Recent Progress in Lattice QCD

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Why Lattices?

- Gauge sector of SM fully fixed by symmetry (local gi)
- Yukawa sector free: fermion masses and mixing are parameters in $\mathcal{L}_{\text{SM}}$ → determined through comparison theory vs. experiment
- quarks - main obstacle NP QCD
- BSM: clue about Yukawa’s if assuming 2HDM-Type ??, “GUT" some rep of SU(5), SO(10), E6..., alignment . . .
- QCD is still mysterious in NP-regime
- Any/every observable involving quarks suffers from NP QCD effects. Sometimes factorization helps (but is rarely a theorem):
  - Either look for observables with minimized hadronic uncertainties
  - use ab initio method to compute hadronic quantities - LQCD
What is Lattice QCD?

Regularization scheme of the path integral formulation of (Euclidean) QCD

\[ Z = \int \mathcal{D} \psi_f \mathcal{D} \bar{\psi}_f \mathcal{D} U_{\mu} e^{-S[\psi, \bar{\psi}, U]} = \int \mathcal{D} U_{\mu} \Pi_f \det(\mathcal{D} + m_f) e^{-S[U]} \]

that can let you compute NP QCD effects

\[ \langle O \rangle = \frac{1}{Z} \int \mathcal{D} \psi_f \mathcal{D} \bar{\psi}_f \mathcal{D} U_{\mu} \ O[\psi, \bar{\psi}, U] \ e^{-S[\psi, \bar{\psi}, U]} \]

- discretization of hypercubic lattice with (anti-)PBC
- \(a\)-lattice spacing: UV regulator [momenta \( p \lesssim \pi/a \)]
  \( L = N_La \) size: IR regulator
- Problem treatable by (H)MC \( \Rightarrow \) stat.uncertainties - CL theorem
  arbitrary accuracy achievable in principle
- Ab initio: only parameters (bare) quark masses and gauge coupling!
A physical quantity

\[ \Phi_{\text{latt.}} = \Phi_{\text{cont.}} \left[ 1 + a\Phi' + a^2\Phi'' + \ldots \right] \]

Many discretizations of Dirac operator [WilsonClover, tmQCD, Staggered, HISQ, FLIC, anisotropic...] each with its pros and cons and each designed to \( \Phi' = 0 \)
Realistic calculations

A physical quantity

$$\Phi^{\text{latt.}}(m_q) = \Phi^{\text{cont.}}(m_q) \left[ 1 + a\Phi'(m_q) + a^2\Phi''(m_q) + \ldots \right]$$

Lattice QCD Challenge:

(i) work at several lattice spacing to go to continuum limit
(ii) include $q\bar{q}$-loops in the vacuum fluctuations $n_f = 2, 2 + 1, 2 + 1 + 1$
(iii) work with light quark masses as close to the physical $m_{ud}$ (i.e. $m_{\pi}^{\text{phys.}}$) as possible
(iv) keep physical volume large - check for the FV corrections if they can be treated by FVChPT
(v) Matrix elements - renormalization constants should be computed non-perturbatively

Several collaborations - physical light quark masses → check on SU(2) ChPT.
Careful: FV corrections exacerbate physical chiral logs [Dürr, Lattice 2014]
Euclidean space $\Rightarrow$ Minkowski physics information (FSI, $\delta_S$, ...) not accessed directly, but in some cases useful information can be extracted.

Best suited for $\textit{hadron} \rightarrow 0$ and $\textit{hadron} \rightarrow \textit{hadron}$ processes
Tree level: leptonic and semileptonic decays
Loop induced: meson mixing, penguin induced decays...

N.B. UTfit and CKMfitter: UT reconstructed from the tree level and loop induced processes are the same (within current accuracy)
In ’00–’10 we checked the CKM unitarity (1st row) \([G_F = G_\mu]\)

\[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \overset{?}{=} 1\]

Using \(f_{+K\pi}(0) = 0.956(6)(6)_{n_f=2}, 0.966(3)_{n_f=2+1}\)

\[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9993(5)\]

Using \(f_K/f_\pi = 1.192(5)_{n_f=2+1}\)

\[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0000(6)\]

Huge coordinated effort!
What quantities?
FLAG- Flavor Lattice Averaging Group [1310.8555]

