Les Rencontre de Physic de Valee d'Aosta (LaThuile2015),
1 – 7 March 2015, La Thuile, Italy

Exotic quarkonium-like states at B factories

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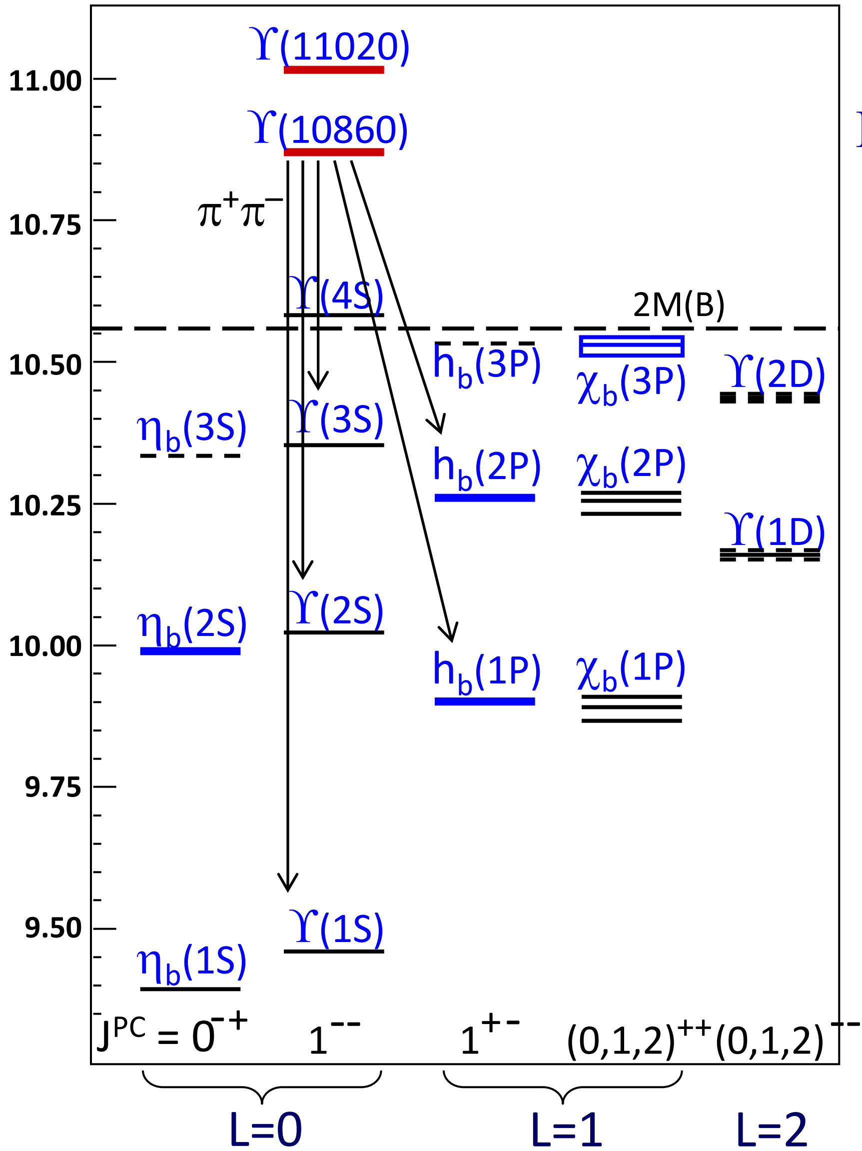
Outline

Charged quarkonium-like states

Vector $J^{PC} = 1^{--}$ states

in bottomonium & charmonium sectors

Hadronic transitions from Υ states



PRL100,112001(2008)

$\Gamma[\Upsilon(5S) \to \Upsilon(1S/2S/3S)\pi^+\pi^-] = 260/430/290$ keV

PRL108,032001(2012)

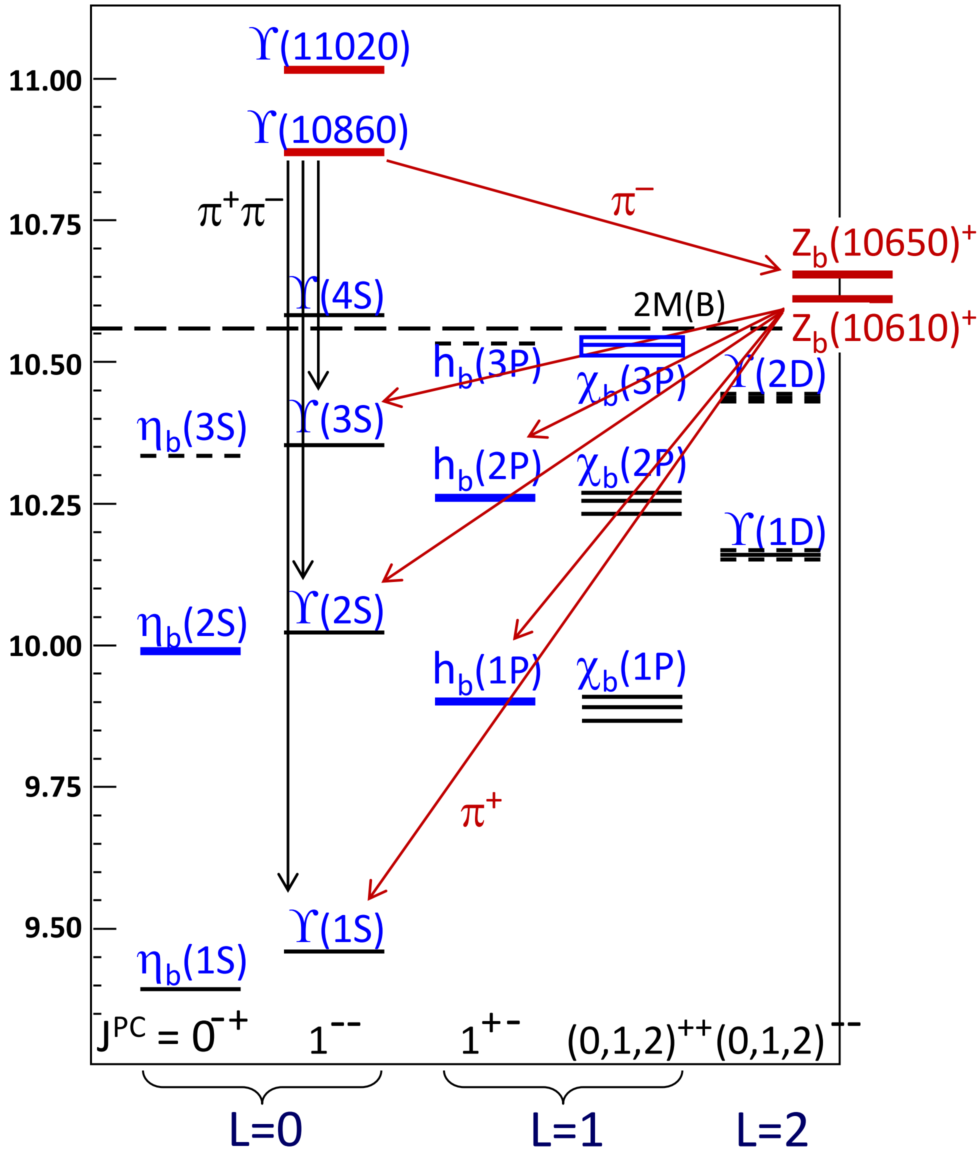
$\Gamma[\Upsilon(5S) \to h_b(1P/2P)\pi^+\pi^-] = 190/330$ keV

involves spin flip of heavy quark

no strong suppression

Observation of charged $Z_b^{\pm}$ states

PRL108,112001(2012)



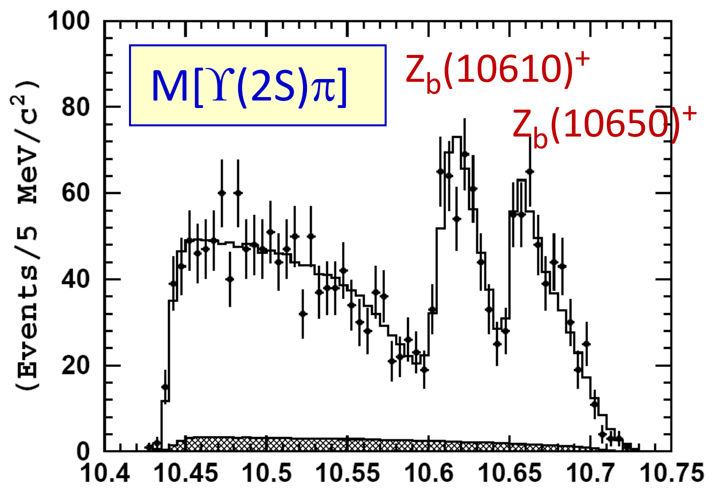
$\Upsilon(5S) \to Z_b(10610/10650)^+\pi^-$

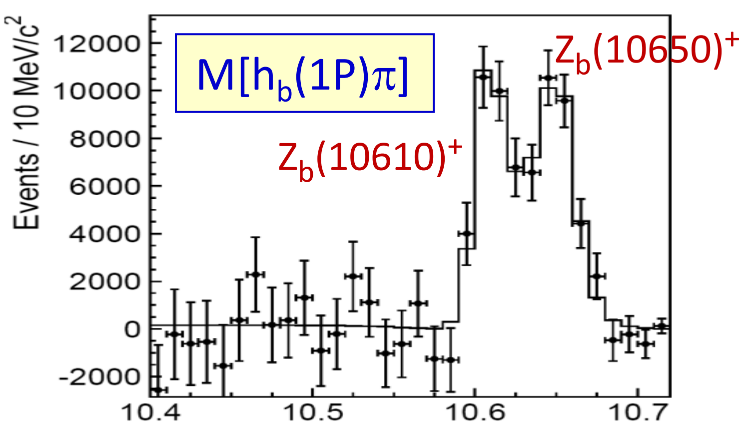
$\quad \to \Upsilon(1S/2S/3S)\pi^+$

$\quad \to h_b(1P/2P)\pi^+$

$|\, b\bar{b}u\bar{d}\,\rangle$

exotic quark content





Experimental summary on Z_b^+ states

PRL108,122001(2012)

$M_{Zb(10610)} - (M_B+M_{B*}) = +2.6 \pm 2.1$ MeV $\Gamma_{Zb(10610)} = 18.4 \pm 2.4$ MeV

$M_{Zb(10650)} - 2M_{B*} = +1.8 \pm 1.7$ MeV $\Gamma_{Zb(10650)} = 11.5 \pm 2.2$ MeV

arxiv:1209.6450

$J^P = 1^+$

6D amplitude analysis

arxiv:1403.0992

Channel	$\mathcal{B}$ of $Z_b(10610)$	$\mathcal{B}$ of $Z_b(10650)$
$\pi^+\Upsilon(1S)$	$(0.32 \pm 0.09)\%$	$(0.18 \pm 0.05)\%$
$\pi^+\Upsilon(2S)$	$(4.38 \pm 1.21)\%$	$(1.80 \pm 0.47)\%$
$\pi^+\Upsilon(3S)$	$(2.15 \pm 0.56)\%$	$(1.23 \pm 0.30)\%$
$\pi^+ h_b(1P)$	$(2.81 \pm 1.10)\%$	$(5.6 \pm 2.0)\%$
$\pi^+ h_b(2P)$	$(4.34 \pm 2.07)\%$	$(11.1 \pm 4.7)\%$
$B^+\bar{B}^{*0} + \bar{B}^0 B^{*+}$	$(86.0 \pm 3.6)\%$	$(25 \pm 10)\%$
$B^{*+}\bar{B}^{*0}$	–	$(55.1 \pm 5.3)\%$

suppressed despite larger PHSP

dominant

Assumption:

$|\, Z_b(10610) \rangle = |\, B\ \bar{B}^* \rangle$

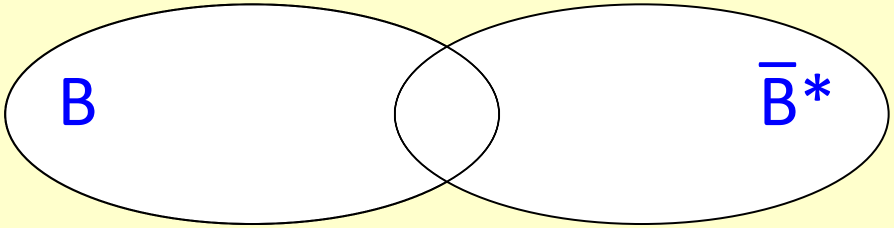
$|\, Z_b(10650) \rangle = |\, B^*\bar{B}^* \rangle$

$\delta M \sim 0 \Rightarrow$ loosely bound or virtual

Decays into constituents dominate

$J^P=1^+ \Rightarrow B^{(*)}\bar{B}^*$ in S-wave

Structure of Z_b^+ : molecule



Bondar, Garmash, Milstein, RM, Voloshin, PRD84,054010(2011)

In the $I^G(J^P) = 1^+(1^+)$ $B^{(*)}\bar{B}^*$ molecule total spin of heavy $b\bar{b}$, S_{bb}, is not definite.

