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Solving the $(g-2)_\mu$ Anomaly in Two Higgs Doublet Models

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arXiv:1409.3199

Introduction

- “Can the muon $g-2$ be accounted for in 2HDMs?”

$$\Delta a_\mu \equiv a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = +262(85) \times 10^{-11}$$

a lot of studies in SUSY, much less in 2HDMs...

0102145, 146, 147, ... vs. Dedes, Haber, 0102297
Cheung, Chou, Kong, 0103183

- Constraints from the 126 GeV Higgs data at LHC.
- Extra Higgs spectrum constrained by EWPD, vacuum stability & perturbativity, and also by $B_q \rightarrow X_s \gamma, \mu^+ \mu^-$, etc.
- The unique choice: Type X with Barr-Zee two loop in the ranges of $m_A < 80 \text{ GeV}$, $m_H \sim m_{H^\pm} < 230 \text{ GeV}$ & $t_\beta > 40$.

2HDMs

- Two Higgs Doublets:

$$\Phi_{1,2} = (\phi_{1,2}^+, \frac{1}{\sqrt{2}}[v_{1,2} + \rho_{1,2} + i \eta_{1,2}]) \rightarrow h, H, A, H^\pm; \quad t_\beta = \frac{v_1}{v_2}$$

$$G^0 = \eta_1 c_\beta + \eta_2 s_\beta \quad h = \rho_1 c_\alpha - \rho_2 s_\alpha$$

$$A = \eta_1 s_\beta - \eta_2 c_\beta \quad H = \rho_1 s_\alpha + \rho_2 c_\alpha$$

- Gauge coupling: $\mathcal{L}_g = g_V m_V (s_{\beta-\alpha} h + c_{\beta-\alpha} H) VV + \dots$

- FCNC: Mass diagonalization \neq Yukawa diagonalization

$$\mathcal{L}_{Yuk} = y_{ij}^{u2} \Phi_2 q_i u_j^c + y_{ij}^{dp} \Phi_p q_i d_j^c + y_{ij}^{eq} \Phi_q l_i e_j^c + h.c.$$

$$M_{ij}^f = y_{ij}^{f1} \frac{v_1}{\sqrt{2}} + y_{ij}^{f2} \frac{v_2}{\sqrt{2}} \rightarrow y_{ij}^d h d_i d_j^c + \dots$$

4 types of 2HDMs

Natural flavor conservation imposed by Z_2 :
only one Higgs to each Yukawa type (d, e)

Nb) Type III : aligned Yukawa
 $y_{ij}^1 \sim y_{ij}^2$

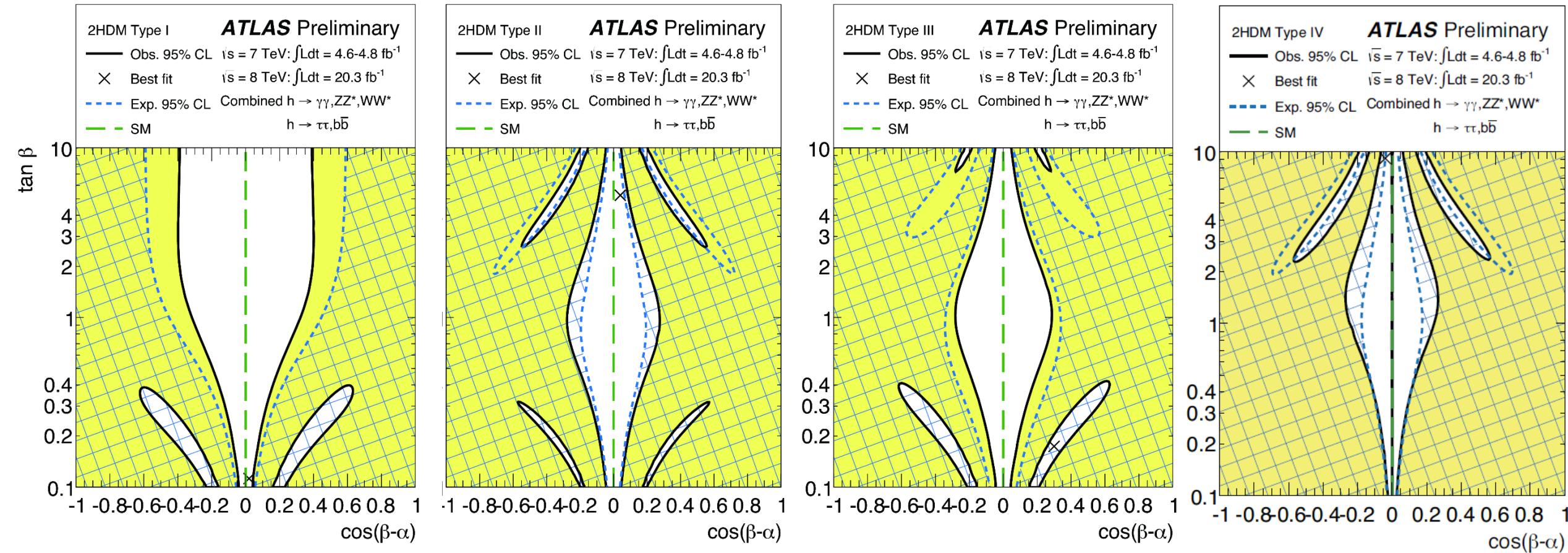
$$\Phi_2(+), \Phi_1(-); t_R(+), d_R(\pm), e_R(\pm)$$

