Beaming Neutrino across the Earth to test neutrino mixing, and mass hierarchy.

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BEAMING NEUTRINOS AND ANTI-NEUTRINOS ACROSS THE EARTH TO DISENTANGLE NEUTRINO MIXING PARAMETERS

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ABSTRACT

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Summary

- ***Neutrino tau and anti-tau birth needs tens GeV energy threshold***
- ***At 20s GeV the oscillation lenght is twice Earth radius***
- ***Muon Neutrinos into tau were too rare at OPERA-CERN***
- ***20s GeV neutrino detection at Deep Core is within***
- ***4-8 Mton thousands times OPERA-Kton detector***
- ***Oscillation at Opera is 1.5% while 100% at ICECUBE***
- ***Size, oscillation distances and rate makes CERN or***

Fermilab beaming across Earth to ICECUBE

Ideal to disentangle CPT violation and mixing parameters and MSW

As tau and anti tau appearence, Hierarchi mass..mixing angles

At least 4 order of magnitude more abundant in tau than

CERN-OPERA experiment

SIMILAR PROPOSALS

Mass hierarchy discrimination
with **atmospheric neutrinos**
in large volume ice/water Cherenkov detector
D Franco, C Jollet, A Kouchner, V Kulikovskiy

Neutrino mass hierarchy determination with
IceCube-**PINGU**
W Winter - Physical Review D, 2013 - APS

Mass hierarchy, 2-3 mixing and CP-phase
with Huge **Atmospheric Neutrino** Detectors

EK Akhmedov, S Razzaque, AY Smirnov

MAIN DIFFERENCES

When you receive a random atmospheric muons at a few (3-6 GeV), you do not know:

- The initial atm.neutrino energy
- Its nature (matter-antimatter)
- The arrival direction because low energy-wi

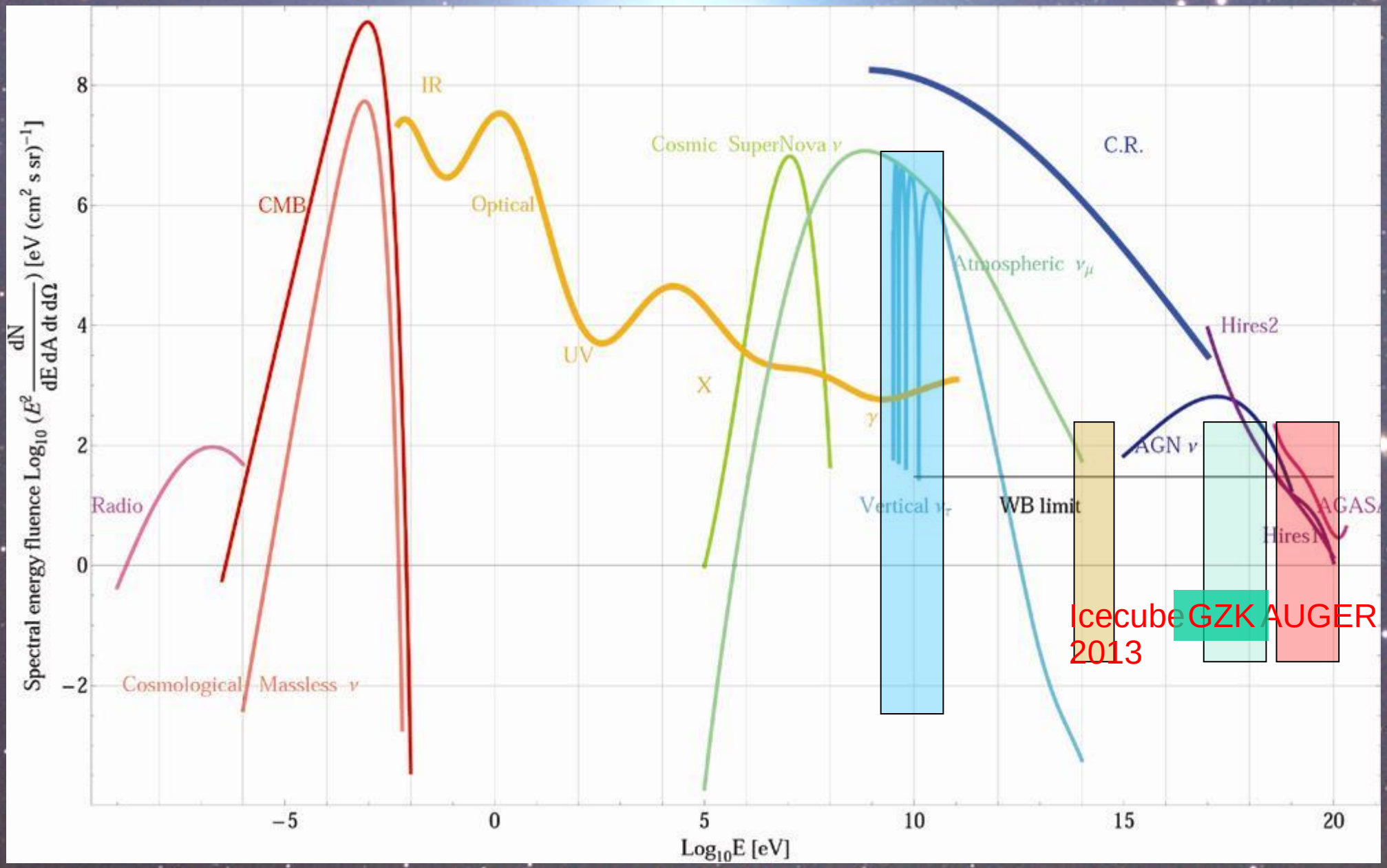
Scattering angle

1) When beaming by 50 GeV pions you choose matter and anti matter by Lorentz bending forces

2) <u>When sending the 20 GeV neutrino you control the energy (by 10-20%)</u>

3) When you cross the Earth you do know in detail the path and the distances

Cosmic Ray Spectra background



Neutrino Mixing flavor Matrix

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k}|\nu_k\rangle, \quad (\alpha = e, \mu, \tau \quad k = 1, 2, 3)$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Usual Mixing in vacuum

$$P_{\nu_\alpha \to \nu_\beta}(t) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U^*_{\alpha j} U_{\beta j} \; e^{-i(E_k - E_j)t}$$

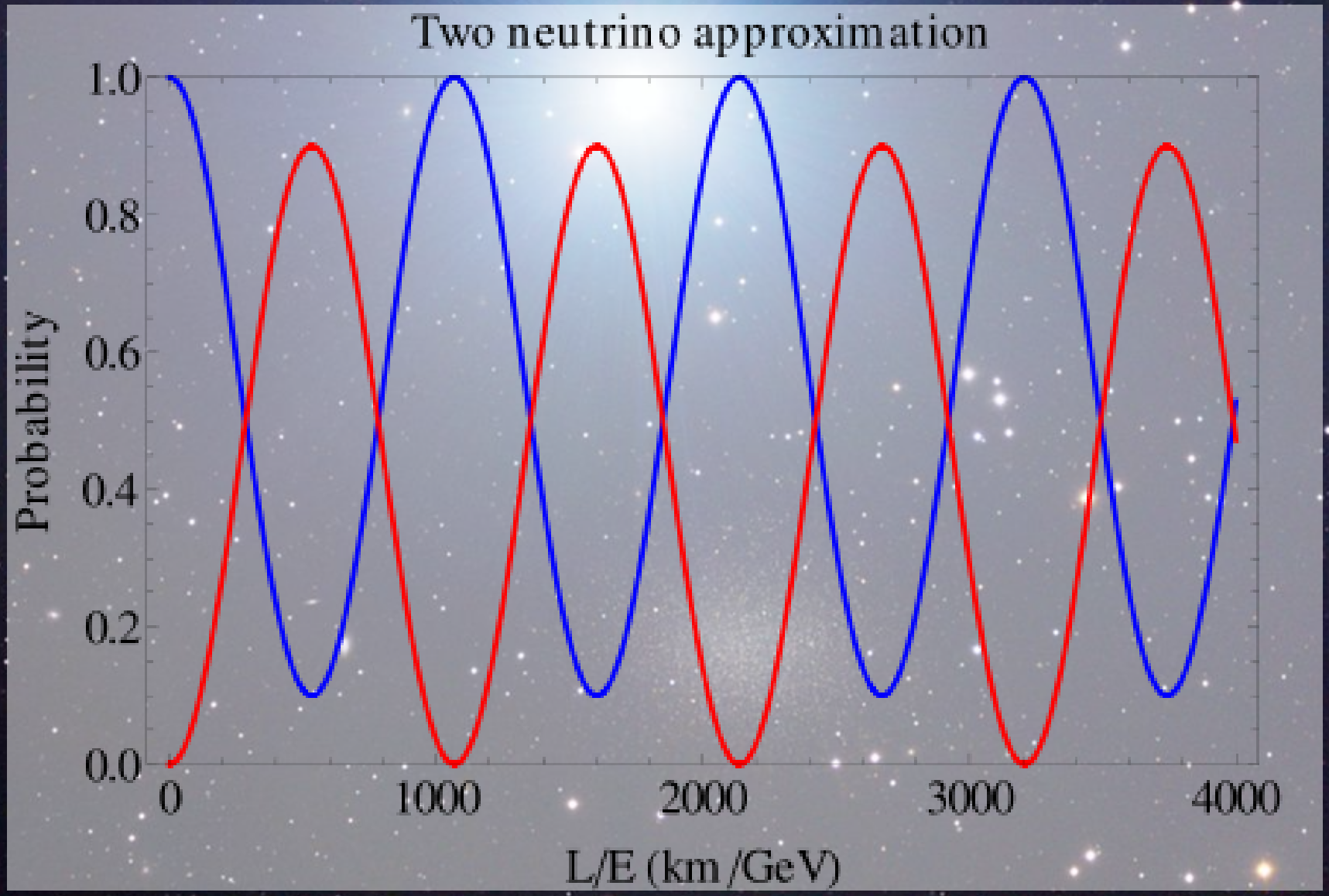
$$= \sum_{k,j} U^*_{\alpha k} U_{\beta k} U^*_{\alpha j} U_{\beta j} \; \exp\left(-i \; 2.54 \; \frac{\Delta m^2_{kj}}{(eV^2)} \frac{(GeV)}{E} \frac{L}{(km)}\right)$$

$$P^{incoerente}_{\nu_\alpha \to \nu_\beta}(t) = \sum_k \left| \langle \nu_\beta | \nu_k \rangle e^{-iE_k t} \langle \nu_k | \nu_\alpha \rangle \right|^2 = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2$$

