A goldstino at the bottom of the cascade

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based upon:

Argurio, Musso, Redigolo hep-th/1411.2658
Argurio, Bertolini, Musso, Porri, Redigolo hep-th/1412.6499
Bertolini, Musso, Papadimitriou, Raj hep-th/1509.03594

Outline through *keywords*

- **Supersymmetry breaking** at **strong coupling**
- **non-SUSY** vacua of **cascading gauge theories**
- **Ward identities** without **UV fixed point**
- **Holography** and the **goldstino**

Conclusion *(flash forward)*

- *General* derivation of **Ward identities** with non-trivial sources
- Insensitivity to the **IR**
- Vacua with **spontaneous** **$\not{\text{SUSY}}$** $\Leftrightarrow$ $\overline{D}$-brane at the tip of the conifold
- Successful, **necessary** check for the existence of **$\not{\text{SUSY}}$** vacua in the **Klebanov-Strassler** cascading gauge theory

(super)symmetry breaking at strong coupling

Klebanov Witten hep-th/9905104

- **Dynamical SUSY breaking $\leftrightarrow$ hierarchy problem**
- **Chiral symmetry in QCD**
- **High T_c superconductors and strange metals**
- ...

At **strongly coupling**, fields either responsible or resulting from symmetry breaking are typically **composite**

Goldstino at strong coupling

- **Goldstone theorem** ensures the presence of a **massless mode in the spectrum**

- **Spontaneously** broken **SUSY** leads to a massless fermion mode, the **Goldstino**

- The **Goldstino** appears (e.g.) in the 2-point correlator of the supercurrent. Its residue is proportional to the ~~**SUSY**~~ order parameter

$$\langle \partial^\mu S_{\mu\alpha}(\sigma^\nu \bar{S}_\nu)_\beta \rangle = 2\,\epsilon_{\alpha\beta}\langle T \rangle$$

- **GOAL**: ***Explicitly check the above framework in holography***

Two questions

- **Is the solution gravitationally (meta)stable?**
- **Is the supergravity mode dual to the Goldstino present?**

→ A positive answer to the first guarantees that the solution is describing an actual QFT vacuum

→ The second ensures that in such a vacuum **SUSY** is broken spontaneously

One answer

*In **QFT** the two questions can be answered independently!*

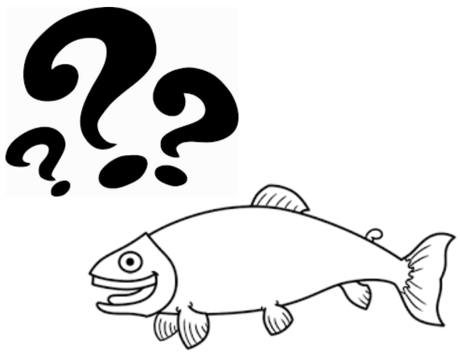
- Consider the **supercurrent** 2-pt function

$$\langle S_{\mu\alpha} \bar{S}_{\nu\dot{\beta}} \rangle$$

- The information regarding the **Goldstino** pole is encoded in the term implied (*upon integration*) by the **supersymmetry Ward identity**

$$\langle \partial^\mu S_{\mu\alpha}(x) \bar{S}_{\nu\dot{\beta}}(0) \rangle = -2\, \sigma^\mu_{\alpha\dot{\beta}} \langle T_{\mu\nu} \rangle \delta^4(x)$$

- **Ward Identities** depend on **UV** data only
- **Vacuum stability** is an **IR** property



Need to break conformal symmetry

- **Lorentz invariant** theory and vacuum

$$T_{\mu\nu} \propto \eta_{\mu\nu}$$

- **Spontaneous** **SUSY** breaking only allowed when conformal symmetry is broken **explicitly**.

$$\cancel{T^\mu_\mu = 0}$$

- Chase for **SUSY**-preserving terms which break conformality **explicitly**

Klebanov-Strassler theory

Klebanov, Strassler hep-th/0007191

Gauge Group	$SU(N+M) \times SU(N)$
Global Symmetry	$SU(2) \times SU(2) \times U(1)_B \times Z_{2M}$
Bifundamental (Chiral) Matter	$A_i\,,\, B_k \; (i,k = 1,2)$
Superpotential	$W = \lambda\, \mathrm{tr}(A_i B_k A_j B_l)\epsilon^{ij}\epsilon^{kl}$