Model	u_R^i	d_R^i	e_R^i	y_u^A	y_d^A	y_l^A	y_u^H	y_d^H	y_l^H	y_u^h	y_d^h	y_l^h
Type I	Φ_2	Φ_2	Φ_2	$\cot\beta$	$-\cot\beta$	$-\cot\beta$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$
Type II	Φ_2	Φ_1	Φ_1	$\cot\beta$	$\tan\beta$	$\tan\beta$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$	$\frac{\cos\alpha}{\cos\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$	$-\frac{\sin\alpha}{\cos\beta}$
Lepton-specific	Φ_2	Φ_2	Φ_1	$\cot\beta$	$-\cot\beta$	$\tan\beta$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$
Flipped	Φ_2	Φ_1	Φ_2	$\cot\beta$	$\tan\beta$	$-\cot\beta$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\cos\alpha}{\sin\beta}$
Type I												
Type II												
Type X												
Type Y												

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}}^{\text{2HDM}} &= - \sum_{f=u,d,\ell} \frac{m_f}{v} \left(\xi_h^f \bar{f} f h + \xi_H^f \bar{f} f H - i \xi_A^f \bar{f} \gamma_5 f A \right) \\ &\quad \text{126 GeV} \\ &\quad - \left\{ \frac{\sqrt{2} V_{ud}}{v} \bar{u} (m_u \xi_A^u P_L + m_d \xi_A^d P_R) d H^+ + \frac{\sqrt{2} m_\ell \xi_A^\ell}{v} \bar{\ell}_R \ell_R H^+ + \text{H.c.} \right\} \end{aligned}$$

126 GeV SM Higgs: $g_{hVV} = \sin(\beta - \alpha) \approx 1$

ATLAS-CONF-2014-010



Aligned/decoupled 2HDMs

$$\begin{aligned}
V_H = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) \\
& + \frac{1}{2} \lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \\
& + \left[\frac{1}{2} \lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \cancel{\lambda_6} \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_1^\dagger \Phi_2 \right) + \cancel{\lambda_7} \left(\Phi_2^\dagger \Phi_2 \right) \left(\Phi_1^\dagger \Phi_2 \right) + \text{H.c.} \right],
\end{aligned}$$

Model	u_R^i	d_R^i	e_R^i	y_u^A	y_d^A	y_l^A	y_u^H	y_d^H	y_l^H	y_u^h	y_d^h	y_l^h	
Type I	Φ_2	Φ_2	Φ_2	Type I	$\cot \beta$	$-\cot \beta$	$-\cot \beta$	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\sin \beta}$
Type II	Φ_2	Φ_1	Φ_1	Type II	$\cot \beta$	$\tan \beta$	$\tan \beta$	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\cos \beta}$	$\frac{\cos \alpha}{\cos \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$-\frac{\sin \alpha}{\cos \beta}$	$-\frac{\sin \alpha}{\cos \beta}$
Lepton-specific	Φ_2	Φ_2	Φ_1	Type X	$\cot \beta$	$-\cot \beta$	$\tan \beta$	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\cos \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$-\frac{\sin \alpha}{\cos \beta}$
Flipped	Φ_2	Φ_1	Φ_2	Type Y	$\cot \beta$	$\tan \beta$	$-\cot \beta$	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\cos \beta}$	$\frac{\sin \alpha}{\sin \beta}$	$\frac{\cos \alpha}{\sin \beta}$	$-\frac{\sin \alpha}{\cos \beta}$	$\frac{\cos \alpha}{\sin \beta}$