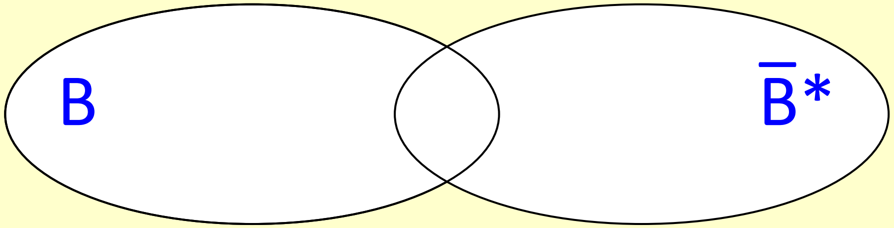
Decomposition in S_{bb} eigenstates ⇒

$$|Z_b'\rangle = (0^-_{b\bar{b}} \otimes 1^-_{q\bar{q}} - 1^-_{b\bar{b}} \otimes 0^-_{q\bar{q}})/\sqrt{2}$$

$$|Z_b\rangle = (0^-_{b\bar{b}} \otimes 1^-_{q\bar{q}} + 1^-_{b\bar{b}} \otimes 0^-_{q\bar{q}})/\sqrt{2}$$

$h_b(mP)\pi$ $\Upsilon(nS)\pi$

Structure of Z_b^+ : molecule



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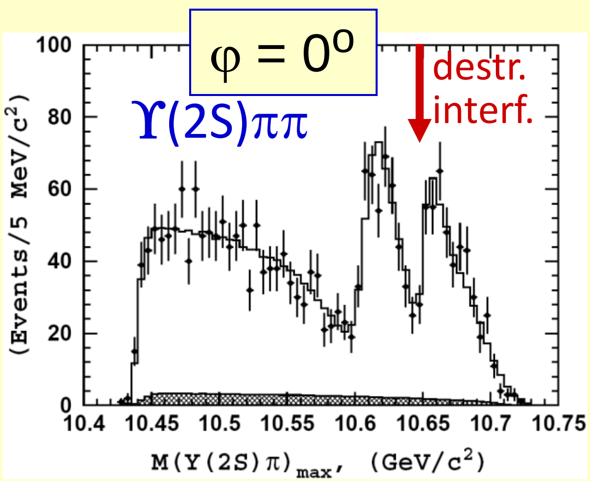
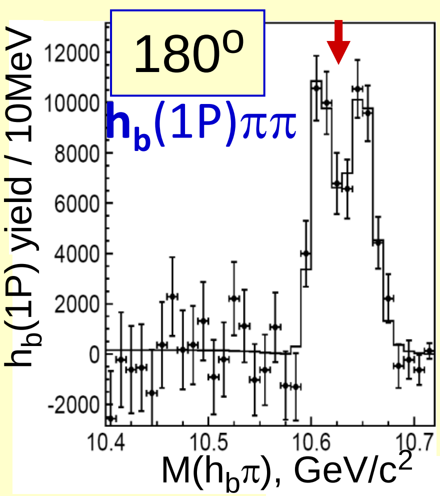
$$|Z_b\rangle = (0^-_{b\bar{b}} \otimes 1^-_{q\bar{q}} + 1^-_{b\bar{b}} \otimes 0^-_{q\bar{q}})/\sqrt{2}$$

$h_b(mP)\pi$ — relative phase — $\Upsilon(nS)\pi$

$$BW(s, M_1, \Gamma_1) + ae^{i\phi}BW(s, M_2, \Gamma_2)$$

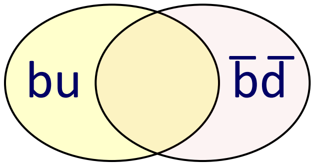




Assumption of molecular structure allows to explain all properties of Z_b states.

Structure of Z_b^+ : diquark-antidiquark

Ali et al, PRD85,054011(2012)



$$|\tilde{Z}_b\rangle = (0_{[bq]} \otimes 1_{[\bar{b}\bar{q}]} - 1_{[bq]} \otimes 0_{[\bar{b}\bar{q}]})/\sqrt{2},$$
$$|\tilde{Z}'_b\rangle = 1_{[bq]} \otimes 1_{[\bar{b}\bar{q}]}.$$

Decomposition:

$$|\tilde{Z}_b\rangle = (-1^-_{b\bar{b}} \otimes 0^-_{q\bar{q}} + 0^-_{b\bar{b}} \otimes 1^-_{q\bar{q}})/\sqrt{2} = 1^-_{b\bar{q}} \otimes 1^-_{q\bar{b}},$$ $B^*\bar{B}^*$

$$|\tilde{Z}'_b\rangle = (1^-_{b\bar{b}} \otimes 0^-_{q\bar{q}} + 0^-_{b\bar{b}} \otimes 1^-_{q\bar{q}})/\sqrt{2}$$
$$= (1^-_{b\bar{q}} \otimes 0^-_{q\bar{b}} + 0^-_{b\bar{q}} \otimes 1^-_{q\bar{b}})/\sqrt{2}.$$ $B\bar{B}^*$

Predictions:

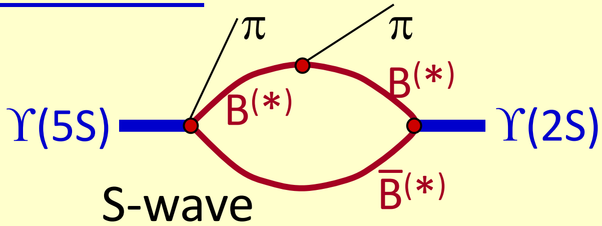
$Z_b \to B^*\bar{B}^*$
$Z_b' \to B\ \bar{B}^*$

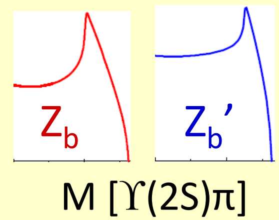
$\Gamma(Z_b \to \Upsilon\pi) \sim \Gamma(Z_b \to h_b\pi) \sim \Gamma(Z_b \to B^{(*)}\bar{B}^*)$ $\Leftarrow$ diquark is destroyed in all cases

Decay pattern of Z_b and Z_b'
disfavors diquark-antidiquark interpretation.

Dynamical model for Z_b states

1. Threshold effect





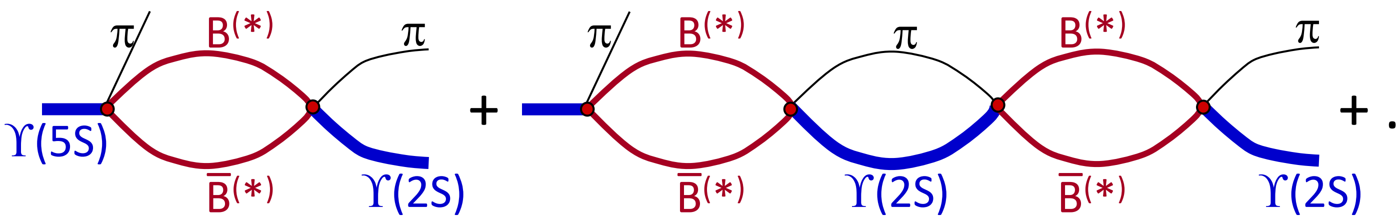
Chen, Liu, PRD84,094003(2011)

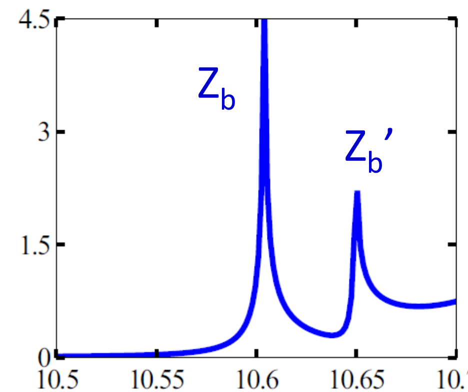
Selfconsistency?

Guo, Hanhart, Wang, Zhao, 1411.5584

Danilkin, Orlovsky, Simonov, PRD85,034012(2012)

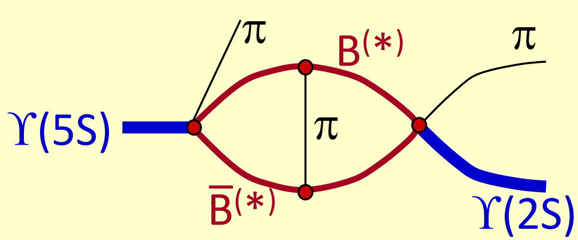
2. Coupled-channels resonance multiple rescattering ⇒ pole





3. Deutron-like molecule

π,ρ,ω,σ exchange



Ohkoda et al. PRD86,014004(2012)

⇒ Predictions to fit data and discriminate models?

X(3872)

topcited Belle paper: 1000+

Produced in B decays and high energy pp collisions

Belle, CDF, LHCb

$M_{X(3872)} - (M_{D^0} + M_{D^{*0}}) = -0.11 \pm 0.22$ MeV $\Gamma_{X(3872)} < 1.2$ MeV $J^{PC} = 1^{++}$

Known decays (BF relative to $J/\psi\, \pi^+\pi^-$ channel)

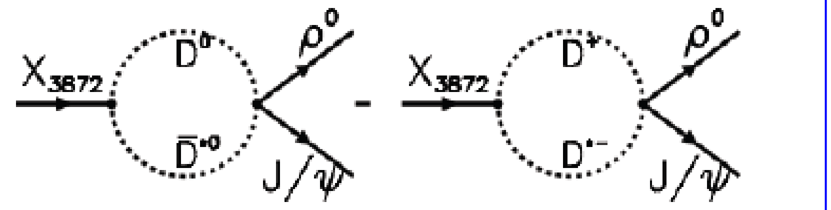
$J/\psi\, \pi^+\pi^-$ (=ρ^0)	1
$J/\psi\, \omega$	0.8 ± 0.3
$J/\psi\, \gamma$	0.21 ± 0.06
$\psi(2S)\, \gamma$	0.50 ± 0.15
$D^0\bar{D}^{*0}$	~10

isospin violation (between $J/\psi\, \pi^+\pi^-$ and $J/\psi\, \omega$)

Favored interpretation:
$D\bar{D}^*$ molecule with admixture of $\chi_{c1}(2P)$

- $D\bar{D}^*$ molecule → mass at threshold, isospin violation
- $\chi_{c1}(2P)$ → production at high energy

Isospin Violation in X(3872) decay:



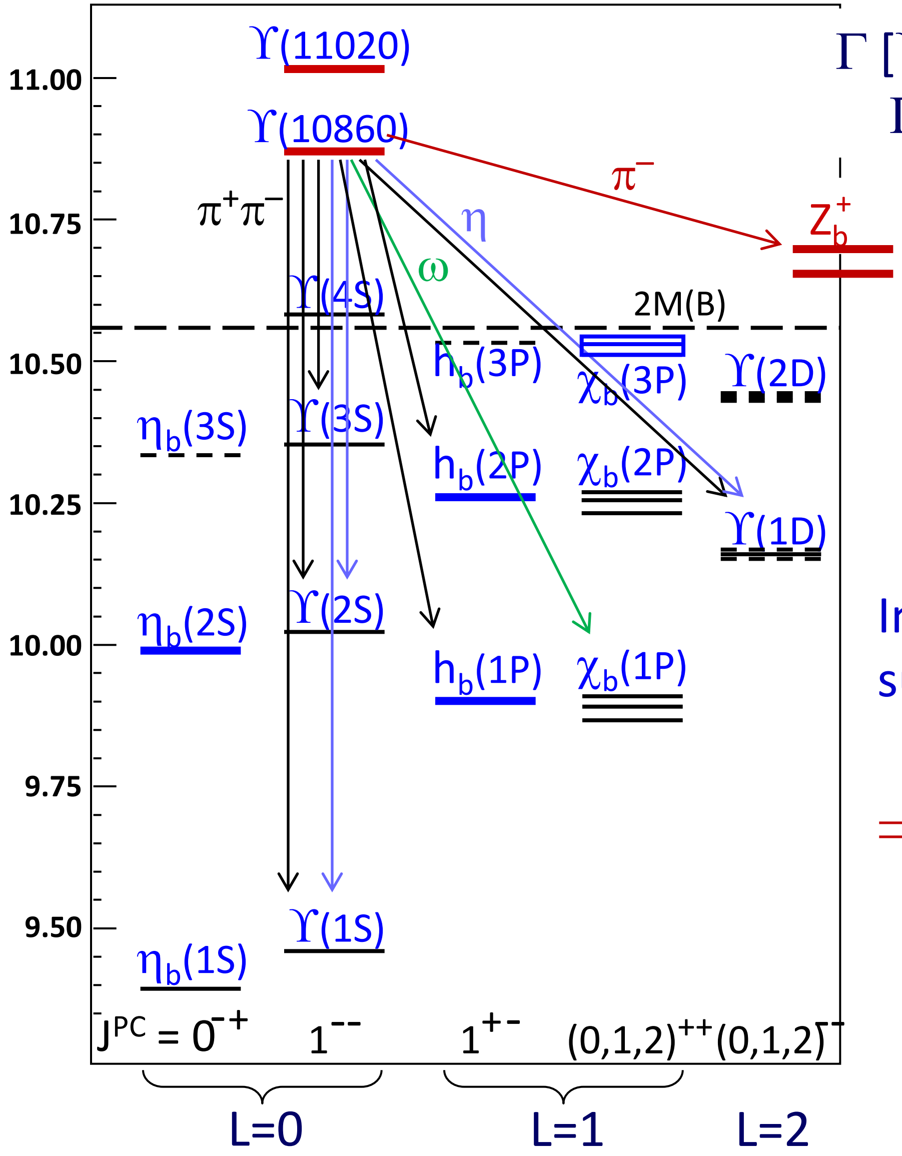
Dynamical model: boson exchange is not strong enough, need $c\bar{c}$-$D\bar{D}^*$ rescattering.

Fraction of $D\bar{D}^*$ component?
Bound or virtual?

Accuracy in line shapes ($J/\psi\, \pi^+\pi^-$, $D\bar{D}^*$)
⇐ LHCb, Belle-II, PANDA.

