Sestri Levante, 17/9/2015

Boost-invariant flow of non-conformal plasmas

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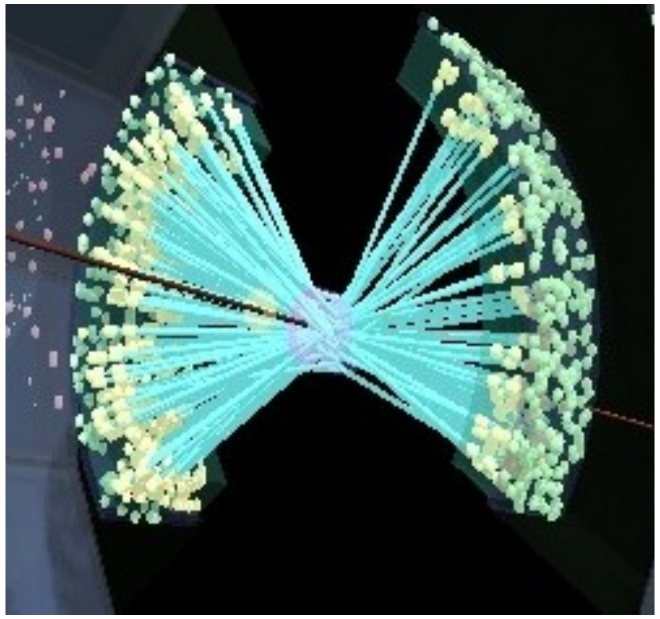
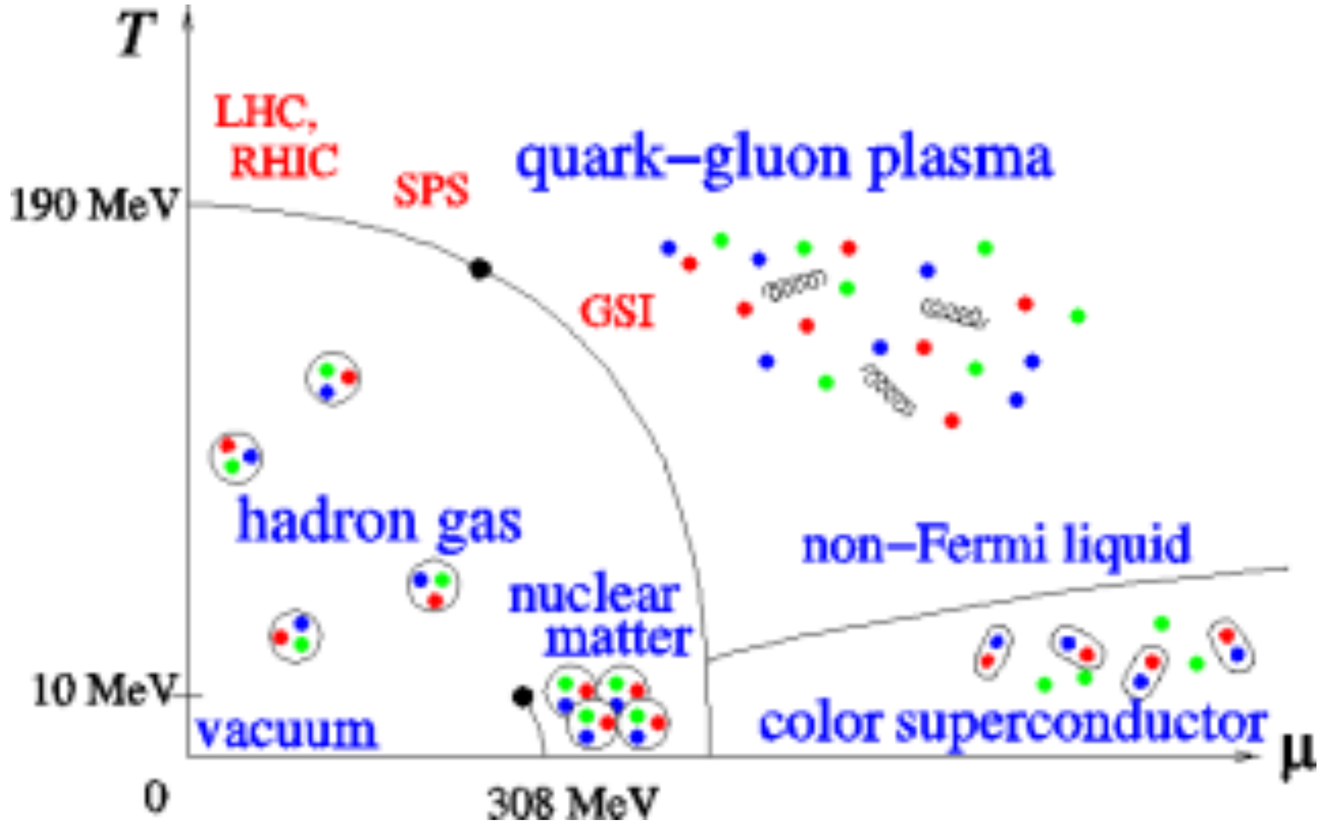
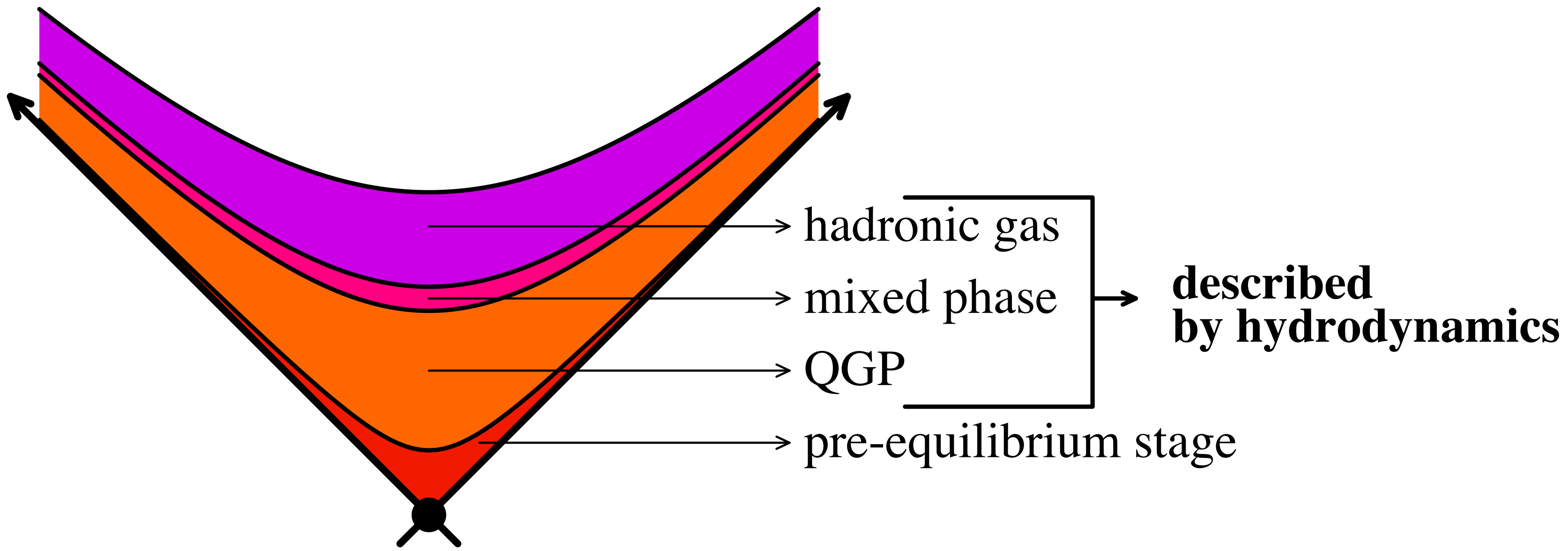
Outline

