Recent Progress in Lattice QCD

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Why Lattices?

- Gauge sector of SM fully fixed by symmetry (local gi)
- Yukawa sector free: fermion masses and mixing are parameters in $\mathcal{L}_{\rm SM}$
  $\to$ determined through comparison theory vs. experiment
  quarks - main obstacle NP QCD
- BSM: clue about Yukawa's if assuming 2HDM-Type ??, “GUT"
  some rep of SU(5), SO(10), E6..., alignment …
- QCD is still mysterious in NP-regime
- Any/every observable involving quarks suffers from NP QCD effects.
  Sometimes factorization helps (but is rarely a theorem):
  ☐ Either look for observables with minimized hadronic uncertainties
  ☐ use ab initio method to compute hadronic quantities - LQCD

What is Lattice QCD?

Regularization scheme of the path integral formulation of (Euclidean) QCD

$$Z = \int \mathcal{D}\psi_f \mathcal{D}\bar{\psi}_f \mathcal{D}U_\mu e^{-S[\psi,\bar{\psi},U]} = \int \mathcal{D}U_\mu \ \Pi_f \det[\not{D} + m_f] e^{-S[U]}$$

that can let you compute NP QCD effects

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi_f \mathcal{D}\bar{\psi}_f \mathcal{D}U_\mu \ \mathcal{O}[\psi, \bar{\psi}, U] \ e^{-S[\psi,\bar{\psi},U]}$$

- discretization of hypercubic lattice with (anti-)PBC
- a-lattice spacing: UV regulator [momenta $p \lesssim \pi/a$]
  $L = N_L a$ size: IR regulator
- Problem treatable by (H)MC $\Rightarrow$ stat.uncertainties - CL theorem
  arbitrary accuracy achievable in principle
- Ab initio: only parameters (bare) quark masses and gauge coupling!

Realistic calculations

A physical quantity

$$\Phi^{\text{latt.}} = \Phi^{\text{cont.}} \left[1 + a\Phi' + a^2\Phi'' + \dots \right]$$

Many discretizations of Dirac operator [WilsonClover, tmQCD, Staggered, HISQ, FLIC, anisotropic...] each with its pros and cons and each designed to $\Phi' = 0$

Realistic calculations

A physical quantity

$$\Phi^{\text{latt.}}(m_q) = \Phi^{\text{cont.}}(m_q)\left[1 + \cancel{a\Phi'(m_q)} + a^2\Phi''(m_q) + \ldots\right]$$

Lattice QCD Challenge:

(i) work at several lattice spacing to go to continuum limit

(ii) include $q\bar{q}$-loops in the vacuum fluctuations $n_f = 2, 2+1, 2+1+1$

(iii) work with light quark masses as close to the physical m_{ud} (i.e. $m_\pi^{\text{phys.}}$) as possible

(iv) keep physical volume large - check for the FV corrections if they can be treated by FVChPT

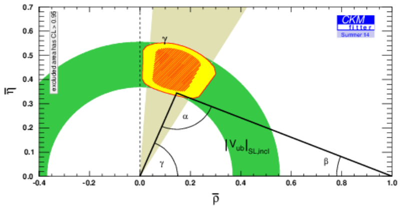
(v) Matrix elements - renormalization constants should be computed non-perturbatively

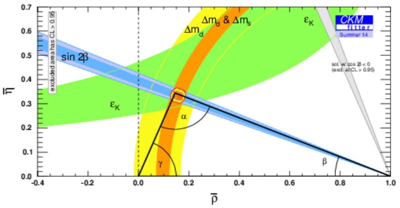
Several collaborations - physical light quark masses → check on SU(2) ChPT.
Careful: FV corrections exacerbate physical chiral logs [Dürr, Lattice 2014]

What quantities?

Euclidean space $\Rightarrow$ Minkowski physics information (FSI, δ_S,...) not accessed directly, but in some cases useful information can be extracted

Best suited for *hadron* $\to$ 0 and *hadron* $\to$ *hadron* processes
Tree level: leptonic and semileptonic decays
Loop induced: meson mixing, penguin induced decays...





N.B. UTfit and CKMfitter: UT reconstructed from the tree level and loop induced processes are the same (within current accuracy)

What quantities?