$M \neq 0 \Rightarrow$ conformal symmetry is broken

(for $M = 0$ we have the Klebanov-Witten model)

Klebanov, Witten hep-th/9807080

non-trivial running $\Rightarrow$ cascade

(two nodes oppositely) (sequence of Seiberg duality)

Lack of a UV fixed point ***(formally KW with infinite rank)***

Cascading theories from 5d supergravity

$5d\ \mathcal{N}=2$ **SUGRA** from $10d$ type IIB **SUGRA** on $T^{1,1}$

Cassani Faedo hep-th/1008.0883

- $SU(2)\times SU(2)(\times U(1))$ **truncation**
- **UV asymptotic analysis only (*up to* z^4)**

⋆ **KT solution** describes the UV of

$SU(N+M)\times SU(N)$ KS model $N=kM$ k integer

⋆ ~~**SUSY**~~ **deformed KT solution** describes the UV of

$SU(N+M)\times SU(N)$ KS model $N=kM-p$ $p\ll M$

Kachru, Pearson, Verlinde hep-th/0112197

$p=\#$ of $\overline{D}$-branes

Spectrum and dictionary

$$\mathrm{Tr}(W_1^2 + W_2^2) + \dots \quad \sim$$

$$\mathrm{Tr}(W_1^2 - W_2^2) + \dots \quad \sim$$

$\mathcal{N} = 2$ multiplet	field fluctuations	AdSmass	Δ
gravity	$A - 2a$	$m^2 = 0$	3
	Ψ	$m = \frac{3}{2}$	$\frac{7}{2}$
	g	$m^2 = 0$	4
universal hyper	$b^\Omega - i\, c^\Omega$	$m^2 = -3$	3
	ζ_ϕ	$m = -\frac{3}{2}$	$\frac{7}{2}$
	$\tau = ie^{-\phi} + C_{(0)}$	$m^2 = 0$	4
Betti hyper	$t\, e^{i\theta}$	$m^2 = -3$	3
	ζ_b	$m = -\frac{3}{2}$	$\frac{7}{2}$
	$b^\Phi, \quad c^\Phi$	$m^2 = 0$	4
massive vector	V	$m^2 = 12$	6
	ζ_V	$m = \frac{9}{2}$	$\frac{13}{2}$
	$A + a$	$m^2 = 24$	7
	$b^\Omega + i\, c^\Omega$	$m^2 = 21$	7
	ζ_U	$m = -\frac{11}{2}$	$\frac{15}{2}$
	U	$m^2 = 32$	8

$$e^{-\phi} \longrightarrow \mathcal{O}_\phi \xrightarrow{\text{sum}} \frac{1}{g_1^2} + \frac{1}{g_2^2}$$

dim 4 operators

$$\widetilde{b}^\Phi = e^{-\phi}\, b^\Phi \longrightarrow \mathcal{O}_{\tilde{b}} \xrightarrow{\text{difference}} \frac{1}{g_1^2} - \frac{1}{g_2^2}$$

Klebanov, Nekrasov hep-th/9911096

Domain wall ansatz and SUSY solution

$$ds^2 = \frac{1}{z^2}\left(e^{2Y(z)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2X(z)}dz^2\right)$$

warped domain-wall geometry

$$e^{2Y} = h^{\frac{1}{3}}(z)\ ,\ e^{2X} = h^{\frac{4}{3}}(z)\ ,\ e^{2U} = h^{\frac{5}{2}}(z)$$

$$b^\Phi(z) = -\frac{9}{2}g_s M\ \log\left(z/z_0\right)$$

$$\phi(z) = \log\, g_s\ ,\ V = 0$$

KT solution

(here obtained in a 5d formalism)

z_0 is a renormalization scale of the dual field theory

$$h(z) = \frac{27\pi}{4g_s}\left(g_s N + \frac{1}{4}a(g_s M)^2 - a(g_s M)^2 \log\left(z/z_0\right)\right)$$