- Quark-gluon plasma and hydro
- Bjorken flow
- AdS dual of Bjorken flow
- Non-AdS dual of Bjorken flow
- Conclusions




T
LHC,
RHIC
SPS
quark–gluon plasma
190 MeV
GSI
hadron gas
non–Fermi liquid
nuclear
matter
10 MeV
vacuum
color superconductor
μ
0
308 MeV

Conventional picture of QGP dynamics



[Heller, Janik, Peschanski]

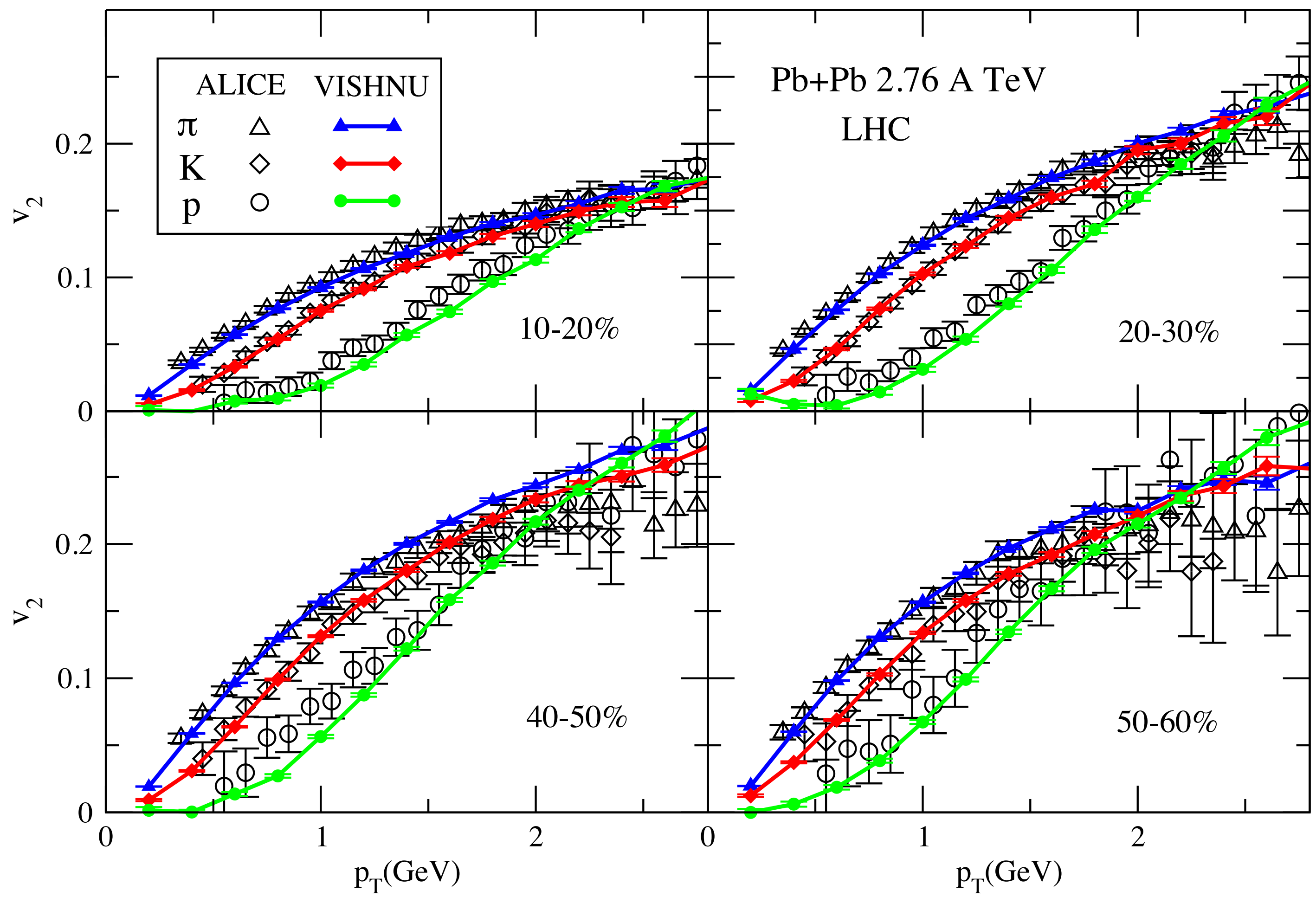
Early stages: Glauber, CGC, problem of inital conditions