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}}^{\text{2HDM}} = & - \sum_{f=u,d,\ell} \frac{m_f}{v} \left(\xi_h^f \bar{f} f h + \xi_H^f \bar{f} f H - i \xi_A^f \bar{f} \gamma_5 f A \right) \\
& \quad \text{126 GeV} \\
& - \left\{ \frac{\sqrt{2} V_{ud}}{v} \bar{u} (m_u \xi_A^u P_L + m_d \xi_A^d P_R) d H^+ + \frac{\sqrt{2} m_\ell \xi_A^\ell}{v} \bar{\nu}_L \ell_R H^+ + \text{H.c.} \right\}
\end{aligned}$$

LHC Higgs data
 $\cos(\beta - \alpha) \approx 0$

EWPD

$$M_W^2 = \frac{M_Z^2}{2} \left[1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}} \frac{1}{1 - \Delta r} \right]$$

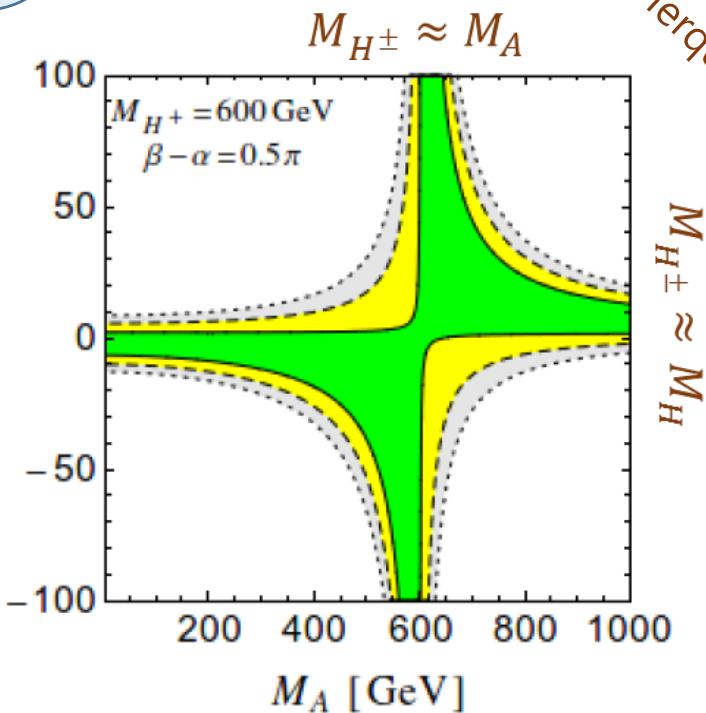
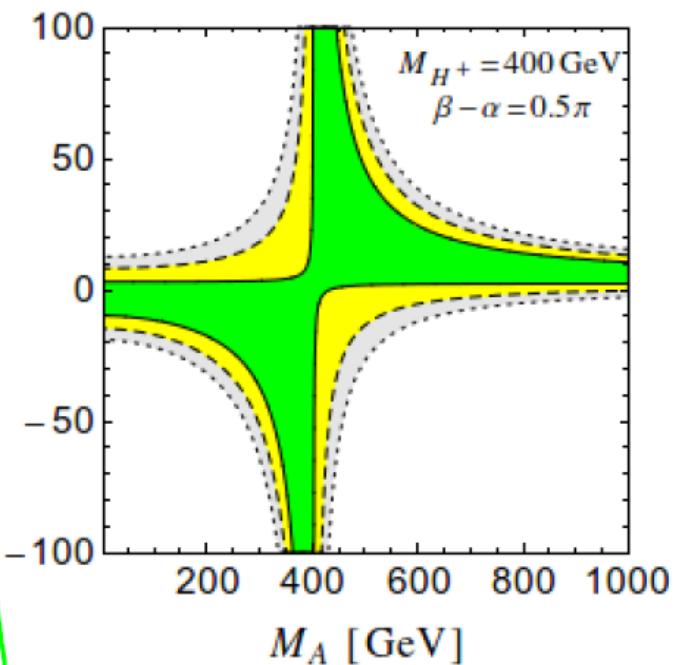
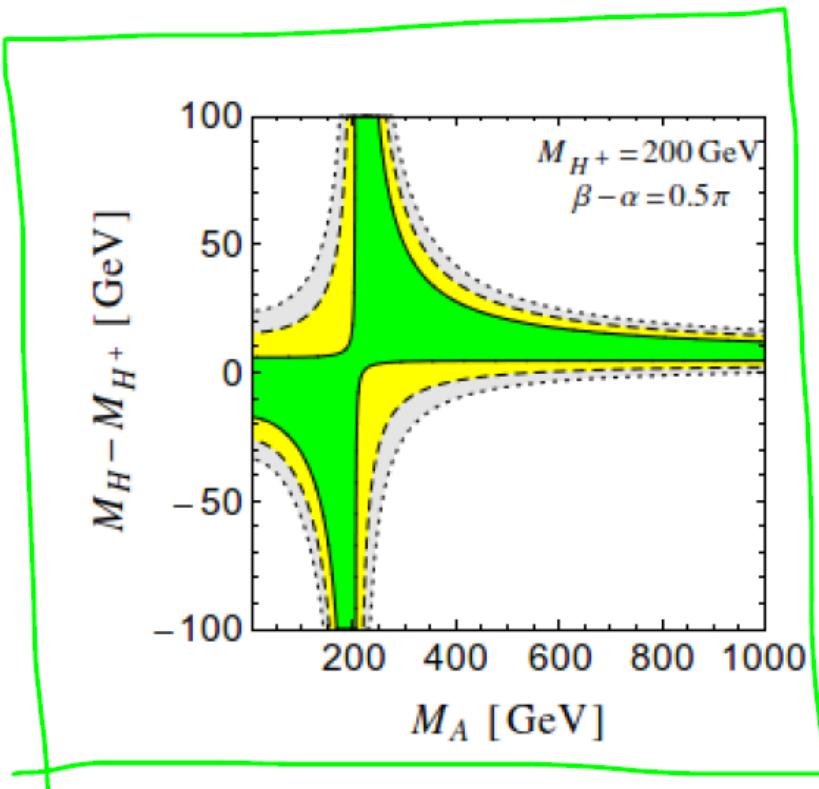
$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = k_l (M_Z^2) \sin^2 \theta_W$$

