

Solid applications of Massive Gravity

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**Based on [arXiv:1411.1003 \(PRL\)](https://arxiv.org/abs/1411.1003) with Oriol Pujolàs
+ work in progress with L.Alberte, A.Khmelnitsky,
D.Musso, A.Amoretti, A.Braggio, N.Magnoli**

**IFAE
IFAE**

Sestri Levante, September 2015

UAB

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de Barcelona

Motivations

**CONDENSED
MATTER**



OF PUZZLES

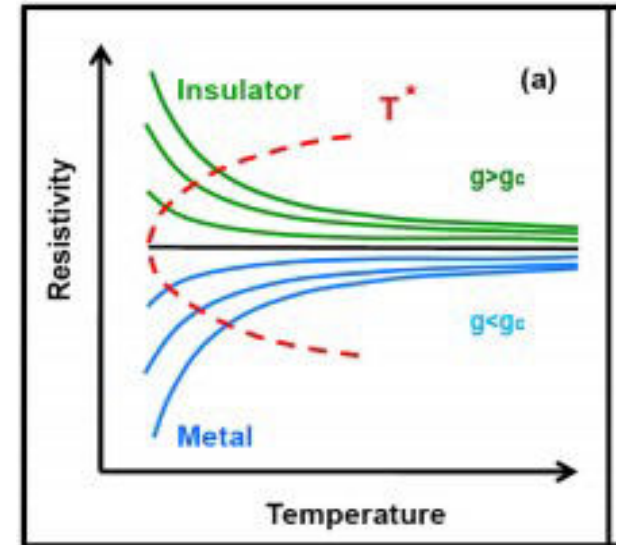
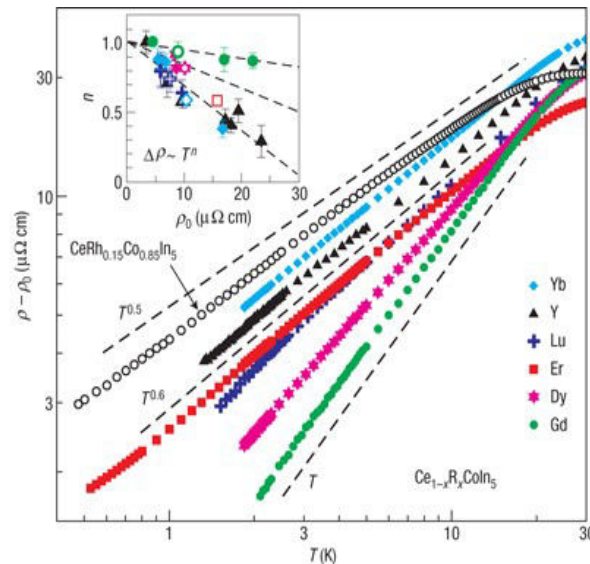
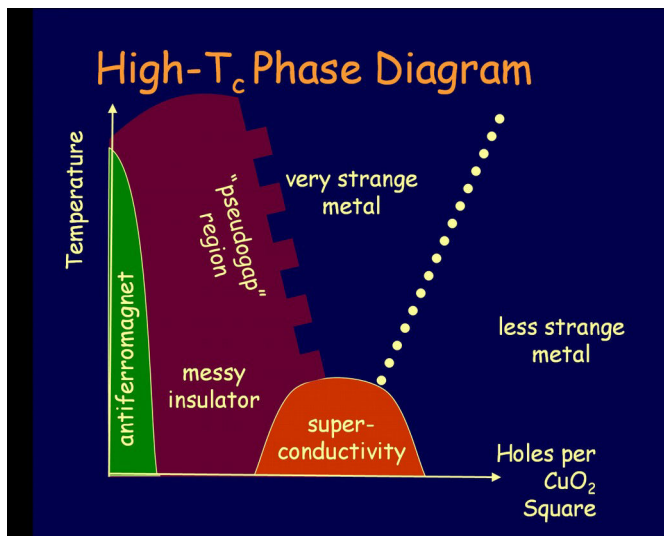


Finite density
+
Strong coupling

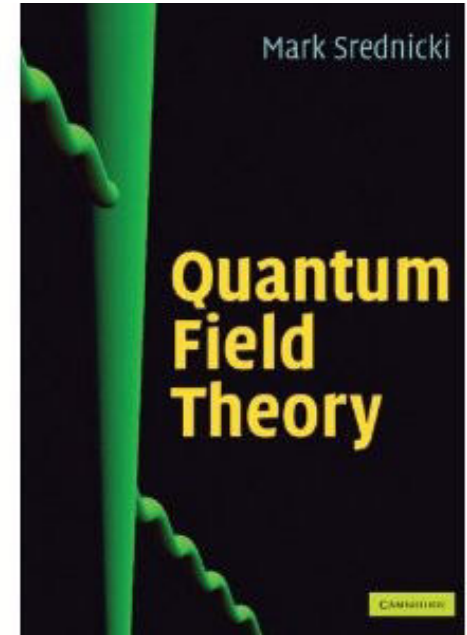
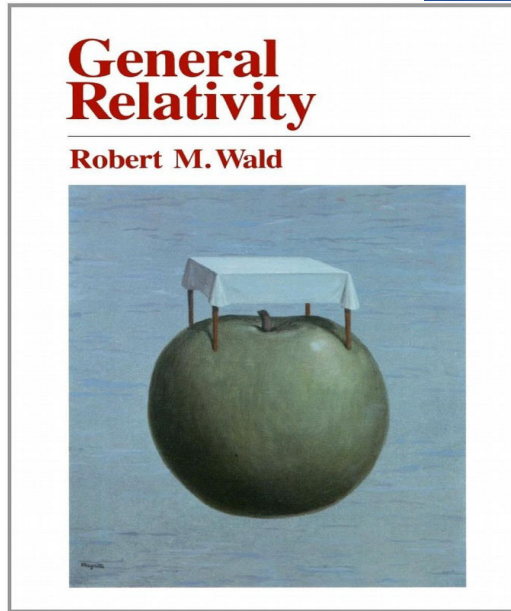


**HARD
PROBLEM**

HTSCs , Strange Metals, Mott-Anderson Insulators & MIT

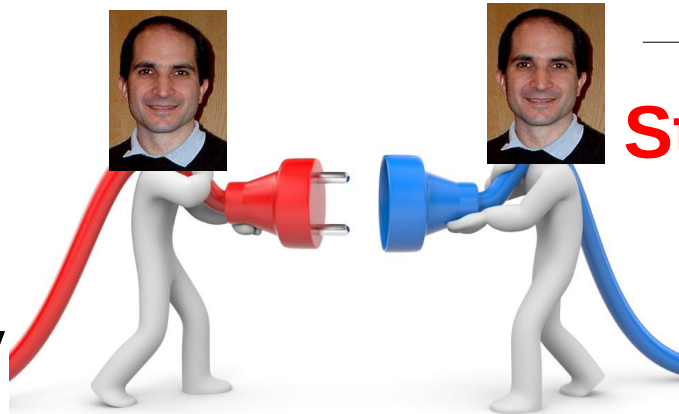


Tools



AdS-CFT

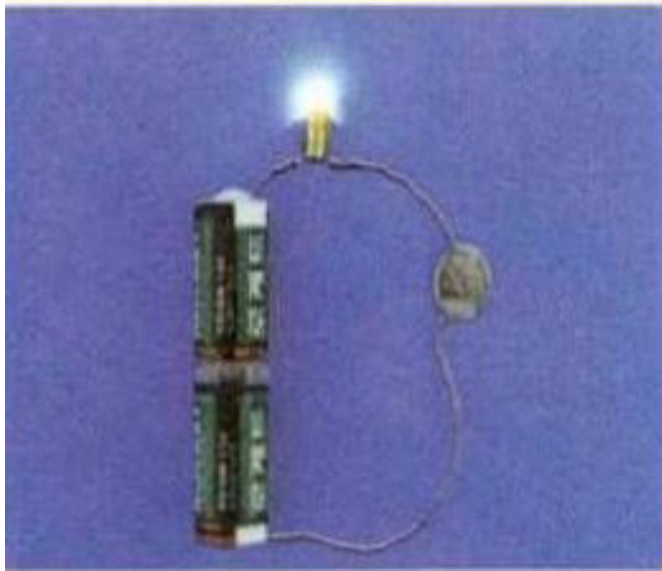
**Weakly coupled
Classical
General Relativity**



**Strongly coupled
Quantum
Field theory**

Metal or Insulator - and why?