Transitions from $\Upsilon(5S)$

Partial widths of hadronic transitions from $\Upsilon(5S)$ are anomalously large:



$\Gamma\,[\Upsilon(5S) \to \Upsilon(1S/2S/3S)\pi^+\pi^-]$ = 260/430/290 keV

$\Gamma\,[\Upsilon(5S) \to Z_b(10610/10650)^+\pi^-]$ = 7/3 MeV

preliminary

$\Gamma\,[\Upsilon(5S) \to \Upsilon(1S/2S)\,\eta]$ = 40/200 keV

$\Gamma\,[\Upsilon(5S) \to \Upsilon(1D)\,(\pi^+\pi^-)/\eta]$ = 60/140 keV

$\Gamma[\Upsilon(5S) \to \chi_{b1/2}(1P)\,\omega]$ = 80/30 keV

$\Gamma\Gamma[\Upsilon(5S) \to \Upsilon(1S)\,K^+K^-]$ = 30 keV

PRL113,142001(2014)

In bottomonium hadronic transitions are OZI suppressed: $\Gamma[\Upsilon(3S/2S) \to \Upsilon(1S)\pi\pi]$ = 1/6 keV

$\Rightarrow$ Evidence for light d.o.f. inside "$\Upsilon(5S)$":

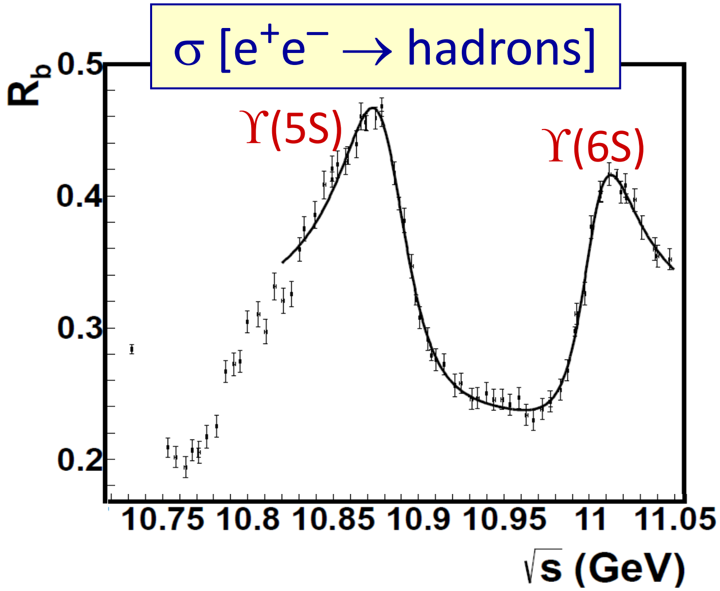
"$\Upsilon(5S)$" = $|b\bar{b}\rangle + |B\bar{B}\rangle + |b\bar{b}g\rangle + |(bq)(\bar{b}\bar{q})\rangle$...

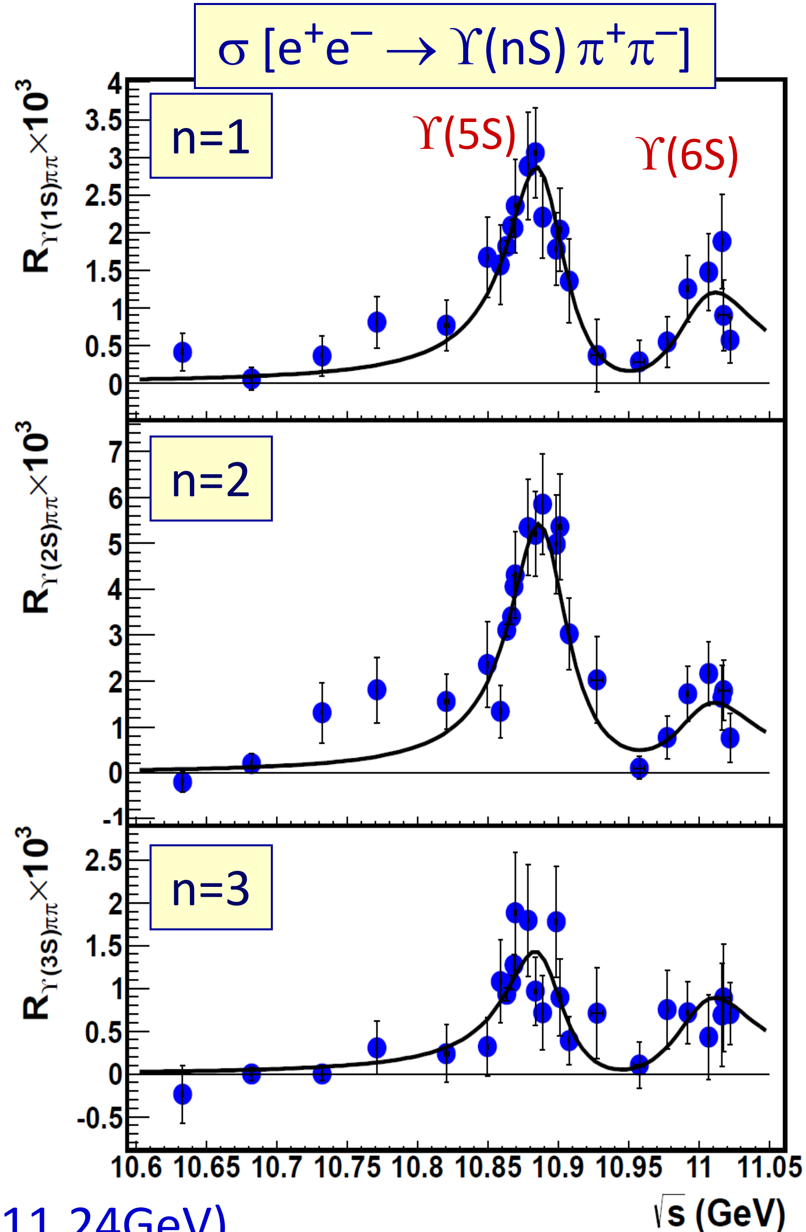
$\swarrow$ $Z_b\pi$

Production of Z_b – evidence for molecular component inside "$\Upsilon(5S)$"?

Energy scan

arxiv:1501.01137





First observation *anomalously big*

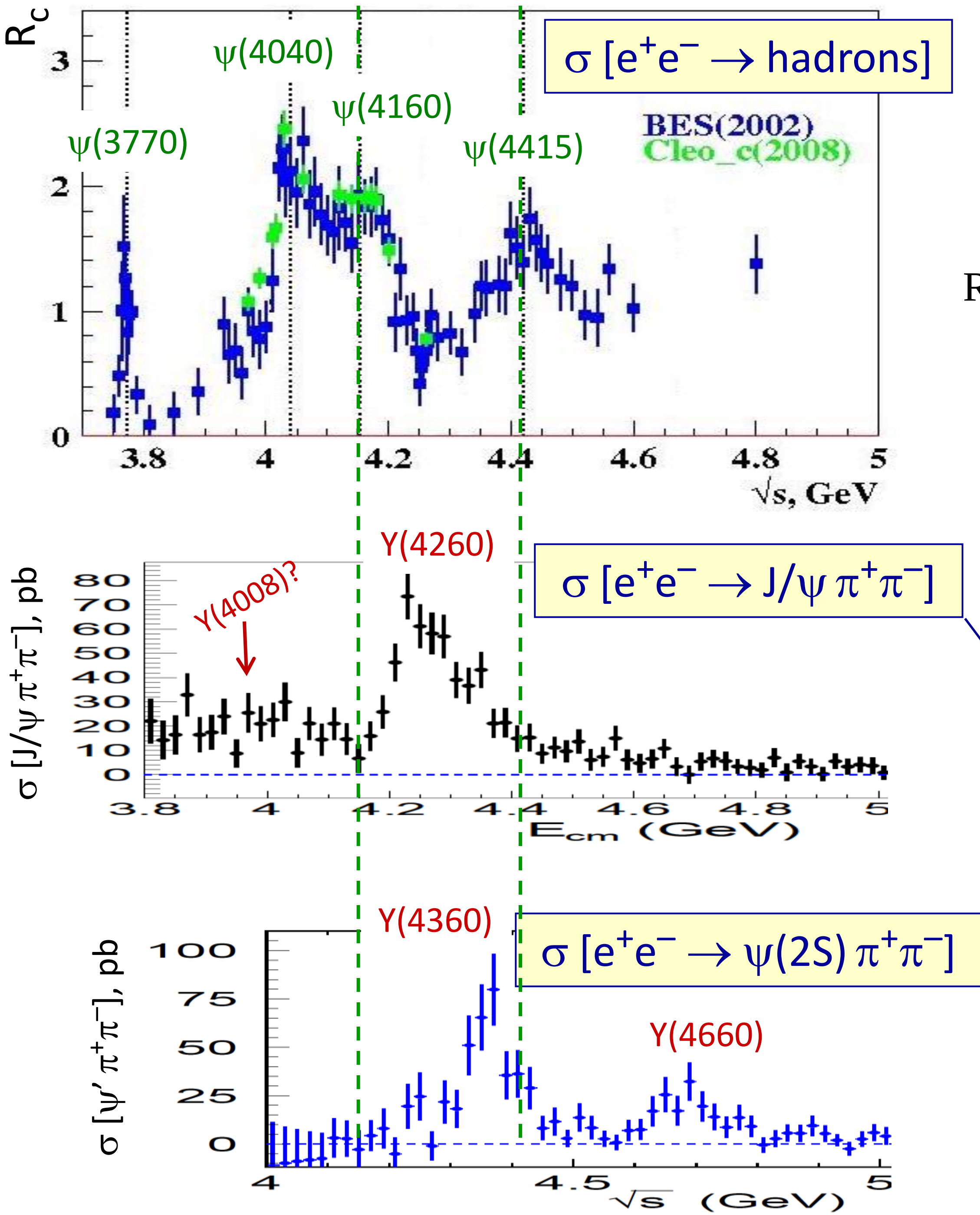
$\Gamma[\Upsilon(6S)\to\Upsilon(1S/2S/3S)\pi^+\pi^-]$ = 120/140/200 keV

Fit results to R_b and $R_{\Upsilon\pi\pi}$ agree, $R_{\Upsilon\pi\pi}$ is more reliable:

	M, MeV	Γ, MeV
Υ(5S)	$10891.1 \pm 3.2^{+0.6}_{-1.5}$	$53.7^{+7.1}_{-5.6}{}^{+0.9}_{-5.4}$
Υ(6S)	$10987.5^{+6.4}_{-2.5}{}^{+9.0}_{-2.1}$	$61^{+9}_{-19}{}^{+2}_{-20}$

Scan to higher energy is planned at Belle-II (E_{max}~11.24GeV).

Cross-sections in charmonium region



$$R_c = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma_0(e^+e^- \to \mu^+\mu^-)} - R_{uds}$$

using ISR

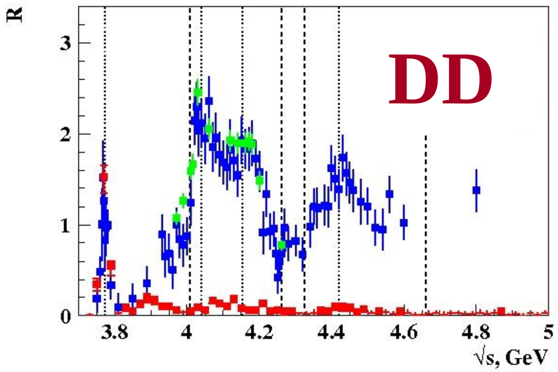
BaBar PRD86,051102R(2012)
Belle PRL110,252002(2013)
BaBar PRD89,111103(2014)
Belle arxiv:1410.7641

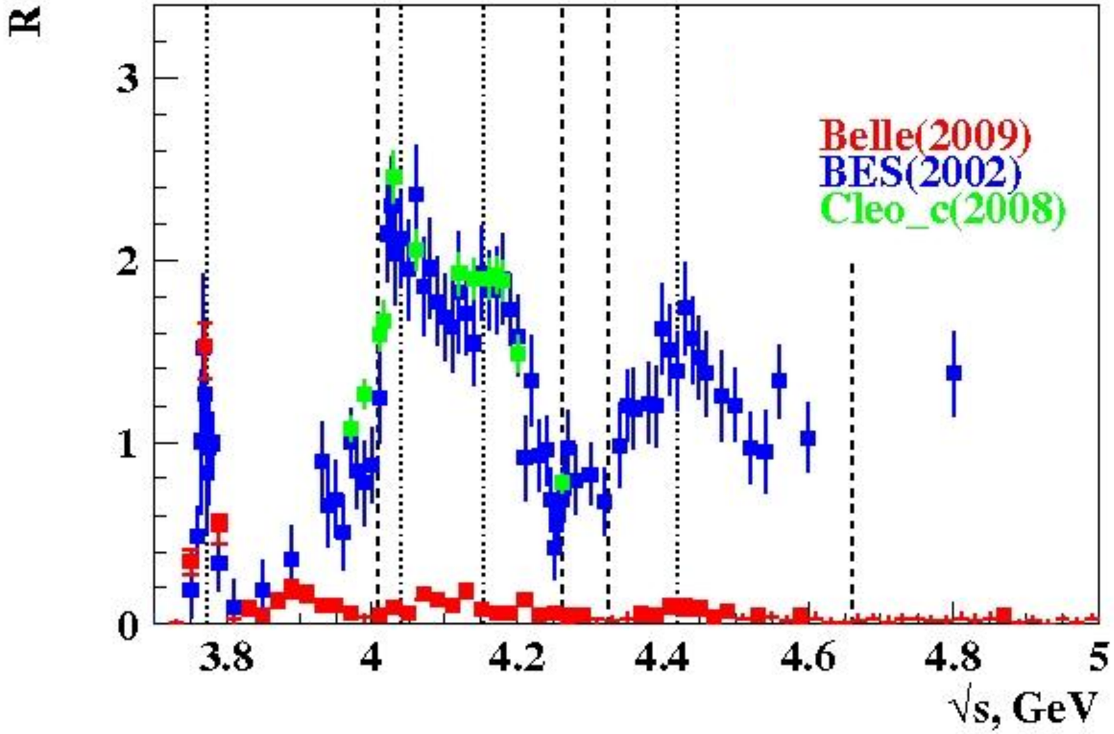
No signals of ψ states

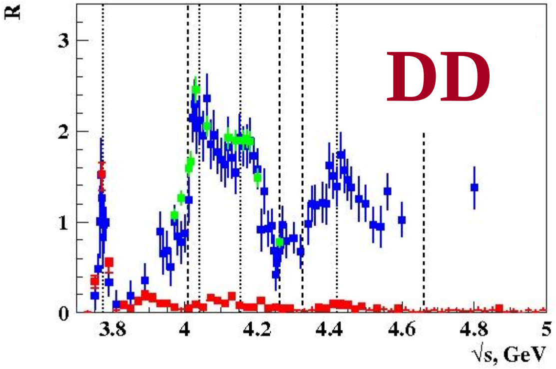
New states:

Y(4260), Y(4360), Y(4660)

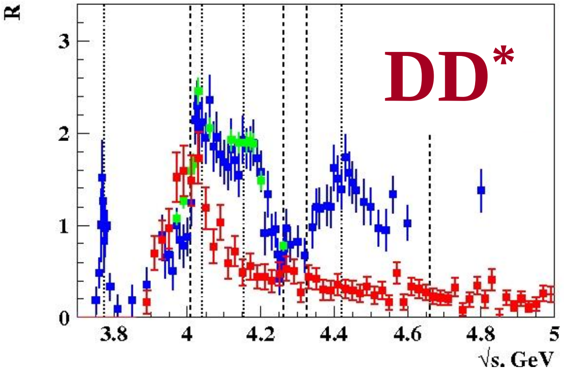
Y states are not seen in total hadronic cross-section.


