

Dynamical relaxation to critical points

**A new approach
to the hierarchy problem**

Alex Pomarol, UAB (Barcelona)

Purpose of my talk:

- Discuss a recently proposed new approach to tackle the Hierarchy Problem in particle physics:

“Relaxation” mechanism

P.W. Graham, D.E. Kaplan, S.Rajendran
arXiv:1504.07551

(see also earlier work by
Abbott 85, G.Dvali, A.Vilenkin 04, G.Dvali 06)

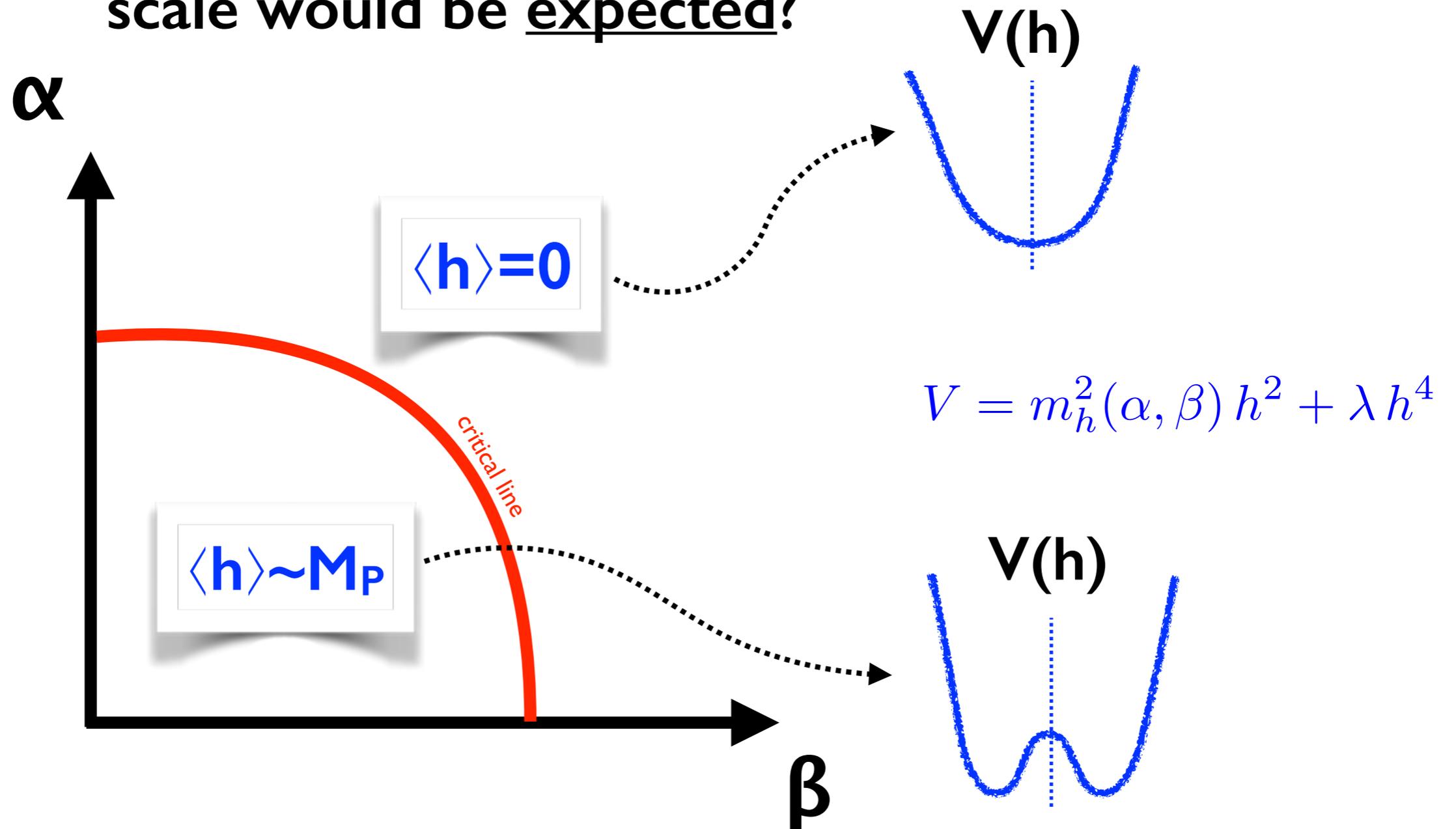
- Point out the impressive successes:
 - First example of natural solutions
 - in which **No New-Physics required at TeV**
- Drawbacks and reasons for improvement

work based on

J.R.Espinosa, C.Grojean, G.Panico, A.P.,
O.Pujolàs, G.Servant
arXiv:1506.09217

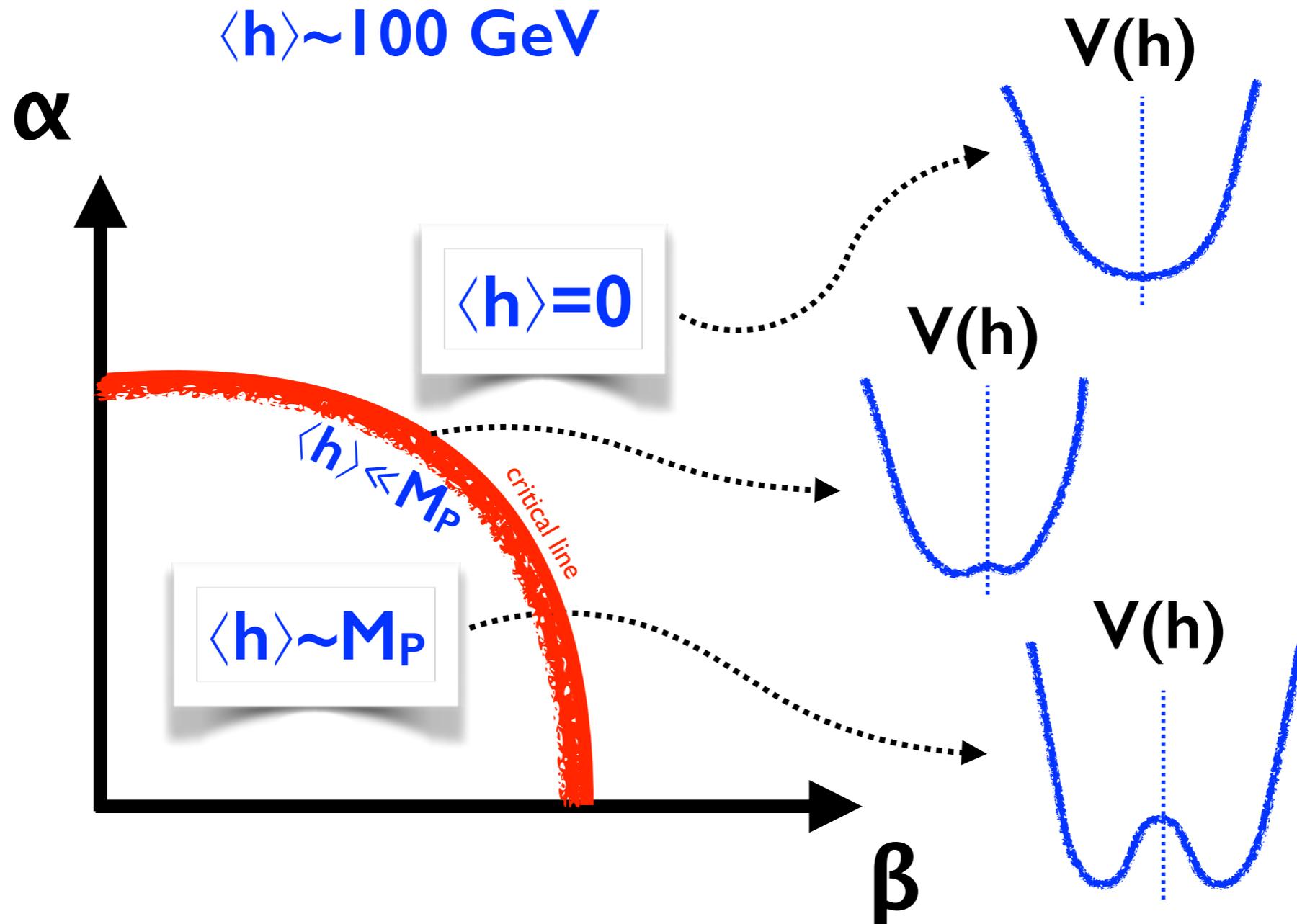
The SM: an EFT below M_P (sets the mass scale)

- Where the Electroweak Symmetry Breaking (EWSB) scale would be expected?



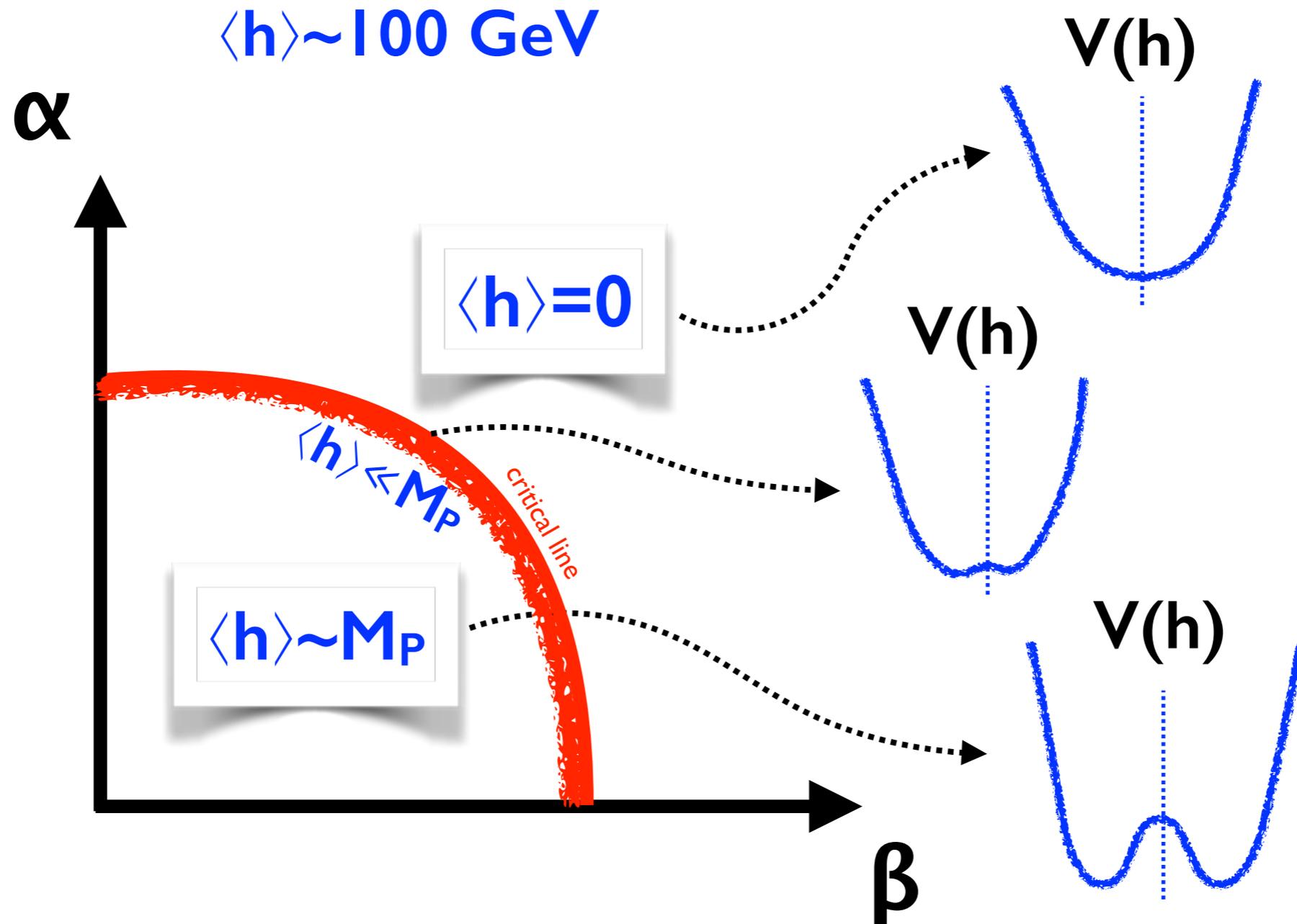
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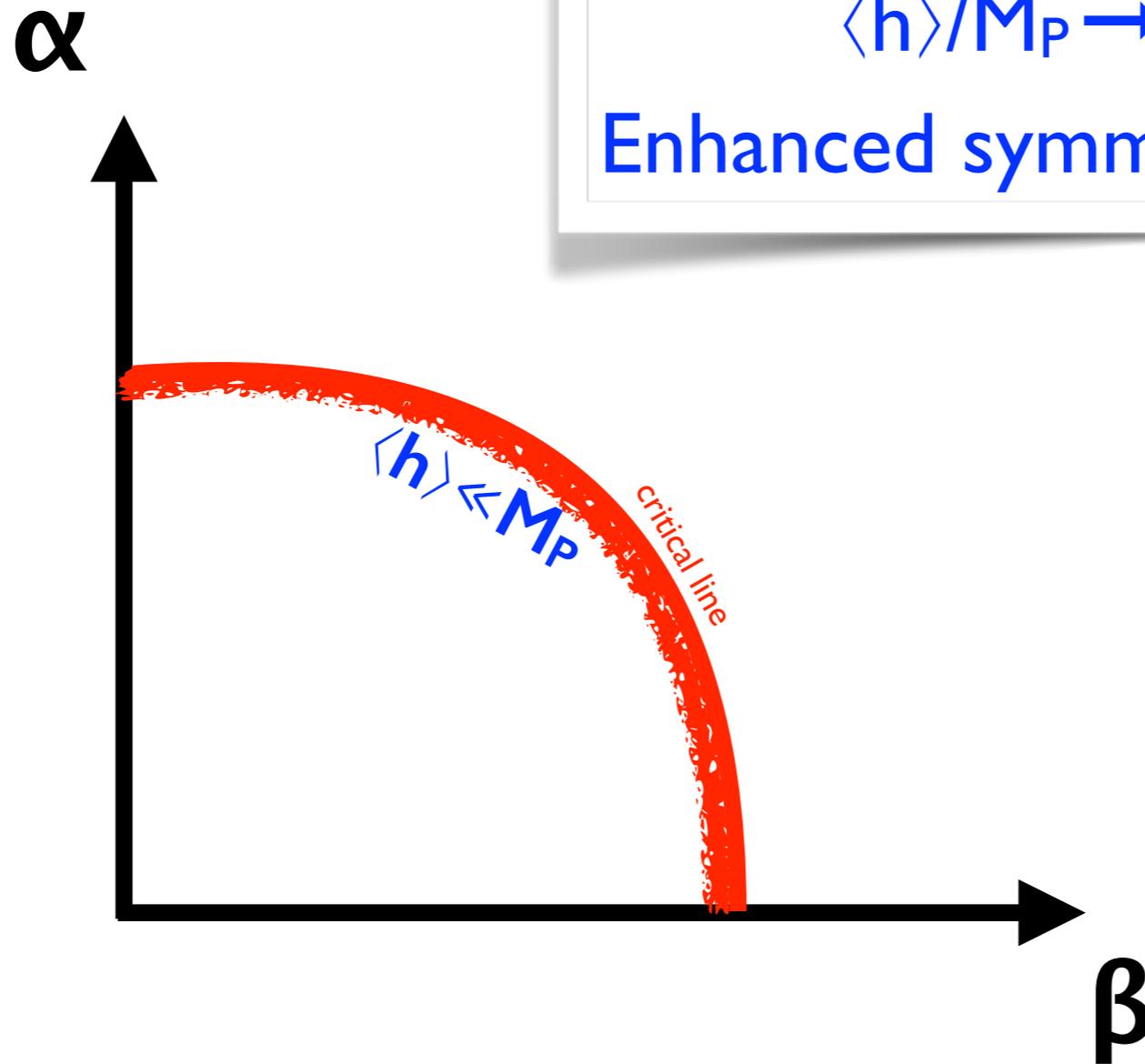
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Hierarchy problem: Why nature is so close to the critical line?