In '00-'10 we checked the CKM unitarity (1st row) $[G_F = G_\mu]$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$$

Using $\underline{f_+^{K\pi}(0) = 0.956(6)(6)_{n_f=2}, 0.966(3)_{n_f=2+1}}$

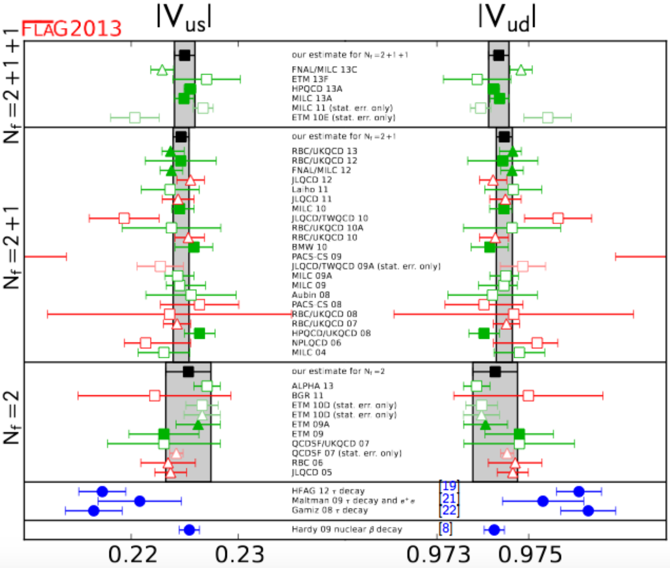
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9993(5)$$

Using $\underline{f_K/f_\pi = 1.192(5)_{n_f=2+1}}$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0000(6)$$

Huge coordinated effort!

What quantities?



FLAG

FLAG- Flavor Lattice Averaging Group [1310.8555]

Major impact of Lattice QCD

- $\alpha_s^{\overline{\rm MS}}(M_Z) = 0.1184(12)$ (!!!)
- $m_s^{\overline{\rm MS}}(2\ \text{GeV}) = 101(3)\ \text{MeV}_{n_f=2}, 94(2)(2)\ \text{MeV}_{n_f=2+1}$
- $m_s/m_{ud} = 28(1)_{n_f=2}, 27.5(5)_{n_f=2+1}$
- $B_K^{\rm rgi} = 0.73(3)(2)_{n_f=2}, 0.77(1)_{n_f=2+1}$

2nd row Unitarity

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

Charm quark is nowadays perfectly treatable like any other light quark (no recourse to effective theories and no troubles with matching with QCD in continuum)

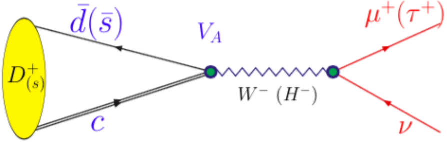
- From axial Ward identities (tmQCD) 1403.4504 (NPR):

$$m_c^{\overline{\rm MS}}(m_c) = 1.35(4)_{n_f=4}\text{GeV} \qquad [1.28(4)_{n_f=2}\text{GeV } 1010.3659]$$

- From ratios of moments (stagQCD) 1408.4169:

$$m_c^{\overline{\rm MS}}(m_c) = 1.27(1)_{n_f=4}\text{GeV}$$

Need a third calculation!

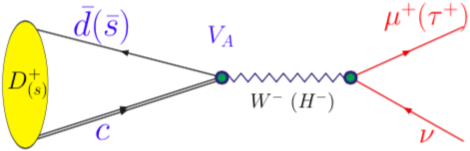


$$\langle 0|\bar{q}\gamma_\mu\gamma_5 c|D_q(p)\rangle = i f_{D_q} p_\mu$$
$$(m_c + m_q)\langle 0|\bar{q} i\gamma_5 c|D_q(p)\rangle = f_{D_q} m^2_{D_q}$$

From 2pt correlation functions $\sum_{\vec{x}} \langle \mathcal{O}_(\vec{x},t)\mathcal{O}(0)\rangle \propto |\langle 0|\mathcal{O}|D_q\rangle|^2 \exp[-m_{D_q} t]$