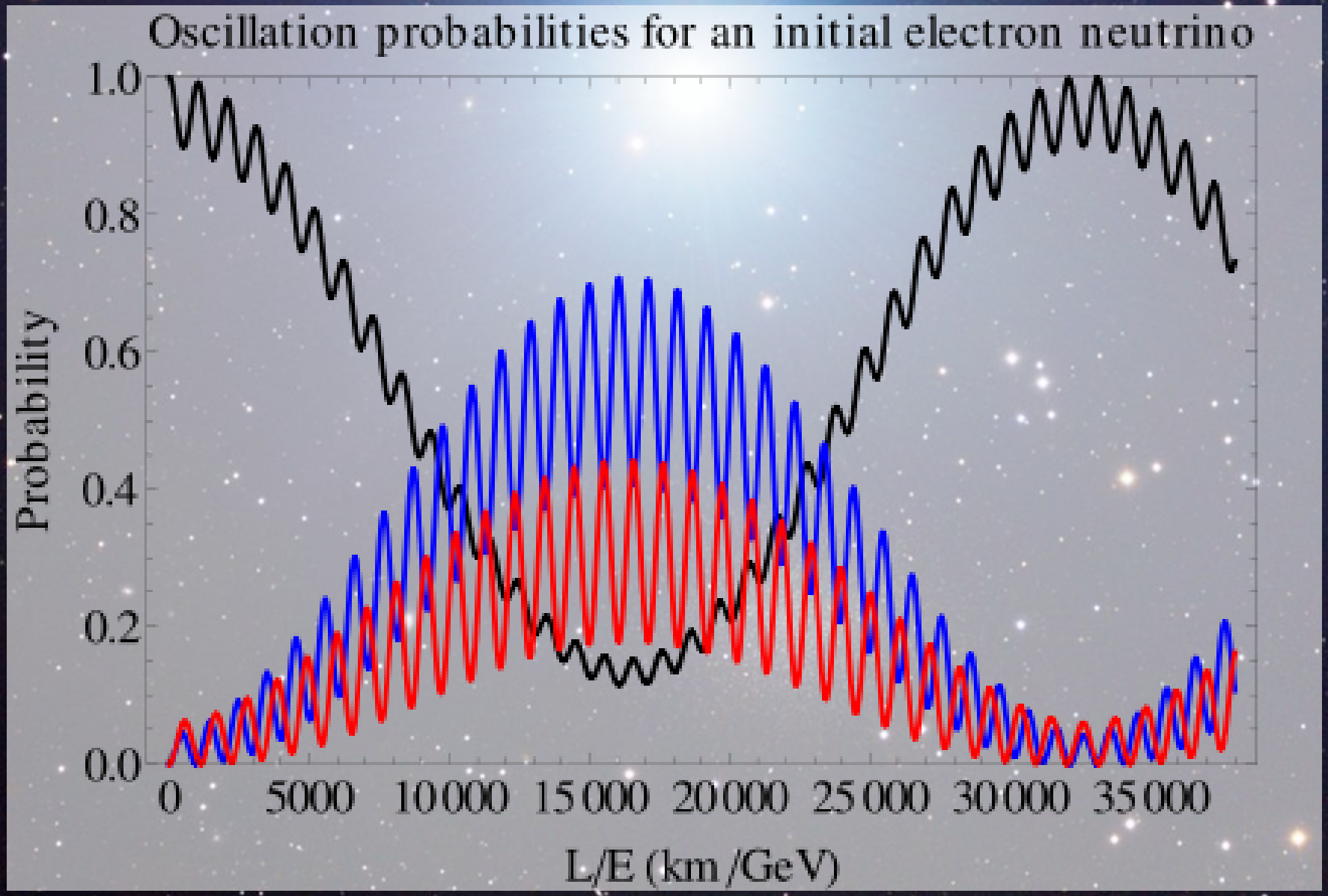
All 3 Oscillations In vacuum

$\alpha\beta$	kj	$\Re\mathrm{e}\left[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\right]$
$e\mu$	21	$-\frac{1}{4}\sin 2\vartheta_{12}\, c_{13}^2\, \left[\sin 2\vartheta_{12}\, \left(c_{23}^2 - s_{23}^2 s_{13}^2\right) + \cos 2\vartheta_{12}\, \sin 2\vartheta_{23}\, s_{13} \cos\delta_{13}\right]$
	32	$-s_{12} s_{23} c_{13}^2 s_{13}\, \left(s_{12} s_{23} s_{13} - c_{12} c_{23} \cos\delta_{13}\right)$
	31	$-c_{12} s_{23} c_{13}^2 s_{13}\, \left(c_{12} s_{23} s_{13} + s_{12} c_{23} \cos\delta_{13}\right)$
$\mu\tau$	21	$\frac{1}{16}\sin^2 2\vartheta_{12}\, \sin^2 2\vartheta_{23}\, \left(1 + s_{13}^2\right)^2 - \frac{1}{4}\left(\sin^2 2\vartheta_{12} + \sin^2 2\vartheta_{23}\right) s_{13}^2 - \frac{1}{16}\sin 4\vartheta_{12}\, \sin 4\vartheta_{23}\, \left(1 + s_{13}^2\right) s_{13} \cos\delta_{13} + \frac{1}{4}\sin^2 2\vartheta_{12}\, \sin^2 2\vartheta_{23}\, s_{13}^2 \cos^2\delta_{13}$
	32	$-\frac{1}{4}\sin 2\vartheta_{23}\, c_{13}^2\, \left[\sin 2\vartheta_{23}\, \left(c_{12}^2 - s_{12}^2 s_{13}^2\right) + \sin 2\vartheta_{12}\, \cos 2\vartheta_{23}\, s_{13} \cos\delta_{13}\right]$
	31	$\frac{1}{4}\sin 2\vartheta_{23}\, c_{13}^2\, \left[\sin 2\vartheta_{23}\, \left(c_{12}^2 s_{13}^2 - s_{12}^2\right) + \sin 2\vartheta_{12}\, \cos 2\vartheta_{23}\, s_{13} \cos\delta_{13}\right]$
τe	21	$\frac{1}{4}\sin 2\vartheta_{12}\, c_{13}^2\, \left[\sin 2\vartheta_{12}\, \left(c_{23}^2 s_{13}^2 - s_{23}^2\right) + \cos 2\vartheta_{12}\, \sin 2\vartheta_{23}\, s_{13} \cos\delta_{13}\right]$
	32	$-s_{12} c_{23} c_{13}^2 s_{13}\, \left(s_{12} c_{23} s_{13} + c_{12} s_{23} \cos\delta_{13}\right)$
	31	$-c_{12} c_{23} c_{13}^2 s_{13}\, \left(c_{12} c_{23} s_{13} - s_{12} s_{23} \cos\delta_{13}\right)$
ee	21	$\frac{1}{4}\sin^2 2\vartheta_{12}\, c_{13}^4$
	32	$\frac{1}{4} s_{12}^2\, \sin^2 2\vartheta_{13}$
	31	$\frac{1}{4} c_{12}^2\, \sin^2 2\vartheta_{13}$
$\mu\mu$	21	$\frac{1}{4}\sin^2 2\vartheta_{12}\, \left(c_{23}^4 + s_{23}^4 s_{13}^2\right) + \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2\vartheta_{12}\right) \sin^2 2\vartheta_{23}\, s_{13}^2 + \frac{1}{4}\sin 4\vartheta_{12}\, \sin 2\vartheta_{23}\, \left(c_{23}^2 - s_{23}^2 s_{13}^2\right) s_{13} \cos\delta_{13} - \frac{1}{4}\sin^2 2\vartheta_{12}\, \sin^2 2\vartheta_{23}\, s_{13}^2 \cos^2\delta_{13}$
	32	$s_{23}^2 c_{13}^2\, \left(c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - \frac{1}{2}\sin 2\vartheta_{12}\, \sin 2\vartheta_{23}\, s_{13}^2 \cos^2\delta_{13}\right)$
	31	$s_{23}^2 c_{13}^2\, \left(s_{12}^2 c_{23}^2 + c_{12}^2 s_{23}^2 s_{13}^2 + \frac{1}{2}\sin 2\vartheta_{12}\, \sin 2\vartheta_{23}\, s_{13}^2 \cos^2\delta_{13}\right)$
$\tau\tau$	21	$\frac{1}{4}\sin^2 2\vartheta_{12}\, \left(s_{23}^4 + c_{23}^4 s_{13}^2\right) + \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2\vartheta_{12}\right) \sin^2 2\vartheta_{23}\, s_{13}^2 + \frac{1}{4}\sin 4\vartheta_{12}\, \sin 2\vartheta_{23}\, \left(s_{23}^2 - c_{23}^2 s_{13}^2\right) s_{13} \cos\delta_{13} - \frac{1}{4}\sin^2 2\vartheta_{12}\, \sin^2 2\vartheta_{23}\, s_{13}^2 \cos^2\delta_{13}$
	32	$c_{23}^2 c_{13}^2\, \left(c_{12}^2 s_{23}^2 + s_{12}^2 c_{23}^2 s_{13}^2 + \frac{1}{2}\sin 2\vartheta_{12}\, \sin 2\vartheta_{23}\, s_{13}^2 \cos^2\delta_{13}\right)$
	31	$c_{23}^2 c_{13}^2\, \left(s_{12}^2 s_{23}^2 + c_{12}^2 c_{23}^2 s_{13}^2 - \frac{1}{2}\sin 2\vartheta_{12}\, \sin 2\vartheta_{23}\, s_{13}^2 \cos^2\delta_{13}\right)$

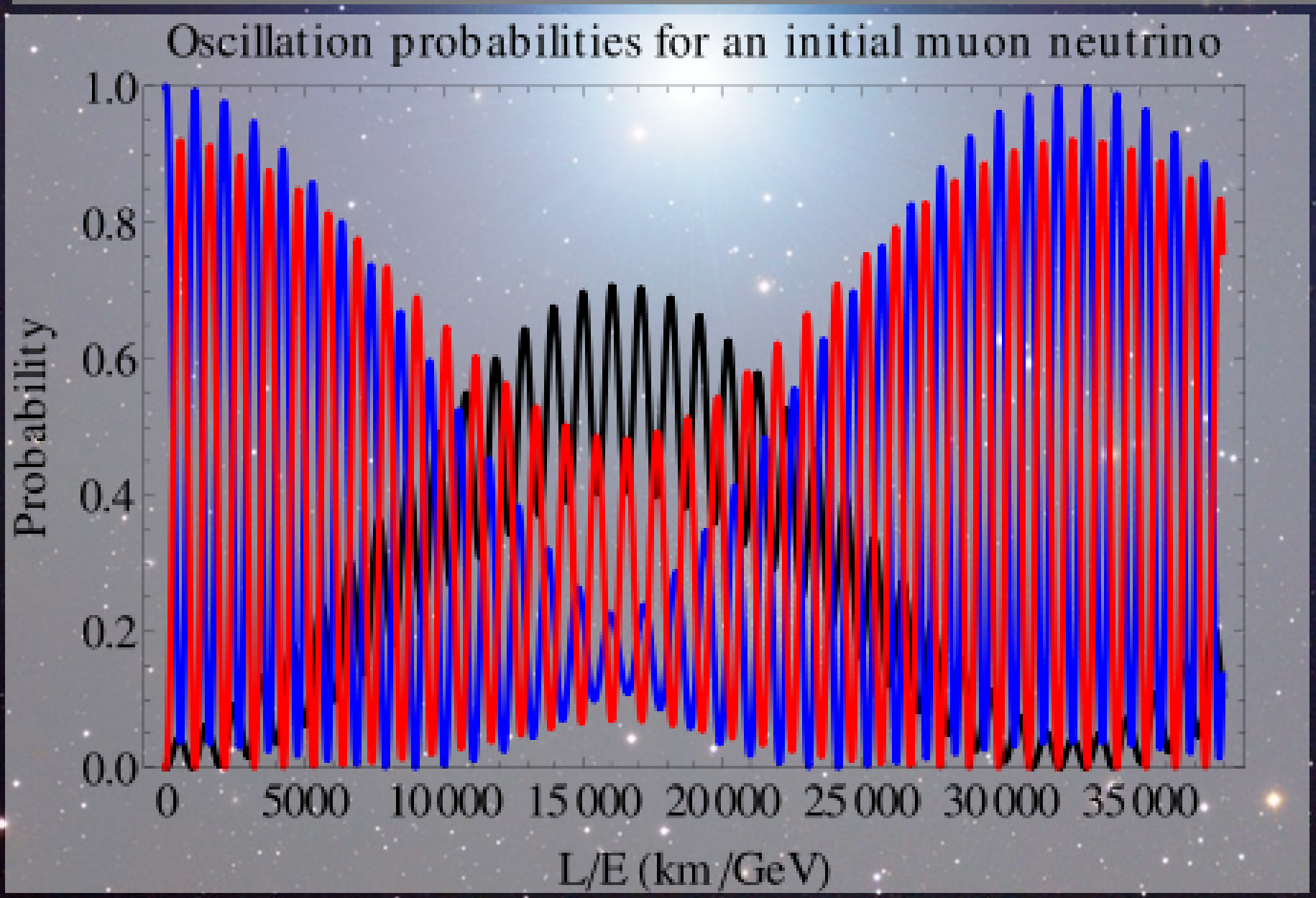
OSCILLATION: Two flavor



Three flavor, Vacuum



http://en.wikipedia.org/wiki/Neutrino_oscillation#mediaviewer/File:Oscillations_muon_long.svg

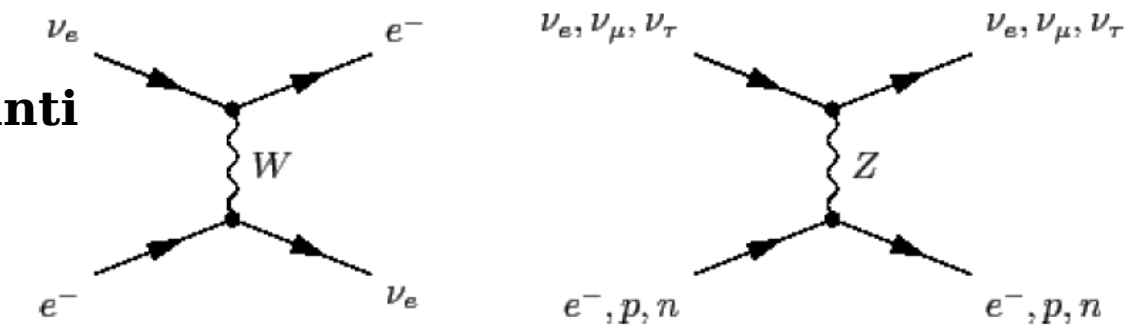


Mixing inside matter: the MSW

mikheyev-smirnov-wolfenstein (MSW) mechanism of amplification of neutrino oscillations in matter

or perturbation

Scattering coerente in avanti



$$i\frac{d}{dx}\Psi_\alpha = \mathcal{H}\Psi_\alpha \ , \quad \text{con} \quad \mathcal{H} = \frac{1}{2E}\left(UMU^\dagger + A\right)$$

Meccanismo MSW

$$A = \begin{pmatrix} A_{\rm CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix}, \quad \Psi_\alpha = \begin{pmatrix} \psi_{\alpha e} \\ \psi_{\alpha\mu} \\ \psi_{\alpha\tau} \end{pmatrix}$$

$$A_{\rm CC} = 2\sqrt{2}E\ G_F\ N_e = 0.76 \cdot 10^{-4} eV^2 \frac{E}{GeV}\frac{\rho}{g\ cm^{-3}}$$

Oscillations across Earth

- ***3 flavours, inside the matter keeping care of:***
  - ***1) ideal constant average density***
  - ***2) precise mass density profile***
- ***$P_{\nu_\alpha - \nu_\beta}$ mixing probability as a function of***
- ***distance and energy,***

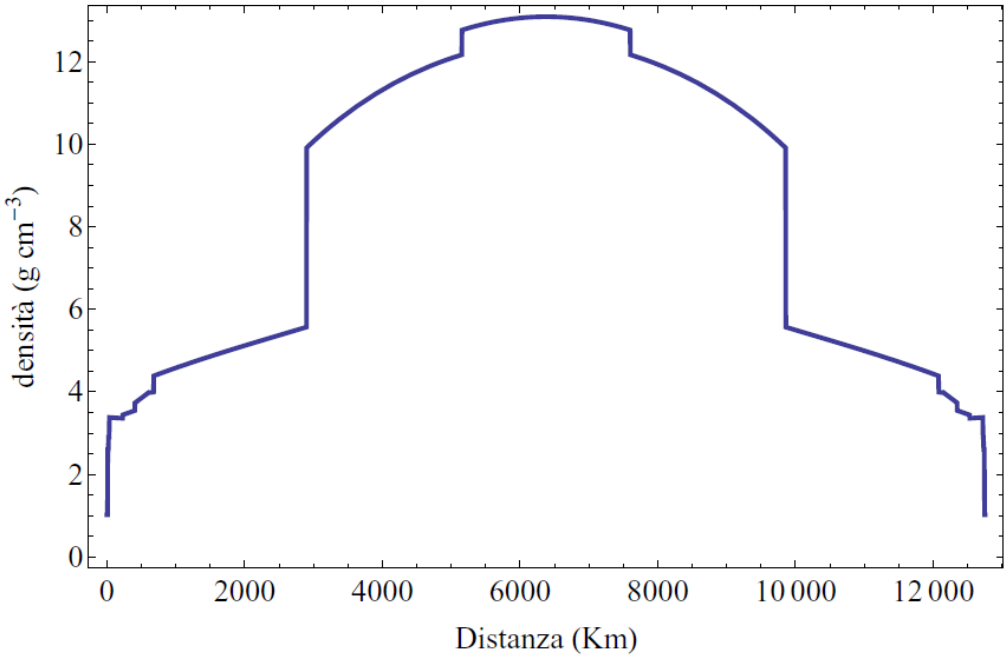
-common popular analitical solutions :

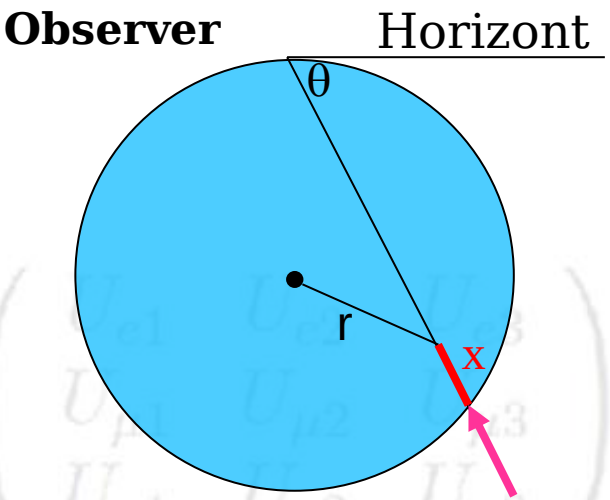
- ***only for average density of Earth,***
- ***-our solution also semi-analitical and exact numerical keeping***
- ***both exact density profile and***
- ***perturbation MSW step by step along the Earth***

Matter Profile along the Earth
(at each angle a different one)

Earth density function:

$\rho(r)$

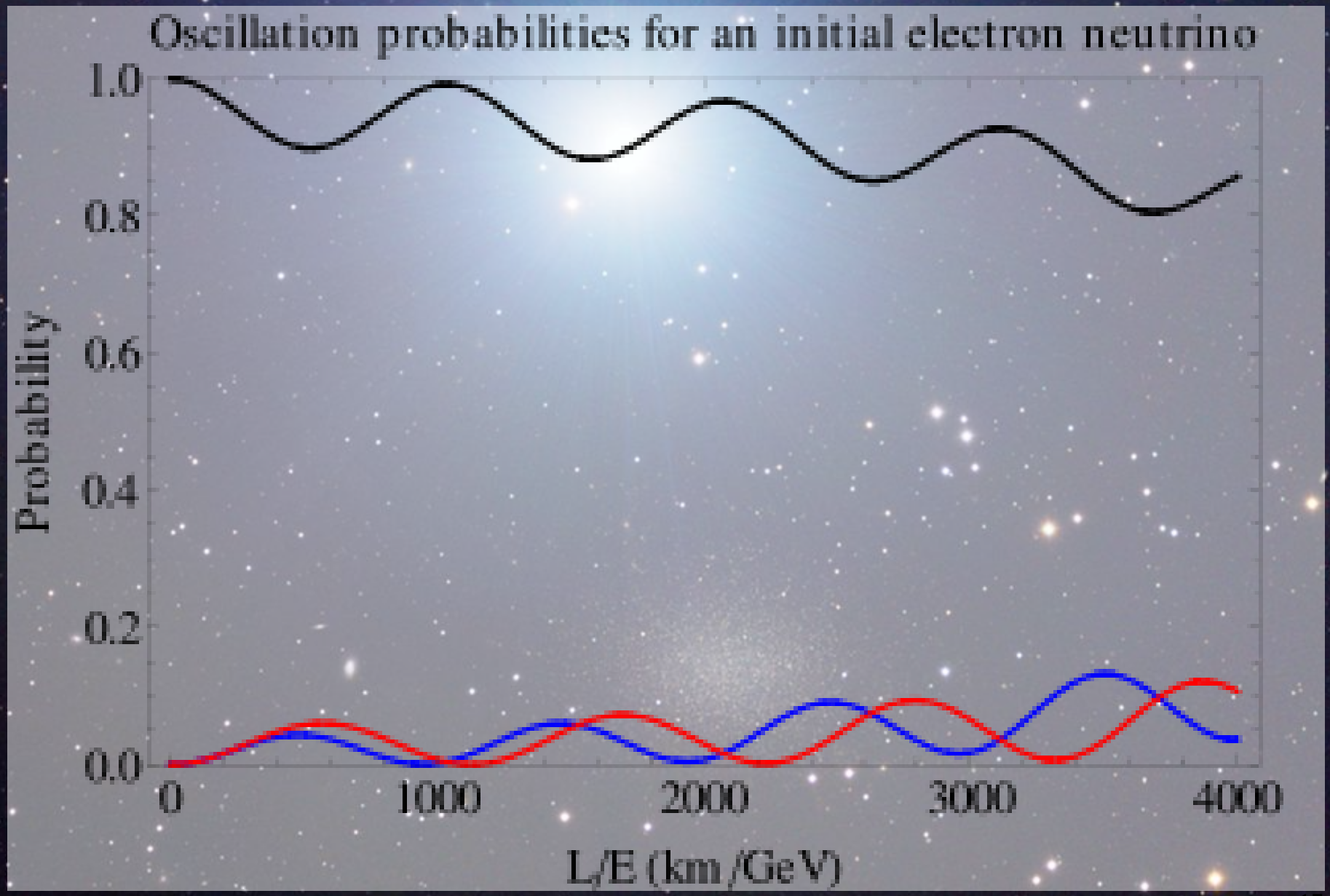




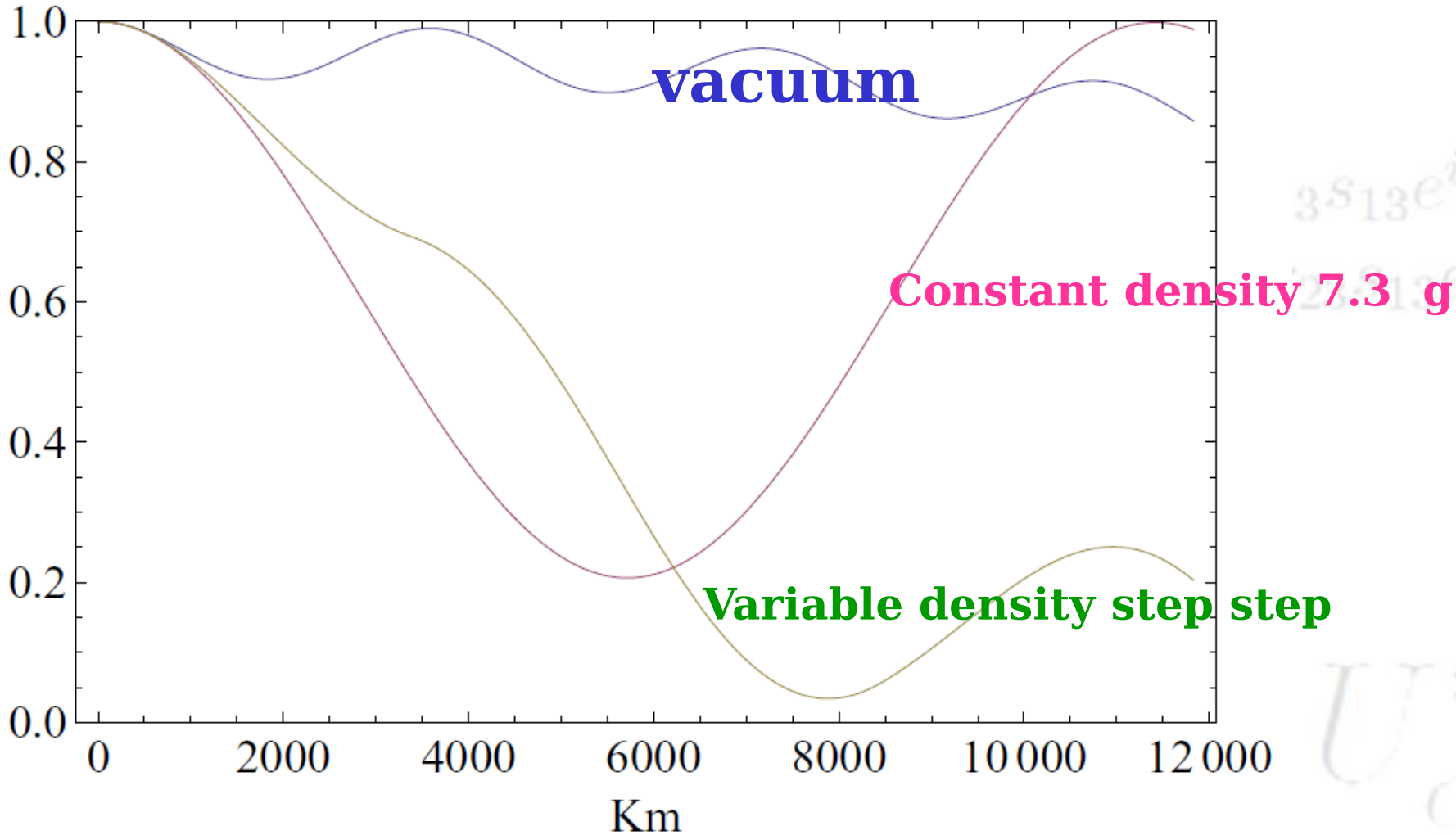
$$r(\theta, x) = \sqrt{(R_\oplus \cdot \cos\theta)^2 + (R_\oplus \cdot \sin\theta - x)^2}$$

$\rho(\, r(\theta,x)\,)$

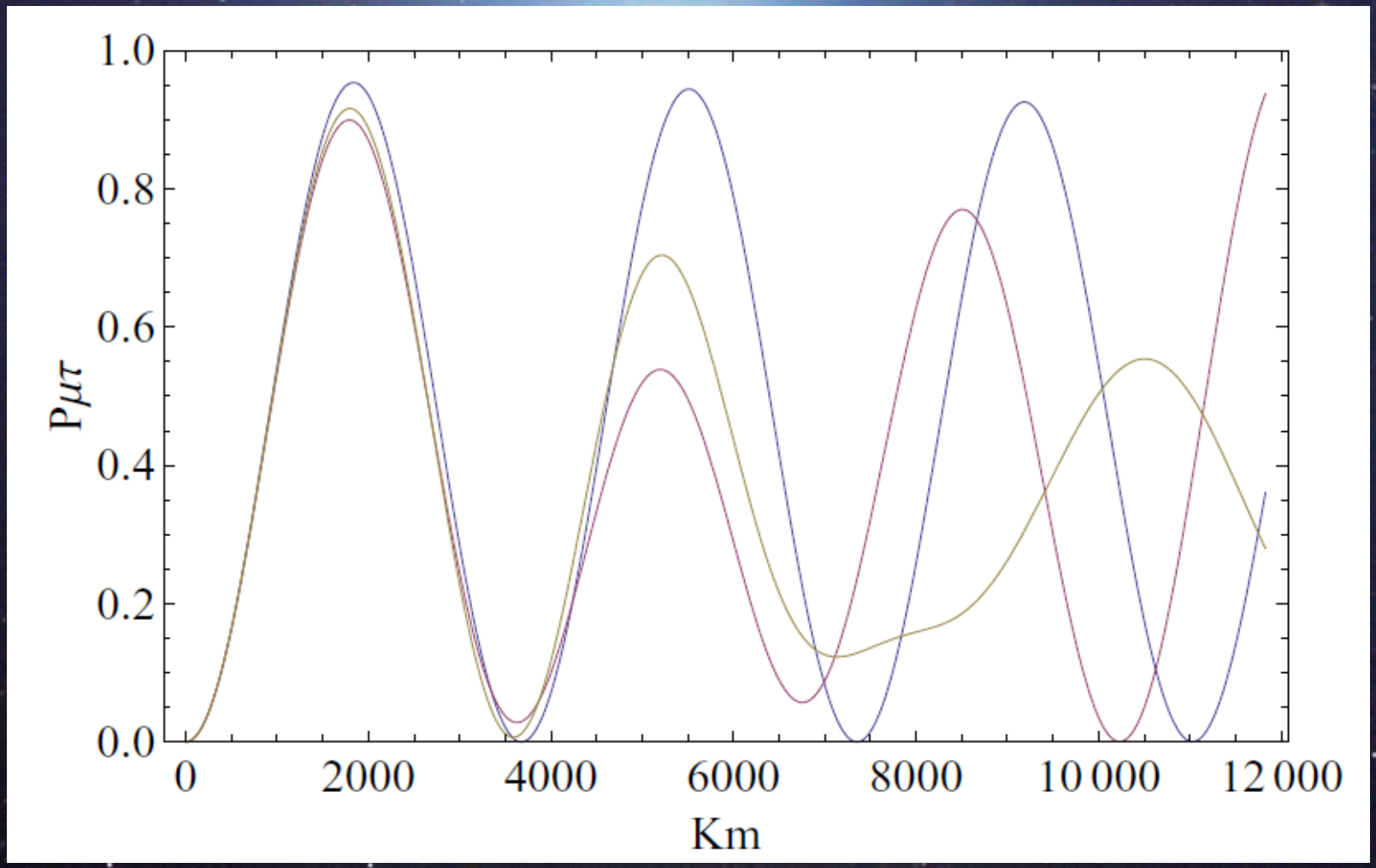
VUOTO


Oscillation probabilities for an initial electron neutrino
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1000
2000
3000
4000
L/E (km/GeV)

Different estimate: Most tuned $P_{ee}(x)$



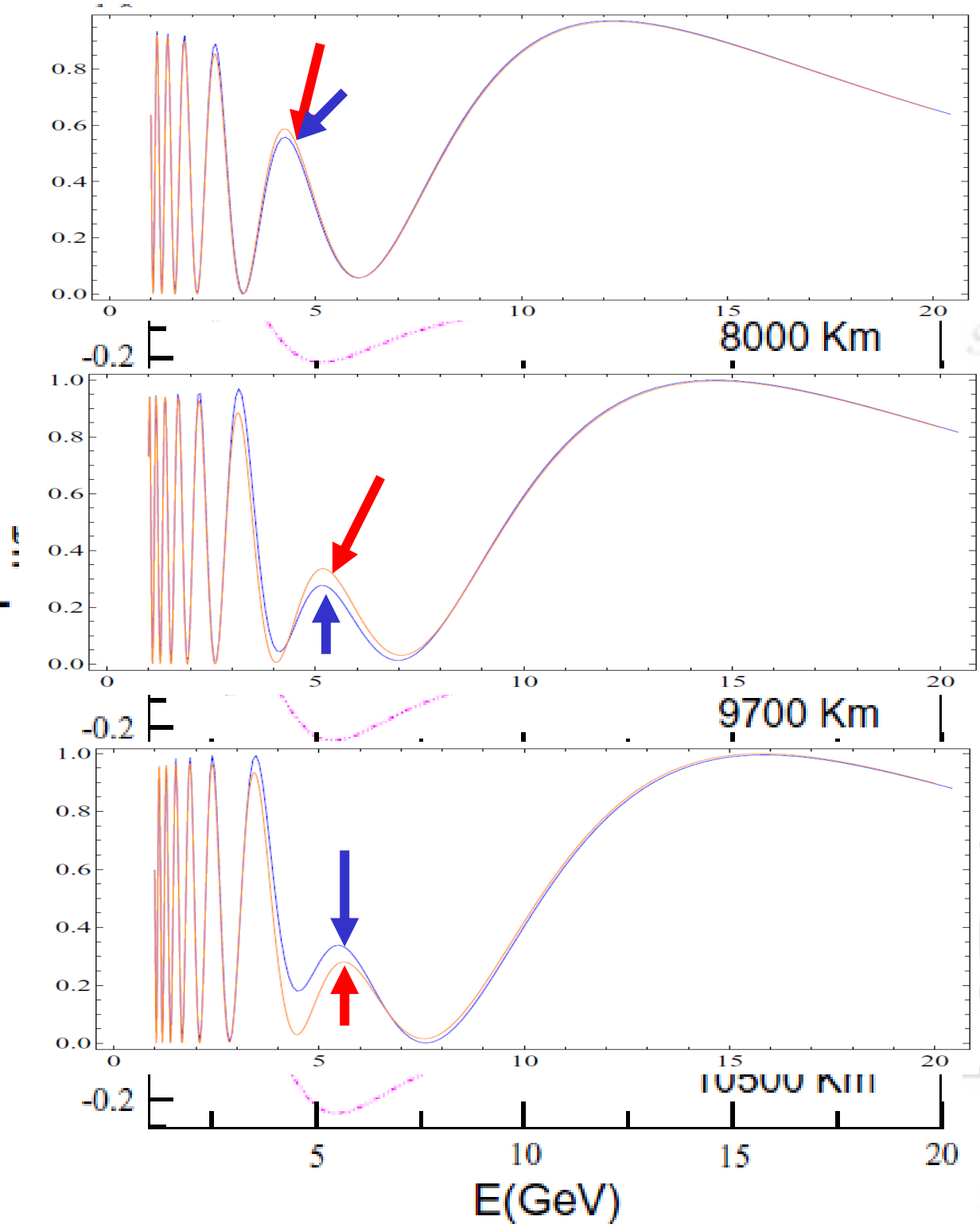
<u>E = 3.4 GeV, θ = 68° below the Horizons: CERN-ICECUBE</u>



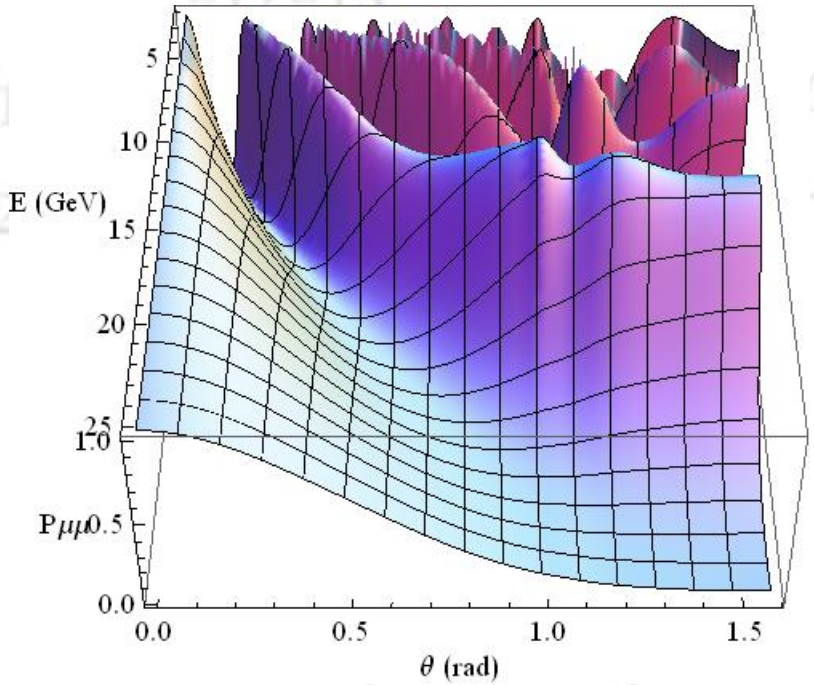
E = 3.4 GeV, θ = 68° sotto l'orizzonte

Quality test: versus-Mu-Tau R. Gandhi et al., Physical Review D73 053001 arXiv:astro-ph/0411252

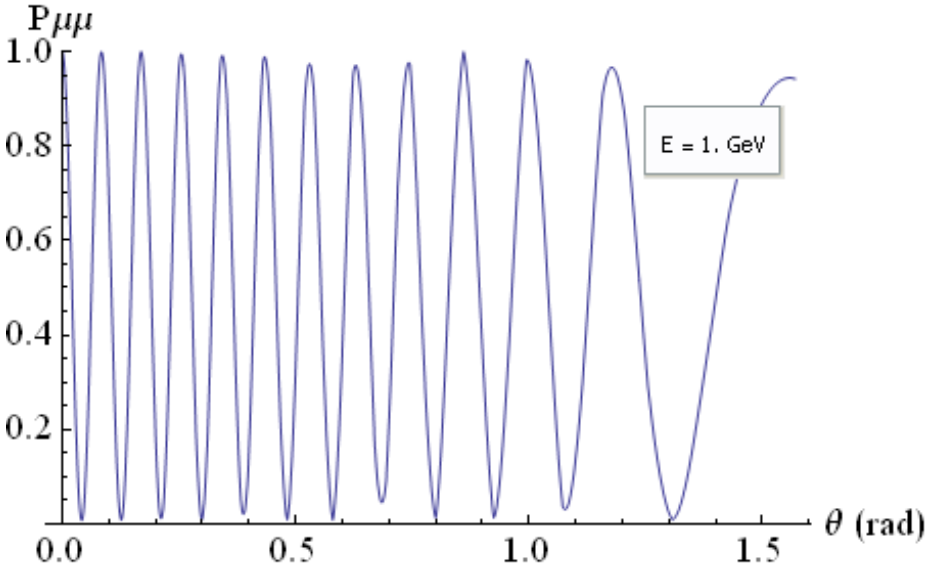
Black-vacuum; Orange-Gandhi; Blue: Our exact Earth profile .

