

Sep. 18, “Physics on the Riviera 2015”

Semi-natural Gauge Mediation from Product Group Unification

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with

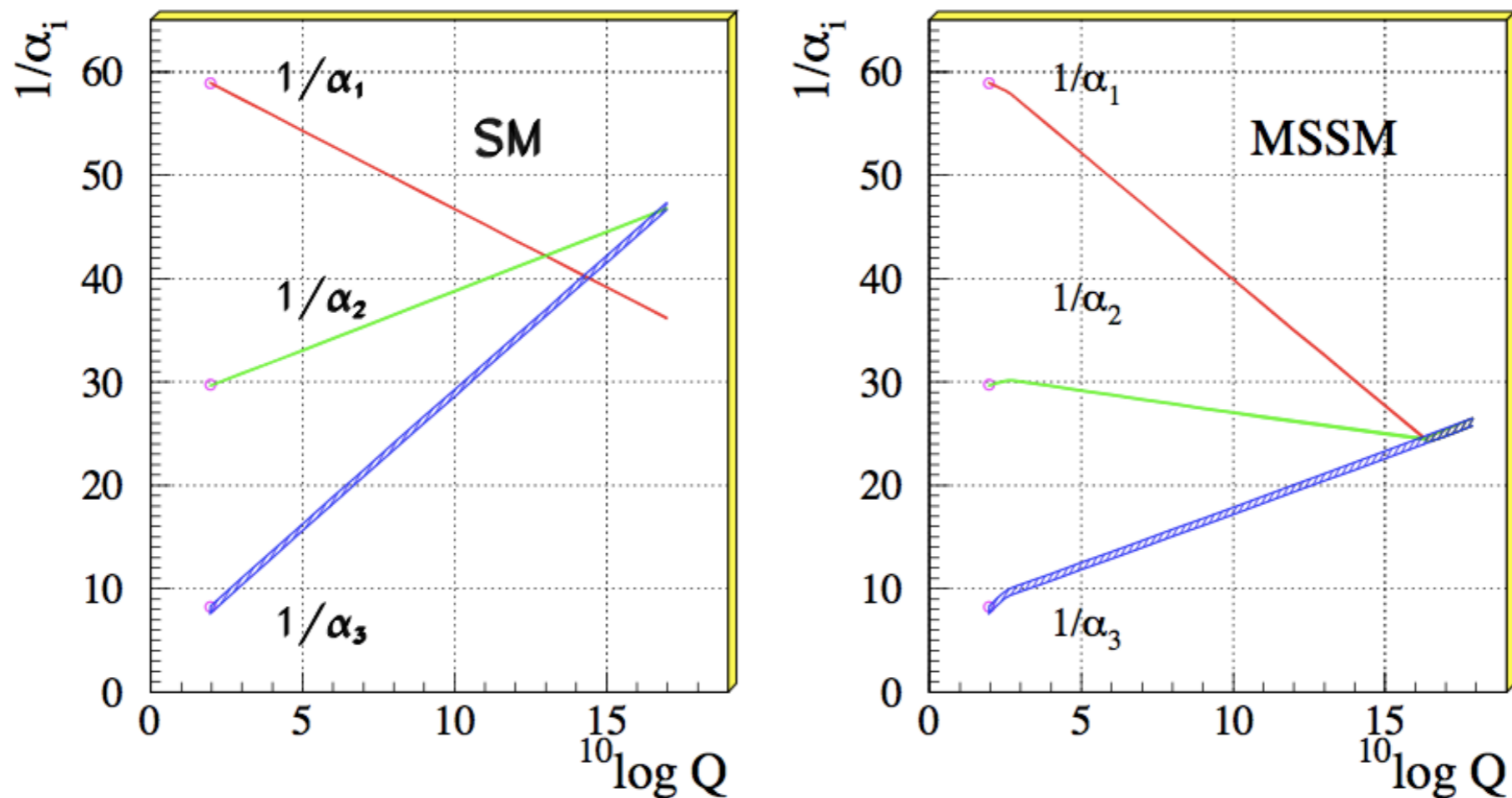
H. Fukuda, H. Murayama and T. Yanagida

[PRD, arXiv:1508.00445](#)

I would like to thank the organizers!

Supersymmetric SM

- Gauge couplings unify around 10^{16} GeV.



[hep-ph/0012288]

- **The muon $g-2$ anomaly ($>3\sigma$)** is explained if weakly interacting SUSY particles are light. (with a help of **$\tan\beta$ enhancement**)

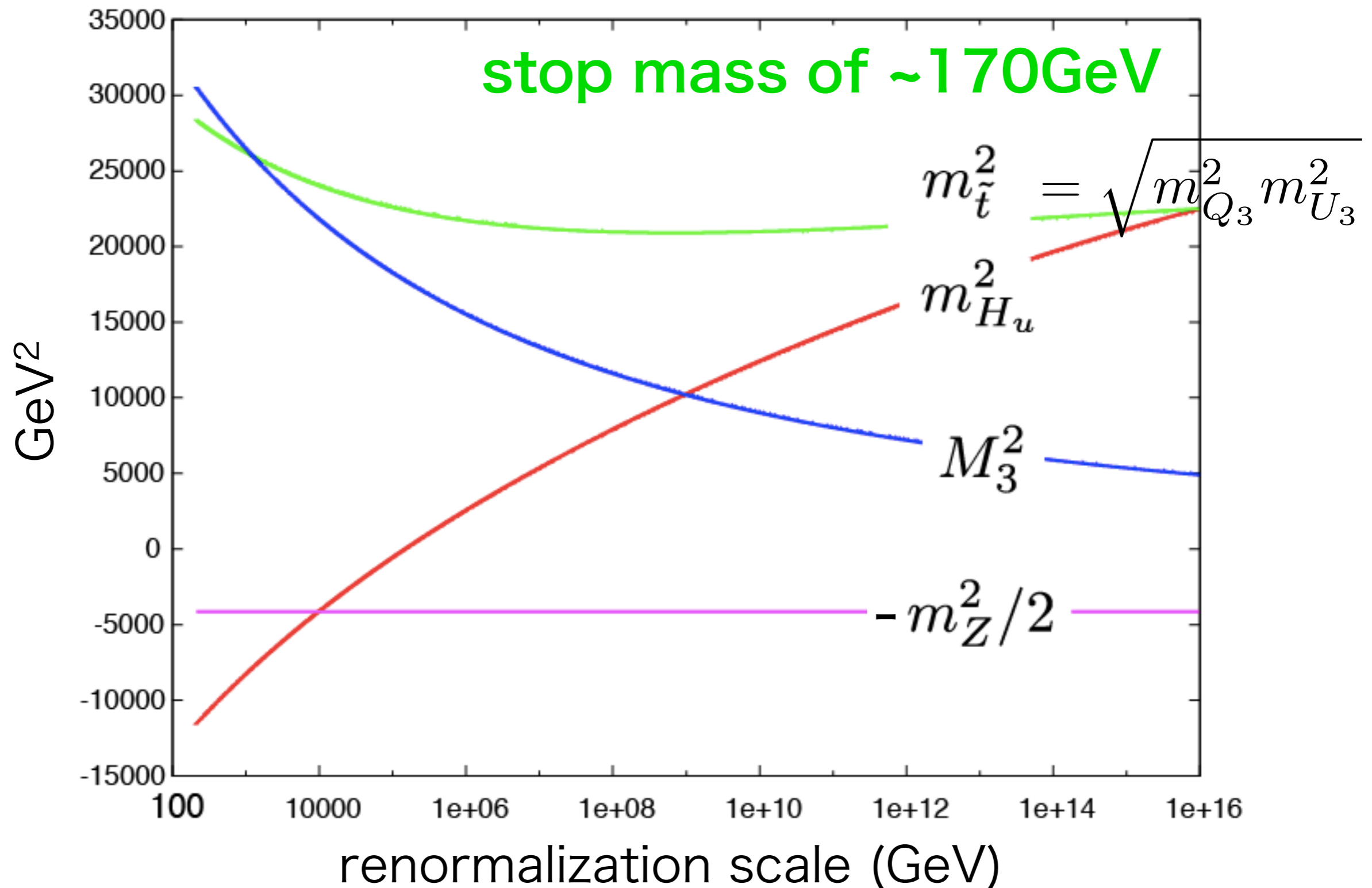
- Stabilizing the large hierarchy between the Planck/GUT/PQ breaking scale and the EWSB scale
- **The EWSB scale is possibly understood from the SUSY particle mass scale**
- Negative quadratic term in the Higgs potential is induced from loops involving top-squark and gluino. (because of $Y_t \sim 1$: if $Y_t \sim 0.5$ radiative EWSB breaking is very difficult.)

