“Universality” (and its limitations) in Cosmic Ray shower development

Paolo Lipari, INFN Roma “Sapienza”

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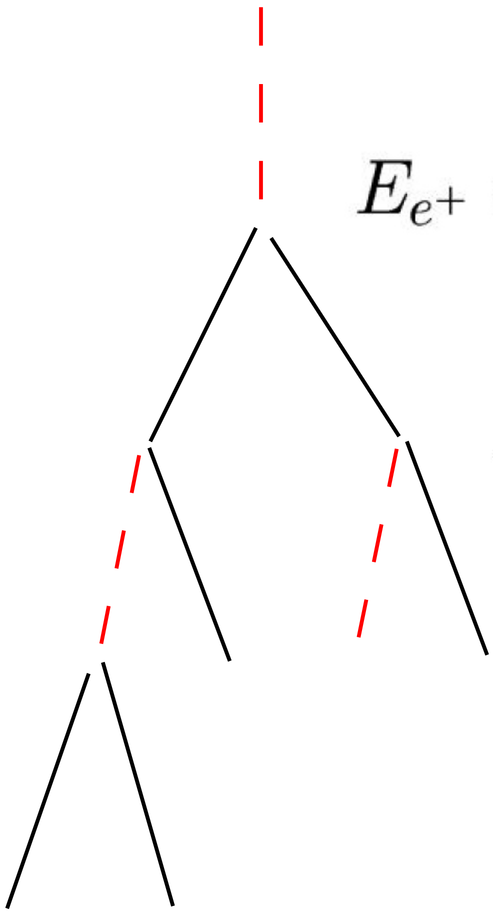
Definition of *"universality"* in the development of very high energy showers:

In showers of the same *"age"*
[that is at the same stage of development]
in a broad range of depth around shower maximum,
(and for E not too large):

the energy spectra of electrons/positrons and photons
have (in good approximation) the same
[age dependent] shape.

This is independent from the nature of the
primary particle (photon/proton/nucleus)
and of the properties of hadronic interactions.

ELECTROMAGNETIC SHOWER



$$E_{e^+} = E_\gamma \, u$$

$$E_\gamma = E_e \, v$$

$\psi(u)$ Pair Production

$$\gamma + Z \to e^+ + e^- + Z$$

$\varphi(v)$ Bremsstrahlung

$$e + Z \to e + \gamma + Z$$

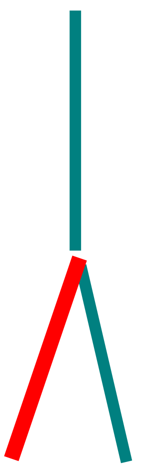
Radiation Length
(Energy independent)

Vertices :
Energy scaling
(splitting functions)

The "SPLITTING FUNCTIONS"

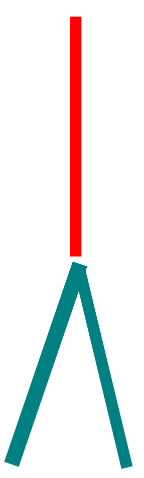
$$\varphi(v) = \left[\frac{d\sigma}{dv}(v)\right]_{\rm brems} \left(\frac{N_A}{A}\,\lambda_{\rm rad}\right)$$

$$\varphi(v) = \frac{1}{v}\,\left[1 - \left(\frac{2}{3} - 2\,b\right)\,(1-v) + (1-v)^2\right]$$



$$\psi(u) = \left[\frac{d\sigma}{du}(u)\right]_{\rm pair} \left(\frac{N_A}{A}\,\lambda_{\rm rad}\right)$$

$$\psi(u) = (1-u)^2 + \left(\frac{2}{3} - 2\,b\right)\,(1-u)\,u + u^2$$



$$b \simeq \frac{1}{18 \log(183\, Z^{-1/3})} \simeq 0.0135$$

$$\sigma_0 = \int_0^1 du\ \psi(u) = \frac{7}{9} - \frac{b}{3}$$

Very Simple Approximation:
ENERGY INDEPENDENT LOSS

$$\left.\frac{dE}{dt}\right|_{\text{collisions}} = -\varepsilon$$

SYSTEM of
INTEGRO-DIFFERENTIAL EQUATIONS

that describe the evolution with **t** of

$n_e(E,t)$ $n_\gamma(E,t)$

Average development of
an electromagnetic shower

$$\frac{\partial n_e(E,t)}{\partial t} = -\int_0^1 dv \;\; \varphi(v) \left[n_e(E,t) - \frac{1}{1-v} n_e\left(\frac{E}{1-v}, t\right) \right]$$

$$+2 \int_0^1 \frac{du}{u} \psi(u) \, n_\gamma \left(\frac{E}{u}, t\right)$$

$$+ \varepsilon \frac{\partial n_e(E,t)}{\partial E}$$

$$\frac{\partial n_\gamma(E,t)}{\partial t} = \int_0^1 \frac{dv}{v} \varphi(v) \, n_e \left(\frac{E}{v}, t\right) - \sigma_0 \, n_\gamma(E,t)$$

"Approximation B"

Solutions for the shower equations.

Physically relevant initial conditions:

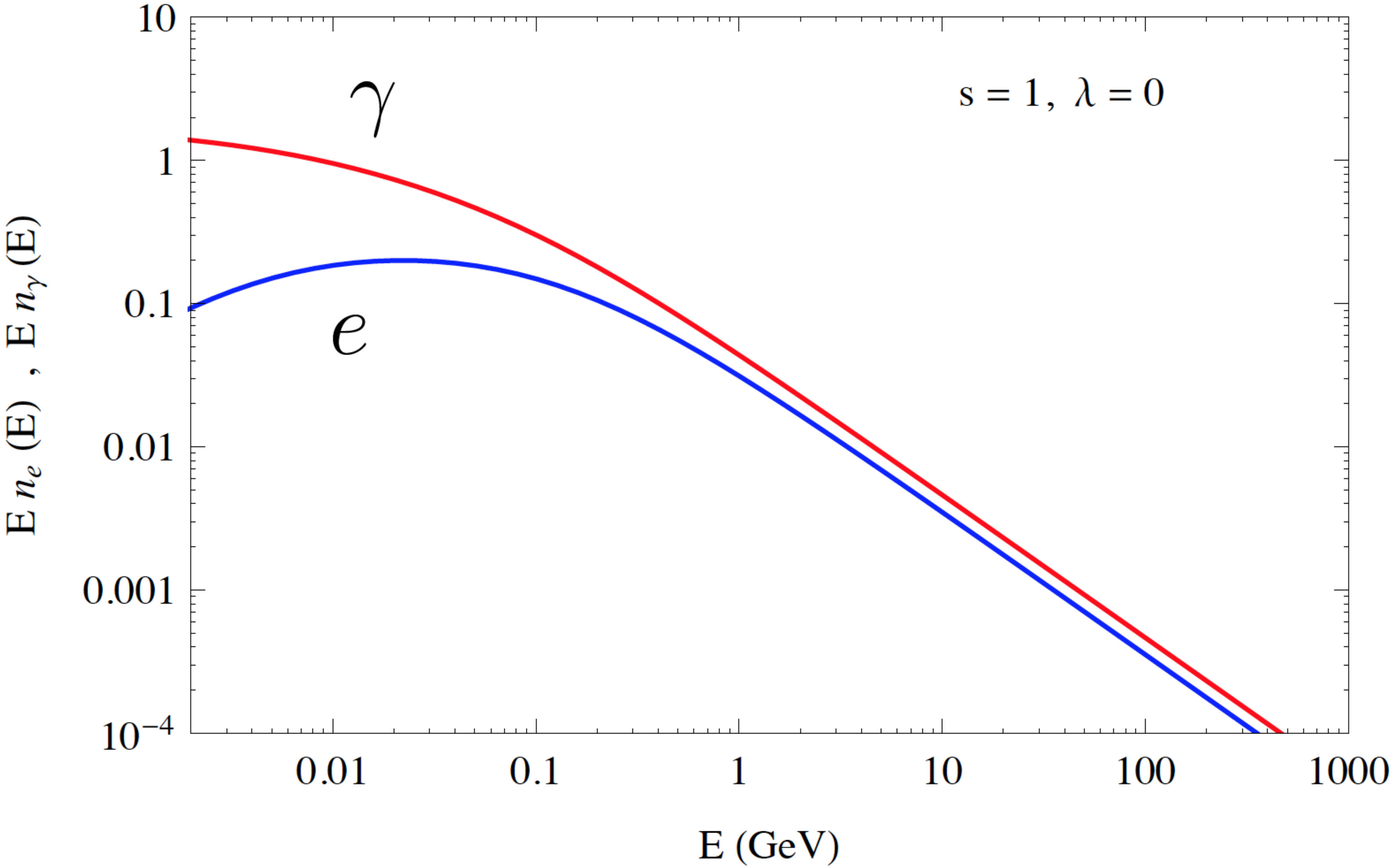
$$\begin{cases} n_e(E,0) = 0 \\ n_\gamma(E,0) = \delta[E - E_0] \end{cases}$$

Photon of energy E_0

$$\begin{cases} n_e(E,0) = \delta[E - E_0] \\ n_\gamma(E,0) = 0 \end{cases}$$

Electron of energy E_0

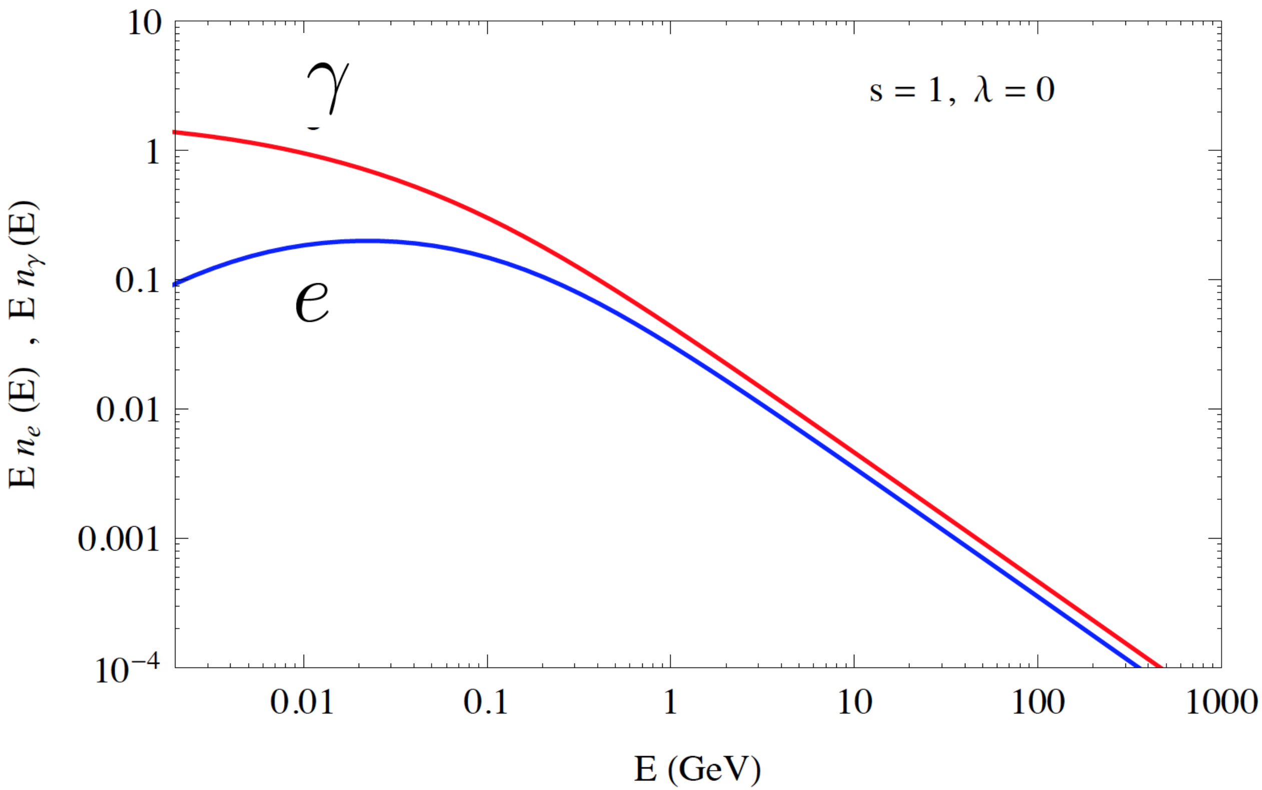
“Elementary solutions” of the electromagnetic shower equations:



Asymptotic form (large energy)