DD
R
3
2
1
0
3.8
4
4.2
4.4
4.6
4.8
5
√s, GeV

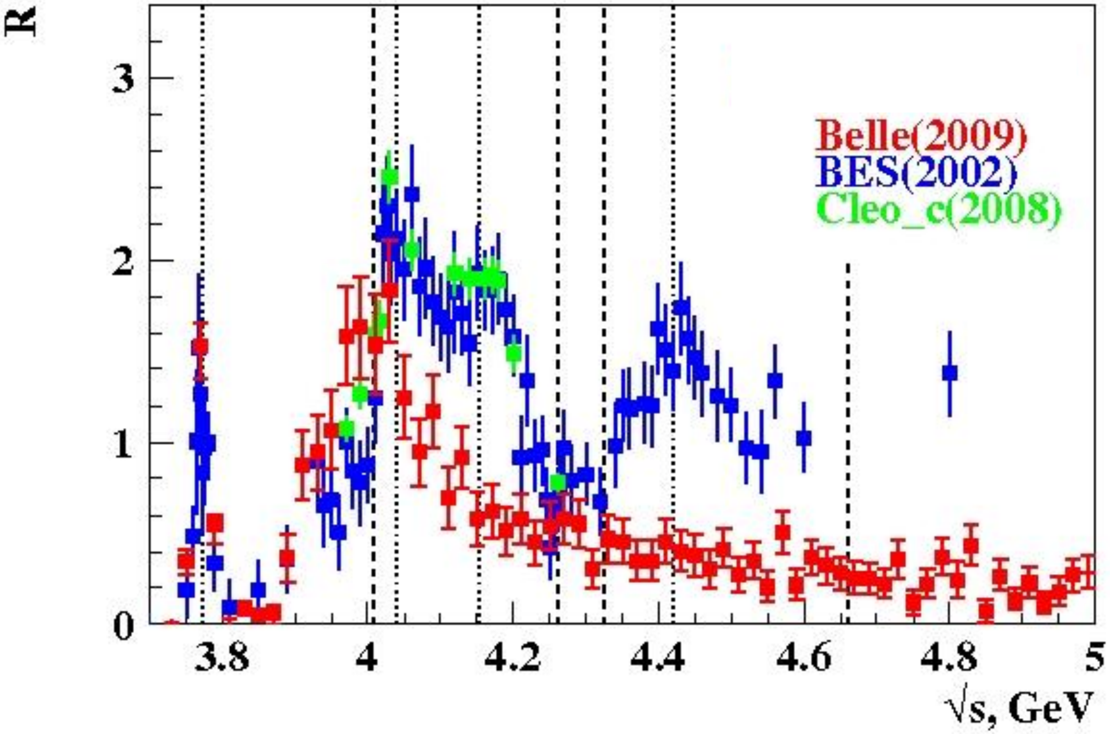

R
Belle(2009)
BES(2002)
Cleo_c(2008)
3
2
1
0
3.8
4
4.2
4.4
4.6
4.8
5
√s, GeV



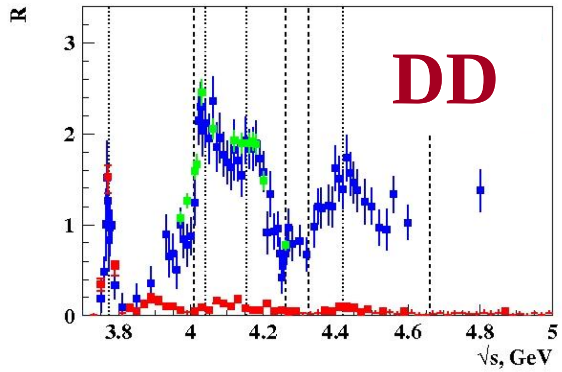
DD
R
√s, GeV

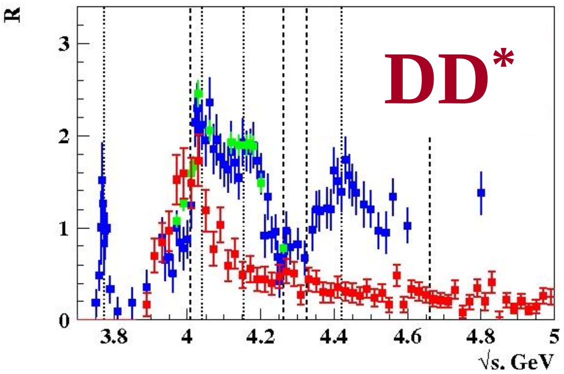


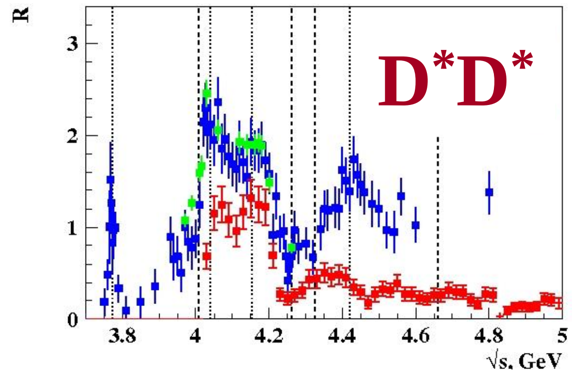
DD*
R
√s, GeV

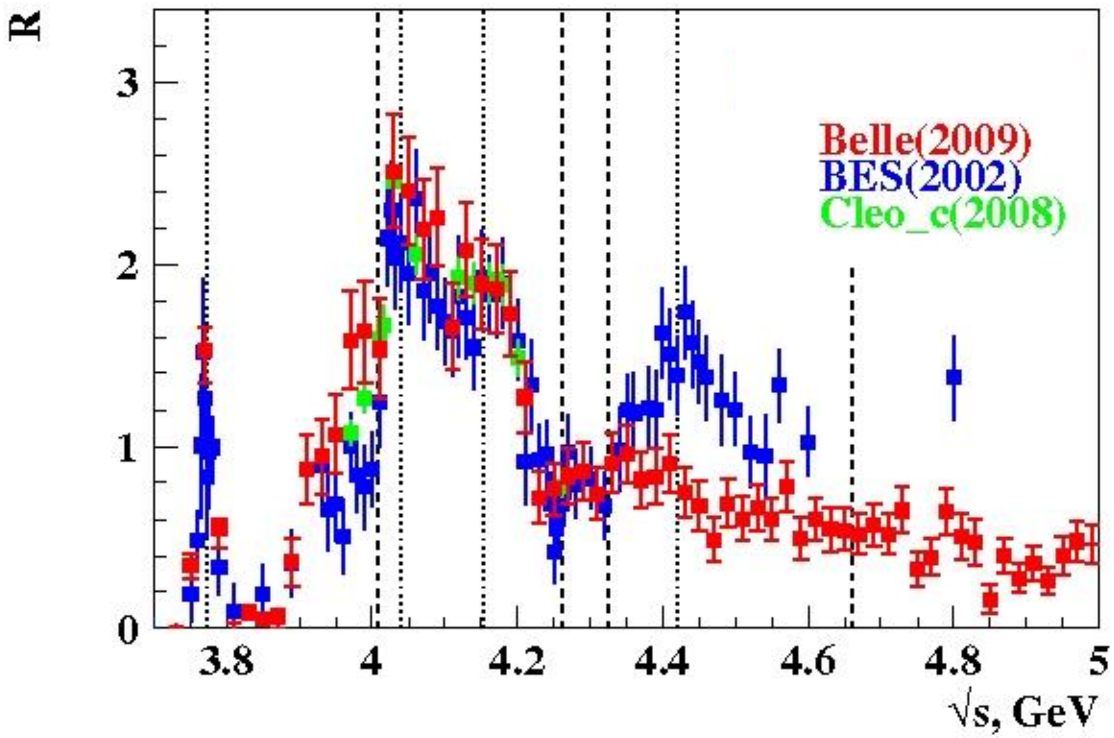


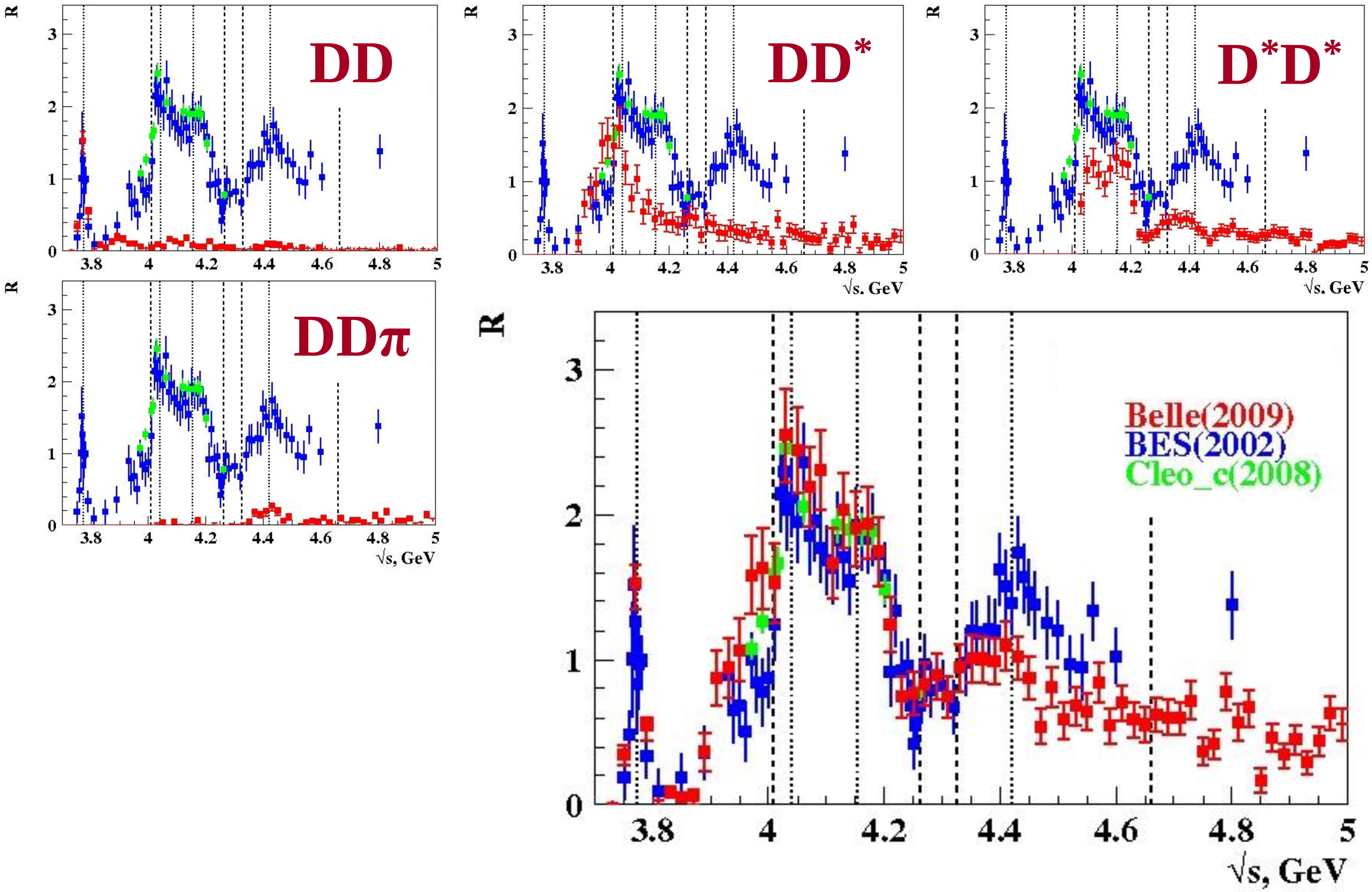
R
Belle(2009)
BES(2002)
Cleo_c(2008)
√s, GeV