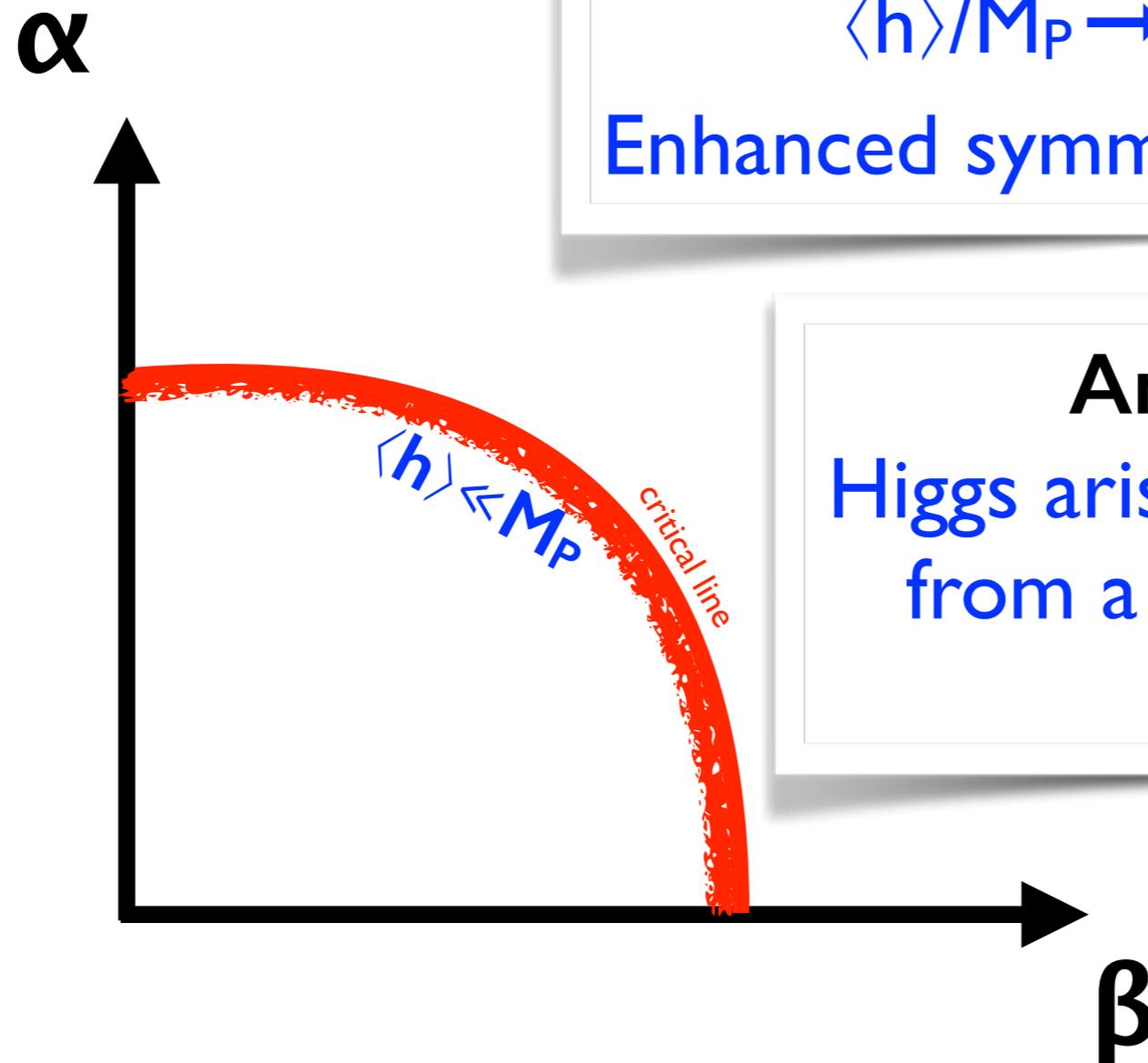
Needs a tuning of parameters to get $\langle h \rangle \ll M_P$

Hierarchy problem: Why nature is so close to the critical line?



One solution:
 $\langle h \rangle / M_P \rightarrow 0$ is a special line
Enhanced symmetry \rightarrow *Supersymmetry*

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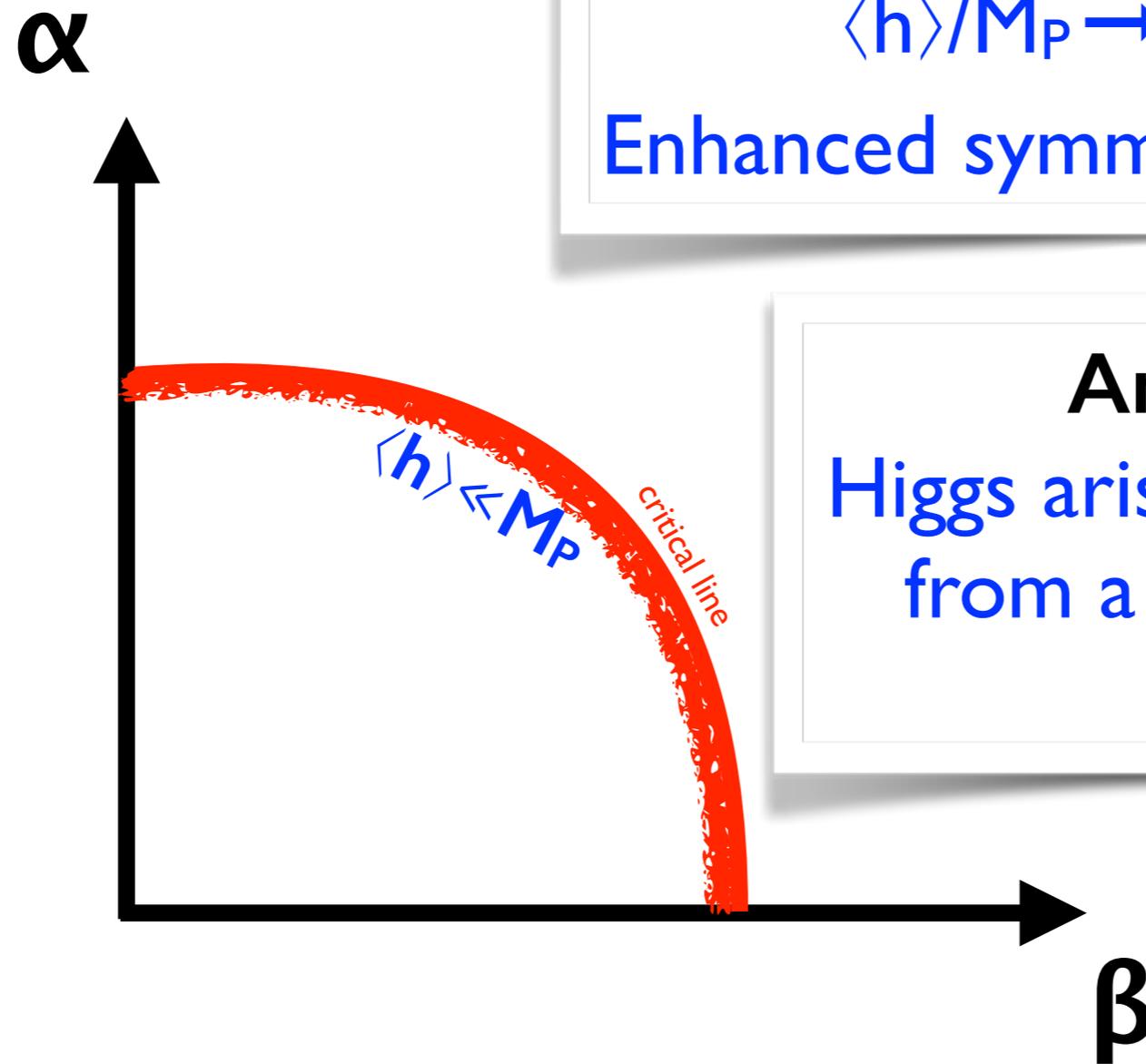
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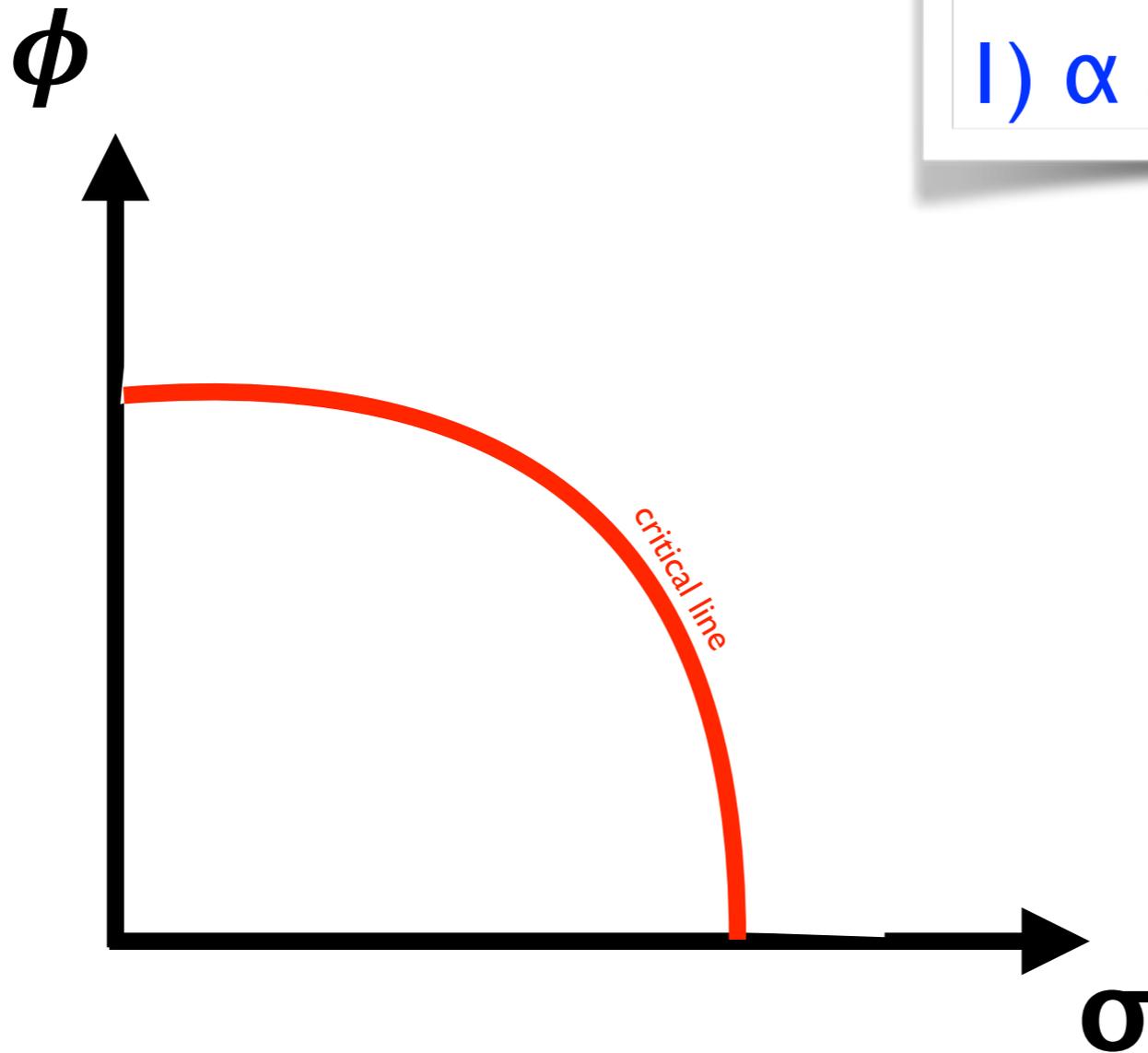
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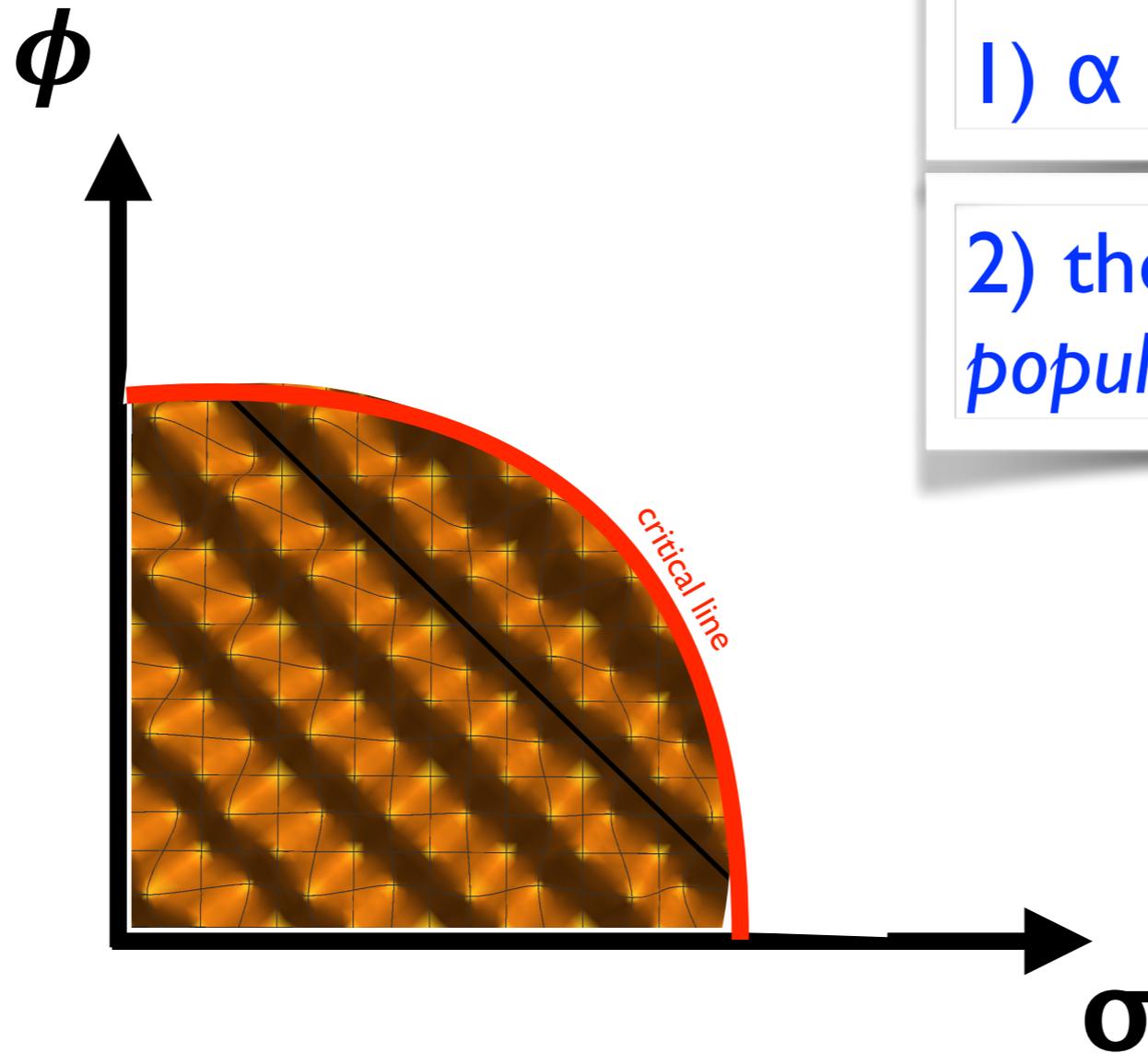
In both cases, TeV new-physics expected!