Middle: low-viscosity hydrodynamics

Late: hadronization

Evidence for QGP phase: elliptic flow

$$v_2 = \langle cos(2\phi) \rangle$$



$$\frac{\eta}{s} = \frac{2}{4\pi}$$

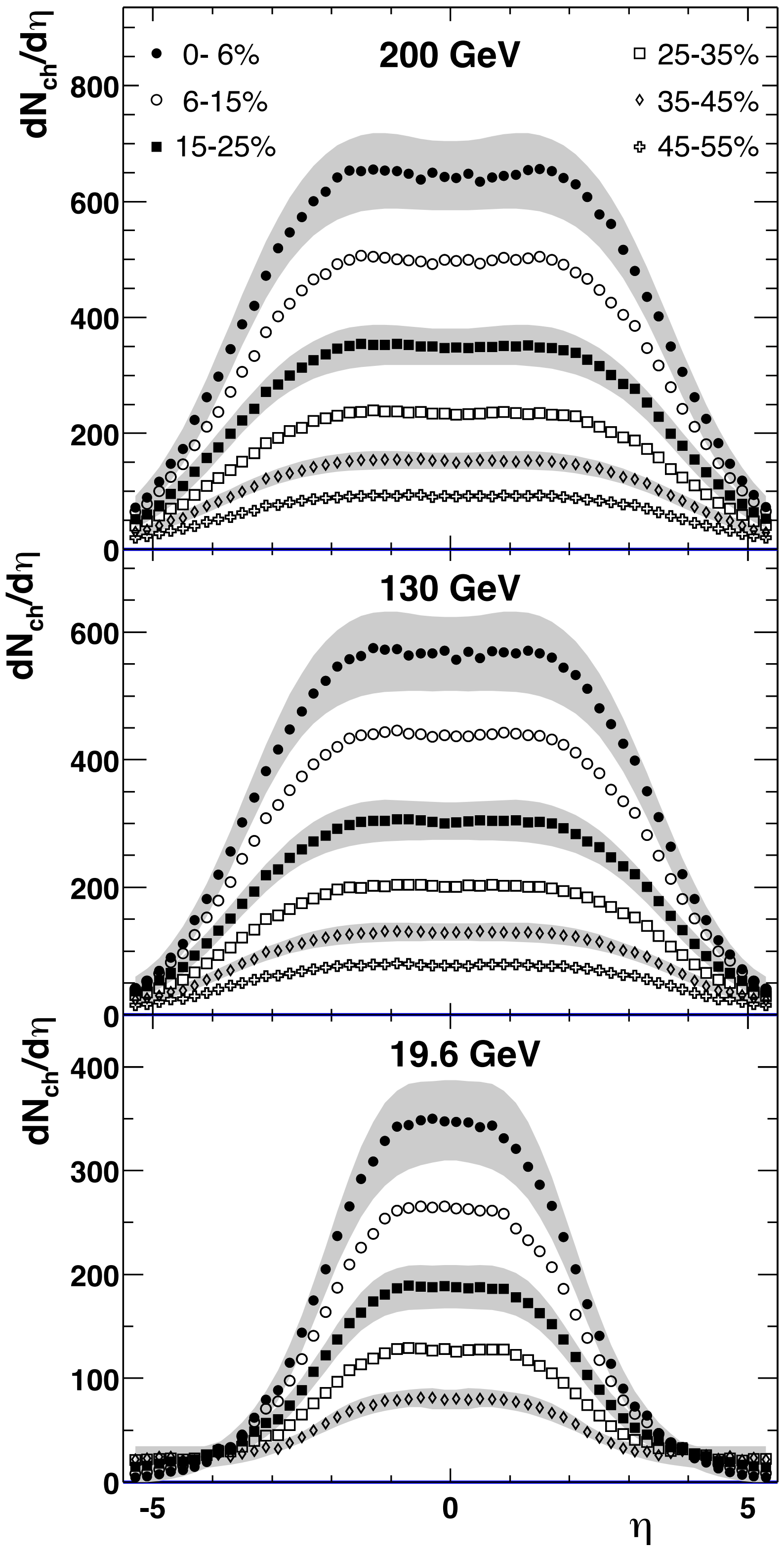
[arXiv:1311.0157]

This scenario requires fast thermalization ~ 1 fm

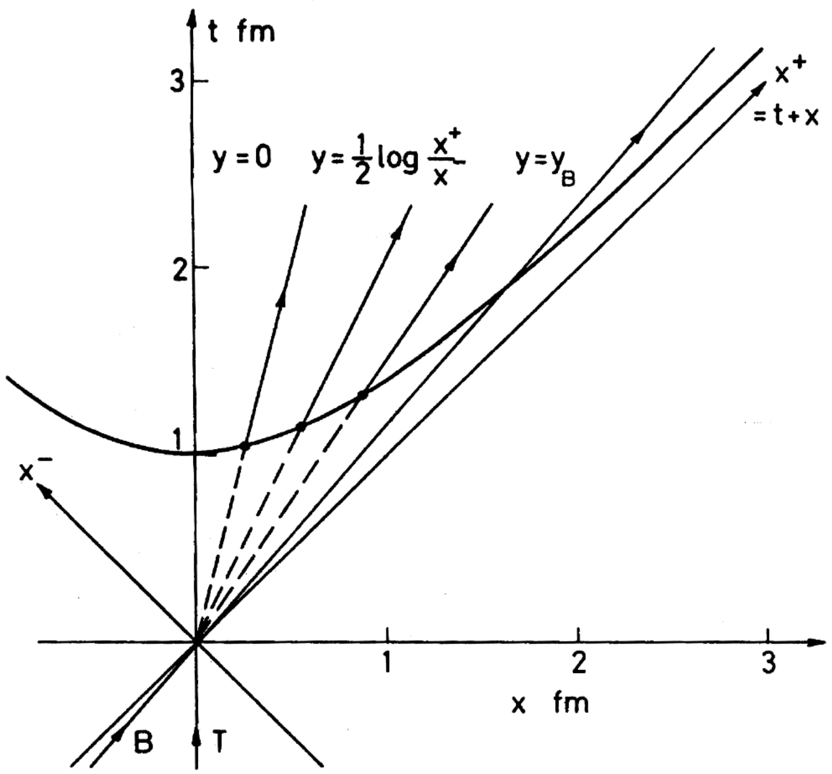
Recent simulations cast some doubt and allow for slower build-up of the flow