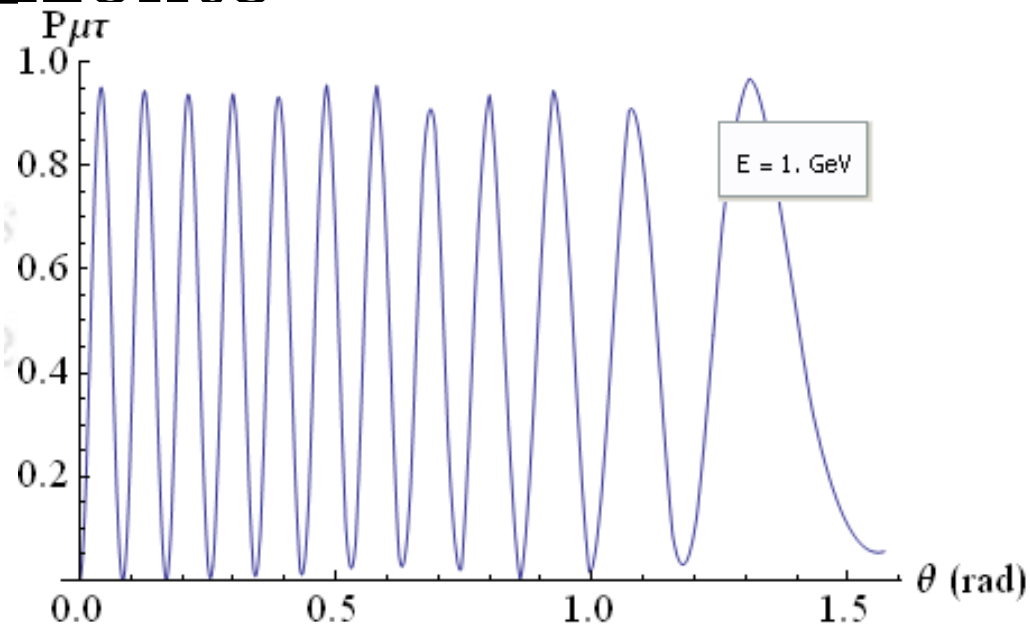


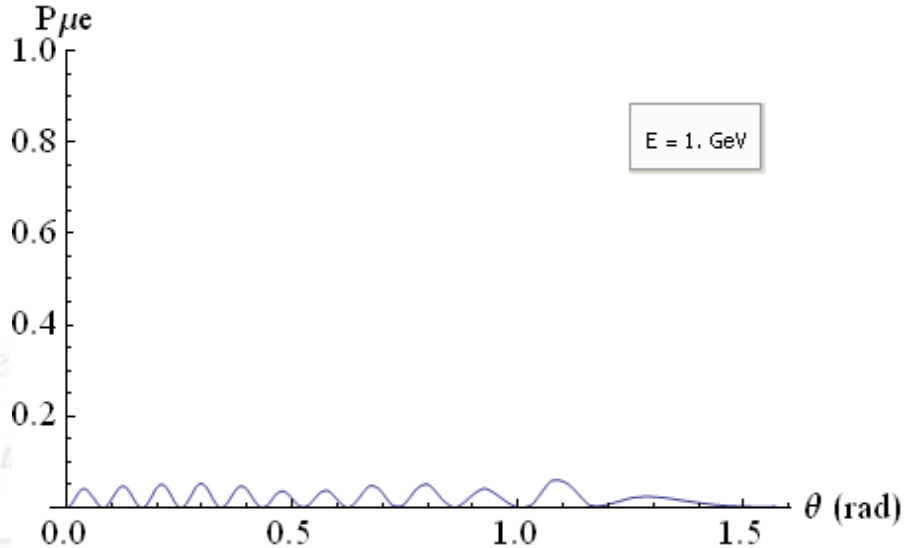
Probabilità di sopravvivenza ν_μ in funzione dell'angolo sotto l'orizzonte e dell'energia



Probabilità di sopravvivenza ν_μ MSW in funzione dell'angolo sotto l'orizzonte



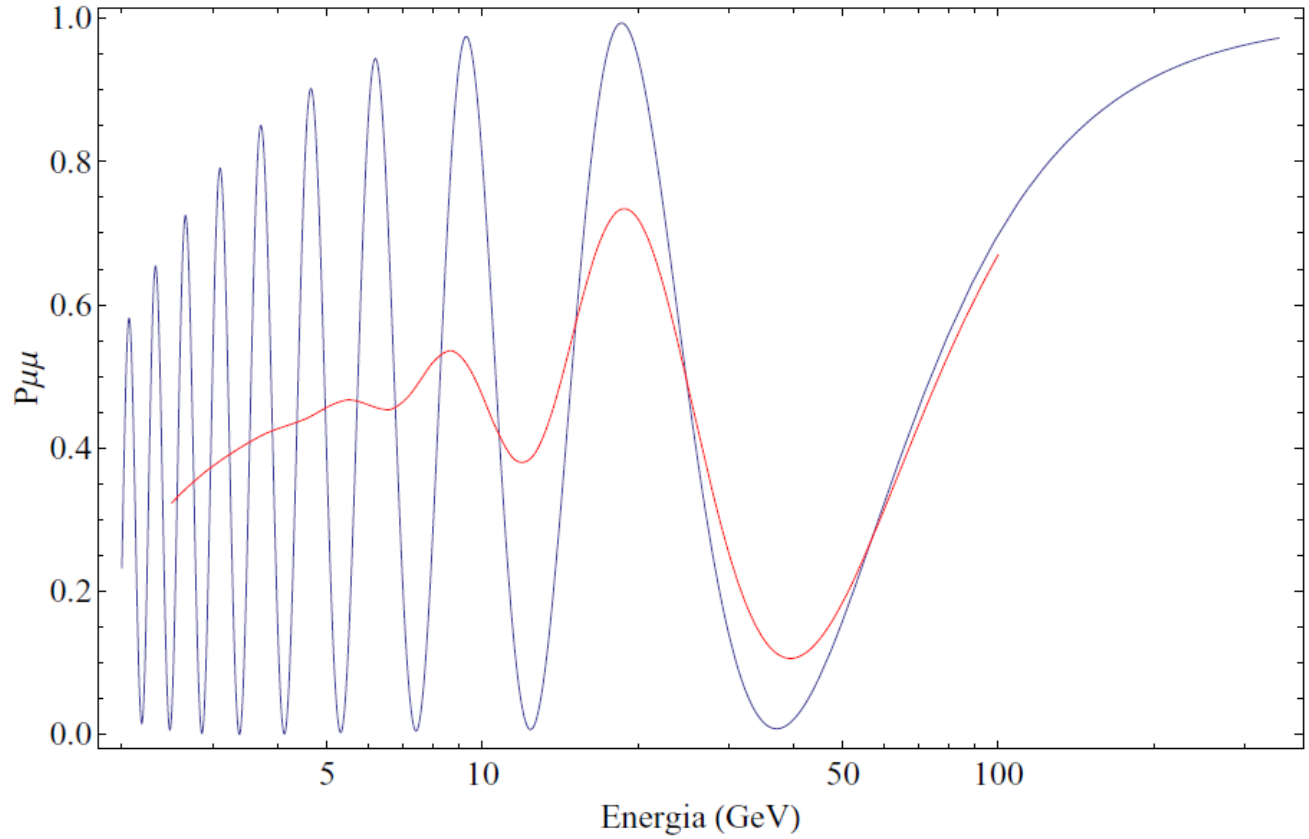




Probabilità mediata sull'incertezza dell'energia

- E' necessario mediare la probabilità di oscillazione sulle distribuzioni di incertezza di energia e lunghezza

$$\frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{(\frac{L}{E} - \langle\frac{L}{E}\rangle)^2}{2\sigma^2}\right)$$

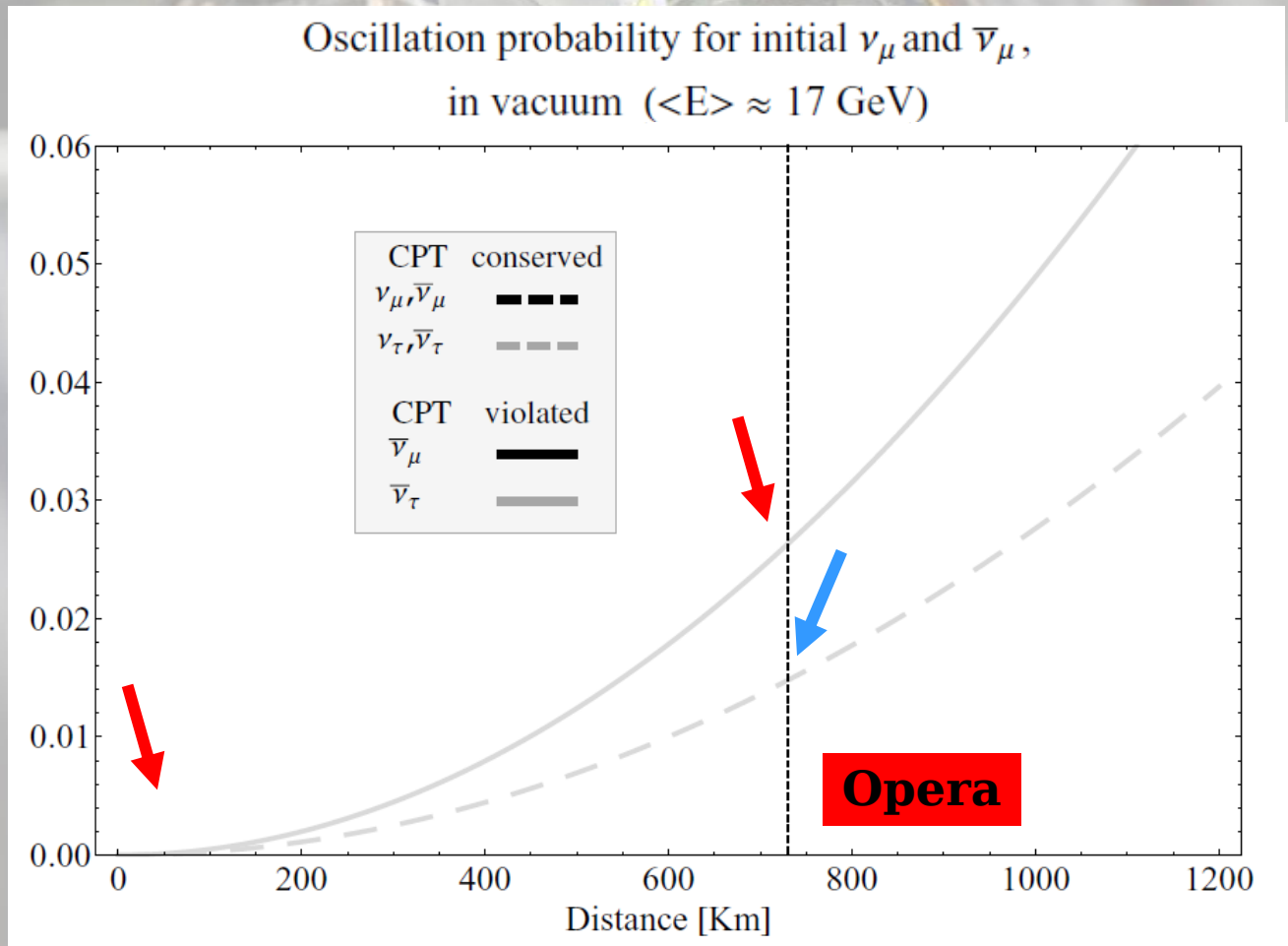


Test proposto per la misura della violazione di CPT

Una prima osservazione semplice, con i parametri che violano CPT

Una seconda osservazione per ottimizzare la conversione

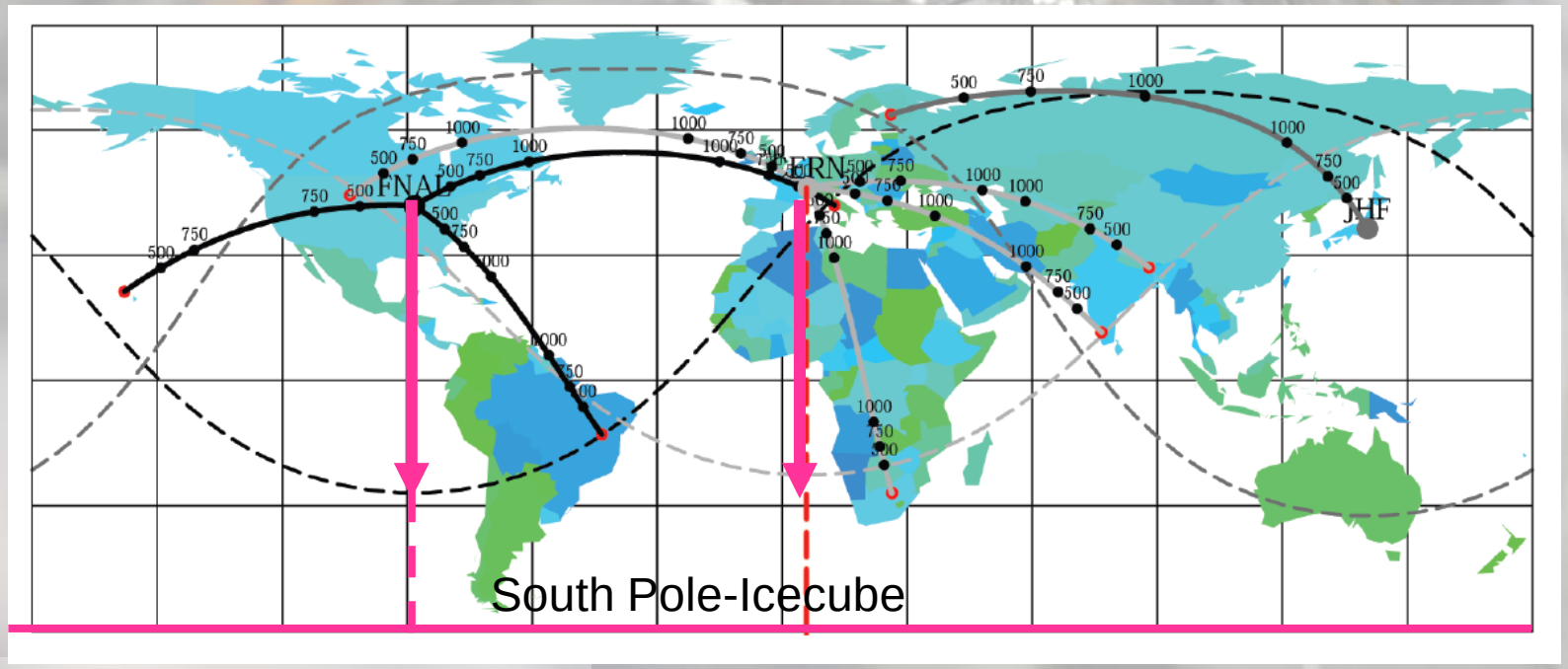
$\nu_\mu \rightarrow \nu_\tau$



Test per la violazione di CPT: Beam attraverso la Terra

La nostra proposta: un esperimento long-baseline per verificare più facilmente i parametri di oscillazione

Gandhi 2006



2010

Abbiamo analizzato 4 baselines:

CERN – SuperKamiokande

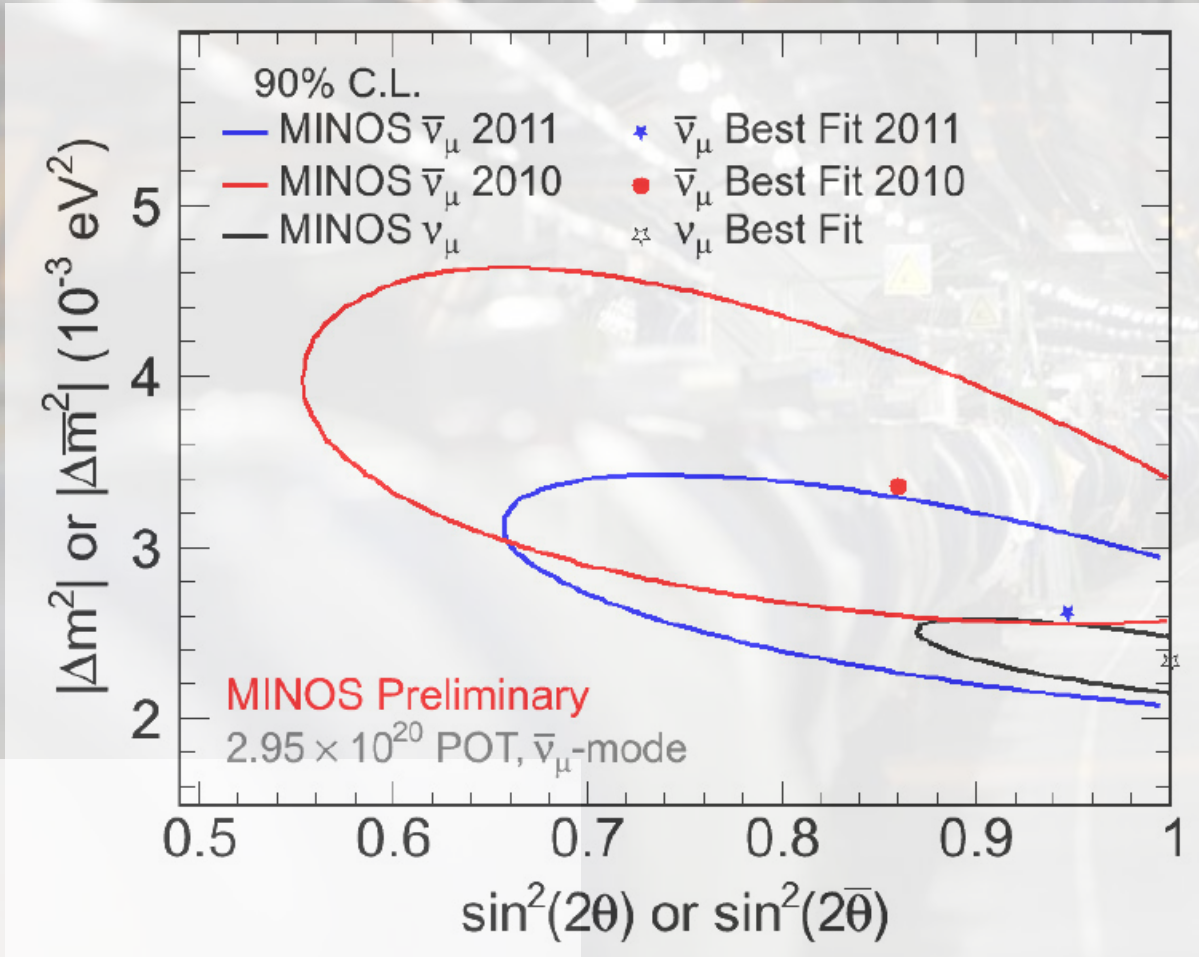
FNAL – SuperKamiokande

FNAL – IceCube

CERN - IceCube

Tang & Winter (2011) riprendono l'idea della baseline verso il polo sud

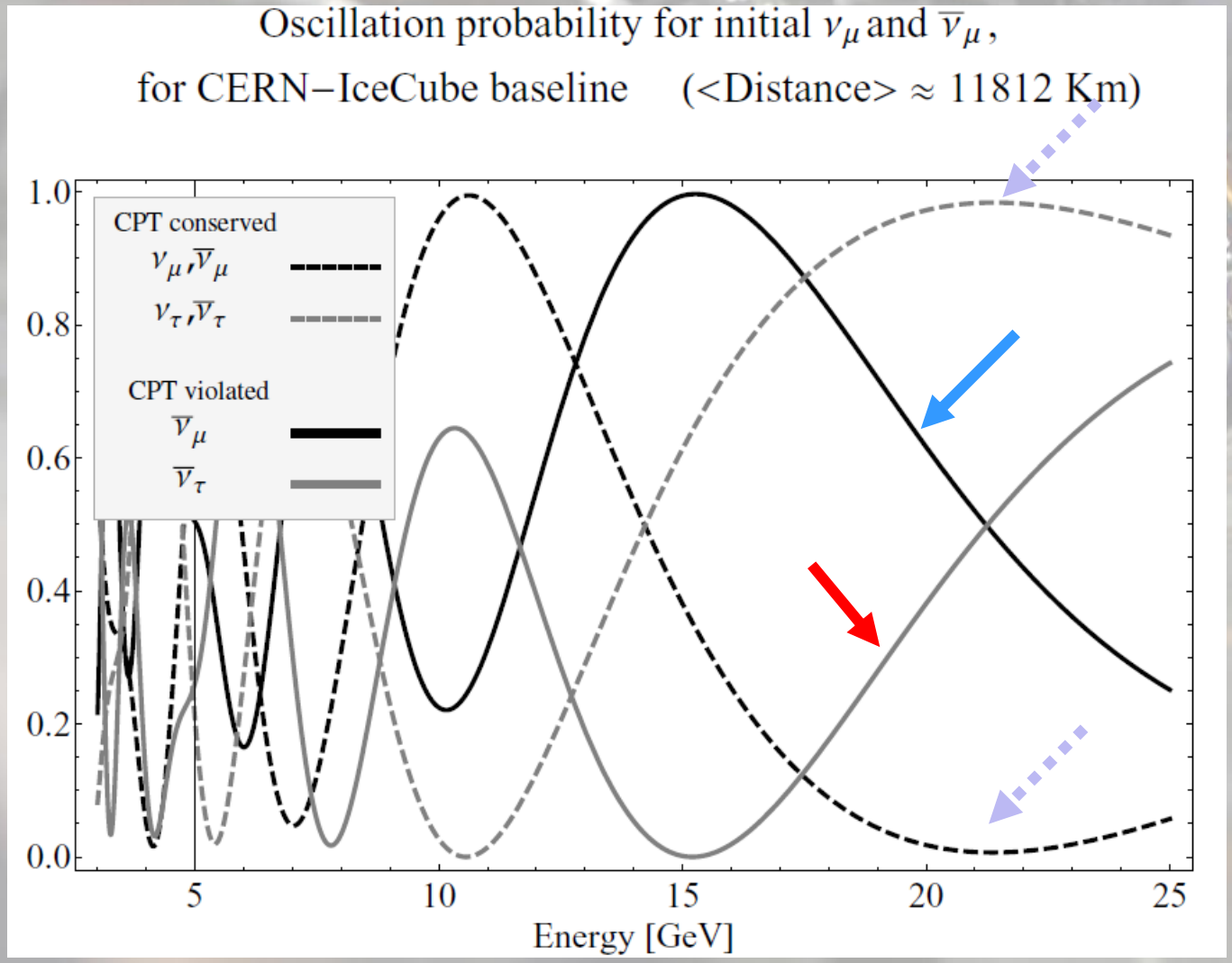
Possibile violazione di CPT in MINOS e influenza sul mixing



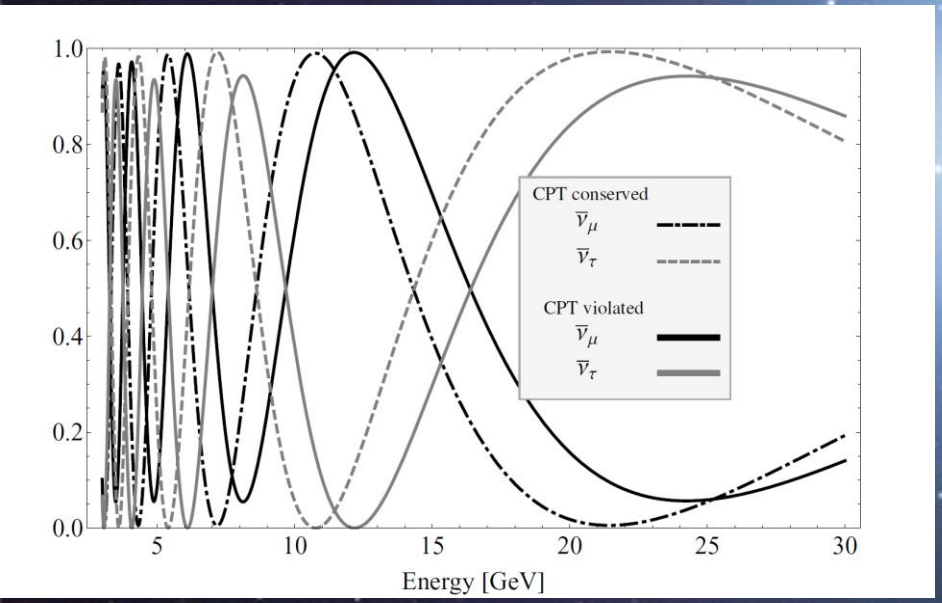
Minos Collaboration, Phys.Rev.Lett.107: 021801, (2011) Arxiv:1104.0344

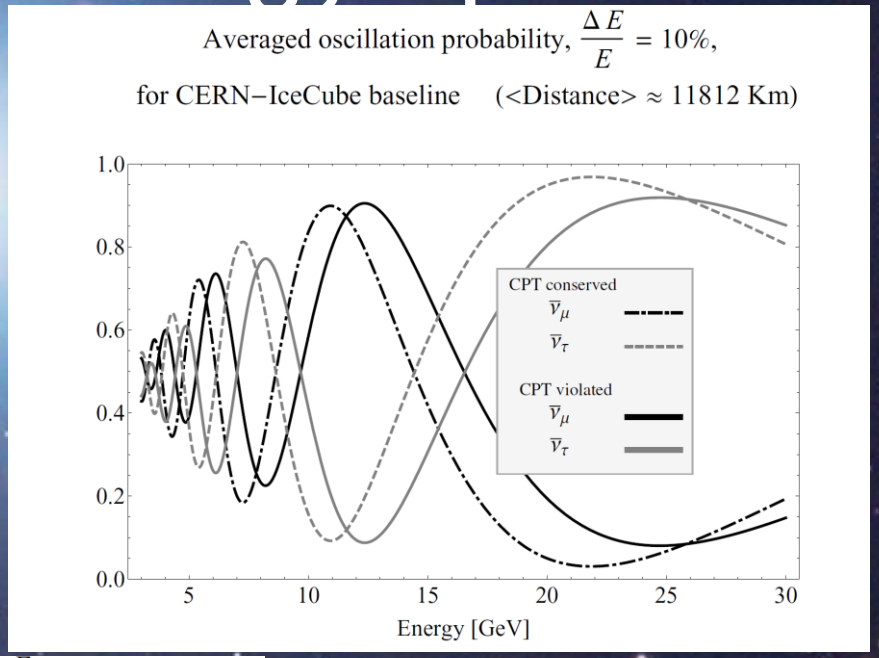
Minos Collaboration, http://www-numi.fnal.gov/PublicInfo/forscientists.html

Energie del fascio di ν



Without and with energy spread





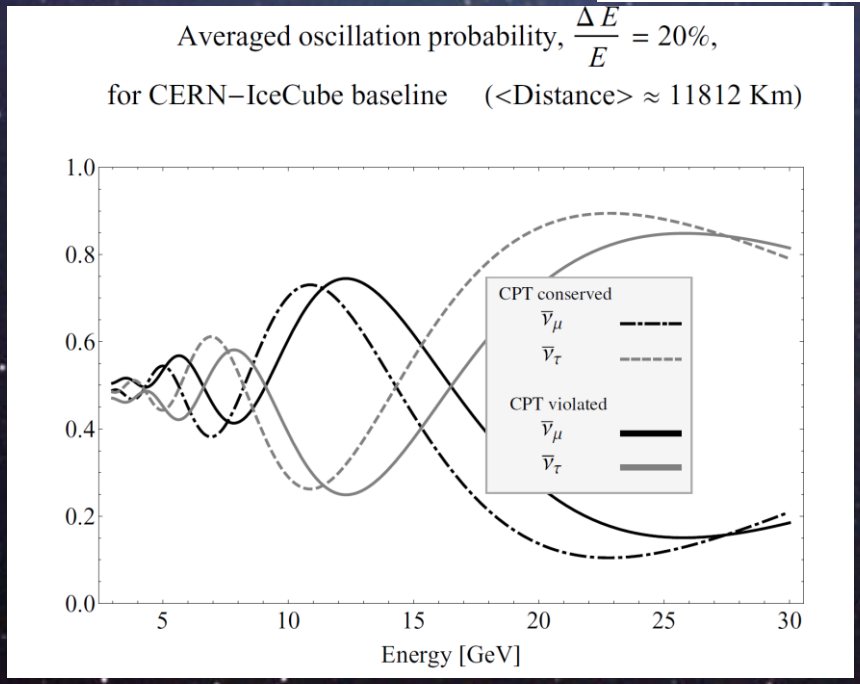


TABLE 7

ν_μ AND $\bar{\nu}_\mu$ EVENT RATES CONSIDERING REDUCED SIZE EXPERIMENT, AND $\frac{\Delta E}{E} = 20\%$ ENERGY SPREAD. THE FIRST ROW SHOWS EVENT NUMBERS TO 10%, BECAUSE OF SHORTER DECAY TUNNEL (50% RESPECT OPERA ONE) AND REDUCED π FLUX INTENS NARROW NEUTRINO SPECTRA (20% RESPECT OPERA FLUX) DUE TO SUGGESTED SETUP FOR PION BENDING MAGNETIC FIELD SECOND ROW SHOWS A VERY ECONOMIC SCENARIO, WHERE EVENT RATE IS REDUCED TO 1%, HAVING CONSIDERED 20% DECAY AND 5% π FLUX INTENSITY. IN THE LAST ROW, N_{μ_τ} AND $N_{\bar{\mu}_\tau}$ ARE THE μ, $\bar{\mu}$ BY τ AND $\bar{\tau}$ DECAY.

Baseline	N_{ν_μ} after osc. year^{-1} $\exists$ CPT	$N_{\bar{\nu}_\mu}$ after osc. year^{-1} $\exists$ CPT	$N_{\bar{\nu}_\mu}$ after osc. year^{-1} $\nexists$ CPT	Statistical Significance σ
CERN–SK	19	**9**	**18 ± 4**	2
Fermilab–SK	20	**10**	**18 ± 4**	2
CERN–IceCube	1516	**758**	**1517 ± 39**	19
Fermilab–IceCube	1515	**757**	**1544 ± 39**	20
1% Baseline	N_{ν_μ} after osc. year^{-1} $\exists$ CPT	$N_{\bar{\nu}_\mu}$ after osc. year^{-1} $\exists$ CPT	$N_{\bar{\nu}_\mu}$ after osc. year^{-1} $\nexists$ CPT	Statistical Significance σ
CERN–SK	1.9	**0.9**	**1.8 ± 1**	0.6
Fermilab–SK	2	**1**	**1.8 ± 1**	0.6
CERN–IceCube	152	**76**	**152 ± 12**	**6.1**
Fermilab–IceCube	151	**76**	**154 ± 12**	**6.3**
1% Baseline	$\langle E_{\mu_\tau} \rangle$(Gev)	$N^*_{\mu_\tau}$ CPT conserved	$N^*_{\bar{\mu}_\tau}$ CPT conserved	$N^*_{\bar{\mu}_\tau}$ CPT violated
CERN–SK	5.4	0.2	0.1	0.5
Fermilab–SK	5.6	0.2	0.1	0.5
CERN–IceCube	7.3	**67**	**33**	**16.5**
Fermilab–IceCube	7.23	**64**	**32**	**15.3**

* Considering a 17.4% branching ratio.

We remind that the μ_τ energy is nearly $\simeq \frac{1}{3}$ of primary tau.

The name μ_τ (in (Fargion 2000,2004)), has been much later renamed Tautsie-pop in (Cowen 2007).