FLAG 1310.8555

$$f_D = [208(7)_{n_f=2}, 209(3)_{n_f=2+1}] \text{ MeV}$$
$$f_{D_s} = [250(7)_{n_f=2}, 249(3)_{n_f=2+1}] \text{ MeV}$$
$$f_{D_s}/f_D = [1.20(2)_{n_f=2}, 1.19(1)_{n_f=2+1}]$$



NEW Fermilab/MILC 1407.3772

$$f_D = 212.6(4)\left({}^{+1.0}_{-1.2}\right)\ \mathrm{MeV}_{n_f=4} \qquad f_{D_s} = 249.0(3)\left({}^{+1.1}_{-1.5}\right)\ \mathrm{MeV}_{n_f=4}$$

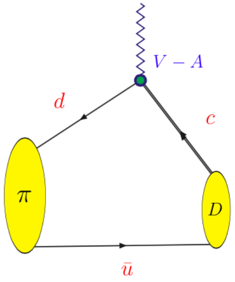
$$f_{D_s}/f_D = 1.171(1)(3)_{n_f=4}$$

NEW ETMC 1411.0484 (update of 1308.1851)

$$f_D = 209(5)\ \mathrm{MeV}_{n_f=4} \qquad f_{D_s} = 248(4)\ \mathrm{MeV}_{n_f=4}$$

$$f_{D_s}/f_D = 1.19(2)_{n_f=4}$$

DWF catching up, cf. eg. 1404.3648



Three-point functions - harder but doable

$$\langle \pi(k)|\bar{q}\gamma_\mu c|D_q(p)\rangle \propto f_+(q^2), f_0(q^2)$$

$m_\ell^2 \leq q^2 \leq (m_D - m_\pi)^2$, and $f_+(0) = f_0(0)$
Need to inject momenta either in Fourier transform or via TBC

FLAG 1310.8555: $f_+^{D\pi}(0) = 0.67(3)$ $f_+^{DK}(0) = 0.75(2)$

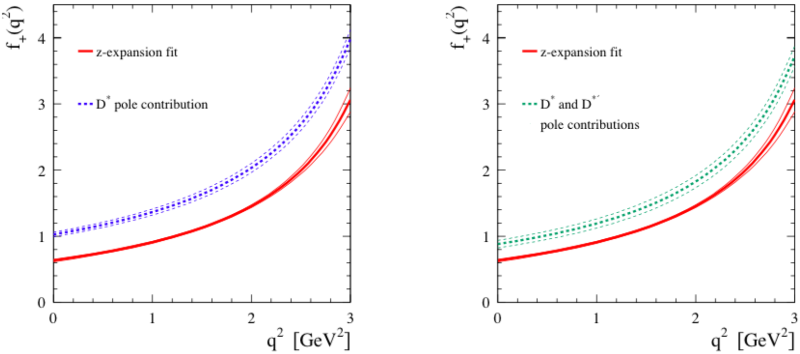
$D \to \pi/K\ell\nu_\ell$

q^2-dependence from lattice consistent with experiment (BaBar, Belle, BES, CLEO-c)
Compare now with nearest pole dominance:

$$f_+^{D\pi}(q^2) = \frac{\gamma_0}{m_{D^*}^2 - q^2}\,, \qquad \gamma_0 \equiv \underset{q^2=m_{D^*}^2}{\mathrm{Res}} f_+(q^2) = \frac{1}{2} m_{D^*} f_{D^*} g_{D^*D\pi}$$

$$\begin{aligned} g_{D^*D\pi} &= 16.9(2)_{\rm exp} \quad \text{BaBar, 1304.5657,} \\ &\quad 16(1)_{\rm latt} \quad \text{DB, Sanfilippo, 1210.5410} \\ f_{D^*}/f_D &= 1.21(3)_{\rm latt} \quad 1407.1019 \end{aligned}$$

Compare now with the nearest pole dominance:



Different from actual exp. data!

Info on second pole can be deduced from experiment - does not saturate FF.