Hierarchy problem: Why nature is so close to the critical line?

New 3rd possibility:
1) α & β are fields $\rightarrow \phi$ & σ



Hierarchy problem: Why nature is so close to the critical line?

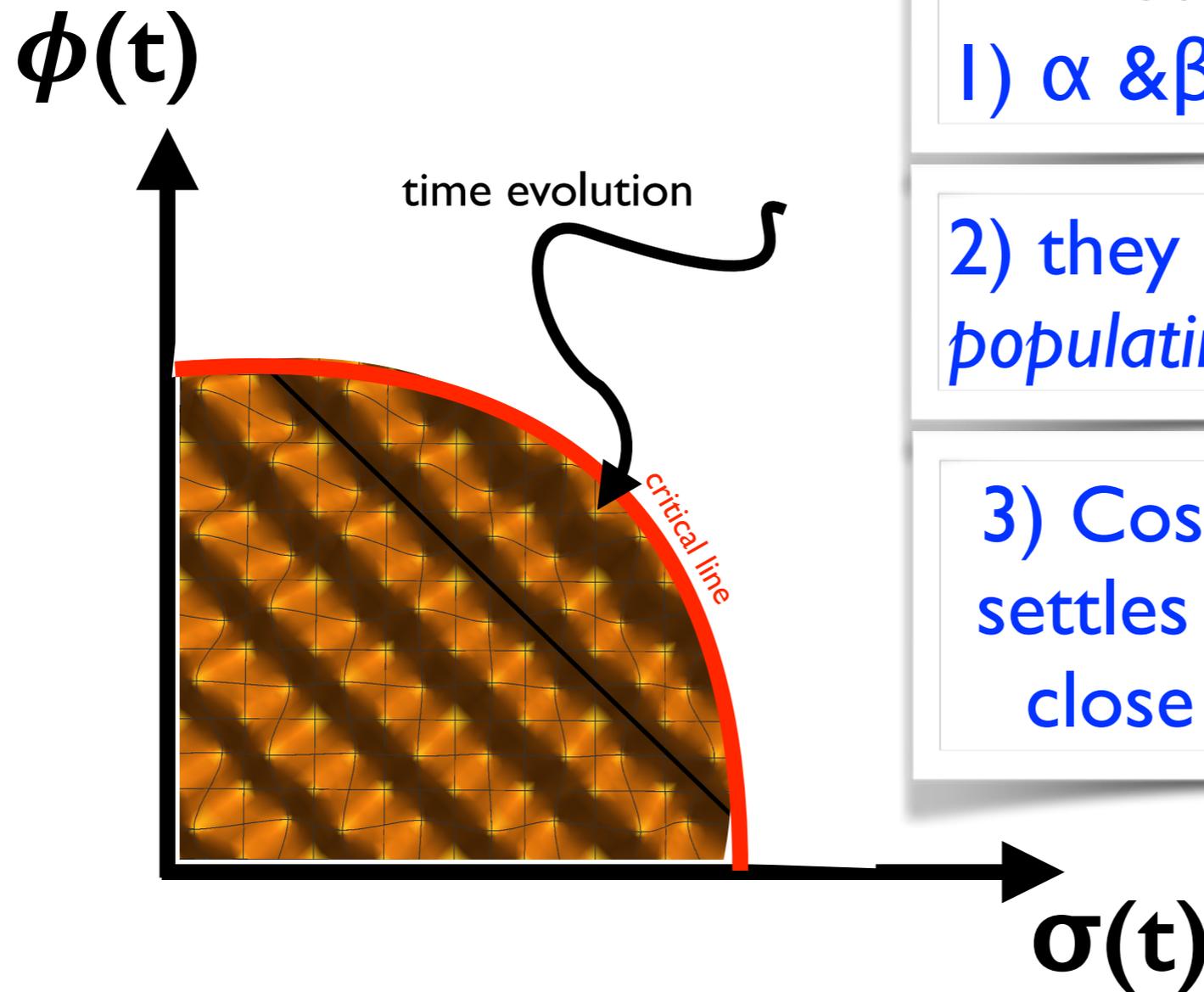


New 3rd possibility:

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2) they have local minima
populating the broken phase

Hierarchy problem: Why nature is so close to the critical line?



New 3rd possibility:

1) α & β are fields $\rightarrow \phi$ & σ

2) they have local minima *populating* the broken phase

3) Cosmological evolution settles them in a minimum close to the critical line

Explicit example:

Higgs-mass parameter \longrightarrow Field-dependent Higgs mass

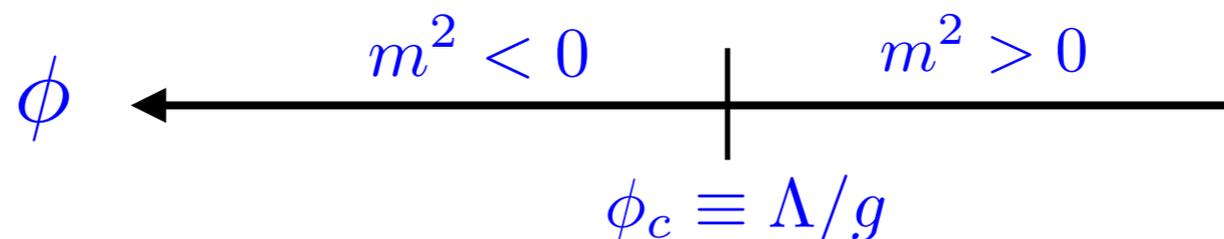
$$\frac{1}{2}m^2 h^2$$

$$\frac{1}{2}m^2(\phi)h^2$$

e.g. $m^2(\phi) = \Lambda^2 \left(1 - \frac{g\phi}{\Lambda} \right)$

Λ = sets the UV cut-off scale of the SM (M_P ?)

ϕ must be stabilized where $m^2(\phi)$ is negative and $\ll \Lambda^2$:



Notice that large field excursions for ϕ needed: $\phi \sim \Lambda/g \gg \Lambda$

Higgs (h) & axion-like (ϕ) potential:

P.W. Graham, D.E. Kaplan, S.Rajendran
arXiv:1504.07551

$$V(\phi, h) = \Lambda^3 g \phi - \frac{1}{2} \Lambda^2 \left(1 - \frac{g\phi}{\Lambda} \right) h^2 + \epsilon \Lambda_c^4 \left(\frac{h}{\Lambda_c} \right)^n \cos(\phi/f)$$

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“Kicking” term

Slope for ϕ to move forward

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ϕ “scans” the Higgs-mass

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$n=1,2,\dots$

**term affording local minima for ϕ
in the broken phase (when $h \neq 0$)**

periodic-function of ϕ as for axion-like states

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Λ : cutoff of the theory

Λ_c : scale that originates the periodic term

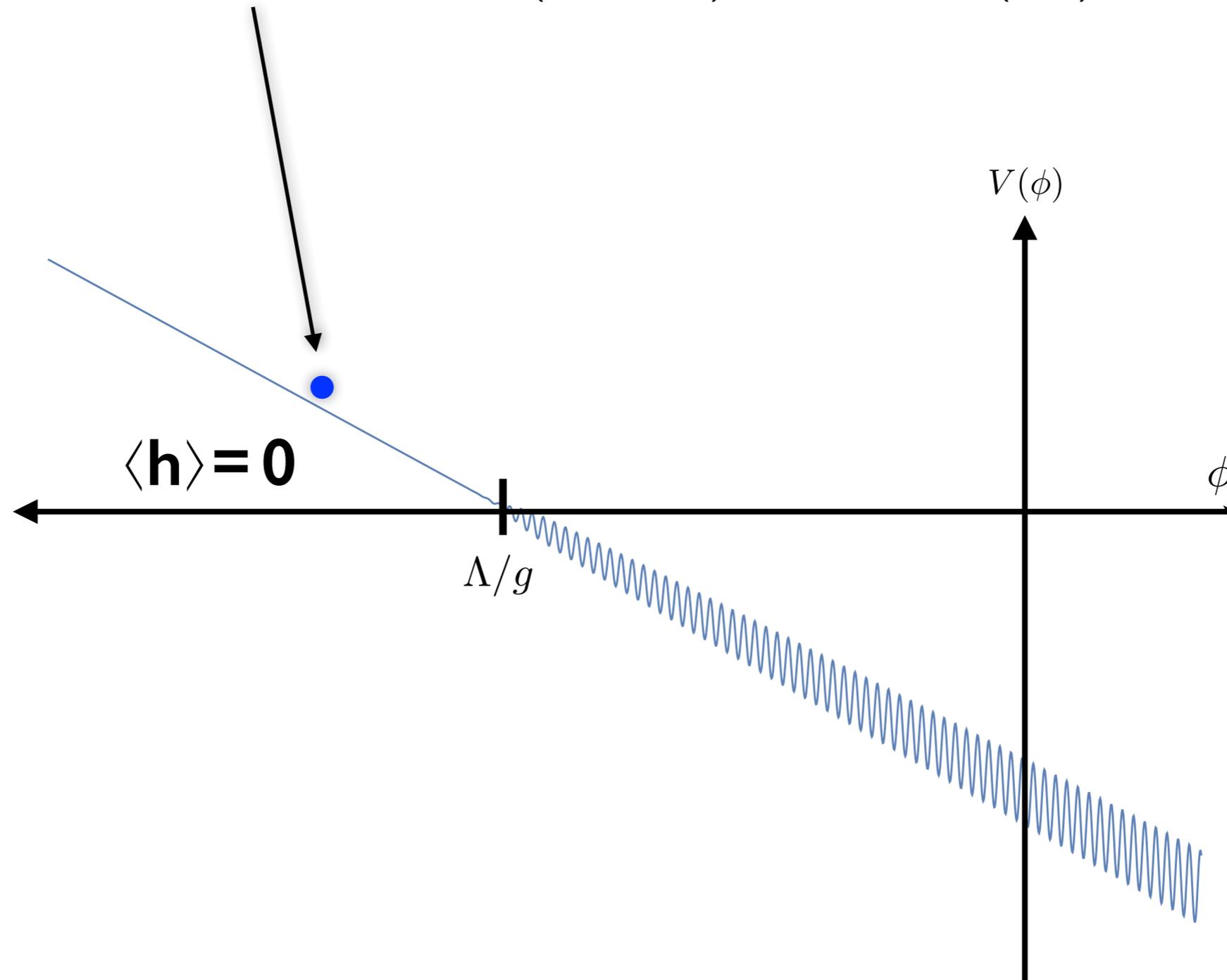
Spurions:

$g \ll 1$: breaking shift symmetry $\phi \rightarrow \phi + c$

$\epsilon \ll 1$: breaking of shift symmetry, respecting $\phi \rightarrow \phi + 2\pi f$, $\phi \rightarrow -\phi$