Plateau of particle production
in the central rapidity region

[nucl-ex/0210015]



Bjorken’s mechanism



assumption: $y = \eta$

$$y = \frac{1}{2}\log\left(\frac{E+p}{E-p}\right) \;\; ; \;\; \eta = \frac{1}{2}\log\left(\frac{x_0+x_1}{x_0-x_1}\right)$$

$$\tau = \sqrt{(x_0)^2 - (x_1)^2}$$

The boost-invariance (y-independence) of the fluid translates into the η - independence of the hadron distribution

Bjorken flow in CFT

In proper time coordinates $ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$

Tracelessness and conservation of $T_{\mu\nu}$

$$-T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} + 2T_{xx} = 0$$

$$\tau\frac{d}{d\tau}T_{\tau\tau} + T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} = 0$$

$$\Downarrow$$

$$T_{\mu\nu} = \begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3\frac{d}{d\tau}f(\tau) - \tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau) + \frac{1}{2}\tau\frac{d}{d\tau}f(\tau) & 0 \\ 0 & 0 & 0 & f(\tau) + \frac{1}{2}\tau\frac{d}{d\tau}f(\tau) \end{pmatrix}$$

Positive energy $-\frac{4}{\tau} \leq \frac{f'}{f} \leq 0$ $f \sim \tau^{-\alpha}\,, \quad 0 < \alpha < 4$

Perfect conformal fluid $T_{\mu\nu} = (\epsilon + p)u_\mu u_\mu + p\eta_{\mu\nu}$ $p = \frac{\epsilon}{3}$

$\alpha = \frac{4}{3}$ $T \sim \tau^{-1/3}$

Free-streaming fluid $p_L = 0$ $\alpha = 1$

(weak-coupling phase)

Viscous conformal fluid $T_{\mu\nu} = (\epsilon + p)u_\mu u_\mu + p\,\eta_{\mu\nu} + \eta\,\partial_{\langle\mu}u_{\nu\rangle}$

$f(\tau) \sim \frac{1}{\tau^{4/3}}(1 - \frac{2\eta_0}{\tau^{2/3}})$ $\eta_0 = \frac{\eta}{\tau}$

Gravity dual of conformal Bjorken flow [Janik, Peschanski]

Most general Ansatz with the symmetries of the flow in FG gauge

$$ds^2 = \frac{dz^2}{z^2} + \frac{-e^{a(\tau,z)}d\tau^2 + \tau^2 e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2}{z^2}$$

Assuming a scaling form $\quad v = \dfrac{z}{\tau^{s/4}}$

Einstein's eqs allow a late-time expansion

$$\begin{aligned} a(\tau,z) &= a(v) + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right) \\ b(\tau,z) &= b(v) + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right) \\ c(\tau,z) &= c(v) + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right) \end{aligned}$$

Late-time equations

$$v(2a'(v)c'(v)+a'(v)b'(v)+2b'(v)c'(v))-6a'(v)-6b'(v)-12c'(v)+vc'(v)^2=0$$

$$3vc'(v)^2+vb'(v)^2+2vb''(v)+4vc''(v)-6b'(v)-12c'(v)+2vb'(v)c'(v)=0$$

$$2vsb''(v)+2sb'(v)+8a'(v)-vsa'(v)b'(v)-8b'(v)+vsb'(v)^2+$$
$$4vsc''(v)+4sc'(v)-2vsa'(v)c'(v)+2vsc'(v)^2=0\ .$$

Constraint

$$(4-3s)a(v)+(s-4)b(v)+2sc(v)=0$$

Basis of solutions

$$\begin{aligned} a(v) &= A(v)-2m(v) \\ b(v) &= A(v)+(2s-2)m(v) \\ c(v) &= A(v)+(2-s)m(v) \end{aligned}$$

$$\begin{aligned} A(v) &= \frac{1}{2}\left(\log(1+\Delta(s)\,v^4)+\log(1-\Delta(s)\,v^4)\right) \\ m(v) &= \frac{1}{4\Delta(s)}\left(\log(1+\Delta(s)\,v^4)-\log(1-\Delta(s)\,v^4)\right) \end{aligned}$$