Eventi di ν_τ e $\bar{\nu}_\tau$ al rivelatore con oscillazione

1% Baseline	$N^{CC}_{\nu_\tau} + N^{NC}_{\nu_i}$ $\exists$ CPT $year^{-1}$	σ for ν_τ	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\exists$ CPT $year^{-1}$	σ for $\bar{\nu}_\tau$	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\nexists$ CPT $year^{-1}$	σ
CERN–SK	2	0.8	1.1	0.5	0.8	0.3
Fermilab–SK	2	0.8	1.2	0.6	0.9	0.3
CERN–IceCube	658	14	**329**	10	**245**	**5.3**
Fermilab–IceCube	665	14	**333**	10	**248**	**5.4**

Potenzialità esperimento: 1% rispetto al tunnel di decadimento e flusso di *Opera*

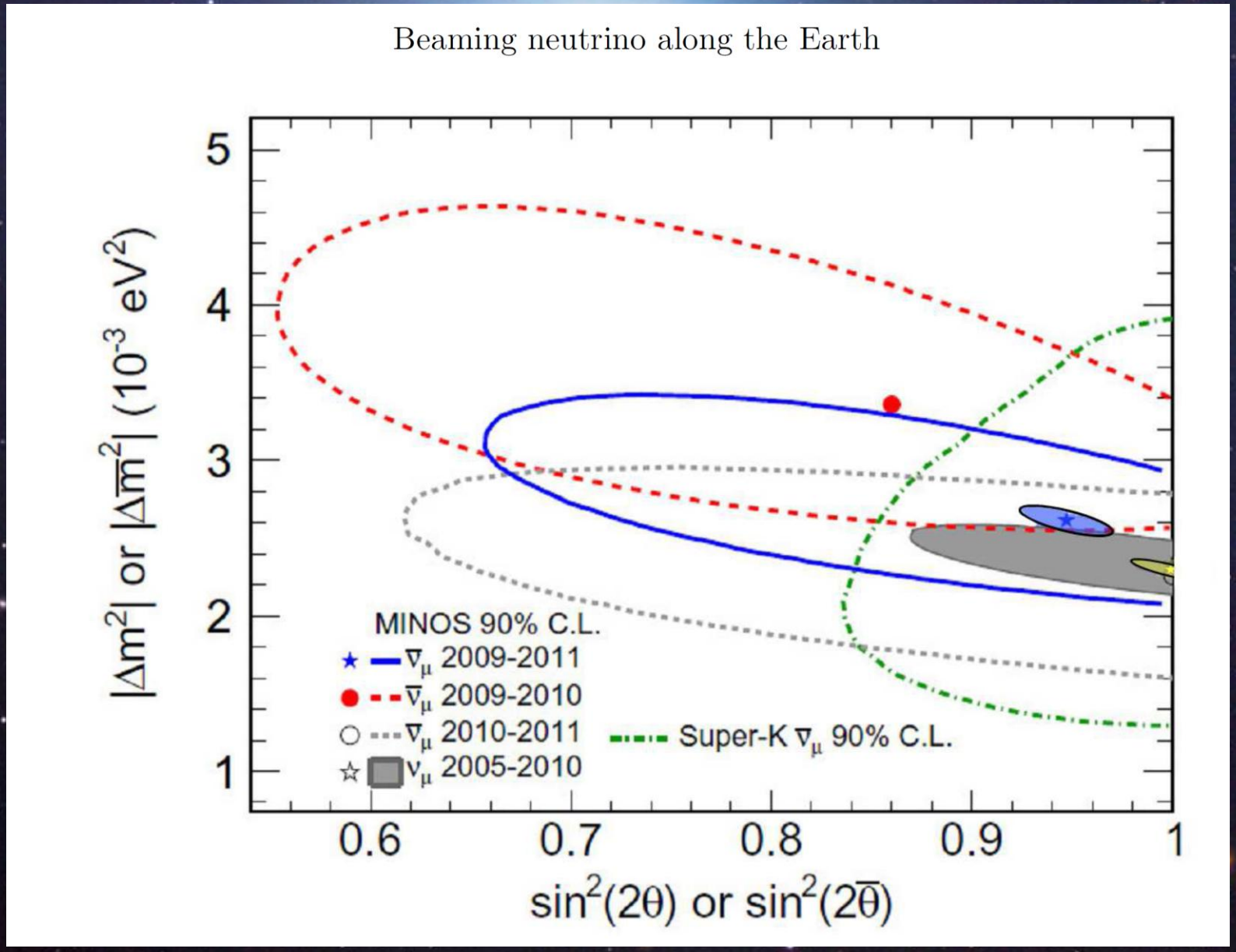
Tau and Anti Tau appearence

TABLE 9

TAU-ANTITAU NEUTRINOS EVENT RATES FOR REDUCED EXPERIMENT, AS BEFORE, TO OVERALL 10% AND TO 1%. STATISTICAL SIGNIFICANCE IS REFERRED BOTH FOR ν_τ, $\bar{\nu}_\tau$ ONLY DETECTION, AND FOR CPT CASES DETECTION.

10% Baseline	$N^{CC}_{\nu_\tau} + N^{NC}_{\nu_i}$ $\exists$ CPT $year^{-1}$	σ for ν_τ	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\exists$ CPT $year^{-1}$	σ for $\bar{\nu}_\tau$	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\nexists$ CPT $year^{-1}$	σ
CERN–SK	22	3	**11**	2	**10**	0.2
Fermilab–SK	23	3	**12**	2	**11**	0.2
CERN–IceCube	6529	45	**3264**	32	**3031**	**4.2**
Fermilab–IceCube	6568	45	**3284**	32	**3053**	**4.2**
1% Baseline	$N^{CC}_{\nu_\tau} + N^{NC}_{\nu_i}$ $\exists$ CPT $year^{-1}$	σ for ν_τ	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\exists$ CPT $year^{-1}$	σ for $\bar{\nu}_\tau$	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\nexists$ CPT $year^{-1}$	σ
CERN–SK	2	0.8	1.1	0.6	1	0
Fermilab–SK	2	0.8	1.2	0.6	1.1	0
CERN–IceCube	653	14	**326**	10	**303**	**1.3**
Fermilab–IceCube	657	14	**328**	10	**305**	**1.3**

What we may achieve in 1 year at 1% of OPERA like experiment



Confronto e calibrazione dei N_{ev} Opera-Minos

Abbiamo effettuato un confronto tra i p.o.t. e gli eventi osservati o osservabili in futuro

$\Phi(\nu) \propto 1 / r^2$ >>>> ~ 1/260

Massa >>>> ~ 4000

$\Omega \propto E_\pi^2$, $\sigma \propto E_\nu$ >>>> ~ 2

Mixing >>>> ~ 60

Efficency on detection ~ 15

per IceCube-DeepCore

Muon Neutrino beam events by
$3.5 \cdot 10^{19}$ proton on target(p.o.t) a year from CERN or FNL

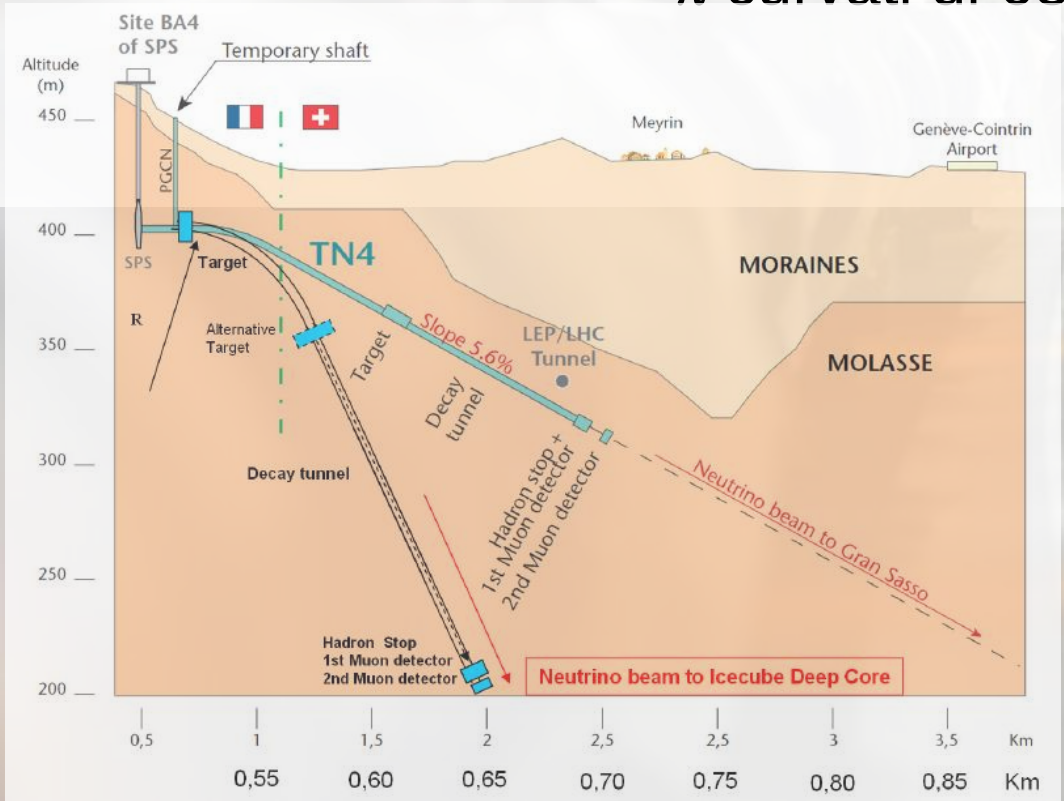
Baseline	*distanza* (km)	E_ν (GeV)	$\left(\frac{L'}{L}\right)^2$	Mass detector *kton*	$N_{ev_{CC+NC}}$ $M_{in}^{-1} year^{-1}$	$N_{ev\mu_{CC}}$ no osc. $kton^{-1} year^{-1}$	$N_{ev\mu_{CC}}$ no osc. $year^{-1}$
CERN–OPERA	$L = 732$	17	1	1.2	3500	2370	2847
CERN–SK	$L' = 8737$	16.2	142.5	22.5	398.5	14.41	324
Fermilab–SK	$L' = 9140$	17	155.9	22.5	420	15.18	341
CERN-IceCube	$L' = 11812$	21.9	260.4	4800	114750	19.45	93343
Fermilab–IceCube	$L' = 11623$	21.7	252.1	4800	115500	19.57	93951

Come lanciare un fascio di ν attraverso la Terra

- Campo magnetico 1 Tesla
- $E^{max}_{\nu} \sim 22$ GeV
- $E_{\pi} \sim 50$ GeV

$$E_\nu^{\max} = \frac{m_\pi^2 - m_\mu^2}{2m_\pi^2}(E_\pi + p_\pi) \approx \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) E_\pi = 0.427 E_\pi$$

π curvati di 68°



Metodo off-axis per aumentare la monocromaticità del fascio

Un nuovo tunnel…

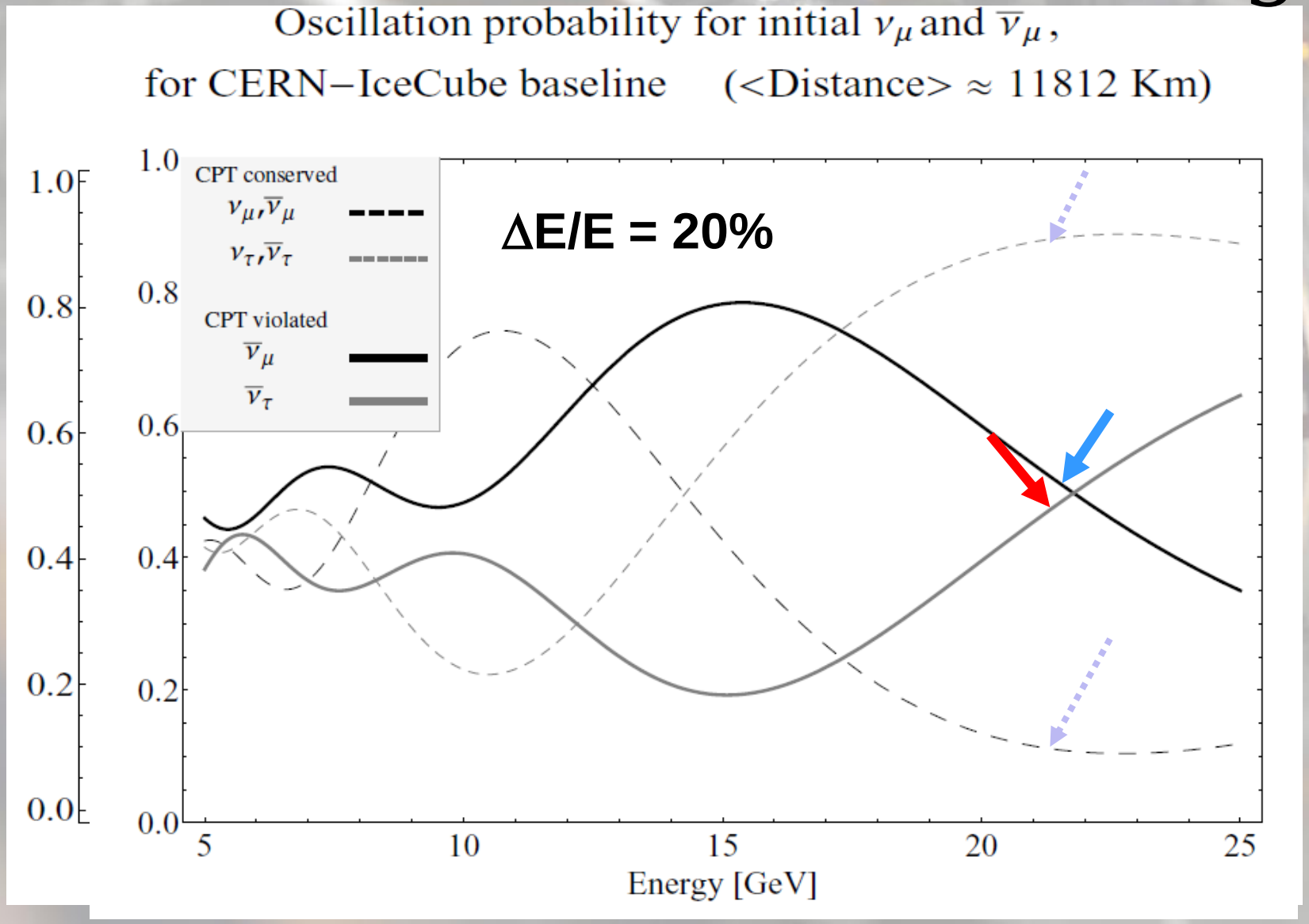
Lanciare un fascio di ν attraverso la Terra

Baseline	E_ν (GeV)	E_π (GeV)	Angolo gradi	R (m)	L_{arc} (m)	Arc Depth (m)
CERN–SK	16.2	37.9	43.19°	126.5	95.3	34.2
Fermilab–SK	17	39.8	45.77°	132.7	106	40
CERN-IceCube	21.9	51.3	67.82°	171	202.3	106.3
Fermilab–IceCube	21.7	50.8	65.67°	169.4	194.2	99.5

Baseline	Tunnel length $L^b_{20\%}$ (m)	Tunnel depth $H^b_{20\%}$ (m)	Tunnel length $L^b_{5\%}$ (m)	Tunnel depth $H^b_{5\%}$ (m)
CERN–SK	202	138	50	34
Fermilab–SK	222	159	55	40
CERN-IceCube	369	342	92	85
Fermilab–IceCube	362	330	90	82

Rispetto a Opera

Probabilità mediata di conversione a causa indeterminazione dell'energia



Eventi di ν_μ e $\bar{\nu}_\mu$ al rivelatore con oscillazione

Baseline	$\langle P_{\mu\mu} \rangle$ $\frac{\Delta E}{E} = 20\%$ $\exists$ CPT	N_{ν_μ} after osc. year^{-1} $\exists$ CPT	$N_{\bar{\nu}_\mu}$ after osc. year^{-1} $\exists$ CPT	$\langle P_{\bar{\mu}\bar{\mu}} \rangle$ $\frac{\Delta E}{E} = 20\%$ $\nexists$ CPT	$N_{\bar{\nu}_\mu}$ after osc. year^{-1} $\nexists$ CPT
CERN–OPERA	0.985	20494	10247	0.972	10125
CERN–SK	0.096	187	93	0.454	441
Fermilab–SK	0.096	197	98	0.457	469
CERN–IceCube	0.106	12415	6207	0.494	28808
Fermilab–IceCube	0.106	12496	6248	0.491	28831

Eventi di ν_μ e anti ν_μ al rivelatore con oscillazione

1% Baseline	N_{ν_μ} after osc. year^{-1} $\exists$ CPT	$N_{\bar{\nu}_\mu}$ after osc. year^{-1} $\exists$ CPT	$N_{\bar{\nu}_\mu}$ after osc. year^{-1} $\nexists$ CPT	Statistical Significance σ
CERN–SK	1.8	**0.9**	**4.4 ± 2**	1.7
Fermilab–SK	1.9	**1**	**4.7 ± 2**	1.7
CERN–IceCube	124	**62**	**288 ± 17**	13.3
Fermilab–IceCube	125	**62**	**288 ± 17**	13.3

Potenzialità esperimento: 1% rispetto al tunnel di decadimento e flusso di *Opera*

$$\sigma = \frac{N_{segnale}}{\sqrt{N_{segnale} + N_{rumore}}} = \frac{N_{\bar{\nu}_\mu}^{\nexists CPT} - N_{\bar{\nu}_\mu}^{\exists CPT}}{\sqrt{N_{\bar{\nu}_\mu}^{\nexists CPT}}}$$

Essendo emessi in finestre del nano secondo, non avrebbero rumore atm.