$$a = 3/2\pi$$

SUSY breaking solutions

DeWolfe, Kachru, Mulligan hep-th/0801.1520

2-parameter family of ~~SUSY~~ solutions

$$e^{2Y} = h^{\frac{1}{3}}(z)\, h_2^{\frac{1}{2}}(z)\, h_3^{\frac{1}{2}}(z) \quad , \quad e^{2X} = h^{\frac{4}{3}}(z)\, h_2^{\frac{1}{2}}(z)$$

$$e^{2U} = h^{\frac{5}{2}}(z)\, h_2^{\frac{3}{2}}(z) \quad , \quad e^{2V} = h_2^{-\frac{3}{2}}(z)$$

$$b^{\Phi}(z) = -\frac{9}{2} g_s M \ \log\left(z/z_0\right) + z^4 \left[\left(\frac{9\pi N}{4M} + \frac{99}{32} g_s M - \frac{27}{4} g_s M \log\left(z/z_0\right) \right) \mathcal{S} - \frac{9}{8} g_s M \varphi \right] + \mathcal{O}(z^8)$$

$$\phi(z) = \log g_s + z^4 \left(3\mathcal{S} \log\left(z/z_0\right) + \varphi \right) + \mathcal{O}(z^8)$$

$$h(z) = \frac{27\pi}{4 g_s} \left(g_s N + \frac{1}{4} a (g_s M)^2 - a (g_s M)^2 \log\left(z/z_0\right) \right) + \frac{z^4}{g_s} \left[\left(\frac{54\pi g_s N}{64} + \frac{81}{4}\frac{13}{64} (g_s M)^2 - \frac{81}{16} (g_s M)^2 \log\left(z/z_0\right) \right) \mathcal{S} - \frac{81}{64} (g_s M)^2 \varphi \right] + \mathcal{O}(z^8)$$

$$h_2(z) = 1 + \frac{2}{3} \mathcal{S} z^4 + \mathcal{O}(z^8) \qquad h_3(z) = 1 + \mathcal{O}(z^8)$$

We need to focus just on up to z^4 order, however the parameterization allows for a deeper solution

Comments on the solutions

$\mathcal{S}$ →

- Correct parameter associated to $\bar{D}$ perturbation
- Determined by **IR** conditions
- Total mass proportional (only!) to it

φ →

In the confromal case ($M = 0$) it corresponds to an independent fluctuation of the dilaton *(in spirit similar to dilaton driven confinement)*

Gubser hep-th/9902155; Kuperstein, Truijen, Van Riet hep-th/1411.3358

- KS and KT differ at order z^3 by terms proportional to the conifold deformation parameter ϵ
- ϵ terms dominate over z^4 **~~SUSY~~** terms, *however* it does not affect the **~~SUSY~~** pattern

Bena, Giecold, Grana, Halmagyi, Massai hep-th/1106.6165

1-point functions

Functional variation of the renormalized action w.r.t. a source at a time

$$\langle T^{\mu\nu}\rangle = \frac{2}{\sqrt{-\widetilde{\gamma}}}\,\frac{\partial S_{\rm ren}}{\partial \widetilde{\gamma}_{\mu\nu}}\bigg|_{\phi,\widetilde{b}^{\Phi},\widetilde{U},\widetilde{\Psi}^{+},\zeta_{\phi}^{-},\widetilde{\zeta}_{b}^{-},\widetilde{\zeta}_{U}^{-}}\,, \qquad \langle \overline{S}^{-\mu}\rangle = \frac{-2i}{\sqrt{-\widetilde{\gamma}}}\,\frac{\partial S_{\rm ren}}{\partial \widetilde{\Psi}^{+}_{\mu}}\bigg|_{\widetilde{\gamma},\phi,\widetilde{b}^{\Phi},\widetilde{U},\zeta_{\phi}^{-},\widetilde{\zeta}_{b}^{-},\widetilde{\zeta}_{U}^{-}}$$

