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Holographic Yang-Mills at finite θ angle

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- *F.B., Aldo Cotrone, Roberto Sisca, JHEP 1508 (2015) 090*
- *F.B., Aldo Cotrone, JHEP 1501 (2015) 104*

Plan

- Motivations: θ-angle in Yang-Mills
- Holographic Yang-Mills at finite θ-angle
- Conclusions

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Effects of the θ parameter interesting but challenging.

The θ-angle in Yang-Mills

- Euclidean Lagrangian

$$\mathcal{L}_\theta = \frac{1}{2g^2_{YM}} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{i}{32\pi^2} \theta \, \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} F_{\mu\nu} F_{\rho\sigma}$$

- θ term breaks P,T and hence CP
- θ multiplies topological charge density q(x), whose 4d integral is integer.

$$q(x) = \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} F_{\mu\nu} F_{\rho\sigma} \, , \qquad \int dx^4 q(x) = n \in \mathbb{Z}$$

- Hence θ is an angle. Physics invariant under θ ⟶ θ + 2π

$$Z[\theta] = \int D[A] e^{-S_\theta}$$

- θ-dependence due to instantons . Not visible in perturbation theory.

The θ-angle in Yang-Mills

- Euclidean Lagrangian

$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2}\theta\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

- Effects of θ interesting:
  - vacuum structure
  - CP violating effects in QGP [Kharzeev et al]
  - mass and interactions of η' meson in QCD [Witten-Veneziano]
  - cosmology (axions)

- In real world QCD, $|\theta| < 10^{-10}$ (from neutron EDM). Strong CP problem.

The θ-angle in Yang-Mills

- Euclidean Lagrangian

$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{i}{32\pi^2} \theta \, \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} F_{\mu\nu} F_{\rho\sigma}$$

- Real θ-angle challenging for Lattice (sign problem: imaginary term)
- Go to imaginary θ, then analytically continue to real around θ=0
- Alternatively, compute n-point correlators of topological charge at θ=0
- Lattice results obtained in this way: first few terms in θ expansion

The θ-angle in Lattice Yang-Mills

- Ground state energy density [Vicari, Panagopoulos, 08; Bonati, D'Elia, Vicari, Panagopoulos, 13]

$$f(\theta) - f(0) = \frac{1}{2}\chi_g \theta^2 \left[1 + \bar{b}_2 \frac{\theta^2}{N_c^2} + \bar{b}_4 \frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right] \quad \chi_g = f''(\theta)|_{\theta=0} \neq 0$$ topological susceptibility

$\bar{b}_2 \approx -0.2$ (from N_c=3,…,6 data) $|b_4| < 0.001$ (N_c=3)

- String tension [Del Debbio, Manca, Panagopoulos, Skouropathis, Vicari, 06]

$$T_s(\theta) = T_s(0)\left[1 + \bar{s}_2 \frac{\theta^2}{N_c^2} + \mathcal{O}(\theta^4)\right], \quad \bar{s}_2 \approx -0.9$$ (from N_c=3,…,6 data)

- Lowest 0^{++} glueball mass [Del Debbio, Manca, Panagopoulos, Skouropathis, Vicari, 06]

$$M_{0^{++}}(\theta) = M_{0^{++}}(0)\left[1 + g_2\theta^2 + \mathcal{O}(\theta^4)\right], \quad g_2 \approx -0.06(2), \quad (N_c = 3)$$

- Deconfinement temperature [D'Elia, Negro, 12, 13]

$$T_c(\theta) = T_c(0)\left[1 + R_\theta \theta^2 + \mathcal{O}(\theta^4)\right], \quad R_\theta \approx -0.0175(7), \quad (N_c = 3)$$

The θ-angle in large N Yang-Mills

$$\mathcal{L} = \frac{N_c}{2\lambda}\left[\mathrm{Tr}F^2 - i\frac{\lambda}{8\pi^2}\frac{\theta}{N_c}\mathrm{Tr}F\tilde{F}\right]$$

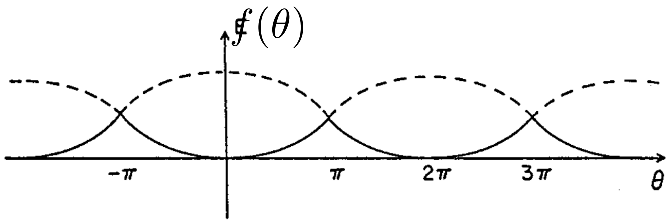
- ‘t Hooft limit: N>>1, $\lambda = g^2_{YM} N_c$ fixed
- Non trivial θ dependence (large N solution of $U(1)_A$ problem) if θ/N fixed.
- Ground state energy density should scale as

$$f(\theta) \equiv \varepsilon(\theta) - \varepsilon(0) = N^2 h\left[\frac{\theta}{N}\right]$$

- Puzzle: should also be periodic in θ with 2π periodicity.