ELECTRIC RESPONSE



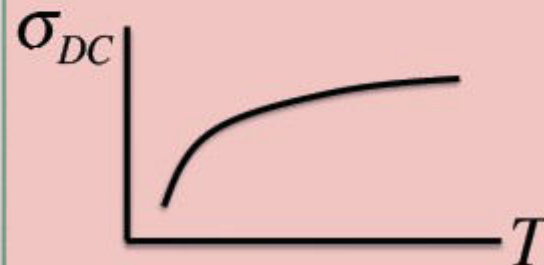
Metal: σ large

$$\frac{d\sigma}{dT} < 0$$



Insulator: σ small

$$\frac{d\sigma}{dT} > 0$$



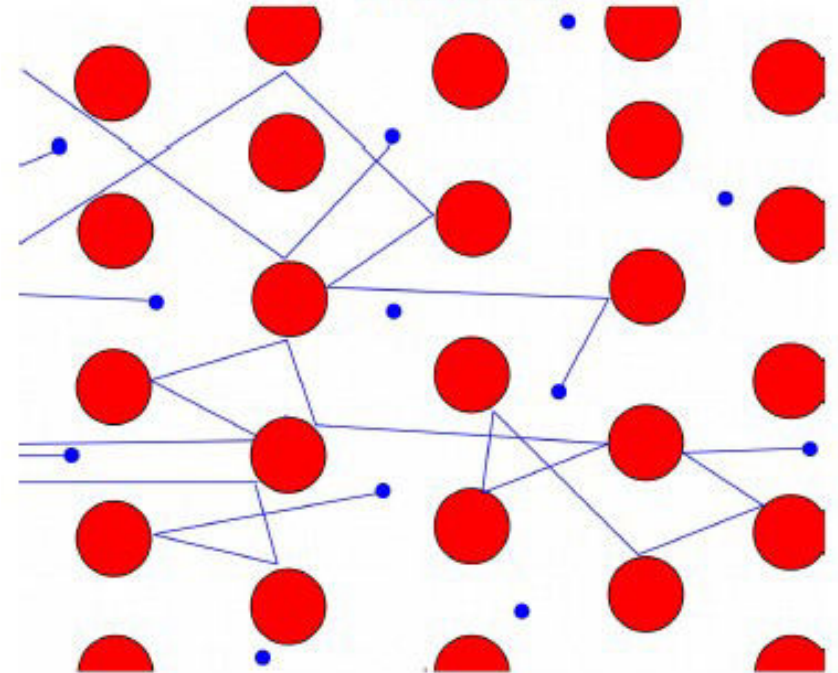
Electric Response à la Drude

Drude Model (1900)

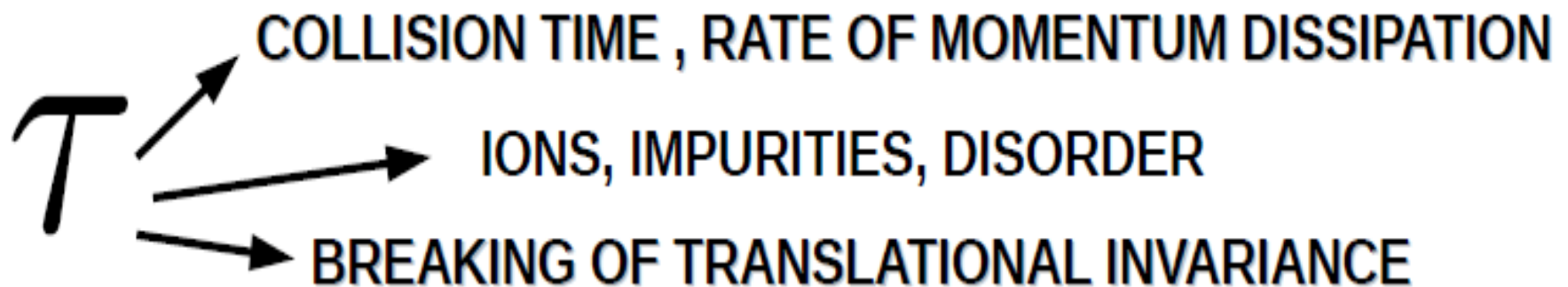
$$\frac{d}{dt} p(t) = qE - \frac{p(t)}{\tau}$$

$$\sigma_{DC} = \frac{n q^2 \tau}{m}$$

Weakly coupled logic:
"Pinball"



A lot of simplifications but a very good phenomenological model



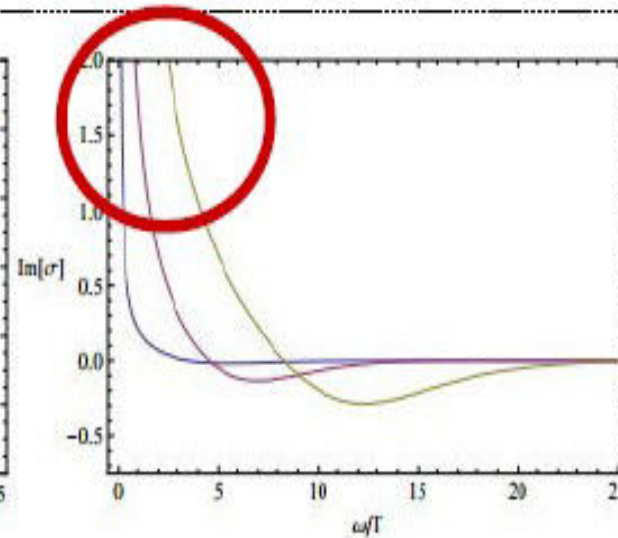
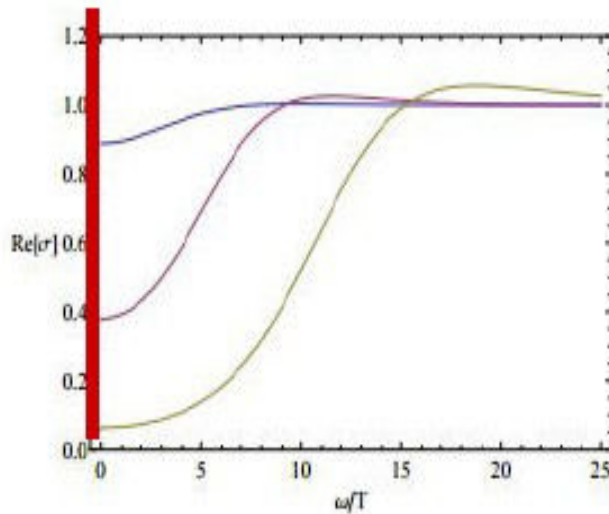
Unrealistic AdS-CMT

Reissner Nordstrom Black Hole

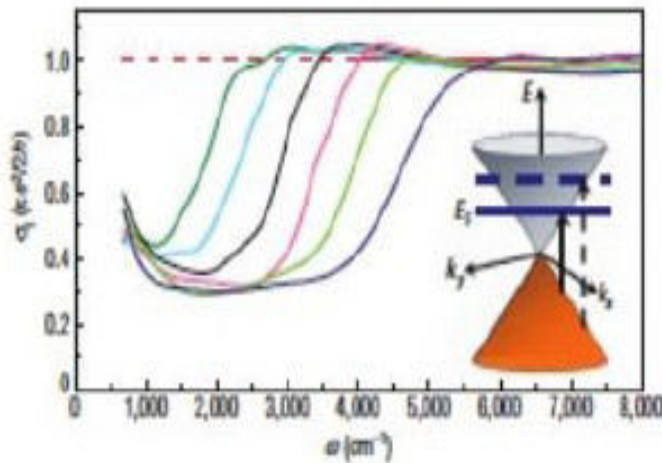
$$S = \int d^{d+1}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{d(d-1)}{L^2} \right) - \frac{1}{4g^2} F^2 \right]$$

- Temperature
- Charge density
- **Translational invariance**

Holography:



Graphene:



There is an infinite DC
Conductivity!

\mathcal{T} Is infinite !

Targets & Open questions

REALISTIC AdS-CMT



MOMENTUM DISSIPATION

Mechanisms :

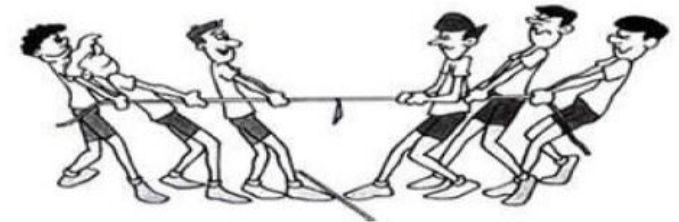
- Umklapp and electron- phonon interactions
- Impurities & Disorder
- Electron- Electron interactions



Phenomenology & Puzzles :

- **1st : Finite DC conductivity**
- Strongly Correlated Insulators
- Metal- Insulator transitions

Mott Insulators



Anderson Insulators



(More) Realistic AdS-CMT

MASSIVE GRAVITY

$$S = \int \sqrt{-g} \left\{ R - 2\Lambda - F_{\mu\nu}^2 + m^2 V(g^{ij}) \right\}$$

$$\begin{aligned} t &\rightarrow \tilde{t}(t, x) \\ \cancel{x^i \rightarrow \tilde{x}^i(t, x)} \end{aligned}$$

