Fermionic T-duality in superstring models

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Dualities = crucial tool to understand the deep nature of AdS/CFT

INTEGRABILITY

Key–properties related to integrability

- **CFT side** → dual superconformal symmetry in the on-shell sector of the theory → Amplitudes/Wilson loop duality
- **String theory side** → Invariance of the $AdS_5 \times S^5$ string model under bosonic and fermionic T-dualities associated to (anti)commuting isometries (Berkovits, Maldacena 0807.3196)

Fundamental Questions

B/f duality can be used to disclose integrability of the dual SCFT.

- How b/f duality works in other dimensions?
- How b/f duality acts on less supersymmetric backgrounds?
- How b/f self-duality works in AdS4 x CP3 (dual to ABJM model)? Still open problem.

We have considered $\mathrm{AdS}_3 \times S^3 \times T^4$ $\mathrm{AdS}_2 \times S^2 \times T^6$

$\mathrm{AdS}_3 \times S^3 \times S^3 \times T^1$ $\mathrm{AdS}_2 \times S^2 \times S^2 \times T^4$

PLAN OF THE TALK

- Bosonic/Fermionic T-duality in coset sigma-models.
- Status of the Art: Discussion on kappa–symmetry gauge–fixing.
- Self-duality of $\mathrm{AdS}_3 \times S^3 \times T^4$, $\mathrm{AdS}_2 \times S^2 \times T^6$ superstring models
- Conclusions and Perspectives

BOSONIC/FERMIONIC T-DUALITY: GENERALITIES

Supercoset description of superstring models on $\mathrm{AdS}_d \times S^d$ geometries.

Supercoset $G/H \;= \{gH \,|\, g \in G\}$

Z_4-automorphism $\Omega : G \to G$ $\;(\Omega^4 = 1$ and $\Omega(H) = H)$

Mapping $g : \Sigma \to G$ $\qquad \Sigma =$ 2-dimensional Riemann surface

Maurer–Cartan form $\qquad J = g^{-1}\mathrm{d}g = J_{(0)} + J_{(1)} + J_{(2)} + J_{(3)}$

Action $\qquad S \;=\; -\frac{T}{2}\int_\Sigma \mathrm{Str}(*J_{(2)} \wedge J_{(2)} + J_{(1)} \wedge J_{(3)})$

$$\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)} = \mathrm{AdS}_5\times S^5 \quad + \quad 32 \text{ fermionic dir.}$$

$$\frac{PSU(1,1|2)\times PSU(1,1|2)}{SU(1,1)\times SU(2)} = \mathrm{AdS}_3\times S^3 \quad + \quad 16 \text{ fermionic dir.}$$

$$\frac{PSU(1,1|2)}{SO(1,1)\times U(1)} = \mathrm{AdS}_2\times S^2 \quad + \quad 8 \text{ fermionic dir.}$$

Z_4-automorphism insures integrability of the supercoset model

(Bena, Polchinski, Roiban, 0305116)

Bosonic/Fermionic T-dualization

Standard procedure (Busher, Phys. Lett. B 201, 466 (1988); Berkovits-Maldacena, 0807.3196)

- Coset representative $g \in G$

$$g = \mathrm{e}^{xP+\theta Q}\mathrm{e}^{B}\,\mathrm{e}^{\xi S} \quad , \quad \mathrm{e}^{B} = \mathrm{e}^{\hat{\theta}\hat{Q}+\hat{\xi}\hat{S}}|y|^{D}\,\Lambda(y^{\hat{a}})$$

$$\langle P, Q\rangle \to \text{maximal abelian subalgebra}$$

- Substitute $(\mathrm{d}x, \mathrm{d}\theta) \mapsto (A_{\mathrm{b}}, A_{\mathrm{f}})$ and introduce dual variables $(\tilde{x}, \tilde{\theta})$

$$S_{\mathrm{gen}} = S[(\mathrm{d}x, \mathrm{d}\theta) \to (A_{\mathrm{b}}, A_{\mathrm{f}})] + \int_{\Sigma} \left(\tilde{x}\mathrm{d}A_{\mathrm{b}} + \tilde{\theta}\mathrm{d}A_{\mathrm{f}}\right)$$

- Integrate $(\tilde{x}, \tilde{\theta})$. Go back to the original Green–Schwarz action S.
- Integrate $(A_{\mathrm{b}}, A_{\mathrm{f}})$. Obtain the dual action $\tilde{S}$.