$$\Delta(s)=\sqrt{\frac{3s^2-8s+8}{24}}\ .$$

$$A\sim v^8\,,\ m\sim v^4$$

Potentially singular geometry at $v^4 = \Delta(s)$

$$\mathrm{Riem}^2 = \frac{P(v,s)}{(1-\Delta(s)^2 v^8)^4}$$

$P(v,s)$ cancels the pole for $s = \frac{4}{3}$

The geometry becomes a black hole with a time-dependent horizon

$$z_h \sim z_0 \tau^{1/3}$$

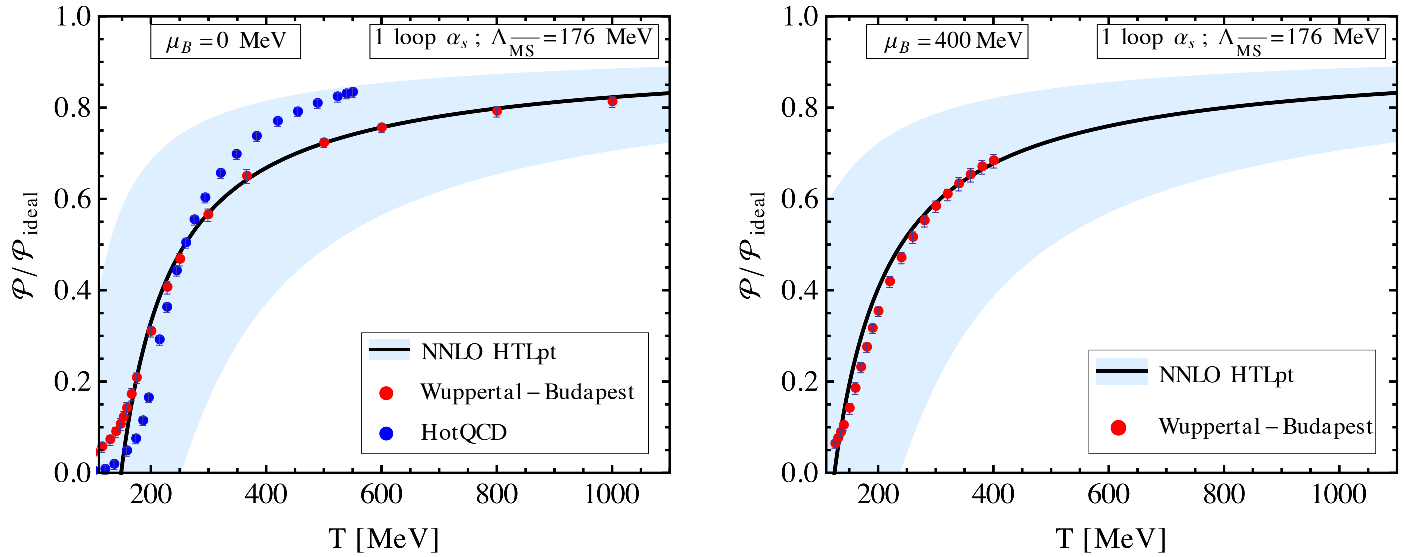
At higher orders $a(z,\tau) = a_0(v) + \frac{1}{\tau^r} a_r(v) + \frac{1}{\tau^{2r}} a_{2r}(v) + \frac{1}{\tau^{\frac{4}{3}}} a_2(v) + \dots$

Regularity implies $r = \frac{2}{3}$ $\frac{\eta}{s} = \frac{1}{4\pi}$

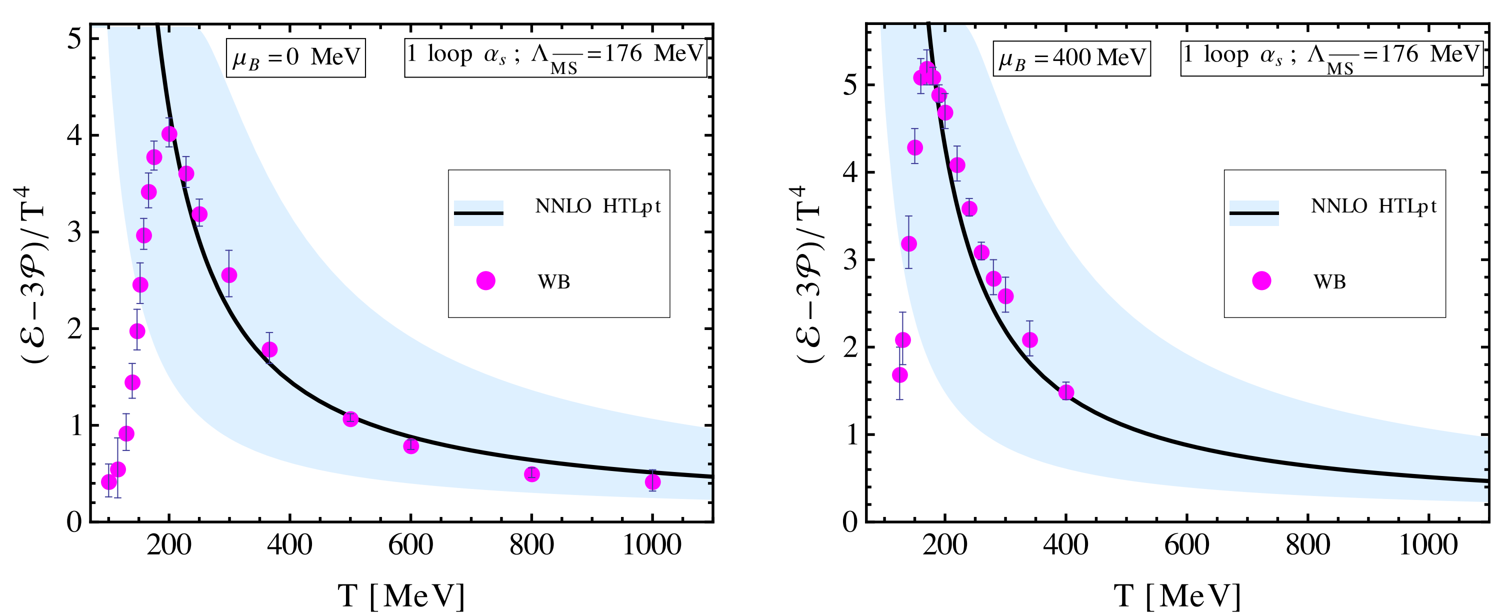
Deviation from conformality

[arXiv:1402.6907]