$$n_e(E,t) = K\ E^{-2}$$

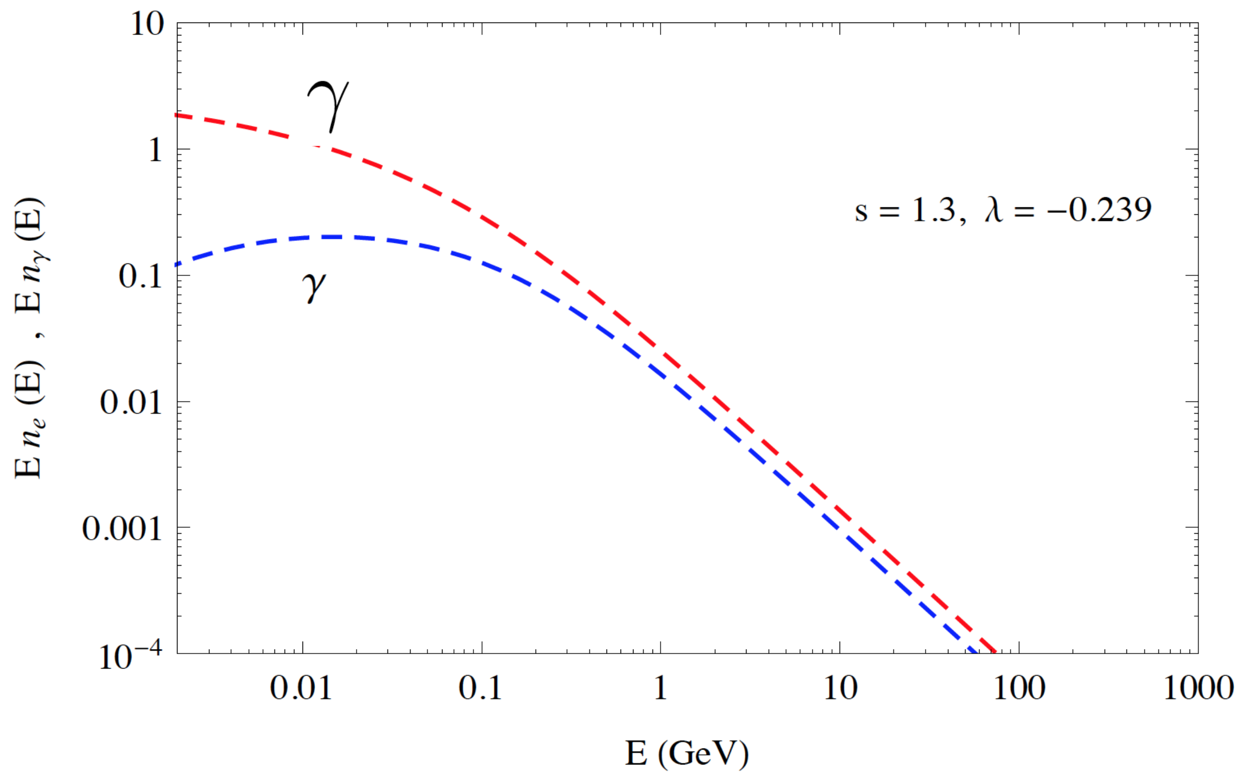
$$n_\gamma(E,t) = 1.310\ K\ E^{-2}$$



Spectra of electrons and photons remain constant at all t

$$n_e(E,t) = K\ E^{-2.3}\ e^{-0.239\,t}$$

$$n_\gamma(E,t) = 1.419\ K\ E^{-2.3}\ e^{-0.239\,t}$$

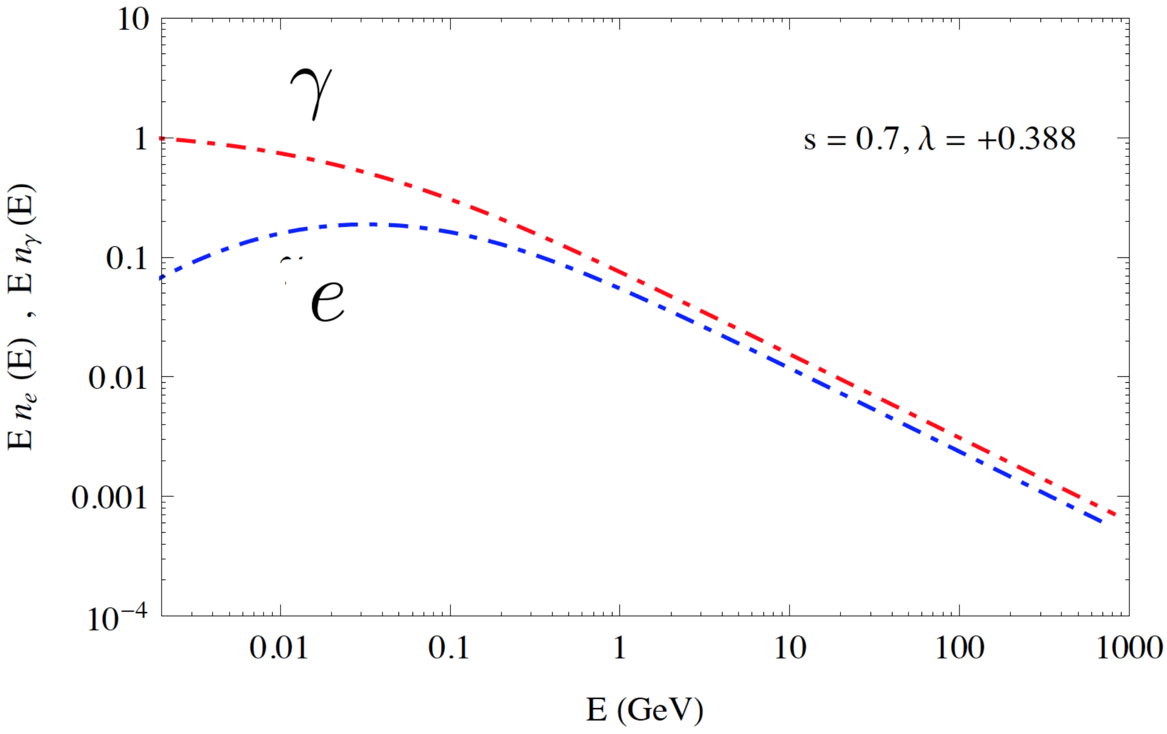


ectra of electrons
d photons have
nstant shape at all t

ponentially
creasing
rmalization

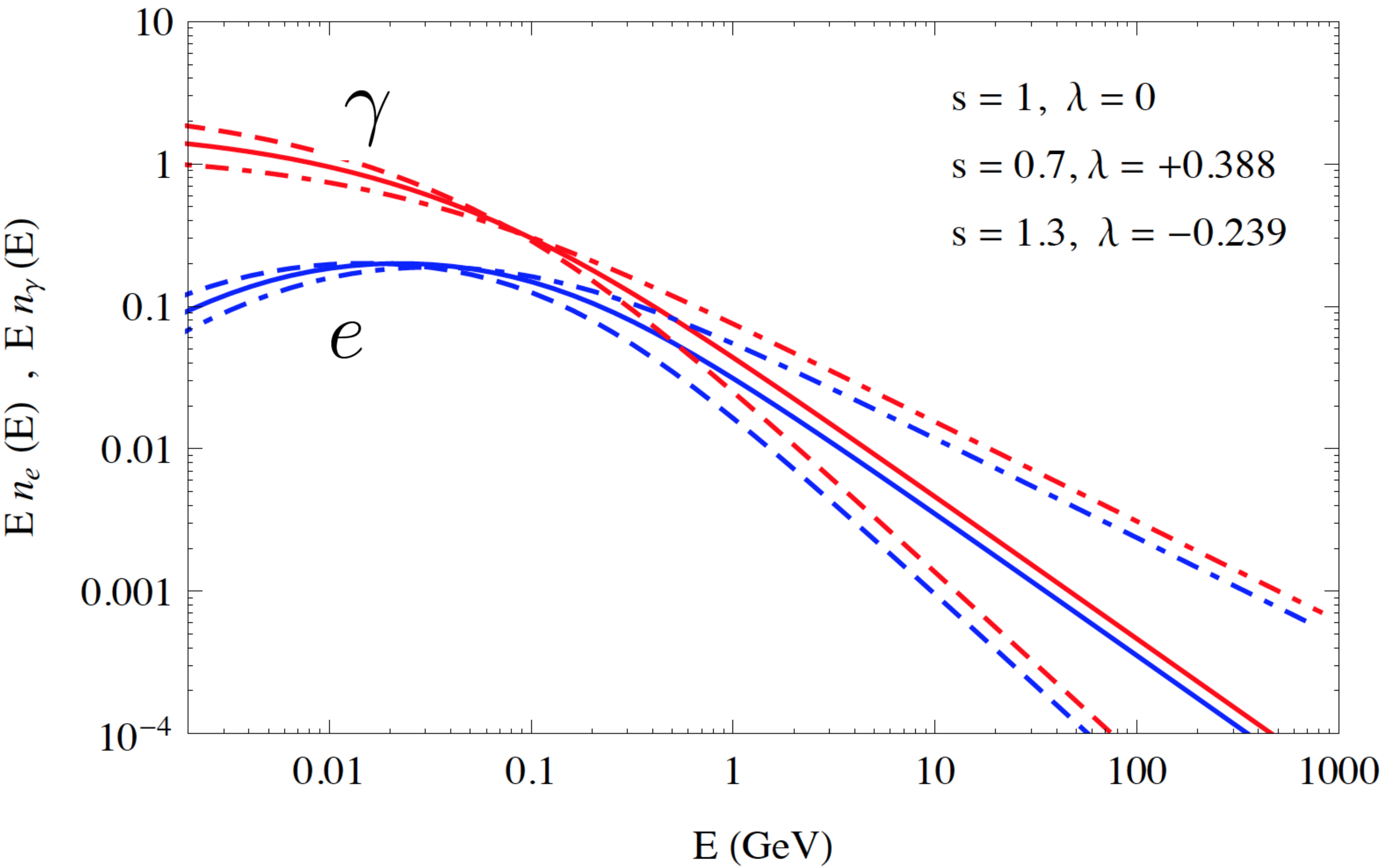
$$n_e(E,t) = K\ E^{-1.7}\ e^{+0.388\,t}$$

$$n_\gamma(E,t) = 1.303\ K\ E^{-1.7}\ e^{+0.388\,t}$$



Spectra of electrons and photons have constant shape at all t

Exponentially increasing normalization



γ
e
s = 1, λ = 0
s = 0.7, λ = +0.388
s = 1.3, λ = −0.239
10
1
0.1
0.01
0.001
10^−4
0.01
0.1
1
10
100
1000
E (GeV)
E n_e (E) , E n_γ (E)

Elementary solutions of the electromagnetic shower equations: [parametrized by real number s]

t-independent shape. Exponential t-dependence

$$n_e(E,t) = K \; f_s\left(\frac{E}{\varepsilon}\right) \; E^{-(s+1)} \; e^{\lambda(s)\,t}$$

$$n_\gamma(E,t) = r_\gamma(s) \; K \; g_s\left(\frac{E}{\varepsilon}\right) \; E^{-(s+1)} \; e^{\lambda(s)\,t}$$

$$\lim_{x\to\infty} f_s(x) = 1$$

$$\lim_{E\to 0} n_e(E) \to \text{const}$$

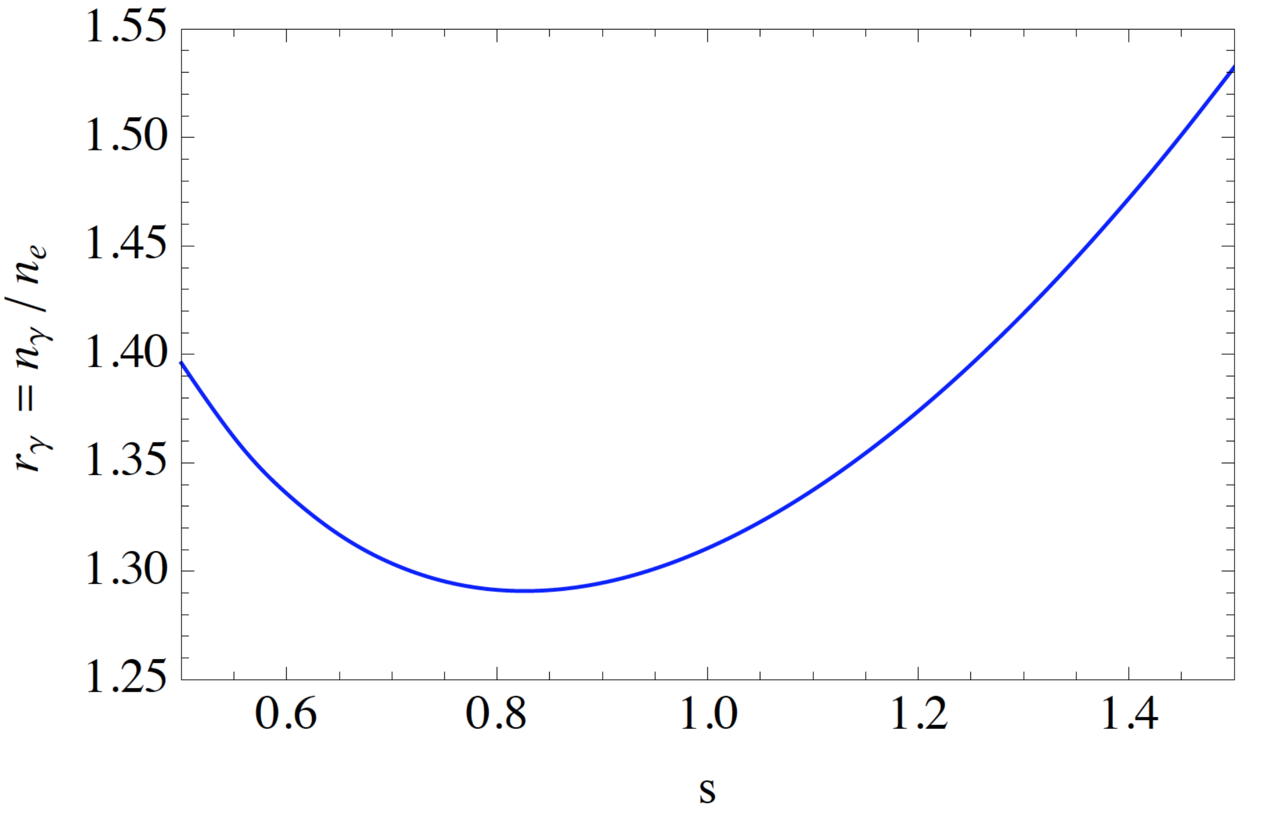
$$\lim_{x\to\infty} g_s(x) = 1$$

$$\lim_{E\to 0} n_\gamma(E) \propto E^{-1}$$

Elementary solutions can be calculated analytically,
[Unphysical - extend to Infinite energy]
Stable, in some sense are “attractors” for any initial condition.

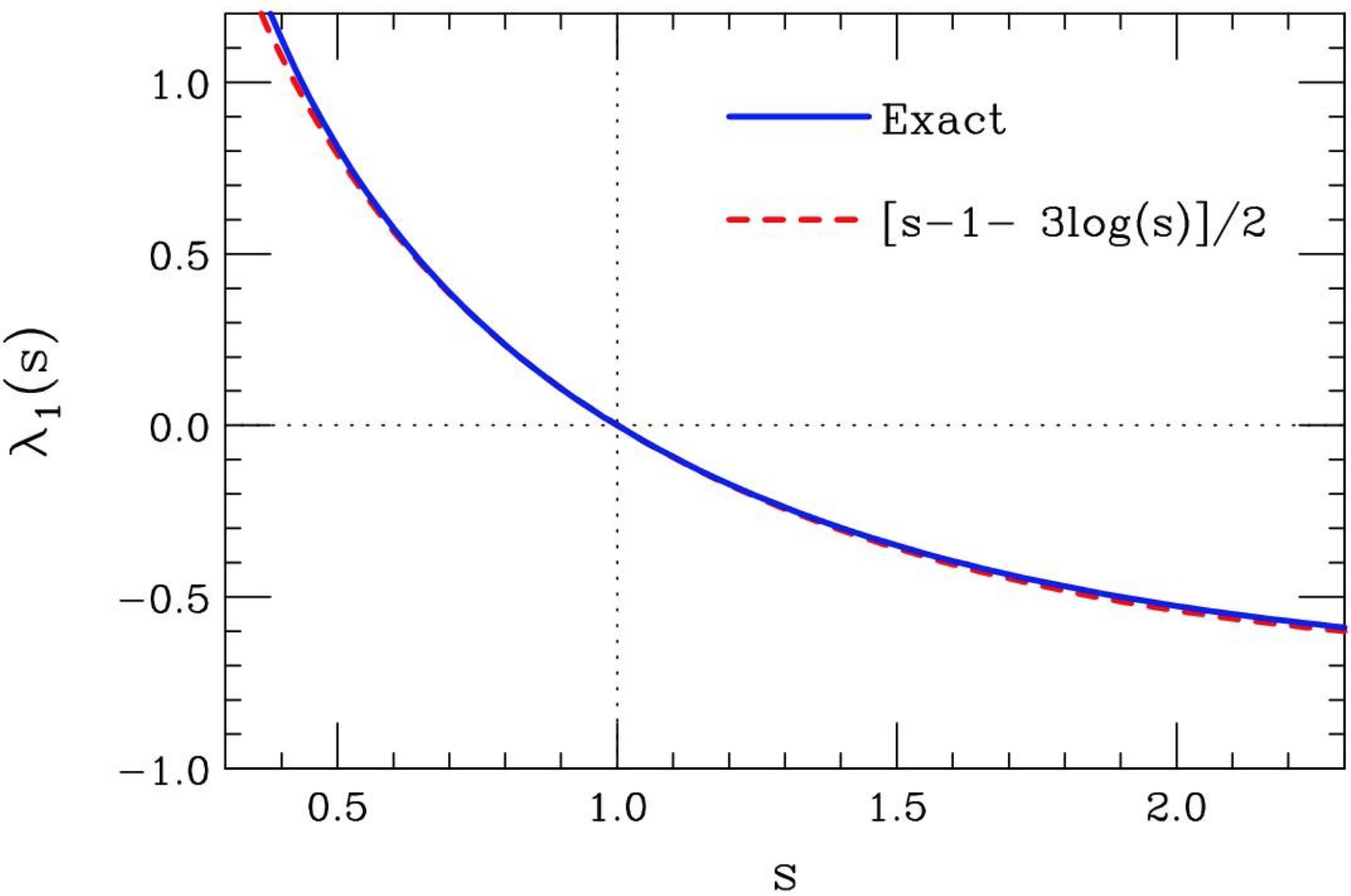
Two functions: $r_\gamma(s) \qquad \lambda(s)$

ratio photon/electrons at large energy



$$\lambda_{1,2}(s) = -\frac{1}{2}\left(A(s) + \sigma_0\right) \pm \frac{1}{2}\sqrt{\left(A(s) - \sigma_0\right)^2 + 4\,B(s)\,C(s)}$$

$$\begin{aligned} A(s) &= \int_0^1 dv\ \varphi(v)\ \left[1 - (1-v)^s\right] \\ &= \left(\frac{4}{3} + 2\,b\right)\ \left(\frac{\Gamma'(1+s)}{\Gamma(1+s)} + \gamma\right) + \frac{s\ (7 + 5\,s + 12\,b\ (2+s))}{6\ (1+s)\ (2+s)} \\ B(s) &= 2\int_0^1 du\ u^s\ \psi(u) = \frac{2\ \left(14 + 11\,s + 3\,s^2 - 6\,b\ (1+s)\right)}{3\ (1+s)\ (2+s)\ (3+s)} \\ C(s) &= \int_0^1 dv\ v^s\ \varphi(v) = \frac{8 + 7\,s + 3\,s^2 + 6\,b\ (2+s)}{3\,s\ (2 + 3\,s + s^2)} \end{aligned}$$



Exact
[s−1− 3log(s)]/2
1.0
0.5
0.0
−0.5
−1.0
0.5
1.0
1.5
2.0
$\lambda_1(s)$
s

Longitudinal development

df very high energy showers

$$N(t) = \int dE \; n_{\text{charged}}(E, t)$$

$$\simeq \int dE \; n_{e^\pm}(E, t)$$

$$t = \frac{X}{\lambda_{\text{rad}}}$$

Quasi-Model independent estimate of the Energy of the primary particle:

$$\varepsilon \int_0^\infty dt \; N(t) \simeq E_0 - E_\mu - E_\nu - E_{\text{had}}$$

$$\frac{dN(t)}{dt} = \lambda \ N(t)$$

Shower *"age"*
]stage of development]

In shower of the same "age"
the shape of the electron and photon energy spectra
are approximately equal to the elementary solutions

$$n_e(E,t) \simeq k \ \overline{n}_e^{s(\lambda)}(E)$$

$$n_\gamma(E,t) \simeq k \ \overline{n}_\gamma^{s(\lambda)}(E)$$

$$E \lesssim E_{\max}$$

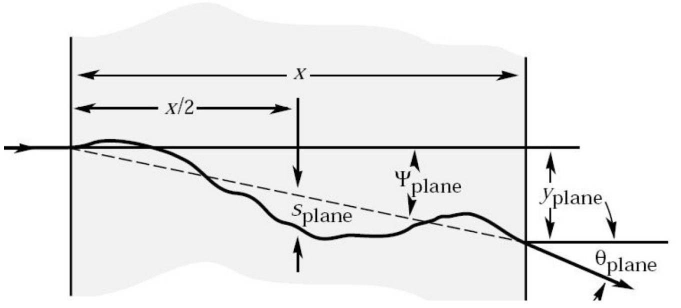
The relation
is valid only up
to a maximum
Energy

Lateral, angular, time distributions of particles in a shower:

$$n_e(X, E, \vec{r}, \vec{\theta}, t)$$

$$n_\gamma(X, E, \vec{r}, \vec{\theta}, t)$$

These distributions are also in reasonably good approximation "universal", that is determined by the shower age [because very strongly correlated by the energy distributions and multiple scattering].



In principle possibility of determining the shower age from a measurement at a single shower level

How can one use the property of "universality" ?

1. Correct calculation of the average energy losses in the shower. $\langle dE/dX \rangle$

2. Correct calculation of the Cherenkov emission in the emission [angular + energy distributions]

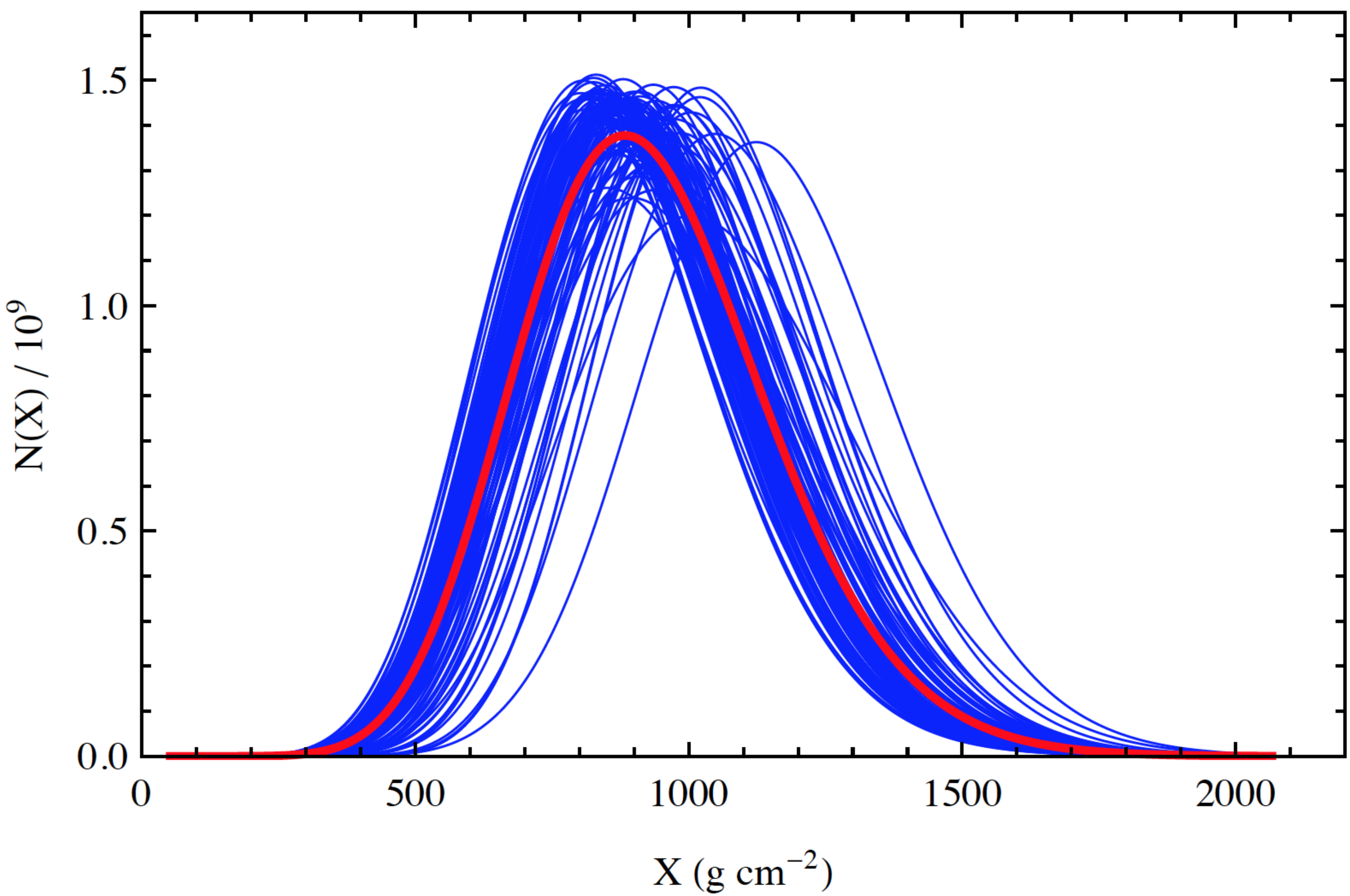
3. Determining the age of a shower in a surface detector [lateral/time distributions of the particles]

Implications of "Universality" for the *shape* of the shower Longitudinal Development.

