Open String Field Theory and D-branes: Classical Solutions and Topological Defects

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1406.3021, JHEP 1410 (2014) 029 (w/ Ted Erler)
To Appear, (w/ Toshiko Kojita, Toru Masuda and Martin Schnabl)

related works
1506.03723, JHEP 1508 (2015) 149 (w/ Martin Schnabl)
1402.3546, JHEP 1405 (2014) 004
1207.4785, JHEP 1307 (2013) 033 (w/ Matej Kudrna and Martin Schnabl)
1201.5122, JHEP 1206 (2012) 084 (w/ Ted Erler)
1201.5119, JHEP 1204 (2012) 107 (w/ Ted Erler)

Physics on the Riviera, Sestri Levante
16/09/2015
Open String Field Theory (OSFT) is a microscopic theory for D-branes, formulated as a field theoretic description of open strings.

Helpful analogy:

- OSFT
- Open strings
- D-branes
- Closed strings

- Yang-Mills
- Gauge fields
- Saddle points (instantons,...)

Gauge/Gravity?
OPEN STRING FIELD THEORY

• Fix a bulk CFT (closed string background)
• Fix a reference BCFT_0 (open string background, D-brane’s system)
• The string field is a state in BCFT_0

\[ |\psi\rangle = \sum_i t_i \psi^i(0)|0\rangle_{SL(2,R)} \]

• There is a non-degenerate inner product (BPZ)

\[ \langle \psi, \phi \rangle = \langle \psi(-1)\phi(1) \rangle_{\text{Disk}}^{\text{BCFT}_0} \]

• The bpz-inner product allows to write a target-space action

\[ S[\psi] = -\frac{1}{2} \langle \psi, Q\psi \rangle_{\text{BCFT}_0} - \frac{1}{3} \langle \psi, \psi * \psi \rangle_{\text{BCFT}_0} = S_{\text{eff}}[t_i] \]

• Witten product *: associative product between states (OPE+conf. map)

• Equation of motion

\[ Q\Psi + \Psi * \Psi = 0 \]
OSFT CONJECTURE (once known as Sen’s Conjectures)

• Key tool for connecting the two sets is the OSFT construction of the boundary state (Kiermaier, Okawa, Zwiebach (2008), Kudrna, CM, Schnabl (2012))

• The (KMS) boundary state is constructed from gauge invariant quantities starting from a given solution

\[ Q\Psi_\ast + \Psi_\ast^2 = 0 \quad \Rightarrow \quad |B_\ast\rangle = \sum_\alpha n_\ast^\alpha |V_\alpha\rangle \]

\[ n_\ast^\alpha = \langle V^\alpha | B_\ast \rangle = \langle V^\alpha(0) \rangle_{\text{BCFT}_\ast}^{\text{disk}} = W_{V^\alpha}[\Psi_\ast - \Psi_{\text{tv}}] \]

• Intriguing possibility of relating BCFT consistency conditions (Cardy-Lewellen, Pradisi-Sagnotti-Stanev) with OSFT equation of motion!
SOLUTION FOR ANY BACKGROUND

Erler, CM (2014)

- A change in boundary conditions is encoded in a bcc operator (*Cardy, ’86-’89*)

- OSFT: describe the dof of a target BCFT* using the dof of a reference BCFT₀

\[
\phi_*(0)|0>* \rightarrow \sigma(1)\phi_*(0)\bar{\sigma}(-1)|0>₀ = \sum_i t_i \phi_0^i(0)|0>_₀
\]

Idea back from Vacuum SFT (*Rastelli, Sen, Zwiebach, 2000*)
• Connect two generic backgrounds by passing through the **tachyon vacuum** (*simplest universal solution: no D-branes*)

\[
\Psi = \Psi^0_{tv} - \Sigma \Psi^*_{tv} \overline{\Sigma}
\]

\[
\Sigma \in \mathcal{H}_{0*}
\]

\[
\overline{\Sigma} \in \mathcal{H}_{*0}
\]

\[
Q\Psi + \Psi^2 = 0
\]

\[
Q_{tv} \overline{\Sigma} = Q_{tv} \Sigma = 0
\]
• The Sigma’s can be constructed due to the trivial cohomology at the Tachyon Vacuum, using bcc’s

\[ Q_{tv} A \equiv QA + [\Psi_{tv}, A] = 1 \quad \text{No open strings at TV} \]

\[ Q_{tv} \Sigma = Q_{tv} \bar{\Sigma} = 0 \quad \rightarrow \quad \Sigma = Q_{tv}(A\sigma) \]

\[ \Sigma \bar{\Sigma} = Q_{tv}(A\bar{\sigma}\sigma) = 1 \quad \text{IF} \quad \bar{\sigma}\sigma = 1 \]

