Short-range correlation effects on the nuclear matrix element of neutrinoless double-β decay

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Outline

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Motivation

★ The results of existing studies of correlation effects on the NME of $0\nu\beta\beta$-decay show a striking model dependence

Nucleus	Bare	FNS	SRC		FNS + SRC	
			CCM	Miller-Spencer	CCM	Miller-Spencer
^{76}Ge $\to$ ^{76}Se	7.39	6.14	5.86	4.46	5.91	4.54
^{100}Mo $\to$ ^{100}Ru	6.15	4.75	4.40	2.87	4.46	2.96
^{130}Te $\to$ ^{130}Xe	5.62	4.49	4.22	2.97	4.27	3.04

▹ F. Šimkovic *et al*, PRC **79**, 055501 (2009)

SRC	$M^{0\nu}_{GT}$	$M^{0\nu}_{F}$	$M^{0\nu}_{T}$	$M^{0\nu}$
None	0.556	−0.219	−0.015	0.711
Miller-Spencer	0.465	−0.141	−0.014	0.570
CD-Bonn	0.688	−0.222	−0.014	0.845
AV18	0.634	−0.204	−0.014	0.779

▹ M. Horoi and S, Stoica, PRC **81**, 024321 (2010)

	Bare	Jastrow	UCOM Bonn-A
$M^{(0\nu)}_{GT}$	−6.755	−4.681	−6.265
$M^{(0\nu)}_{F}$	2.474	1.778	2.310
Total	−8.328	−5.811	−7.734

▹ M. Kortelainen *et al*, PLB **647**, 128 (2007)

Half-life of $0\nu\beta\beta$-decay

★ The half-life associated with $0\nu\beta\beta$-decay of a nucleus of mass A and charge Z

$$(\mathrm{A},\mathrm{Z}) \to (\mathrm{A},\mathrm{Z}-2) + 2e^- \ ,$$

blue τ , can be written in the form

$$\frac{1}{\tau} = G|M|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 \ ,$$

where G is a phase-space factor, m_e is the electron mass and the so called effective neutrino mass is defined in terms of neutrino mass eigenvalues and elements of the mixing matrix according to

$$\langle m_{\beta\beta}\rangle = \left|\sum_k U_{ek}^2 m_k\right|^2 \ .$$

Structure of NME

- ⋆ The NME can be cast in the form

$$M = M_{\rm GT} - \left(\frac{g_V}{g_A}\right)^2 M_{\rm F} \ ,$$

where g_V and g_A are the vector and axial-vector coupling constant, respectively, while $M_{\rm F}$ and $M_{\rm GT}$ denote the Fermi (F) and Gamow-Teller (GT) transition matrix elements.

- ⋆ Within the closure approximation $M_{\rm F}$ and $M_{\rm GT}$ can be written in the general form

$$M_\alpha = \langle \Psi_f, \mathcal{J}_f^\pi | \sum_{jk} \tau_j^+ \tau_k^+ O_{jk}^\alpha(r) \, | \Psi_i, \mathcal{J}_i^\pi \rangle \ ,$$

where $\alpha = {\rm F}, {\rm GT}$, τ_i^+ is the charge-raising operator acting in the isospin space of the i-th nucleon and Ψ_i and Ψ_f are the intial and final nuclear states, the total angular momentum and parity of which are labeled $\mathcal{J}_i^\pi$ and $\mathcal{J}_f^\pi$.

- Fermi (F) and Gamow-Teller (GT) transition operators

$$O^{F}_{jk}(r) = \mathbb{1}\, H(r_{jk}) \quad , \quad O^{GT}_{jk}(r) = (\boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k)\, H(r_{jk}) \ ,$$

where $H(r_{jk})$ is the so-called neutrino potential, given by

$$H(r_{jk}) = R_A \, \frac{2}{\pi} \int_0^{+\infty} \frac{j_0(q r_{jk})}{q + \langle E \rangle} \, q dq \ ,$$

with $j_0(x) = \sin x / x$, $r_{jk} = |\mathbf{r}_j - \mathbf{r}_k|$, R_A is the nuclear radius and $\langle E \rangle$ is the average energy of the virtual intermediate states employed in the closure approximation.

