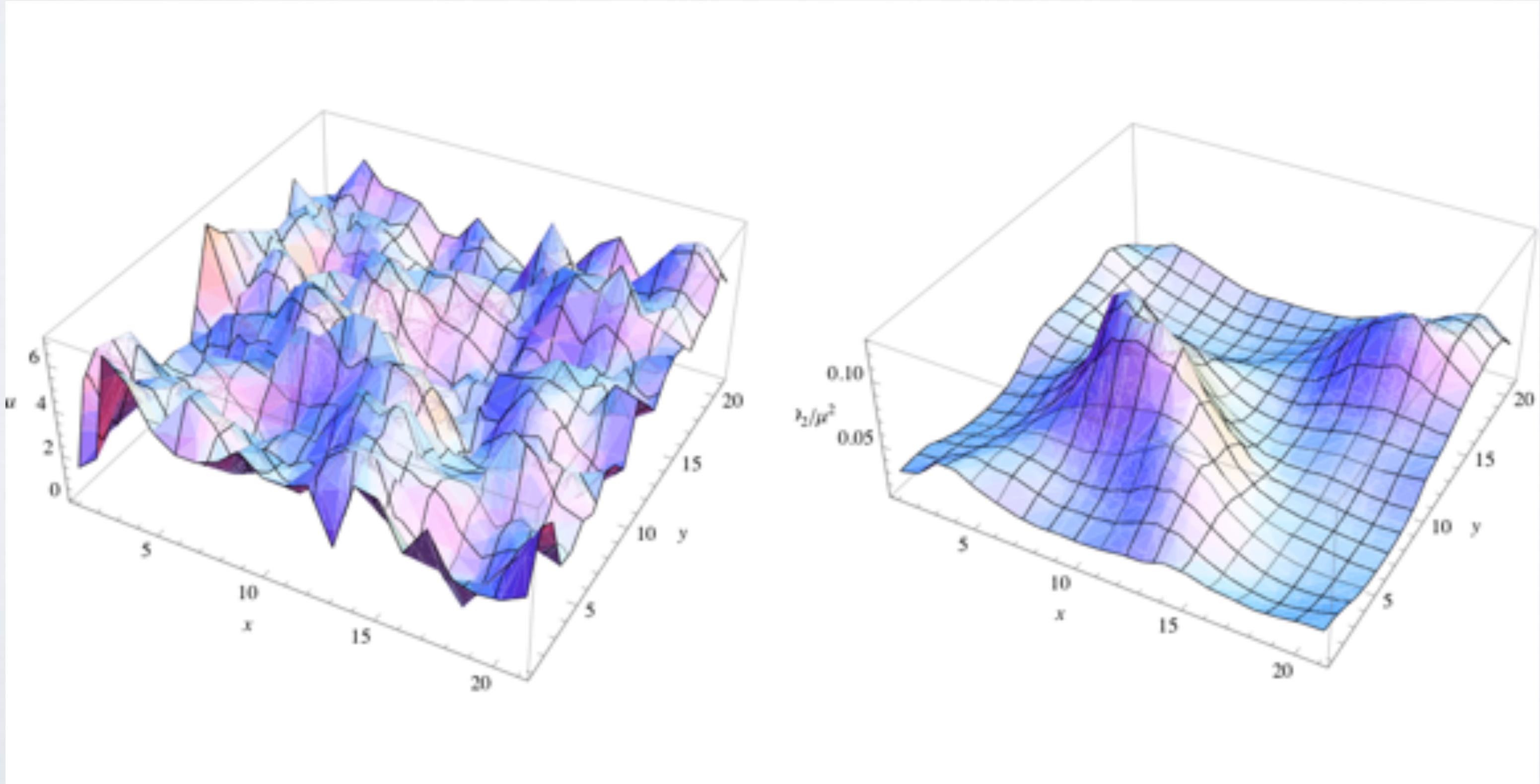
>Disordered holo SCs: both s- and p-wave [1308.1920, 1407.7526]

>1D Islands of Superfluidity

> 'Disordered' phase transition (non mean field)

>Conductivity: superfluid density ⇨ phase diagram, ~ Higgs mode

>Future Thin Films, backreaction (insulator?), . . .

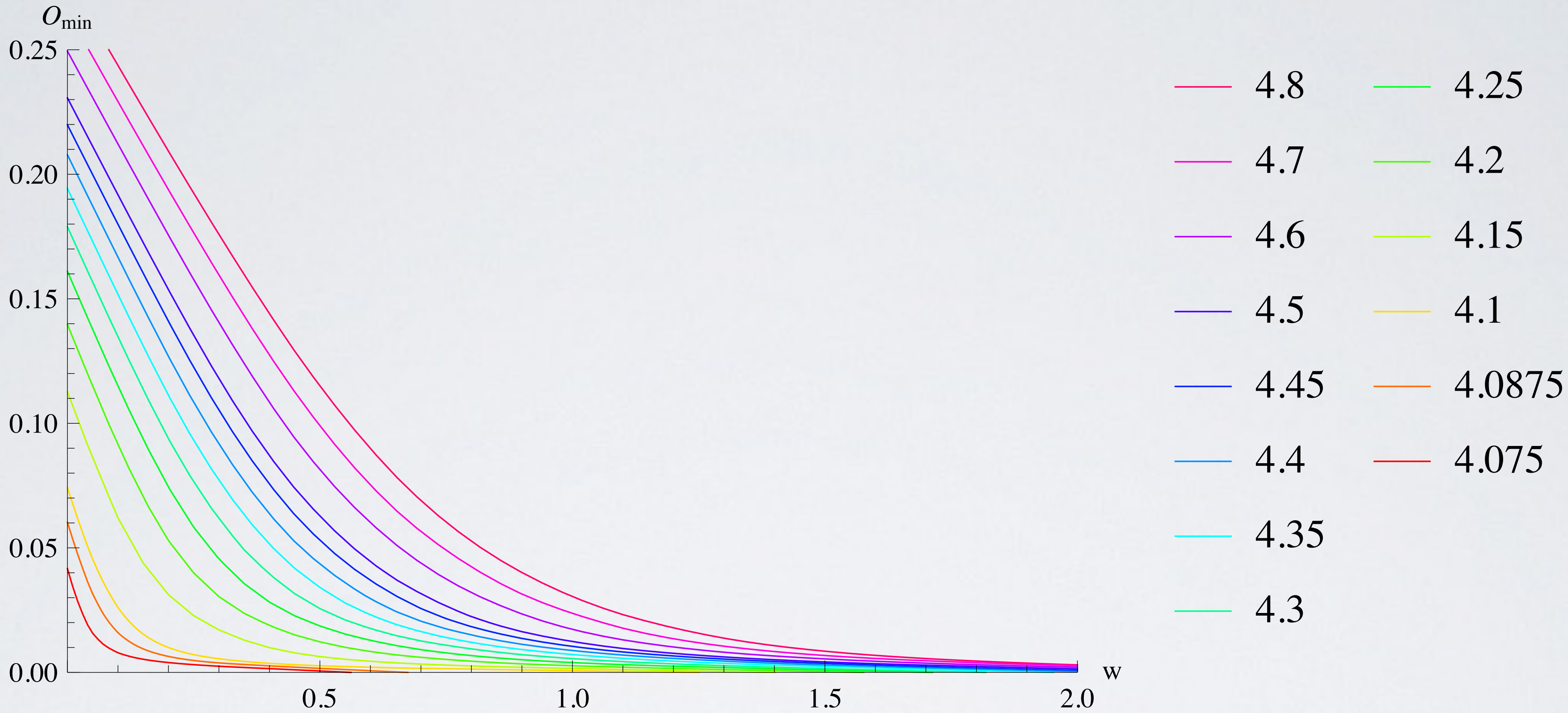


> And now, some additional slides...