Broken diffeos

Broken CFT Transl INV

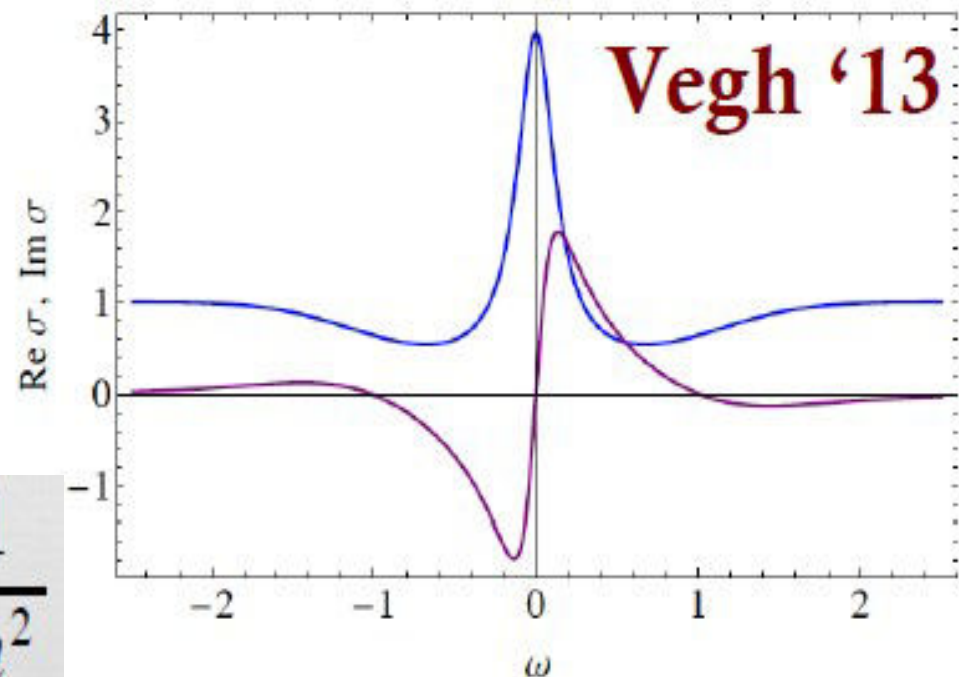
Momentum Dissipation

Conductivity:

$$\sigma(\omega) \approx \frac{\sigma_{DC}}{1 - i\omega\tau}$$

$$\sigma_{DC} = 1 + \frac{\rho^2 z_H^2}{m^2}$$

$$\tau \sim \frac{1}{m^2}$$



EFT for Solids and Massive Gravity

Comoving coordinates of the volume elements: $\Phi^I(\vec{x}, t)$

EFT for Phonons (Goldstone bosons of SB-translations)

equilibrium

$$\Phi^i = \underbrace{x^i} + \underbrace{\phi^i}$$

Leutwyler '93

Classification in *Nicolis Penco Rosen '14* & *Nicolis Penco Piazza Rattazzi '15*

SOLIDS =

Internal Shift symmetry : $L_{\text{eff}} \simeq \partial\phi^i\partial\phi^i + (\vec{\partial}\cdot\vec{\phi})^2 \longrightarrow$ **Elastic response**

EFT of fluids/solids \Leftrightarrow LV Massive Gravity

Gauging SSB of translations \longrightarrow graviton 'eats up' phonons

$$L = V(X, Y)$$

$$X \equiv \partial_\mu \Phi^I \partial_\nu \Phi^I g^{\mu\nu}$$

SOLIDS:

$$\langle \Phi^I \rangle = \delta^I_i x^i$$

$$Y \equiv \partial_\mu \Phi^J \partial^\mu \Phi^K \partial_\nu \Phi^J \partial^\nu \Phi^K$$

(More²) Realistic AdS-CMT

$$S = \int \sqrt{-g} \left\{ R - 2\Lambda - F_{\mu\nu}^2 + m^2 V(X) \right\}$$

Massive Gravity in Stuckelbergs form; healthy theory

$$X \equiv \partial_\mu \Phi^I \partial_\nu \Phi^I g^{\mu\nu}$$

$$\langle \Phi^I \rangle = \delta^I_i x^i$$

$$V'(X) > 0 \quad XV''(X) + V'(X) > 0$$

$$\Phi^I = \langle \Phi^I \rangle + \delta\Phi^I$$

$$M_{\delta\Phi}^2 \approx -\frac{V''(z_H^2)}{V'(z_H^2)} < 0$$

POLARON
FORMATION

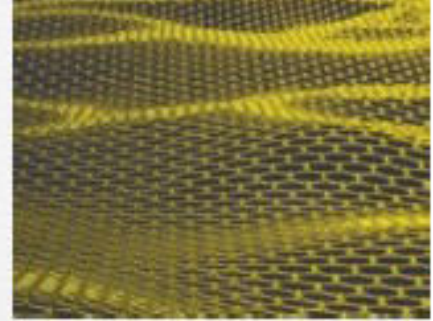
$$\sigma_{DC} = 1 + \frac{\rho^2 z_H^2}{m^2 V'(z_H^2)}$$

Quite generic T dependence
Of the Drude-Like part

+

Gapped phonons & Polaron formation

Strongly coupled solids



$\langle \Phi^I \rangle = \delta_i^I x^i$ realizes **spontaneous and explicit breaking**

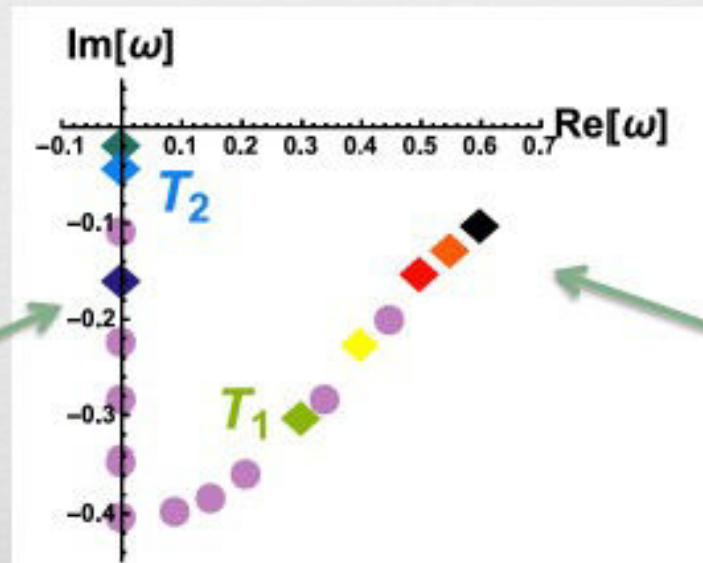
$\delta \Phi^I$ contains a **phonon pole** for $ml \ll 1$

$$w(k) \simeq w_0 - i\Gamma_0 + c_s k$$

disorder (expl breaking)

elastic moduli (w.i.p.)