Major impact of Lattice QCD

- $\alpha_s^{\overline{\text{MS}}}(M_Z) = 0.1184(12)$ (!!!)
- $m_s^{\overline{\text{MS}}}(2 \text{ GeV}) = 101(3) \text{ MeV}_{n_f=2}, 94(2)(2) \text{ MeV}_{n_f=2+1}$
- $m_s/m_{ud} = 28(1)_{n_f=2}, 27.5(5)_{n_f=2+1}$
- $B_{rgi}^K = 0.73(3)(2)_{n_f=2}, 0.77(1)_{n_f=2+1}$
\[ |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \]

Charm quark is nowadays perfectly treatable like any other light quark (no recourse to effective theories and no troubles with matching with QCD in continuum)

- From axial Ward identities (tmQCD) 1403.4504 (NPR):
  \[ m_c^{\overline{MS}}(m_c) = 1.35(4)_{n_f=4}\text{GeV} \quad [1.28(4)_{n_f=2}\text{GeV} 1010.3659] \]

- From ratios of moments (stagQCD) 1408.4169:
  \[ m_c^{\overline{MS}}(m_c) = 1.27(1)_{n_f=4}\text{GeV} \]

Need a third calculation!
\[ D_{(s)} \rightarrow \ell \nu_{\ell} \]

\[ \langle 0 | \bar{q} \gamma_\mu \gamma_5 c | D_q(p) \rangle = i f_{D_q} p_\mu \]

\[ (m_c + m_q) \langle 0 | \bar{q} i \gamma_5 c | D_q(p) \rangle = f_{D_q} m_{D_q}^2 \]

From 2pt correlation functions

\[ \sum_{\vec{x}} \langle \mathcal{O}(\vec{x}, t) \mathcal{O}(0) \rangle \propto |\langle 0 | \mathcal{O} | D_q \rangle|^2 \exp[-m_{D_q} t] \]

**FLAG 1310.8555**

\[ f_D = [208(7)_{n_f=2}, 209(3)_{n_f=2+1}] \text{ MeV} \]

\[ f_{D_s} = [250(7)_{n_f=2}, 249(3)_{n_f=2+1}] \text{ MeV} \]

\[ f_{D_s}/f_D = [1.20(2)_{n_f=2}, 1.19(1)_{n_f=2+1}] \]
\[ D_{(s)} \rightarrow \ell \nu_\ell \]

**NEW Fermilab/MILC 1407.3772**

\[ f_D = 212.6(4) \left( \frac{+1.0}{-1.2} \right) \text{ MeV}_{n_f=4} \quad f_{D_s} = 249.0(3) \left( \frac{+1.1}{-1.5} \right) \text{ MeV}_{n_f=4} \]

\[ f_{D_s}/f_D = 1.171(1)(3)_{n_f=4} \]

**NEW ETMC 1411.0484 (update of 1308.1851)**

\[ f_D = 209(5) \text{ MeV}_{n_f=4} \quad f_{D_s} = 248(4) \text{ MeV}_{n_f=4} \]

\[ f_{D_s}/f_D = 1.19(2)_{n_f=4} \]

DWF catching up, cf. eg. 1404.3648
Three-point functions - harder but doable

\[ \langle \pi(k)|\bar{q}\gamma_\mu c|D_q(p)\rangle \propto f_+(q^2), f_0(q^2) \]

\[ m_\ell^2 \leq q^2 \leq (m_D - m_\pi)^2, \text{ and } f_+(0) = f_0(0) \]

Need to inject momenta either in Fourier transform or via TBC

FLAG 1310.8555: \( f^D_+(0) = 0.67(3) \quad f^{DK}_+(0) = 0.75(2) \)
$q^2$-dependence from lattice consistent with experiment (BaBar, Belle, BES, CLEO-c)

Compare now with nearest pole dominance:

$$f_{+}^{D\pi}(q^2) = \frac{\gamma_0}{m_{D^*}^2 - q^2}, \quad \gamma_0 \equiv \text{Res}_{q^2 = m_{D^*}^2} f_+(q^2) = \frac{1}{2} m_{D^*} f_{D^*} g_{D^* D\pi}$$

$g_{D^* D\pi} = 16.9(2)_{\exp} \quad \text{BaBar, 1304.5657,}$

$16(1)_{latt} \quad \text{DB, Sanfilippo, 1210.5410}$

$f_{D^*}/f_D = 1.21(3)_{latt} \quad 1407.1019$
Compare now with the nearest pole dominance:

Different from actual exp. data!
Info on second pole can be deduced from experiment - does not saturate FF.
- Vector current made of (mass) non-degenerate quarks - VMD fails
- Elastic case: pion form factor, $\eta_c$ form factor - consistent with VMD
More recent $f_{+,0}^{D\to K/\pi}(q^2)$ by Fermilab/MILC, ETMC, HPQCD. Vector meson in the final state?