EWSB condition:
$$m_Z^2 \simeq -2 \left[m_{H_u}^2 + \mu^2 + \frac{1}{2v_u} \frac{\partial \Delta V_{CW}}{\partial v_u} \right] + \mathcal{O}(1/\tan^2 \beta) + \dots$$

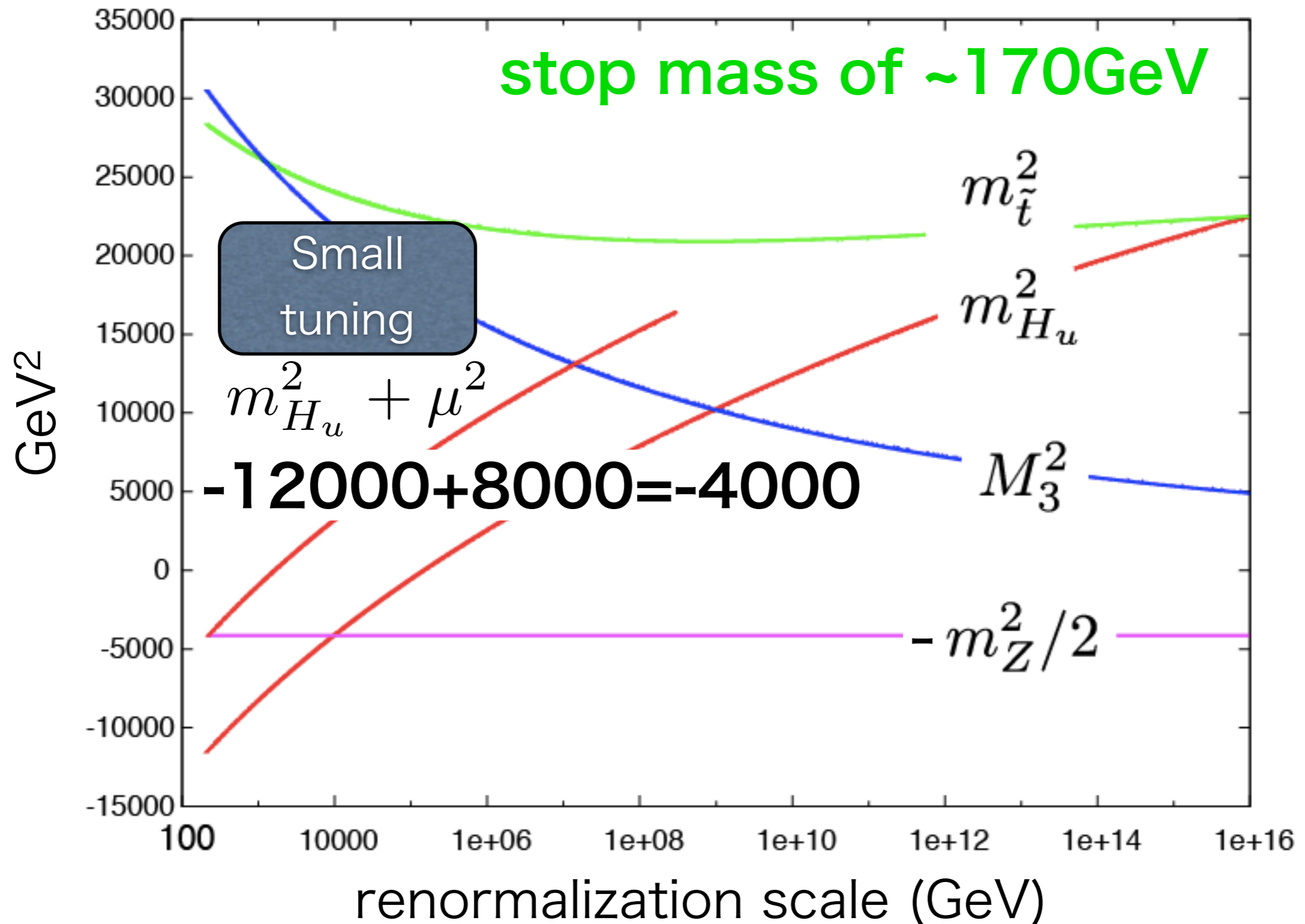
($\tan \beta = v_u/v_d$ is determined by the other EWSB condition)

- The size of negative $m_{H_u}^2$ is around the stop mass
 → the natural size of m_Z is the stop mass