$$\langle \mathcal{O}_{\phi}\rangle = \frac{1}{2\sqrt{-\widetilde{\gamma}}}\,\frac{\partial S_{\rm ren}}{\partial \phi}\bigg|_{\widetilde{\gamma},\widetilde{b}^{\Phi},\widetilde{U},\widetilde{\Psi}^{+},\zeta_{\phi}^{-},\widetilde{\zeta}_{b}^{-},\widetilde{\zeta}_{U}^{-}}\,, \qquad \langle \overline{\mathcal{O}}^{+}_{\zeta_{\phi}}\rangle = \frac{1}{\sqrt{-\widetilde{\gamma}}}\,\frac{i}{\sqrt{2}}\,\frac{\partial S_{\rm ren}}{\partial \zeta_{\phi}^{-}}\bigg|_{\widetilde{\gamma},\phi,\widetilde{b}^{\Phi},\widetilde{U},\widetilde{\Psi}^{+},\widetilde{\zeta}_{b}^{-},\widetilde{\zeta}_{U}^{-}}$$

$$\langle \mathcal{O}_{\widetilde{b}}\rangle = \frac{1}{2\sqrt{-\widetilde{\gamma}}}\,\frac{\partial S_{\rm ren}}{\partial \widetilde{b}^{\Phi}}\bigg|_{\widetilde{\gamma},\phi,\widetilde{U},\widetilde{\Psi}^{+},\zeta_{\phi}^{-},\widetilde{\zeta}_{b}^{-},\widetilde{\zeta}_{U}^{-}}\,, \qquad \langle \overline{\mathcal{O}}^{+}_{\widetilde{\zeta}_{b}}\rangle = \frac{1}{\sqrt{-\widetilde{\gamma}}}\,\frac{i}{\sqrt{2}}\,\frac{\partial S_{\rm ren}}{\partial \widetilde{\zeta}_{b}^{-}}\bigg|_{\widetilde{\gamma},\phi,\widetilde{b}^{\Phi},\widetilde{U},\widetilde{\Psi}^{+},\zeta_{\phi}^{-},\widetilde{\zeta}_{U}^{-}}$$

$\tilde{\gamma}$ is the field theory metric

$S_{\rm ren} = S_{\rm reg} + S_{\rm ct}$ ***renormalized action at finite cut-off***

boundary/bulk coupling at finite radial cut-off → sources identified with **induced fields** on cut-off shell

Supersymmetry transformations

SUGRA transformations
in the *bulk*

$$\delta_\epsilon \zeta^I = -\frac{i}{2}\left(\partial\!\!\!/\varphi^I - \mathcal{G}^{IJ}\partial_J \mathcal{W}\right)\epsilon$$

$$\delta_\epsilon \Psi_A = \left(\nabla_A + \frac{1}{6}\mathcal{W}\Gamma_A\right)\epsilon$$

$$\delta_\epsilon \varphi^I = \frac{i}{2}\bar{\epsilon}\zeta^I + \text{h.c.}$$

$$\delta_\epsilon e^a_A = \frac{1}{2}\bar{\epsilon}\Gamma^a\Psi_A + \text{h.c.}$$

Axial gauge *(radial)* $\Psi_r = 0$

$$\left(\nabla_r + \frac{1}{6}\mathcal{W}\Gamma_r\right)\epsilon = 0$$

asymptotic large radii
analysis

$$\epsilon = \epsilon^+ + \epsilon^-$$

parameters of the boundary
SUSY and **Superconformal**
transformations respectively

$$\epsilon^+(z,x) = z^{-1/2}h(z)^{1/12}\epsilon_0^+(x) + \mathcal{O}(z^4)$$

$$\epsilon^-(z,x) = z^{1/2}h(z)^{-1/12}\epsilon_0^-(x) + \mathcal{O}(z^4)$$

Supersymmetry Ward identities

Invariance of $S_{\rm ren}$ under a generic SUSY transformation

$$\delta_{\epsilon^+} S_{\rm ren} = \int d^4x \sqrt{-\widetilde{\gamma}} \left(\frac{i}{2} \langle \overline{S}^{-\mu} \rangle \delta_{\epsilon^+} \widetilde{\Psi}^+_\mu + \frac{1}{2} \langle T^{\mu\nu} \rangle \delta_{\epsilon^+} \widetilde{\gamma}_{\mu\nu} + 2 \langle \mathcal{O}_\phi \rangle \delta_{\epsilon^+} \phi + 2 \langle \mathcal{O}_{\widetilde{b}} \rangle \delta_{\epsilon^+} \widetilde{b}^\Phi \right)$$