- Simplifying assumption: the decay process only involves two neutrons of the initial state nucleus, while all other nucleons act as spectators

★ Factorisation of the two-nucleon matrix elements

$$M_\alpha = \sum_{j_1,j_2,j'_1,j'_2,J^\pi} TBTD\,(j_1,j_2,j'_1,j'_2;J^\pi)\langle j'_1 j'_2;\, J^\pi\ T|\,\tau_1^+\tau_2^+ O_{12}^\alpha(r)\,|j_1 j_2;\, J^\pi\ T\rangle_a \,.$$

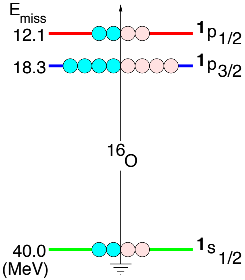
- The coefficients $TBTD\,(j_1,j_2,j'_1,j'_2;J^\pi)$ describe how the spectator nucleons rearrange themselves as a result of the decay process. They are computed in a model space using an effective nucleon-nucleon interaction.
- the two-body matrix element is decomposed into products of reduced matrix elements of operators acting in spin and coordinate space
- The coordinate-space two-nucleon state is rewritten in terms of relative and center of mass coordinates using the Talmi-Moshinski transformation of the harmonic oscillator basis $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{R}_{12} = (\mathbf{r}_1 + \mathbf{r}_2)/2$

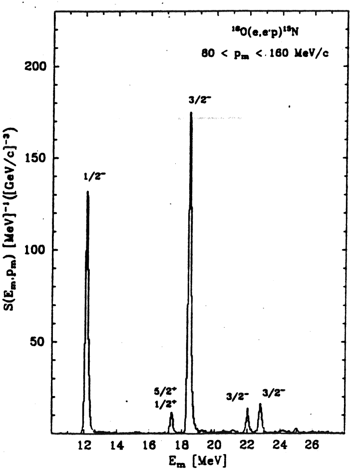
$$\langle \mathbf{r}_1|k_1 l_1\rangle\langle \mathbf{r}_2|k_2 l_2\rangle = \sum_{k,l,K,L} \langle kl,\ KL|k_1 l_1,\ k_2 l_2\rangle_\Lambda \langle \mathbf{R}_{12}|KL\rangle\langle \mathbf{r}_{12}|kl\rangle \,,$$

Limits of the shell-model picture

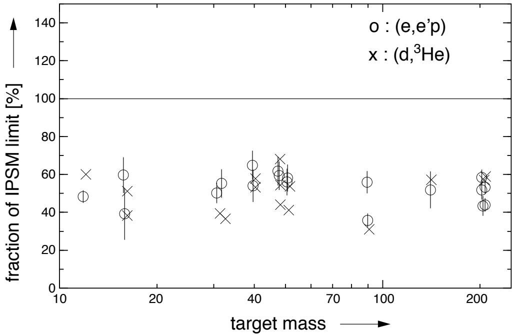
- ⋆ The spectral lines corresponding to the shell model states clearly seen in the missing energy spectra of measured by

$$e + \mathrm{A} \to e' + p + X$$





★ The spectroscopic factors (i.e. the residues of the Green's function at the quasiparticle poles, obtained integrating the spectra in the region of the correponding peak) turn out to be significantly below the shell model prediction, independently of A



★ Correlation effects are known to be large, and significantly affect any processes driven by two-nucelon operators

Correlated basis functions

- Correlated states are obtained from the shell-model states through the transformation

$$|\Psi_n\rangle = \frac{F|\Phi_n\rangle}{\langle\Phi_n|F^\dagger F|\Phi_n\rangle^{1/2}} \ ,$$

with the correlation operator F is defined as (note that $[f_{ij}, f_{ik}] \neq 0$)

$$F = \mathcal{S} \prod_{ij} f_{ij} \ .$$

- The operator structure of the two-body correlation functions f_{ij} reflects the complexity of the nucleon-nucleon potential (spin-isospin dependence, rotational symmetry breaking ...)

$$f_{ij} = \sum_{m=1}^{6} f^{(m)}(r_{ij}) O_{ij}^{(m)} \quad , \quad O_{ij}^{(m)} = [1, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), S_{ij}] \otimes [1, (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)]$$

$$S_{ij} = 3(\boldsymbol{\sigma}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\sigma}_j \cdot \mathbf{r}_{ij})/r_{ij}^2 - (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \ .$$

Correlation function

- The correlation function has been written in the form

$$f_{12} = f(r_{12}) + g(r_{12})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

$$f(r_{12}) = f^{(1)}(r_{12}) + f^{(2)}(r_{12}) \ , \ \ g(r_{12}) = f^{(3)}(r_{12}) + f^{(4)}(r_{12})$$

- The radial dependence of $f(r)$ and $g(r)$ is determined through functional minimization of the expectation value of a realistic nuclear hamiltonian in the correlated ground state of isospin-symmetric nuclear matter

$$\frac{\delta}{\delta\{f^{(m)}\}} \langle 0|H|0\rangle = 0$$

with the boundary conditions

$$\lim_{r\to\infty} f^{(1)}(r) = 1 \quad , \quad \lim_{r\to\infty} f^{(m>1)}(r) = 0$$

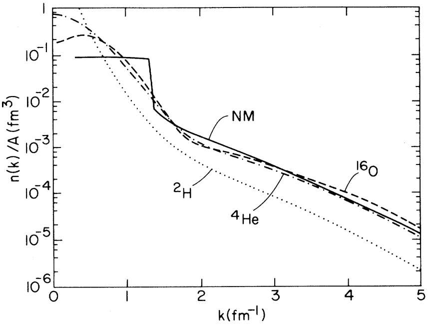
Why nuclear matter?