• Convenient universal choice

\[ \sigma = e^{i\sqrt{h}X^0}(c=25) \quad \sigma_* \]

\[ \bar{\sigma} = e^{-i\sqrt{h}X^0}(c=25) \bar{\sigma}_* \quad \text{Explicitly possible for time independent backgrounds! (this adds a pure gauge time-like Wilson line) other possible constructions??} \]
Background independence

- Action for fluctuations

\[
S[\Psi + \phi] = \frac{1}{2\pi^2} (g_0 - g_*) - \frac{1}{2} \text{Tr}[\phi Q_\Psi \phi] - \frac{1}{3} \text{Tr}[\phi^3]
\]

- Consider the peculiar BCFT\(_0\) states

\[
\phi = \sum \phi_\ast \overline{\Sigma}, \quad \phi_\ast \in \text{Fock}_{\text{BCFT}_\ast}
\]

- Remarkably

\[
Q_\Psi \left( \sum \phi_\ast \overline{\Sigma} \right) = \sum (Q \phi_\ast) \overline{\Sigma}
\]

- And we get the theory DIRECTLY formulated in BCFT\(^*\)!

\[
S[\Psi + \phi] = \frac{1}{2\pi^2} (g_0 - g_*) - \frac{1}{2} \text{Tr}[\phi_\ast Q \phi_\ast] - \frac{1}{3} \text{Tr}[\phi^3_\ast]
\]

- BCFT\(^*\) can be composite (multibranes) and this construction gives rise to Chan-Paton’s factors, out of a single D-brane!
So we are now in a new phase for OSFT

- Tantalizing conjecture

**OSFT EOM implies BCFT constraints (bootstrap)**

- Can we find the most generic solution to OSFT?
- All known D-branes give rise to solutions, can we reverse the argument to DISCOVER new D-branes?
- Long-standing problem in CFT!
...As a first step in this challenge let’s see how to generate new solutions from known ones:

*Topological Defects in OSFT*
Open string defect operators in OSFT
(Kojita, Masuda, CM, Schnabl, 2015)

\[ \mathcal{D} : H_{\text{open}} \rightarrow H'_{\text{open}} \]

\[ [\mathcal{D}, Q] = 0 \]

\[ \mathcal{D}(\Psi_1 \ast \Psi_2) = (\mathcal{D}\Psi_1) \ast (\mathcal{D}\Psi_2) \]

- They map solutions to solutions

\[ Q\Psi + \Psi \ast \Psi = 0 \quad \rightarrow \quad Q(\mathcal{D}\Psi) + (\mathcal{D}\Psi) \ast (\mathcal{D}\Psi) = 0 \]

- Generalization of symmetries (which are group-like defects)
• An operator $D$ can be explicitly constructed starting from a **closed** topological defect line $D_{cl}$.

\[
[D_{cl}, T(z)] = [D_{cl}, \bar{T}(\bar{z})] = 0
\]

\[
D_{cl}^d = \sum_{i, \bar{i} \in H_{cl}} D_{\bar{i} i}^d P_{\bar{i} i}
\]  
*Petkova-Zuber (2000)*

\[
|\Psi\rangle_{\text{closed}}
\]

• In (diagonal) RCFT: as many fundamental defects as irreps. The fusion rules govern their composition and the action on boundary states

\[
[\phi_a] [\phi_b] = \sum_i N_{ab}^i [\phi_i]
\]

\[
D^d_i = \frac{S_{di}}{S_{1i}}
\]

\[
D^d D^c = \sum_i N_{cd}^i D^i
\]

\[
D^c |a\rangle = \sum_i N_{ca}^i |i\rangle
\]  
*Graham-Watts (2003)*
• In the open string sector we must have

\[ D^d \psi_i^{(ab)} = \sum_{a' \in d \times a} \sum_{b' \in d \times b} X_{i a' b'}^{d a b} \psi_i^{(a' b')} = \sum_{a' \in d \times a} \sum_{b' \in d \times b} X_{i a' b'} \]

• Determine \( X \) coeff. imposing the star algebra homomorphism

\[ D^d \left( \phi_i^{(ab)}(x) \phi_j^{(bc)}(y) \right) = \left( D^d \phi_i^{(ab)}(x) \right) \left( D^d \phi_j^{(bc)}(y) \right) \]

\[ X_{k a' c'}^{a b c} C_{i j}^{(abc) k} = \sum_{b' \in d \times b} C_{i j}^{(a' b' c') k} X_{k a' b'}^{d a b} X_{k b' c'}^{d b c}, \]