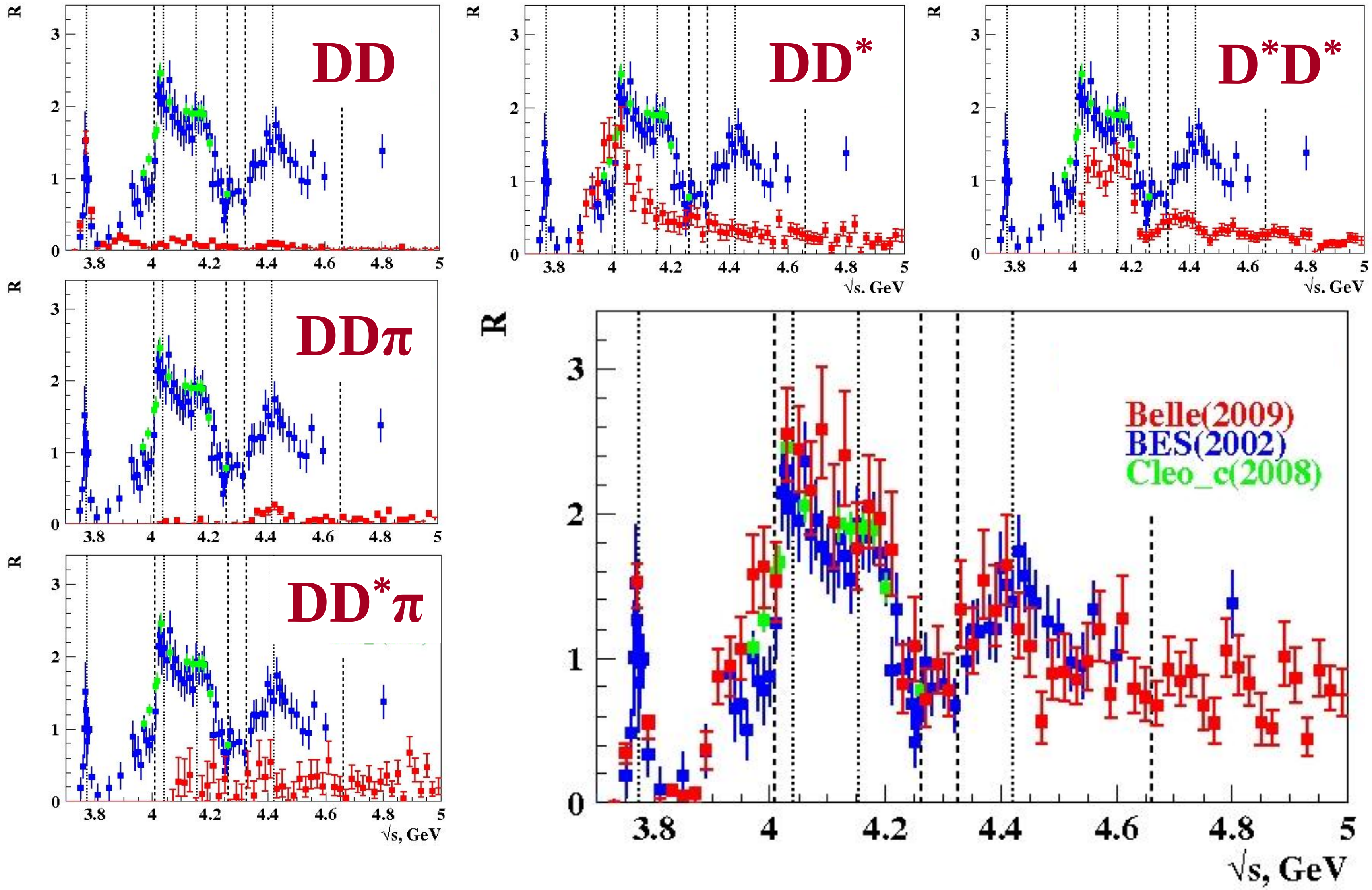

DD
R
√s, GeV

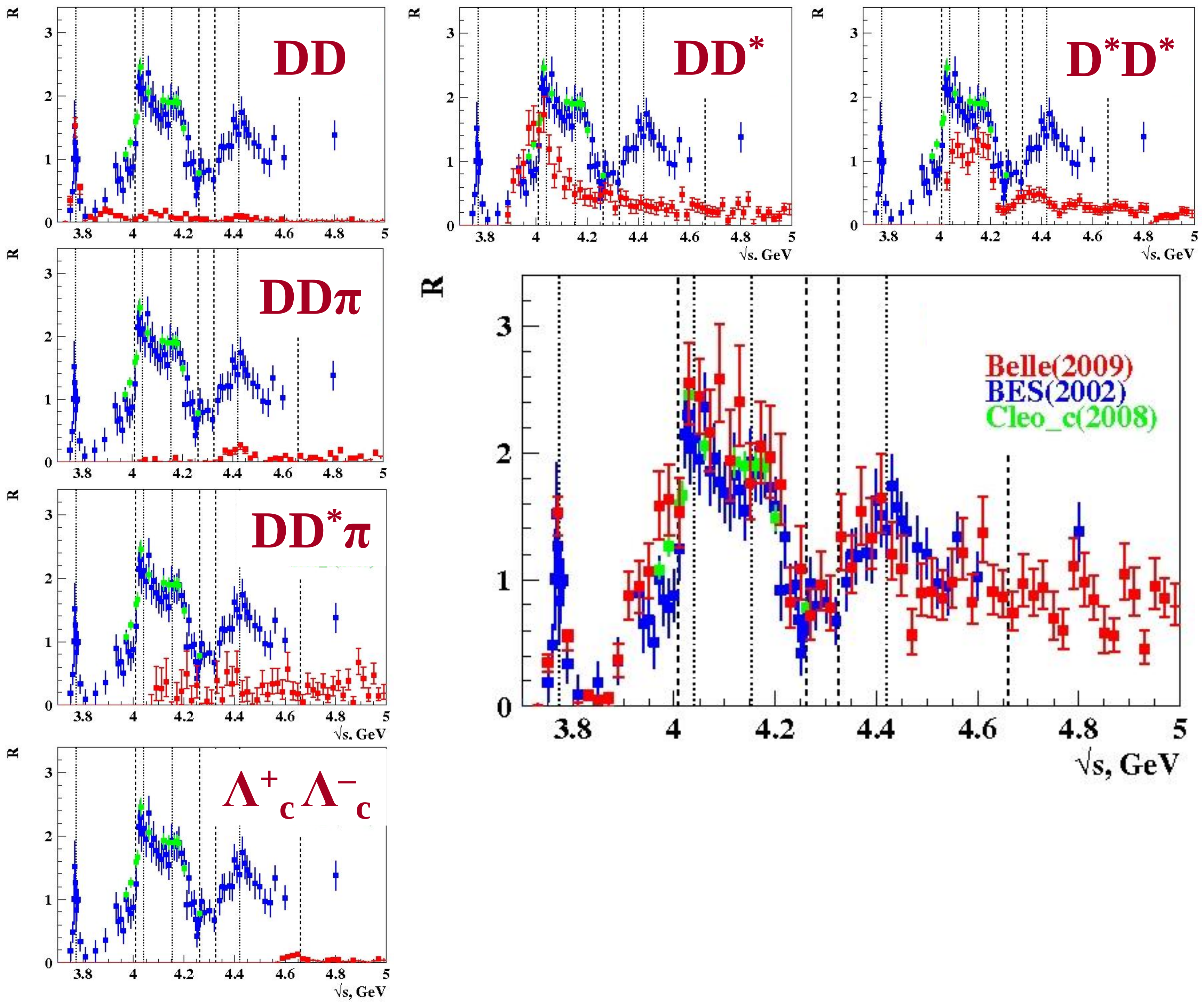

DD*
R
√s. GeV

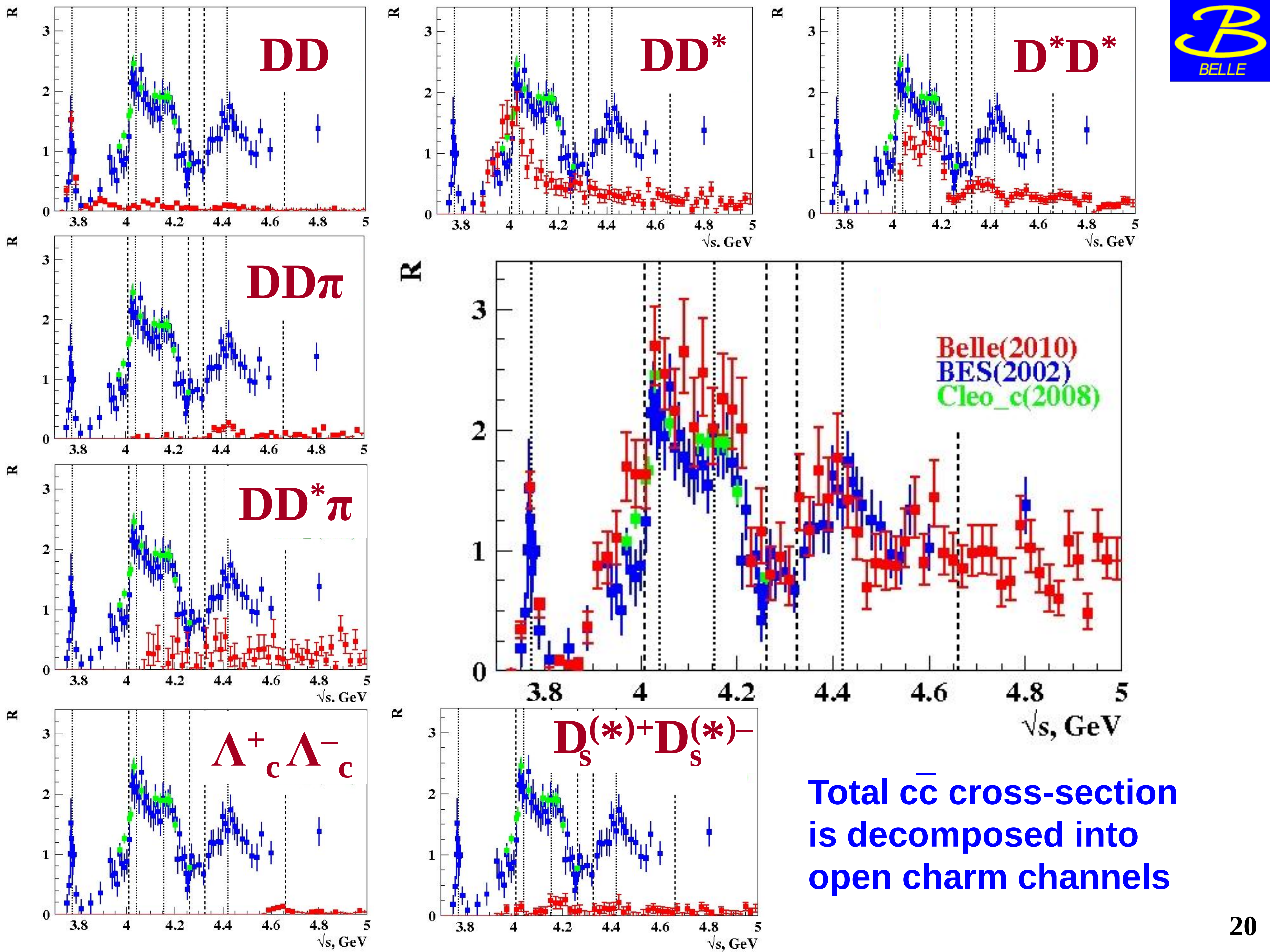

D*D*
R
√s. GeV


R
Belle(2009)
BES(2002)
Cleo_c(2008)
√s, GeV


DD
DD*
D*D*
DDπ
Belle(2009)
BES(2002)
Cleo_c(2008)
R
√s, GeV

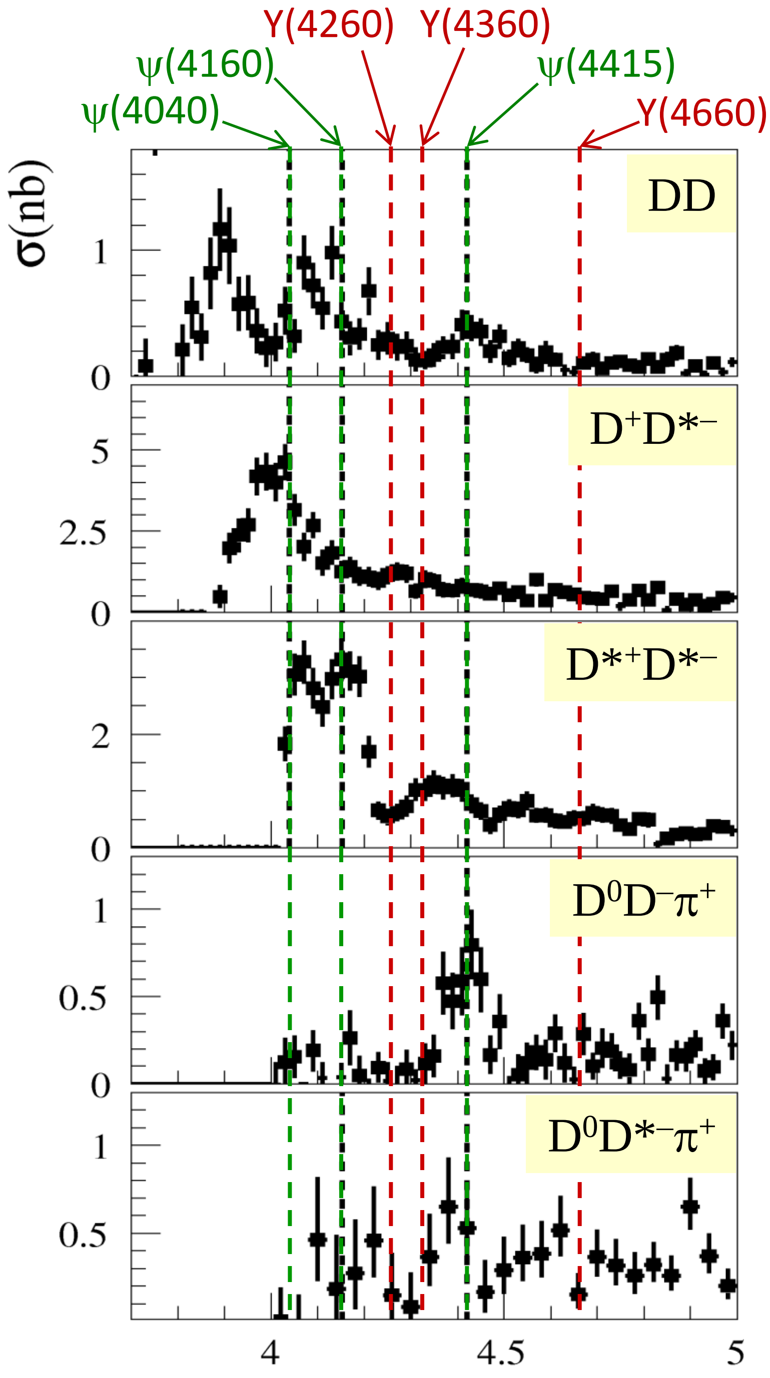

DD
DD*
D*D*
DDπ
DD*π
Belle(2009)
BES(2002)
Cleo_c(2008)
R
√s, GeV


DD
DD*
D*D*
DDπ
DD*π
Λ+c Λ−c
Belle(2009)
BES(2002)
Cleo_c(2008)
R
√s, GeV



Total $c\bar{c}$ cross-section is decomposed into open charm channels

$e^+e^- \to$ open charm using ISR



EPJC71,1534(2011)