The θ-angle in large N Yang-Mills

- Solution [Witten, 1980]: f(θ) multi-branched



$$f(\theta) = N^2 \mathrm{min}_k h\left[\frac{\theta + 2k\pi}{N}\right]$$

- At θ=(2k+1)π: expect first order transitions. CP spontaneously broken
- Minimum at θ=0 (integrand of Euclidean path int. real and positive), k=0.
- Large N solution of $U(1)_A$ problem

$$\chi_\infty = \frac{f_{\eta'}^2 m_{\eta'}^2}{4N_f} + O(1/N) \qquad \chi_\infty = f''_{YM}(\theta)|_{\theta=0} \neq 0 \text{ topological susceptibility}$$

$$f(\theta) = b\, N^2 \mathrm{min}_k \left(\frac{\theta + 2k\pi}{N}\right)^2 + O\,(\theta^4/N^4)$$

Plan

- Motivations: θ-angle in Yang Mills
- Holographic Yang-Mills at finite θ-angle
- Conclusions

Exact θ-dependence in a non-susy large N Yang-Mills model
with dual gravity description. Explicit realization of expected YM features.
Qualitative matching with lattice YM at small θ. Benchmark beyond small θ ?

Witten's holographic Yang-Mills

- N_c D4-branes in IIA string theory
- Low energy physics: 5d $SU(N_c)$ Super-Yang-Mills theory.

- N_c D4-branes on S^1_{x4} of radius $R_4 = 1/M_{KK}$ with antiperiodic fermions.
- Low energy: 4d non-susy $SU(N_c)$ Yang-Mills + adjoint KK modes [Witten 1998]

- Holography: Large N_c, strong coupling regime mapped into dual gravity description (open/closed string duality)

- Can add θ term to the model, no sign problem [Witten 1998]

Witten's holographic Yang-Mills

- Gravity action (closed string description), sourced by the N D4-branes

$$S = \frac{1}{2k_0^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}\left(\mathcal{R}+4(\partial\phi)^2\right)-\frac{1}{2}|F_4|^2-\frac{1}{2}|F_2|^2\right]$$

- Gauge theory action (open string description), wrapped D4-branes (IR expansion)

$$S = \frac{1}{8\pi g_s l_s M_{KK}}\int d^4x\ \mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2 l_s}\int_{S_{x^4}} C_1 \int d^4x\ \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

$F_2 = d\,C_1$ $\qquad \lambda_4 = g_{YM}^2 N_c = 2\pi g_s l_s M_{KK} N_c$ $\qquad \boxed{\theta + 2\pi k = \frac{1}{l_s}\int_{S_{x_4}} C_1}$

- Holography: gravity picture dual to gauge theory at $\lambda_4 \gg 1,\ N_c \gg 1$

The θ-backreacted gravity solution

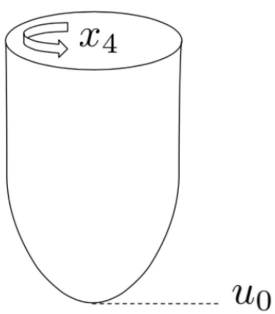
[Barbon, Pasquinucci 99; Dubovsky, Lawrence, Roberts 2011] $(x_4 \sim x_4 + 2\pi/M_{KK})$

$$ds^2_{10} = \left(\frac{u}{R}\right)^{3/2}\left[\sqrt{H_0}\, dx_\mu dx^\mu + \frac{f}{\sqrt{H_0}} dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\sqrt{H_0}\left[\frac{du^2}{f} + u^2 d\Omega_4^2\right]$$

$$f = 1 - \frac{u_0^3}{u^3}\,, \qquad H_0 = 1 - \frac{u_0^3}{u^3}\frac{\Theta^2}{1+\Theta^2} \qquad e^\phi = g_s\left(\frac{u}{R}\right)^{3/4} H_0^{3/4}\,, \qquad C_1 = \frac{\Theta}{g_s}\frac{f}{H_0} dx^4\,, \qquad F_4 = 3R^3\omega_4$$

$$\int_{S^4} F_4 = 8\pi^3 l_s^3 g_s N_c\,, \quad R = (\pi g_s N_c)^{1/3} l_s \qquad \theta + 2\pi k = \frac{1}{l_s}\int_{S_{x_4}} C_1$$

$$\Theta \equiv \frac{\lambda_4}{4\pi^2}\left(\frac{\theta + 2k\pi}{N_c}\right) \qquad u_0 = \frac{4R^3}{9} M_{KK}^2 \frac{1}{1+\Theta^2}$$