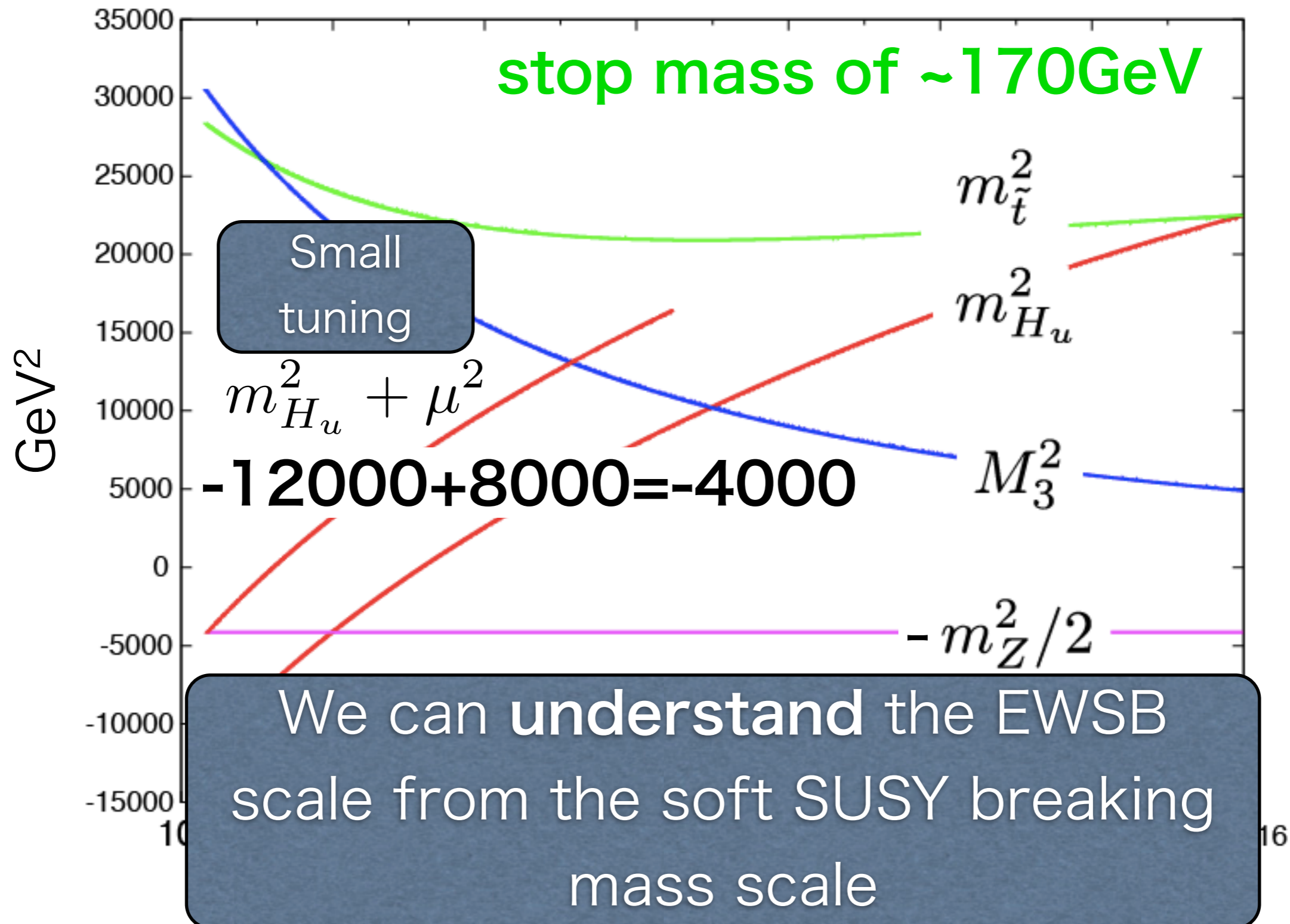
Radiative Electroweak Symmetry Breaking



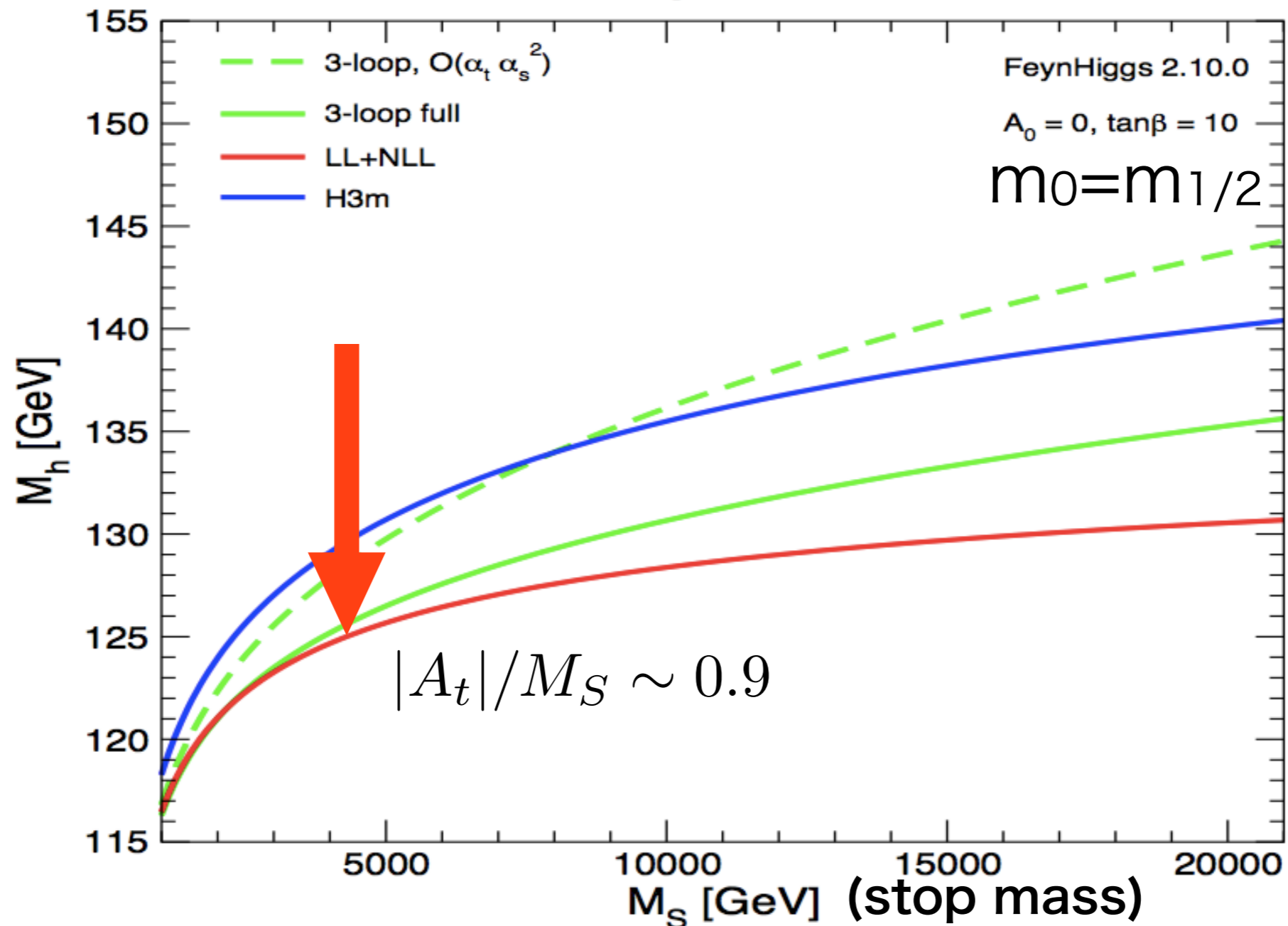
Radiative Electroweak Symmetry Breaking



Radiative Electroweak Symmetry Breaking

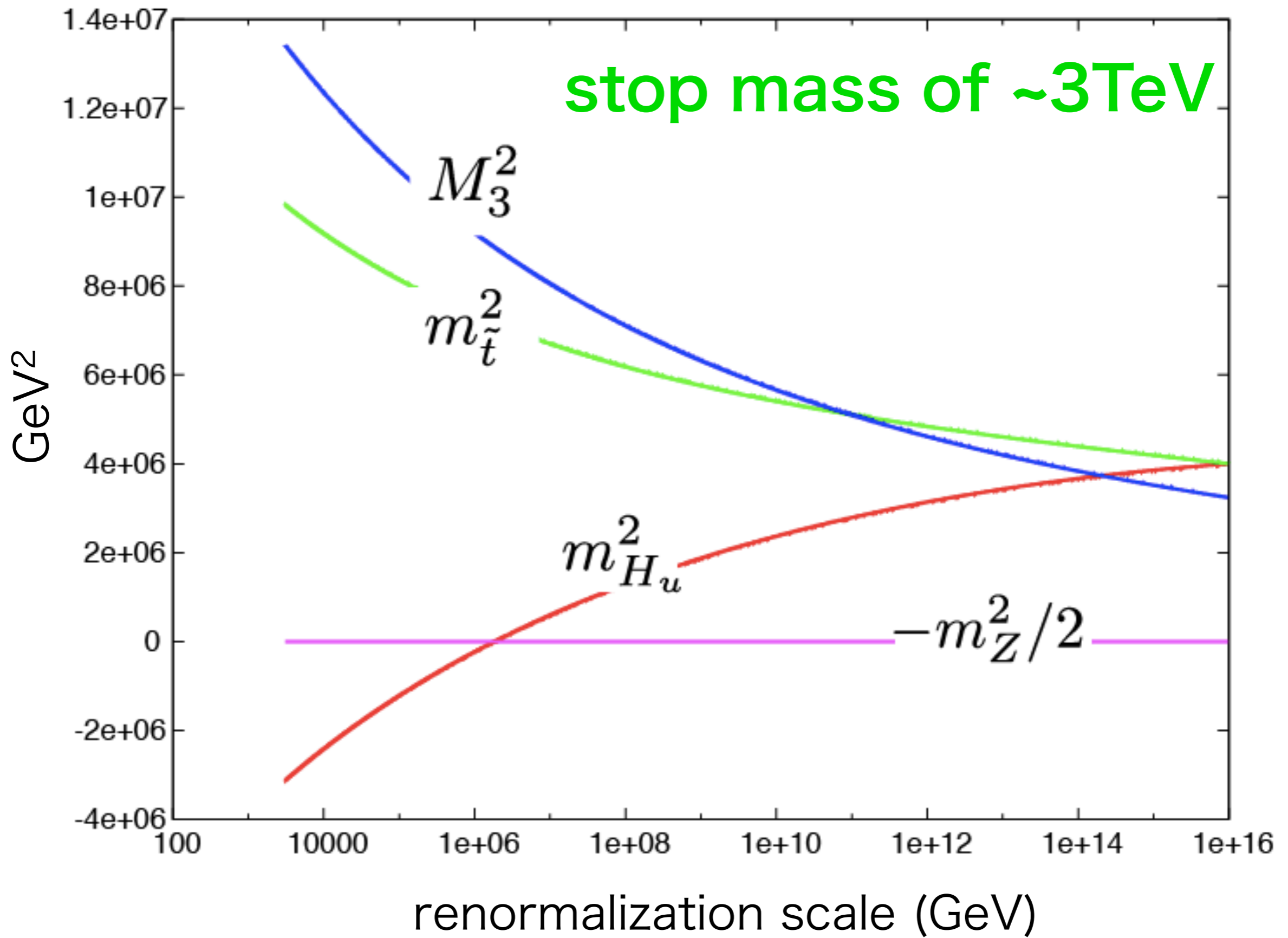


- However, the LHC excludes, top-squark (stop) < 600 GeV, gluino < 1.4 TeV
- **The observed Higgs boson mass of 125 GeV requires, stop > 3-5 TeV**



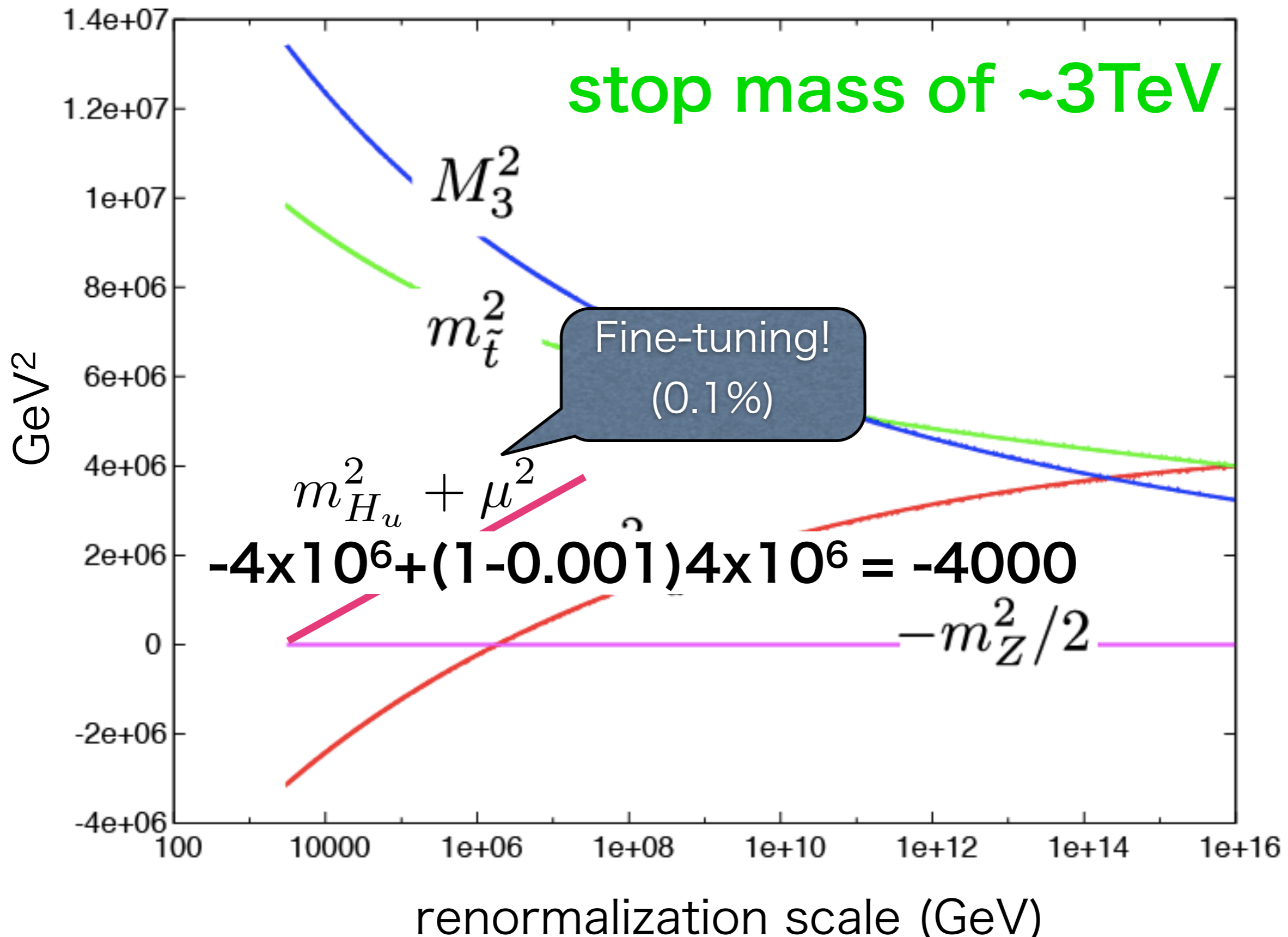
[T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein, 2013]

stop mass of $\sim 3\text{TeV}$



We need an elaborate choice of μ -parameter to explain the EWSB scale.

stop mass of ~3TeV



We need an elaborate choice of μ -parameter to explain the EWSB scale.

Approaches to the origin of the EWSB scale

- The best way is to consider the SUSY mass scale as well as the mediation scale is low, **though difficult.**
- Never mind (just a little hierarchy)

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Focus point!

- **Special relation(s) among parameters at UV physics, reducing the fine-tuning.**

Original Focus Point

universal scalar mass

gaugino mass

m_0

\gg

$M_{1/2}$

Arises from
minimal Kahler

input parameters at the
GUT scale

$m_{H_u}^2(m_Z)$ becomes much smaller than the expected value m_0^2 , and does not sensitive to the change of m_0

[Feng, Matchev, Moroi, '99]