- Vector current made of (mass) non-degenerate quarks - VMD fails
- Elastic case: pion form factor, η_c form factor - consistent with VMD

$D \to \pi/K\ell\nu_\ell$

More recent $f_{+,0}^{D\to K/\pi}(q^2)$ by Fermilab/MILC, ETMC, HPQCD.
Vector meson in the final state?

- $D \to \rho\ell\nu_\ell$
  difficult $\Gamma_\rho/m_\rho = 0.19$
  resonant vs. nonresonant contribution; treatment of the FSI
  (complicated but possible - accuracy not competitive)
- $D \to K^*\ell\nu_\ell$
  borderline $\Gamma_{K^*}/m_{K^*} = 0.05$
  S-wave $K\pi$ difficult (feasible)
- $D_s \to \phi\ell\nu_\ell$
  NRA should be OK, $\Gamma_\phi/m_\phi = 0.004$

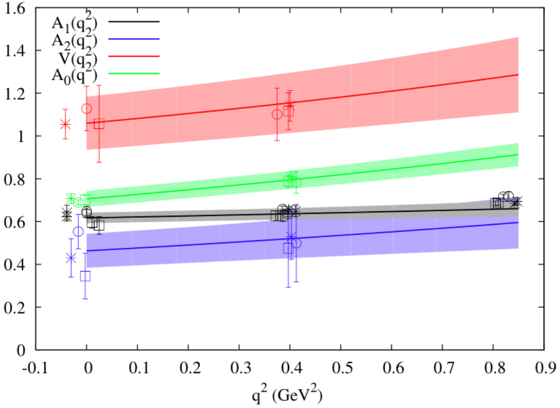
HPQCD [1311.6669] - 2 lattices

$$\langle\phi|\bar{s}\gamma_\mu c|D_s\rangle \propto V(q^2) \quad \langle\phi|\bar{s}\gamma_\mu\gamma_5 c|D_s\rangle \propto A_1(q^2), A_2(q^2), A_0(q^2)$$

Smooth q^2-behavior

$D_s \to \phi \ell \nu_\ell$ HPQCD [1311.6669]

$$\langle \phi | \bar{s}\gamma_\mu c | D_s \rangle \propto V(q^2) \quad \langle \phi | \bar{s}\gamma_\mu \gamma_5 c | D_s \rangle \propto A_1(q^2), A_2(q^2), A_0(q^2)$$



$$V(0) = 1.06(12), \qquad A_1(0) = 0.62(2),$$
$$A_2(0) = 0.46(8), \qquad A_0(0) = 0.71(4)$$

$B_{(s)}$ mesons

b quark is still too heavy to be directly simulated on the lattice.
$\rightarrow$ Step scaling or use effective theories

FLAG 1310.8555

$$f_B = [189(8)_{n_f=2}, 191(4)_{n_f=2+1}]\ \mathrm{MeV}$$
$$f_{B_s} = [228(8)_{n_f=2}, 228(5)_{n_f=2+1}]\ \mathrm{MeV}$$
$$f_{B_s}/f_B = 1.21(2)_{n_f=2}, 1.20(2)_{n_f=2+1}$$

RBC/UKQCD, 1404.4670 [DWF light $n_f = 2 + 1$, Fermilab' heavy]

$$f_B = 195(16)\ \mathrm{MeV}, \quad f_{B_s} = 235(12)\ \mathrm{MeV}_{n_f=2+1} \quad f_{B_s}/f_B = 1.22(8)$$

See also 1406.6192 [DWF + HQET (Pert. match!)]

ETMC 1411.0484, tmQCD ($n_f = 4$) + Step Scal. [update of 1308.1851]

$$f_B = 196(9)\ \mathrm{MeV} \quad f_{B_s} = 235(9)\ \mathrm{MeV} \quad f_{B_s}/f_B = 1.20(3)$$

Alpha 1404.3590 [Wilson-impr $n_f = 2$ + HQET $+1/m_b$]

$$f_B = 186(13)\ \mathrm{MeV} \quad f_{B_s} = 224(14)\ \mathrm{MeV} \quad f_{B_s}/f_B = 1.20(7)$$