- (u,x_4) subspace is a cigar
- $g_{00}(u_0) \neq 0$ (regular) : confinement
- KK modes NOT decoupled
- Small curvature if $\Theta \ll \lambda_4^{1/4}$

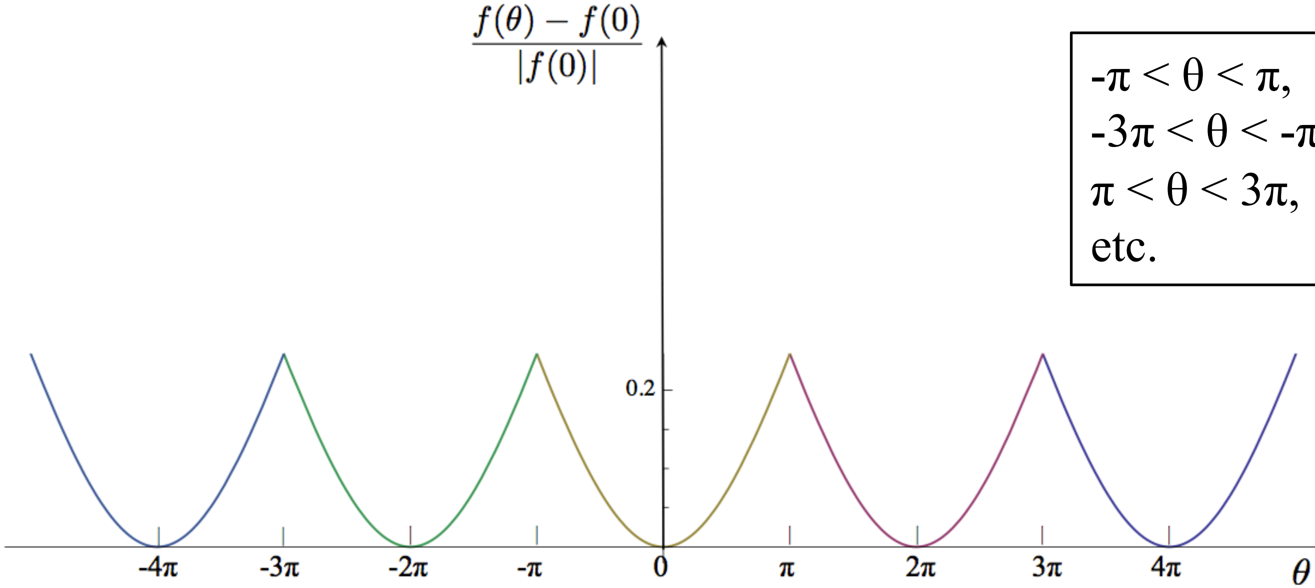
The ground-state energy density

From holographic relation $Z = e^{-V_4 f(\theta)} \approx e^{-S^{\rm ren}_{E\,\rm on-shell}}$

$$f(\Theta) = -\frac{2N_c^2\lambda_4}{3^7\pi^2}\frac{M_{KK}^4}{(1+\Theta^2)^3}$$

$$\Theta \equiv \frac{\lambda_4}{4\pi^2}\left(\frac{\theta+2k\pi}{N_c}\right)$$

$$f(\theta) = \min_k f(\Theta)$$



$-\pi < \theta < \pi$, k=0
$-3\pi < \theta < -\pi$, k=1
$\pi < \theta < 3\pi$, k=-1
etc.