Eventi di ν_τ e $\bar{\nu}_\tau$ al rivelatore con oscillazione

Baseline	$N^{CC}_{\nu_\tau}$ with osc. $\exists$ CPT $year^{-1}$	$N^{CC}_{\bar{\nu}_\tau}$ with osc. $\exists$ CPT $year^{-1}$	$N^{CC}_{\bar{\nu}_\tau}$ with osc. $\nexists$ CPT $year^{-1}$	$N^{NC}_{\nu_i}$ with osc. noise τ like $year^{-1}$	$N^{NC}_{\bar{\nu}_i}$ with osc. noise τ like $year^{-1}$
CERN–OPERA	16	0.50	0.90		
				RUMORE	
CERN–SK	119	59	31	101	51
Fermilab–SK	125	63	33	107	53
CERN–IceCube	36713	18357	9958	29123	14562
Fermilab–IceCube	37191	18596	10117	29313	14656

Baseline	$N^{CC}_{\nu_\tau} + N^{NC}_{\nu_i}$ $\exists$ CPT $year^{-1}$	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\exists$ CPT $year^{-1}$	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\nexists$ CPT $year^{-1}$
CERN–SK	220	110	66
Fermilab–SK	232	116	70
CERN–IceCube	65836	32918	19541
Fermilab–IceCube	66504	33252	19715

Eventi di ν_τ e $\bar{\nu}_\tau$ al rivelatore con oscillazione

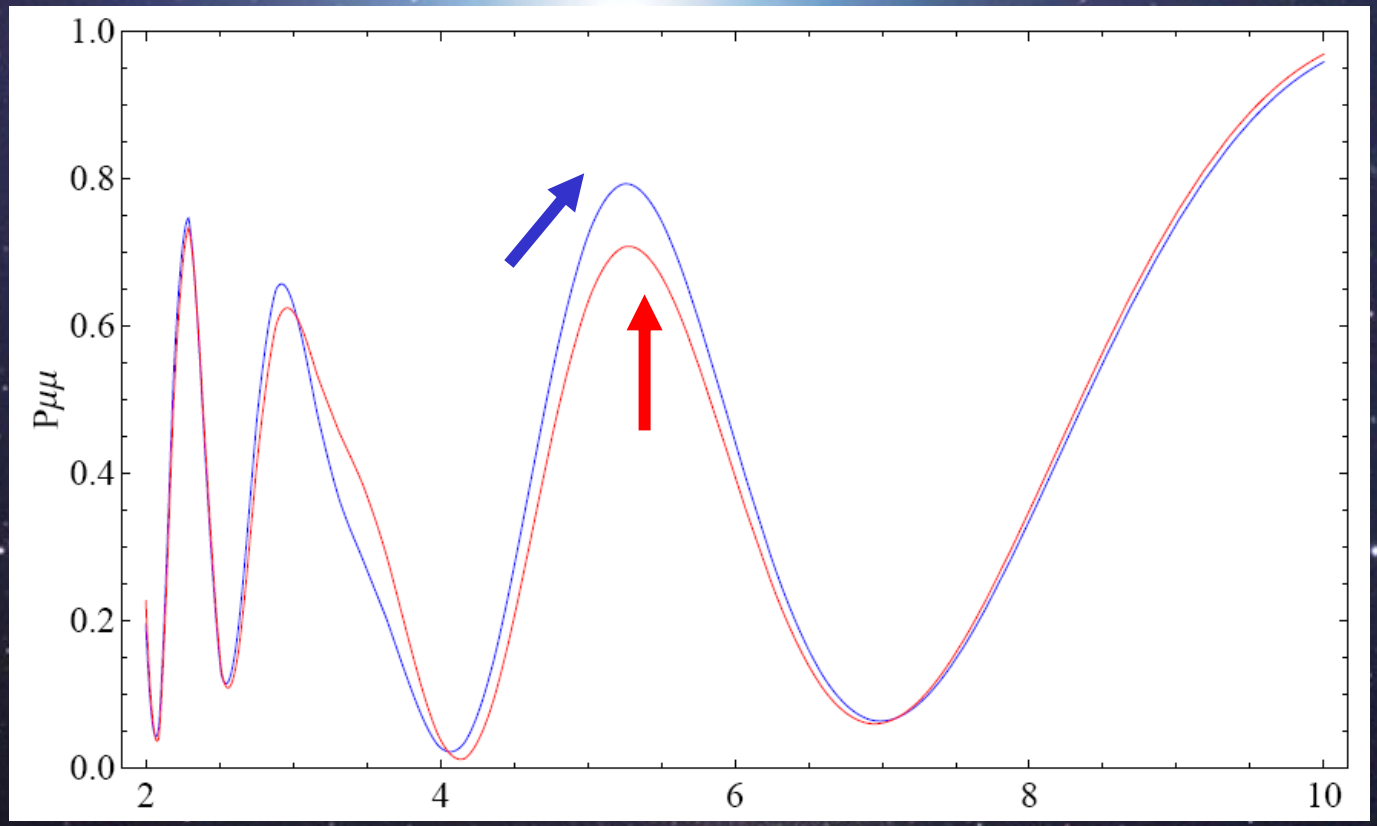
1% Baseline	$N^{CC}_{\nu_\tau} + N^{NC}_{\nu_i}$ $\exists$ CPT $year^{-1}$	σ for ν_τ	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\exists$ CPT $year^{-1}$	σ for $\bar{\nu}_\tau$	$N^{CC}_{\bar{\nu}_\tau} + N^{NC}_{\bar{\nu}_i}$ $\nexists$ CPT $year^{-1}$	σ
CERN–SK	2	0.8	1.1	0.5	0.8	0.3
Fermilab–SK	2	0.8	1.2	0.6	0.9	0.3
CERN–IceCube	658	14	**329**	10	**245**	**5.3**
Fermilab–IceCube	665	14	**333**	10	**248**	**5.4**

Potenzialità esperimento: 1% rispetto al tunnel di decadimento e flusso di *Opera*

Requirements for a New Detector at the South Pole Receiving an Accelerator Neutrino Beam

Authors: Jian Tang, Walter Winter

(Submitted on **26 Oct 2011** (v1), last revised **23 Jan 2012** (this version, v2))



Theta 13

16

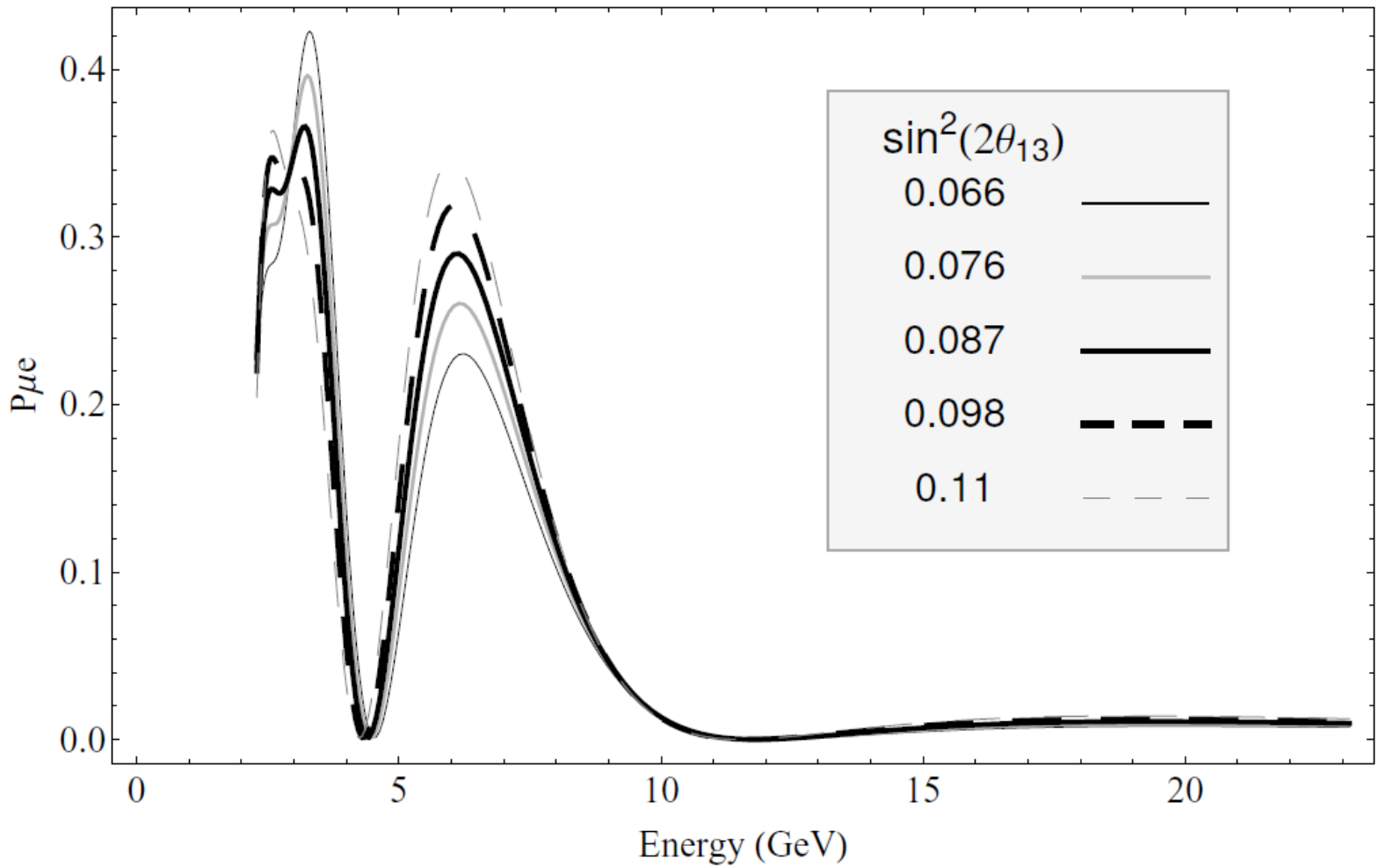


FIG. 12.— The conversion probability of muon neutrino into electron, for different $\sin(2\theta_{13})^2$ values, a probability deviations that may test at low energy the disappearance of muons tracks and the appearance of electron showers (similar to tau ones). The present Deep Core array can hardly be able to reveal such a small energy signals (muon tracks versus electron showers), while future more dense PINGU array may be a better tuned detector.

Oscillation probability NH and IH, for initial ν_μ,

for CERN–IceCube baseline (<Distance> ≈ 11812 Km

Muon to electron

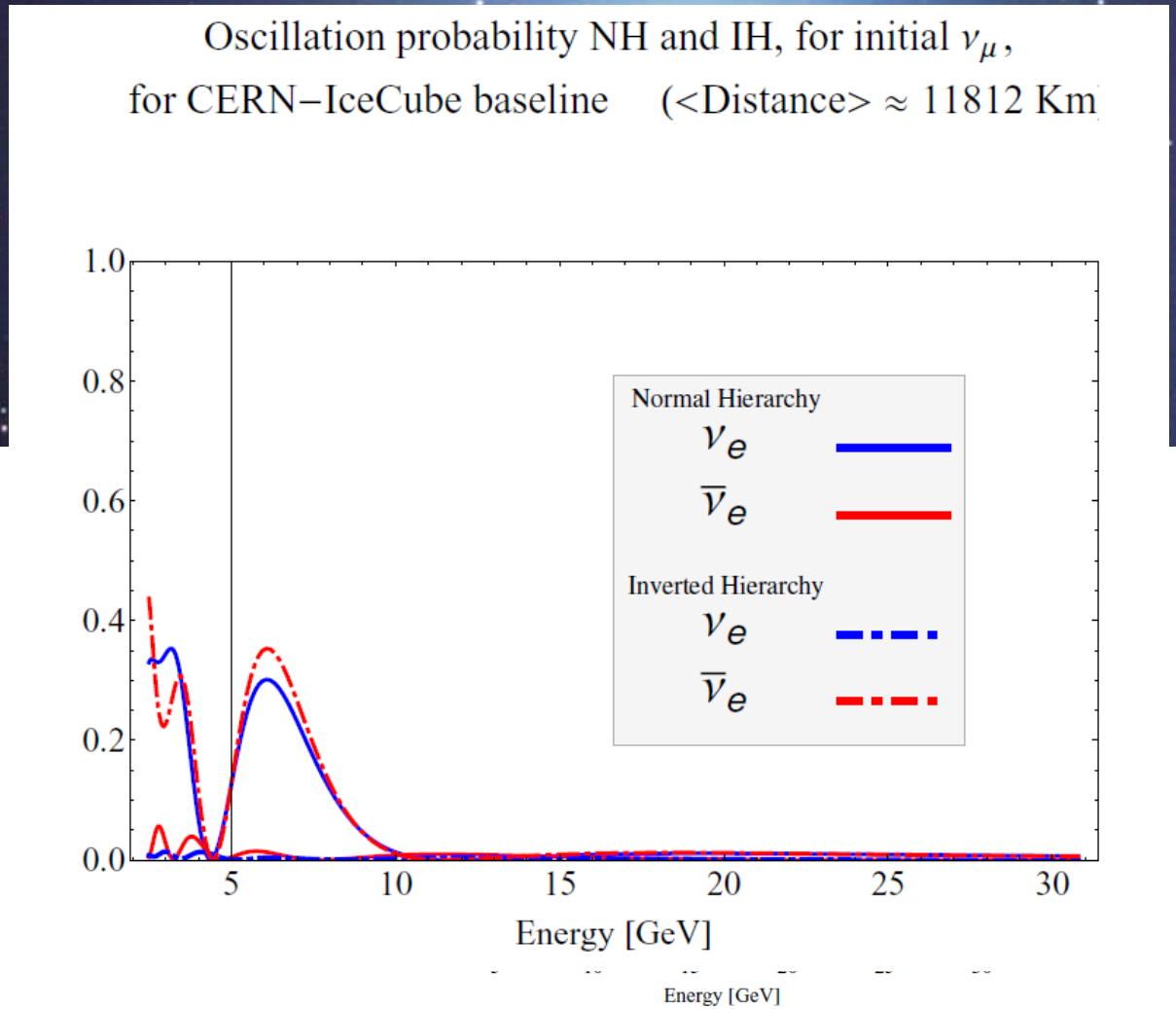


FIG. 13.— The conversion probability of muon neutrino into electron,in normal or inverted neutrino mass hierarchy. As shown in the figure there is a remarkable deviation from normal neutrino mass hierarchy and inverted one mostly in the low energy region ($\simeq$ 6 GeV), where the $\bar{\nu}_e$ appears (in the inverted case) at a 38% probability rate (born by a $\bar{\nu}_\mu$ conversion), while it is nearly absent $\simeq$ 2% probability rate in normal neutrino mass hierarchy.