No signals of Y(2460/4360/4660) $\to D^{(*)}D^{(*)}(\pi)$

CLEO PRD80,072001(2009)

$$\frac{\Gamma\,[Y(4260) \to D^*D^*]}{\Gamma\,[Y(4260) \to J/\psi\,\pi^+\pi^-]} < 11$$

C.f.

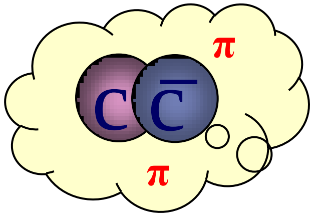
$$\frac{\Gamma\,[\psi(4160) \to D^*D^*]}{\Gamma\,[\psi(4160) \to J/\psi\,\pi^+\pi^-]} > 300$$

Y and ψ states have different structure?

Dubynskiy, Voloshin, PLB666,344(2008)

Y states: hadrocharmonium?

charmonium embedded into light hadron



Explains "selection rules":

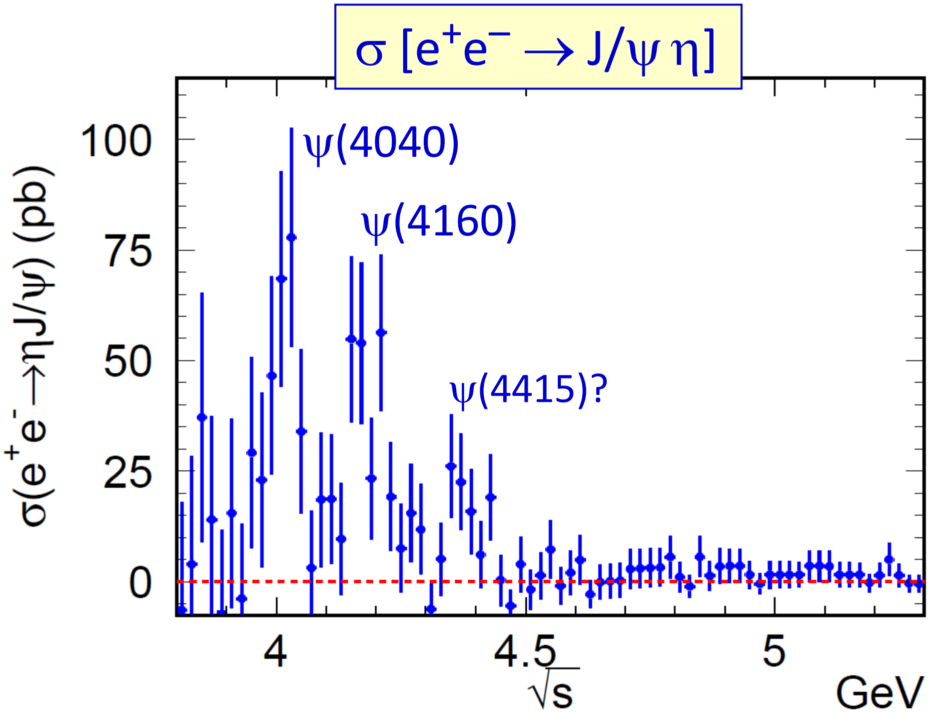
Y(4260) $\to J/\psi\,\pi^+\pi^-$

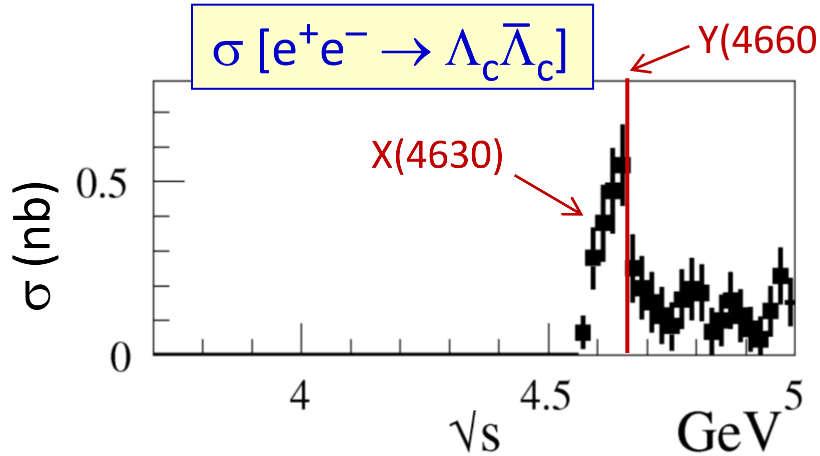
Y(4360), **Y(4660)** $\to \psi(2S)\,\pi^+\pi^-$

Decays to open flavor channels are suppressed.

PRD87,051101R(2013)

PRL101,172001(2008)





X(4630) – peak at $\Lambda_c\bar{\Lambda}_c$ threshold.

$\Lambda_c\bar{\Lambda}_c$ molecule? X(4630) =Y (4660)?

$\Gamma[\,\psi(4040,4160) \to J/\psi\ \eta] \sim 1$ MeV

ψ states also have anomalous transitions.
Why J/ψη channel?

Decay pattern of vector charmonium-like states remains puzzling.

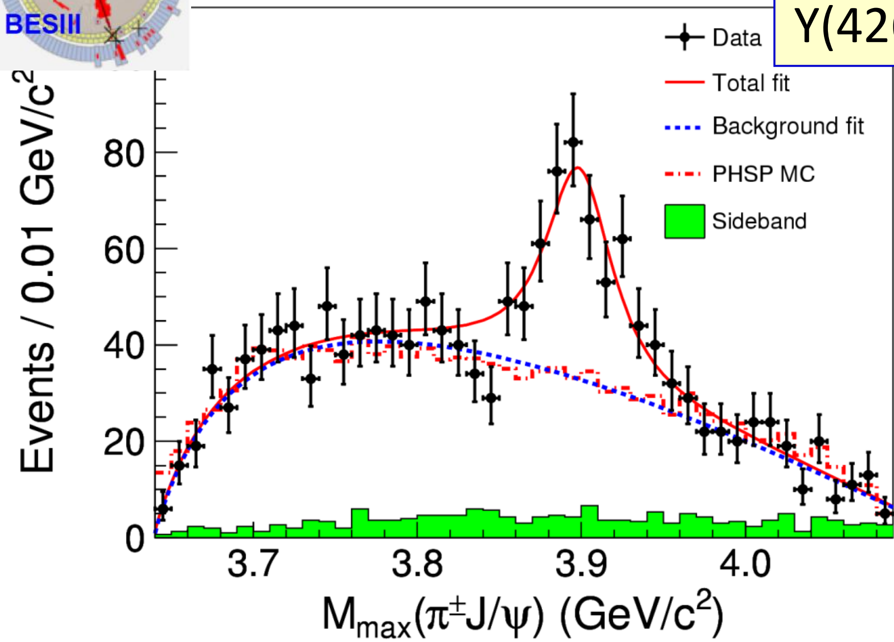
Need further studies: Belle/BaBar, BESIII, Belle-II, Super- cτ-Factory.

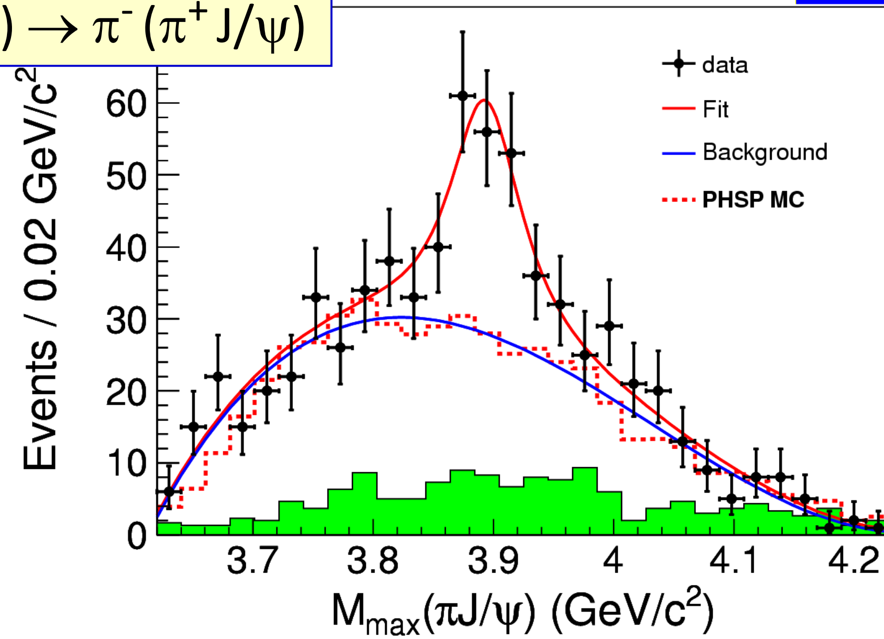
$Z_c(3900)^+$

PRL110, 252001 (2013)

PRL110, 252002 (2013)

$Y(4260) \to \pi^- (\pi^+ J/\psi)$

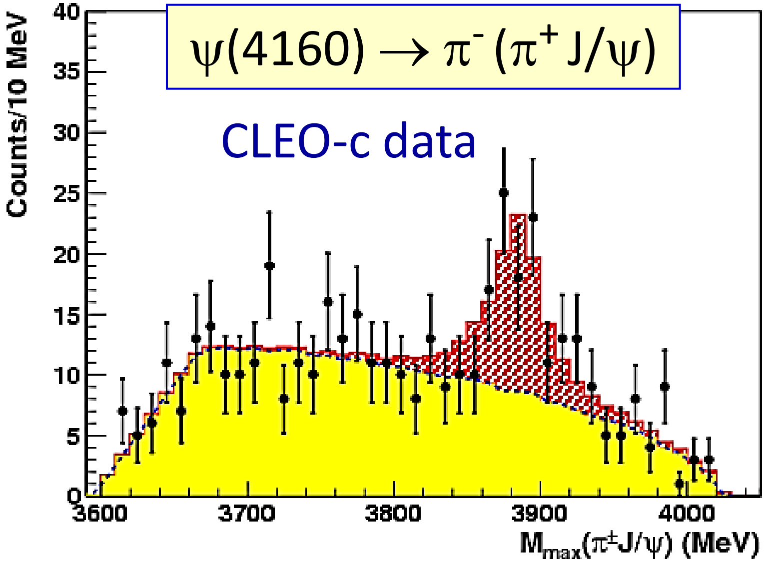




PLB727, 366 (2013)

$\psi(4160) \to \pi^- (\pi^+ J/\psi)$

CLEO-c data



PRL 111, 242001 (2013)

DD* threshold

$Z_c(3900) \longrightarrow J/\psi\ \pi$

no

$Z_c(4020) \longrightarrow h_c\ \pi$

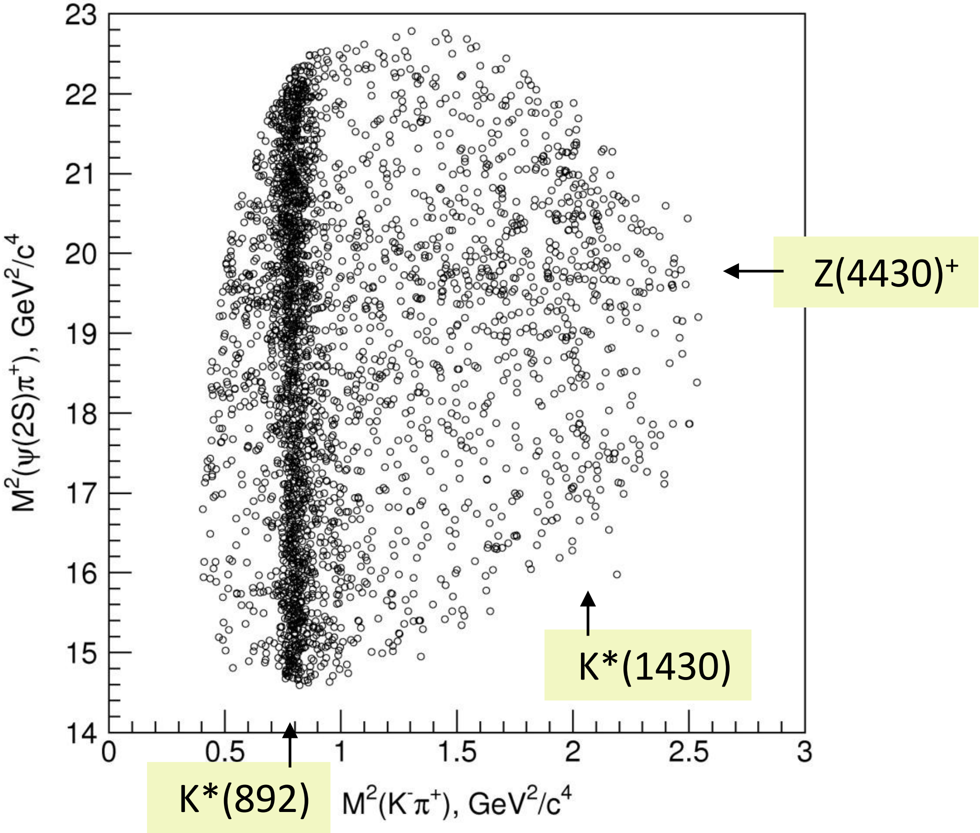
D*D* threshold

Z_c states are different from Z_b. Puzzling.