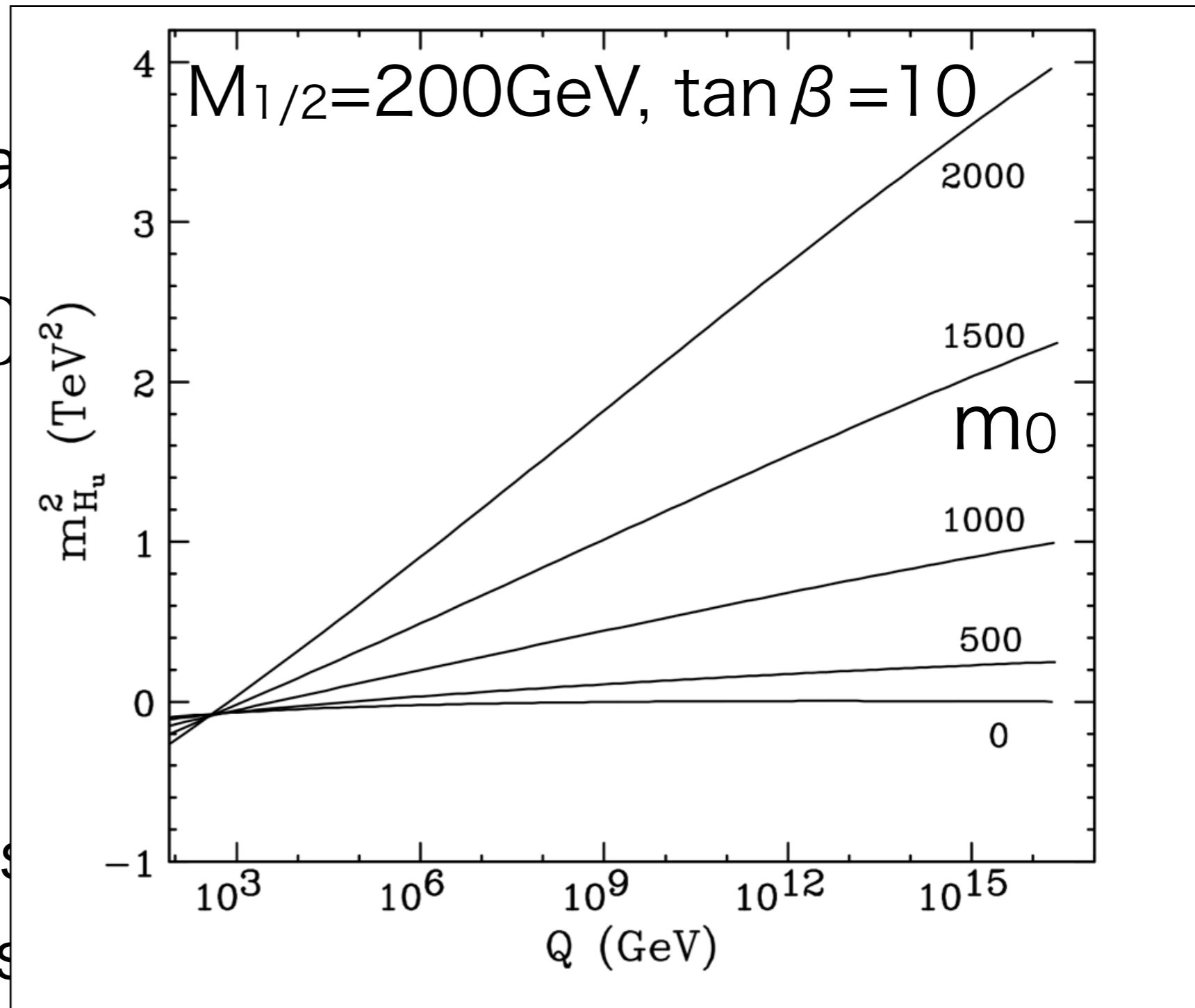
Original Focus Point

universal scalar

m_0

Arises from
minimal Kahler

$m_{H_u}^2(mz)$ becomes
value m_0^2 , and does



[Feng, Matchev, Moroi, '99]

- Usually, fine-tuning of the EWSB scale is estimated using the following measure:

$$\Delta = \max(\Delta_a) \quad \Delta_a = \left| \frac{\partial \ln m_{\hat{Z}}}{\partial \ln a} \right|_{m_{\hat{Z}} = 91.2 \text{ GeV}}$$

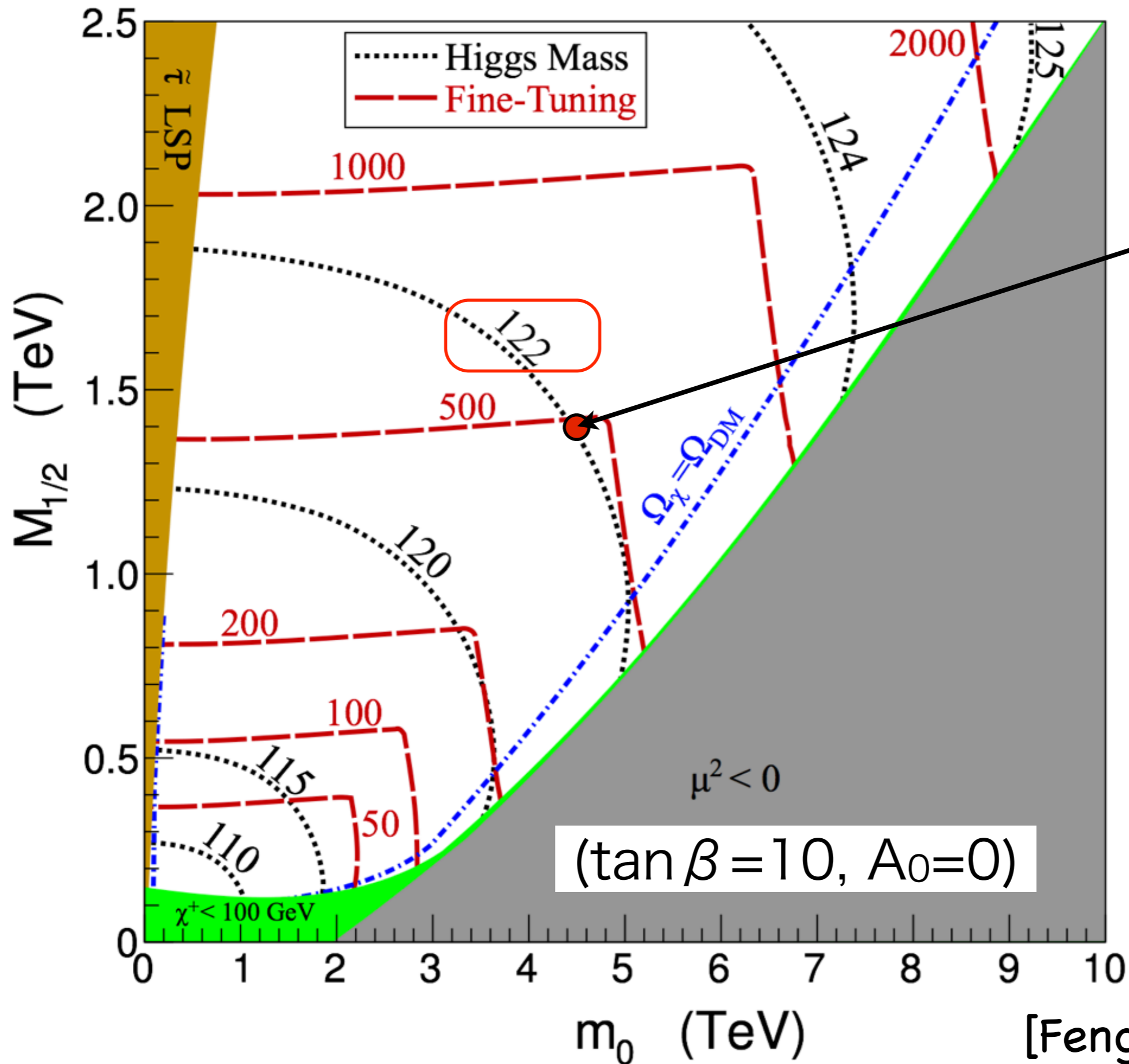
[Barbieri and Giudice, 1988]

“a” is a fundamental parameter

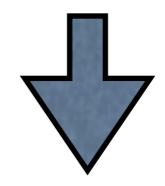
e.g., mSUGRA

$$\{a_i\} = m_0, M_{1/2}, A_0, \mu, B_0$$

(The Higgs potential can be written as a function of these fundamental parameters)



$m_h = 123.7 \text{ GeV}$
 (using FeynHiggs)
 $\text{stop} = 3.8 \text{ TeV}$
 $\text{gluino} = 3.2 \text{ TeV}$



$m_h = 124.7 \text{ GeV}$
 for $\tan \beta = 25$

Fine-tuning and Focus Point

- In the original scenario, the fine-tuning is reduced, but it is not significant.