Expected structure explicitely realized

The ground-state energy density

- Expansion around θ=0

$$f(\theta) - f(0) = \frac{1}{2}\chi_g \theta^2 \left[1 + \bar{b}_2 \frac{\theta^2}{N_c^2} + \bar{b}_4 \frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right]$$

$$\chi_g = \frac{\lambda_4^3 M_{KK}^4}{4(3\pi)^6} \qquad \bar{b}_2 = -\frac{\lambda_4^2}{8\pi^4}\,, \quad \bar{b}_4 = \frac{5\lambda_4^4}{384\pi^8}$$

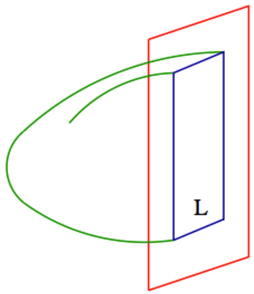
- Cfr. with Lattice Yang-Mills: $\bar{b}_2 \approx -0.2 \qquad |b_4| < 0.001$

- Qualitative agreement with Lattice
- Prediction: $b_4 > 0$ (it would be nice to check it on the lattice)

- Curiosity: qualitative disagreement with 2d CP^{N-1} model where b_2 and b_4 are both negative [Del Debbio, Manca, Panagopoulos, Skouroupathis, Vicari, 2006]

The string tension

Rectangular Wilson loop: from minimal open string surface [Maldacena; Rey, Yee '98]



$$W(\mathcal{C}) = \mathrm{Tr}\left[P \exp\left(i \oint_{\mathcal{C}} A\right)\right]$$

$$\langle W[\mathcal{C}]\rangle \sim e^{-S^r_{NG}}$$

$$\langle W[\mathcal{C}]\rangle \approx e^{-TV(L)} \qquad V(L) = T_s L$$

$$S^r_{NG} \approx -\frac{1}{2\pi\alpha'} T \sqrt{-g_{00}g_{xx}}|_{u=u_0} L \qquad T_s = \frac{1}{2\pi\alpha'}\sqrt{-g_{00}g_{xx}}|_{u=u_0}$$

$$T_s = \frac{2\lambda_4}{27\pi} M^2_{KK} \frac{1}{(1+\Theta^2)^2}$$

$$T_s = \frac{2\lambda_4}{27\pi} M^2_{KK}\left(1 - \frac{\lambda_4^2}{8\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{256\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right)$$

$$T_{s\,lat} = T_{s\,lat}(0)\left[1 + \bar{s}_2 \frac{\theta^2}{N_c^2} + \dots\right], \qquad \bar{s}_2 \approx -0.9$$

‘t Hooft loop and oblique confinement

- Monopole-antimonopole potential from minimal action of D2 wrapping S_{x4}

$$S_{D2} = -T_2 \int d^3\xi e^{-\hat{\phi}} \sqrt{-\det(g+\mathcal{F})} + T_2 \int \hat{C}_1 \wedge \mathcal{F}$$

- ‘t Hooft loop has an area law at finite theta (magnetic screening only at theta=0)

$$T_m = \frac{1}{27\pi^2} M_{KK}^2 \lambda_4 \frac{|\theta + 2k\pi|}{(1+\Theta^2)^2} \equiv T_s \frac{|\theta + 2k\pi|}{2\pi}$$

- Dyons are screened under certain conditions (oblique confinement)

$$T_{dy} = -pT_s + qT_m = \left(-p + \frac{\theta}{2\pi} q\right) T_s \qquad \theta = 2\pi(p/q)$$

See also [Gross, Ooguri, 98]

The scalar glueball mass

- 0^{++} glueball spectrum → $\langle \mathrm{Tr}F^2(x)\mathrm{Tr}F^2(y)\rangle = \sum_n c_n\, e^{-M_n|x-y|}$
- Solve e.o.m of dual gravity scalar field (a metric fluctuation in the 11d completion)

$$h_{ab} = H_{ab}(u)e^{-ik\cdot x}$$

$$H''(u) + \frac{4u^3-u_0^3}{u(u^3-u_0^3)}H'(u) - \frac{M^2R^3}{u^3-u_0^3}H(u) = 0$$

- Regularity at $u=u_0$ and UV normalizability only if $M^2>0$ and discrete

$$M(\Theta) = \frac{M(\Theta=0)}{\sqrt{1+\Theta^2}}$$

$$M(\theta) = M(\theta=0)\left(1 - \frac{\lambda_4^2}{32\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{2048\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right)$$