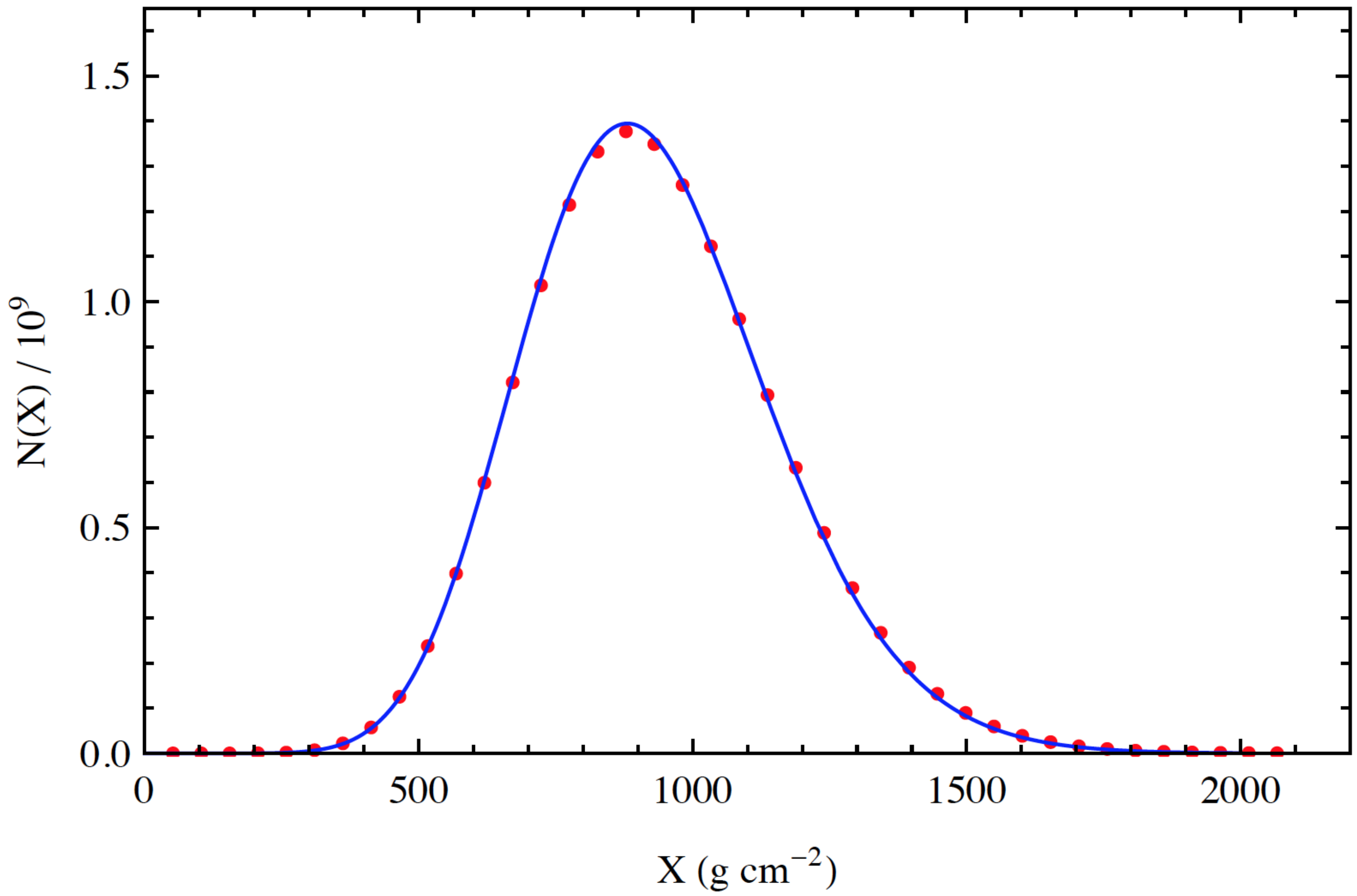
High energy showers not too far from maximum
["mature" but not "too old"]

have a longitudinal development
with a simple shape determined by
only few parameters.

Montecarlo calculation of the development of individual photon showers $E_0 = 10^{18.25}$ eV



Points: Numerical average of Montecarlo calculations:
Line: analytic solutions ("Greisen formula")



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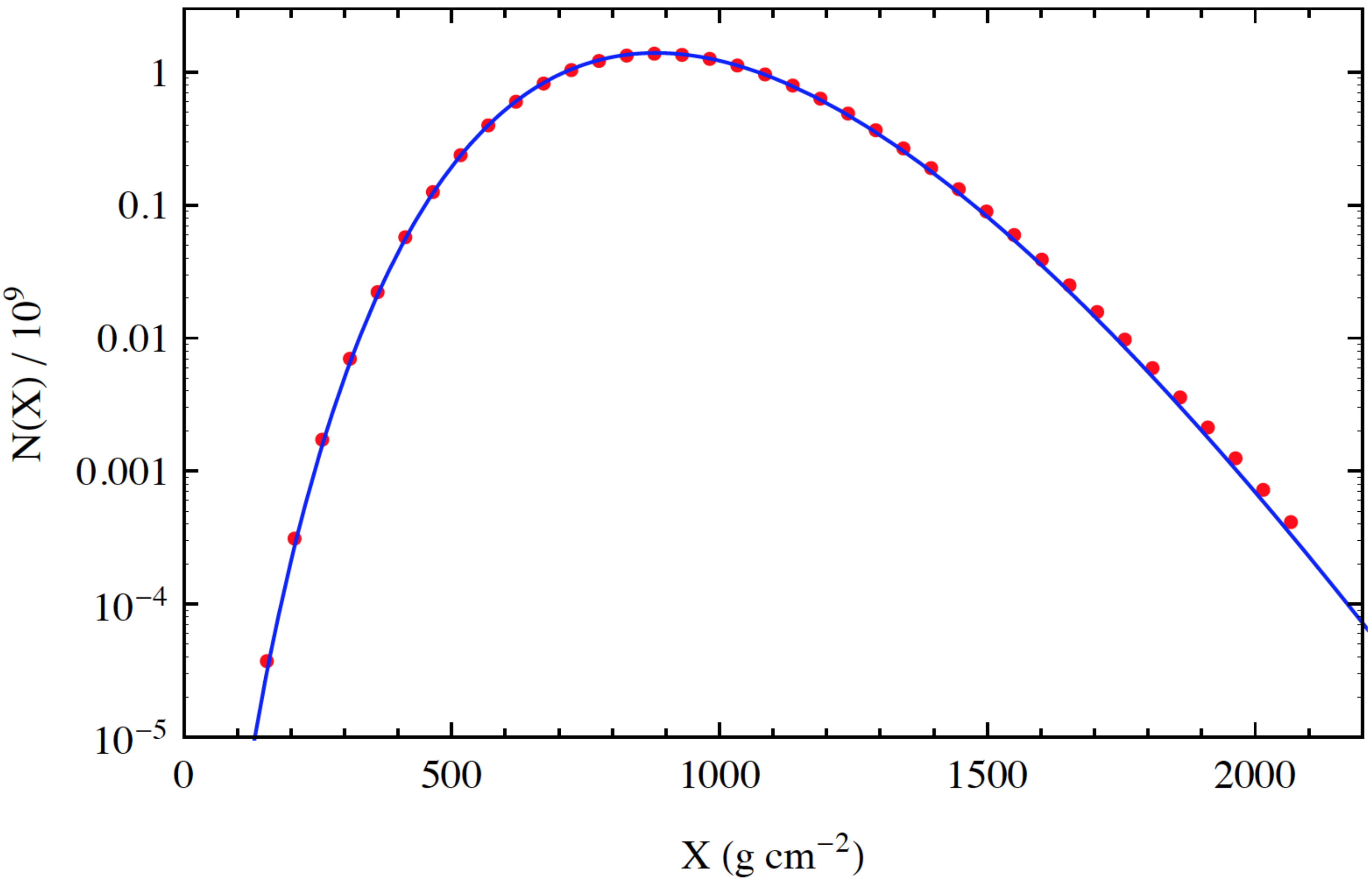
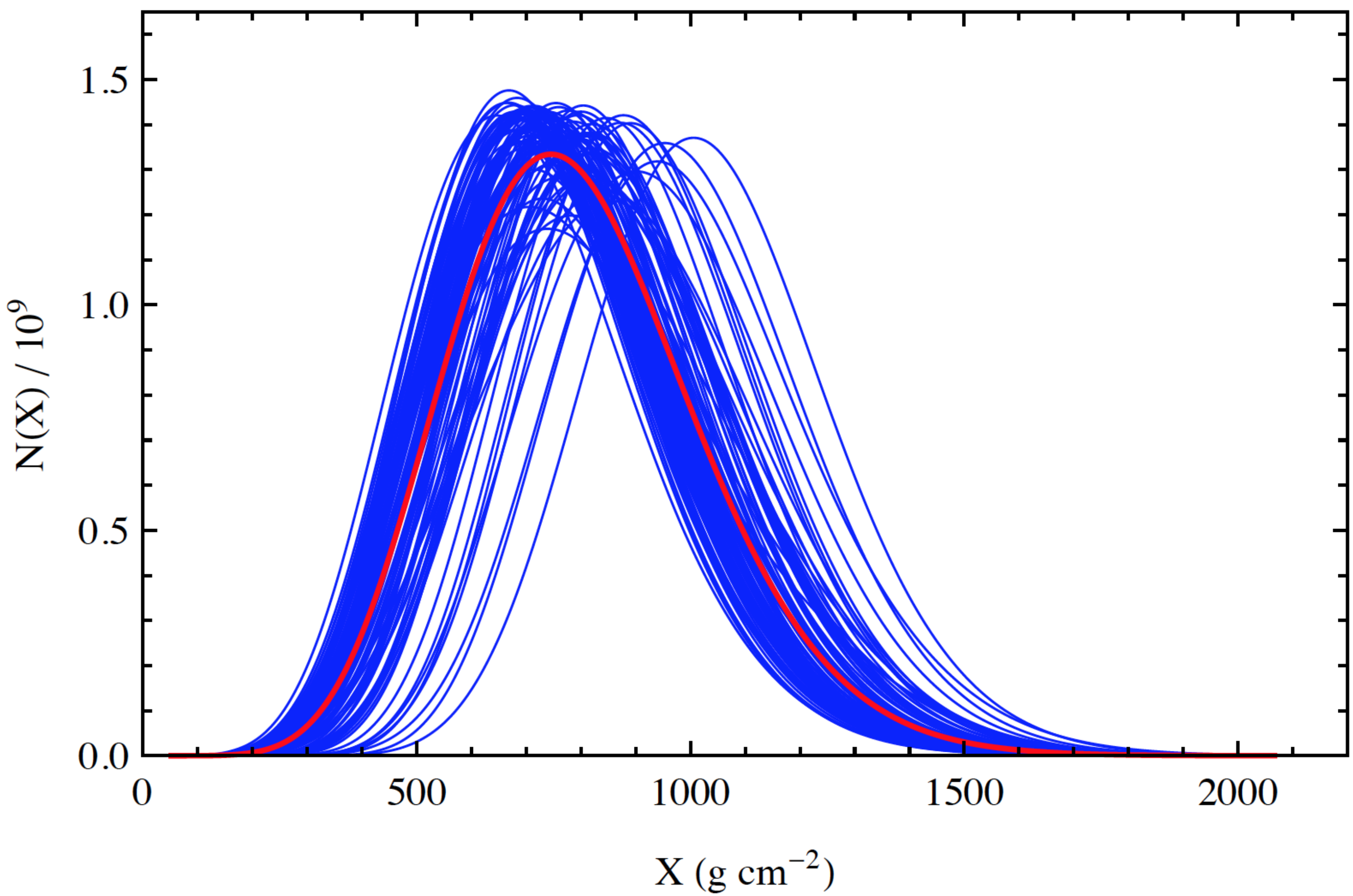
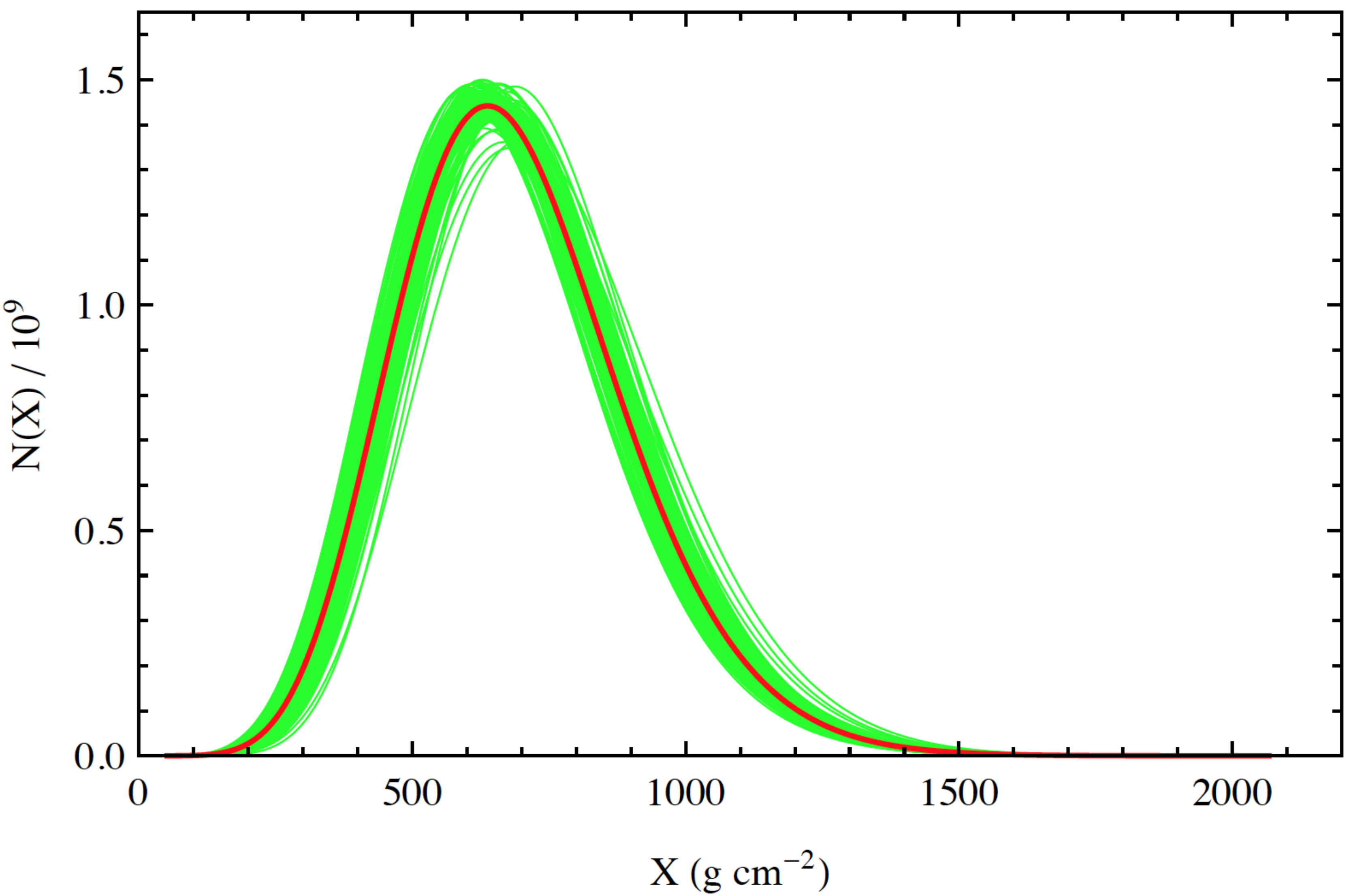


Montecarlo calculation of the development of individual proton showers $E_0 = 10^{18.25}$ eV