(If the ratio of $m_0/M_{1/2}$ (=4-5) is fixed by the UV physics, it will be improved.)

[Harigaya, Yanagida, Yokozaki, 2015]

- Generally, the SUSY Flavor problem is **severe** in gravity mediation.
- It is better to consider the scenario with a convincing solution to the SUSY flavor problem.

Focus point gauge mediation

Good things in GMSB

- The masses of the SUSY particles are generated from **the messenger loops**.
- **The absence of the FCNC processes** is guaranteed.
(The masses of the SUSY particles are generated via gauge interactions.)
- A SUSY mass spectrum is unambiguously calculated in minimal gauge mediation.

Focus point gauge mediation

$$W_{\text{mess}} = \lambda_D Z \Psi_D^a \bar{\Psi}_D^a + \lambda_T Z \Psi_T^A \bar{\Psi}_T^A$$

$$a = 1 \dots N_D \quad A = 1 \dots N_T$$

N_D pairs of SU(2) doublet messengers

N_T pairs of SU(3) triplet messengers

Z is a SUSY breaking field $Z = \langle Z \rangle + \langle F_Z \rangle \theta^2$

If messengers are SU(5) complete multiplets, $N_D = N_T$

If $N_D/N_T \sim 5/2$, the fine-tuning can be significantly reduced

[In High scale GMSB, Brummer and Buchmuller, 2012; Brummer, Ibe and Yanagida, 2013]

Focus point gauge mediation

weakly interacting particles (charged under SU(2))

$$M_{\tilde{W}} \simeq \frac{\alpha_2}{4\pi} \overbrace{N_D}^{\Delta b_2} \Lambda \quad m_L^2 = m_{H_u}^2 \simeq \frac{3}{2} \left(\frac{\alpha_2}{4\pi} \right)^2 \overbrace{N_D} \Lambda^2$$

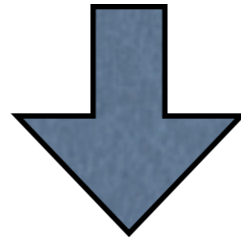
strongly interacting particles (charged under SU(3))

$$M_{\tilde{g}} \simeq \frac{\alpha_3}{4\pi} \overbrace{N_T}^{\Delta b_3} \Lambda \quad m_{\text{squark}}^2 \simeq \frac{8}{3} \left(\frac{\alpha_3}{4\pi} \right)^2 \overbrace{N_T} \Lambda^2$$

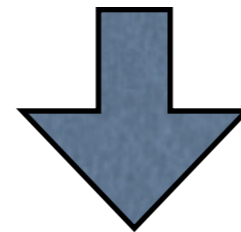
$$(16\pi^2) \frac{dg_i}{dt} = [(b_i)_{\text{MSSM}} + \Delta b_i] g_i^3 \quad \Lambda = \langle F_Z \rangle / \langle Z \rangle$$

The masses **do not depend on** the Yukawa couplings, λ_D and λ_T
(squark corresponds to a RH squark, omitting U(1) contributions)

$$m_{H_u}^2(4\text{TeV}) \simeq \left[-0.236m_Q^2 - 0.140m_{\bar{U}}^2 - 0.253M_{\tilde{g}}^2 \right. \\ \left. + 0.789m_{H_u}^2 + 0.139M_{\tilde{W}}^2 + \dots \right]_{M_{\text{inp}}=10^9\text{GeV}} \\ (=M_{\text{mess}})$$



$$m_{H_u}^2(4\text{TeV}) = [-0.253N_T^2 - 0.011N_D N_T - 1.073N_T \\ + 0.058N_D^2 + 0.380N_D] \left(\frac{\alpha_3}{4\pi} \Lambda \right)^2,$$



Putting $(N_D, N_T)=(5,2)$

$$m_{H_u}^2(4\text{TeV}) = \underline{0.020} \cdot M_{\tilde{g}}^2(M_{\text{inp}})$$

If $N_D=N_T$, $m_{H_u}^2(4\text{TeV}) \sim -M_{\tilde{g}}^2(M_{\text{inp}})$

Good things

- The focus point behavior is controlled by the messenger numbers. (not continuous variables)
- The behavior is stable against radiative correction.

Bad things

- No reason for $(N_D, N_T)=(5, 2)$.
- **GUT is messy!**

- As shown later, $N_D/N_T=5/2$ is explained in the product group unification model, **based on $SU(5) \times U(3)_H$**
- Why do we need the product group unification?

- As shown later, $N_D/N_T=5/2$ is explained in the product group unification model, **based on $SU(5) \times U(3)_H$**
- Why do we need the product group unification?

The product group unification provides a simple way to solve the **doublet-triplet splitting problem**

Severe fine-tuning in the minimal SU(5)

Doublet-Triplet splitting problem
(another fine-tuning problem!)

In the minimal SU(5), the Higgs doublets belong to 5 and 5* representations

$$H = H_5 = (H_C, H_u)^T$$
$$\bar{H} = H_{\bar{5}} = (\bar{H}_C, H_d)^T$$

The colored Higgs should have the GUT scale mass, otherwise the proton decays quickly

Severe fine-tuning in the minimal SU(5)

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Triplet $\sim 10^{16-17}\text{GeV}$

Doublet $\sim 100\text{GeV}$

$$10^{17}\text{GeV} - (1-10^{-15})10^{17}\text{GeV} \sim 100\text{GeV}$$

- It is **not easy** to solve this doublet-triplet (DT) splitting problem in GUT models based on simple gauge groups (e.g. SU(5) and SO(10))

There is no solution **based on a symmetry**, which is not broken at the GUT scale.
(If only MSSM particles exist below the GUT scale)

- There even exists the **no-go theorem** in simple cases [Witten, 2002; Fallbacher, Ratz, Vaudrevange, 2011; Ibe, Harigaya, Suzuki, 2015]

$$\frac{\langle W_0 \rangle}{M_P^2} H_u H_d, \quad \frac{\langle PQ \rangle^2}{M_P} H_u H_d \quad \longleftrightarrow \quad M_{\text{GUT}} H_C \bar{H}_C \quad ?$$

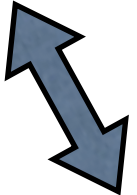
R-sym. PQ-sym.

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R-sym.  PQ-sym.

$$K = H_u H_d + h.c. \quad [\text{Inoue, Kawasaki, Yamaguchi and Yanagida, 1992}]$$

[For the mu-term from the PQ sym. breaking, Kim and Nilles, 1984]

$SU(5) \times U(3)_H$ product group unification

- The DT-splitting is easily achieved
- Gauge coupling unification is still **hold** approximately if the coupling of $U(3)_H$ is strong
- Charge quantization of the MSSM particles is **hold**

[Hotta, Izawa, Yanagida, 1995; Watari, Yanagida, 2002]

- $SU(5) \times U(3)_H$ is broken to the SM gauge group, by the VEVs of bi-fundamental fields

$$B = (\mathbf{5}, \bar{\mathbf{3}})$$

$$\bar{B} = (\bar{\mathbf{5}}, \mathbf{3})$$

$$\langle B_\alpha^i \rangle = v_{GUT} \delta_\alpha^i$$

$$\langle \bar{B}_i^\alpha \rangle = v_{GUT} \delta_i^\alpha$$

$$i=1..5$$

$$\alpha=1..3$$

Then,

$$W = H^i \bar{B}_i^\alpha \bar{T}_\alpha + \bar{H}_i B_\alpha^i T^\alpha$$



$$v_{GUT} H_C \bar{T} + v_{GUT} \bar{H}_C T$$

Colored Higgs multiplets become heavy, while doublets remain massless.