Observation of $Z(4430)^{\pm} \rightarrow \psi' \pi^{\pm}$

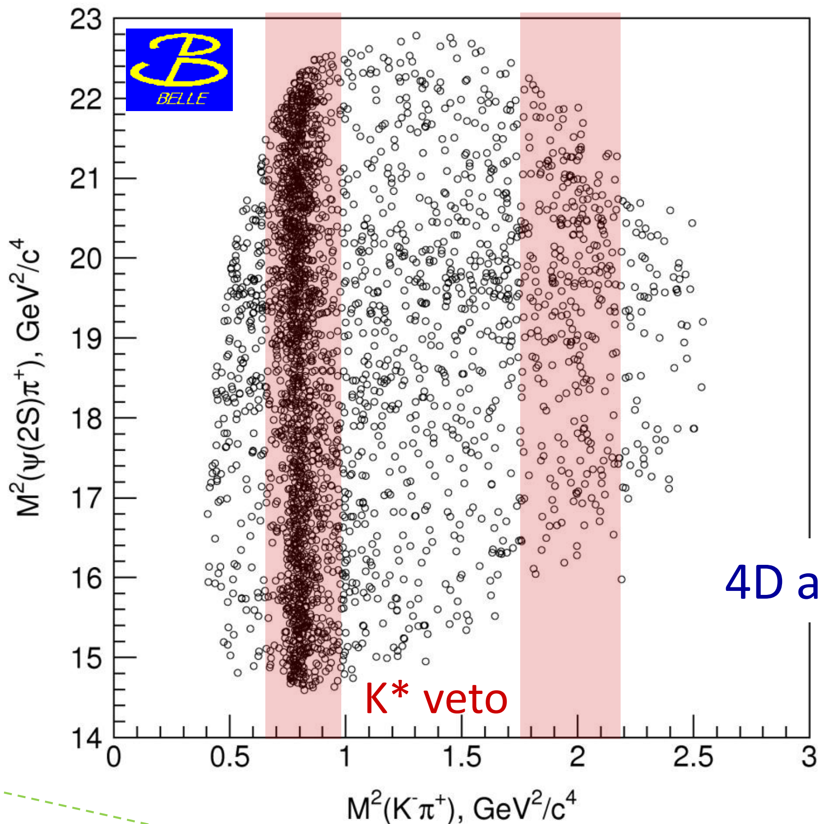
PRL100,142001(2008)

$B^{0,+} \rightarrow K^{-,0} \pi^{+} \psi'$ Dalitz plot



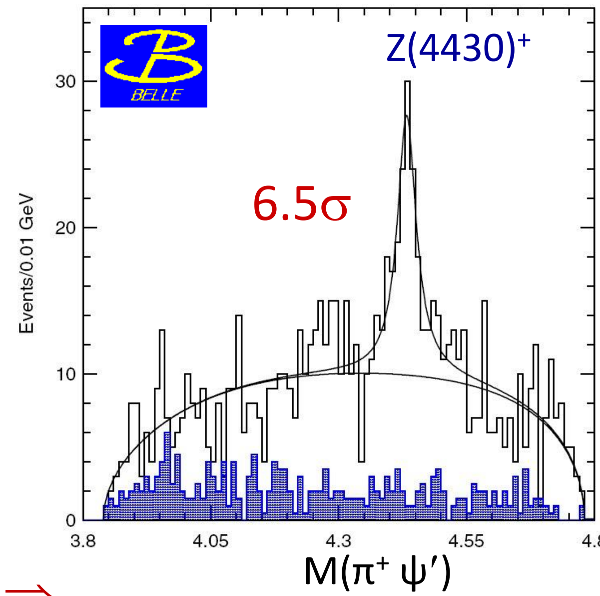
Observation of $Z(4430)^{\pm} \to \psi' \pi^{\pm}$

Dalitz plot of of $B \to \psi' \pi^+ K$



projection with K* veto applied

PRL100,142001(2008)



PRD88,074026(2013)

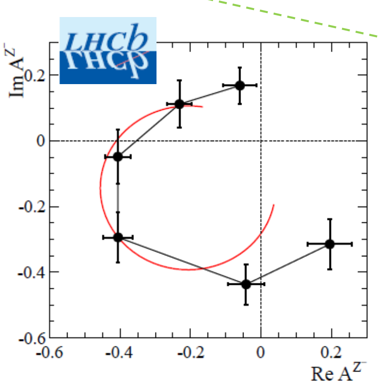
4D amplitude analysis ⇒

$$M = 4485^{+22+28}_{-22-11} \text{ MeV}$$
$$\Gamma = 200^{+41+26}_{-46-35} \text{ MeV.}$$

including syst.

$J^P = 1^+$

Exclusion levels of other hypotheses:
- 0^- (3.4σ)
- 1^- (3.7σ)
- 2^- (4.7σ)
- 2^+ (5.1σ)



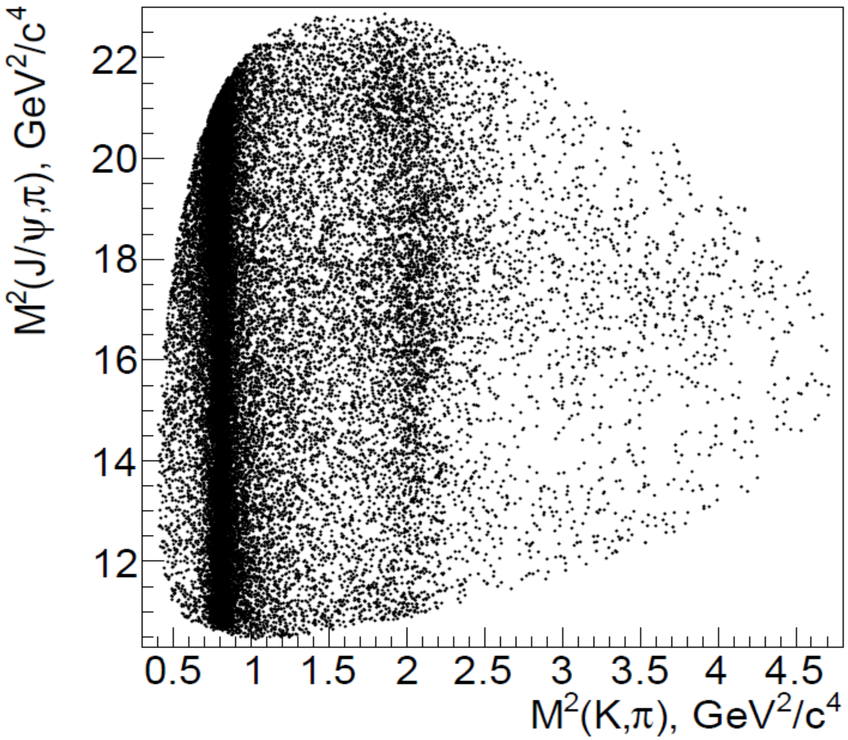
PRL112,222002(2014)

LHCb confirmed all Belle results.

Argand plot ⇒ resonant behavior of amplitude.

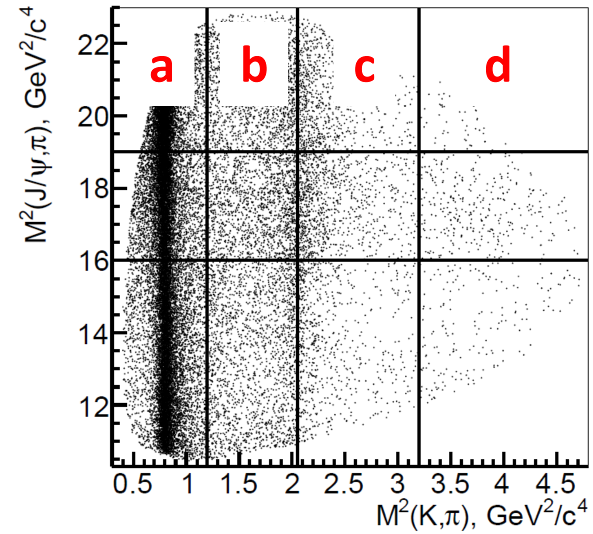
Full amplitude analysis of $B \to J/\psi\, \pi^+ K^-$

PRD90,112009(2014)

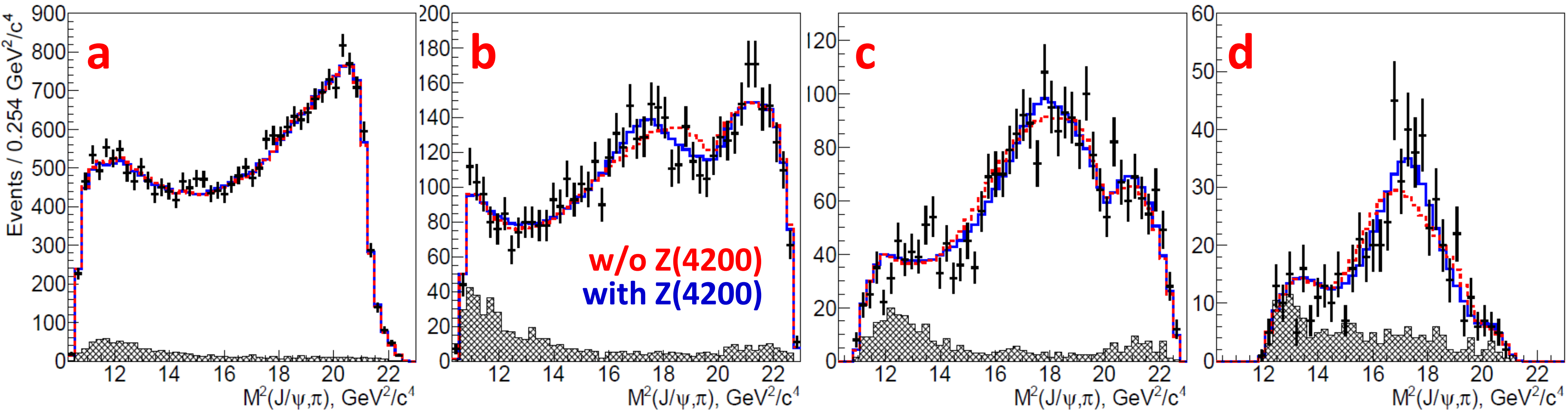


Model:

κ, $K^*(892)$, $K^*(1410)$, $K_0(1430)$, $K_2(1430)$, $K^*(1680)$, $K_3(1780)$, $K_0(1950)$, $K_2(1980)$, $K_4(2045)$, $Z(4430)^+$, new Z^+



Fit results in slices:



$Z(4430)^{\pm} \rightarrow J/\psi\, \pi^{\pm}$ & new $Z(4200)^{\pm}$

PRD90,112009(2014)

Significance of $Z(4430)^{\pm} \rightarrow J/\psi\, \pi^{\pm}$ is 5.1σ (4.0σ including systematics)

$$\frac{\Gamma\,[\,Z(4430) \rightarrow J/\psi\, \pi^{+}\,]}{\Gamma\,[\,Z(4430) \rightarrow \psi'\, \pi^{+}\,]} = 0.09\,^{+0.08}_{-0.05}$$

suppressed despite larger phase space

expected for hadrocharmonium

Significance of new $Z(4200)^{\pm}$ in $J/\psi\, \pi^{\pm}$ channel:

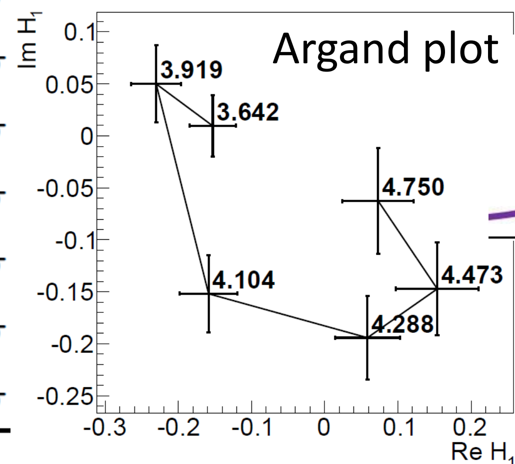
Model	0^-	1^-	1^+	2^-	2^+
default	3.9σ	2.3σ	8.2σ	3.9σ	1.9σ
Without $K^*(1680)$	3.2σ	3.1σ	8.4σ	3.7σ	1.9σ
Without $K_0^*(1950)$	3.6σ	2.8σ	8.6σ	5.0σ	2.6σ
LASS	3.8σ	1.0σ	6.6σ	5.2σ	2.3σ
Free masses and widths	2.4σ	1.6σ	7.3σ	4.6σ	1.9σ
Free r	5.0σ	2.6σ	8.4σ	4.5σ	0.9σ
Nonresonant ampl. (S)	3.8σ	2.9σ	7.9σ	4.1σ	2.0σ
Nonresonant ampl. (S,P)	3.7σ	2.4σ	7.7σ	3.7σ	1.4σ
Nonresonant ampl. (S,P,D)	4.1σ	2.3σ	7.7σ	3.8σ	1.3σ

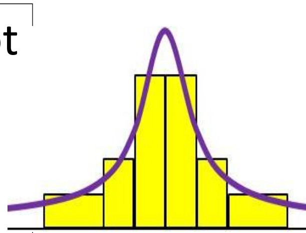
"Look elsewhere" effect ⇐ toy MC

$$M = 4196^{+31+17}_{-29-13}\ \text{MeV}$$
$$\Gamma = 370^{+70+70}_{-70-132}\ \text{MeV}.$$

$J^P = 1^+$

Exclusion levels:
- 0^- (6.1σ)
- 1^- (7.4σ)
- 2^- (4.4σ)
- 2^+ (7.0σ)





⇒ Resonant behavior of amplitude.