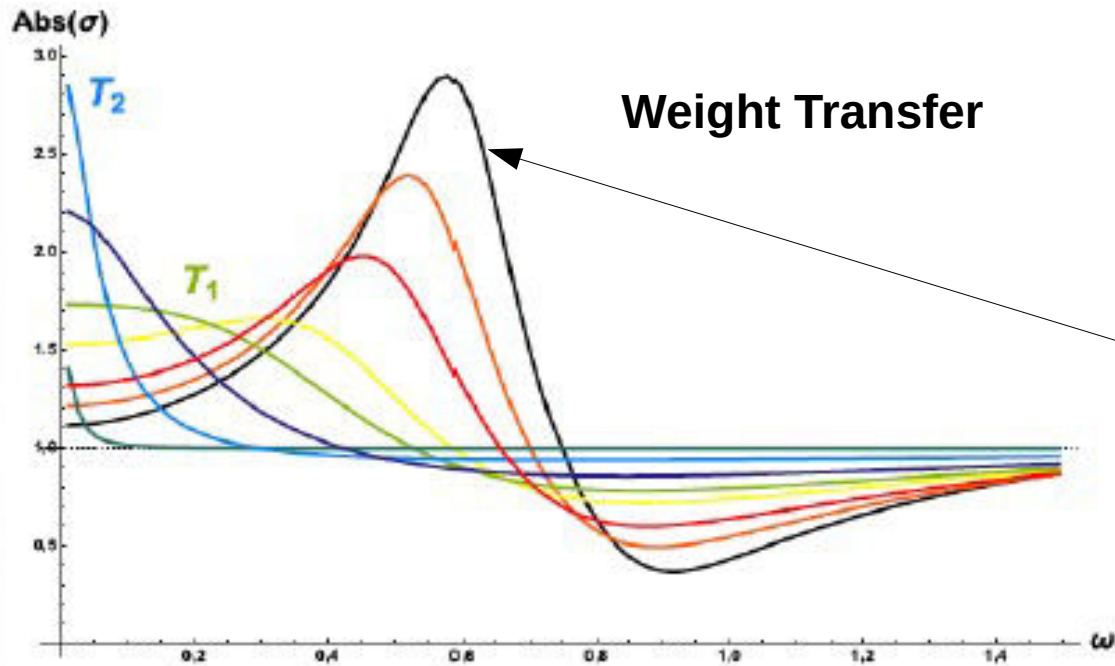
over-damped



Work in Progress

under-damped
energy gap

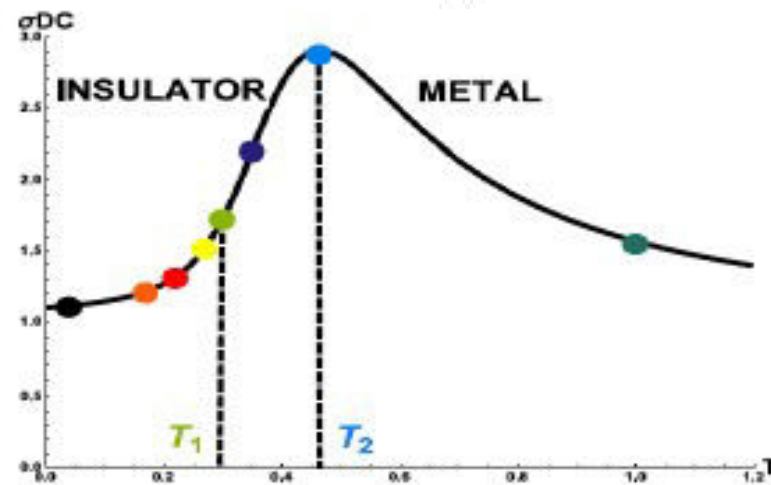
Metal – Insulator transition



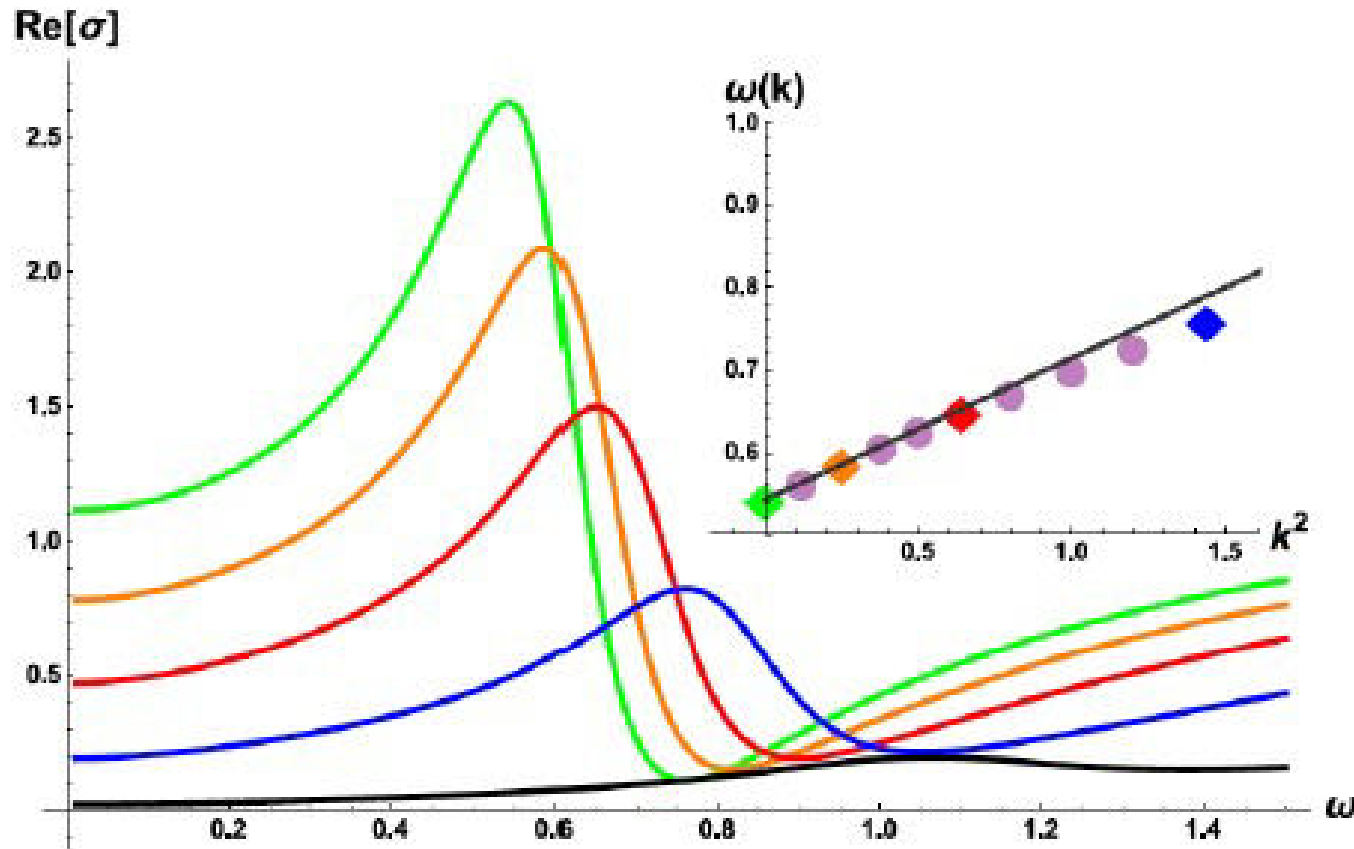
phonons gapped

electrons trapped into **polarons** with large m_*

$$m_*^2 = m^2 V'(z_h^2)$$



Emergent Polaron-like dofs



POLARON

There is a localized and propagating excitation with well defined mass, width and dispersion relation...

Disorder Driven Metal-Insulator transition from holography

$$S_{\text{bulk}} = \int d^4x \sqrt{-g} \left(\mathcal{R} - 2\Lambda - \frac{F^2}{4e^2} + \mathcal{D}(\phi^I, \dots) \right) \longrightarrow \sigma_{DC} = \frac{1}{e^2} (\underline{1} + \dots)$$

There is a lower bound on the DC conductivity $\rightarrow \sigma_{DC} \geq \frac{1}{e^2}$

Coupling Massive Gravity to Charged Sector

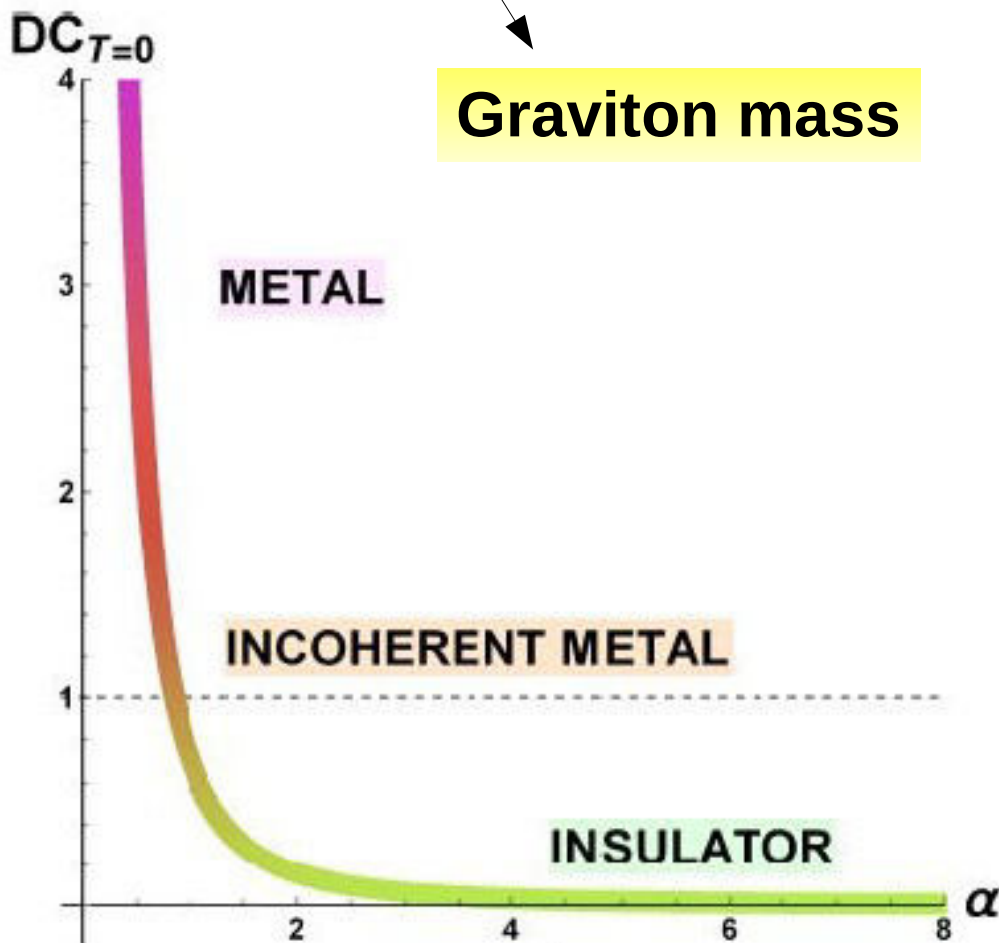
$$S = \int d^4x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{2e^2} Y(X) F^2 - 2m^2 V(X) \right], \quad \begin{aligned} X &= g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^I \\ \phi^I &= \alpha \delta_i^I x^i \end{aligned}$$