- Self-duality requires that, under a suitable number of T-duality transformations, $\tilde{S}$ is still of the Green–Schwarz form respect to a dual coset representative

$$\tilde{g} \;= \mathrm{e}^{\tilde{x}K+\tilde{\theta}M^{-1}S}\,\mathrm{e}^{B}\,\mathrm{e}^{F(\xi)} \qquad\qquad M = \mathrm{Str}(QS)$$

$$\downarrow$$

$$F(\xi) \;=\; -\big[\xi + \mathcal{O}(\xi^5)\big]Q + \big[\mathcal{O}(\xi^3) + \mathcal{O}(\xi^7)\big]S$$

$$P_m \;\leftrightarrow\; K_m \;, \quad D \;\to\; -D \;, \quad R_{\hat{a}} \;\to\; -R_{\hat{a}} \;,$$
$$S \;\to\; -\mathrm{i}Q \;, \quad \hat{S} \;\to\; -\mathrm{i}\hat{Q} \;, \quad Q \;\to\; -\mathrm{i}S \;, \quad \hat{Q} \to\; -\mathrm{i}\hat{S}$$

- If the model is self–dual

superc. invariance ↔ dual superconf. invariance

AMPLITUDES ↔ WILSON LOOPS

Self-duality of supercoset sigma models proved ONLY in a (partially) fixed kappa-simmetry gauge: $\xi = 0 \quad (g = e^{xP+\theta Q} e^{B} e^{\xi S})$

Berkovits-Maldacena, 0807.3196

Beisert-Ricci-Tseytlin-Wolf, 0807.3228

Adam-Dekel-Oz, 0902.3805; Dekel-Oz, 1101.0400

In general, it cannot be extended to give self–duality of a superstring model for the following reasons:

- When kappa–symmetry is used to set non–susy fermions to zero, we cannot use it anymore to remove some coset fermions ($\xi = 0$).

- We would like to investigate b/f T–duality in cosets with rank(k-symmetry) = 0. No way to set $\xi = 0$.

- $\frac{PSU(1,1|2)\times PSU(1,1|2)}{SU(1,1)\times SU(2)}$ supercoset plus four extra T^4 coordinates describes a kappa–symmetry fixed superstring model on $AdS_3 \times S^3 \times T^4$ with the non–susy fermions $\upsilon = 0$. This gauge–fixing is not always compatible with all string solutions (Ex: strings only in $AdS_3 \times S^3$, Rughoonauth-Sundin-Wulff, 1204.4742)

- $\frac{PSU(1,1|2)}{SO(1,1)\times U(1)}$ supercoset plus six extra T^6 coordinates does NOT describe a superstring model, as it contains only 8 fermions. Non–susy fermions υ (at least 8 of them) need to be included.

Two directions of generalization of previous studies

1. Investigate self T–duality of supercosets without setting $\xi = 0$ (no k–symmetry gauge fixing)

2. Apply b/f T–duality to the complete superstring action (coset action PLUS extra bosonic coordinates PLUS extra non–susy fermions)

RESULTS I: Self–duality of Supercosets

We have considered $\frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}$ superstring sigma model with

$$F_{01234} = -F_{56789} = 4$$

and NO k-symmetry gauge fixing

We have proved self–duality under 4 bosonic and 8 fermionic T–dualities. Dual action = supercoset action with

$$g = \mathrm{e}^{x^m P_m + \theta^\alpha Q_\alpha}\, \mathrm{e}^B\, \mathrm{e}^{\xi^\alpha S_\alpha}$$

$$\downarrow$$

$$\tilde{g} = \mathrm{e}^{\tilde{x}^n K_n - \mathrm{i}\tilde{\theta}\Gamma^4 S}\, \mathrm{e}^B\, \mathrm{e}^{-(\xi Q + S_\alpha \mathcal{S}^\alpha{}_\beta \xi^\beta)(1-\frac{1}{4}\mathcal{M}_{\hat{a}\hat{b}}\mathcal{M}^{\hat{a}\hat{b}})^{-1}}$$