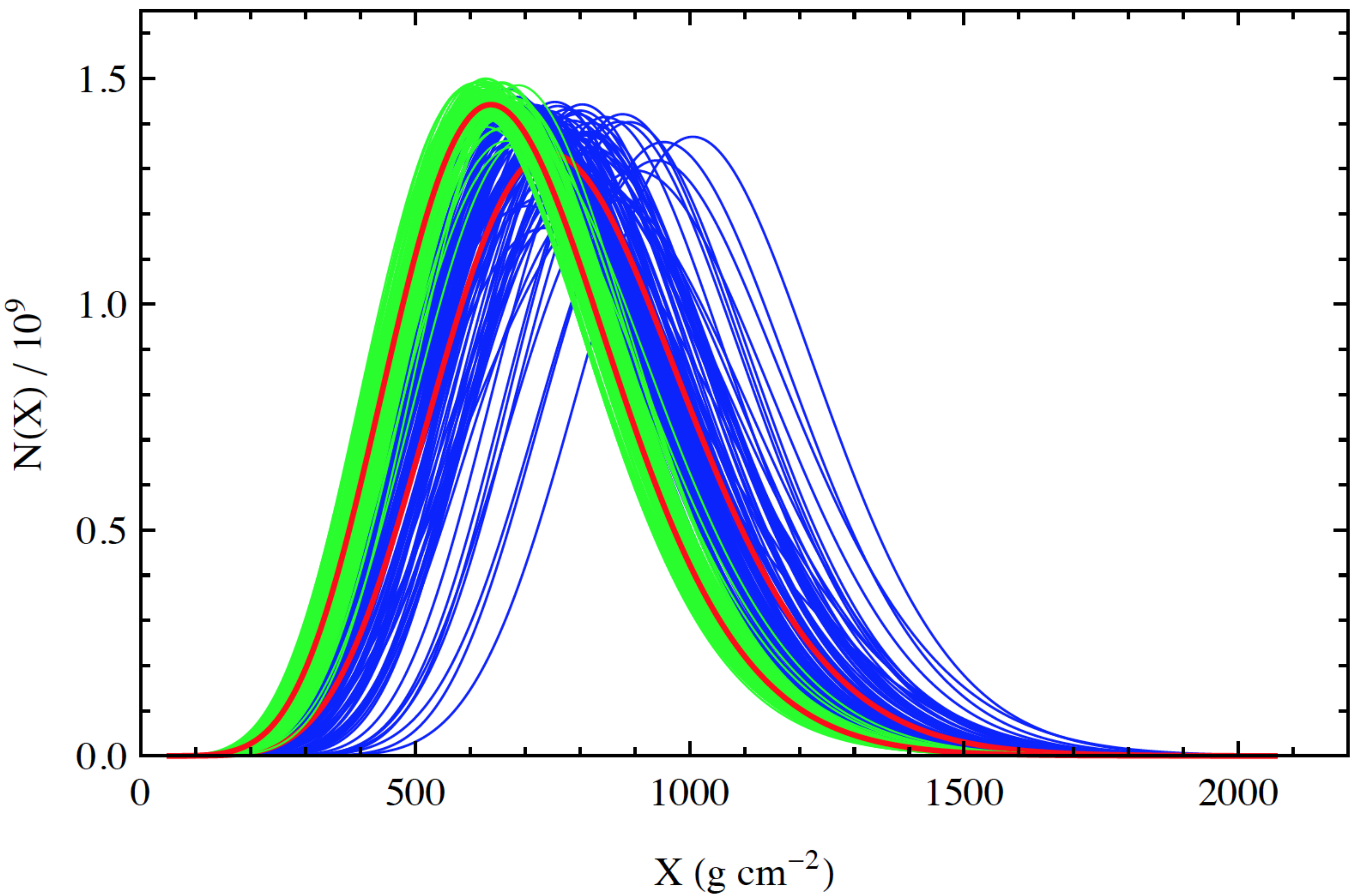


Montecarlo calculation of the development of individual iron nuclei showers $E_0 = 10^{18.25}$ eV

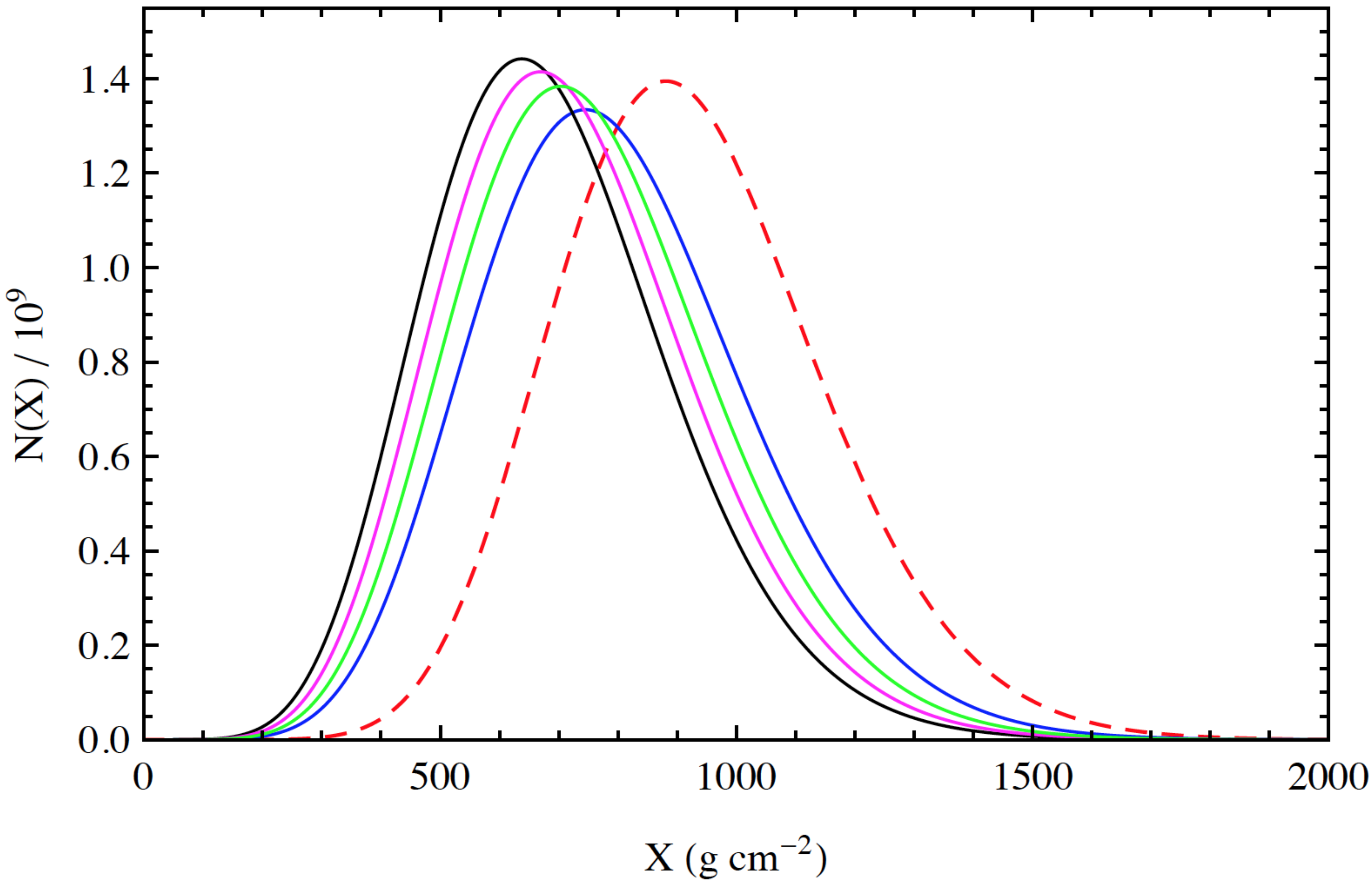


Compare proton and Iron nuclei showers

$$E_0 = 10^{18.25}\ \text{eV}$$



Average development of:
photon,
proton, helium, oygen, iron primary particles



Three analytic expressions to describe the Longitudinal profile:

1. Greisen profile
   [average development of a photon shower] E_0

2. Modified Greisen profile
   [3 parameter function]

   $$\{N_{\rm max},\ \ t_{\rm max},\ \ \sigma\}$$

3. Gaisser-Hillas profile
   [4 parameters function]

   $$\{N_{\rm max},\ \ t_{\rm max},\ \ \sigma,\ \ z\}$$

The “Greisen profile”
[average development of electromagnetic shower]

$$N_{\text{Greisen}}(t, E_0) = N_0 \; \exp\left[t\left(1 - \frac{3}{2}\log\left(\frac{3t}{t + 2\,\ln(E_0/\varepsilon)}\right)\right)\right]$$

$$t_{\max} = \ln\frac{E_0}{\varepsilon}$$

Position of Maximum

$$\sigma = \sqrt{\frac{3}{2}\;\ln\frac{E_0}{\varepsilon}}$$

Width of distribution

$$-\frac{1}{\sigma^2} = \left.\frac{d^2N(t)}{dt^2}\right|_{t=t_{\max}}$$

$$\tau = t - t_{\max}$$

$$\ln \frac{N(t)}{N_{\max}} = -\frac{\tau^2}{2\,\sigma^2} + \frac{5\,\tau^3}{12\,\sigma^4} - \frac{3\,\tau^4}{8\,\sigma^8} + \ldots$$

The Greisen profile can be understood qualitatively as a Gaussian

centered at $t_{\max}$

With width σ

with a small asymmetric distortion (tail at large t) that become negligible with at very large energy

$$\int_0^\infty dt\ N(t) \simeq \sqrt{2\pi}\, N_{\rm max}\, \sigma\ \left[1 + \frac{0.176}{\sigma^2} + \ldots\right]$$

Points at half-maximum

$$N(t_{\rm max} \mp W_{L,R}) = \frac{N_{\rm max}}{2}$$

$$W_{L,R} = \sqrt{2\ \ln 2}\, \sigma \mp \frac{5}{6}\ \ln 2 + \frac{125\,(\ln 2)^{3/2}}{72\,\sqrt{2}}\ \frac{1}{\sigma} + \ldots$$

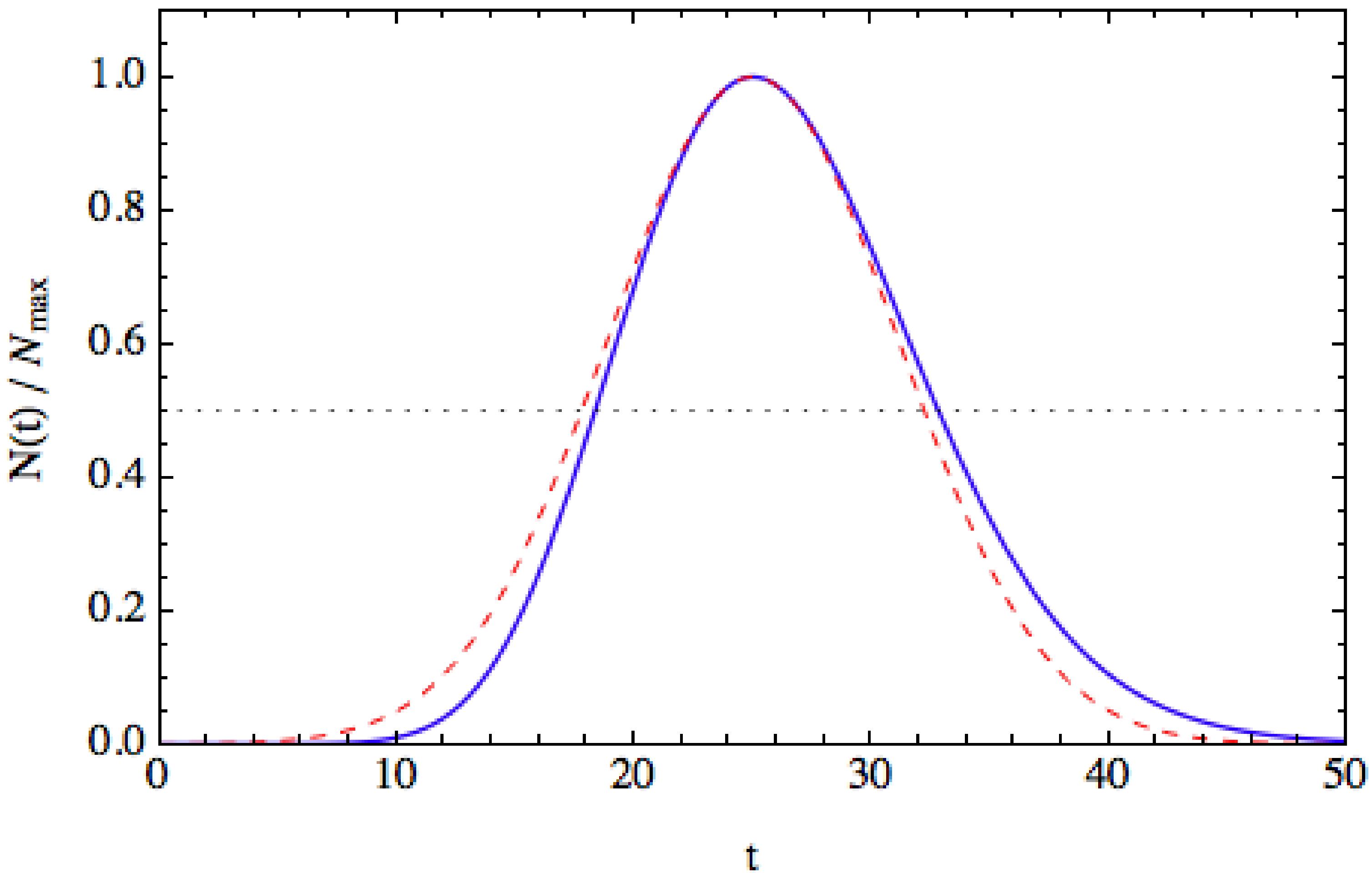
$$= 0.708\, \sigma \mp 0.578 + \frac{0.708}{\sigma} + \ldots$$

$$a = \frac{W_R - W_L}{W_R + W_L}$$

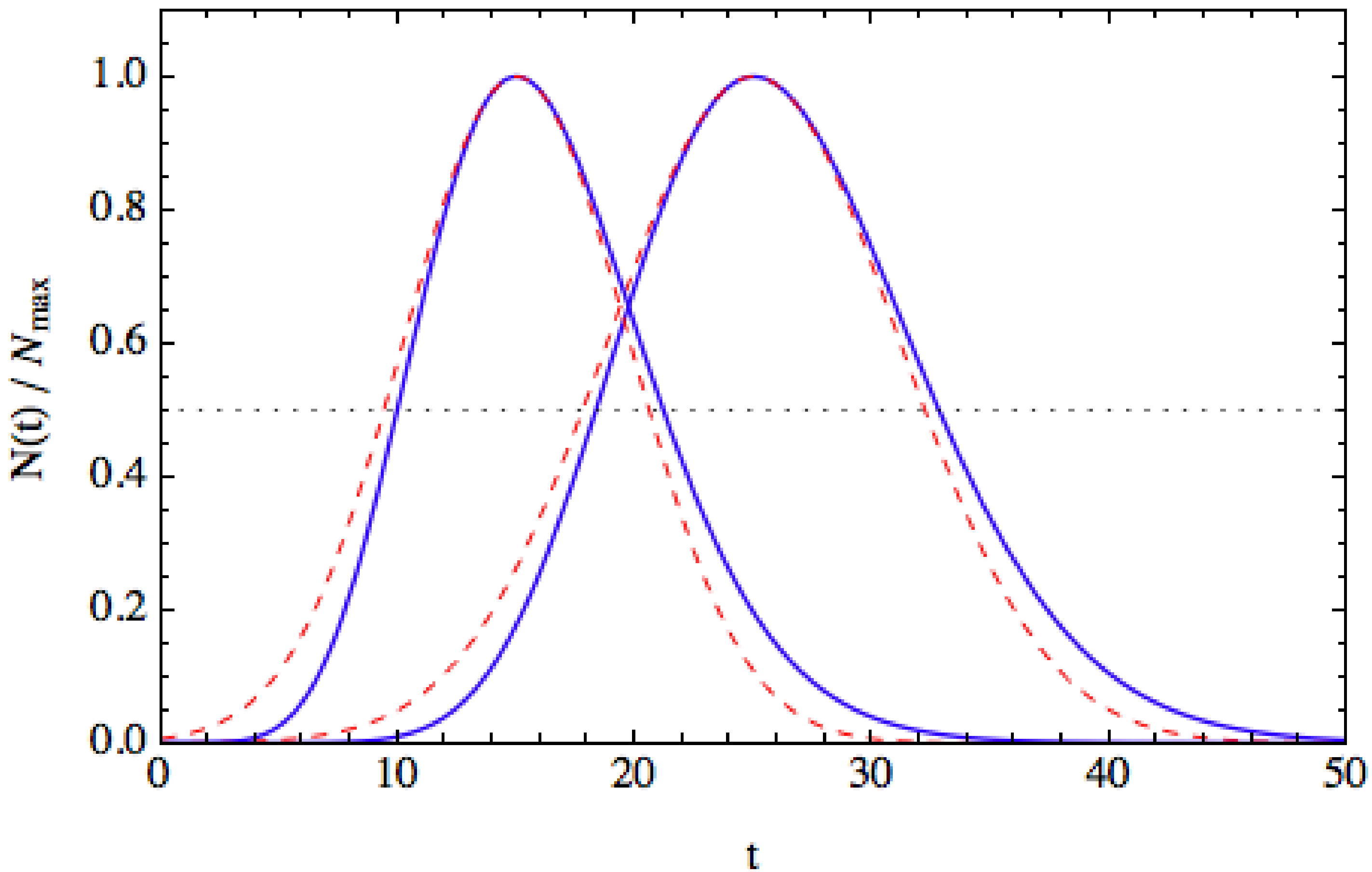
$$a \simeq \frac{5}{6}\sqrt{\frac{\ln 2}{2}}\;\frac{1}{\sigma} + \frac{1375\,(\ln 2)^{3/2}}{864\,\sqrt{2}}\;\frac{1}{\sigma^3} + \ldots$$

$$\simeq \frac{0.490}{\sigma} + \frac{0.649}{\sigma^3} + \ldots$$

$$= \;\; 0.401\frac{1}{\sqrt{\ln E_0/\varepsilon}} - \frac{0.161}{(\ln E_0/\varepsilon)^{3/2}} + \ldots$$

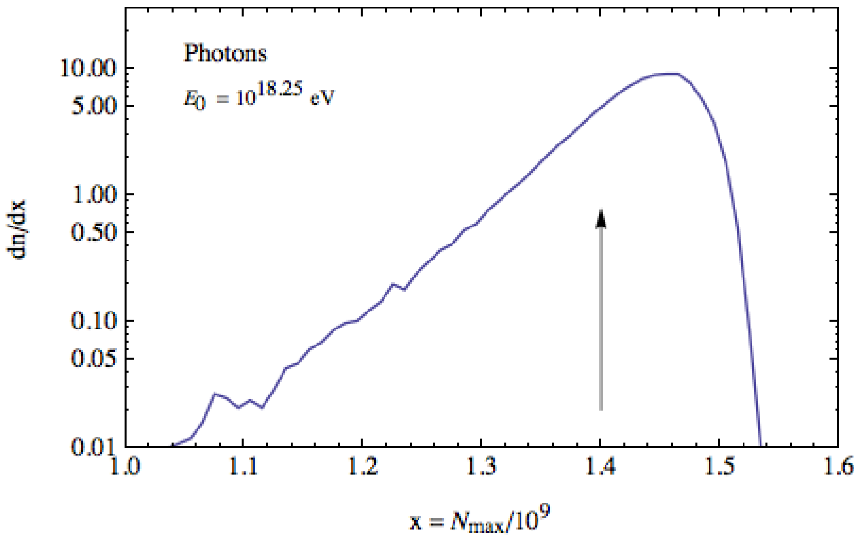


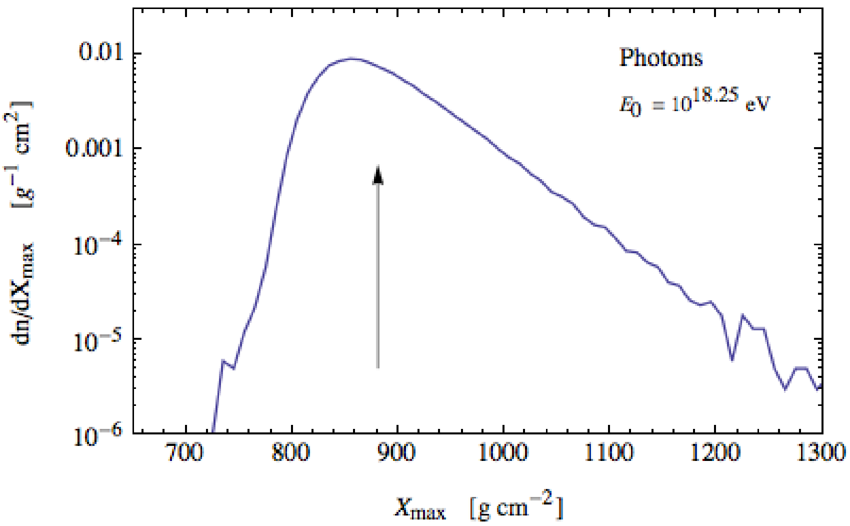
N(t) / N_{max}
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
t