- ⋆ In isospin-symmetric nuclear matter, the simplifications arising from translation invariance allow to carry out very accurate calculations
- ⋆ Short range dynamics, determining the correlation function, is expected to be little affected by surface and shell effects

- ⋆ The momentum distribution

$$n(k) = \langle 0|a_k^\dagger a_k|0\rangle$$

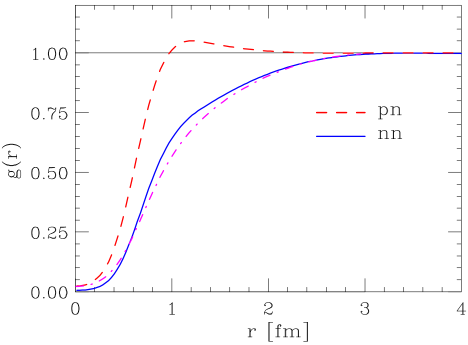
at $k \gtrsim 1.5\ \mathrm{fm}^{-1}$ is nearly independent of A for A> 2



★ Two-nucleon distribution functions

$$g^{nn}(r) = \frac{1}{4\pi r^2}\langle\sum_{j>i}\delta(r - r_{ij})\frac{1}{2}(1 - \tau_i^3)\frac{1}{2}(1 - \tau_j^3)\rangle \ ,$$

$$g^{pn}(r) = \frac{1}{4\pi r^2}\langle\sum_{j>i}\delta(r - r_{ij})\frac{1}{2}(1 + \tau_i^3)\frac{1}{2}(1 - \tau_j^3)\rangle \ ,$$



★ The two-body cluster approximation turns out to be remarkably accurate

Correlations in the two-body $0\nu\beta\beta$-decay matrix element

- Correlations are included modifying the two-nucleon states according to

$$|kl\rangle \to f_{12}|kl\rangle$$

- The above prescription amounts to replacing the F and GT transition operators with effective operators defined as

$$\widetilde{O}^{\alpha}_{12} = f_{12} O^{\alpha}_{12} f_{12} \ .$$

- Because for a neutron-neutron pair $(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) = 1$, neglecting non-central correlations

$$f_{12} = f(r_{12}) + g(r_{12})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

and

$$\widetilde{O}^{F}_{12} = [f^2(r_{12}) + 3g^2(r_{12})]O^{F}_{12} + 2g(r_{12})[f(r_{12}) - g(r_{12})]O^{GT}_{12} \ ,$$

$$\widetilde{O}^{GT}_{12} = [f^2(r_{12}) - 4f(r_{12})g(r_{12}) + 7g^2(r_{12})]O^{GT}_{12} + 6g(r_{12})[f(r_{12}) - g(r_{12})]O^{F}_{12} \ .$$

Numerical calculation

- ⋆ We have considered the reaction

$$^{48}_{20}\mathrm{Ca} \rightarrow \, ^{48}_{22}\mathrm{Ti} + 2e^-$$

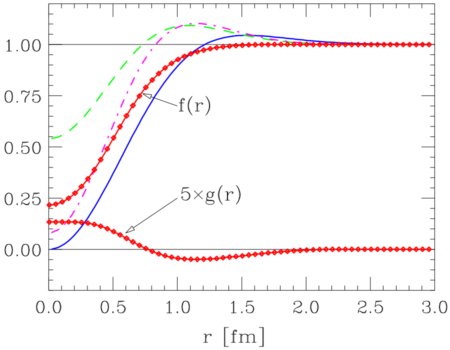
in which both the initial and the final nucleus are in their ground states, having $\mathcal{J}^\pi = 0^+$.