> Results: Tentative Phase Diagram

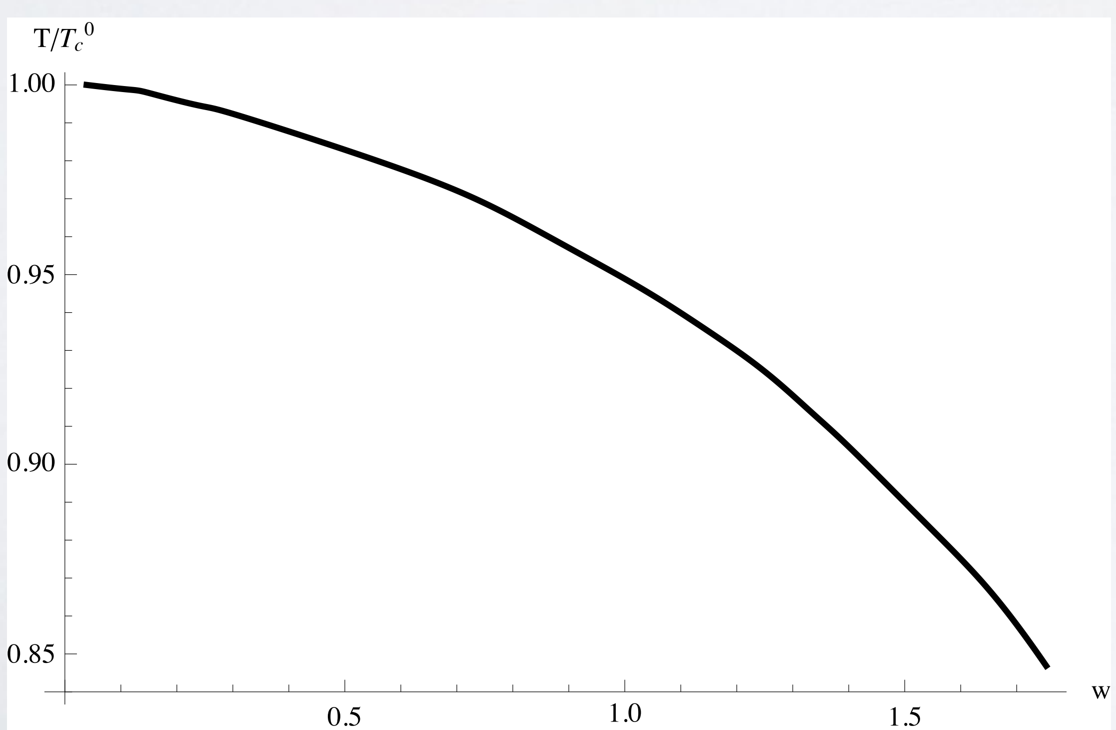
Plotting the minimum vs noise for several values of μ

$O_{\min}$

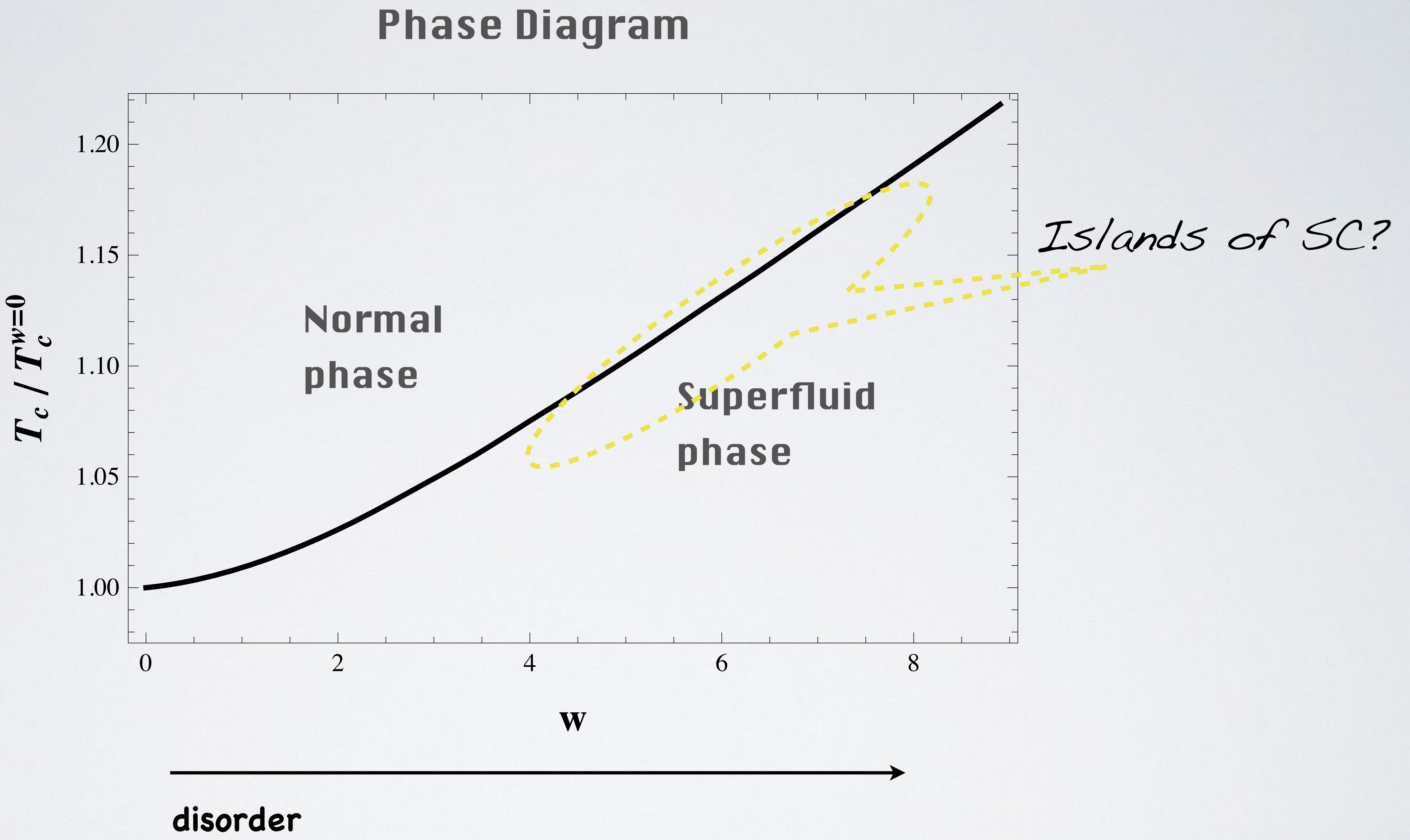


disorder

$\frac{T_c}{T_c^{w=0}}$



⋆ Enhancement and the island menace



Seen before in CM (hard-core bosons)

- 'Disorder-induced superfluidity', Dang et al, Phys. Rev. B 79, 214529

⋆ Spectrum 'renormalization'