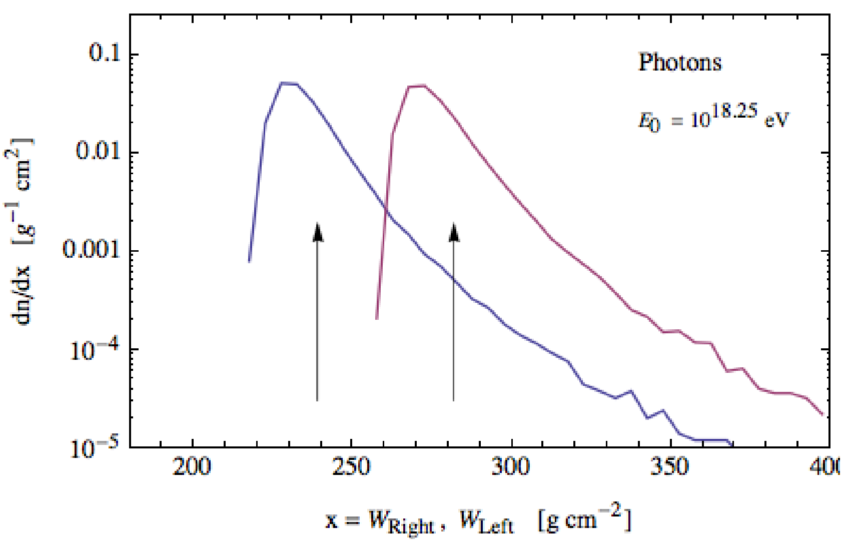


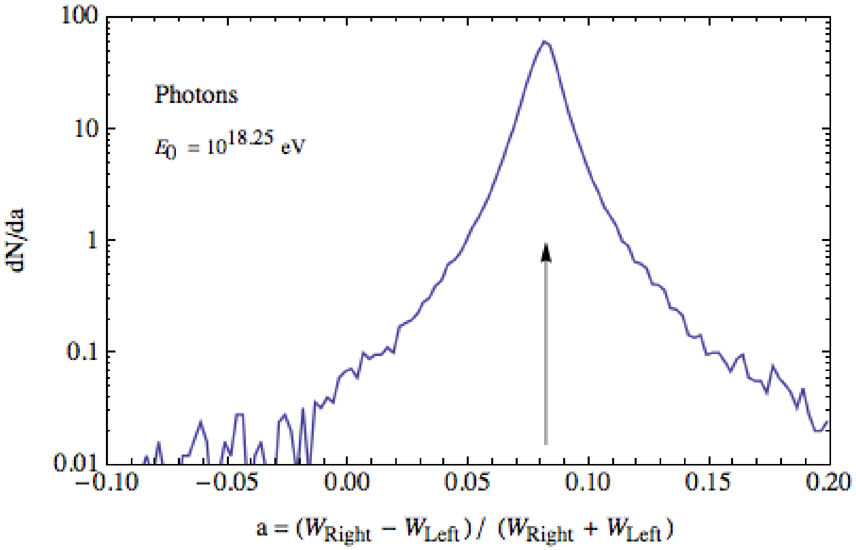
N(t) / N_{max}
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
t

Fluctuations in the development of individual showers:

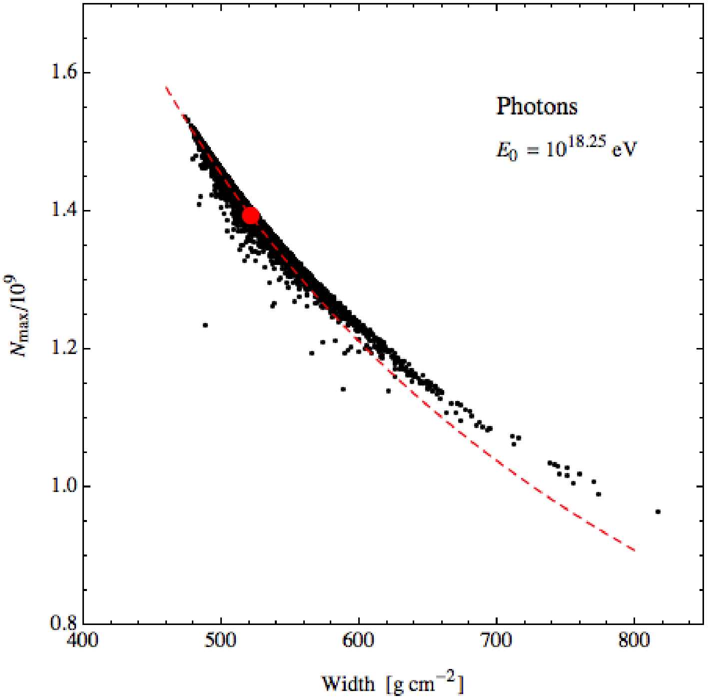


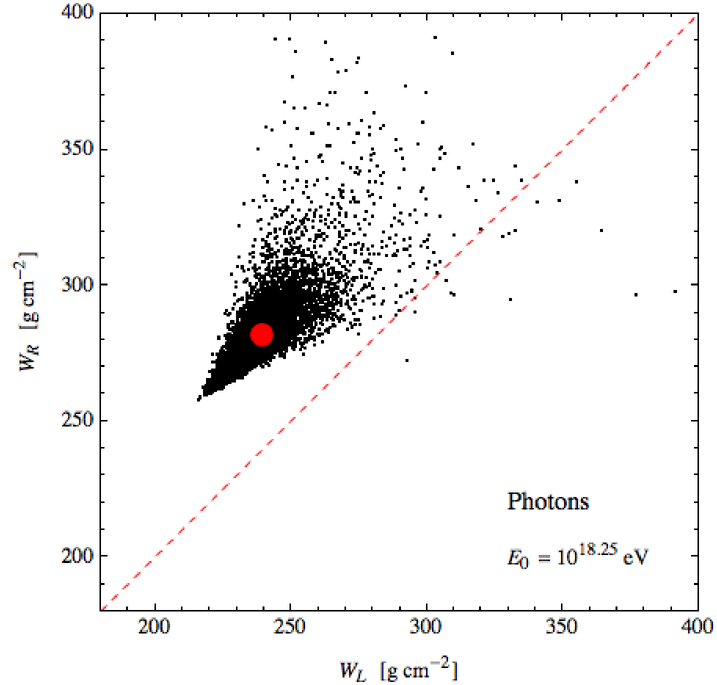


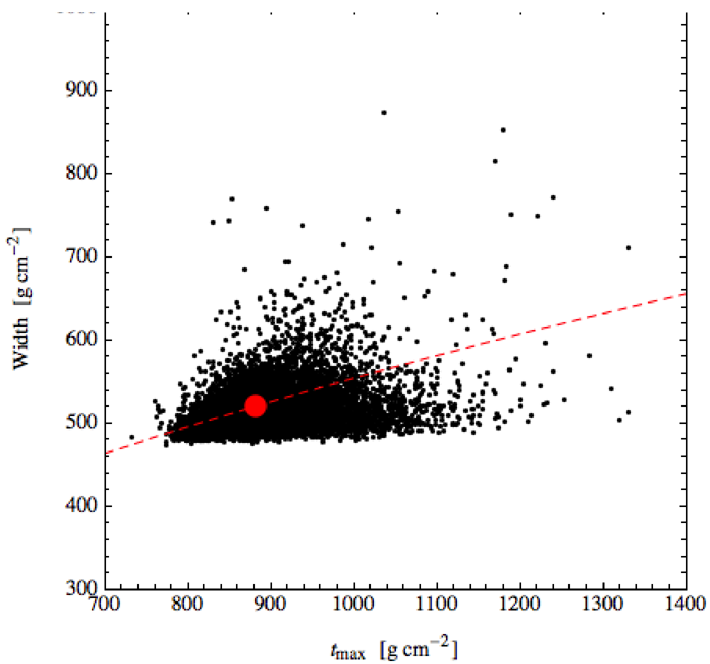


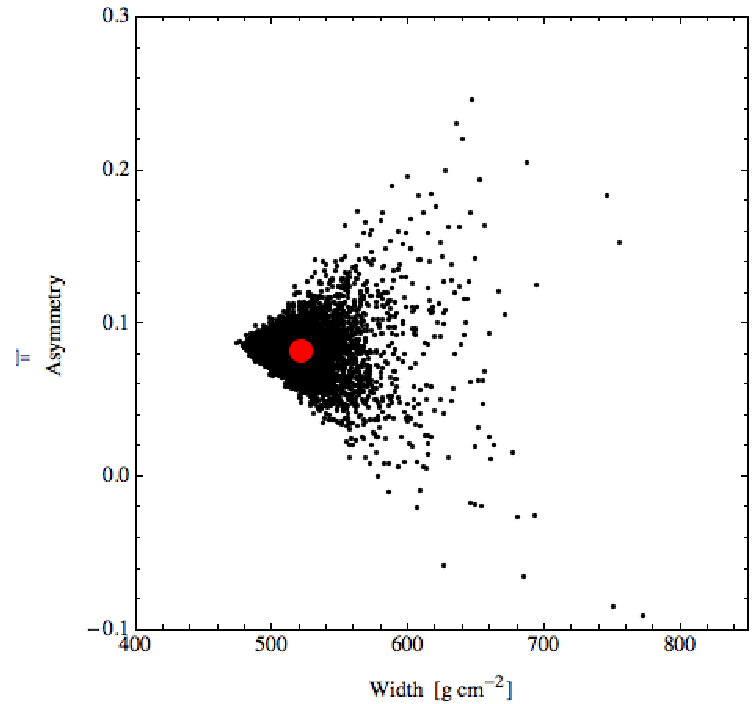


Correlations between parameters:









Euristic derivation of the Greisen formula:
(and connection to “universality”)

$$\frac{dN(E_{\rm min}, t)}{dt} = \lambda(t)\; N(E_{\rm min}, t)$$

$$N(E_{\rm min}, t) \simeq K \left(\frac{E_0}{E_{\rm min}} \right)^{s(t)} e^{\lambda[s(t)] \;\; t}$$

$$N(E_{\rm min}, t) \simeq K \left(\frac{E_0}{E_{\rm min}}\right)^{s(t)} e^{\lambda[s(t)] \;\; t}$$

$$\frac{dN(E_{\rm min}, t)}{dt} = \left[\lambda(t) + \left(\ln\frac{E_0}{E_{\rm min}} + \frac{d\lambda(s)}{ds}\, t\right) \frac{ds(t)}{dt}\right] N(E_{\rm min}, t)$$

To obtain : $\frac{dN(E_{\rm min}, t)}{dt} = \lambda(t)\; N(E_{\rm min}, t)$

one needs the condition :

$$\ln\frac{E_0}{E_{\rm min}} + \frac{d\lambda(s)}{ds}\, t = 0$$

Differential equation that determines $\lambda(t)$

or equivalently $s(t)$

$$\lambda(s) \simeq \frac{1}{2}\,(s - 1 - 3\,\ln s)$$

Approximation of the exact expression

$$\frac{d\lambda(s)}{ds} \simeq \frac{1}{2}\,\left(1 - \frac{3}{s}\right)$$

$$\ln\frac{E_0}{E_{\min}} + \frac{d\lambda(s)}{ds}\,t = 0$$

$$\ln\frac{E_0}{E_{\min}} + \frac{1}{2}\,\left(1 - \frac{3}{s}\right)\,t = 0$$

$$s(t) = \frac{3t}{t + 2\,\ln(E_0/\varepsilon)} = \frac{3t}{t + 2\,t_{\max}}$$

$$\lambda(t) = \lambda[s(t)]$$

$$\simeq \frac{1}{2}\left[\frac{3t}{2\ln E_0/\varepsilon + t} - 1 - 3\ \ln\left(\frac{3t}{2\ln E_0/\varepsilon + t}\right)\right]$$

$$\frac{d\ln N(t)}{dt} = \lambda(t)$$

$$N_{\max} = N(t_{\max})$$

$N(t)$ → Greisen profile

Generalization to a shower that around maximum is dominated by the electromagnetic component.
Developing around shower maximum:

$$\tau = t - t_{\max}$$

3 parameter expression
$\{N_{\max}, \quad t_{\max}, \quad \sigma\}$

$$s(\tau) = \frac{3(\tau + \ln E^*/\varepsilon)}{\tau + 3 \ln E^*/\varepsilon} = \frac{3\,\tau + 2\,\sigma^2}{\tau + 2\,\sigma^2}$$

$$\sigma^2 \simeq \frac{3}{2} \ln \frac{E^*}{\varepsilon}$$

E^* Characteristic energy [maximum energy of validity of the solution]

$$N(\tau) = N_{\max} \exp\left[\tau - \left(\frac{3}{2}\tau + \sigma^2\right) \ln\left(\frac{3\,\tau + 2\,\sigma^2}{\tau + 2\,\sigma^2}\right)\right]$$

$$a = \frac{W_R - W_L}{W_R + W_L}$$

$$a \simeq \frac{5}{6}\sqrt{\frac{\ln 2}{2}}\,\frac{1}{\sigma} + \frac{1375\,(\ln 2)^{3/2}}{864\,\sqrt{2}}\,\frac{1}{\sigma^3} + \ldots$$

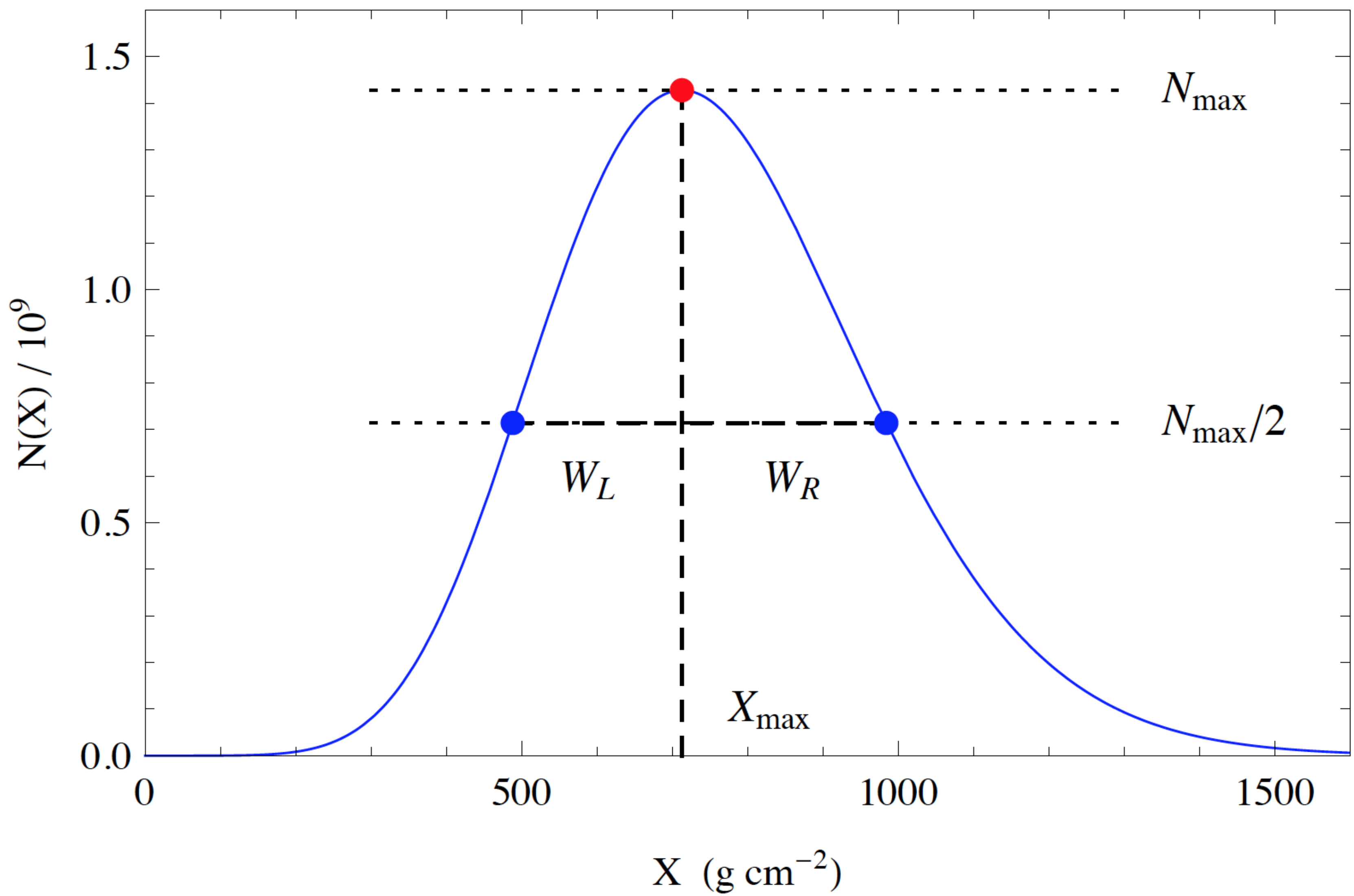
$$\simeq \frac{0.490}{\sigma} + \frac{0.649}{\sigma^3} + \ldots$$

$$= \; 0.401\frac{1}{\sqrt{\ln E_0/\varepsilon}} - \frac{0.161}{(\ln E_0/\varepsilon)^{3/2}} + \ldots$$

Description of individual showers

$$\{N_{\max},\ \ t_{\max},\ \ W_L,\ \ W_R\}$$

$$W = W_L + W_R \qquad a = \frac{W_R - W_L}{W_R + W_L}$$



Gaisser–Hillas profile
for individual showers (around shower maximum)

4 parameter expression: $\{N_{\rm max}, \quad t_{\rm max}, \quad \Lambda, \quad t_0\}$

$$N_{\rm GH}(t) = N_{\rm max} \left(\frac{t - t_0}{t_{\rm max} - t_0}\right)^{\frac{t_{\rm max} - t_0}{\Lambda}} \exp\left[\frac{t_{\rm max} - t_0}{\Lambda}\right]$$

$$N(t) = N_{\rm max} \exp\left[-\frac{\tau}{\sigma\, z} + \frac{1}{z^2} \ln\left(1 + \frac{\tau\, z}{\sigma}\right)\right]$$