$$= \int d^4x \sqrt{-\widetilde{\gamma}} \left(-\frac{i}{2} e^{-\frac{2}{15}U} \langle \partial_\mu \overline{S}^{-\mu} \rangle - \frac{1}{2} \langle T^{\mu\nu} \rangle \overline{\widetilde{\Psi}}{}^+_\mu \widetilde{\Gamma}_\nu + i \langle \mathcal{O}_\phi \rangle \overline{\zeta}^-_\phi + i \langle \mathcal{O}_{\widetilde{b}} \rangle \overline{\widetilde{\zeta}}{}^-_b \right) \epsilon^+ = 0$$

$$\frac{i}{2} e^{-\frac{2}{15}U} \langle \partial_\mu \overline{S}^{-\mu} \rangle = -\frac{1}{2} \langle T^{\mu\nu} \rangle \overline{\widetilde{\Psi}}{}^+_\mu \widetilde{\Gamma}_\nu + i \langle \mathcal{O}_\phi \rangle \overline{\zeta}^-_\phi + i \langle \mathcal{O}_{\widetilde{b}} \rangle \overline{\widetilde{\zeta}}{}^-_b$$

"operator identity" at the cut-off

$$e^{-\frac{2}{15}U} \langle \partial_\mu \overline{S}^{-\mu}(x) S^-_\nu(0) \rangle = 2i\, \widetilde{\Gamma}_\mu \langle T^\mu_\nu \rangle\, \delta^4(x,0)$$

further functional differentiation, then sources put to zero

Eventual **zero cut-off limit**

$$\langle \partial^\mu S_{\mu\alpha}(x)\ \bar{S}_{\nu\dot\beta}(0) \rangle_{\rm QFT} = -2\, \sigma^\mu_{\alpha\dot\beta} \langle T_{\mu\nu} \rangle_{\rm QFT}\ \delta^4(x)$$

SUSY Ward identity

Closing the circle: the *goldstino* mode

Actual evaluation of the bosonic 1-point functions on the **~~SUSY~~** background

$$\langle T^{\mu}_{\mu} \rangle_{\mathrm{QFT}} = -12\,\mathcal{S}$$

$$\langle \mathcal{O}_{\phi} \rangle_{\mathrm{QFT}} = \frac{(3\mathcal{S} + 4\varphi)}{2}$$

$$\langle \mathcal{O}_{\tilde{b}} \rangle_{\mathrm{QFT}} = \frac{4}{3\,M}\mathcal{S}$$

Aharony, Buchel, Yarom hep-th/0506002;
DeWolfe, Kachru, Mulligan hep-th/0801.1520

$$\langle S_{\mu\alpha}(x)\,\overline{S}_{\nu\dot{\beta}}(0) \rangle = \cdots - \frac{\mathrm{i}}{4\pi^2} \langle T \rangle \left(\sigma_{\mu} \overline{\sigma}^{\rho} \sigma_{\nu}\right)_{\alpha\dot{\beta}} \frac{x_{\rho}}{x^4}$$

corresponding *(upon Fourier transforming)* to the presence of a massless pole whose residue is proportional to $\langle T \rangle$ ...

...THE GOLDSTINO!

Conclusion

- *General* derivation of **Ward identities** with non-trivial sources

- Insensitivity to the **IR**

- Vacua with **spontaneous** **~~SUSY~~** $\Leftrightarrow$ $\overline{D}$-brane at the tip of the conifold

- Successful, **necessary** check for the existence of **~~SUSY~~** vacua in the **Klebanov-Strassler** cascading gauge theory

Outlook

- On with the program of holographic renormalization!
  (higher-point functions, systematic derivation of counter-terms and relation with Hamilton-Jacobi formalism)

- Extension to **~~SUSY~~** deformation of full KS

- F vs. D breaking toy models
  (D-term version of hep-th/1502.02631)

- **Induced fields** and derivation of **SUSY-respecting** counter-terms in generic setups
  *(e.g. renormalization of **SUSY** theories on curved manifolds)*

- Bottom-up and effective field theories for strongly coupled Goldstones (*and Goldstinos!*)


Thanks!