pressure

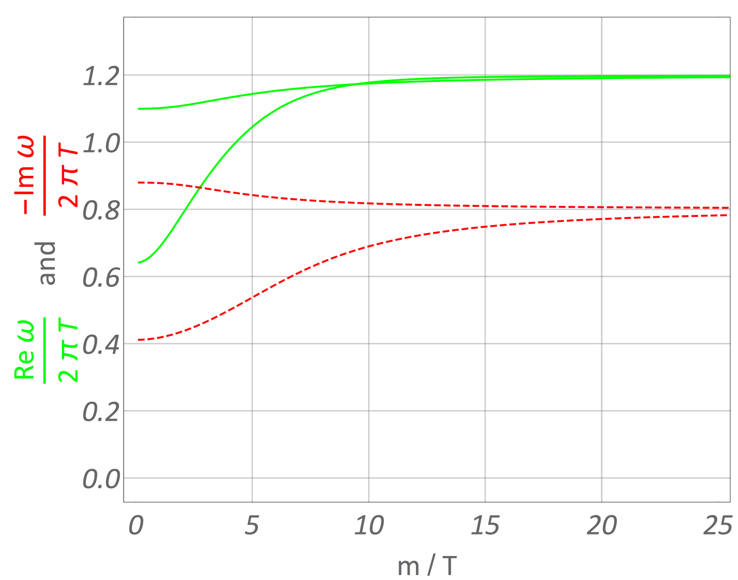


trace anomaly

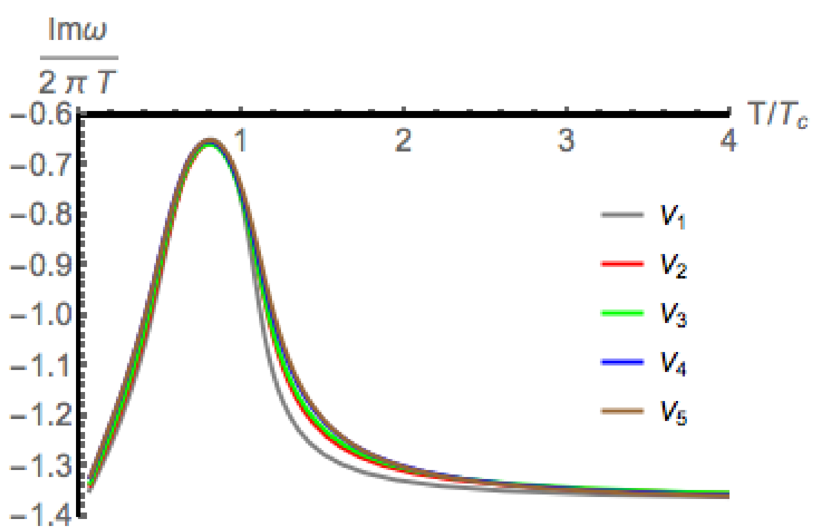


The study of deviations from the conformal behavior in the QGP dynamics has started only recently

[Buchel, Heller, Myers][Janik, Plewa, Soltanpanahi, Spalinski] consider the equilibration rate determined by lowest quasi-normal modes in non-conformal theories