>>> Noisy chemical potential

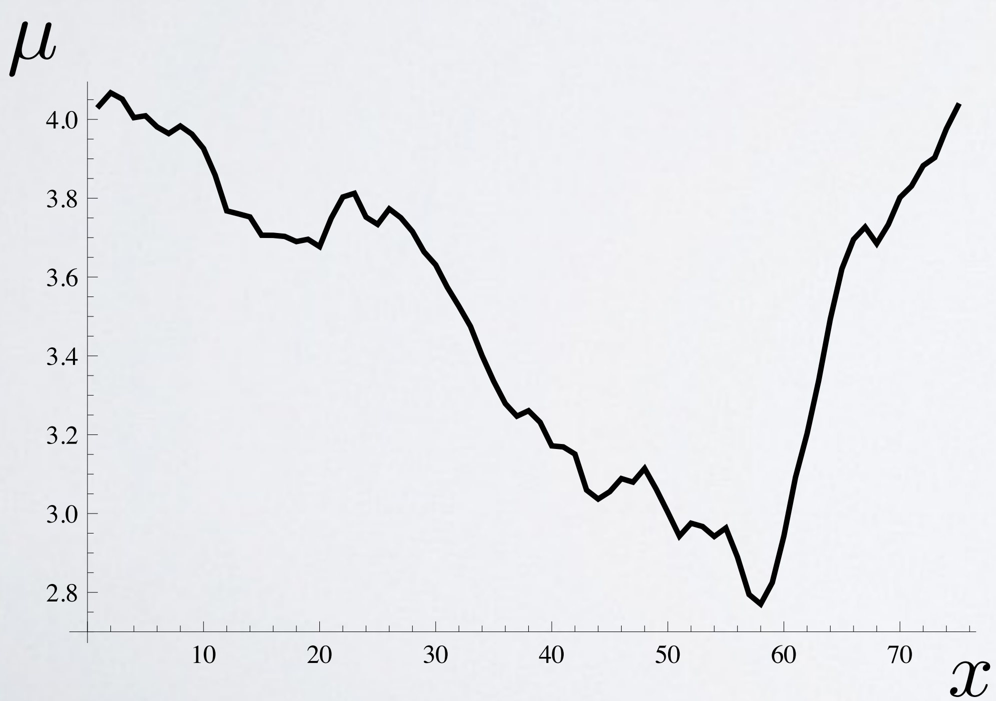
$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k} \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha} \cos(k\,x + \delta_k)$$

Random phases

Power spectrum

Strength of noise

>input spectrum $S_k = \dfrac{1}{k^{2\alpha}}$



>output spectra

Condensate

Charge density

$$S_k = \frac{1}{k^\Gamma} \;?$$

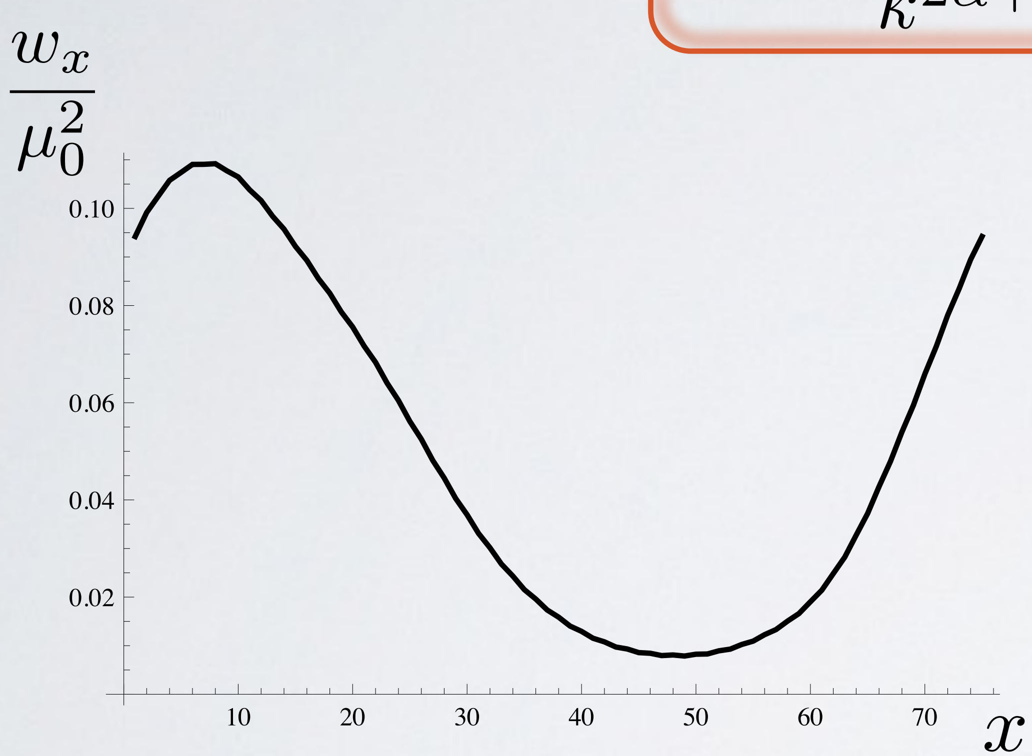
⋆ Spectrum 'renormalization'