Re-parametrization $\{N_{\rm max}, \quad t_{\rm max}, \quad \sigma, \quad z\}$

$$\sigma = \sqrt{\Lambda\, (t_{\rm max} - t_0)}$$

$$z = \sqrt{\frac{\Lambda}{(t_{\rm max} - t_0)}}$$

$$\ln \frac{N(t)}{N_{\max}} = -\frac{\tau^2}{2\,\sigma^2} + \frac{\tau^3\ z}{3\,\sigma^3} - \frac{\tau^4\ z^2}{4\,\sigma^4} + \ldots$$

In the limit $z \to 0$

$N(t) \to \text{Gaussian}$

$$a = \frac{W_R - W_L}{W_R + W_L} = \frac{2\ln 2}{3}\,z - \frac{\sqrt{2}}{54}\,(\ln 2)^{3/2}\,z^3 + \ldots$$

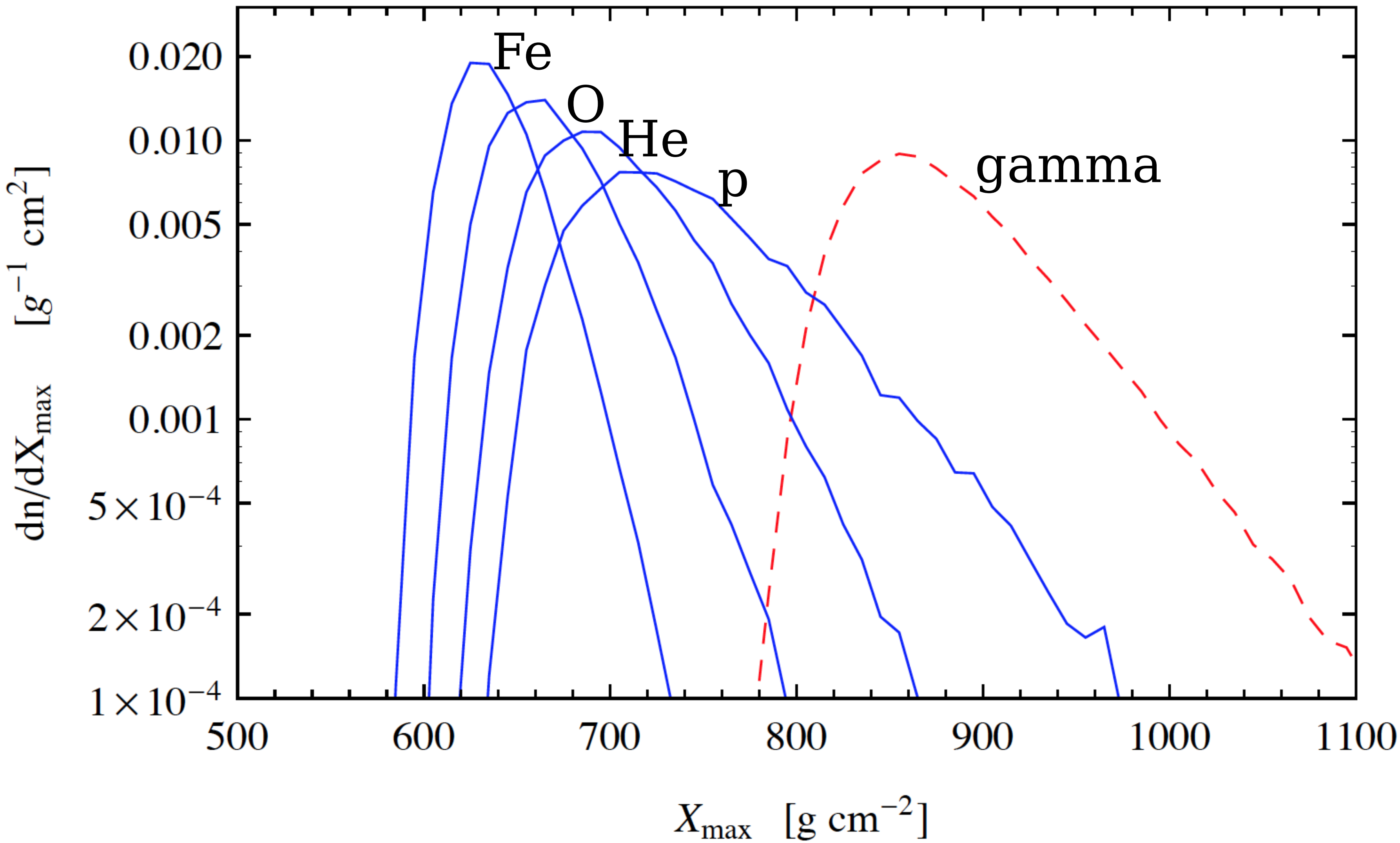
$$= 0.392\,z - 0.0151\,z^3 + \ldots$$

$$W \overset{2*}{=} \sigma\sqrt{2\ln 2}\ \left[1 + \frac{\ln 2}{18}\,z^2 + \ldots\right]$$

The set of the longitudinal profile distributions
for showers of particles of a certain type
(and a fixed energy) does contain some
information (and therefore also depend on)
the properties of hadronic interactions

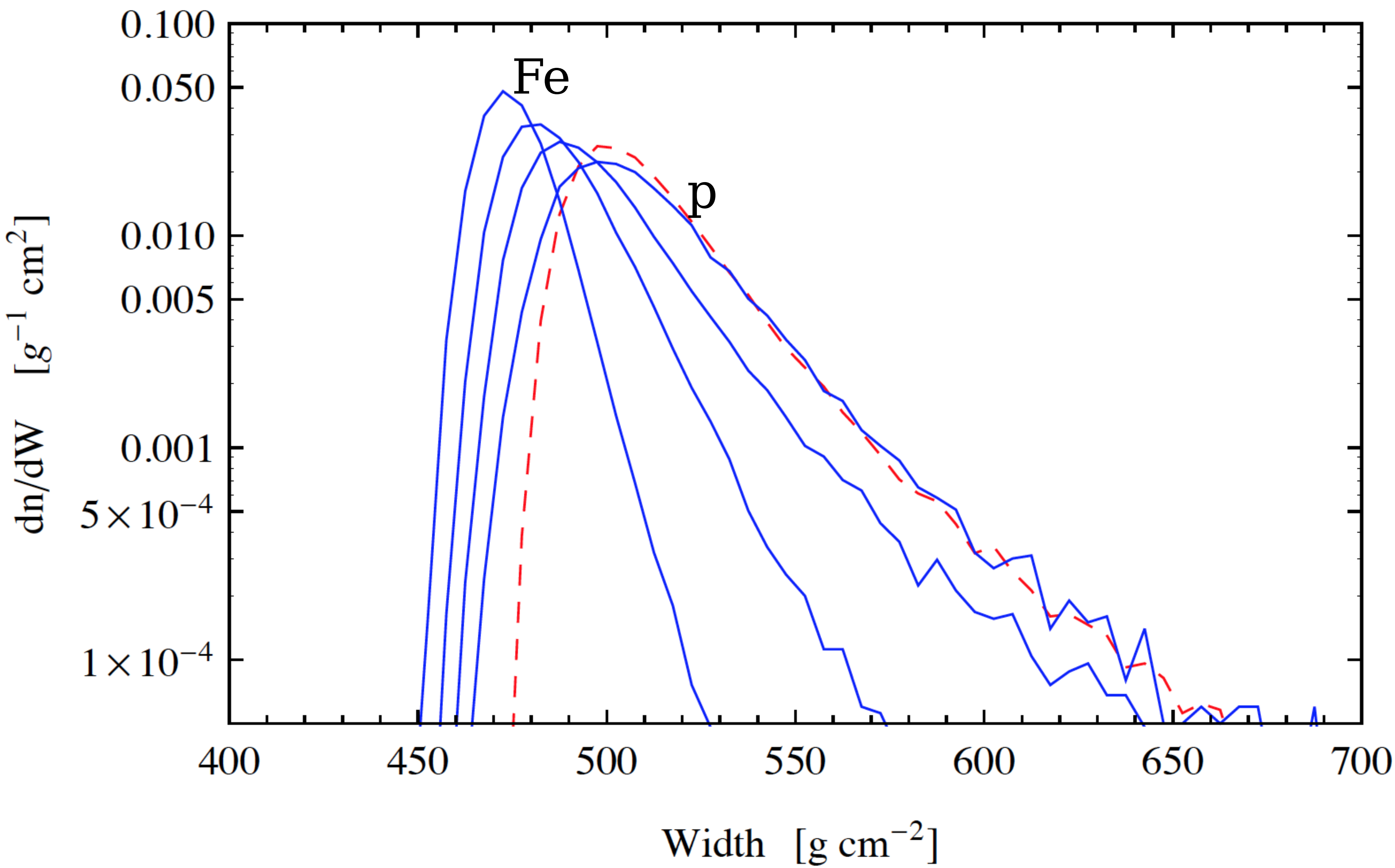
Distributions in $X_{\max}$

$E_0 = 10^{18.25}$ eV



Distributions in the Width

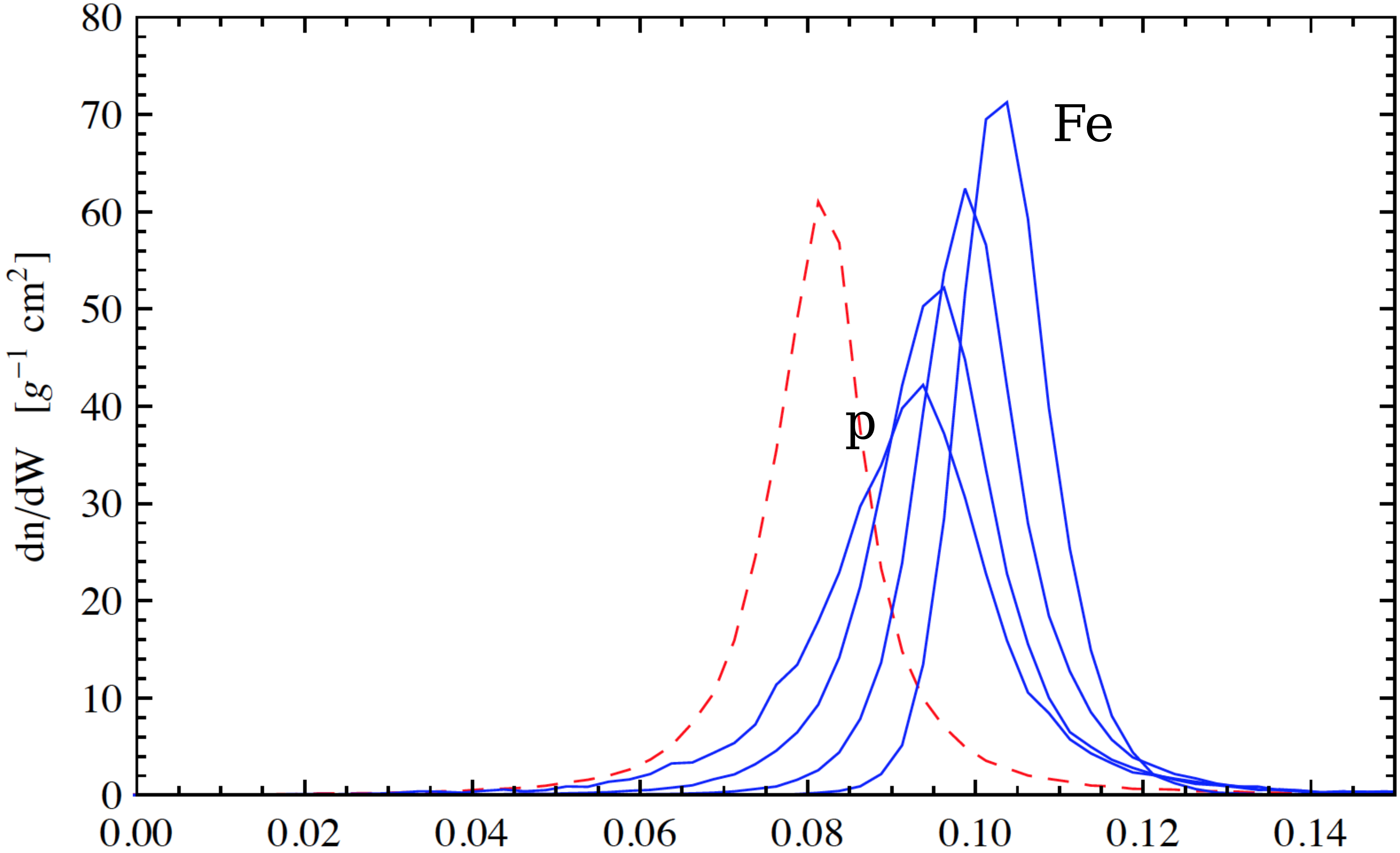
$E_0 = 10^{18.25}$ eV



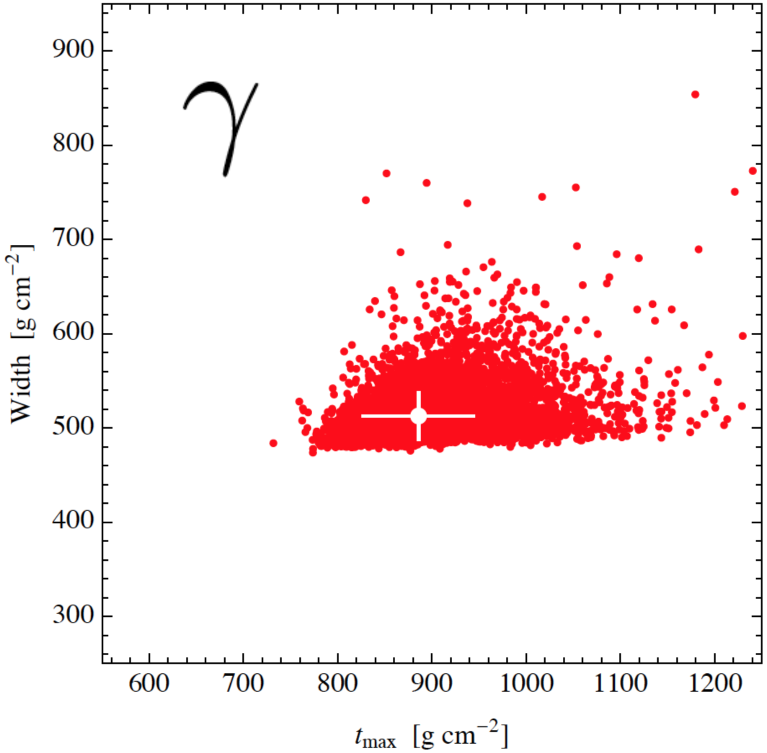
Complementary information about composition

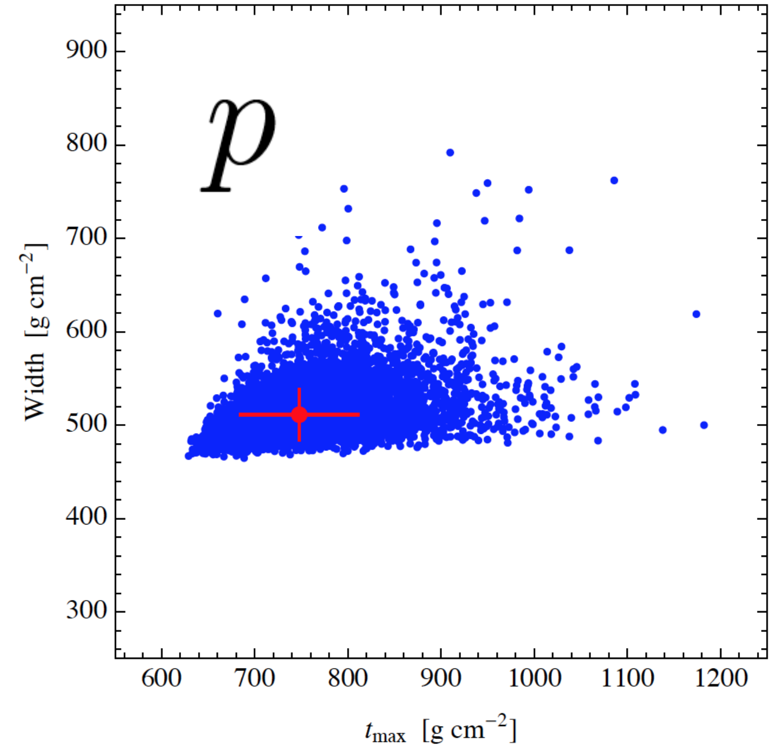
Distributions in Asymmetry

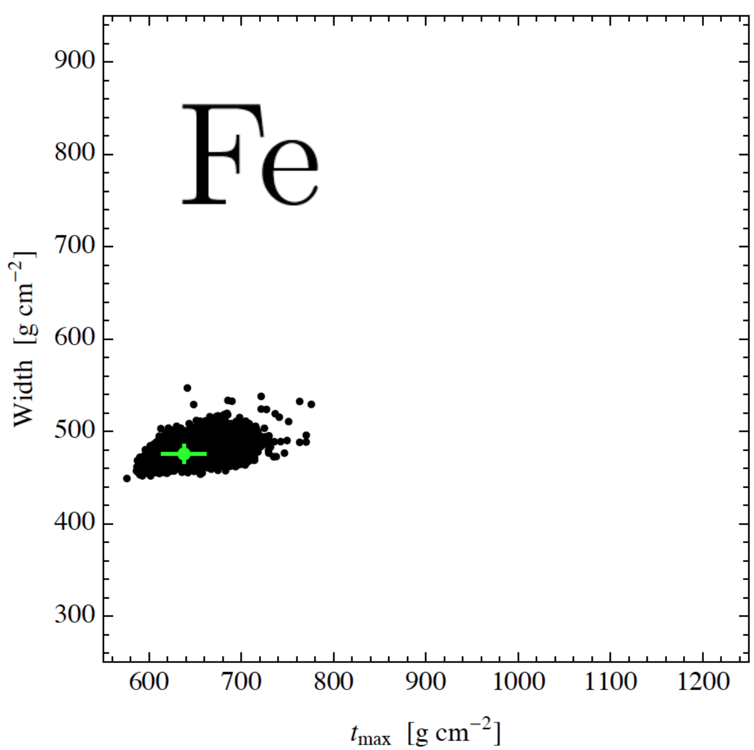
$$E_0 = 10^{18.25}\ \mathrm{eV}$$



Asymmetry parameter (Wr-Wl)/(Wr + Wl)