• In explicit case of diag. minimal models we find (pentagon identity)

\[ X_{i a' b'}^{d a b} = F_{d i} \left[ \begin{array}{c c} a & b \\ a' & b' \end{array} \right] \sqrt{ \frac{F_{1 a'}^{[a \ d]}}{F_{1 b'}^{[b \ d]}} } \frac{F_{1 i}^{[a \ b]}}{F_{1 i}^{[a \ b]}} \]

Generalizes
• The composition (fusion) is trickier than in the bulk case. Naively we would expect

\[ \mathcal{D}^d \mathcal{D}^c \psi = \bigoplus_e N_{dc}^e \mathcal{D}_e \psi \quad (?) \]

• However explicit computation reveals a similarity transformation!

\[ \mathcal{D}^d \mathcal{D}^c \psi = U_{dc} \left( \bigoplus_e N_{dc}^e \mathcal{D}_e \psi \right) U_{dc}^{-1} \]

\[ (U_{dc})^T = (U_{dc})^{-1} \]

\[ (U_{dc})^{\{aa'a''\} \{e; a, a''\}} = \sqrt{\frac{F_{1a''} \begin{bmatrix} d & a' \\ d & a' \end{bmatrix} F_{1a'} \begin{bmatrix} c & a \\ c & a \end{bmatrix}}{F_{1a''} \begin{bmatrix} e & a \\ e & a \end{bmatrix} F_{1e} \begin{bmatrix} d & c \\ d & c \end{bmatrix}} \cdot F_{a' e} \begin{bmatrix} d & c \\ a'' & a \end{bmatrix}} \]
• This becomes transparent using defect-network manipulations

\[
\sum_q F_{pq} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \sum_{a' \in d \times a} \phi_i \begin{bmatrix} a' & b' \end{bmatrix} = D^d \phi_i^{ab} 
\]

• Defect action

\[
D^d(\phi_i^{ab} \ast \phi_j^{bc}) = \sum_{e \in d \times c} F_{ie} \begin{bmatrix} d \mid e \end{bmatrix} = U_{dc} \left( \bigoplus_{e \in d \times c} D^e \phi_i^{ab} \right) U_{dc}^T
\]

• Important to keep track of defect junctions (from which the square roots of F originates), example:

\[
D^d(\phi_i^{ab} \ast (D^d \phi_j^{ab}))
\]

[Frolich, Fuchs, Runkel, Schwieger (2006)]

[These are defect networks, not (a priori) conformal blocks!]

\[
\phi_i \begin{bmatrix} a' & b' \end{bmatrix} = D^d \phi_i^{ab}
\]
OSFT observables and defects

- Change in the (off-shell) action

\[ S_{\text{OSFT}}(\mathcal{D}^d\Psi) = \frac{g d}{g_1} S_{\text{OSFT}}(\Psi) \]

\[ g_a \equiv \langle a\|0\rangle_{SL(2,\mathbb{C})} = \langle 1\rangle_{\text{disk}}^{\text{BCFT}} \]

- Change in the gauge-invariant coupling to closed strings (Ellwood invariant)

\[ \text{Tr}_V[\mathcal{D}^d\Psi] = \text{Tr}_{D_{\text{cl}}^d} V[\Psi] \]
OSFT boundary state and defects

- Given a solution $\Psi_{X \rightarrow Y}$, we can compute its boundary state

\[
|B(\Psi_{X \rightarrow Y})\rangle_{\text{OSFT}} = |X\rangle_{\text{BCFT}}
\]

Kiermaier, Okawa, Zwiebach (2008)
Kudrna, CM, Schnabl (2012)

- Previous slide computation has the important consequence that

\[
|B(D^{d}\Psi_{X \rightarrow Y})\rangle_{\text{OSFT}} = D_{c1}^{d}|Y\rangle_{\text{BCFT}}
\]

Kojita, Masuda, CM, Schnabl (2015)

\[
D\Psi_{X \rightarrow Y} = \Psi_{DX \rightarrow DY}
\]
OSFT as a dynamical “field” theory for BCFT.

All known (time ind.) BCFT’s remarkably give exact analytic solutions of OSFT. The string field is indeed “big enough”!

OSFT is (open) string background independent: background shift + field redefinition \(\ldots\) some subtlety still to address here, (1to1?)

Topological defects give rise to new operators in the open string algebra which map solutions to solutions.

As solution generating operators, they must play an important role in the (so far mysterious) classification of OSFT solutions.

Superstrings??

Thank you.