>input spectrum $S_k = \frac{1}{k^{2\alpha}}$

> OUTPUT

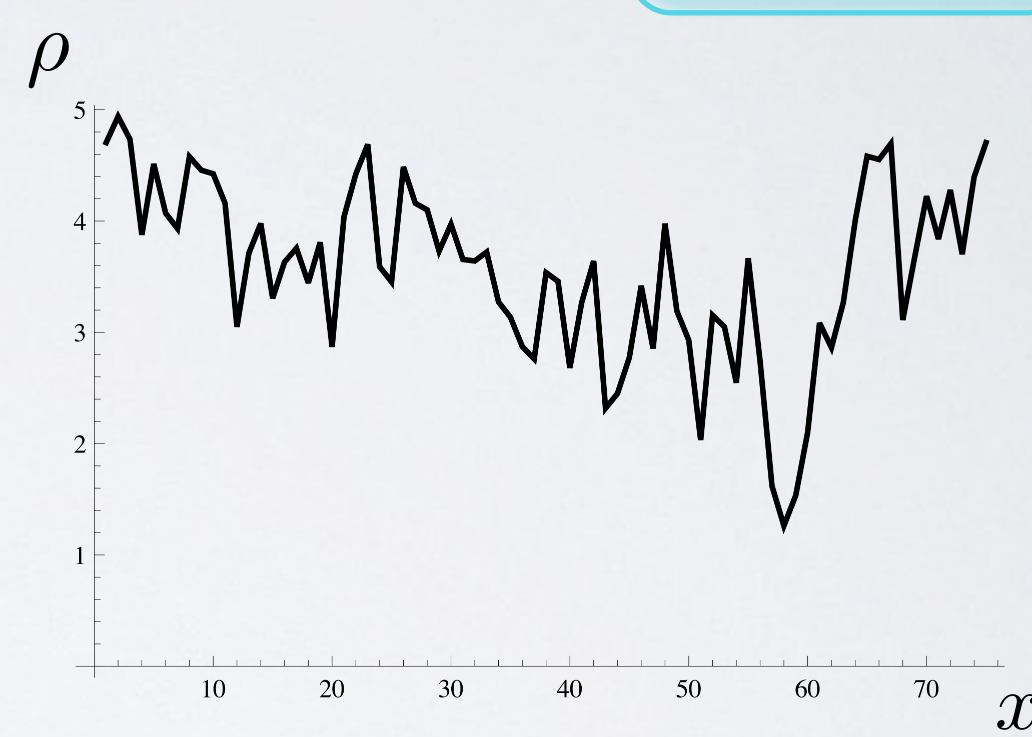
Condensate

$$S_k = \frac{1}{k^{2\alpha+4}}$$



Charge density

$$S_k = \frac{1}{k^{2\alpha-2}}$$

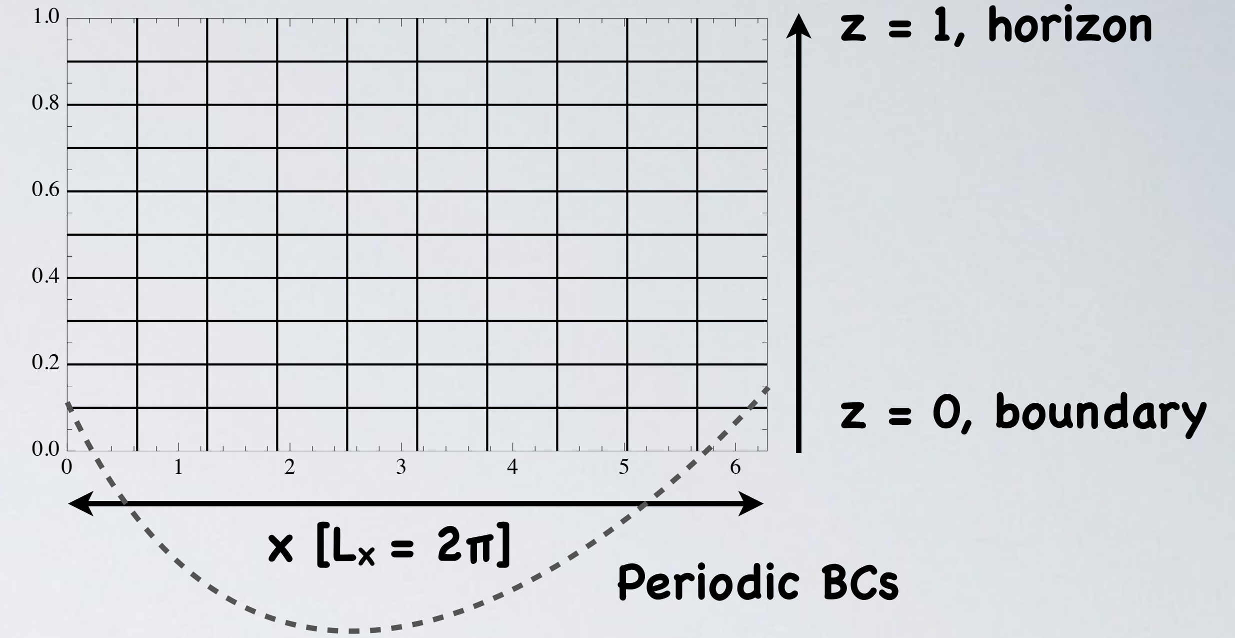


Hints of universality

- S-wave [1308.1920]
- [Hartnoll&Santos 1402.0872]
- Fundamental matter (D3-D5) [w/ M. Araújo, J. Lizana, I.S. Landea]
- FT: noisy U(1) @ finite T [D. Musso, I.S. Landea]

> Noisy chemical potential

- GRID

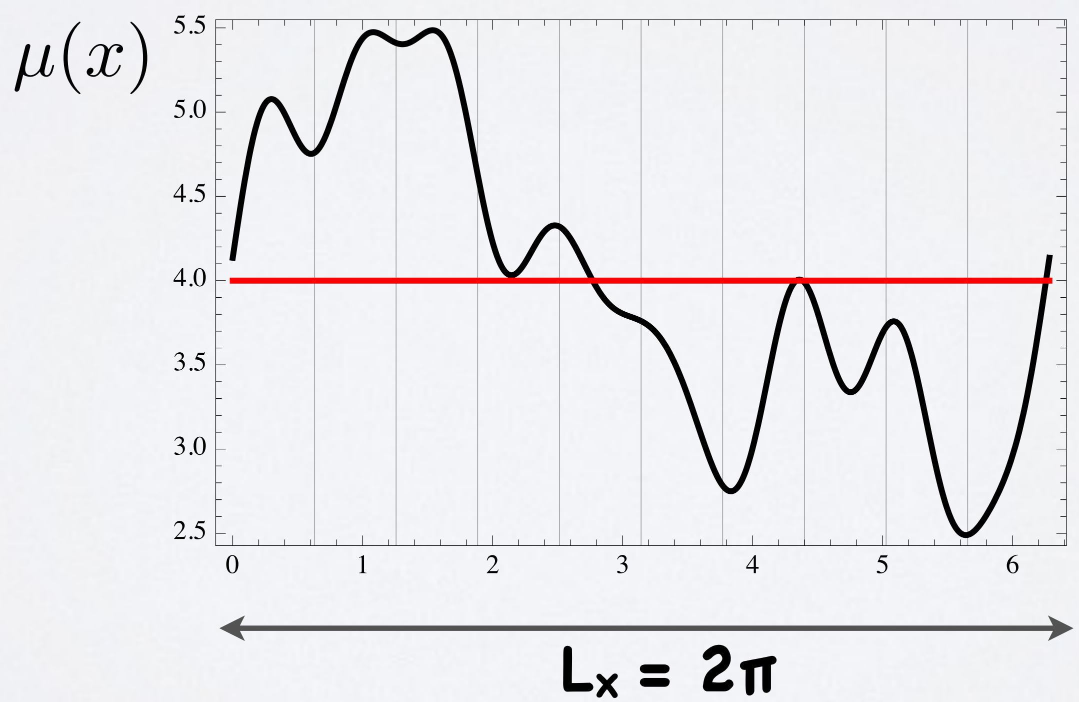


$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k} \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha} \cos(k\,x + \delta_k)$$

Random phases

Power spectrum

Strength of noise



> Noisy chemical potential

- **Noise through random phases**

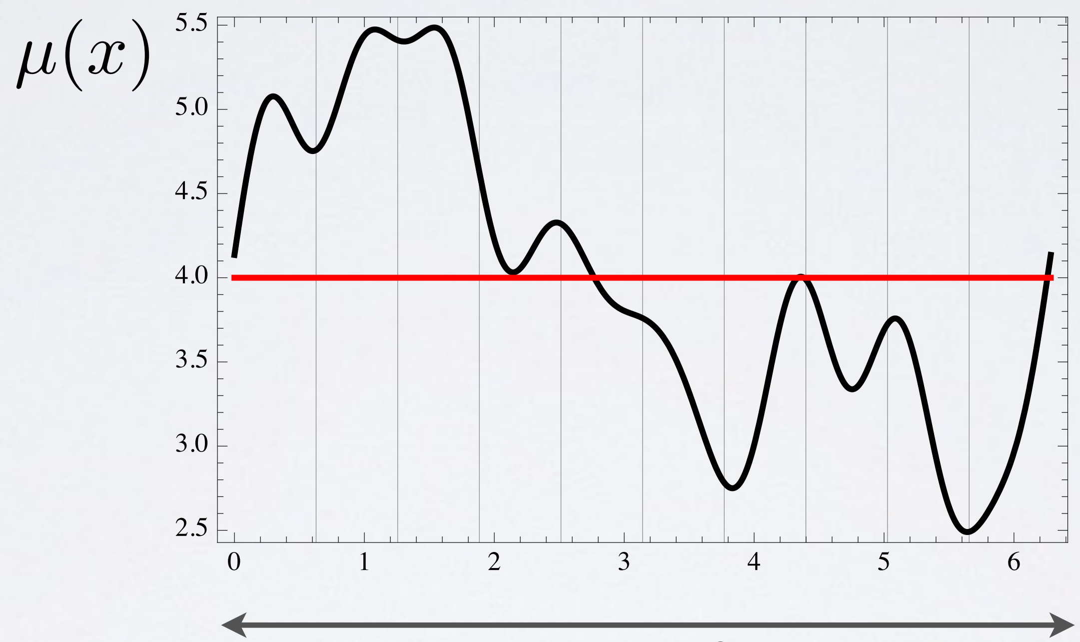
$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