Electrons- Disorder Interactions

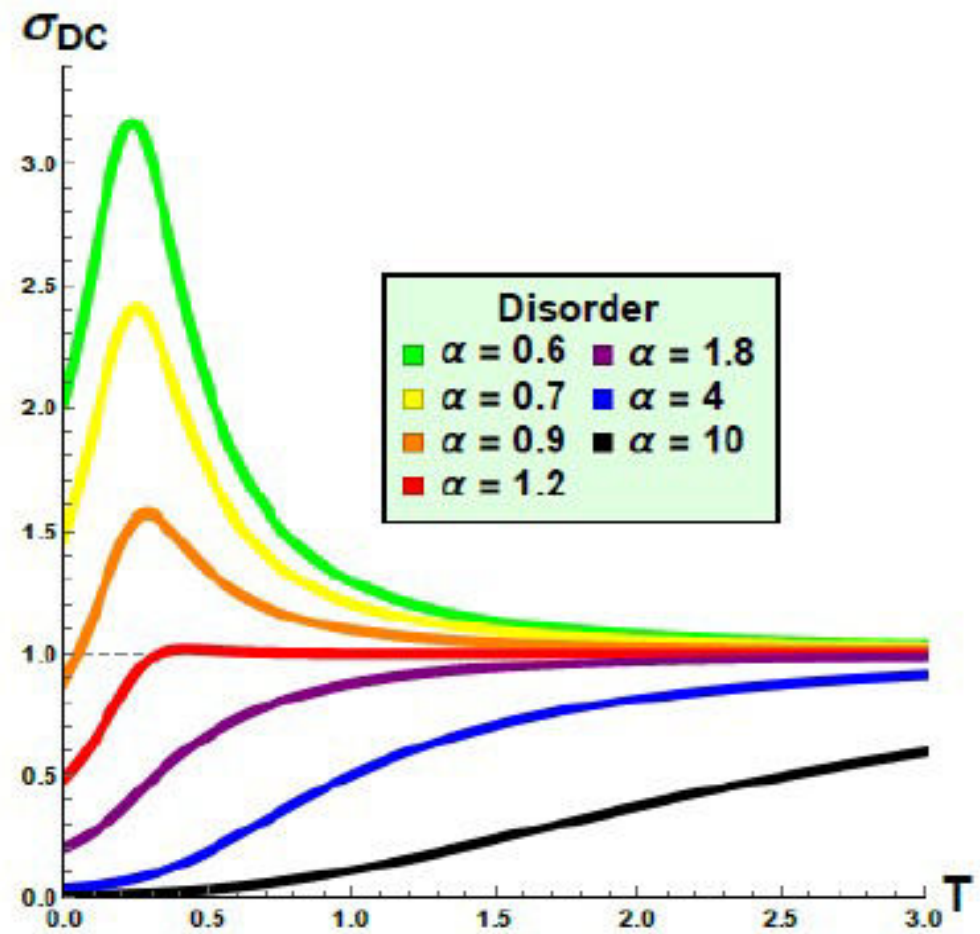
$$\sigma_{DC} = Y(\bar{X}_h) + \dots$$

COMING SOON

Disorder Driven Scenario Results



Disorder Driven Insulator



Disorder Driven
Metal-Insulator Transition

Conclusions

- **MASSIVE GRAVITY** has **REAL WORLD** applications (**CM**)
- There are many **USEFUL** phases of **NR MASSIVE GRAVITY**
- **MASSIVE GRAVITY** encodes **PHONONS** dynamics
- **MASSIVE GRAVITY** can model **DISORDER DRIVEN MIT**
- **AdS – CMT PREDICTABILITY ?**

Several candidate correlations & observables...

Work in progress

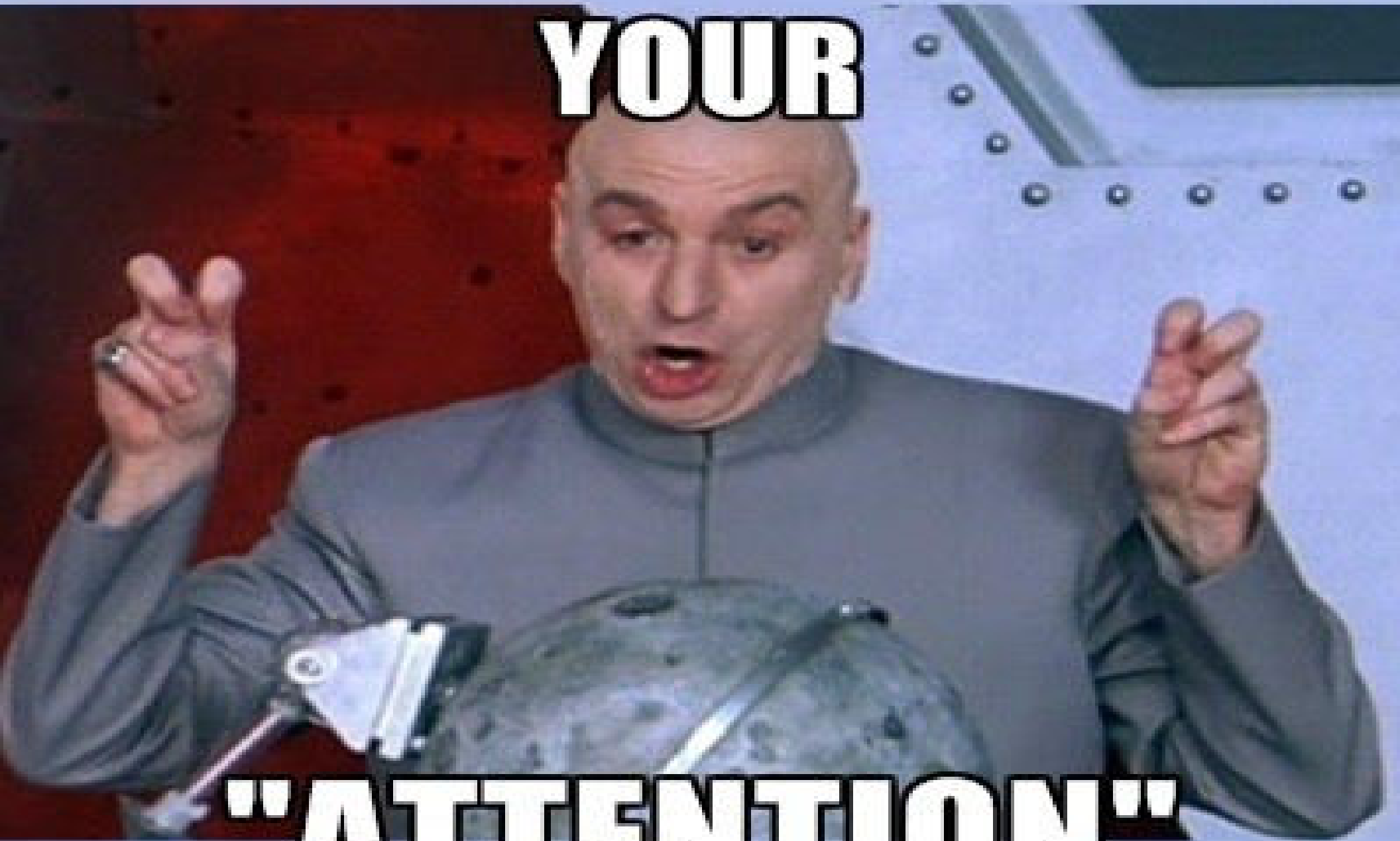


- x **Phases of Non Relativistic Massive gravity**
- x **(with L. Alberte, O. Pujolàs, A. Khmelnitsky)**
- x **Thermoelectric Response and Coherent-Incoherent contributions**
- x **(with A. Amoretti, D. Musso)**
- x **Towards Holographic Mott Insulators**
- x **(with O. Pujolàs)**
- x **Bounds, scalings and phenomenology**
- x **(with N. Magnoli, A. Braggio)**

& much more...



**AND THANKS FOR
YOUR**

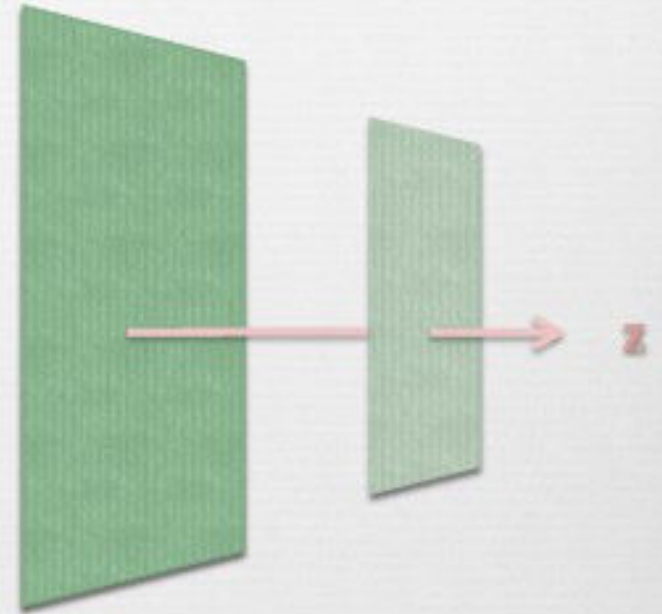


"ATTENTION"

AdS/CFT

$$z \sim \frac{1}{\mu} \quad (\text{RG-scale})$$

$$\Phi(z, x) \simeq \underbrace{\Phi_-(x)}_{J(x)} z^{\Delta_-} + \underbrace{\Phi_+(x)}_{\hat{O}(x)} z^{\Delta_+} + \dots$$



$$S_{on-shell} = \int d^d x \sqrt{h} (\Phi_- \Phi_+ + \dots) = \log [Z(J)]$$

**QFT interpretation in terms of boundary data
for a strongly coupled CFT**

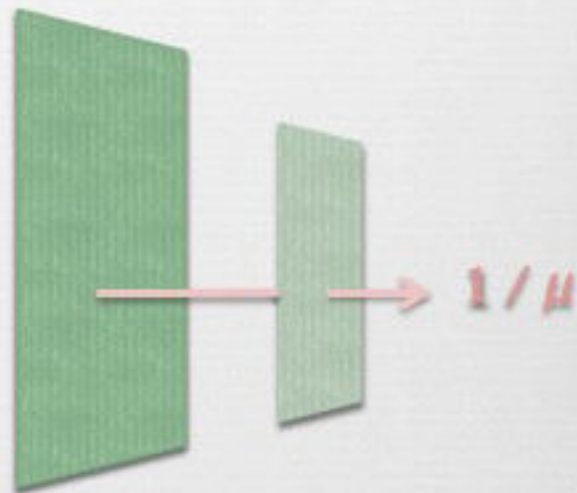
AdS/CFT

- “QFTs with gravity dual” → dynamics simplify enormously

few QFT operators $\{T_{\mu\nu}, J_{\mu}, O, \dots\}$

$$T_{\mu\nu}^{CFT} \subset g_{\mu\nu}$$

$$J_{\mu}^{CFT} \subset A_{\mu}$$



- Many non-trivial QFT effects:

nonperturbative RG flows, collective effects,
emergent symmetries & DOFs, unparticles,
QFT plasmas, dissipation in QFT