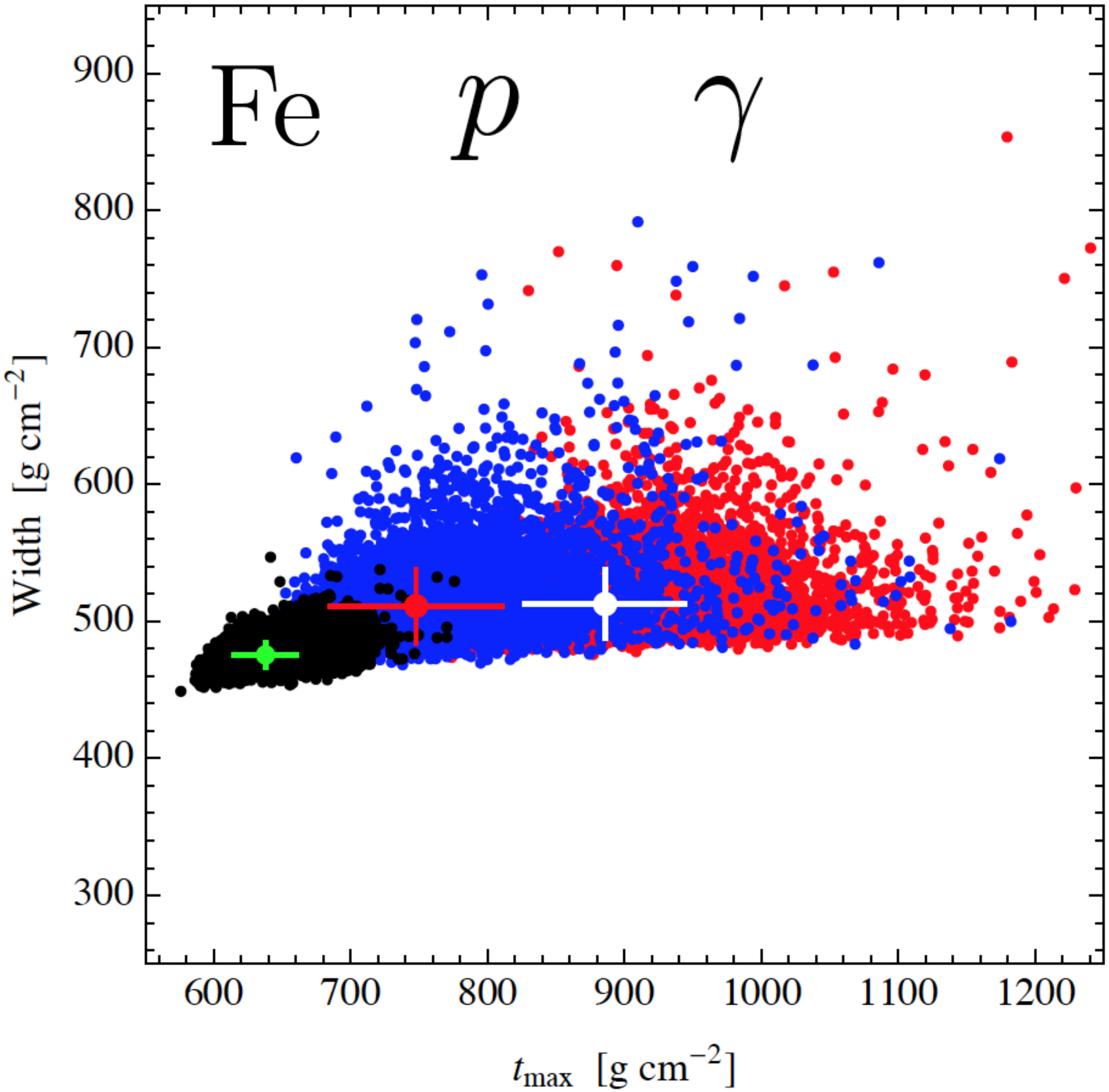




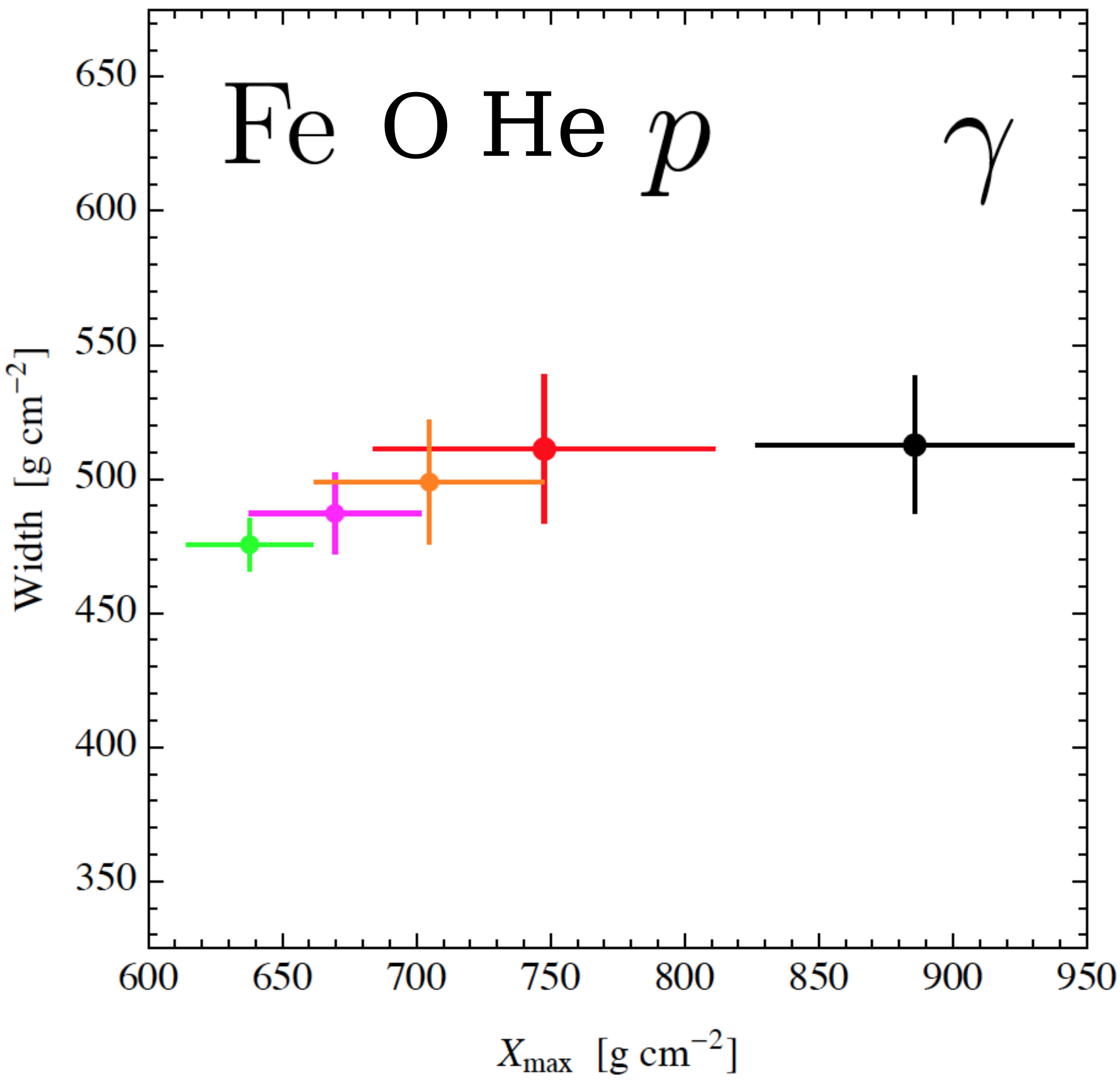


$E_0 = 10^{18.25}$ eV

W versus X_{max}



Fe
p
γ
Width [g cm^−2]
t_max [g cm^−2]
900
800
700
600
500
400
300
600
700
800
900
1000
1100
1200



Additional information in the Width of the showers.

How good is the description of the Gaisser-Hillas form in the description of individual cosmic ray showers ?

$$\{t_1, t_2, \ldots\}$$

Levels for the Montecarlo study of cosmic ray showers.

$$\{N(t_1), N(t_2), \ldots\}$$

One Montecarlo shower event

$$N_{\rm fit}(t; \quad N_{\max}, t_{\max}, W, a)$$

4 parameter function that gives correct quantities

$$Q = \sqrt{\frac{1}{n_{\rm levels}} \sum_j \left[\frac{N(t_j) - N_{\rm fit}(t_j)]}{N(t_j)}\right]^2}$$

Estimate of the "quality' description

$$N(t_j) \geq f \; N_{\max}$$

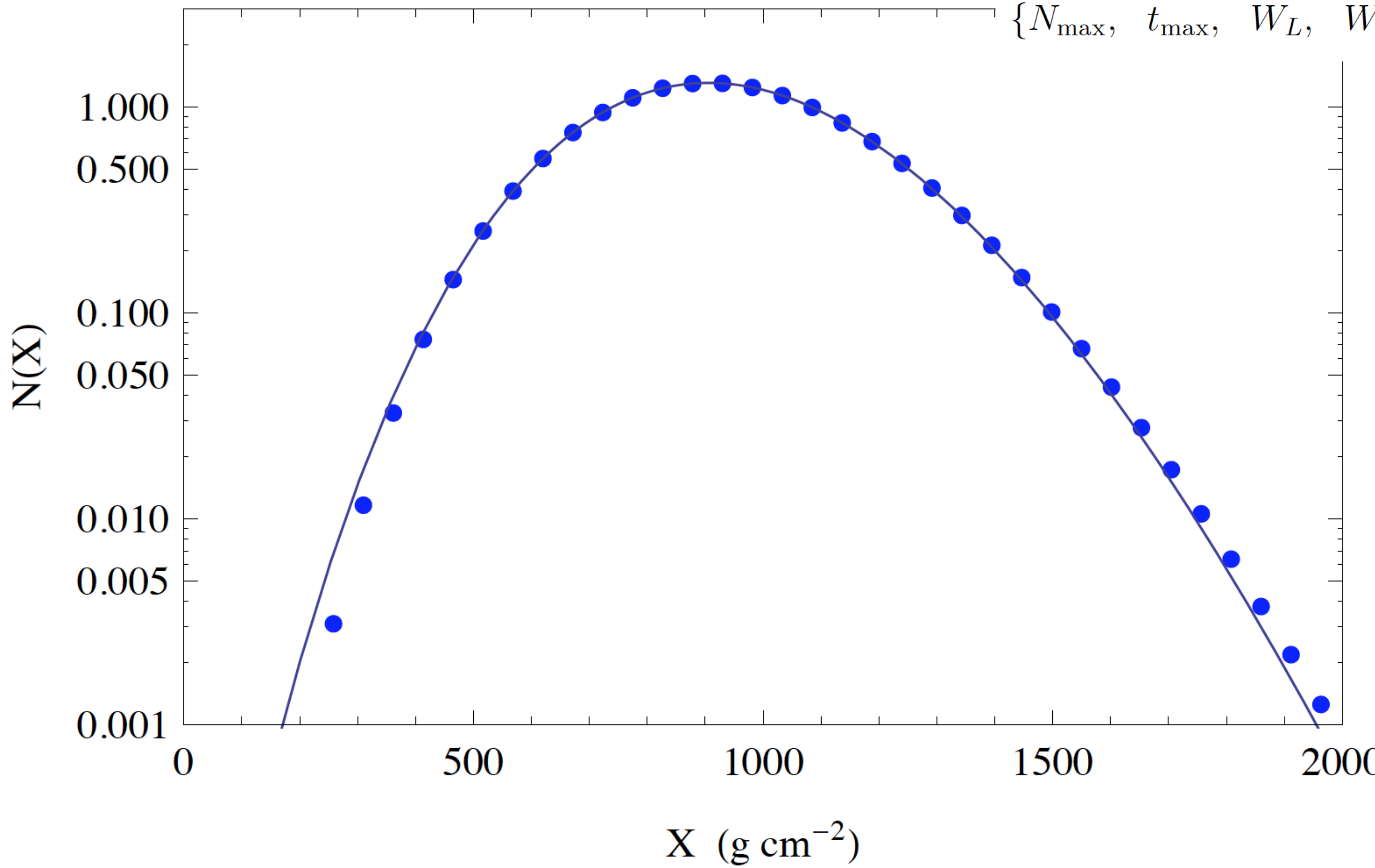
Example (taken at random)
Proton shower

$$E_0 = 10^{18.25} \text{ eV}$$

Points = MC shower

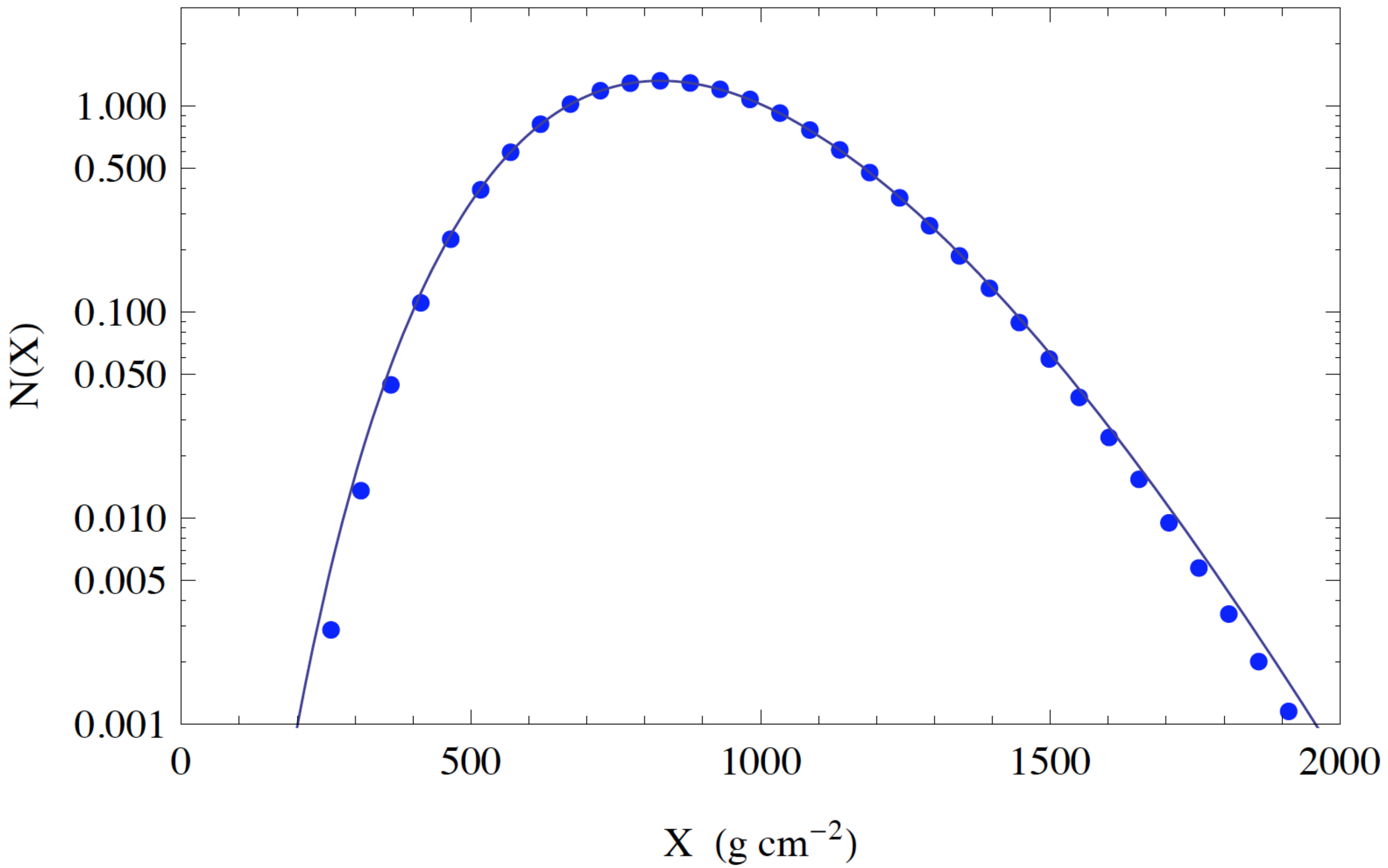
Line = GH line
with correct

$\{N_{\max}, \quad t_{\max}, \quad W_L, \quad W_R\}$



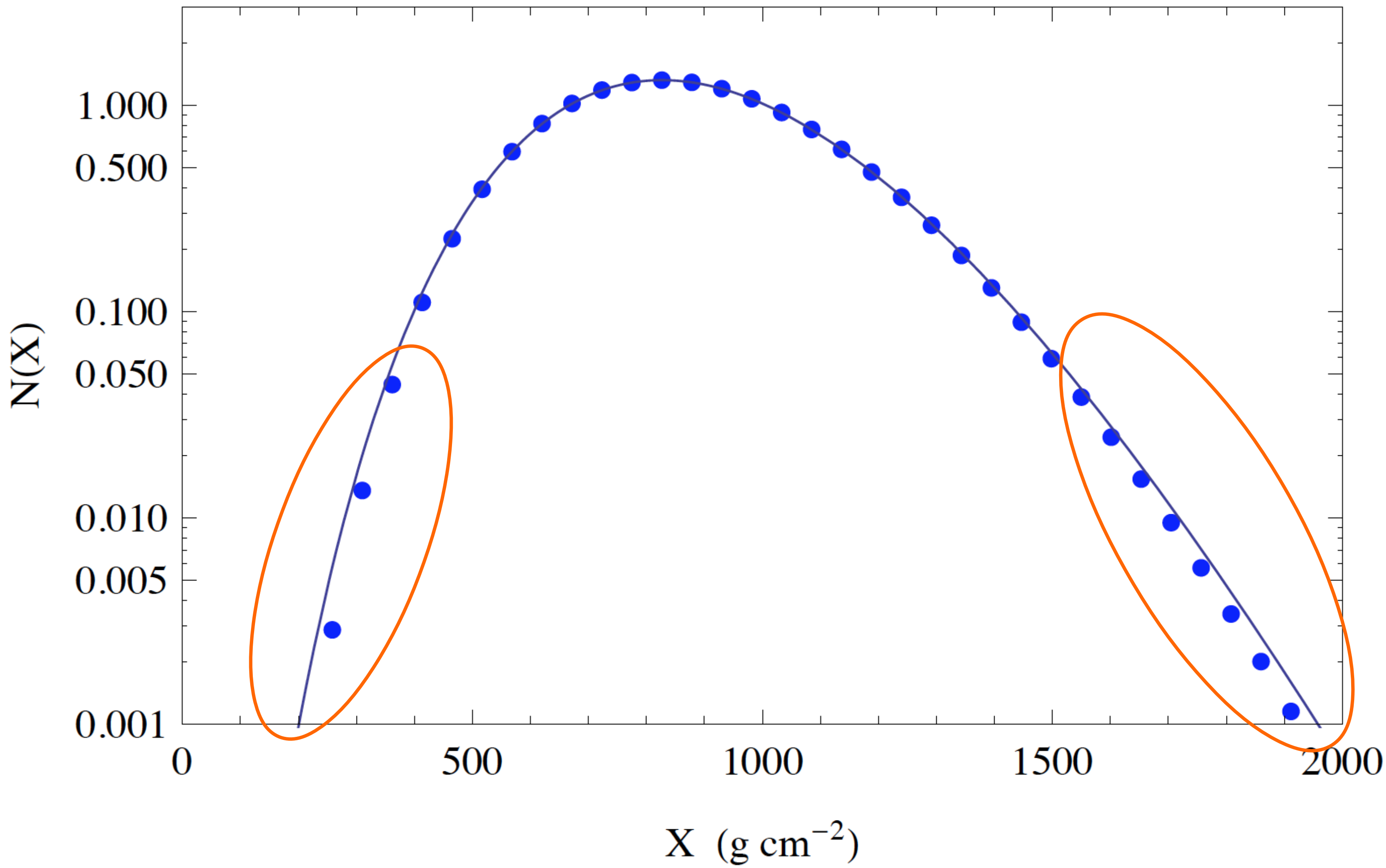
Second example (taken at random)
Proton shower