Random phases

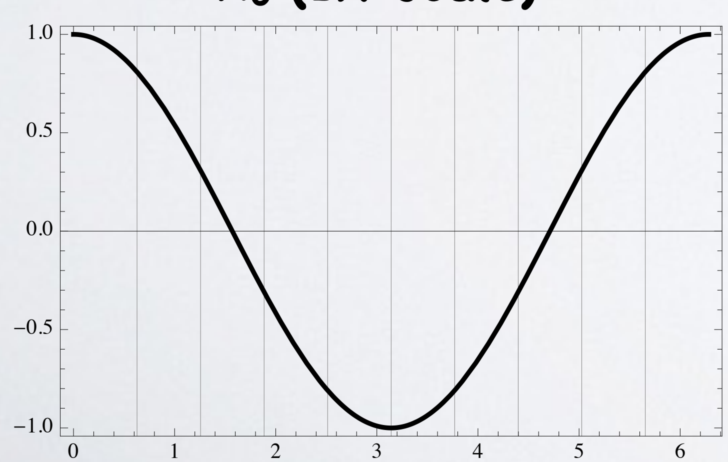
Power spectrum

Strength of noise

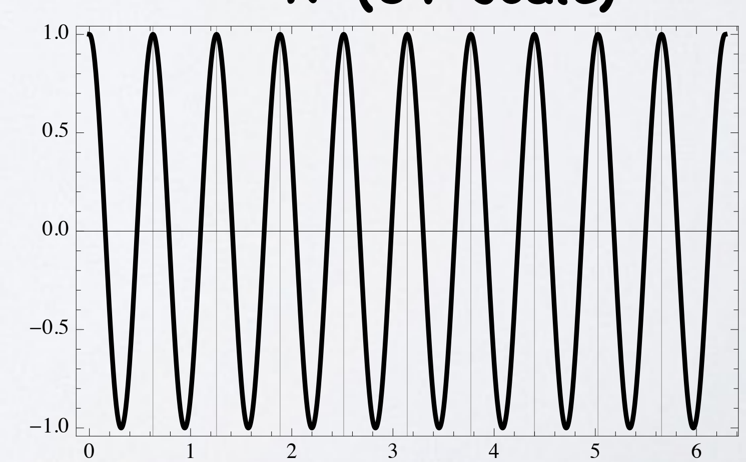
- **System on a grid**



$L_x = 2\pi$



K_0 (IR scale)



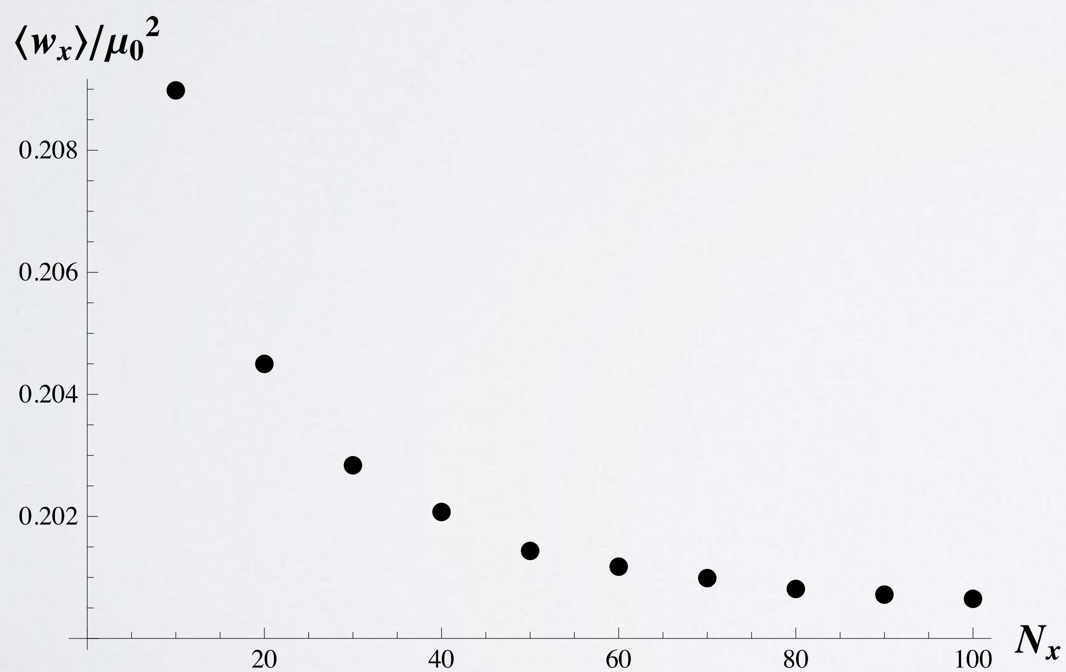
K_* (UV scale)

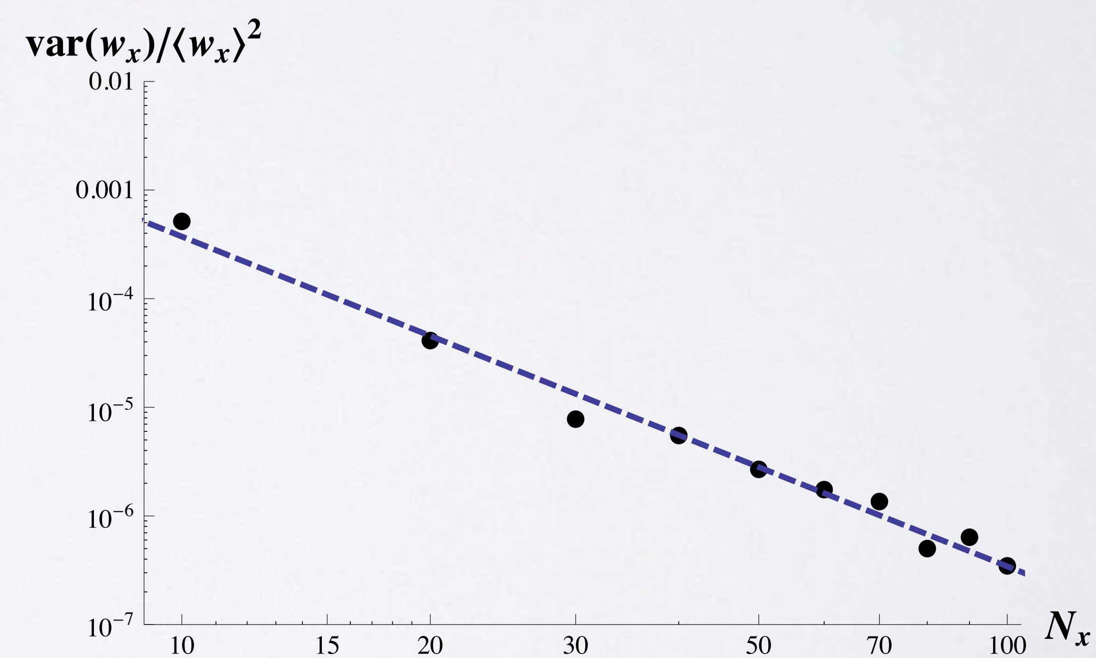
> Thermodynamic limit

- **Thermo limit: Noise correlation length << System length**
  - > Flat spectrum noise: correlation length $\propto$ 1 / (grid size)