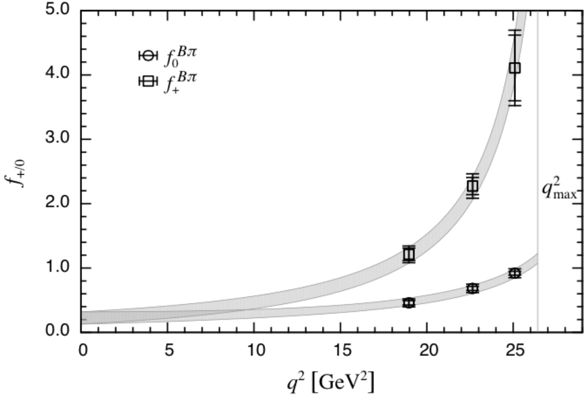
$B \to \pi \ell \nu_\ell$

RBC/UKQCD 1501.05373

[DWF light $n_f = 2+1$, Fermilab' heavy, 2-"a"]
Results directly accessible at large q^2

q^2[GeV^2]	19	22.6	26.1
$f_+^{B\to\pi}(q^2)$	1.21(13)	2.27(19)	4.1(6)
$f_0^{B\to\pi}(q^2)$	0.46(6)	0.68(7)	0.92(7)

RBC/UKQCD 1501.05373
Elsewhere - using "z"-expansion extrapolation formula



In the end $V_{ub} = 3.61(32) \times 10^{-3}$ (comparison with BaBar - large q^2-bin!)

$B_{d,s} - \bar{B}_{d,s}$ mixing

ETMC - tour de force ($n_f = 2$)
full set of matrix elements using step scaling - tmQCD + OS, multi a

$$
\begin{aligned}
\mathcal{O}_1 &= \bar{b}\gamma_\mu(1-\gamma_5)q\ \bar{b}\gamma_\mu(1-\gamma_5)q \\
\mathcal{O}_2 &= \bar{b}(1-\gamma_5)q\ \bar{b}(1-\gamma_5)q \\
\mathcal{O}_3 &= \bar{b}^i(1-\gamma_5)q^j\ \bar{b}^j(1-\gamma_5)q^i \\
\mathcal{O}_4 &= \bar{b}(1-\gamma_5)q\ \bar{b}(1+\gamma_5)q \\
\mathcal{O}_5 &= \bar{b}^i(1-\gamma_5)q^j\ \bar{b}^j\gamma_\mu(1+\gamma_5)q
\end{aligned}
$$

$$
\langle \bar{B}_q^0 | \mathcal{O}_1(\mu) | B_q^0 \rangle = 8/3\ m_{B_q}^2 f_{B_q}^2\ B_1^{(q)}(\mu)
$$

$$
\langle \bar{B}_q^0 | \mathcal{O}_{2-5}(\mu) | B_q^0 \rangle = C_i\ \frac{m_{B_q}^2 f_{B_q}}{m_b(\mu) + m_q(\mu)}\ B_{2-5}^{(q)}(\mu)
$$

$C_{2-5} = (-5/3, 1/3, 2, 2/3)$.

$B_{d,s} - \bar{B}_{d,s}$ mixing

NPR in the RI/MOM scheme - perturbative two-loop matching to $\overline{\rm MS}$ (NDR of Buras et al) at $\mu = m_b$

$$B_1^{(d)} = 0.85(4), B_2^{(d)} = 0.72(3), B_3^{(d)} = 0.88(13), B_4^{(d)} = 0.95(5), B_5^{(d)} = 1.47(12)$$
$$B_1^{(s)} = 0.86(3), B_2^{(s)} = 0.73(3), B_3^{(s)} = 0.89(12), B_4^{(s)} = 0.93(4), B_5^{(s)} = 1.57(11)$$

- $\Delta m_{d,s}$ - Saturated by B_1 (SM)
- Squeeze in B_{2-5}, NP Wilson coeffs $\ell_i Y^i_{bq}/\Lambda_{\rm NP}$.