The SUGRA Model

$$S_b = \frac{1}{2\kappa^2}\int d^5x\,\sqrt{-g}\,\left[R - \mathcal{G}_{IJ}(\varphi)\,\partial_i\varphi^I\partial^i\varphi^J - \mathcal{V}(\varphi)\right]$$

$$\varphi^I = \begin{pmatrix} U \\ V \\ b^\Phi \\ \phi \end{pmatrix}, \quad \zeta^I = \begin{pmatrix} \zeta_U \\ \zeta_V \\ \zeta_b \\ \zeta_b^\Phi \end{pmatrix}, \quad \mathcal{G}_{IJ}(\varphi) = \begin{pmatrix} \frac{8}{15} & 0 & 0 & 0 \\ 0 & \frac{4}{5} & 0 & 0 \\ 0 & 0 & e^{-\frac{4}{5}(U+V)-\phi} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\mathcal{W}(\varphi) = -\frac{9}{2}(3\pi N + M)e^{-\frac{4}{3}U} + 3\,e^{-\frac{4}{15}(2U-3V)} + 2\,e^{-\frac{2}{15}(4U+9V)}$$

$$\mathcal{V}(\varphi) = \mathcal{G}^{IJ}\partial_I\mathcal{W}(\varphi)\partial_J\mathcal{W}(\varphi) - \frac{4}{3}\mathcal{W}(\varphi)^2$$

⋆ σ-**model** description and **superpotential** → efficient account of the action and its **SUSY** structure

$$\text{(e.g.)} \qquad \delta_\epsilon\zeta^I = -\frac{i}{2}(\partial\!\!\!/\varphi^I - \mathcal{G}^{IJ}\partial_J\mathcal{W})\epsilon$$

⋆ M and N **continuous** in **SUGRA** but **quantised** in **type IIB**

$$\frac{1}{4\pi^2\alpha'}\int_{\mathbf{S}^3} F_3 = M \qquad\qquad \frac{1}{(4\pi^2\alpha')^2}\int_{T^{1,1}} F_5 = N$$

SUSY field theories and string theory

- A large class of **SQFT** (*admitting* ~~***SUSY***~~ *vacua*) can be constructed in string theory
  - *e.g. quiver gauge theories arising from stacks of D-branes at Calabi-Yau singularities*
- Relying on the **holographic conjecture**, in the **decoupling limit** one can describe ~~**SUSY**~~ vacua by means of dual gravitational backgrounds (*further effort typically needed to acquire full control on* ***stability properties***, ***dynamics*** *and* ***spectrum***)
- Kachru-Pearson-Verlinde proposal to put $\bar{D}$ on the Klebanov-Strassler conifold

  Kachru, Pearson, Verlinde hep-th/0112197
  - Likely to be a mechanism with more general relevance (i.e. beyond the KS setting)

Argurio, Bertolini, Franco, Kachru hep-th/0610212 hep-th/0703236

$U(1)$ breaking *(holographic toy model)*

Prototypical example of the occurrence of **Goldstone** bosons in holography:
5D **vector** *with an axion-like* **scalar**.

$$S = \int d^5x \sqrt{G} \left[\frac{1}{4} F^{MN} F_{MN} + \frac{1}{2} m(z)^2 (\partial_M \alpha - A_M)(\partial^M \alpha - A^M) \right]$$

$$\Phi = \frac{1}{\sqrt{2}} m\, e^{i\alpha}$$

Higgs mechanism for the minimally-coupled Φ

Bianchi, Freedman, Skenderis hep-th/0105276
Argurio, Musso, Redigolo hep-th/1411.2658

$$A_M \to A_M + \partial_M \lambda \qquad \alpha \to \alpha + \lambda$$

Gauge transformation

Bulk: a non-trivial profile for $m(z)$ breaks *spontaneously* the gauge symmetry

$$(\Delta_{\mathcal{O}_m} = 3) \quad m(z) = m_0\, z + \tilde{m}\, z^3 + \ldots$$

EXPLICIT (m_0) **SPONTANEOUS** ($\tilde{m}$)

Boundary: the breaking of the global $U(1)$ depends on the details of the profile of $m(z)$

Spontaneous vs Explicit: *expectations*

The transverse and longitudinal parts of A_μ are **dual** respectively to the transverse and longitudinal parts of the boundary current J_μ

$$\langle J_\mu(k) J_\nu(-k)\rangle = -(k^2\delta_{\mu\nu} - k_\mu k_\nu)\, C(k^2) - m_0^2 \frac{k_\mu k_\nu}{k^2} F(k^2)$$