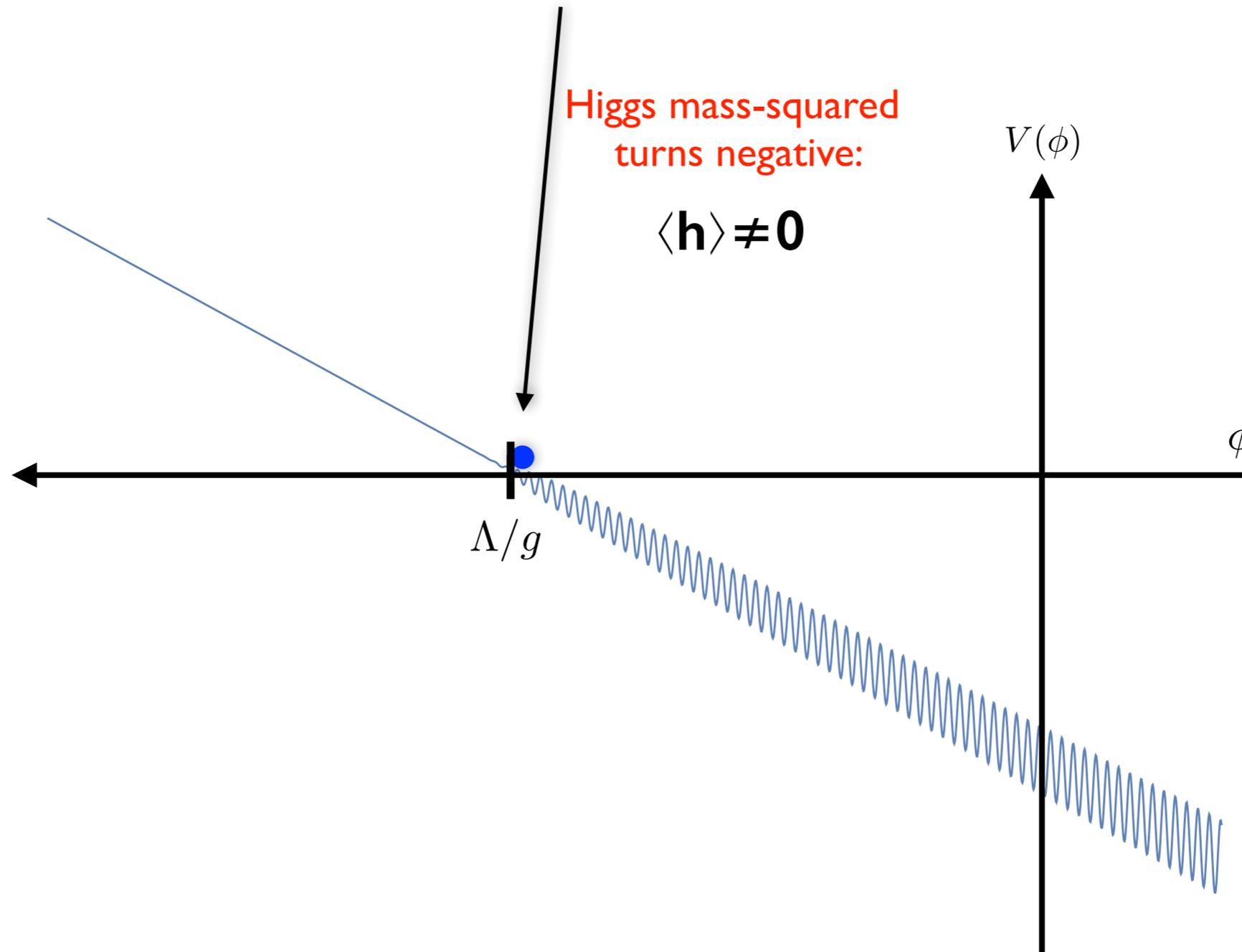
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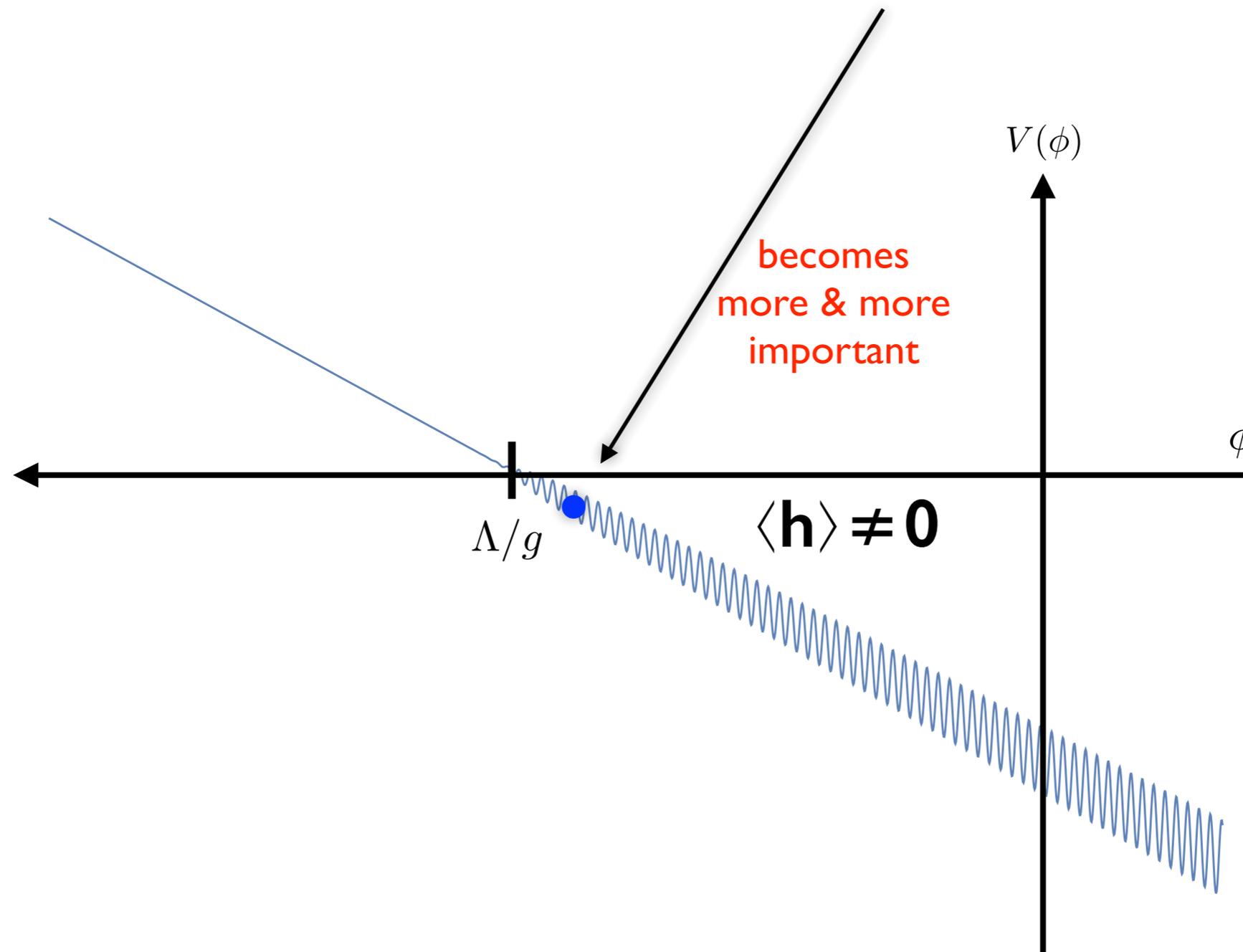
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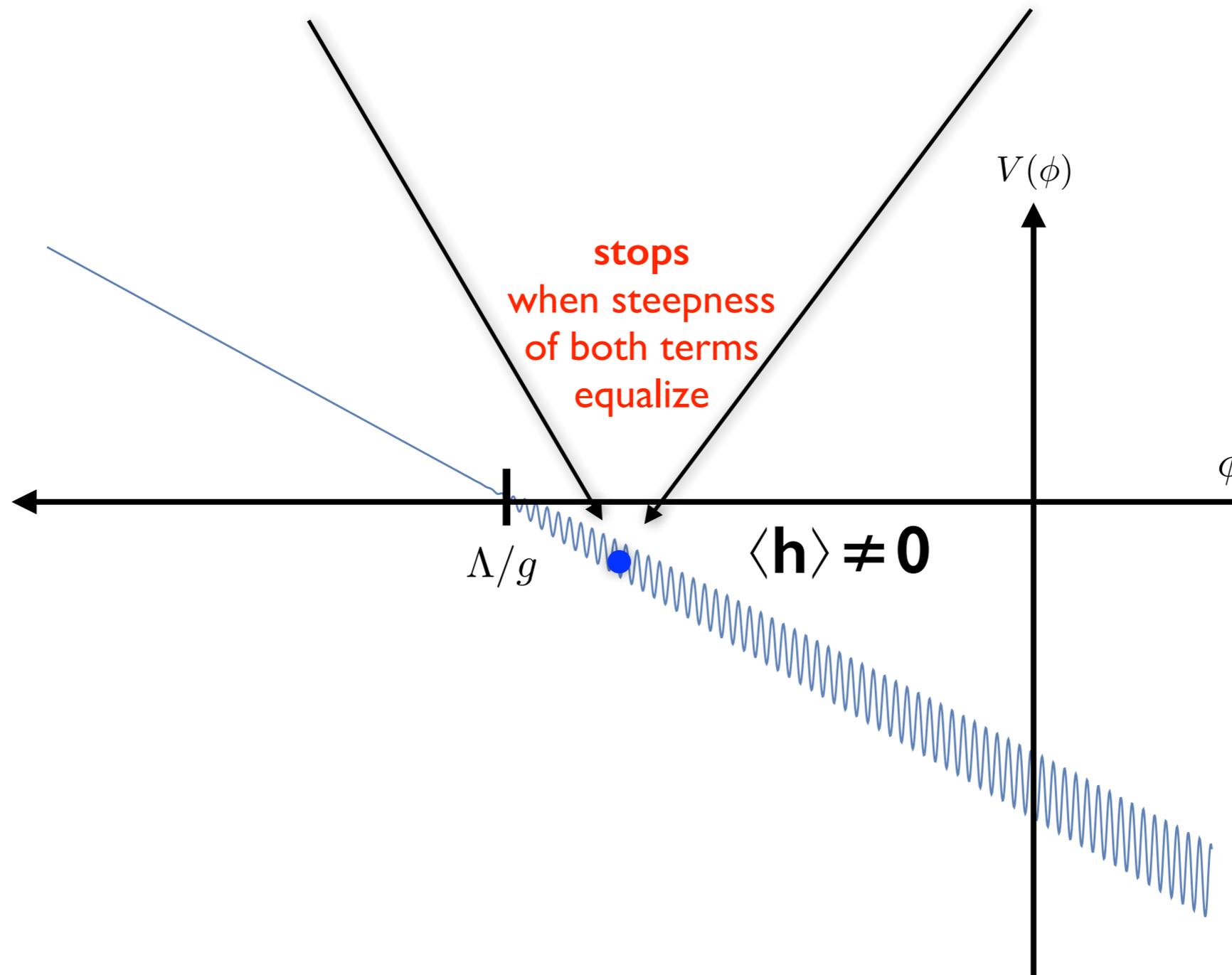
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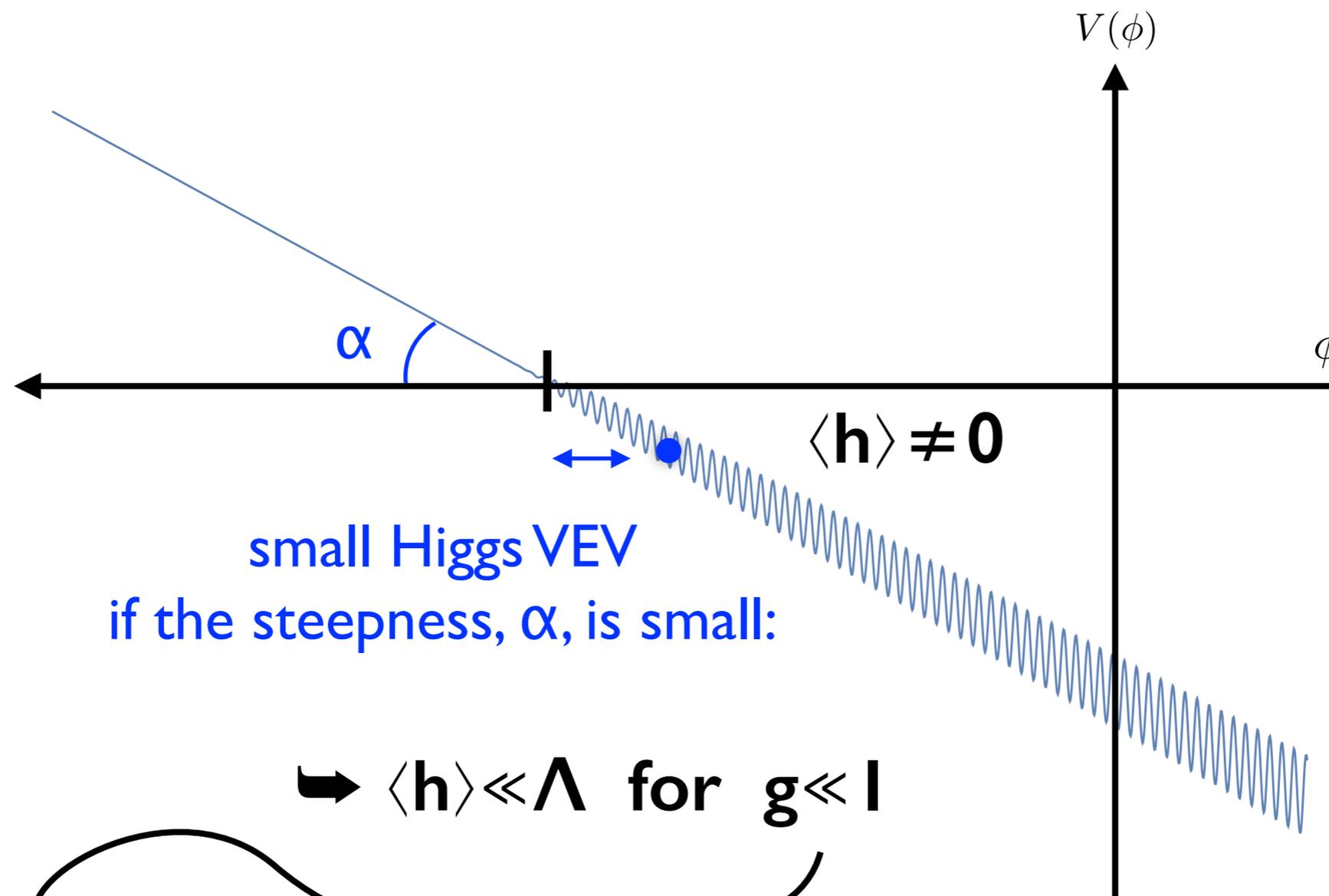
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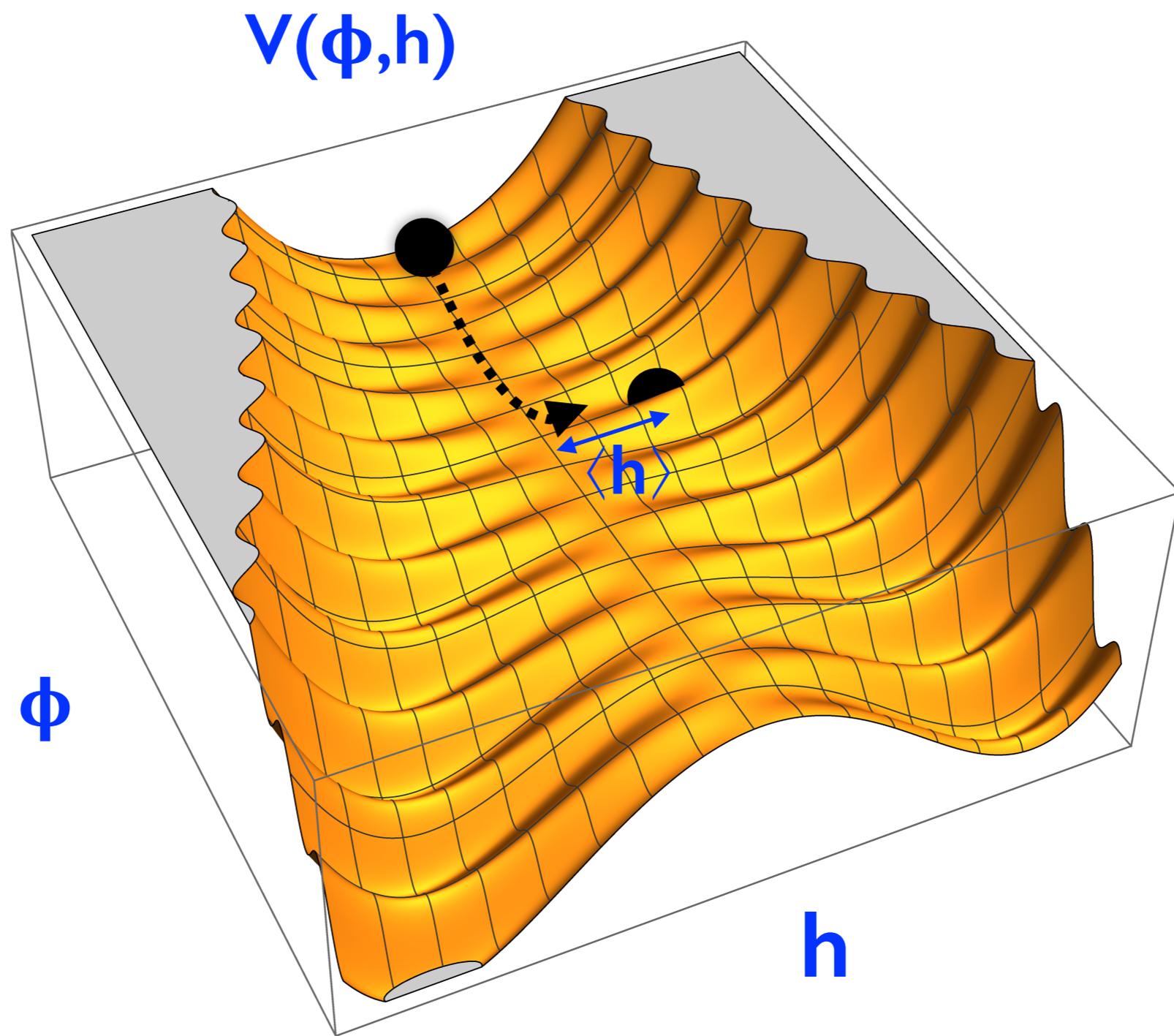


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technically natural since $g=0$ is a point of enhanced symmetry



Tuning the initial conditions?