$\mathcal{S}^\alpha{}_\beta = \frac{\mathrm{i}}{4}\xi^\alpha(\xi\Gamma^4)_\beta + \frac{\mathrm{i}}{4}(\Gamma^4\Gamma_{\hat{a}}\xi)^\alpha(\xi\Gamma^{\hat{a}})_\beta + \frac{\mathrm{i}}{8}(\Gamma_{mn}\xi)^\alpha(\xi\Gamma^{mn}\Gamma_4)_\beta - \frac{\mathrm{i}}{8}(\Gamma_{\hat{a}\hat{b}}\xi)^\alpha(\xi\Gamma^{\hat{a}\hat{b}}\Gamma^4)_\beta$

$\mathcal{M}_{\hat{a}\hat{b}} = \mathrm{i}\xi\Gamma_{\hat{a}\hat{b}}\Gamma_4\xi$

By dimensional truncation of the algebra it is easy to derive self–duality of $\frac{PSU(1,1|2)\times PSU(1,1|2)}{SU(1,1)\times SU(2)}$ and $\frac{PSU(1,1|2)}{SO(1,1)\times U(1)}$ supercosets.

- $\mathrm{AdS}_3 \times S^3$ coset with F_5 RR flux $\rightarrow$ self–dual under

  2 bosonic and 4 fermionic T–dualities

- $\mathrm{AdS}_2 \times S^2$ coset with F_5 RR flux $\rightarrow$ dual to $\mathrm{AdS}_2 \times S^2$ coset with F_2, F_4 RR flux under

  1 bosonic and 2 fermionic T–dualities

- Generalization to $\mathrm{AdS}_3 \times S^3 \times S^3$ and $\mathrm{AdS}_2 \times S^2 \times S^2$. Self–duality works, as well.

- By T-dualizing along torus directions we generate other kind of backgrounds (different flux content). Self–duality is not affected.

RESULTS II: Self–duality of Superstring models

Superstring sigma–model actions in $\mathrm{AdS}_3 \times S^3 \times T^4$ and $\mathrm{AdS}_2 \times S^2 \times T^6$ include non–susy fermions.

Start from the GS action written in terms of supervielbeins $\mathcal{E}^A(X, \vartheta, \upsilon)$ and NSNS $B_2(X, \vartheta, \upsilon)$ form. Expand perturbatively in powers of υ. (Wulff, 1304.6422)

We have studied self–duality up to order–2 in υ.

- $\mathrm{AdS}_3 \times S^3 \times T^4$ self–dual under 2b + 4f + 2 T^4 coordinates
- $\mathrm{AdS}_2 \times S^2 \times T^6$ self–dual under 1b + 2f + 3 T^6 coordinates

CONCLUSIONS AND PERSPECTIVES

We have studied self–duality of GS superstring actions up to second order in the non–susy fermions and no k-symmetry gauge fixing.
Results consistent with the classical integrability of the models.
What's next:

- Reconsider $\mathrm{AdS}_4 \times \mathrm{CP}^3$ case (role of the non-susy fermions)

$$\frac{\mathrm{Osp}(6|4)}{SO(1,3) \times U(3)} = \mathrm{AdS}_4 \times CP^3 \; + \; 24 \text{ fermionic dir.}$$

- Backgrounds with $H \neq 0$ or $\mathrm{AdS}_d \times S^{d'}$
- B/f T-duality on the Lax pair
- TsT dualities which produce less susy theories commutes with b/f T-duality?
- Dualities which produce N(A)C theories commutes with b/f T-duality?

$$\frac{D(2,1;\alpha)\times D(2,1;\alpha)}{SO(1,2)\times SO(3)\times SO(3)} \;=\; \mathrm{AdS}_3\times S^3\times S^3 \;+\; 16 \text{ fermionic dir.}$$

$$\frac{D(2,1;\alpha)}{SO(1,1)\times SO(2)\times SO(2)} \;=\; \mathrm{AdS}_2\times S^2\times S^2 \;+\; 8 \text{ fermionic dir.}$$