The longitudinal part of A_μ mixes with α which is **dual** to the imaginary part of the operator breaking the symmetry, $\mathrm{Im}\,\mathcal{O}_m$

$$\int d^4x\, \Phi_0\, \mathcal{O}_m + c.c. = \sqrt{2} \int d^4x\, (m_0\, \mathrm{Re}\mathcal{O}_m - m_0\, \alpha_0\, \mathrm{Im}\mathcal{O}_m)$$

- SPONTANEOUS: $F(k^2) = 0$ (*purely transverse*), massless pole in $C(k^2)$, massless pole in the correlator $\langle J_\mu(k)\, \mathrm{Im}\mathcal{O}_m(-k)\rangle = \sqrt{2}\, \tilde{m}_2\, (k_\mu/k^2)$

- EXPLICT: $F(k^2) \neq 0$, no massless pole in $C(k^2)$, Ward identity

$$\partial_\mu J^\mu = m_0\, \mathrm{Im}\, \mathcal{O}_m$$

Spontaneous vs Explicit: *outcome*

SPONTANEOUS

$$S_{\rm ren} = -\int d^4k\ \left[a^t_{0\mu}\tilde{a}^t_{2\mu} + \tilde{m}^2\,\alpha_0(\tilde{\alpha}_2 + a^l_0) + {\rm local}\right]$$

- $\tilde{m}_2^2\,\alpha_0\,a^l_0$ term leads to $\langle\partial_\mu J^\mu\,{\rm Im}\,\mathcal{O}_m\rangle \sim \tilde{m}_2$

$t \to$ transverse

$l \to$ longitudinal

EXPLICIT

$$S_{\rm ren} = -\int d^4k\ \left[a^t_{0\mu}\tilde{a}^t_{2\mu} + m_0^2\,(\alpha_0 - a^l_0)\tilde{\alpha}_2 + {\rm local}\right]$$

- The Ward identity (coming from $S_{\rm ren}$) depends only on the combination $\alpha_0 - a^l_0$ (*bulk gauge symmetry*)

The massless pole appears in $a^t_{0\mu}\tilde{a}^t_{2\mu}$ because of bulk dynamics and in the **explicit** case it cancels from the full correlator $\langle J_\mu J_\nu\rangle$ *(and it could be canceled by local terms* $\propto m_0^2$*).*

Lessons from the toy model

- In general the UV analysis **does not** commute with the zero-source limit

- When the breaking is **spontaneous** we have a **Goldstone** in $\langle J_\mu J_\nu \rangle$ (*which is transverse*) and in $\langle \mathrm{Im}\mathcal{O}_m \mathrm{Im}\mathcal{O}_m \rangle$ arising from bulk dynamics and insensitive to the UV details (*renormalization*)

- The **Goldstone** appears also in $\langle J_\mu(k)\, \mathrm{Im}\mathcal{O}_m(-k) \rangle = \sqrt{2}\, \tilde{m}_2\, (k_\mu / k^2)$ because the VEV of $\mathrm{Re}\mathcal{O}_m \propto \tilde{m}_2$ produces a Schwinger term whose holographic manifestation depends **only** on the renormalization

- Same kind of reasoning (*modulo technicalities*) can be adopted for other symmetries, e.g. R-symmetry, conformal, ...

Computation of the T 1-point function

$$\langle T^{\mu\nu} \rangle = e^{\frac{4}{15}U} e^{X} \left(-2 \left(K\gamma^{\mu\nu} - K^{\mu\nu} \right) + \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\rm ct}}{\delta \gamma_{\mu\nu}} \right)$$

$$K_{\mu\nu} = \frac{1}{2}\dot{\gamma}_{\mu\nu} = -\frac{1}{2} z e^{-X} \partial_z \left(\frac{e^{2Y}}{z^2} \right) \eta_{\mu\nu}$$

$$S_{\rm ct} = -\int d^4x \sqrt{-\gamma}\, 2\mathcal{W}$$

$$\langle T^{\mu}_{\nu} \rangle = -2 \left[3z\partial_z \log\left(\frac{e^{Y}}{z} \right) + e^{X}\mathcal{W} \right] \delta^{\mu}_{\nu}$$