N=2*

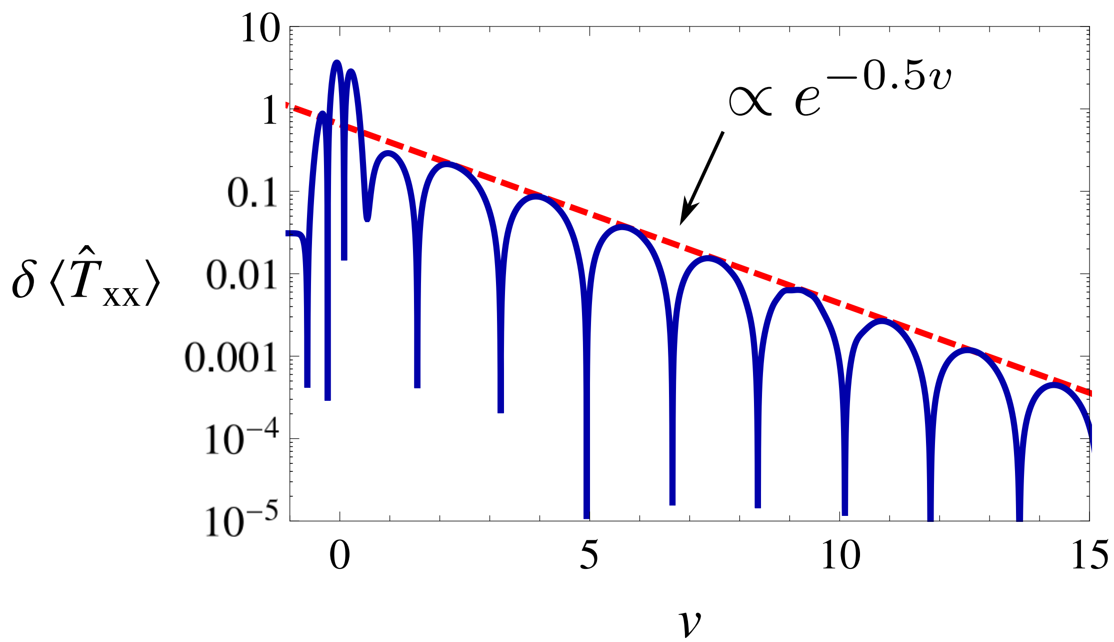


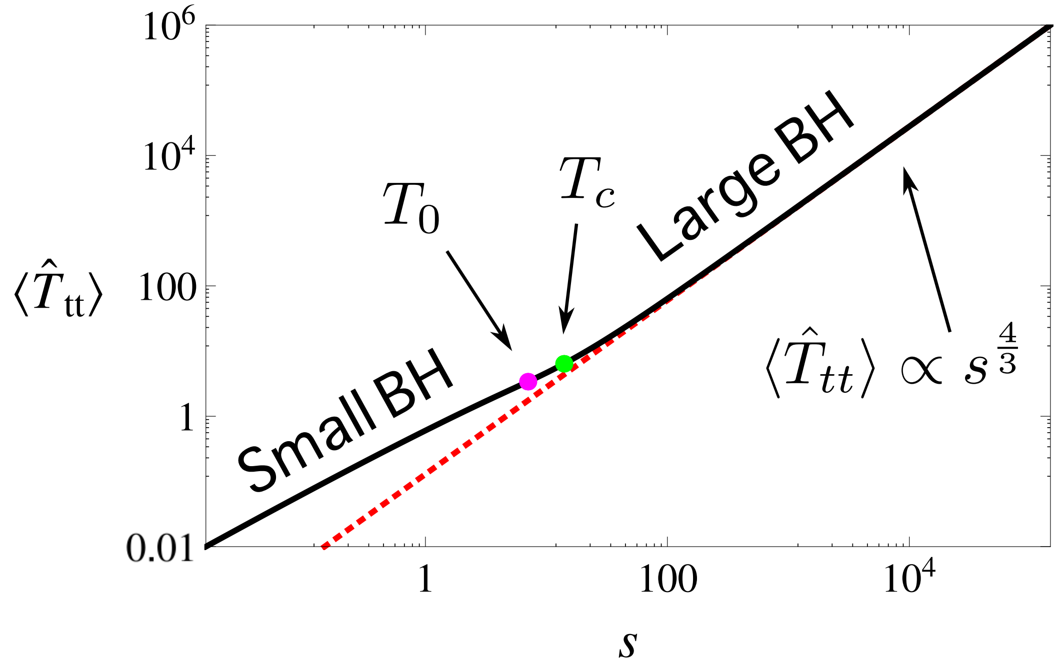
Einstein-scalar with

$$V(\phi) = cosh(\phi) + \phi^2 + \phi^4 + \phi^6$$

Variation of the imaginary part = attenuation rate by factor of ~ 2

[Ishii, Kiritsis, Rosen] consider thermalization after a quench and are mostly interested in the dependence on the quench parameters





Bottom-up non-conformal models [Gursoy, Kiritsis…]

Einstein-dilaton gravity

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^5x\sqrt{-g}\left(R - \frac{4}{3}(\partial\varphi)^2 + V(\varphi)\right) - \frac{1}{\kappa^2}\int_\partial \mathrm{d}^4x\sqrt{-\gamma}\,\mathcal{K}$$

The potential can be tuned to reproduce the beta-function

For asymptotically AdS UV $\quad V = V_0 + v_1\lambda + v_2\lambda^2 + \ldots$

For confinement in the IR $\quad V \sim \lambda^Q(\log\lambda)^P$

$$Q > 4/3 \,\mathrm{or}\, Q = 4/3\,,\, P \geq 0$$

Confinement $\Leftrightarrow$ finite-T transition between thermal gas and BH

We consider a simple setup with an exponential potential

$$V = V_0(1 - X^2)e^{-\frac{8}{3}X\,\phi}. \qquad X < 0 \quad \text{(confining for} \quad X < -\frac{1}{2}\text{)}$$

For $X > -\frac{1}{2}$ analytic BH solution [Chamblin,Reall]

$$ds^2 = e^{2A(u)}\left(-f(u)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{du^2}{f(u)}$$

$$e^A = e^{A_0}\lambda^{\frac{1}{3X}} \qquad f = 1 - C_2\lambda^{-\frac{4(1-X^2)}{3X}}$$

$$\lambda \equiv e^{\phi} = \left(C_1 - 4X^2\frac{u}{\ell}\right)^{\frac{3}{4X}}$$

Thermodynamics

$$\beta = \pi\ell\frac{e^{-A_0}C_2^{-\frac{\frac{1}{4}-X^2}{1-X^2}}}{1 - X^2}$$

for $X < -\frac{1}{2}$
negative specific heat

$$-T^{\mu}_{\mu} = E + 3F = 3c_s\frac{X^2}{1 - X^2}\left(T\ell\right)^{\frac{4(1-X^2)}{1-4X^2}}$$

Boost-invariant CR flow

Trace condition $-T_{\tau\tau} + \frac{1}{\tau^2} T_{yy} + 2T_{xx} = -c\, T^{\xi}$ $\xi = \frac{4(1-X^2)}{1-4X^2}$

$$T_{\mu\nu} = \mathrm{d}iag\left(\epsilon(\tau),\ -\tau^3\partial_\tau\epsilon - \tau^2\epsilon,\ \epsilon + \frac{\tau}{2}\partial_\tau\epsilon - \frac{c}{2}T^{\xi}, \epsilon + \frac{\tau}{2}\partial_\tau\epsilon - \frac{c}{2}T^{\xi}\right)$$

Assuming $T = T_0\tau^{-\alpha}$ the energy is determined

$$\epsilon(\tau) = \epsilon_0\tau^{-\frac{4}{3}} + \frac{c\, T_0^{\xi}}{4 - 3\alpha\xi}\tau^{-\alpha\xi}$$

If $\alpha\xi < \frac{4}{3}$ the trace anomaly determines the late time behaviour

Ansatz for metric and dilaton

$$ds^2 = z^{-\frac{2}{1-4X^2}} \left(dz^2 - e^{a(v)}d\tau^2 + e^{b(v)}\tau^2 dy^2 + e^{c(v)}dx_\perp^2\right)$$

$$\lambda = z^{-\frac{3X}{1-4X^2}} e^{\lambda_1(v)} \qquad\qquad v = \frac{z}{\tau^{s/4}}$$

Complicated system of equations for late time…

Basis of solutions

$$\begin{aligned}
a(v) &= A(v) - 2\left(1-4X^2\right)m(v) + 2Xn(v)\\
b(v) &= A(v) + 2\left(s-1+4X^2\right)m(v) + 2Xn(v)\\
c(v) &= A(v) - \left(s-2+8X^2\right)m(v) - 2Xn(v)\\
\lambda_1(v) &= \frac{3}{2}XA(v) + X\left(1-4X^2\right)m(v) + \left(1-X^2\right)n(v)
\end{aligned}$$

The equations decouple

$$A(w) = \frac{2}{\chi}w - \frac{1}{2}\log m'(w) + \text{const.}\,, \qquad n(w) = \kappa\, m(w) + \text{const.}$$

$$w = \log v\,, \qquad \chi = \frac{1-4X^2}{1-X^2}$$

The remaining equation for $m(v)$ can be integrated and a closed form can be found for $v(m)$