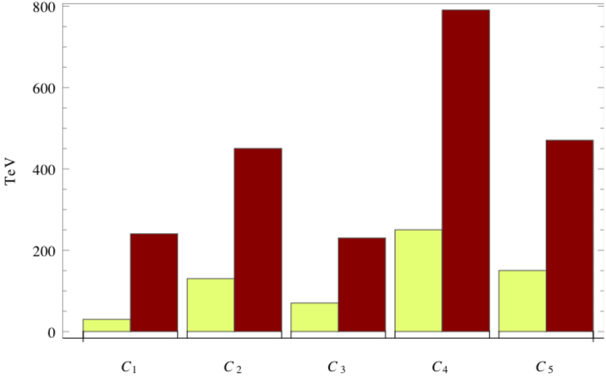
Setting $\ell_i = Y^i_{bq} = 1$ results in

$$\Lambda_{\rm NP}^{B^0-\bar{B}^0} \gtrsim 10^3 \text{ TeV},\ \Lambda_{\rm NP}^{B^0_s-\bar{B}^0_s} \gtrsim 10^2 \text{ TeV},\ \Lambda_{\rm NP}^{K^0-\bar{K}^0} \gtrsim 10^4 \text{ TeV}$$

Flavor problem!

$$\Lambda_{\rm NP}^{B^0-\bar{B}^0} \gtrsim 10^3~{\rm TeV},~\Lambda_{\rm NP}^{B_s^0-\bar{B}_s^0} \gtrsim 10^2~{\rm TeV},~\Lambda_{\rm NP}^{K^0-\bar{K}^0} \gtrsim 10^4~{\rm TeV}$$

Flavor problem!



$(g-2)_\mu$

$a_\mu = (g-2)_\mu/2$, and

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(63)(49)\times 10^{-11}$$

- Major uncertainty hadronic $a_\mu^{\rm hadr.} = a_\mu^{\rm hvp} + a_\mu^{\rm hlbl} + \dots$
- $a_\mu^{\rm hvp}$ DR+exp $\sigma(e^+e^- \to \text{hadrons})$ and from $\Gamma(\tau \to \text{hadrons}\nu_\tau)$:

$a_\mu^{\rm hvp} = (692.3 \pm 4.2)\times 10^{-10}$ Davier et al. 1010.4180

$a_\mu^{\rm hvp(ud)} = (506.4 \pm 3.5)\times 10^{-10}$ Davier et al. 1312.1501

$$\begin{aligned} a_\mu^{\rm hvp} &= \alpha \int_0^\infty \frac{dQ^2}{Q^2} w(Q^2/m_\mu^2)\left(\Pi(Q^2)-\Pi(0)\right) \\ \Pi_{\mu\nu} &= \int d^4x e^{iqx}\langle T[j_\mu(x) j_\nu^\dagger(0)]\rangle = (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2) \\ j_\mu &= \sum_f Q_f \bar\psi_f \gamma_\mu \psi_f \end{aligned}$$

$Q^2 \sim \mathcal{O}(m_\mu^2)$ dominant:
(i) Compute for larger Q^2 and model down to $Q^2 = 0$, (ii) Use TBC noisy!

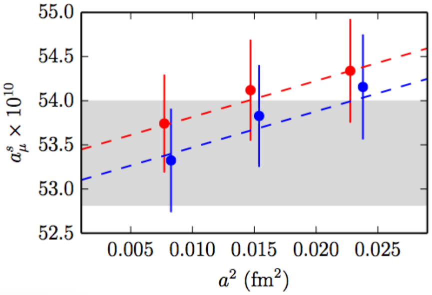
Use Moments

Excellent way to get around the problem! $[\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)]$

$$\Pi_{ii} = q^2\Pi(q^2) = \sum_t e^{iqt} \sum_{\vec{x}} \langle j_i(\vec{x},t) j_i(0)\rangle$$

$$G_{2n} = \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j_i(\vec{x},t) j_i(0)\rangle = (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} [q^2 \hat{\Pi}(q^2)]_{q^2=0}$$

$$\Rightarrow \hat{\Pi}(q^2) = \sum_{n=1}^{\infty} q^2 \left[(-1)^{n+1} \frac{G_{2n+1}}{(2n+2)!} \right]$$



Results a_μ^{hvp}

Results HPQCD [1403.1778 and 1408.5768]

$$a_\mu^{\text{hvp}} = \left[53.4(6)^{(s)}, 14.4(4)^{(c)}, 0.27(4)^{(b)}\right] \times 10^{-10}$$