- ⋆ The neutrons and protons involved in the decay process occupy the $1f_{7/2}$ shell
- ⋆ Numerical calculations have been carried out using the TBTD computed by B.A. Brown and harmonic oscillator wave functions corresponding to $\hbar\omega = 45\mathrm{A}^{-1/3} - 25\mathrm{A}^{-2/3}$ MeV
- ⋆ The vector and axial-vector coupling constant and the average energy have been set to the values $g_V = 1$, $g_A = 1.25$ and $\langle E \rangle = 7.72$ MeV

★ Numerical results: $\sim 20\%$ suppression

	$f(r_{12})$	$f(r_{12}) + g(r_{12})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$
M/M_{SM}	0.77	0.79

★ Comparison between different correlation functions, yielding different results



Alternative approach

- ⋆ Correlation effects can also be included through a renormalization of the shell model states

$$|k_i l_i j_i\rangle \to \sqrt{Z_{k_i l_i j_i}}\, |k_i l_i j_i\rangle$$

- ⋆ The spectroscopic factor $Z_{k_i l_i j_i}$ is the residue of the nucleon Green's function at the single particle pole.
- ⋆ It can be computed from the definition $(\alpha = p,\ n)$

$$Z^{\alpha}_{k_i l_i j_i} = \int d^3x |\phi^{\alpha}_{k_i l_i j_i}(x)|^2$$

$$\phi^{\alpha}_{k_i l_i j_i}(x_1) = \frac{\sqrt{A}}{N^{\alpha}_{k_i l_i j_i}} \langle \Psi^{\alpha}_{k_i l_i j_i}(x_2, \ldots, x_A) | \Psi_0(x_1, \ldots, x_A)\rangle \ .$$

$$N^{\alpha}_{k_i l_i j_i} = \langle \Psi^{\alpha}_{k_i l_i j_i} | \Psi^{\alpha}_{k_i l_i j_i}\rangle^{1/2} \langle \Psi_0 | \Psi_0\rangle^{1/2}$$

Including the spectroscopic factors in the NME

- ⋆ In the case of $^{48}\mathrm{Ca} \to {}^{48}\mathrm{Ti}$ $0\nu\beta\beta$-decay we replace

$$M_\alpha \to \widetilde{M}_\alpha = Z^p_{1f_{7/2}}(^{48}\mathrm{Ti}) Z^n_{1f_{7/2}}(^{48}\mathrm{Ca})\, M_\alpha$$

- ⋆ Under the further assumption (the validity of which is strongly supported by nuclear matter calculations)

$$Z^p_{1f_{7/2}}(^{48}\mathrm{Ti}) \approx Z^n_{1f_{7/2}}(^{48}\mathrm{Ca})$$

one finds

$$\widetilde{M}_\alpha = [Z^n_{1f_{7/2}}(^{48}\mathrm{Ca})]^2 M_\alpha$$

- ⋆ The spectroscopic factor of the $f_{7/2}$ state of $^{48}\mathrm{Ca}$ calculated using nuclear wave functions, including correlations as well as surface and shell effects, can be used to obtain the NME

$$Z^n_{1f_{7/2}}(^{48}\mathrm{Ca}) = 0.91 \Rightarrow M/M_{SM} = 0.83$$

Connecting correlation function and spectroscopic factor

- ⋆ Note that using renormalised single particle states is conceptually equivalent to using correlated states.
- ⋆ In the absence of correlations, $Z_{k_i l_i j_j} = 1$ for all occupied shell model states, and $Z_{k_i l_i j_i} = 0$ otherwise.
- ⋆ Formally, the connection can be shown at leading order in the cluster expansion of the overlap

$$\phi_n(x_1) = \langle \Psi_n(x_2, \ldots, x_A) | \Psi_0(x_1, \ldots, x_A) \rangle$$

with the correlated wave functions defined as

$$\Psi_0(x_1, \ldots, x_A) = \prod_{j>i=1}^{A} f_{ij} \, |\Phi_0^{\rm SM}(x_1, \ldots, x_A)\rangle$$

$$\Psi_0(x_1, \ldots, x_A) = \prod_{k>j=2}^{A} f_{jk} \, |\Phi_n^{\rm SM}(x_2, \ldots, x_A)\rangle$$

Summary & Outlook

- ⋆ Bottom line: whatever the theoretical approach taken, the consistent inclusion of short range correlations in the shell model picture is a long standing and still very elusive issue.
- ⋆ The results of our calculations indicate that inclusion of correlations leads to a ∼20% decrease of the NME
- ⋆ Comparison between our results and those available in the literature show that the shape of the correlation function plays a critical role.
- ⋆ The ∼ 20% suppression is supported by the results of an alternative calculation based on the use of sprctroscopic factors
- ⋆ While our results appear to be encouraging, further studies are needed:
  - analysis of higher order contributions to the cluster expansion of the spectroscopic factor
  - use of a correlation function obtained from the minimisation of the ground state energy of ^{48}Ca
  - inclusion of the full operator structure of the correlation function