Using m as radial coordinate yields a simple form for the metric

$$
\begin{aligned}
ds^2 \simeq \tau^{-\frac{s}{2(1-4X^2)}} \Bigg\{ & \tau^{s/2} \left(\frac{S\chi}{2}\right)^2 \left(e^{2Sm}-1\right)^{-\frac{2}{1-X^2}} e^{\frac{2S+2K}{1-X^2}m} dm^2 \\
& + \left(e^{2Sm}-1\right)^{-\frac{1}{2(1-X^2)}} \Bigg[- e^{\frac{S+4K}{2(1-X^2)}m} e^{-2\chi m} d\tau^2 \\
& + \tau^2 e^{\frac{S+4K}{2(1-X^2)}m} e^{2(s-\chi)m} dy^2 + e^{\frac{S-2K}{2(1-X^2)}m} e^{(2\chi-s)m} dx_\perp^2 \Bigg] \Bigg\},
\end{aligned}
$$

S, K constants depending on s, κ

IR regularity at $m \to \infty$ requires

$$\boxed{s = \frac{4}{3}\left(1-4X^2\right)}, \qquad \kappa = 0$$

For these values the metric is that of a BH with a moving horizon

The dual stress-energy tensor can be obtained by holographic renormalization in 5d, or more easily lifting the solution by a generalized dimensional reduction

$$S = \frac{1}{16\pi \tilde{G}_N} \int d^{d+1}x \, d^{2\sigma - d}y \, \sqrt{-\tilde{g}} \left(\tilde{R} - 2\Lambda \right)$$

Reducing on $\mathbb{R}^{d+1} \times T^{2\sigma - d}$ $\qquad \widetilde{ds}^2 = e^{-\delta_1 \phi(x)} dx^2 + e^{\delta_2 \phi(x)} dy^2$

$$\delta_1 = \frac{4\sqrt{2\sigma - d}}{\sqrt{3(d-1)(2\sigma - 1)}}, \quad \delta_2 = \frac{4\sqrt{d-1}}{\sqrt{3(2\sigma - 1)(2\sigma - d)}} \qquad 2\sigma - d = \frac{4(d-1)^2 x^2}{3 - 4(d-1)x^2}$$

The uplifted metric is AAdS $\qquad \langle T^{\mu\nu} \rangle_{2\sigma} = \frac{2\sigma l^{2\sigma - 1}}{16\pi \tilde{G}_N} \tilde{g}^{\mu\nu}_{(2\sigma)}$

$T_{\mu\nu}$ consistent with perfect fluid and $\qquad \epsilon(\tau) \sim \tau^{-\frac{4}{3}(1 - X^2)}$

leading w.r.t. the conformal form

The solution appears singular for X=-1/2 but it can be removed by a rescaling

$$\ell' = \frac{1}{1-4X^2} \qquad q = \hat{v}^{\frac{1}{1-4X^2}}$$

$$ds^2 = \frac{\tau^{-\frac{8}{3}X^2} q^{-2(1+4X^2)}}{1-q^{4(1-X^2)}} dq^2 + \tau^{-\frac{2}{3}} q^{-2} \left[-\left(1-q^{4(1-X^2)}\right) d\tau^2 + \tau^2 dy^2 + dx_\perp^2 \right]$$

$$T_{\mu\nu} = t_0(1-4x^2)\tau^{-\frac{4}{3}(1-x^2)}\, diag(-\frac{3}{1-4x^2}, \tau^2, 1, 1)$$

Pressureless gas in the limit X = -1/2

In this form it can be continued to -1< X < - 1/2

However the solution is unphysical, the temperature grows with time and it corresponds to moving on the small (unstable) BH branch

At $X=-\frac{1}{2}$ it is necessary to take into account the subleading behaviour of the potential

$$V \sim e^{-8/3X\phi}\,\phi^P$$

An analytic solution is not available, need to work in an expansion in $1/\phi$

$$ds^2 = e^{2A(\phi)}\left(-f(\phi)dt^2 + \delta_{ij}dx^i dx^j\right) + e^{2B(\phi)}\frac{d\phi^2}{f(\phi)}$$

$$\begin{aligned} e^{2A(\phi)} &= e^{2A_0}e^{2\phi/(3X)}\phi^{P/(4X^2)} \\ e^{2B(\phi)} &= e^{2B_0}e^{\frac{8}{3}X\phi}\phi^{-P} \qquad +\mathcal{O}\left(\frac{1}{\phi}\right) \\ f(\phi) &= 1-e^{a(\phi_h-\phi)}(\phi_h/\phi)^b \end{aligned}$$

$$a=\frac{4(1-X^2)}{3X}, \qquad b=\frac{P(1+X^2)}{2X^2}$$

General solution not available

A working Ansatz is $$A(\phi) \to A(\phi)\,, \quad B(\phi) \to B(\phi)$$

$$f(\phi) \to f(\phi + a_1 \log\tau + a_2 \log\log\tau, 1/\log\tau)$$

$$a_1 = X\,, \quad a_2 = \frac{3P}{8X}$$

Holographic EM tensor
$$\epsilon \sim \frac{3f_0}{2(1-4x^2)}\phi^{\frac{P}{2}(1+\frac{1}{x^2})}\tau^{-\frac{1}{3}(1-4x^2)}(\log\tau)^{\frac{P}{2}(1-\frac{1}{x^2})}\,,$$
$$p \sim \frac{1}{3}(1-4x^2)\,\epsilon\,.$$

In the late-time scaling limit $$\epsilon \sim \tau^{-\frac{1}{3}(1-4X^2)}(\log\tau)^P$$

consistent with thermodynamics

Conclusions

We found an analyitic solution describing the late-time behavior of a class of non-conformal theories

Our results indicate that the deviation from conformality results in a slower relaxation to equilibrium, slightly different than results from quasi-normal modes

The relaxation stops at the critical case $X = -\frac{1}{2}$ separating confining from non-confining theories, beyond this a new Ansatz may be needed, perhaps describing relaxation towards the critical temperature

Extensions

- Higher-order terms and viscosity
- Early-time dynamics
- Corrections to boost-invariance and isotropy
- Models with three potentials (confining, analytic BH solutions known but with T-dependent parameters
- Matching with AdS UV
- Relation with resummation of hydrodynamic series

  [Heller, Janik, Witaszczyk]

Thank you