Metal-Insulator transitions

Debye / plasma mass term

$$\partial_u (f \partial_u a_i) + \left[\frac{\omega^2}{f} - k^2 - 2u^2 \rho^2 \right] a_i = \frac{i \rho u^2 (2\bar{m}^2 + k^2)}{\omega} U_i - \frac{i f \rho k^2}{\omega} \partial_u B_i,$$

bulk linearized equations for vector modes

$$\frac{1}{u^2} \partial_u \left[\frac{f u^2}{\bar{m}^2} \partial_u (\bar{m}^2 U_i) \right] + \left[\frac{\omega^2}{f} - k^2 - 2\bar{m}^2 \right] U_i = -2i \rho \omega a_i + \frac{f' k^2}{u^2} B_i,$$

$$k \left\{ u^2 \partial_u \left(\frac{f}{u^2} \partial_u B_i \right) + \left[\frac{\omega^2}{f} - k^2 - 2\bar{m}^2 \right] B_i = -2 \frac{\bar{m}'}{\bar{m}} U_i \right\},$$

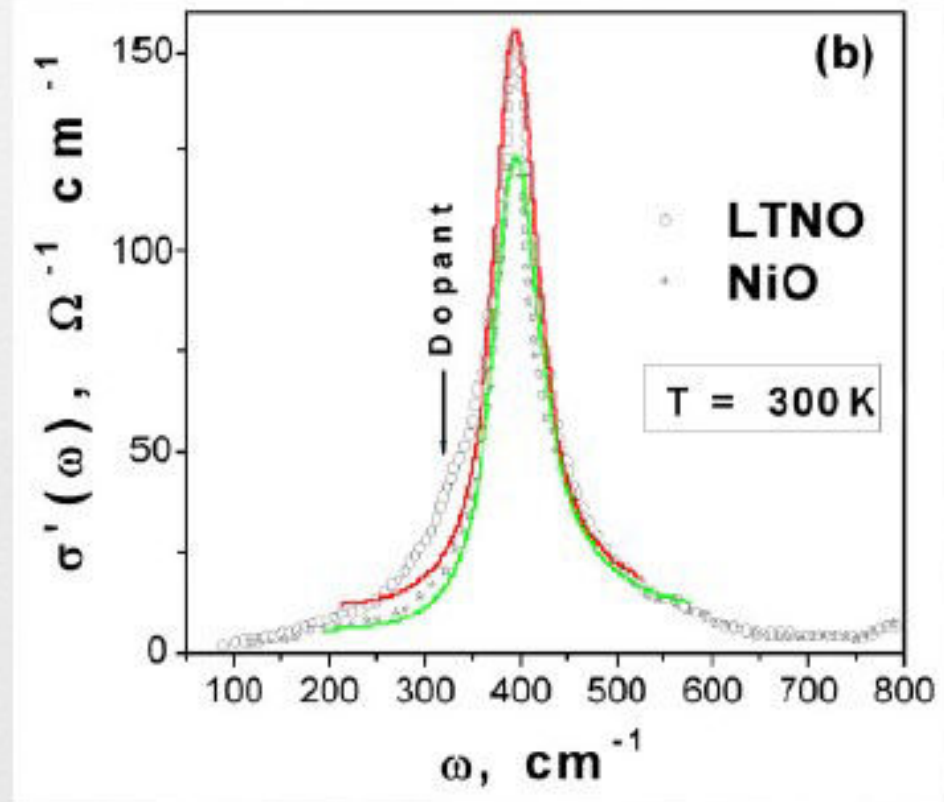
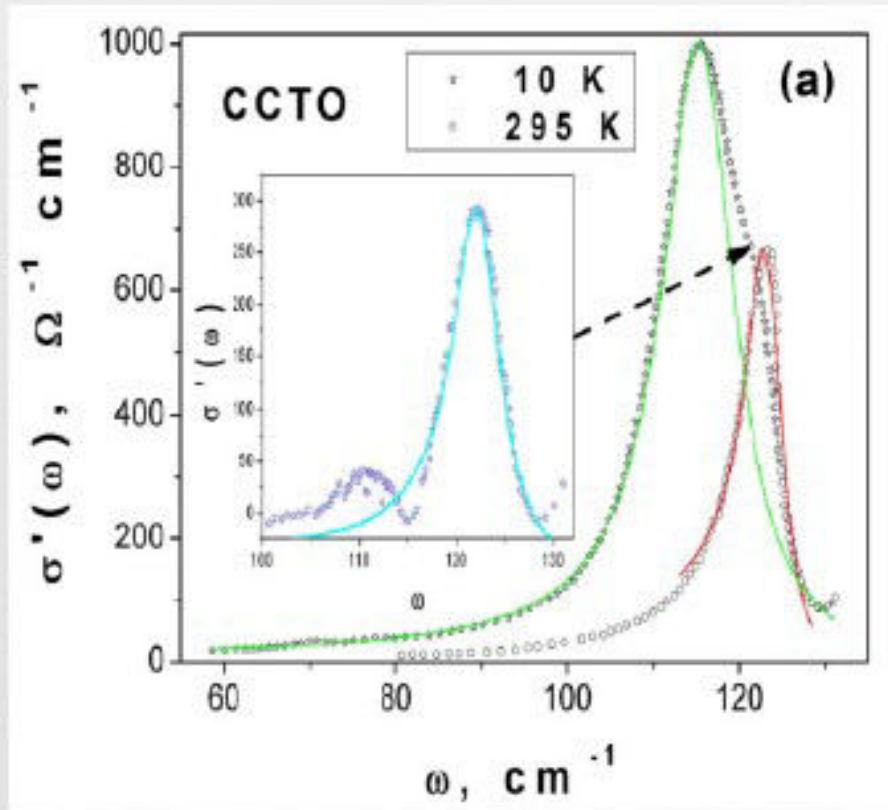
gauge invariant variables

$$U_i \equiv f(u) \left[h_{ui} - \frac{\partial_u \phi_i}{\alpha u^2} \right], \quad B_i \equiv b_i - \frac{\phi_i}{\alpha}$$

$$\bar{m}^2(u) = \alpha^2 m^2 V'(\alpha^2 u^2)$$

Polarons in the real world

ii)

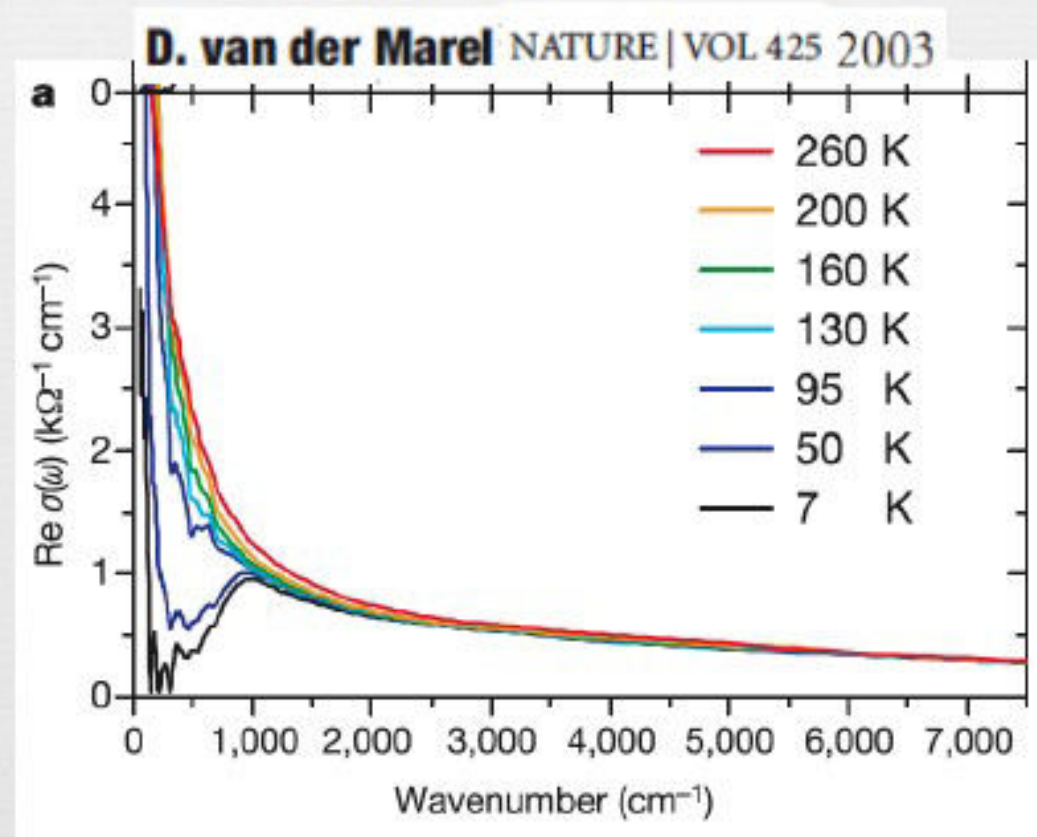


Phys. Status Solidi B 251, No. 3, 569–592 (2014)

Valeri Ligatchev

Polarons in the real world

ii)