$$M_W^{\text{exp}} = 80.385 \pm 0.015 \text{ GeV},$$

$$\sin^2 \theta_{\text{eff}}^{\text{lept, exp}} = 0.23153 \pm 0.00016.$$

$$\Delta r^{\text{2HDM}} = \Delta \alpha^{\text{2HDM}} - \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \Delta \rho^{\text{2HDM}} + \dots,$$

$$\Delta k_l^{\text{2HDM}} = + \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \Delta \rho^{\text{2HDM}} + \dots,$$



Twisted custodial symmetry:
 Gerard-Herquet, 0703051

Vacuum stability & perturbativity

$$\lambda_{1,2} > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}, \quad |\lambda_5| < \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2}$$

$$m_{12}^2(m_{11}^2 - m_{22}^2 \sqrt{\lambda_1/\lambda_2})(\tan \beta - (\lambda_1/\lambda_2)^{1/4}) > 0$$

For $\tan \beta \gg 1$

$$\lambda_2 v^2 \simeq M_h^2 + \lambda_1 v^2 / \tan^4 \beta,$$

$$\lambda_3 v^2 \simeq 2M_{H^\pm}^2 - 2M_H^2 + M_h^2 + \lambda_1 v^2 / \tan^2 \beta,$$

$$m_{12}^2 \simeq M_H^2 / \tan \beta + (M_h^2 - \lambda_1 v^2) / \tan^3 \beta.$$

$$M_A^2 = \frac{m_{12}^2}{\sin \beta \cos \beta} - \lambda_5 v^2,$$

$$M_{H^\pm}^2 = M_A^2 + \frac{1}{2} v^2 (\lambda_5 - \lambda_4).$$

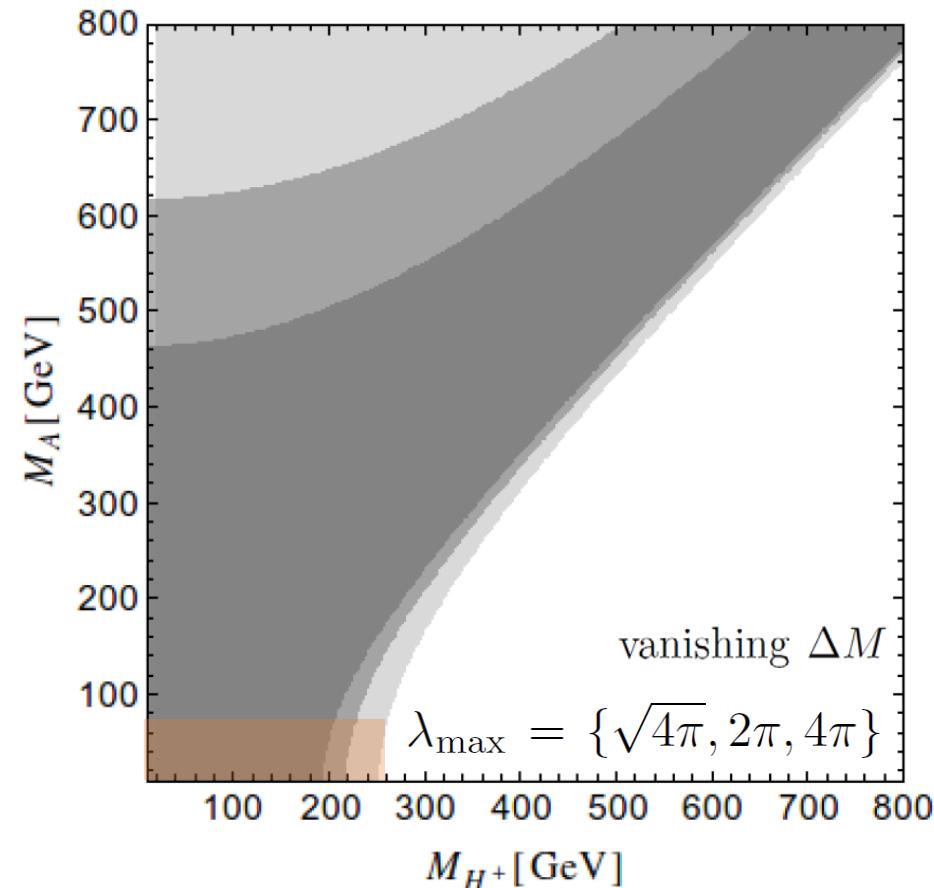
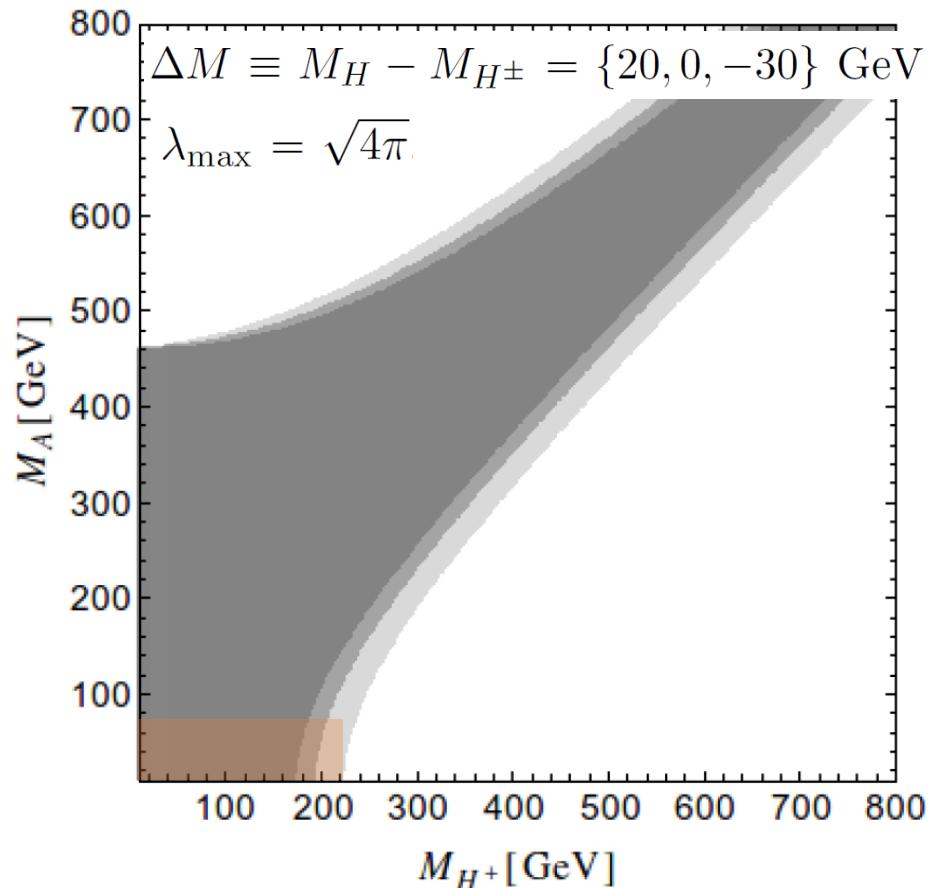
$$\lambda_2 v^2 \approx M_h^2$$