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No, if slow rolling due to a friction:
possible in the **inflationary epoch!** (Hubble friction)

can be neglected \rightarrow
$$\ddot{\phi} + 3H_I \dot{\phi} = -\partial_{\phi} V(\phi)$$

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Long period of inflation needed,
in order for ϕ to “scan” large ranges of the Higgs mass

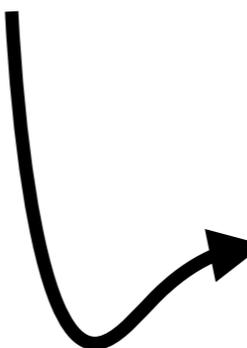
e-folds needed: $N_e \gtrsim \frac{H_I^2}{g^2 \Lambda^2} \sim 10^{40}$

**For simplicity,
we will assume that inflation
is driven by other fields**

Important limitation:

ϕ must roll-down classically and not *wiggle* by quantum effects:

$$\Delta\phi_{class} \sim g \frac{\Lambda^3}{H_I^2} \gtrsim \Delta\phi_{quant} \sim H_I$$


$$g \gtrsim (H_I/\Lambda)^3$$

Origin of $\epsilon \Lambda_c^4 \left(\frac{h}{\Lambda_c} \right)^n \cos(\phi/f) \quad ?$

P.W. Graham, D.E. Kaplan, S.Rajendran
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n=1: axion term from QCD condensate: $\Lambda_c = \Lambda_{QCD}$

$$\frac{\phi}{f} G^{\mu\nu} \tilde{G}_{\mu\nu} \quad \rightarrow \quad m_u(h) \langle q\bar{q} \rangle \cos(\phi/f)$$

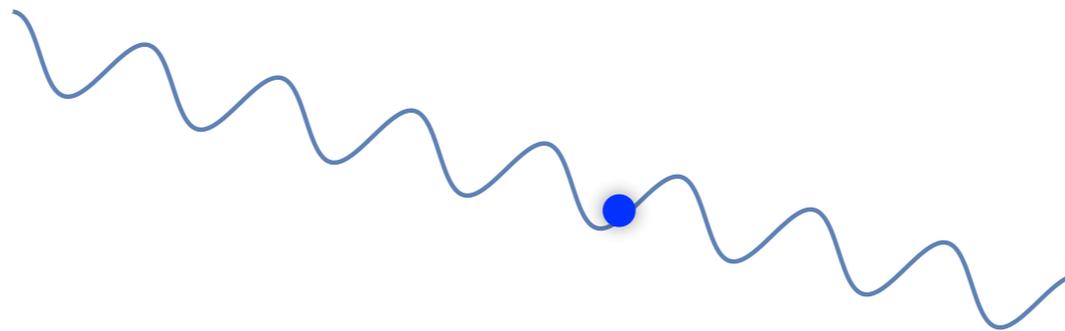
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but leads to $\theta_{QCD} \sim 1$ due to the *tilt* !



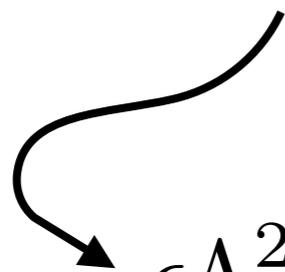
it must be arranged such that at the end of inflation, the *tilt* disappears



one gets: $\Lambda \approx 30 \text{ TeV}$ (1000 TeV if the *tilt* changes sign) ($H_I \sim 10^{-9} \text{ GeV}$)

Origin of $\epsilon \Lambda_c^4 \left(\frac{h}{\Lambda_c} \right)^n \cos(\phi/f)$?

n=2:


$$\epsilon \Lambda_c^2 |H|^2 \cos(\phi/f)$$

gauge-invariant, no need to rely on QCD

($\Lambda_c \sim$ some new-physics scale
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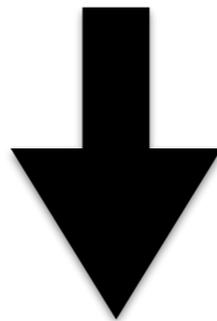
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($\Lambda_c \sim$ some new-physics scale
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at the quantum level,
closing H in a loop $\rightarrow \epsilon \Lambda_c^4 \cos(\phi/f)$

this term gives minima for ϕ in the unbroken phase ($h=0$)



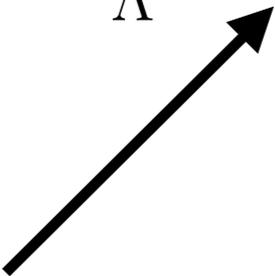
J.R.Espinosa, C.Grojean, G.Panico, A.P.,
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Proposal to go further:

Make the amplitude of the $\cos(\phi/f)$ -term also field dependent

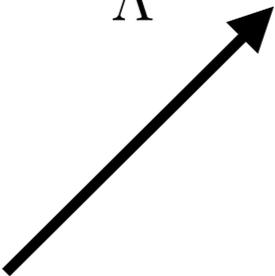
$A \cos(\phi/f)$  **Field-dependent amplitude:**

$$A(\phi, \sigma, H) \equiv \epsilon \Lambda^4 \left(\beta + c_\phi \frac{g\phi}{\Lambda} - c_\sigma \frac{g_\sigma \sigma}{\Lambda} + \frac{|H|^2}{\Lambda^2} \right)$$

 new field σ “scanning” the amplitude

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$$V(\phi, \sigma, H) = \Lambda^4 \left(\frac{g\phi}{\Lambda} + \frac{g_\sigma \sigma}{\Lambda} \right) + m^2(\phi) |H|^2 + A(\phi, \sigma, H) \cos(\phi/f)$$

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spurions

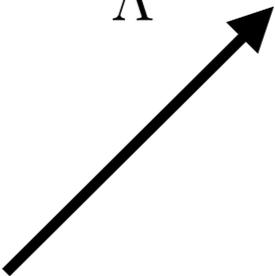
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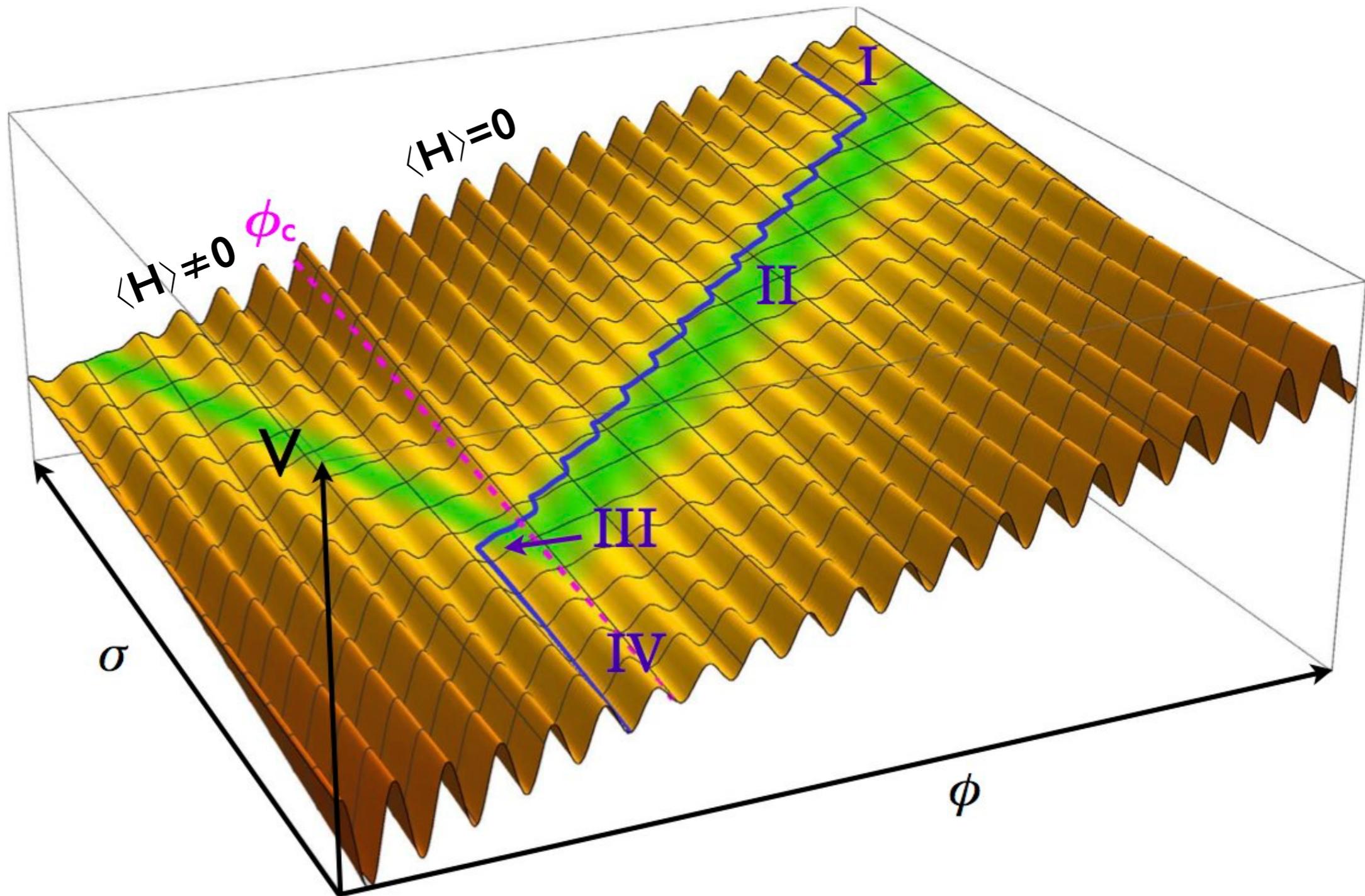
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we'll be taking $\Lambda \sim \Lambda_c$ and try to see how far away can be pushed up

ALPine Cosmology:

$$V(\phi, \sigma, H) = \Lambda^4 \left(\frac{g\phi}{\Lambda} + \frac{g_\sigma\sigma}{\Lambda} \right) + m^2(\phi)|H|^2 + A(\phi, \sigma, H) \cos(\phi/f)$$

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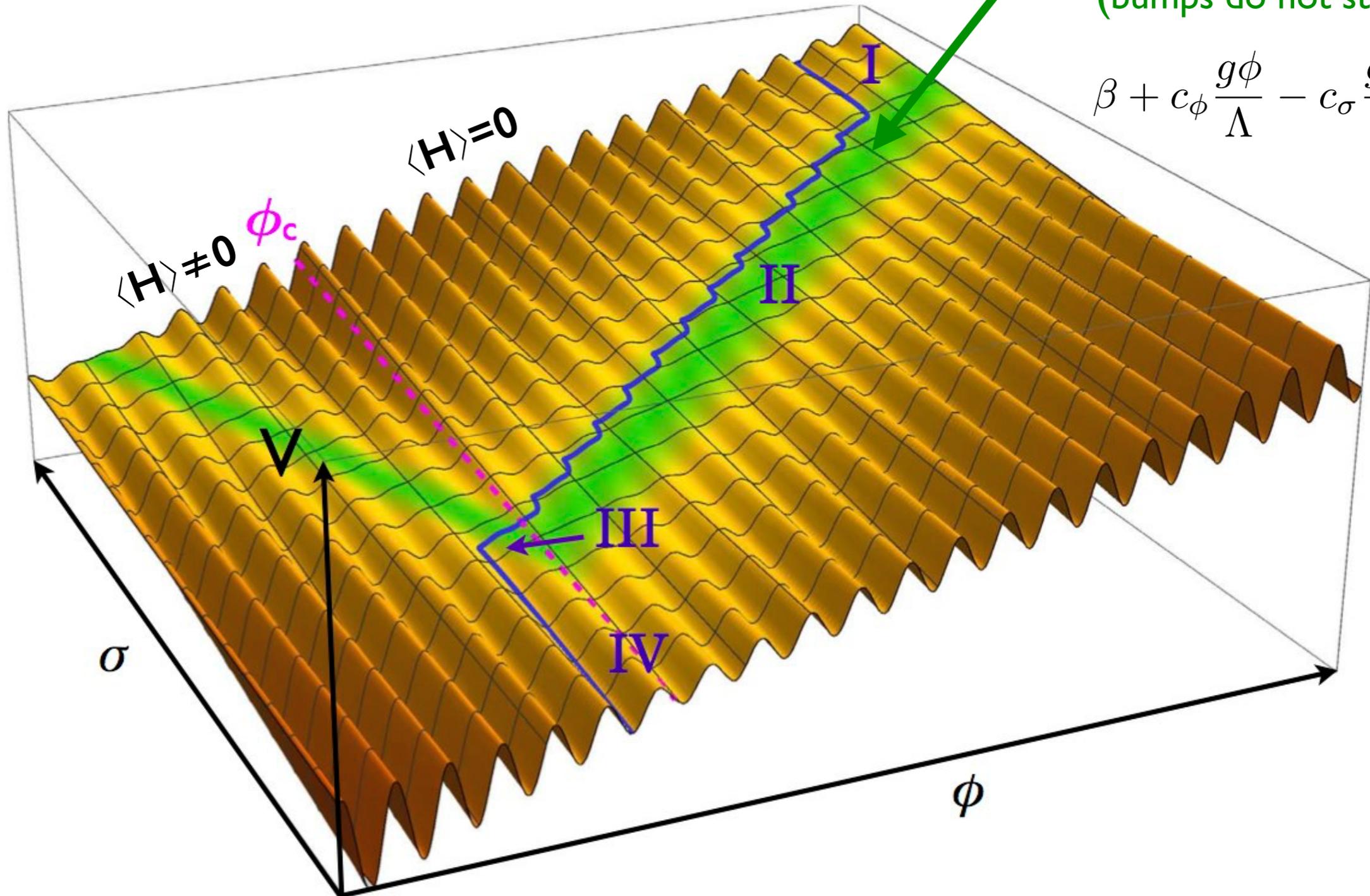
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area where $A \approx 0$

(bumps do not stop ϕ)

$$\beta + c_\phi \frac{g\phi}{\Lambda} - c_\sigma \frac{g_\sigma\sigma}{\Lambda} \approx 0$$