$$E_0 = 10^{18.25}\ \text{eV}$$



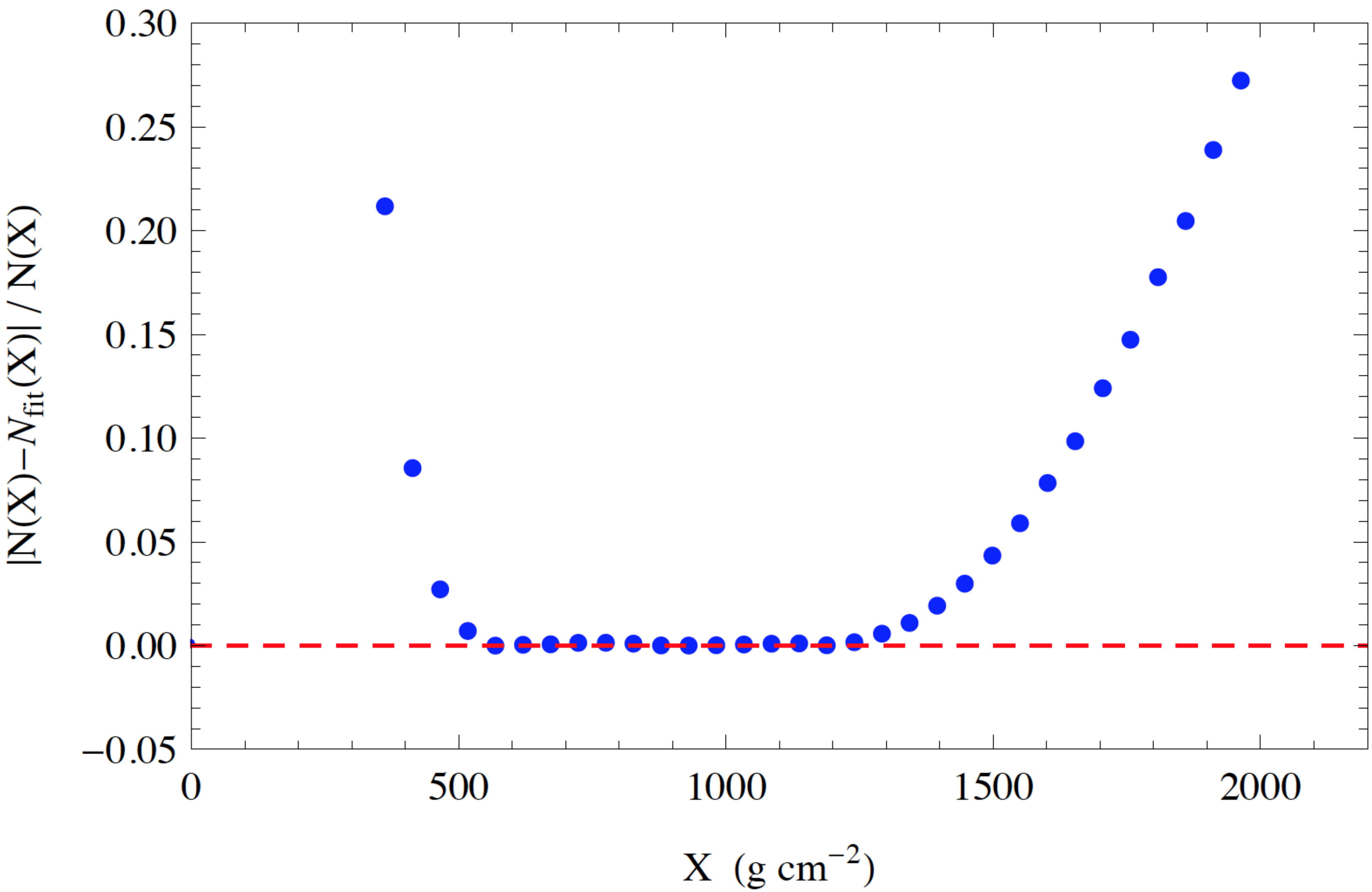
Second example (taken at random)
Proton shower

$$E_0 = 10^{18.25}\ \mathrm{eV}$$

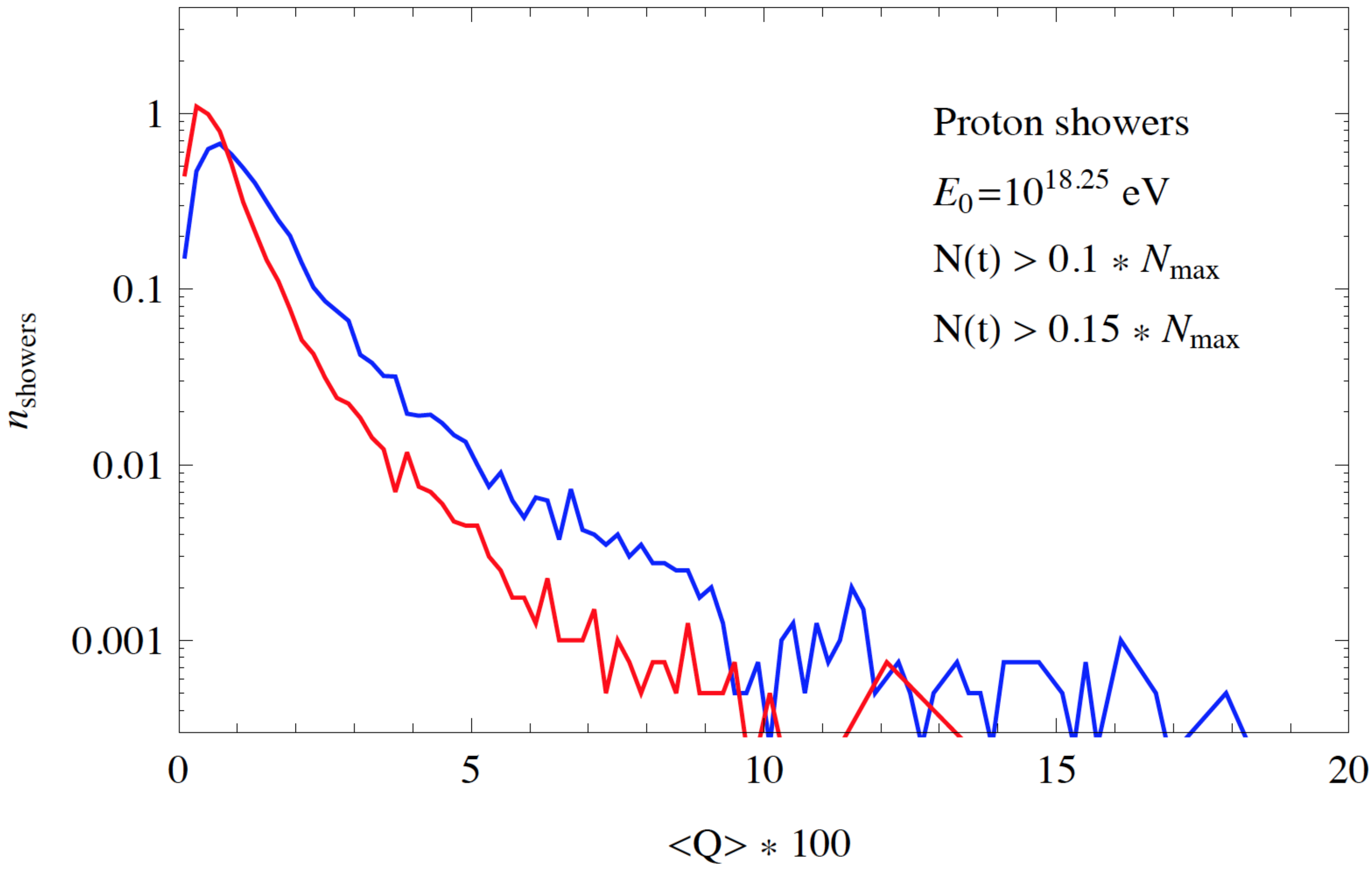


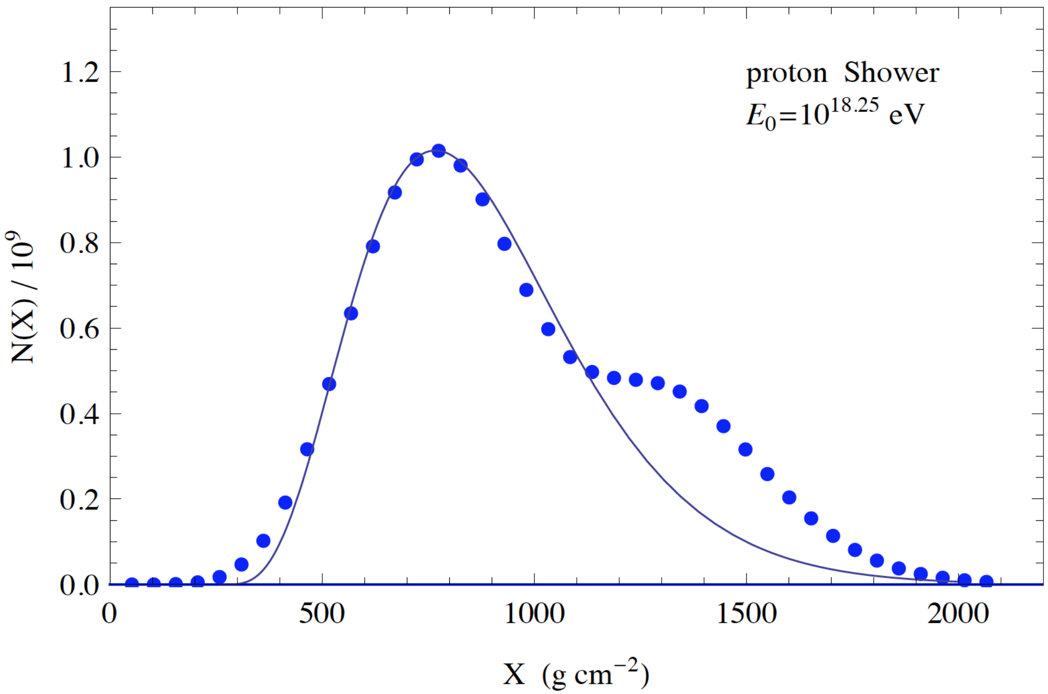
Deviations at low and large depths. Excellent description around maximum

Deviations from the fit

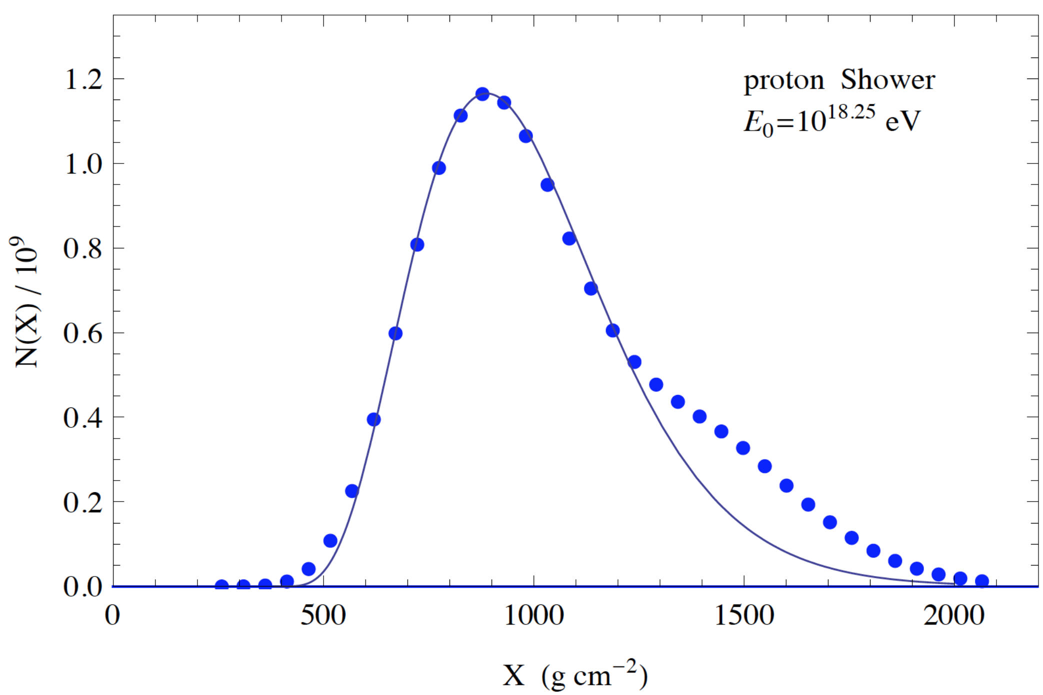


Distribution of the Quality factor for proton showers

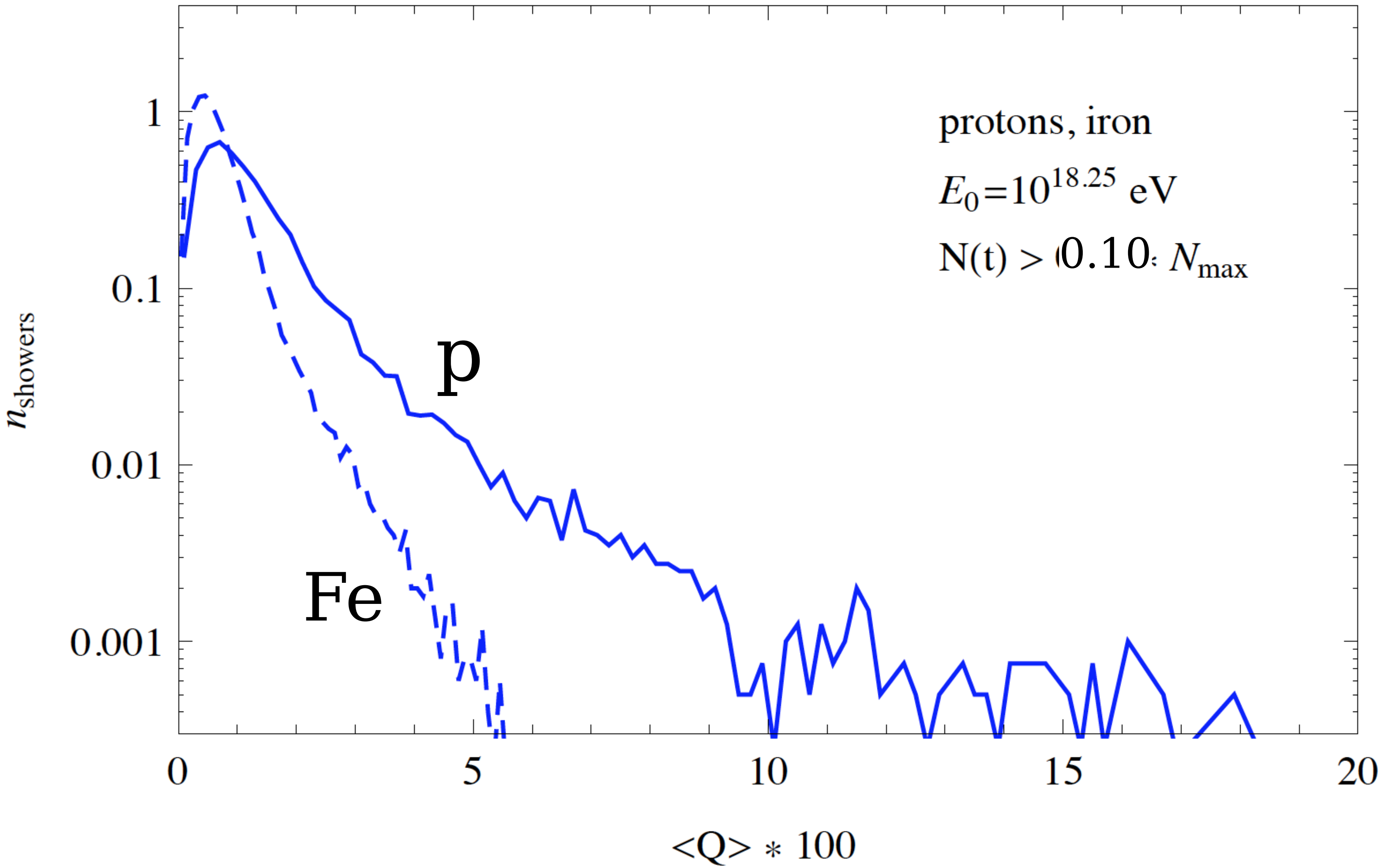




Fluctuations at the level of 10^{-4}



“Irregular showers” are much less likely for heavy nuclei and at larger energy



Concluding remarks :

1. The spectra of electrons and photons in all "mature" [not too young, not too old] cosmic ray showers have shapes that are in good approximation determined by the shower age [logarithmic derivative of the size].

2. The shapes of the longitudinal profile of mature showers have a simple "quasi gaussian" form.

3. Most information about the primary particle identity and the interaction model property are contained in the distribution of Xmax.

4. Additional information is contained in the distributions of the width and asymmetry of the showers.

   Experimental studies in this direction are potentially very interesting

With increasing energy cosmic ray showers become:

[a] More penetrating [X_{max} increases]

[b] Broader [Width increases]

[c] More symmetric [more gaussian like]

If the composition evolves with energy
[Light → Heavy] signatures on the parameter distributions

$$\langle X_{\rm max}(E, A)\rangle \sim D\ \ln(E_0/A)$$

$$\langle W(E, A)\rangle \sim D_W\ \sqrt{\ln(E_0/A)}$$

less sensitivity

What I have neglected in this seminar:

Crucial problem in the field of UHECR:

Estimate of energy and Mass of the primary particle from measurements in a surface array.

[*Auger upgrade:*
Separate measurement of elctromagnetic and muon component]

Best use of the lateral (+ shower front) distributions of the particles.