$$\lambda_3 v^2 \approx 2M_{H^\pm}^2 - 2M_H^2 + M_h^2$$

$$\lambda_4 v^2 \approx M_H^2 - 2M_{H^\pm}^2 + M_A^2$$

$$\lambda_5 v^2 \approx M_H^2 - M_A^2$$

Vacuum stability & perturbativity



Muon $g-2$

- One-loop contribution:

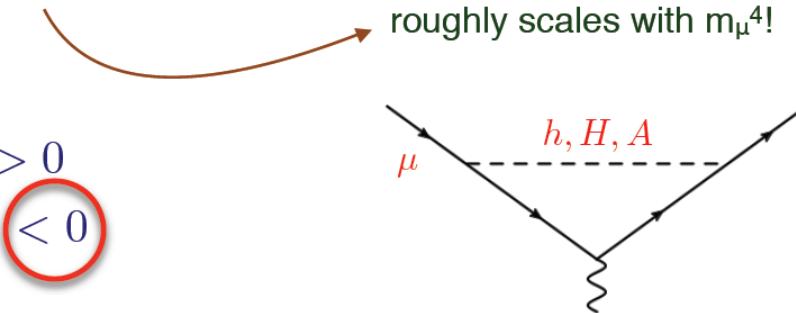
$$\delta a_\mu^{2\text{HDM}}(\text{1loop}) = \frac{G_F m_\mu^2}{4\pi^2 \sqrt{2}} \sum_{j=h,H,A,H^\pm} (y_\mu^j)^2 r_\mu^j f_j(r_\mu^j)$$

For $r_\mu^j = m_\mu^2/M_j^2 \ll 1$:

$$f_{h,H}(r) \sim -\ln r - 7/6 + O(r) > 0$$

$$f_A(r) \sim +\ln r + 11/6 + O(r) < 0$$

$$f_{H^\pm}(r) \sim -1/6 + O(r) < 0$$



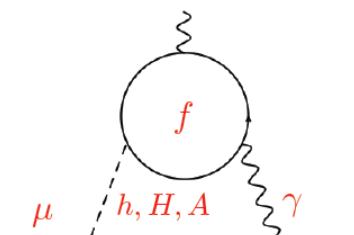
- Two-loop Barr-Zee type diagrams:

$$\delta a_\mu^{2\text{HDM}}(\text{2loop} - \text{BZ}) = \frac{G_F m_\mu^2}{4\pi^2 \sqrt{2}} \frac{\alpha_{\text{em}}}{\pi} \sum_{f; i=h,H,A} N_f^c Q_f^2 y_\mu^i y_f^i r_f^i g_i(r_f^i)$$

$$g_{h,H}(r) < 0$$

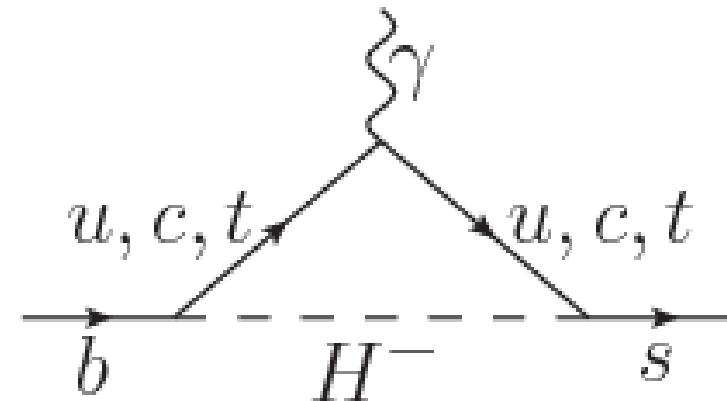
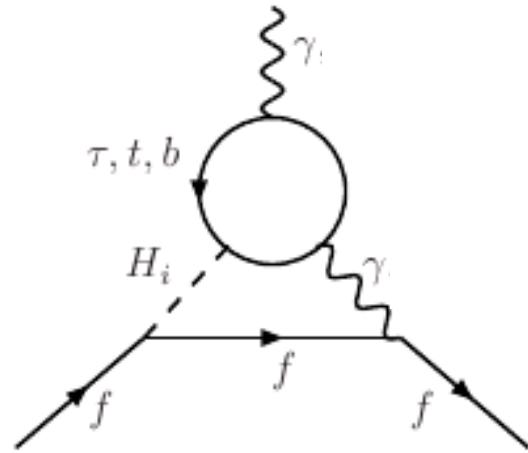
$$g_A(r) > 0$$