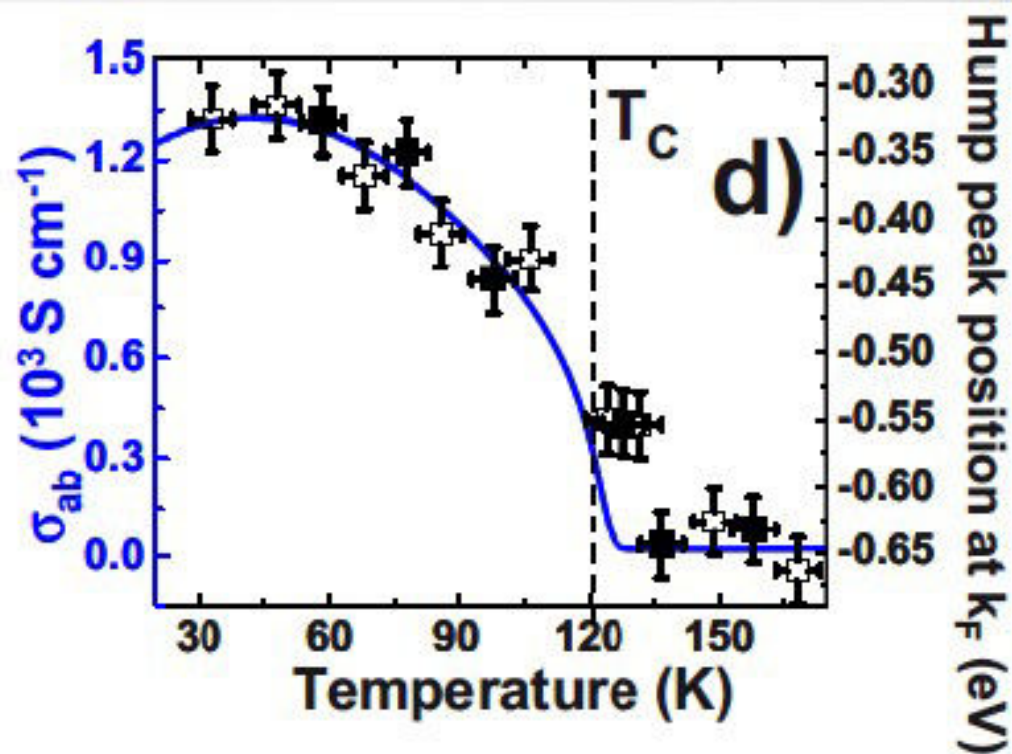
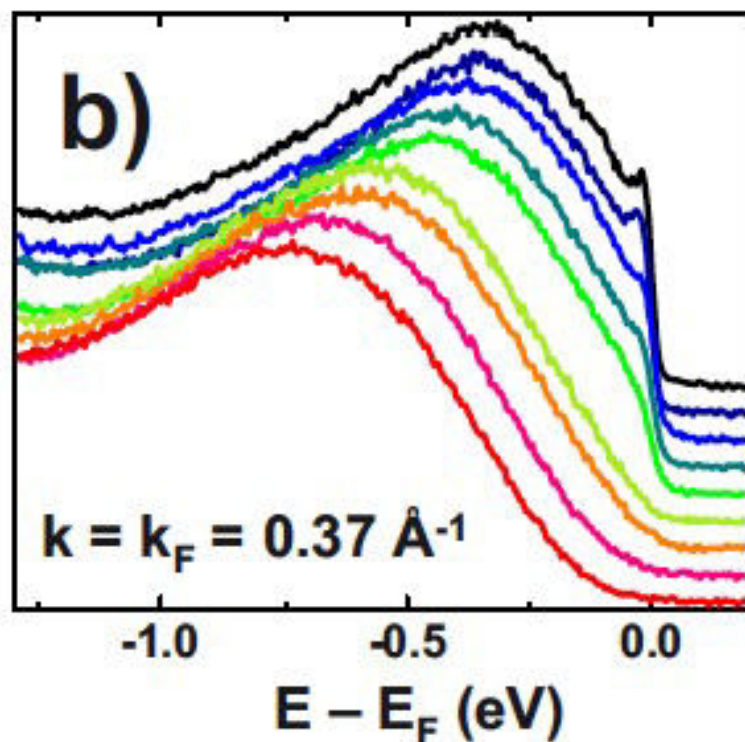
Polarons in the real world

ii) Polaron coherence condensation as the mechanism for colossal magnetoresistance in layered manganites

N. Mannella, PHYSICAL REVIEW B 76, 233102 (2007)

T (K) =

- 33
- 48
- 68
- 86
- 106
- 124
- 132
- 149
- 168



Mott-Wigner localization

iii) $e^- - e^-$ interactions

Natural expectation:
nonlinear electrodynamics

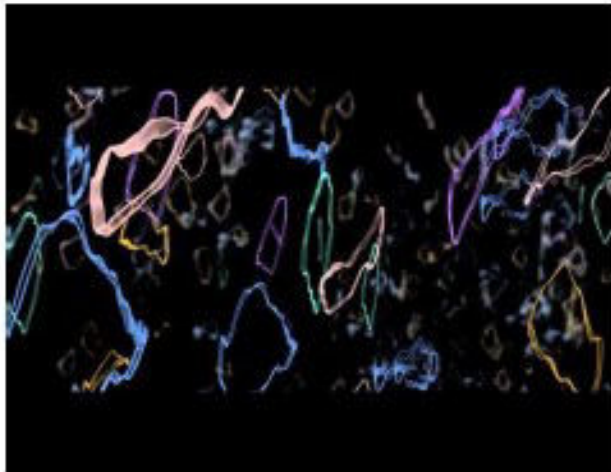


$$S = \int \sqrt{-g} \left\{ R - 2\Lambda - K \left(F_{\mu\nu}^2 \right) + m^2 V(X) \right\}$$

$$\sigma_{DC} = K'(F_H^2) + \frac{\rho^2 z_H^2}{m^2 V'(z_H^2)}$$

- smoother transition
- also forms quasiparticles
→ Heavy Fermions ??

The dilatonic (known) case



String theory inspired (embedding known)

Adding a new (running) scalar degree of freedom

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\underbrace{R - \frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{new dof}} - \frac{Z(\phi)}{4e^2} F^2 + \underbrace{\mathcal{D}_m(\phi, \psi^I, \dots)}_{\text{Dissipative sector}} \right)$$



$$\sigma_{DC} = \frac{1}{e^2} (Z(\phi)_{\text{horizon}} + \dots)$$

Rich phenomenology

Habemus Insulators

An additional gain

STRANGE METALS : $\sigma \propto \frac{1}{T}$ $\Theta_H \propto \frac{1}{T^2}$

Famous and robust LINEAR T RESISTIVITY

From holography: $\sigma \propto \sigma_{ccs} + \sigma_{diss}$ $\Theta_H \propto \frac{B}{Q} \sigma_{diss}$

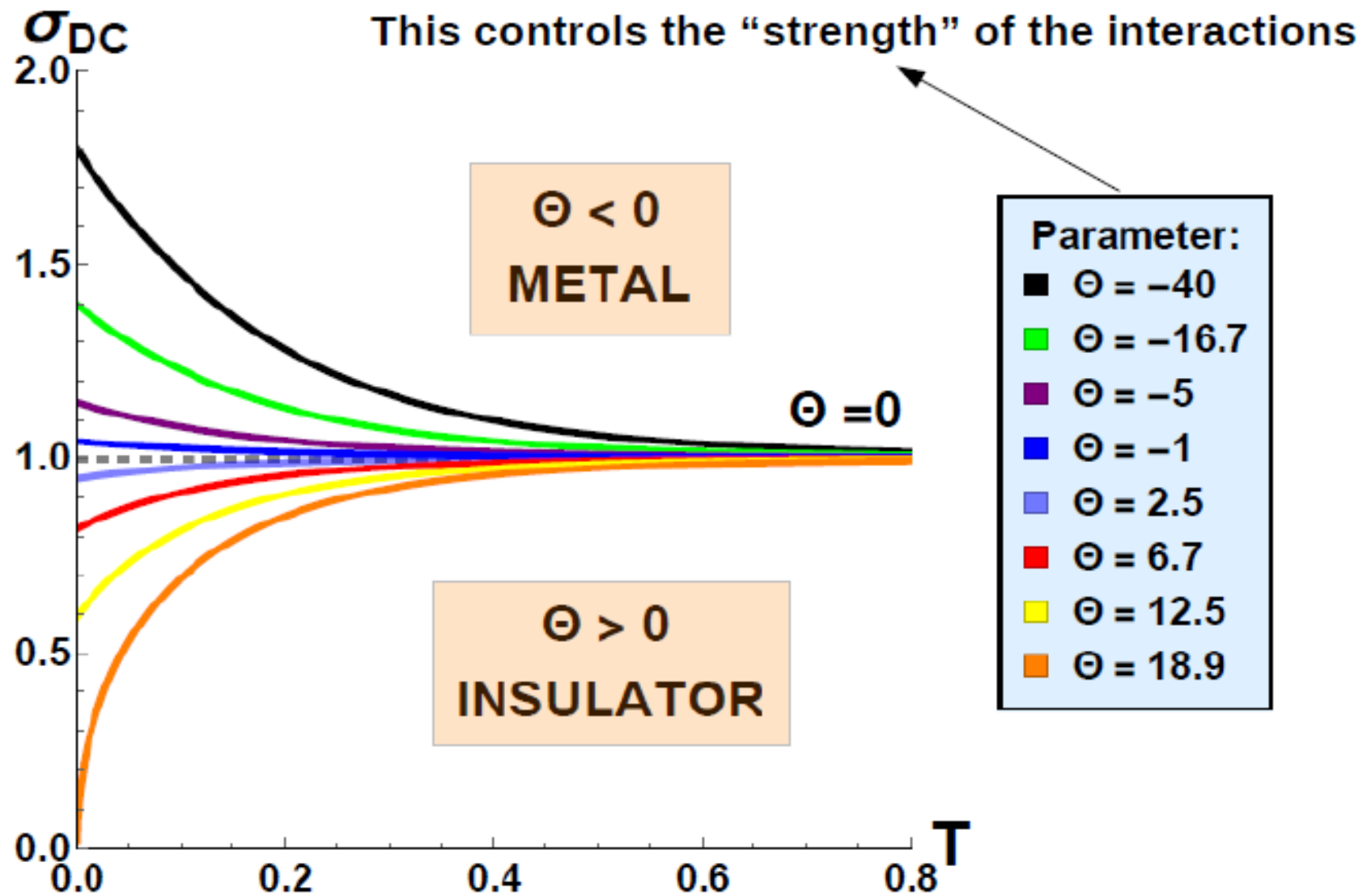
If $\sigma_{ccs} = \frac{1}{e^2}$ like with standard maxwell term →