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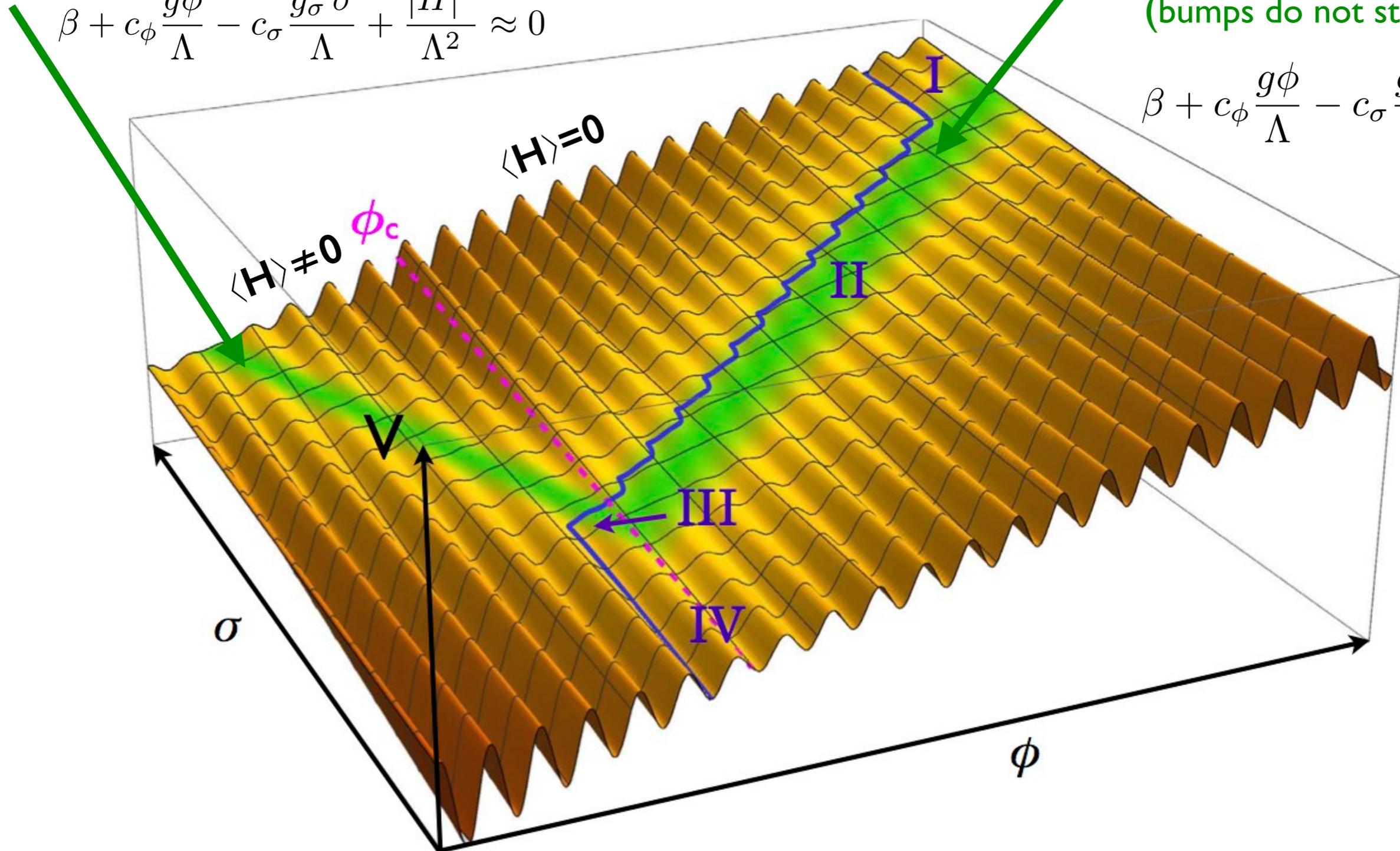
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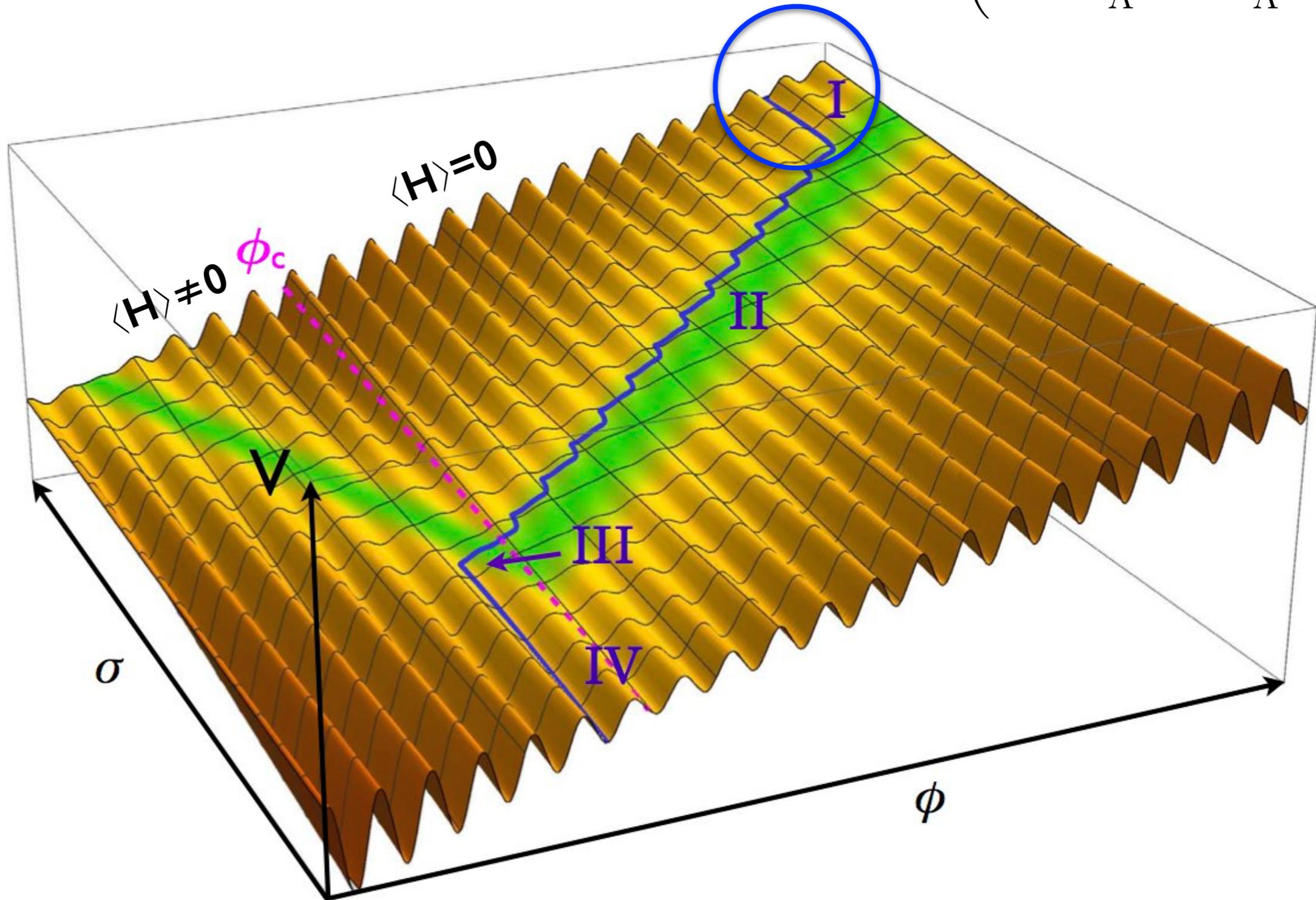
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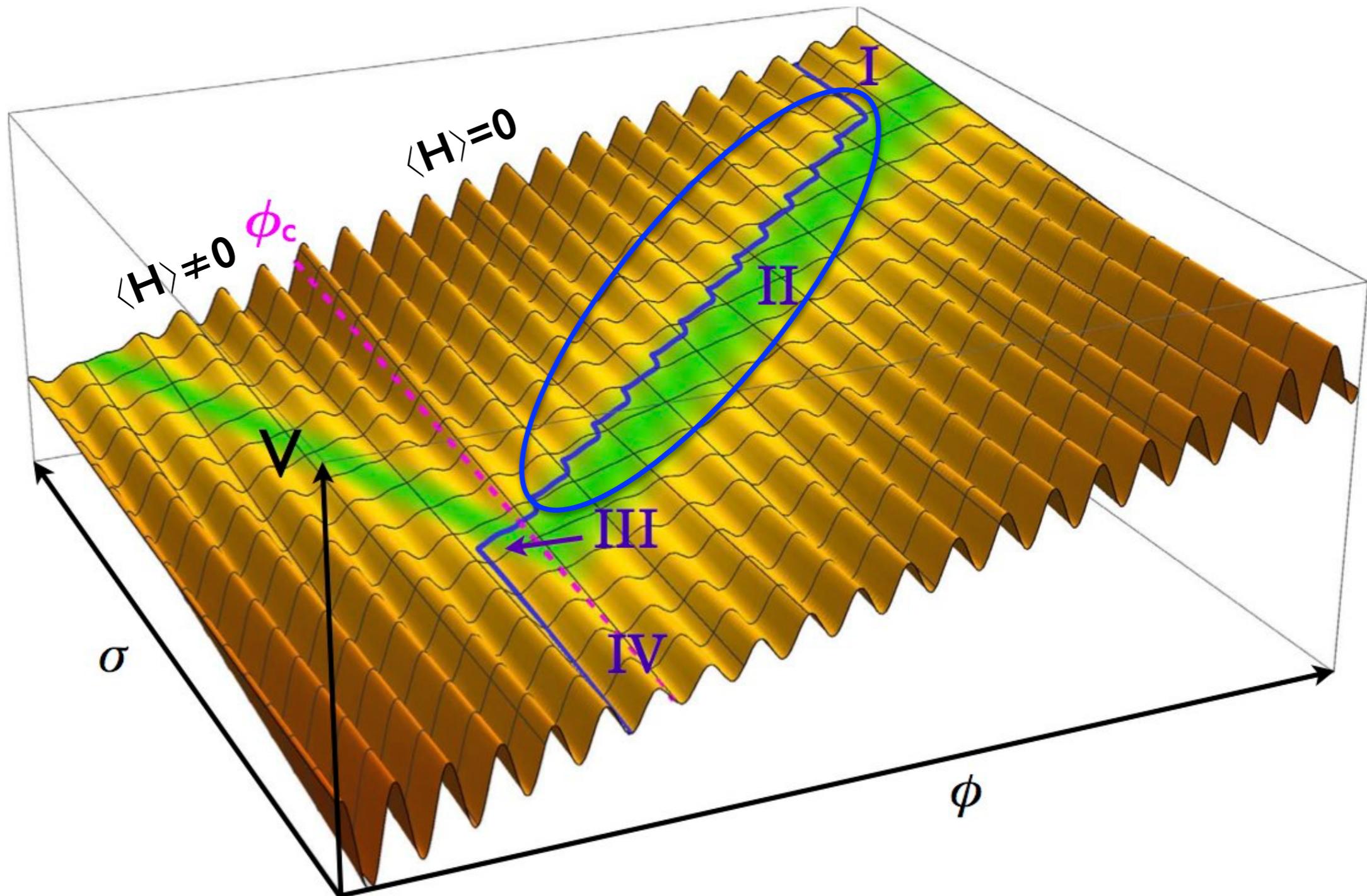
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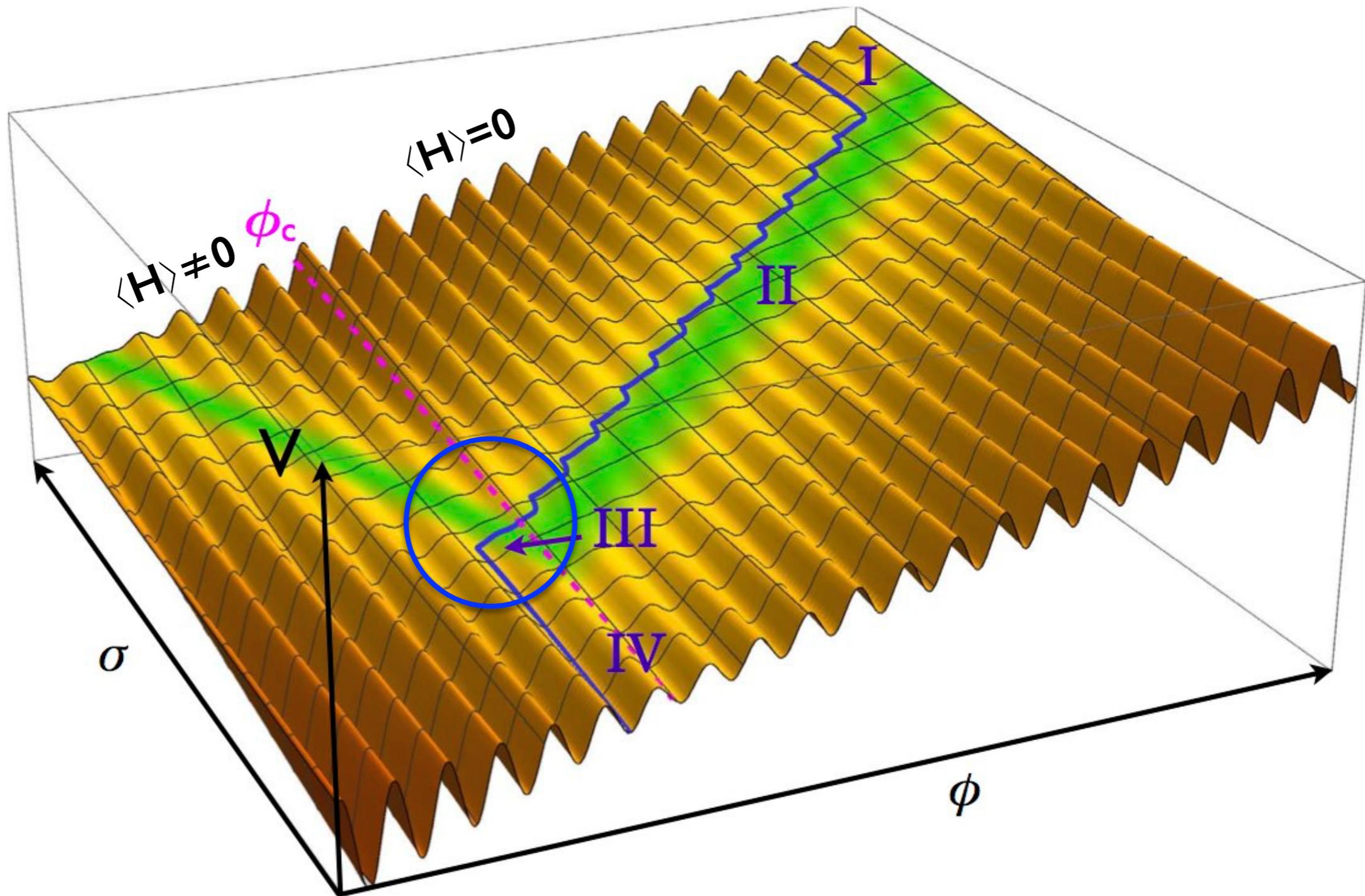