$a_\mu^{\text{hvp(b)}}$ relying on NRQCD on the lattice

Results ETMC 1308.4327

$$a_\mu^{\text{hvp}} = \left[567(11)^{(ud)}, 655(21)^{(uds)}, 674(21)(18)^{(udsc)}\right] \times 10^{-10}$$

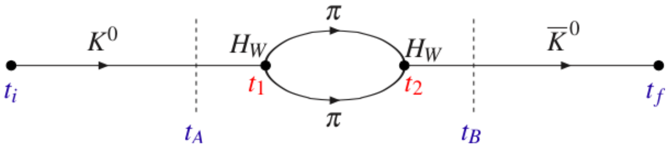
Mainz group preparing their results, cf. 1411.3031, RBC/UKQCD as well.

Preliminary attempt to compute a_μ^{hlbl}, Blum et al.1407.2923

Δm_K on the lattice

$$\Delta m_K = m_{K_L} - m_{K_S} = 2\mathcal{P}\sum_n \frac{\langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{m_K - E_n} = 3.483(6)\times 10^{-12}\ \text{MeV}$$

e.g. for $n = \pi\pi$

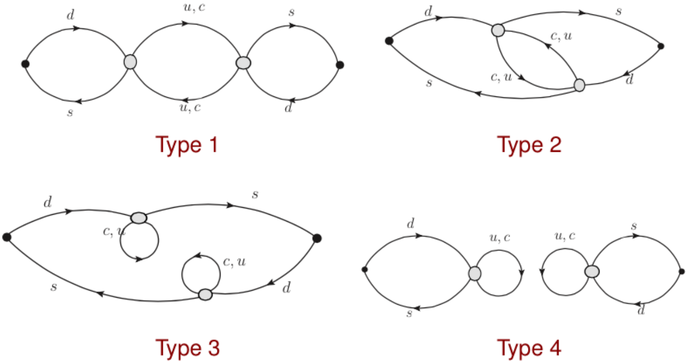


In this setup, it can be done on the lattice - linear coeff. in $T = t_B - t_A + 1$.
1401.1362 Christ, Martinelli, Sachrajda - "magic" formula:

$$\Delta m_K = (\Delta m_K)_V - 2\pi_V\langle \bar{K}^0|H|n_0\rangle_{V\,V}\langle n_0|H|K^0\rangle_V \left[\pi h \frac{dh}{dE}\right]_{m_K}$$

$\pi h(E,L) \equiv \phi(q) + \delta(k)$, $q = kL/2\pi$

1406.0916 computed with “unphysical” parameters at a single lattice spacing



Found: Type 3 small, Type 4 large; Intermediate $(\pi\pi)$ small, and remarkably

$$\Delta m_K = 3.19(41)(96) \times 10^{-12}\ \text{MeV}$$

Summarizing

- Tremendous progress in LQCD over the past 10 years
- Unquenched simulations close to realistic physical situations (almost there!)

Simulations with $n_f = 2$, $2+1$, and even $2+1+1$ at several lattice spacings: better computing architectures, better algorithms, clever ideas

- Many topics that I did not cover (even those that I am working on!)
- Huge progress in charm quark physics [C. Bouchard, Latt2014]

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.064(22)_{\rm QCD}(42)_{\rm exp}$$

- B-physics harder but viable strategies exist

$B \to K^*$ form factors computed in 2013 with Stagg-light and NRQCD-heavy
$B_{(s)} \to D_{(s)}^{(*)} \ell\nu$ with step scaling 2013 (tmQCD!) + FF BSM

Summarizing

- $B^0 - \bar{B}^0$ mixing confirms the flavor problem already noted with $K^0 - \bar{K}^0$
- Solved a long standing problem of $B(J/\psi \to \eta_c \gamma)$

$\Rightarrow$ Extend SM in tree level processes - matrix elements of BSM operators
Look for discrepancies!
$\Rightarrow$ Rare kaon decays - LD physics obstacle
Strategy to compute those exist (similar to Δm_K) Back to kaons to see NP?!

☐ Update of FLAG review underway: check out http://itpwiki.unibe.ch/