- $D \to \rho \ell \nu_\ell$
  difficult $\Gamma_\rho/m_\rho = 0.19$
  resonant vs. nonresonant contribution; treatment of the FSI (complicated but possible - accuracy not competitive)

- $D \to K^* \ell \nu_\ell$
  borderline $\Gamma_{K^*}/m_{K^*} = 0.05$
  S-wave $K\pi$ difficult (feasible)

- $D_s \to \phi \ell \nu_\ell$
  NRA should be OK, $\Gamma_\phi/m_\phi = 0.004$

HPQCD [1311.6669] - 2 lattices

$$\langle \phi | \bar{s} \gamma_\mu c | D_s \rangle \propto V(q^2) \quad \langle \phi | \bar{s} \gamma_\mu \gamma_5 c | D_s \rangle \propto A_1(q^2), A_2(q^2), A_0(q^2)$$

Smooth $q^2$-behavior
\[ \langle \phi | \bar{s} \gamma_\mu c | D_s \rangle \propto V(q^2) \quad \langle \phi | \bar{s} \gamma_\mu \gamma_5 c | D_s \rangle \propto A_1(q^2), A_2(q^2), A_0(q^2) \]

\begin{align*}
V(0) &= 1.06(12), & A_1(0) &= 0.62(2), \\
A_2(0) &= 0.46(8), & A_0(0) &= 0.71(4)
\end{align*}
$B_s$ mesons

$b$ quark is still too heavy to be directly simulated on the lattice.
→ Step scaling or use effective theories

**FLAG 1310.8555**

\[ f_B = [189(8)_{n_f=2}, 191(4)_{n_f=2+1}] \text{ MeV} \]
\[ f_{B_s} = [228(8)_{n_f=2}, 228(5)_{n_f=2+1}] \text{ MeV} \]
\[ f_{B_s}/f_B = 1.21(2)_{n_f=2}, 1.20(2)_{n_f=2+1} \]

RBC/UKQCD, 1404.4670 [DWF light $n_f = 2 + 1$, Fermilab’ heavy]

\[ f_B = 195(16) \text{ MeV}, \quad f_{B_s} = 235(12) \text{ MeV}_{n_f=2+1} \quad f_{B_s}/f_B = 1.22(8) \]

See also 1406.6192 [DWF + HQET (Pert. match!)]

ETMC 1411.0484, tmQCD ($n_f = 4$) + Step Scal. [update of 1308.1851]

\[ f_B = 196(9) \text{ MeV} \quad f_{B_s} = 235(9) \text{ MeV} \quad f_{B_s}/f_B = 1.20(3) \]

Alpha 1404.3590 [Wilson-impr $n_f = 2 + HQET + 1/m_b$]

\[ f_B = 186(13) \text{ MeV} \quad f_{B_s} = 224(14) \text{ MeV} \quad f_{B_s}/f_B = 1.20(7) \]
\[ B \rightarrow \pi \ell \nu_\ell \]

**RBC/UKQCD 1501.05373**

[DWF light \( n_f = 2 + 1 \), Fermilab’ heavy, 2-"a"]

Results directly accessible at large \( q^2 \)

<table>
<thead>
<tr>
<th>( q^2 [\text{GeV}^2] )</th>
<th>19</th>
<th>22.6</th>
<th>26.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{B \rightarrow \pi}^B(q^2) )</td>
<td>1.21(13)</td>
<td>2.27(19)</td>
<td>4.1(6)</td>
</tr>
<tr>
<td>( f_{0}^{B \rightarrow \pi}(q^2) )</td>
<td>0.46(6)</td>
<td>0.68(7)</td>
<td>0.92(7)</td>
</tr>
</tbody>
</table>
In the end $V_{ub} = 3.61(32) \times 10^{-3}$ (comparison with BaBar - large $q^2$-bin!)
ETMC - tour de force ($n_f = 2$)
full set of matrix elements using step scaling - tmQCD + OS, multi $a$