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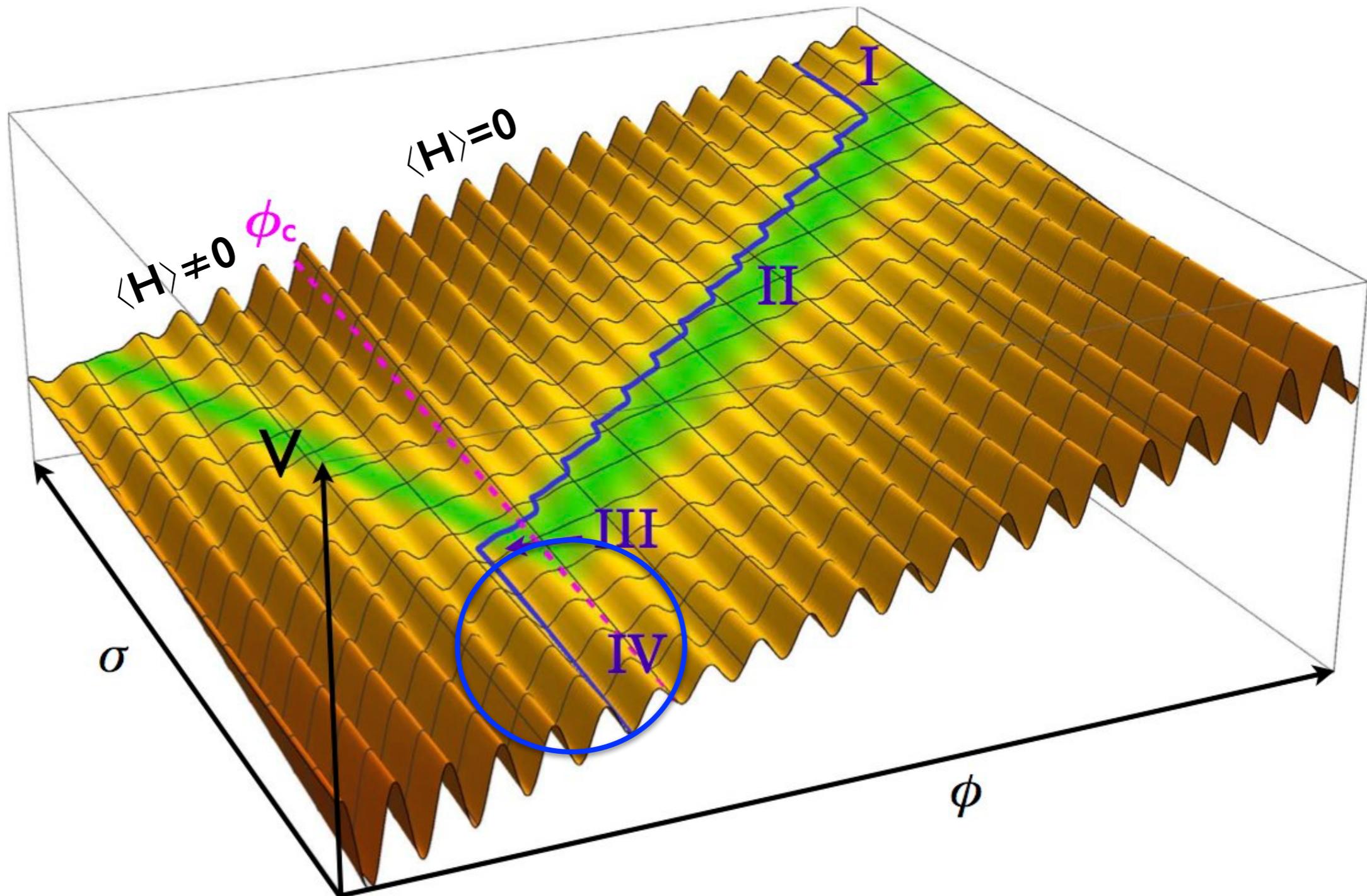
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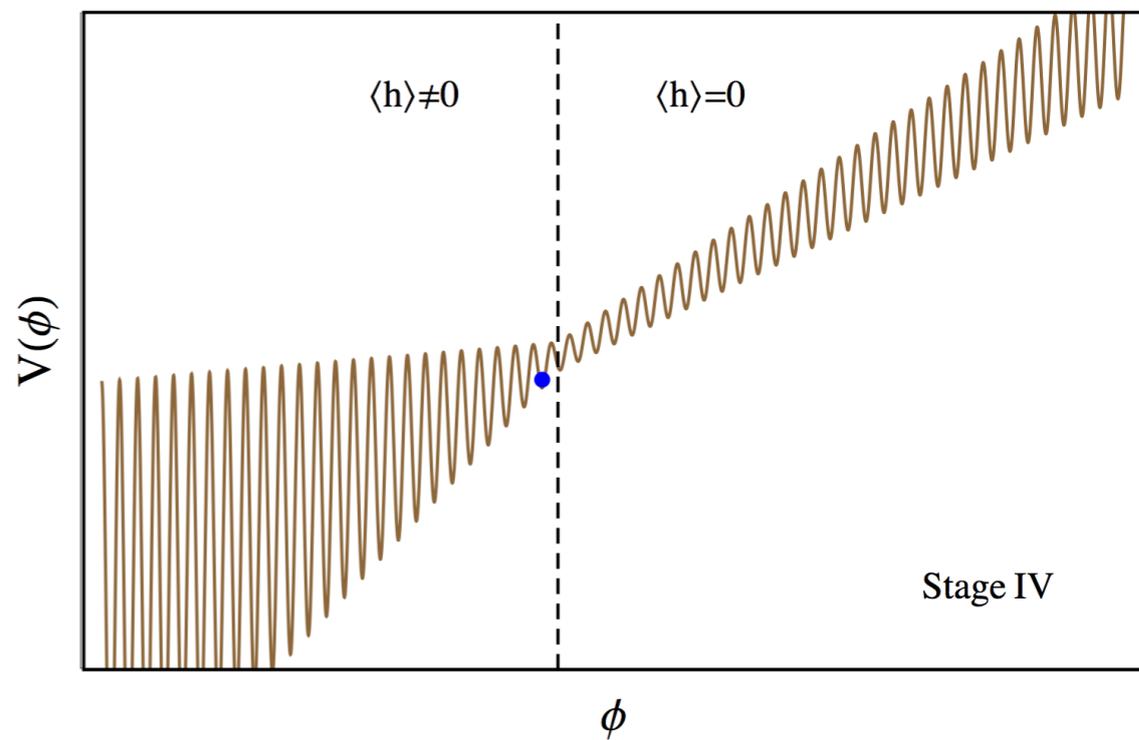
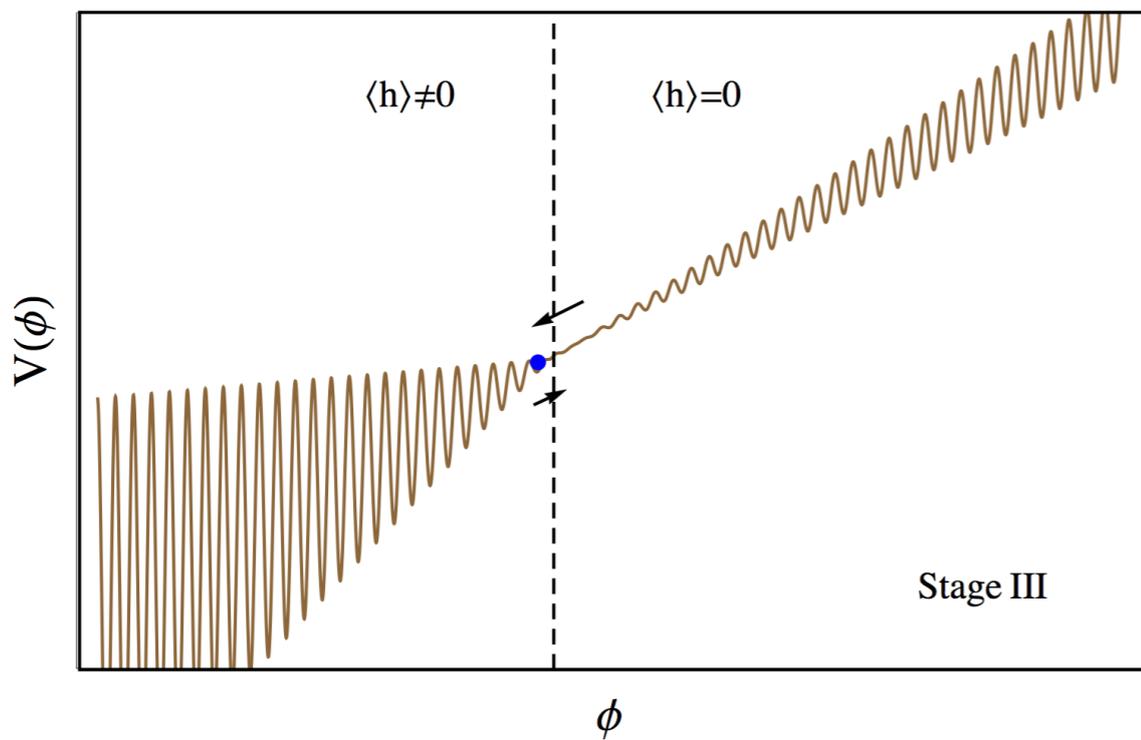
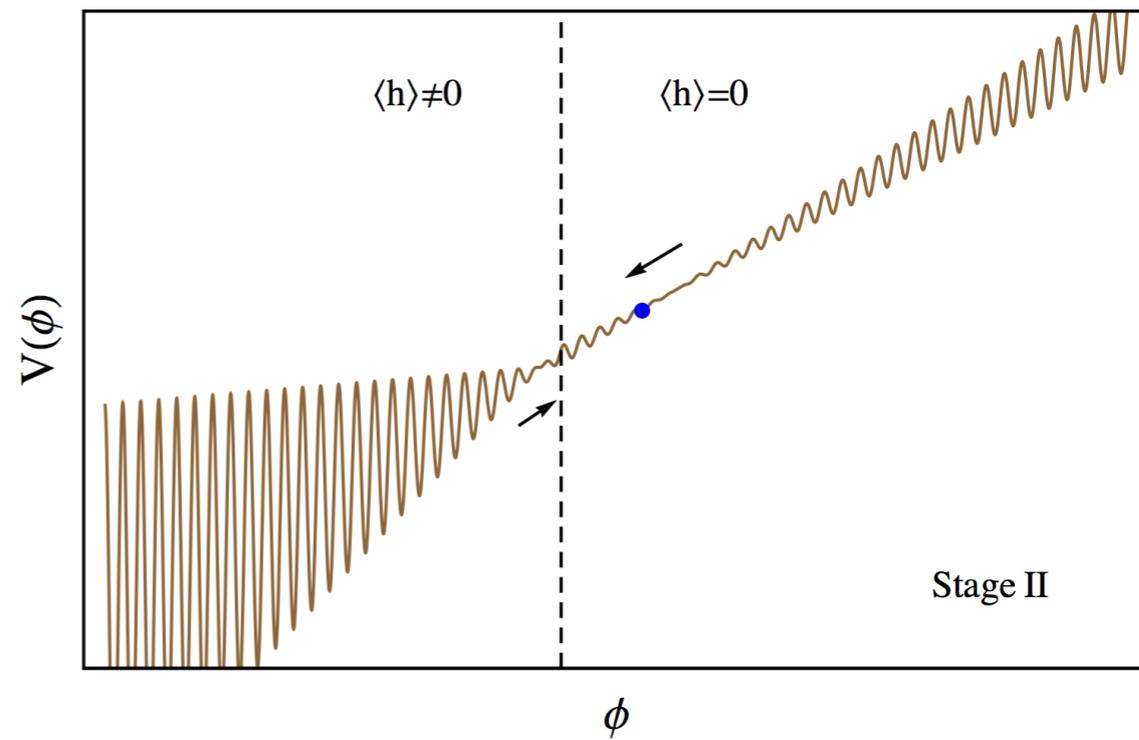
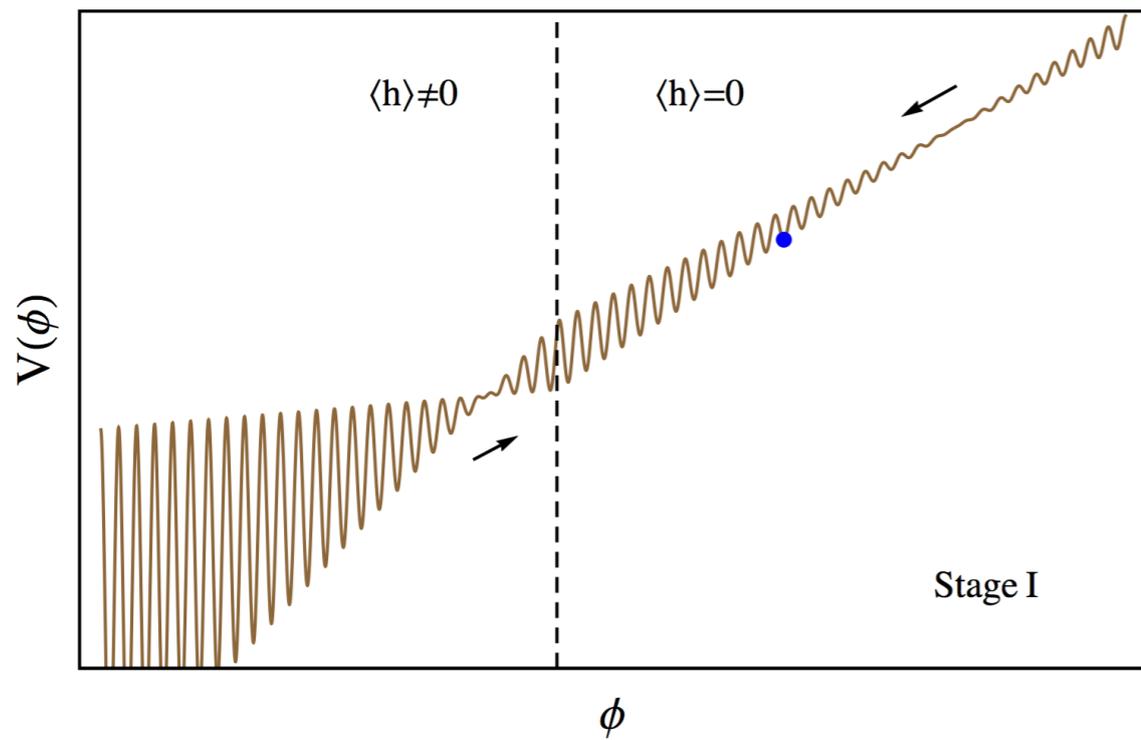
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Two scanner model: “The Movie”