Otherwise we can achieve having two different scales
And reproducing the right phenomenology (scalings) →

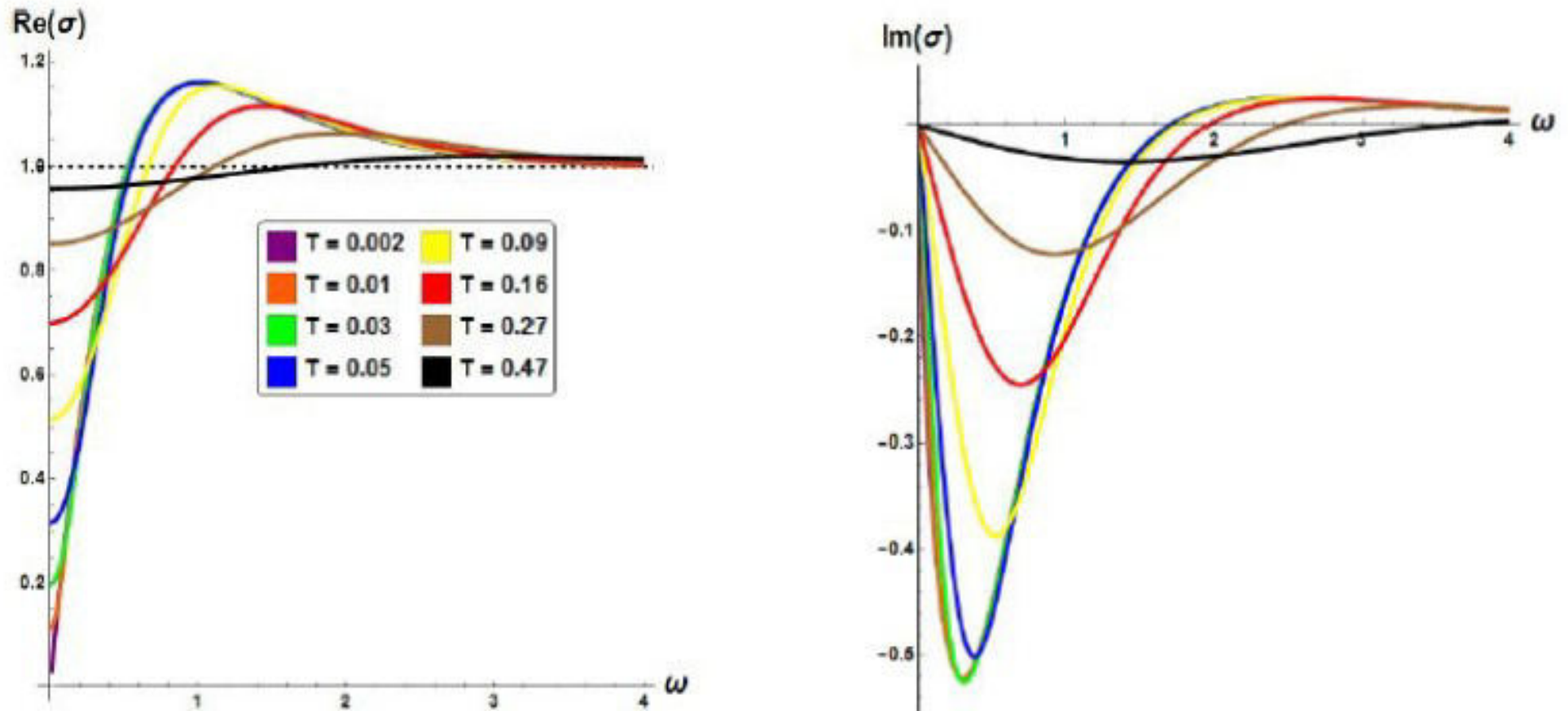


Non Linear Electrodynamics Results



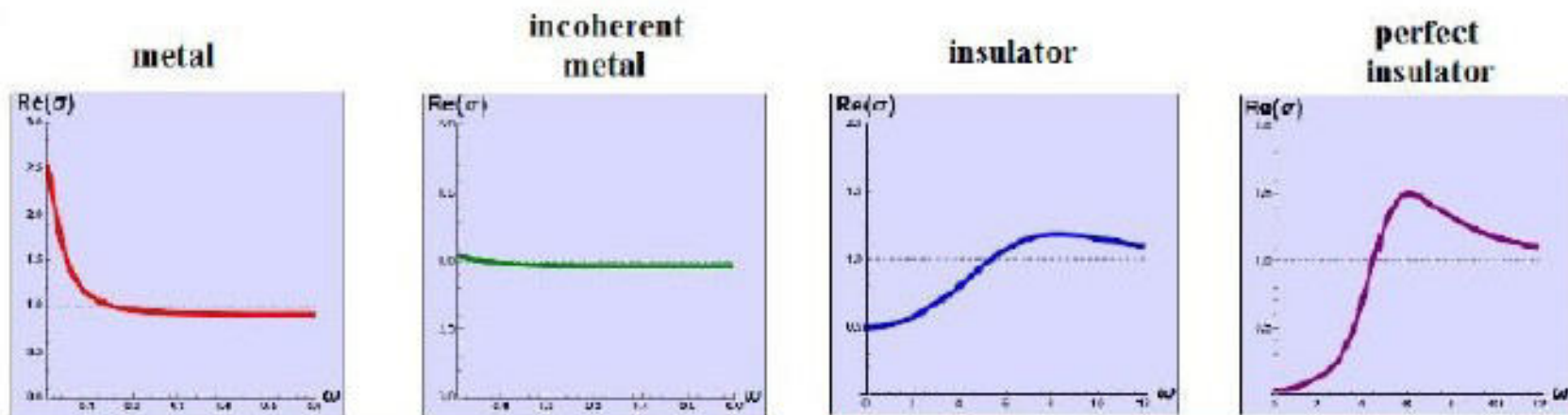
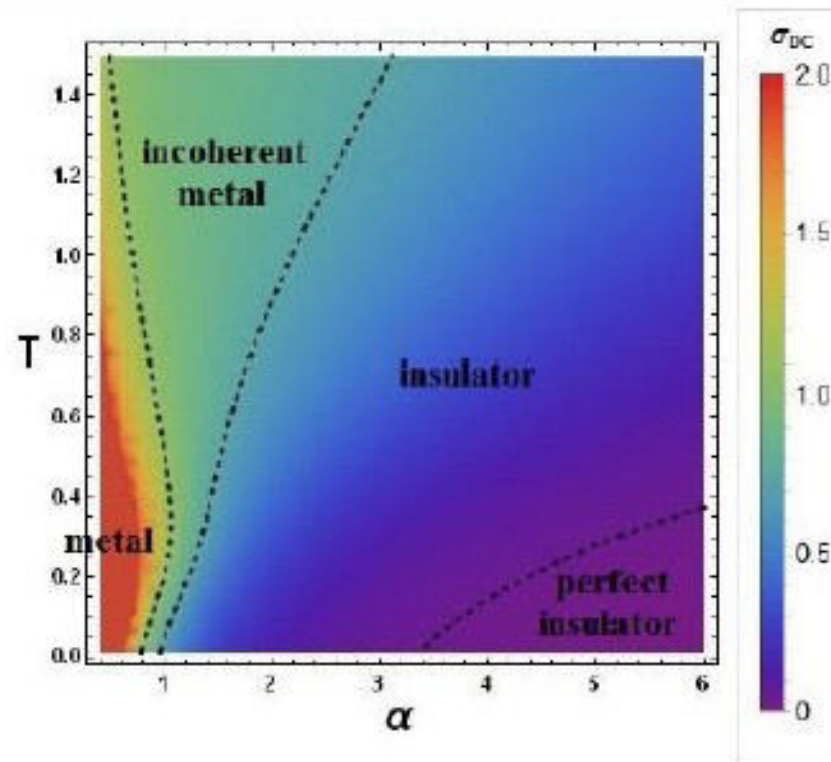
METAL- INSULATOR TRANSITION à LA MOTT

Non Linear Electrodynamics Results



Optical conductivity in the insulating phase

Disorder Driven Scenario Results



Old results...

New results...