- **Condensate and Charge density are self-averaging in the thermo limit:**
  - > X_n is self-averaging when $$\frac{< X_n^2 > - < X_n >^2}{< X_n >^2} \to 0$$

Condensate





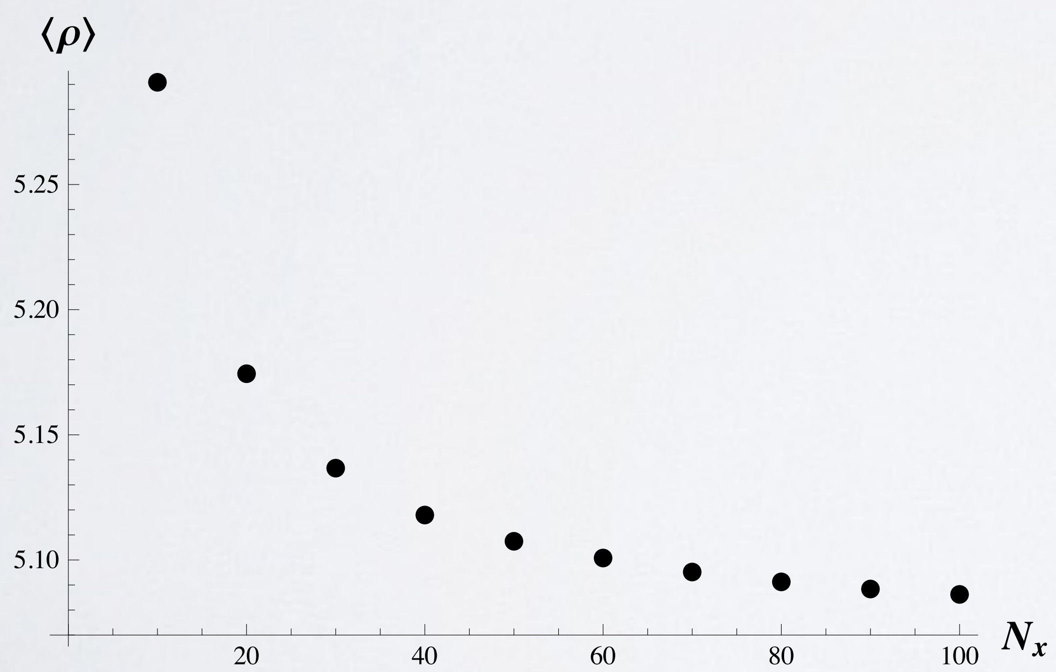
$$\log(\mathrm{var}(w_x)/\langle w_x\rangle^2) = -0.90 - 3.03\,\log(N_x)$$

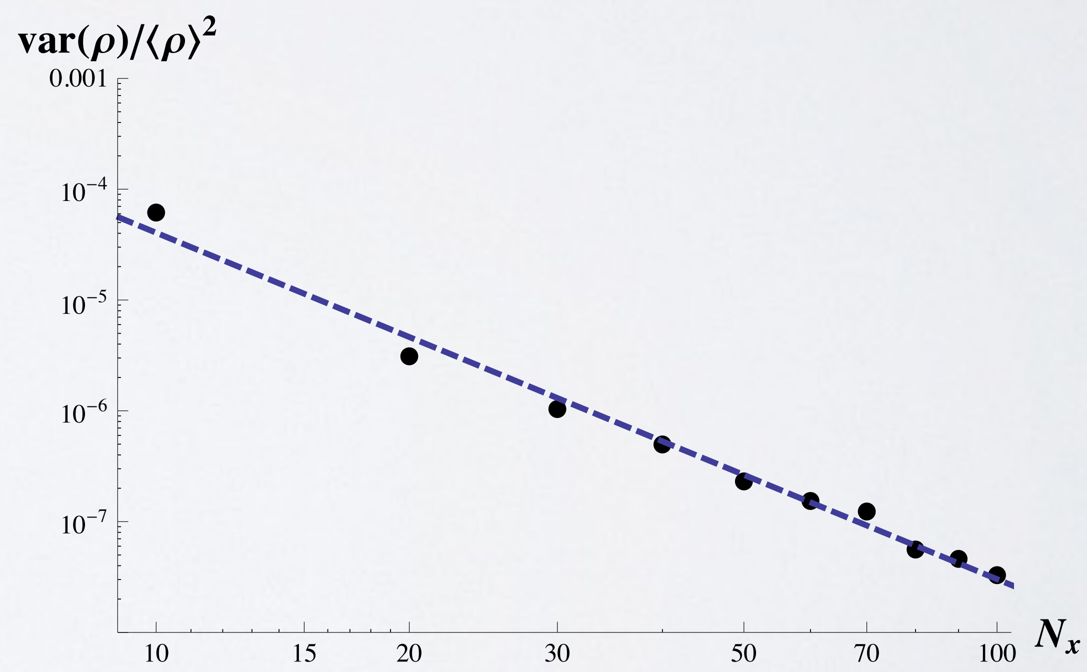
> Thermodynamic limit

- **Thermo limit: Noise correlation length << System length**
  - > Flat spectrum noise: correlation length $\propto$ 1 / (grid size)

- **Condensate and Charge density are self-averaging in the thermo limit:**
  - > X_n is self-averaging when $$\frac{< X_n^2 > - < X_n >^2}{< X_n >^2} \to 0$$