(The doublet mass can arise from a symmetry breaking term.)

Adjoint messenger

- Consider the messenger field in the adjoint representation of SU(5), **24**

SU(3)xSU(2)xU(1)_Y:

$$(\mathbf{8}, \mathbf{1}, 0) \quad \longrightarrow \quad \begin{aligned} \Delta b_2 &= 0 \\ \Delta b_3 &= 3 \end{aligned}$$

if this is heavy!

$$(\mathbf{1}, \mathbf{3}, 0) \quad \longrightarrow \quad \begin{aligned} \Delta b_2 &= 2 \\ \Delta b_3 &= 0 \end{aligned}$$

$$(\mathbf{3}, \mathbf{2}, -5/6) \quad \longrightarrow \quad \Delta b_2 = 3$$

$$(\mathbf{3}^*, \mathbf{2}, 5/6) \quad \longrightarrow \quad \Delta b_3 = 2$$

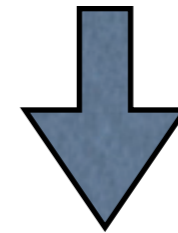
$$\begin{aligned} (\Delta b_2)_{\text{total}} &= 5 \\ (\Delta b_3)_{\text{total}} &= 2 \end{aligned}$$

corresponding to
 $(N_D, N_T) = (5, 2)$

Realization:

$$W = \lambda_{24} Z (\Sigma_{24})_j^i (\Sigma_{24})_i^j + \frac{c_0}{M_P} \bar{B}_i^\alpha (\Sigma_{24})_j^i (\Sigma'_8)_\alpha^\beta B_\beta^j,$$

SU(3)_H adjoint



$$\mu_8 \text{Tr}(\Sigma_8 \Sigma'_8)$$

$$\sim (v_{\text{GUT}}^2 / M_P) \sim 10^{12-14} \text{ GeV}$$

For $\lambda_{24} \langle Z \rangle \ll \mu_8$

the desired SUSY mass spectrum,
corresponding to $(N_D, N_T) = (5, 2)$ is obtained.

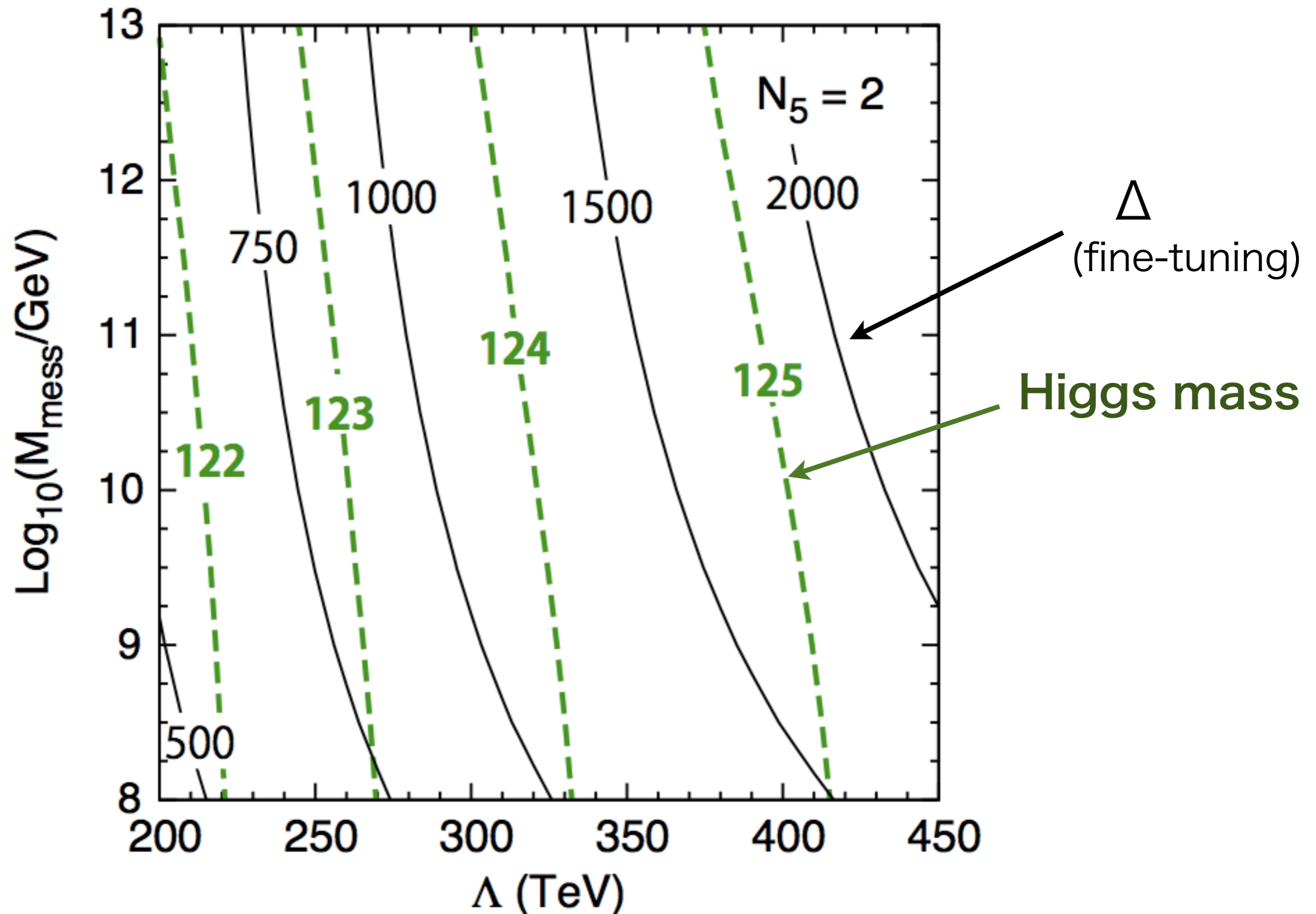
Estimation of the fine-tuning

- Finally, we estimate the fine-tuning using the following measure:

$$\Delta = \max\{\Delta_a\}, \quad \Delta_a = \left\{ \frac{\partial \ln m_{\hat{Z}}}{\partial \ln |\mu|}, \frac{\partial \ln m_{\hat{Z}}}{\partial \ln |F_Z|}, \frac{\partial \ln m_{\hat{Z}}}{\partial \ln |B_{\text{mess}}|} \right\}_{m_{\hat{Z}}=91.2\text{GeV}}$$