Type II, $M_A \gtrsim 3\text{GeV}$ & $\tan\beta \gtrsim 5$:
 $(\text{2loop})_A > (\text{1loop})_A$. Similar for X.



Muon $g-2$ vs. $b \rightarrow s \gamma$

Cheung, Chou, Kong, 0103183
 Cao, et.al, 0909.5148



$$\delta a_\mu^{2\text{loop}}(\gamma\text{BZ}) = \frac{G_F m_\mu^2}{4\pi^2 \sqrt{2}} \frac{\alpha_{\text{em}}}{\pi} \sum_{i,f} N_f^c Q_f^2 y_\mu^i y_f^i r_f^i g_i(r_f^i)$$

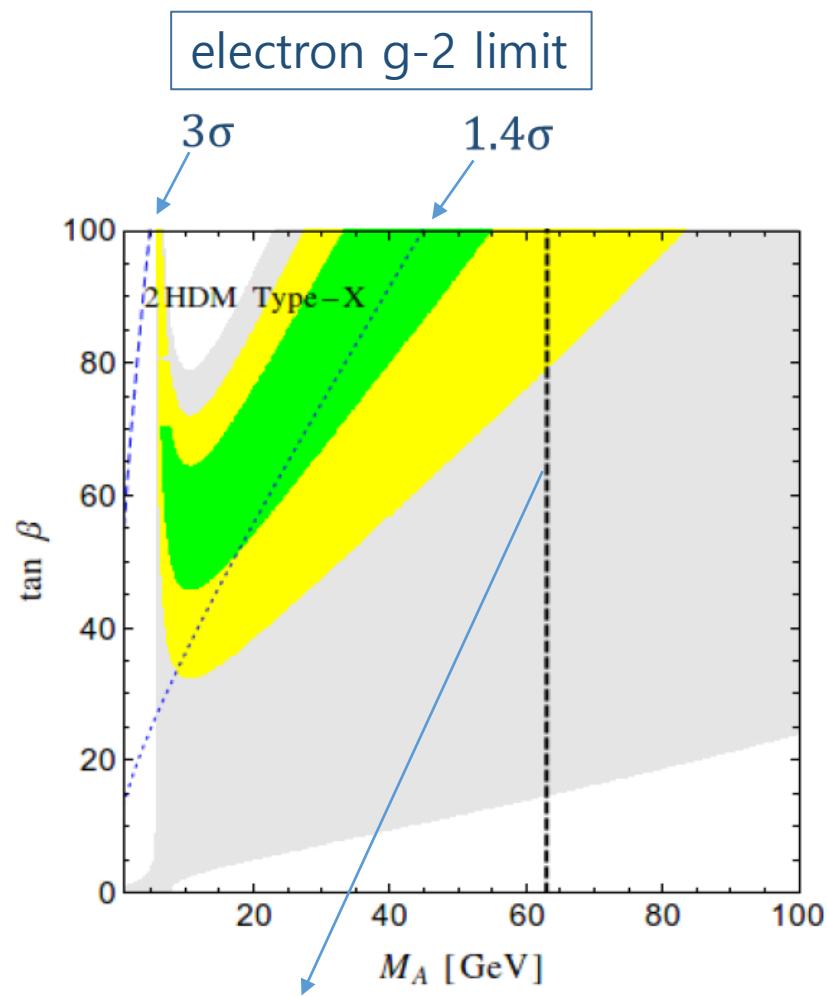
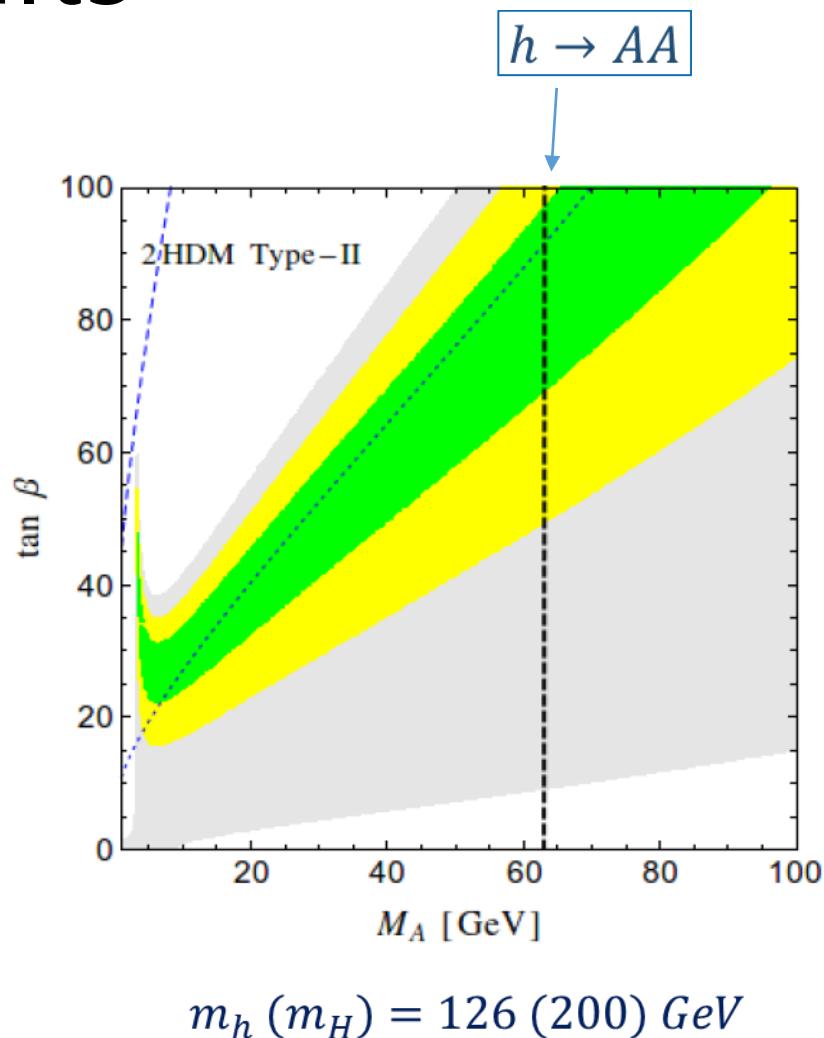
	τ	t	b
I	$1/t_\beta^2$	$-1/t_\beta^2$	$1/t_\beta^2$
II	t_β^2	1	t_β^2
X	t_β^2	1	-1
Y	$1/t_\beta^2$	$-1/t_\beta^2$	-1

H : negative
 A : positive

$$\frac{m_t}{t_\beta} P_L - \frac{m_b}{t_\beta} P_R \quad (I, X) \quad \text{NA} \quad \text{for } t_\beta > 2$$

$$\frac{m_t}{t_\beta} P_L + m_b t_\beta P_R \quad (II, Y) \quad m_{H^\pm} > 380 \text{ GeV}$$

Results



$$\lambda_{hAA} v \approx (1 + \xi_h^\tau) M_H^2 - \xi_h^\tau M_h^2 - 2M_A^2 \ll v^2$$

Conclusion

- Imposing the muon $g-2$ singles out Type X 2HDM in a specific parameter region provable at the LHC2.
- The 1σ region of $t_\beta > 50, m_A < 50 \text{ GeV}$ requires $\lambda_{hAA}/v \ll 1$ to suppress $h \rightarrow AA$.
- At 2σ allowed is $t_\beta > 30, m_A \lesssim 80 \text{ GeV} \& m_H \sim m_{H^\pm} \lesssim 230 \text{ GeV}$.
- LHC13 prospect under investigation:

$$pp \rightarrow h, Z, W \rightarrow AA, AH, AH^\pm \rightarrow 4\tau, 3\tau + \nu.$$

Kanemura, et.al., 1406.3294