Charge density





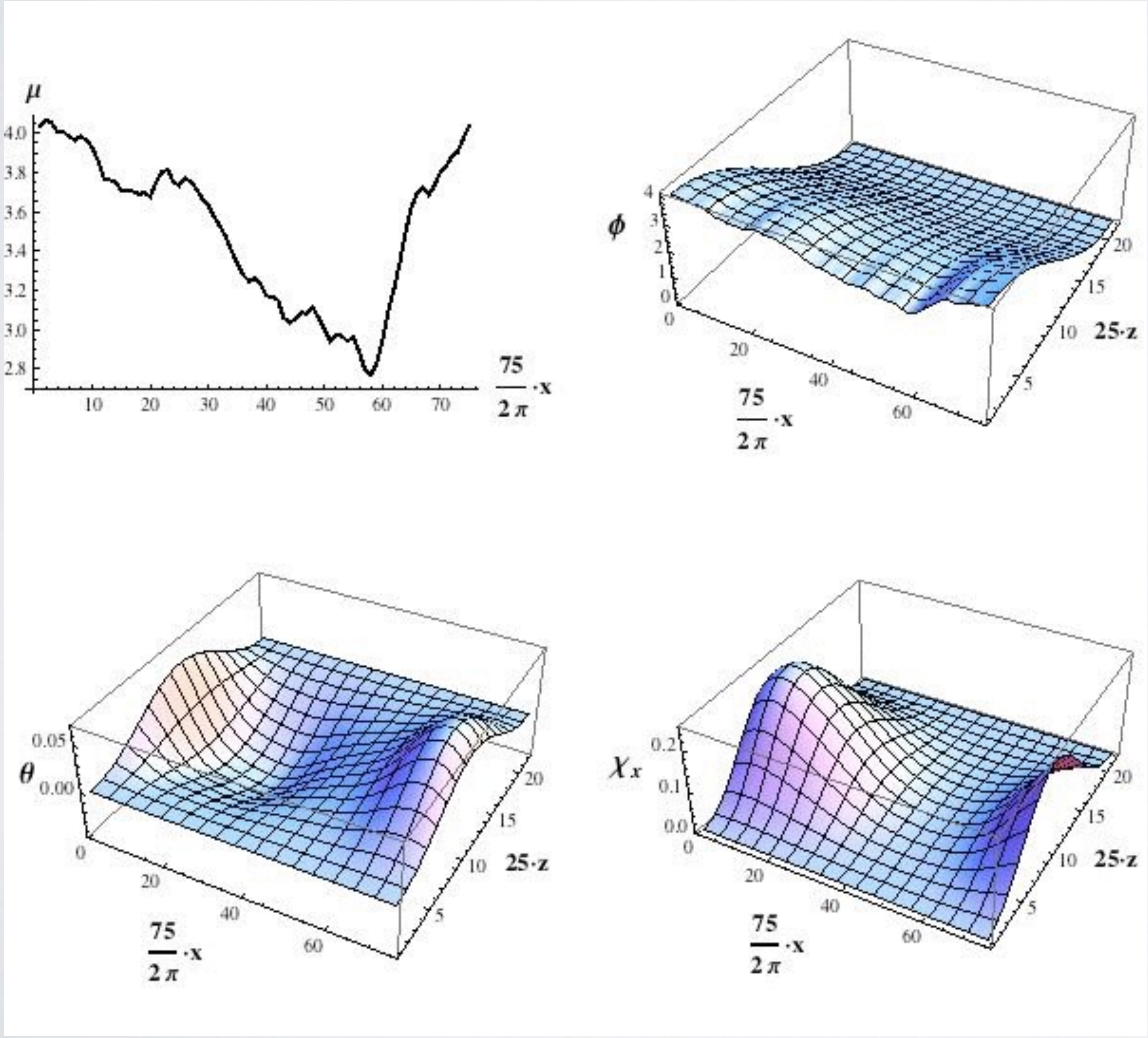
$$\log(\mathrm{var}(\rho)/\langle\rho\rangle^2) = -2.92 - 3.13\,\log(N_x)$$

> **Simulation #1**

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\,\cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\,\cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

- $\mu_0 = 3.50\,,\ \alpha = 1.50\,,\ w = 3.50 \quad [\mu_0 < \mu_c = 3.66]$



$L_x = 2\pi \rightarrow K_0 = 1$

$N_z \times N_x = 25 \times 75$

> Simulation #1

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

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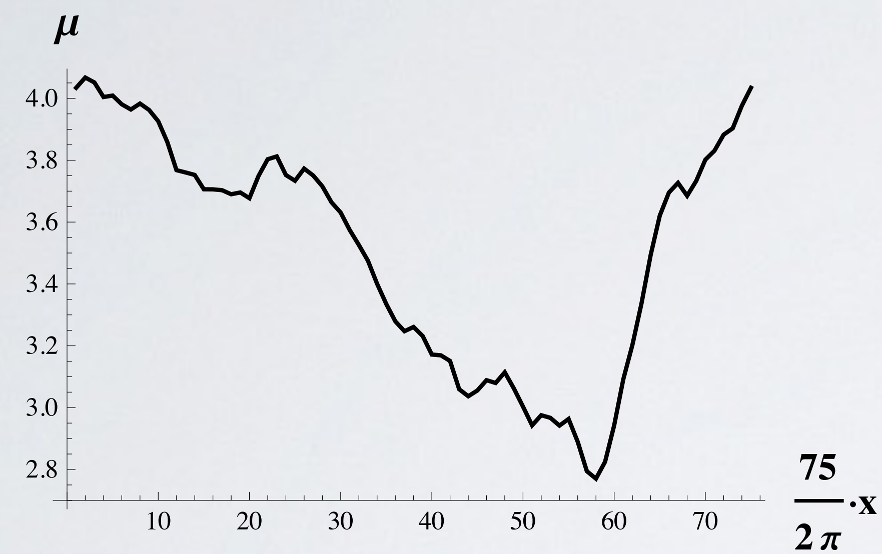
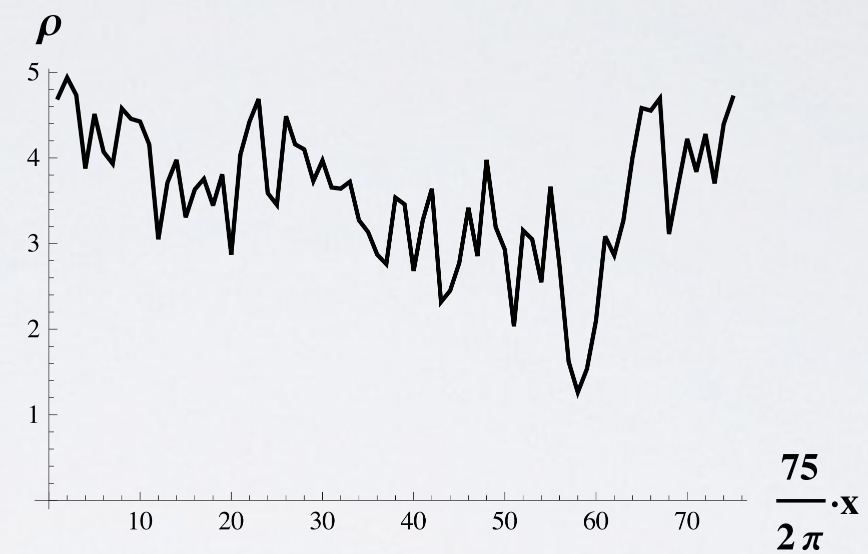
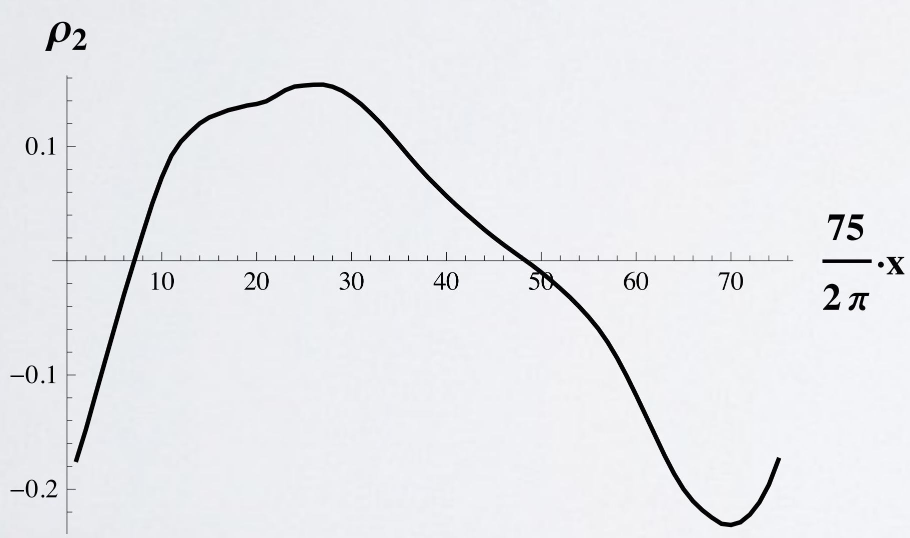
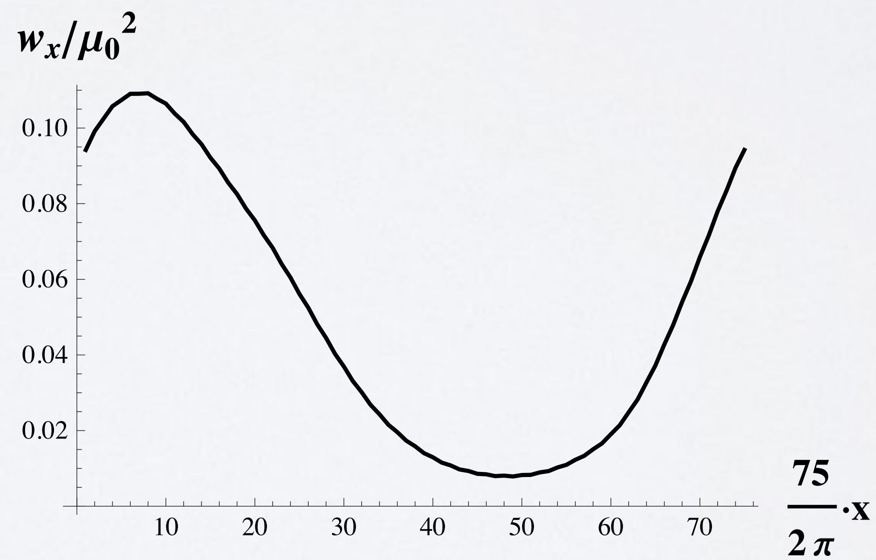