Conclusions

Recent results:

Observation of $\Upsilon(4S) \to h_b(1P)\,\eta$ — U.Tamponi QWG2014

$\Upsilon(5S) \to \Upsilon(1D)\,\eta$

$\Upsilon(5S) \to \chi_{b1/2}(1P)\,\omega$ — PRL113,142001(2014)

$\Upsilon(6S) \to \Upsilon(1S/2S/3S)\,\pi^+\pi^-$ — arxiv:1501.01137

Search for $\Upsilon(5S) \to X_b\,\gamma$ — G.Tatishvili QWG2014

Measurement of $\sigma\,[e^+e^- \to J/\psi\, K^+K^-]$ — PRD89,072015(2014)

Evidence for new state $Z(4050)^+$ in $\psi(2S)\,\pi^+$ channel — arxiv:1410.7641

Observation of new state $Z_c(4200)^+$ in $J/\psi\pi^+$ channel in B decays

Evidence for $Z(4430)^+ \to J/\psi\,\pi^+$ — PRD90,112009(2014)

Properties of Z_b states, $\Upsilon(5S/6S)$ are so far consistent with molecular interpretation.

Charmonium sector is very different from bottomonium. Decay pattern is puzzling.

Presence of “short-distance” exotics (diquarks, valence gluons) is an open question.

Further studies: Belle/BaBar, BESIII, LHC, Belle-II, PANDA, Super-ct-Factory.

Back-up

Search for partners of X(3872)

There could exist molecules
with $J^P = 0^{++}, 1^{+-}, 2^{++}$...

Belle searched for new states in decays

$B^+ \to X K^+$

$X \to \chi_{c1}\gamma, \chi_{c2}\gamma$ PRL111, 032001 (2013)

$\quad \hookrightarrow J/\psi\,\gamma$

$X \to J/\psi\,\eta$ PTEP2014,043C01

$X \to \eta_c\,\eta$ Vinokurova QCD14

$X \to \eta_c\,\pi^0$

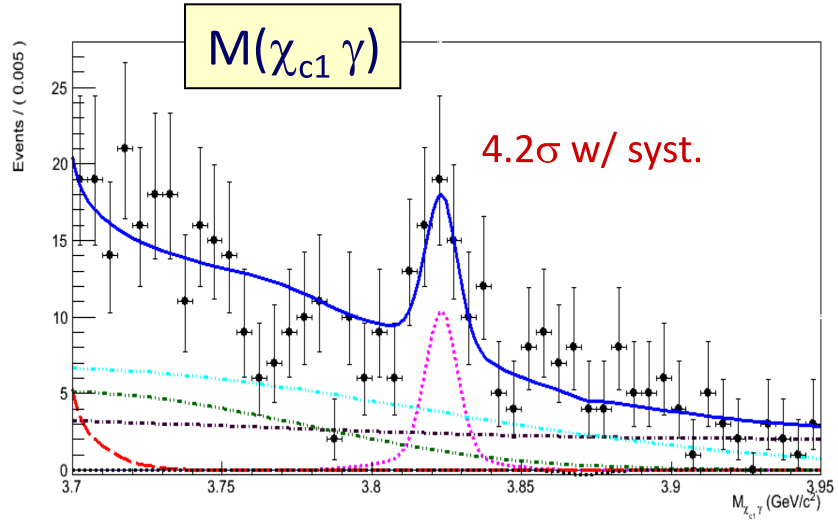
$X \to \eta_c\,\pi^+\pi^-$

$X \to \eta_c\,\omega$

No peaks near DD, DD* or D*D* thresholds.

$\Rightarrow$ Subject for Belle-II.

one new peak



$M = 3823.5 \pm 2.8$ MeV

$\Gamma < 14$ MeV @90% C.L.

Radiative decay seen $\Rightarrow \Gamma \sim O(100\text{keV})$

Above $D\bar{D}$ threshold $\Rightarrow J^P = 0^-, 1^+, 2^-, ..$

Potential models $\Rightarrow \psi_2(1D)$ candidate

2^{--} D-wave spin-triplet

Confirmation, $J^P \Leftarrow$ Belle-II.

X(3940)

Double charmonium production:

$e^+e^- \to J/\psi\ X(3940)$
$\hookrightarrow D\bar{D}^*$

$e^+e^- \to J/\psi\ X(4160)$
$\hookrightarrow D^*\bar{D}^*$

The only states produced in the same process

$\eta_c, \chi_{c0}, \eta_c(2S)$

$J^{PC} = 0^{?+}$

$X \not\to D\bar{D} \Rightarrow J^P = 0^-, 1^+, 2^-, ..$

$X(3940,4160) = \eta_c(3S,4S)$?

masses off by ~300MeV

Belle-II: measure J^P.

Y(3940) $\to$ Y(3915) $\to \chi_{c0}(2P)$

B decays:

$B \to Y(3915)\ K$
$\hookrightarrow J/\psi\ \omega$

Two-photon process:

$\gamma\gamma \to Y(3915) \to J/\psi\ \omega$

PRD86,072002(2012)

BaBar measured $J^P = 0^+$

model dependent assumption

Y(3915) is 200MeV above $D\bar{D}$ threshold and can decay to $D\bar{D}$

$\Rightarrow \Gamma \sim$ 20MeV is extremely small

$\Rightarrow$ exotic structure

Belle-II: confirm J^P,
find other decay channels.

Z(3930) $\to \chi_{c2}(2P)$

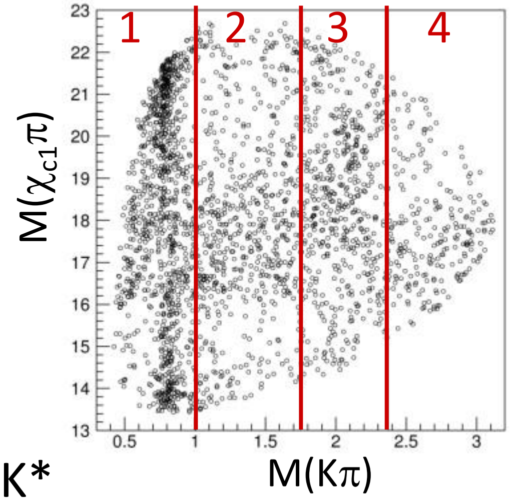
Two-photon process:

$\gamma\gamma \to Z(3930) \to D\bar{D}$

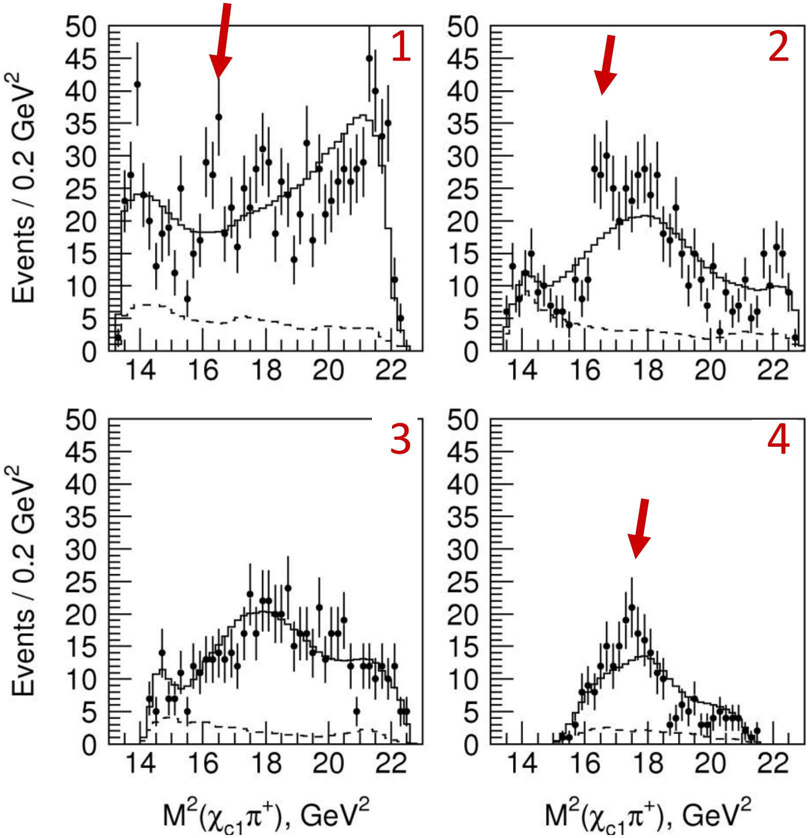
Mass, width, diphoton width agree with expectations.

PRD79,112001 (2009)

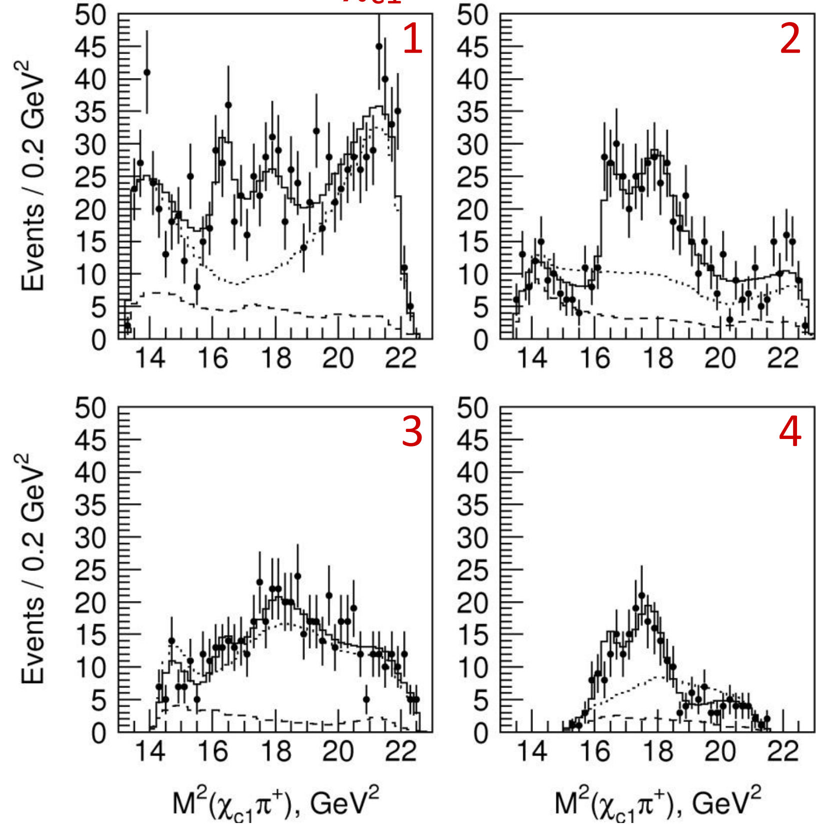
Dalitz analysis of the decay $\bar{B}^0 \to K^- \pi^+ \chi_{c1}$



all low K*



all low K*
+ two ($\pi^+ \chi_{c1}$) resonances



$\Rightarrow$ Observation of $Z(4050)^+$, $Z(4250)^+$

Belle-II: confirmation, measurement of J^P.

Voloshin PRD84, 031502 (2011)

$$\left|Z_b'\right\rangle = \frac{1}{\sqrt{2}} 0^-_{bb} \otimes 1^-_{Qq} - \frac{1}{\sqrt{2}} 1^-_{bb} \otimes 0^-_{Qq}$$

$$\left|Z_b\right\rangle = \frac{1}{\sqrt{2}} 0^-_{bb} \otimes 1^-_{Qq} + \frac{1}{\sqrt{2}} 1^-_{bb} \otimes 0^-_{Qq}$$

$$\left|W_{b0}'\right\rangle = \frac{\sqrt{3}}{2} 0^-_{bb} \otimes 0^-_{Qq} - \frac{1}{2} 1^-_{bb} \otimes 1^-_{Qq}$$

$$\left|W_{b0}\right\rangle = \frac{1}{2} 0^-_{bb} \otimes 0^-_{Qq} + \frac{\sqrt{3}}{2} 1^-_{bb} \otimes 1^-_{Qq}$$

$$\left|W_{b1}'\right\rangle = (1^-_{bb} \otimes 1^-_{Qq})_{J=1}$$

$$\left|W_{b2}\right\rangle = (1^-_{bb} \otimes 1^-_{Qq})_{J=2}$$