Conditions on parameters:

- $\epsilon \lesssim v^2 / \Lambda^2$ to avoid to be dominated by terms like $\epsilon^2 \Lambda^4 \cos^2(\phi/f)$
- $H_I^3 \lesssim g_\sigma \Lambda^3$ to avoid quantum wiggles spoiling classical rolling
- $g_\sigma \lesssim g$ to avoid ϕ not tracking σ
- $\frac{\Lambda^2}{M_P} \lesssim H_I$ to avoid ϕ & σ affect inflation

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Minimization: $v^2 \simeq \frac{g\Lambda f}{\epsilon}$

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- $g_\sigma \lesssim g$ to avoid ϕ not tracking σ
- $\frac{\Lambda^2}{M_P} \lesssim H_I$ to avoid ϕ & σ affect inflation

Minimization: $v^2 \simeq \frac{g\Lambda f}{\epsilon}$

$\frac{\Lambda^3}{M_P^3} \lesssim g_\sigma \lesssim g \lesssim \frac{v^4}{f\Lambda^3}$



$\Lambda \lesssim (v^4 M_P^3)^{1/7} \simeq 2 \times 10^9 \text{ GeV}$

UV origin of the periodic term:

Strong sector
a la QCD
with a light fermion: N

+ **Axion-like ϕ**



$\frac{\phi}{f} G'_{\mu\nu} \tilde{G}'^{\mu\nu}$

UV origin of the periodic term:

Strong sector
a la QCD
with a light fermion: N

+ Axion-like ϕ

$\xrightarrow{\hspace{10em}}$ $\frac{\phi}{f} G'_{\mu\nu} \tilde{G}'^{\mu\nu}$

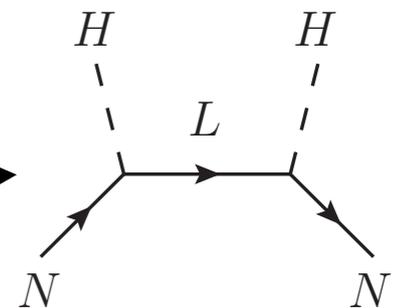
Axion potential:

$$V \simeq \Lambda^3 m_N \cos(\phi/f)$$

Assuming mass of N given by:

$$m_N \simeq \epsilon \left(\Lambda + g_\sigma \sigma + g\phi - \frac{|H|^2}{\Lambda} \right)$$

from integrating
a fermion-doublet L



Phenomenological implications:

- Nothing at the LHC to be discovered!
- Only BSM below Λ :

ϕ & σ : Light scalars weakly-coupled to the SM

$$m_\phi \sim 10^{-20} - 10^2 \text{ GeV}$$

$$m_\sigma \sim 10^{-45} - 10^{-2} \text{ GeV}$$

mixing to the SM through the Higgs: $|H|^2 \cos \phi/f$, $g\phi|H|^2$

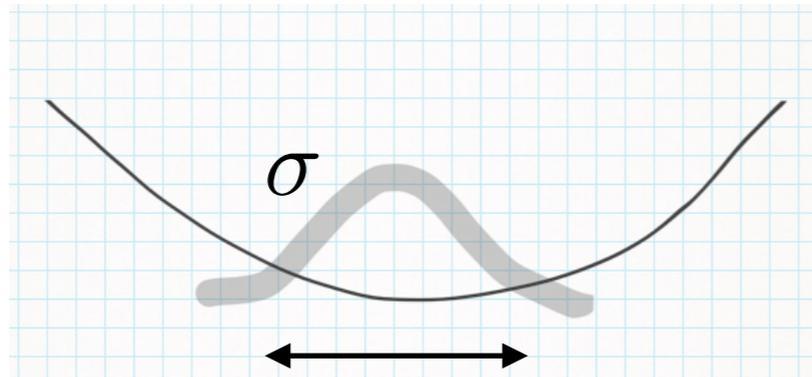
Benchmark values: $\Lambda \sim 10^9 \text{ GeV}$ \rightarrow $m_\phi \sim 100 \text{ GeV}$
 $\theta_{\phi h} \sim 10^{-21}$
 $\phi\phi hh$ -coupling $\sim 10^{-14}$
 $m_\sigma \sim 10^{-18} \text{ GeV}$
 $\theta_{\sigma h} \sim 10^{-50}$

Experimental constraints:

From cosmological overabundances, late decays, BBN bounds, γ -rays, CMB, pulsar timing observations, ...

Interestingly, σ as it oscillates around its minimum can be a good Dark Matter candidate (as axions)

$$V(g_\sigma \sigma / \Lambda)$$



quantum spreading

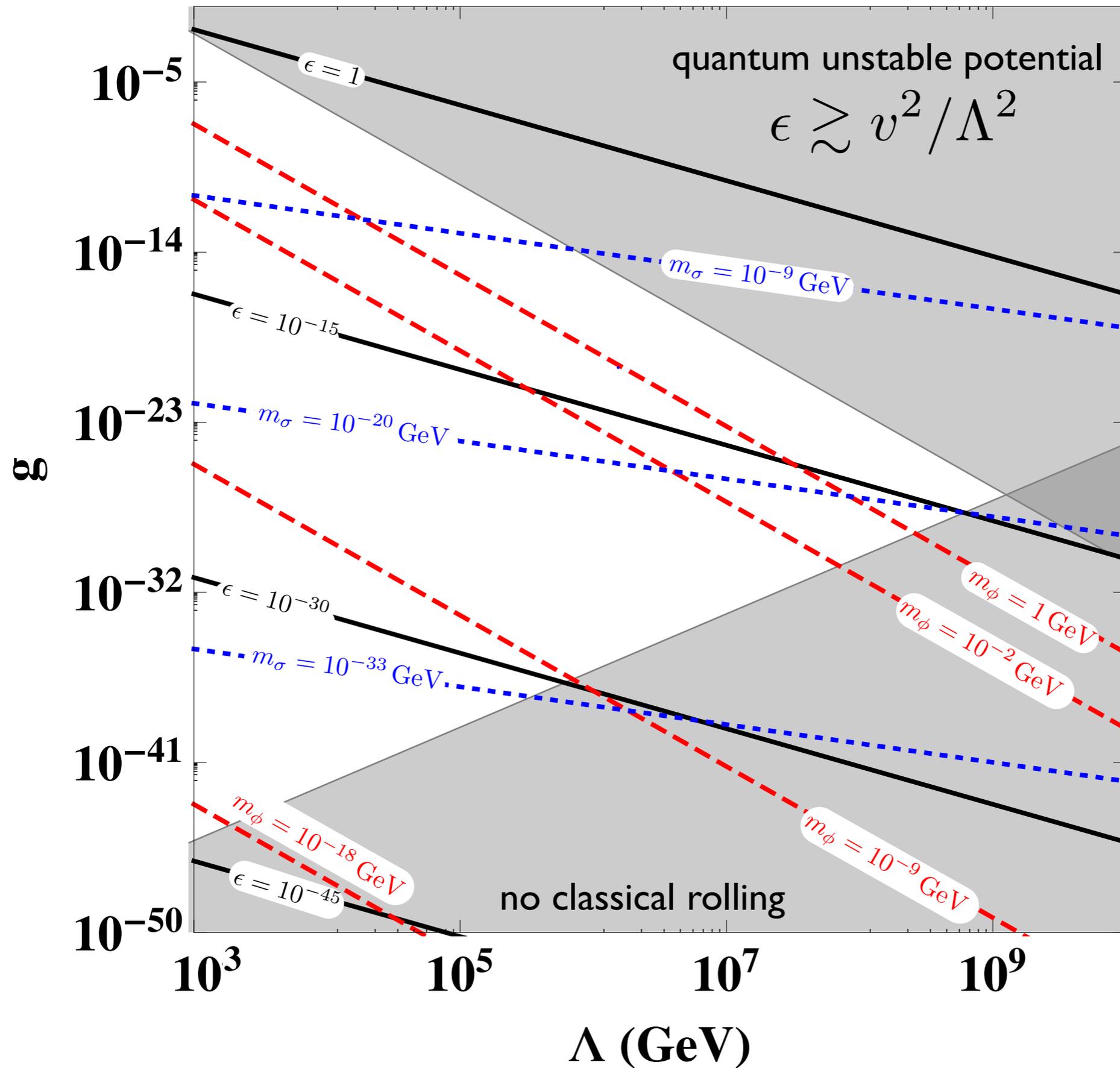
$$\sim \sqrt{N_e} H_I$$



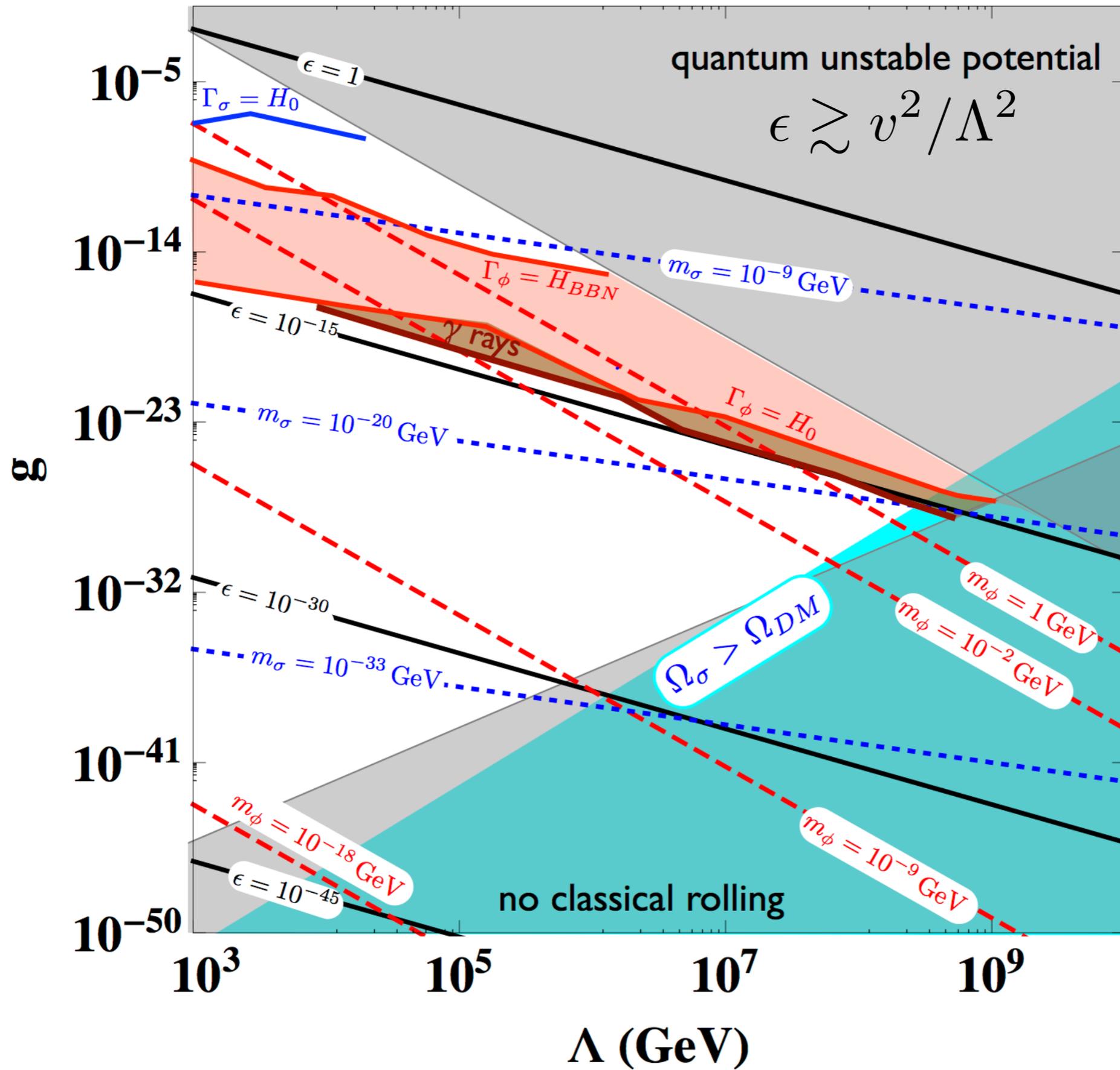
$$\rho_{ini}^\sigma \sim H_I^4$$

$$\rho_\sigma(T) \sim \rho_{ini}^\sigma (T/T_{osc})^3 \rightarrow \Omega_\sigma \gtrsim \left(\frac{10^{-27}}{g_\sigma} \right)^{3/2} \left(\frac{\Lambda}{10^8 \text{ GeV}} \right)^{13/2}$$

Taking $g_\sigma \sim 0.1g$



Taking $g_\sigma \sim 0.1g$



Supersymmetric UV completion (at Λ)

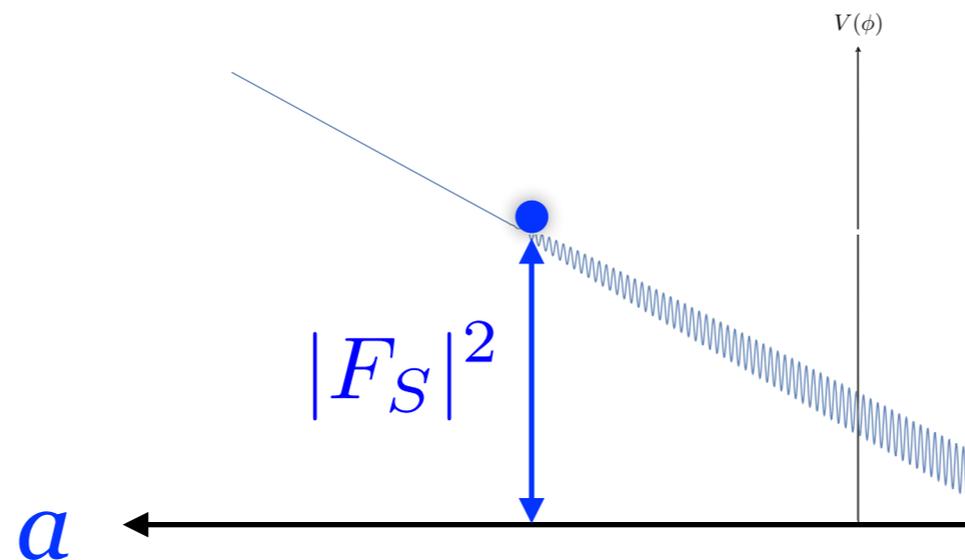
Batell, Giudice, McCullough 15

Fits nicely:

$$S = \frac{s + i a}{\sqrt{2}} + \sqrt{2} \theta \tilde{a} + \theta^2 F$$

axion

$$\text{MSSM} + \int d^2\theta [S W^a W^a + m S^2]$$



For nonzero a ,

supersymmetry is broken,

Higgs mass notice this breaking $\rightarrow M_H(a)$

Conclusions

“Relaxation” mechanism can give a natural explanation for

$$\langle h \rangle \sim 100 \text{ GeV} \ll \Lambda \sim 10^9 \text{ GeV} \quad (\text{not yet } \Lambda \sim M_{\text{P}})$$

based on a cosmological history of the Higgs and axion-like states

The good: Change of paradigm:

- The new-physics are weakly-coupled light states
- No big colliders needed!

Other type of experiments needed:

- Astro (γ -rays, pulsar timing, ...), CMB, table-top (fifth-force searches, EPV), ...

The bad & ugly: $N_e > 10^{38}$ & super-Planckian field excursions