L$_x$ = 2π → K$_0$ = 1

N$_z$ x N$_x$ = 25 x 75





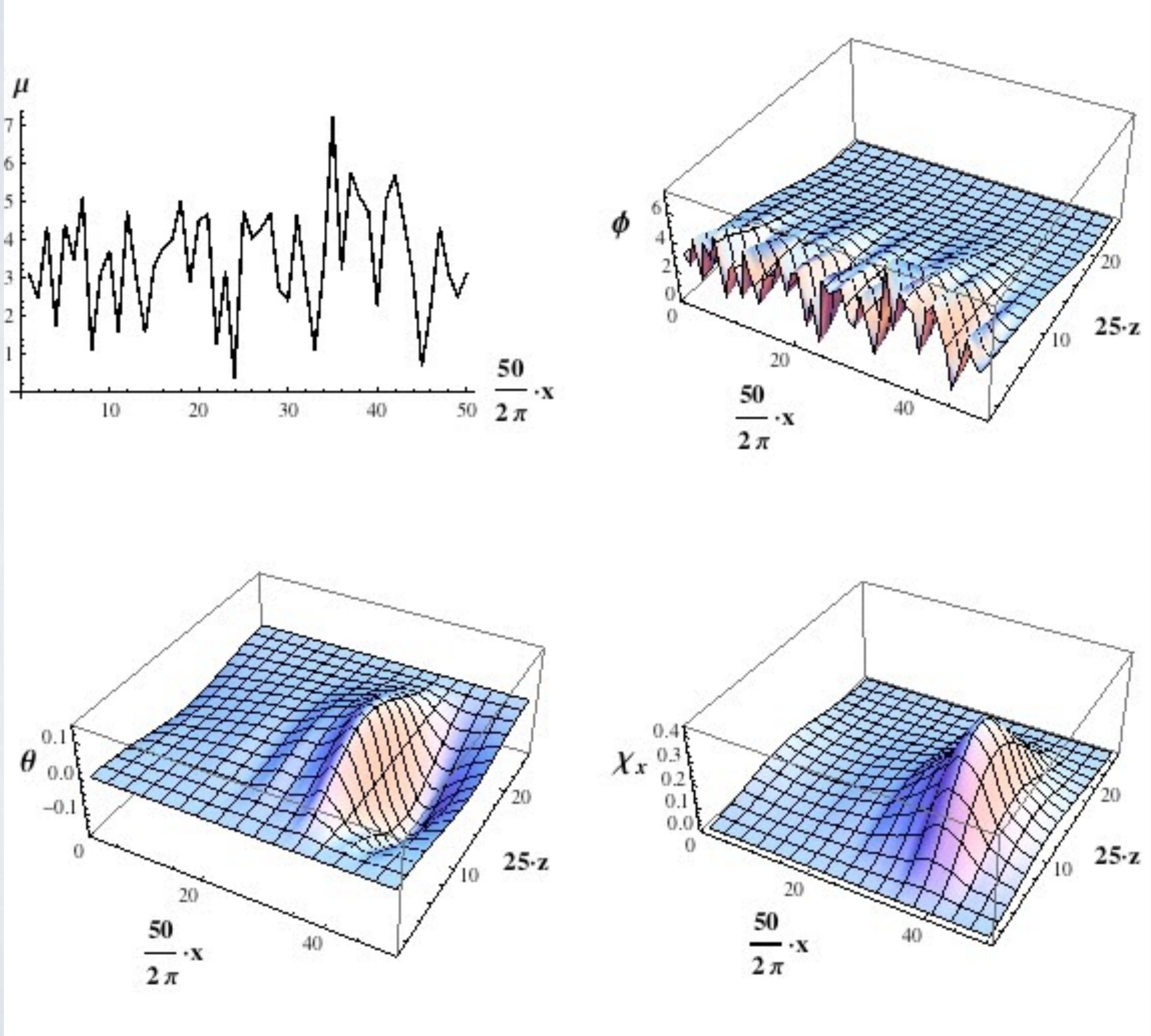
> Simulation #2

Flat Noise

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k} \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha} \cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

- $\mu_0 = 3.50\,,\ \alpha = 0\,,\ w = 3.50\ \ [\mu_0 < \mu_c = 3.66]$



$L_x = 2\pi \rightarrow K_0 = 1$

$N_z \times N_x = 25 \times 75$

> Simulation #2

Flat Noise

$$\mu(x) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \sqrt{S_k}\, \cos(k\,x + \delta_k) = \mu_0 + \epsilon \sum_{k=k_0}^{k_*} \frac{1}{k^\alpha}\, \cos(k\,x + \delta_k)$$

$$w = 25\epsilon/\mu_0$$

- $\mu_0 = 3.50\,,\ \alpha = 0\,,\ w = 3.50\ \ [\mu_0 < \mu_c = 3.66]$

$L_x = 2\pi \rightarrow K_0 = 1$

$N_z \times N_x = 25 \times 75$