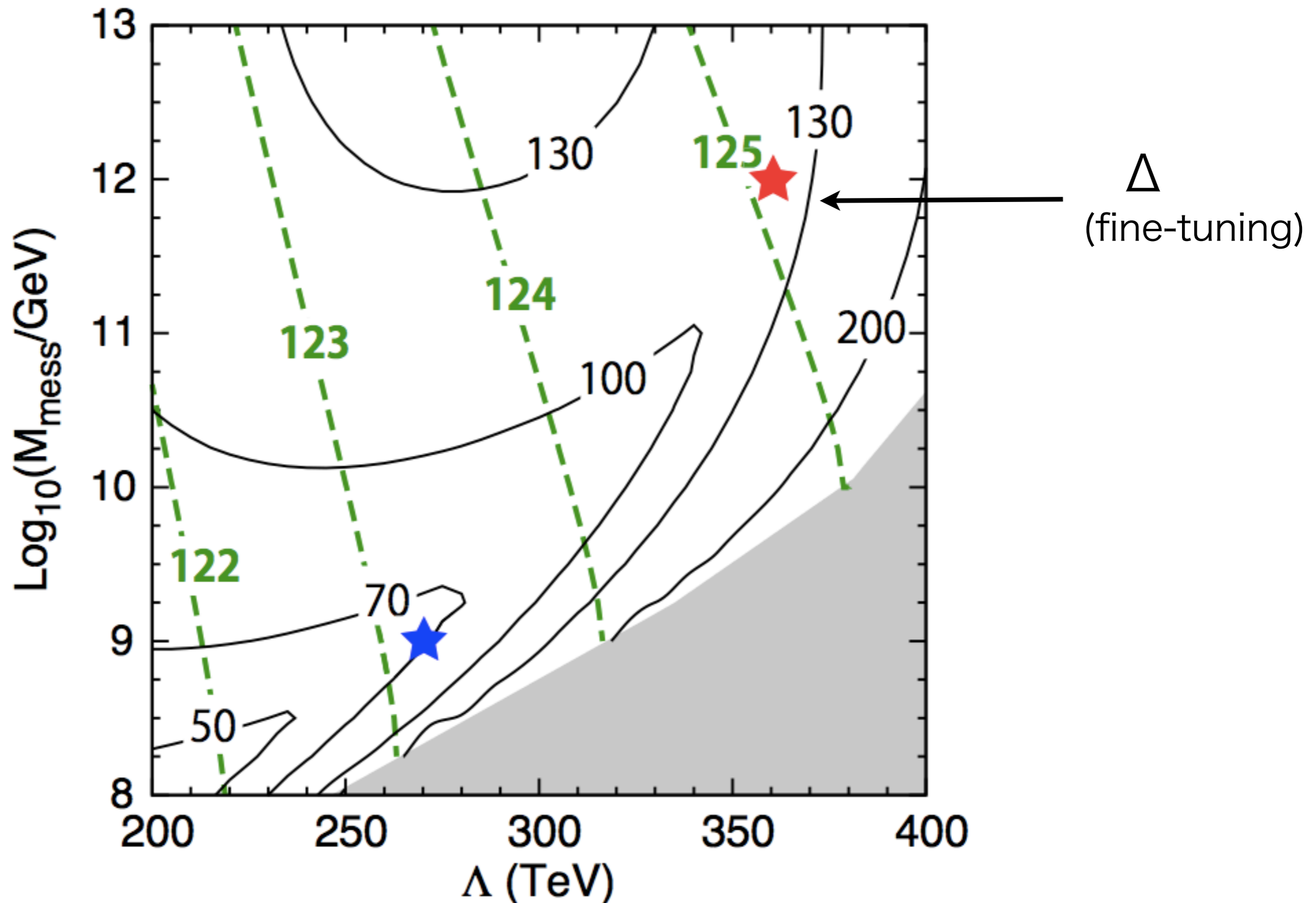
We regard F_Z , μ and B_{mess} as independent fundamental parameters

Minimal GMSB with 5 5^* messengers ($N_T=N_D=2$)



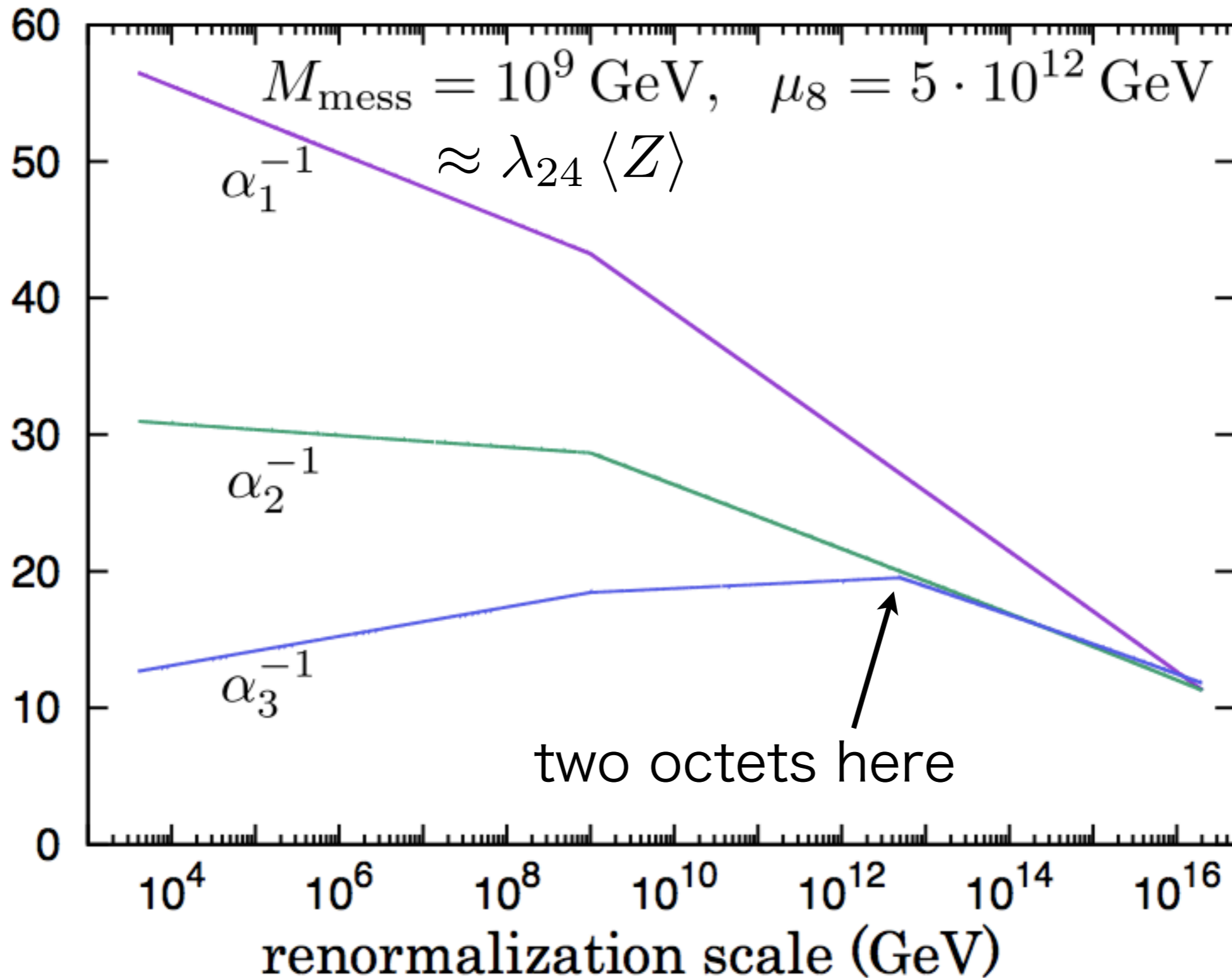
$\tan \beta = 25$, $m_t(\text{pole}) = 173.34 \text{ GeV}$ and $\alpha_s(m_Z) = 0.1185$.

SU(5) \times U(3)_H Product Group Unification



There exist solutions with $B_{\text{mess}}=0$ for $\mu < 0$ and $\tan\beta \sim 25-30$, where the SUSY CP-problem is solved.

The gauge couplings unify consistently



(2-loop RGEs are used)

Summary

- There still exist a hope to understand the origin of the EWSB scale **if the UV physics provides certain relation(s)** among soft SUSY breaking masses.
- The **semi-natural SUSY** may be as a result of the product group unification, which is somewhat natural from the view point of the doublet-triplet splitting.
- To be completed, we need to explain **why the EWSB is sensitive to the top Yukawa coupling**.

Consistency with gravitino cosmology (briefly)

- The thermalized **gravitino can be dark matter without over-closure of the universe.**
- This is achieved with **a late-time entropy production** from the decay of the messenger, diluting the gravitino abundance.
- Baryon number is explained via **the thermal Leptogenesis** with $M_N = 10^{(11-12)}$ GeV and $T_R > M_N$.
(M_N is a RH neutrino mass, T_R is a Reheating temperature)