\[
\mathcal{O}_1 = \bar{b} \gamma_\mu (1 - \gamma_5) q \, \bar{b} \gamma_\mu (1 - \gamma_5) q
\]
\[
\mathcal{O}_2 = \bar{b} (1 - \gamma_5) q \, \bar{b} (1 - \gamma_5) q
\]
\[
\mathcal{O}_3 = \bar{b}^i (1 - \gamma_5) q^j \, \bar{b}^j (1 - \gamma_5) q^i
\]
\[
\mathcal{O}_4 = \bar{b} (1 - \gamma_5) q \, \bar{b} (1 + \gamma_5) q
\]
\[
\mathcal{O}_5 = \bar{b}^i (1 - \gamma_5) q^j \, \bar{b}^j \gamma_\mu (1 + \gamma_5) q
\]

\[
\langle \bar{B}_q^0 | \mathcal{O}_1(\mu) | B_q^0 \rangle = \frac{8}{3} \, m_{B_q}^2 \, f_{B_q}^2 \, B_1^{(q)}(\mu)
\]
\[
\langle \bar{B}_q^0 | \mathcal{O}_{2-5}(\mu) | B_q^0 \rangle = C_i \frac{m_{B_q}^2 \, f_{B_q}}{m_b(\mu) + m_q(\mu)} \, B_{2-5}^{(q)}(\mu)
\]

\[
C_{2-5} = (-5/3, 1/3, 2, 2/3).
\]
NPR in the RI/MOM scheme - perturbative two-loop matching to $\overline{\text{MS}}$ (NDR of Buras et al) at $\mu = m_b$

\[
B_1^{(d)} = 0.85(4), \ B_2^{(d)} = 0.72(3), \ B_3^{(d)} = 0.88(13), \ B_4^{(d)} = 0.95(5), \ B_5^{(d)} = 1.47(12) \\
B_1^{(s)} = 0.86(3), \ B_2^{(s)} = 0.73(3), \ B_3^{(s)} = 0.89(12), \ B_4^{(s)} = 0.93(4), \ B_5^{(s)} = 1.57(11)
\]

- $\Delta m_{d,s}$ - Saturated by $B_1$ (SM)
- Squeeze in $B_2 - 5$, NP Wilson coeffs $\ell_i Y_{bq}/\Lambda_{\text{NP}}$.

Setting $\ell_i = Y_{bq} = 1$ results in

\[
\Lambda_{\text{NP}}^{B^0 - \bar{B}^0} \gtrsim 10^3 \text{ TeV}, \ \Lambda_{\text{NP}}^{B^0_s - \bar{B}^0_s} \gtrsim 10^2 \text{ TeV}, \ \Lambda_{\text{NP}}^{K^0 - \bar{K}^0} \gtrsim 10^4 \text{ TeV}
\]

**Flavor problem!**
$B_{d,s} - \bar{B}_{d,s}$ mixing

\[ \Lambda_{NP}^{B^0 - \bar{B}^0} \gtrsim 10^3 \text{ TeV}, \quad \Lambda_{NP}^{B_s^0 - \bar{B}_s^0} \gtrsim 10^2 \text{ TeV}, \quad \Lambda_{NP}^{K^0 - \bar{K}^0} \gtrsim 10^4 \text{ TeV} \]

Flavor problem!
\( (g - 2)_{\mu} \)

\[ a_{\mu} = (g - 2)_{\mu}/2, \text{ and} \]

\[ \Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 287(63)(49) \times 10^{-11} \]

- Major uncertainty hadronic \( a_{\mu}^{\text{hadr.}} = a_{\mu}^{\text{hvp}} + a_{\mu}^{\text{hlbl}} + \ldots \)
- \( a_{\mu}^{\text{hvp}} \) DR + exp \( \sigma(e^+ e^- \to \text{hadrons}) \) and from \( \Gamma(\tau \to \text{hadrons}\nu_\tau) \):
  \[ a_{\mu}^{\text{hvp}} = (692.3 \pm 4.2) \times 10^{-10} \]
  \[ a_{\mu}^{\text{hvp(ud)}} = (506.4 \pm 3.5) \times 10^{-10} \]

\[ a_{\mu}^{\text{hvp}} = \alpha \int_0^\infty \frac{dQ^2}{Q^2} w(Q^2/m_{\mu}^2) \left( \Pi(Q^2) - \Pi(0) \right) \]

\[ \Pi_{\mu\nu} = \int d^4x e^{iqx} \langle T[j_{\mu}(x)j_{\nu}^\dagger(0)] \rangle = (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2) \]

\[ j_{\mu} = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f \]