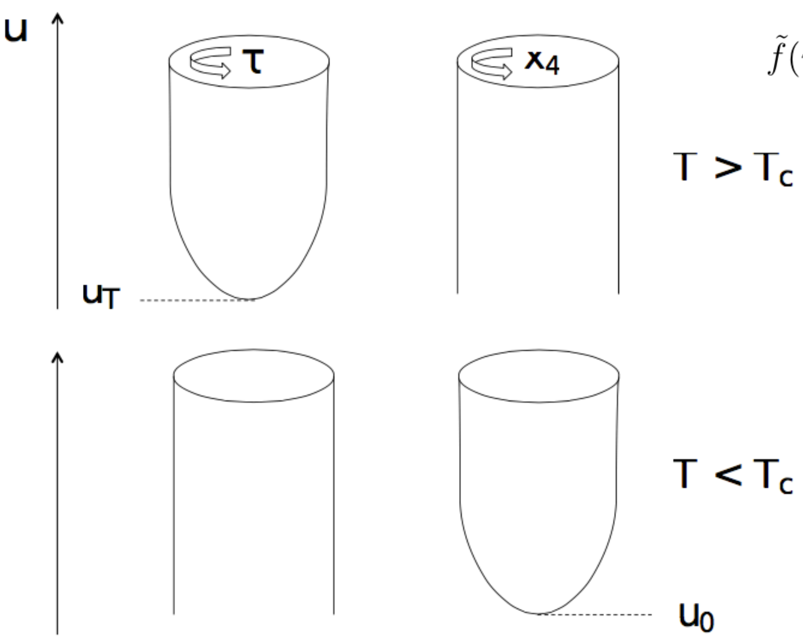
$$M_{lat}(\theta) = M_{lat}(0)\left(1 + g_2\theta^2 + \mathcal{O}(\theta^4)\right)\,, \quad g_2 = -0.06(2)$$

Finite temperature

Take Euclidean time periodic with period 1/T. Two possible gravity solutions

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[\tilde{f}(u)dx_0^2 + dx_a dx^a + dx_4^2\right] + \left(\frac{u}{R}\right)^{-3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right],$$

$$\tilde{f}(u) = 1 - \frac{u_T^3}{u^3}.$$



- black hole solution
- $g_{00}(u_T) = 0$: deconfinement
- no theta dependence

$$C_1 \sim \theta dx_4, \;\; F_2 = 0 \quad [(u, x_4) : \text{cylinder}]$$

- Just Euclidean extension of one discussed before
- Theta-dependence
- confinement

Deconfinement temperature

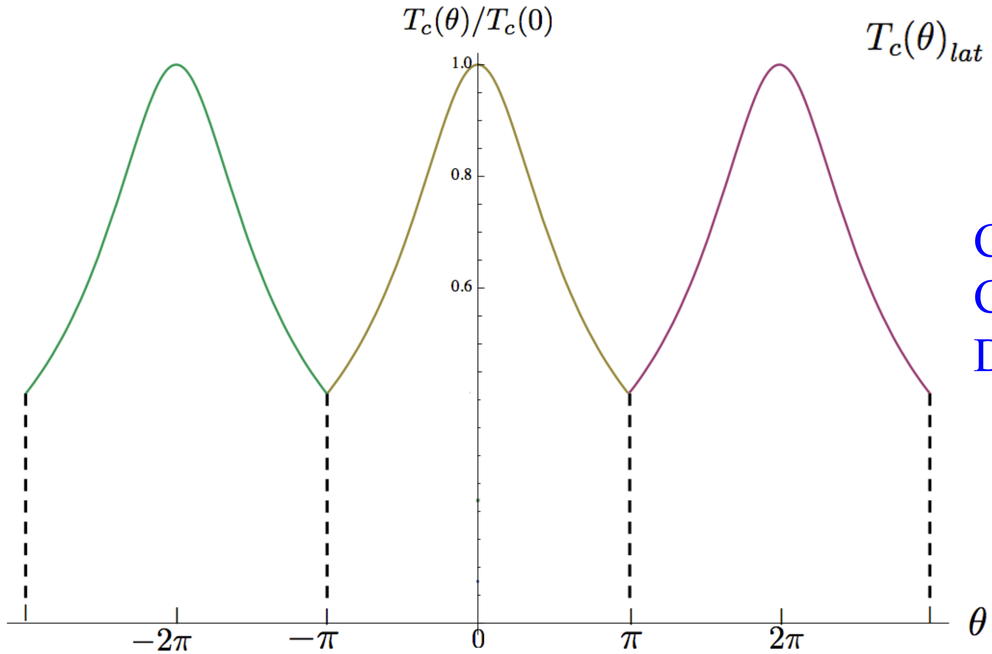
Compare the free energy densities of confined and deconfined phase