\( Q^2 \sim \mathcal{O}(m_{\mu}^2) \) dominant:

(i) Compute for larger \( Q^2 \) and model down to \( Q^2 = 0 \), (ii) Use TBC noisy!
Use Moments

Excellent way to get around the problem! \[ \hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0) \]

\[ \Pi_{ii} = q^2\Pi(q^2) = \sum_t e^{iq_t} \sum_{\vec{x}} \langle j_i(\vec{x}, t) j_i(0) \rangle \]

\[ G_{2n} = \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j_i(\vec{x}, t) j_i(0) \rangle = (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} [q^2 \hat{\Pi}(q^2)]_{q^2=0} \]

\[ \Rightarrow \hat{\Pi}(q^2) = \sum_{n=1}^{\infty} q^2 \left[ (-1)^{n+1} \frac{G_{2n+1}}{(2n + 2)!} \right] \]
Results $a_{\mu}^{hvp}$

**Results HPQCD [1403.1778 and 1408.5768]**

\[
a_{\mu}^{hvp} = \left[ 53.4(6)^{(s)}, 14.4(4)^{(c)}, 0.27(4)^{(b)} \right] \times 10^{-10}
\]

$a_{\mu}^{hvp(b)}$ relying on NRQCD on the lattice

**Results ETMC 1308.4327**

\[
a_{\mu}^{hvp} = \left[ 567(11)^{(ud)}, 655(21)^{(uds)}, 674(21)(18)^{(udsc)} \right] \times 10^{-10}
\]

Mainz group preparing their results, cf. 1411.3031, RBC/UKQCD as well.

Preliminary attempt to compute $a_{\mu}^{hlbl}$, Blum et al.1407.2923
\[ \Delta m_K = m_{K_L} - m_{K_S} = 2\mathcal{P} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} = 3.483(6) \times 10^{-12} \text{ MeV} \]

e.g. for \( n = \pi \pi \)

In this setup, it can be done on the lattice - linear coeff. in \( T = t_B - t_A + 1 \).

1401.1362 Christ, Martinelli, Sachrajda - “magic” formula:

\[ \Delta m_K = (\Delta m_K)_V - 2\pi V \langle \bar{K}^0 | H | n_0 \rangle_V V \langle n_0 | H | K^0 \rangle_V \left[ \pi h \frac{dh}{dE} \right]_{m_K} \]

\[ \pi h(E, L) \equiv \phi(q) + \delta(k), \quad q = kL/2\pi \]
1406.0916 computed with “unphysical” parameters at a single lattice spacing

\[ \Delta m_K \]

\[ \Delta m_K = 3.19(41)(96) \times 10^{-12} \text{ MeV} \]
• Tremendous progress in LQCD over the past 10 years
• Unquenched simulations close to realistic physical situations (almost there!)
  Simulations with $n_f = 2$, 2 + 1, and even 2 + 1 + 1 at several lattice spacings: better computing architectures, better algorithms, clever ideas
• Many topics that I did not cover (even those that I am working on!)
• Huge progress in charm quark physics [C. Bouchard, Latt2014]

\[ |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.064(22)_{\text{QCD}}(42)_{\text{exp}} \]

• $B$-physics harder but viable strategies exist
  $B \to K^*$ form factors computed in 2013 with Stagg-light and NRQCD-heavy
  $B_{(s)} \to D_{(s)}^{(*)} \ell \nu$ with step scaling 2013 (tmQCD!) + FF BSM
• $B^0 - \bar{B}^0$ mixing confirms the flavor problem already noted with $K^0 - \bar{K}^0$
• Solved a long standing problem of $B(J/\psi \rightarrow \eta_c \gamma)$

⇒ Extend SM in tree level processes - matrix elements of BSM operators
Look for discrepancies!
⇒ Rare kaon decays - LD physics obstacle
Strategy to compute those exist (similar to $\Delta m_K$) Back to kaons to see NP?!

☐ Update of FLAG review underway: check out http://itpwiki.unibe.ch/