$$f = -p = -\frac{2N_c^2\lambda_4}{3^7\pi^2}\frac{M_{KK}^4}{(1+\Theta^2)^3} \equiv \frac{f(0)}{(1+\Theta^2)^3}$$

$$f_{dec} = -p_{dec} = -\frac{1}{6}\frac{256N_c^2\pi^4\lambda_4}{729M_{KK}^2}T^6$$

$$\boxed{T_c(\Theta) = \frac{M_{KK}}{2\pi}\frac{1}{\sqrt{1+\Theta^2}}}$$

$$T_c(\theta) = \frac{M_{KK}}{2\pi}\left[1 - \frac{\lambda_4^2}{32\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{2048\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right]$$

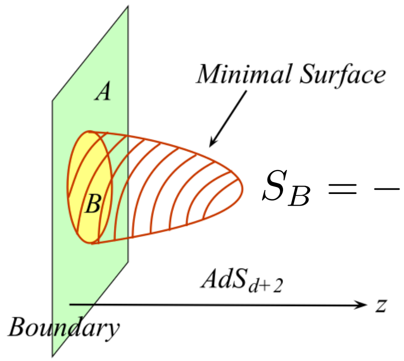


$$T_c(\theta)_{lat} = T_c(0)_{lat}\left[1 - R_\theta\theta^2 + \mathcal{O}(\theta^4)\right], \quad R_\theta = 0.0175(7)$$

Cusps: tri-critical points
Colored: deconf. first order transition
Dashed: CP-breaking first order transition

Entanglement entropy

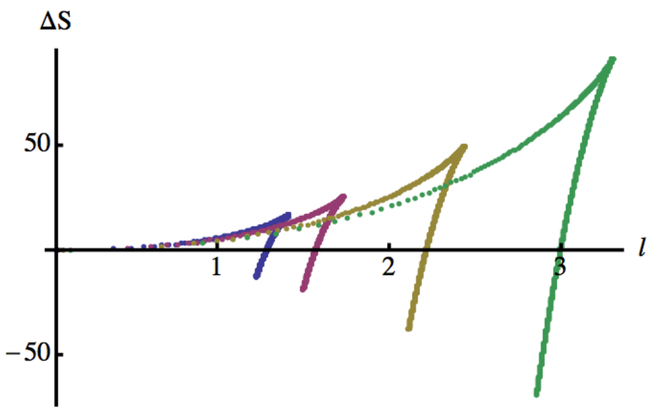
[Ryu, Takayanagi 2006]



$$S_B = -Tr(\rho_B \log \rho_B) \approx \min_{\gamma_B} \frac{A(\gamma_B)}{4G_N}$$

$$S = \frac{1}{4G_N} \int d^8\sigma e^{-2\phi} \sqrt{-\det(G_{\rm ind})}$$

Slab geometry: B={x in [-l/2, l/2]} [Klebanov, Kutasov, Murugan 07 at θ=0]



$$S_{dis} = V_2 \frac{2^2}{\pi 3^5} \frac{\lambda_4 N_c^2}{(1+\Theta^2)^2} M_{KK}^2 \left(\frac{1}{\epsilon} - 1\right)$$

$$S_{conn,fin} = -V_2 \frac{2^6}{3} \frac{\pi^{3/2}\Gamma[\frac{3}{5}]^5}{\Gamma[\frac{1}{10}]^5} \frac{\lambda_4 N_c^2}{(l\ M_{KK})^4} M_{KK}^2$$

$$l_c = l_{c,0}\sqrt{1+\Theta^2}$$

Overview

- Holographic YM results exact in θ, large N
- Observables are those at $\theta=0$ multiplied by powers of $(1+\Theta^2)$
- A factor of $(1+\Theta^2)^{-1/2}$ for each power of M_{KK}
- A factor of $(1+\Theta^2)^{-1}$ for each power of λ_4

- Mass scales reduced by θ (checked also baryon vertex mass)
- Structure of (T,θ) phase diagram explicitely
- Agreement with lattice trends at small θ
- Benchmarks for subleading coefficients in θ expansion

- Development: θ in Holographic QCD (Witten-Sakai-Sugimoto)
- Take massive quarks.
- Deduce topological susceptibility (zero if $m_{quarks}= 0$)
- Deduce θ-dependent observables (neutron electric dipole moment)

Thank you