Hierarchy neutrino mass

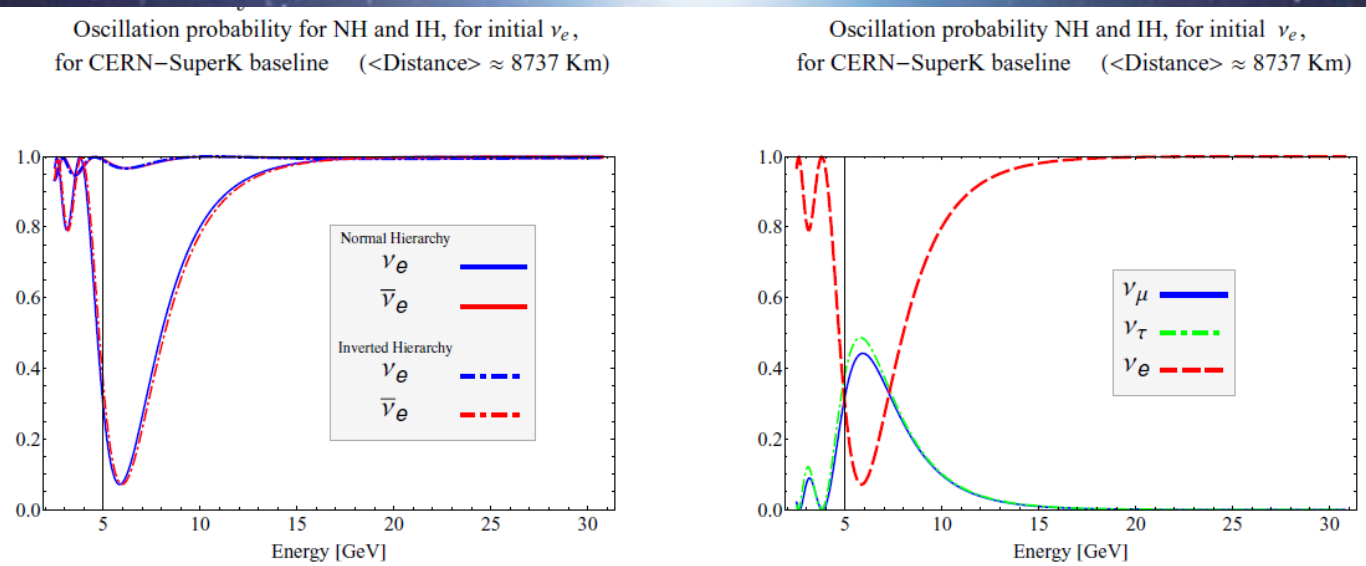
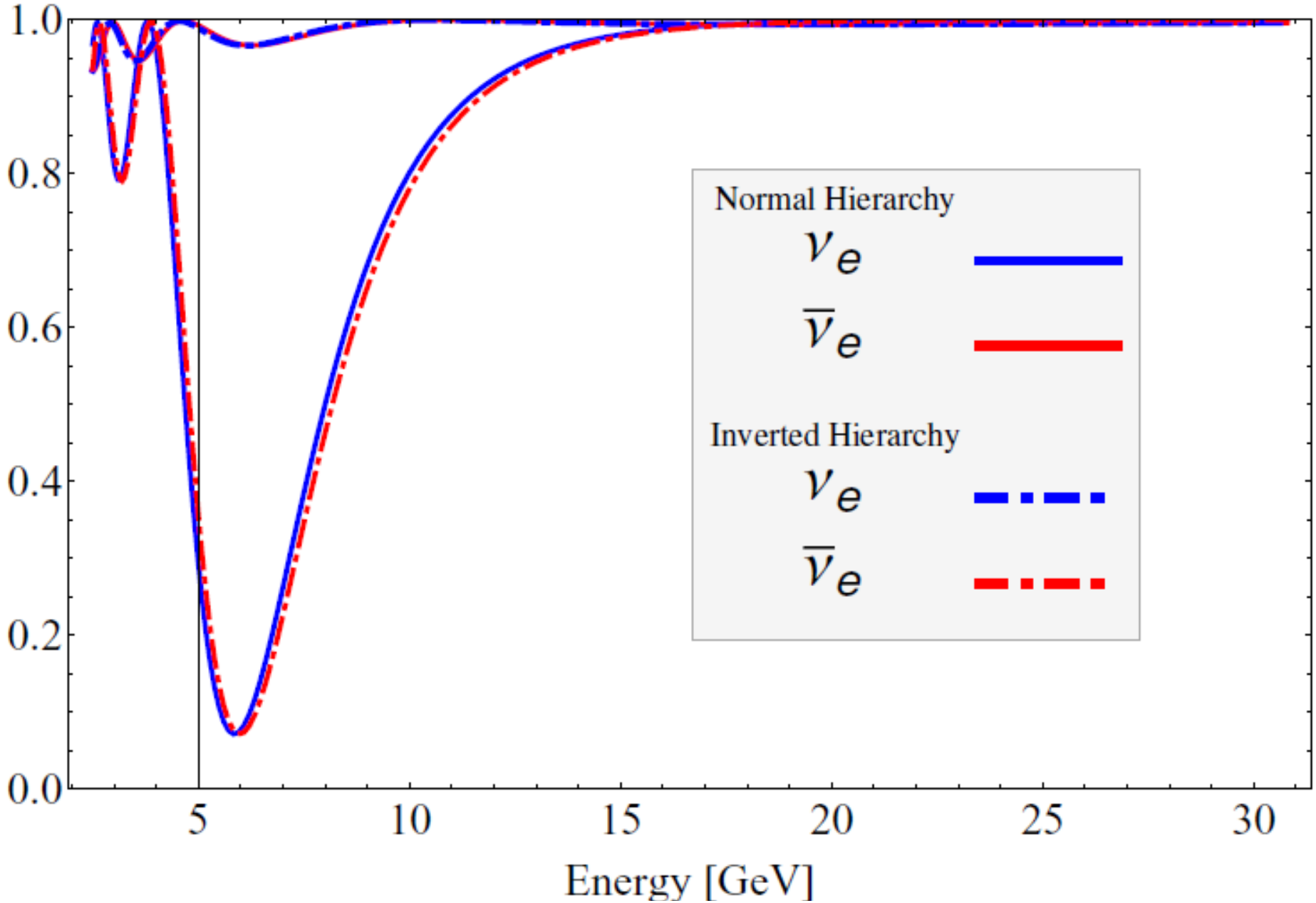
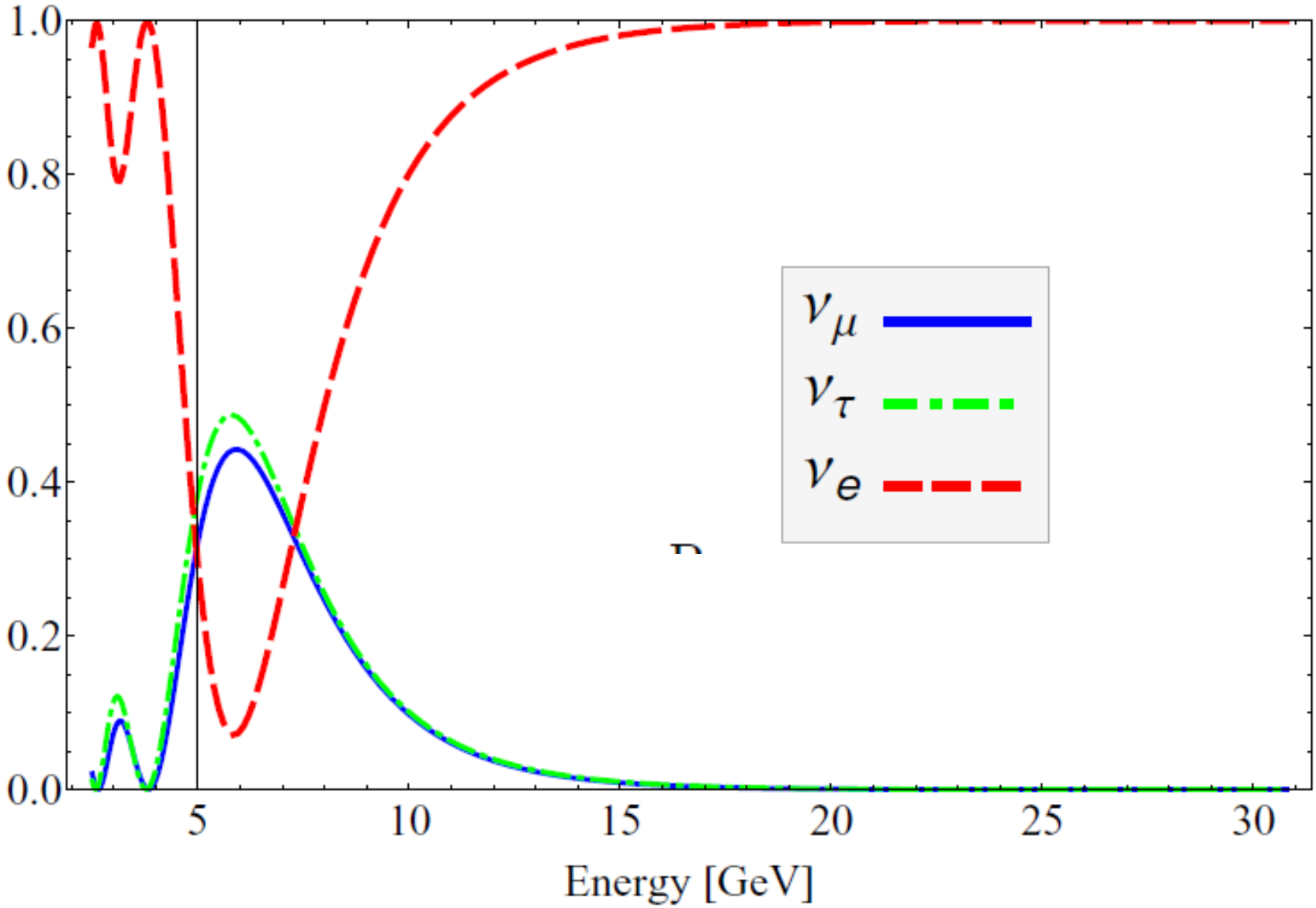


FIG. 14.— Left: the electron neutrino survival probability $P_{\nu_e \to \nu_e}$ and $P_{\bar{\nu}_e \to \bar{\nu}_e}$, in normal or inverted neutrino mass hierarchy. . As shown in the figure there is a remarkable deviation from normal neutrino mass hierarchy and inverted one mostly in the low energy region ($\simeq$ 6 GeV), which turns to be higher in the SuperK baseline respect the IceCube one. A $\nu_e, \bar{\nu}_e$ beam is suitable for best hierarchy model discrimination. Right: the oscillation probability $P_{\nu_e \to \nu_\alpha}$ for the SuperK baseline show that there is non-negligible conversion into muon neutrino, possibly detectable in future ν_e beam.



the electron neutrino survival probability $P_{\nu_e \to \nu_e}$ and $P_{\bar{\nu}_e \to \bar{\nu}_e}$,

in normal or inverted neutrino mass hierarchy.



the oscillation probability $P_{\nu_e \to \nu_\alpha}$ for the SuperK baseline

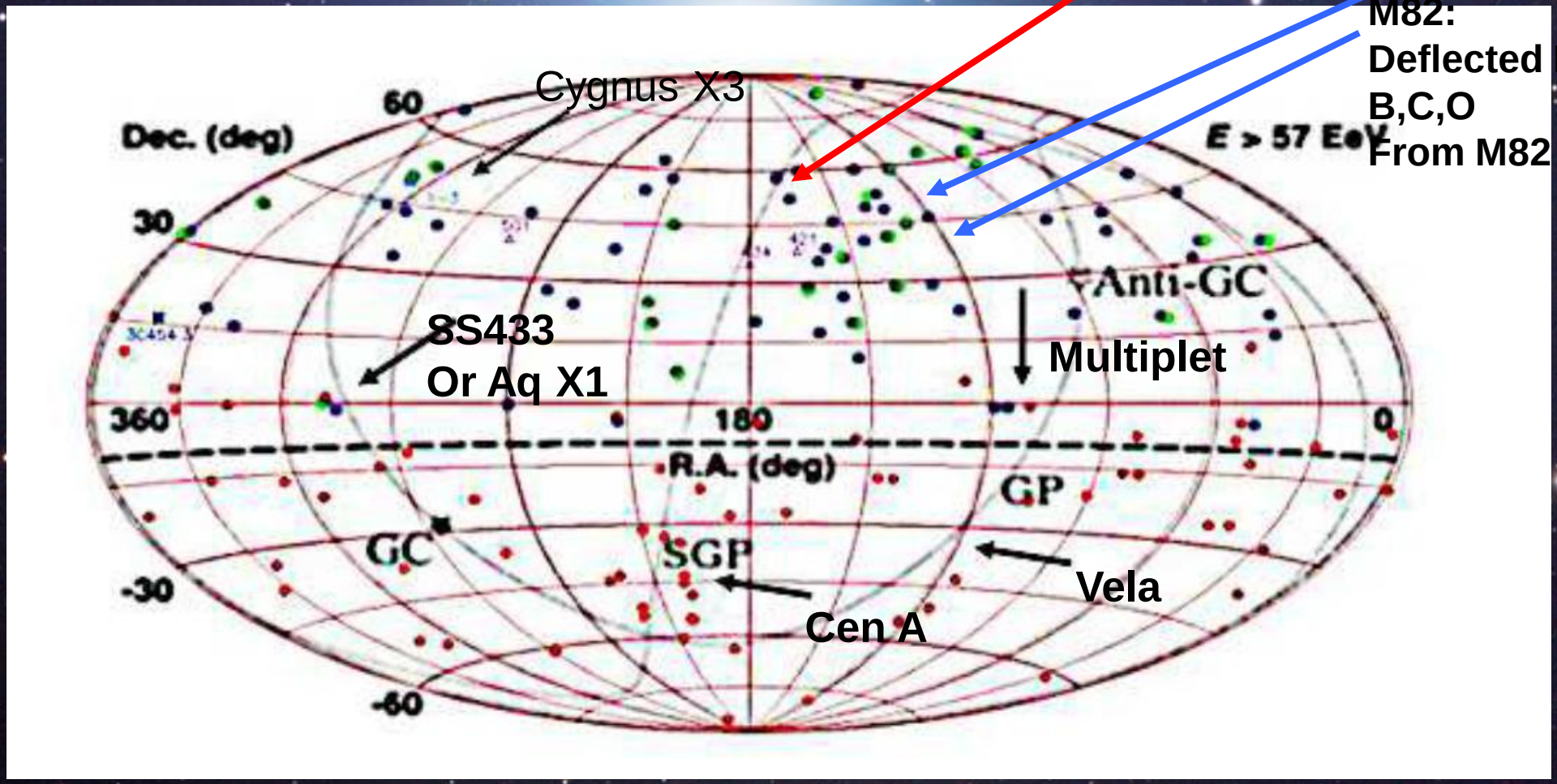
Conclusione:
Beaming Neutrino may act as a fast comunication system
while testing the lepton sector

related to:

UHECR and
Tau neutrino Airshower

Most recent Proposal (in press Nucl.Phys.B)

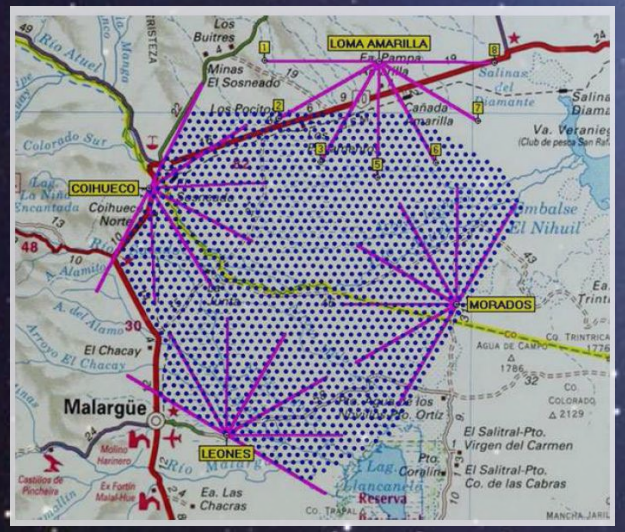
arXiv:1408.0227 DF

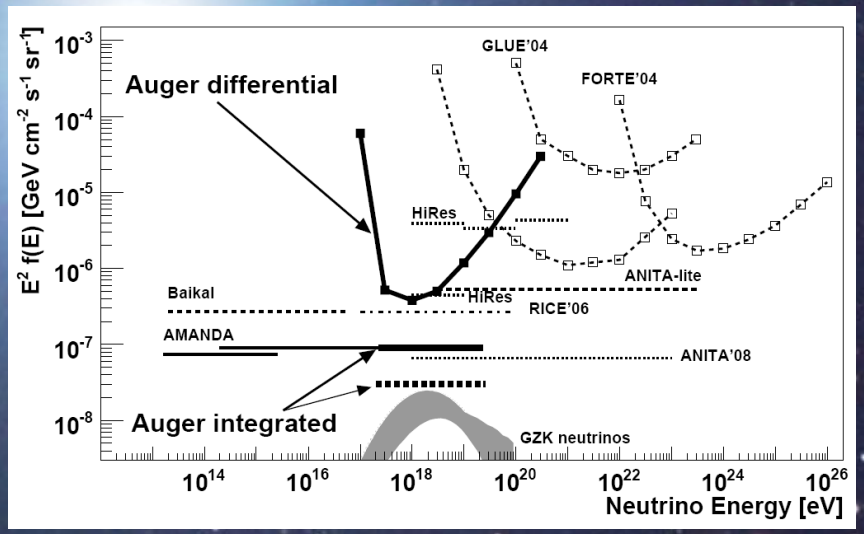


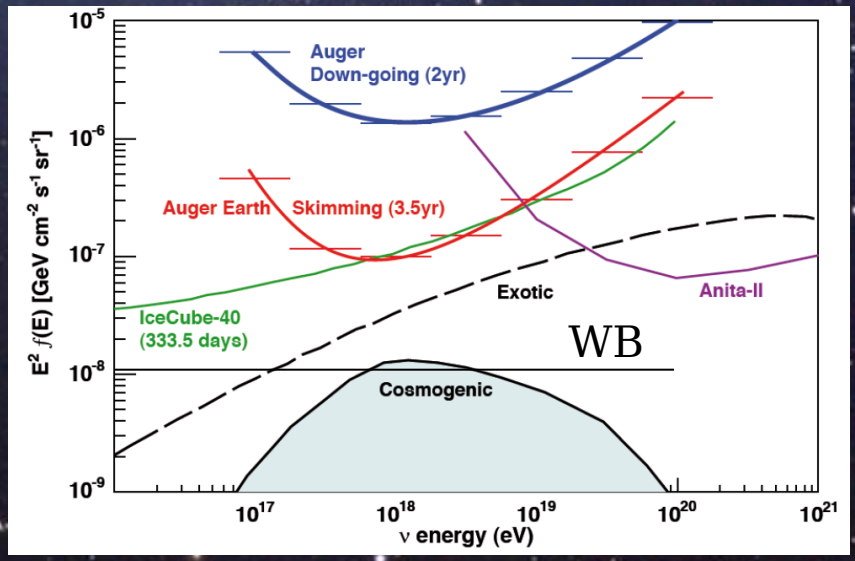
https://indico.cern.ch/event/287474/session/10/contribution/0/mate

UHE ν_τ di origine astrofisica all'orizzonte

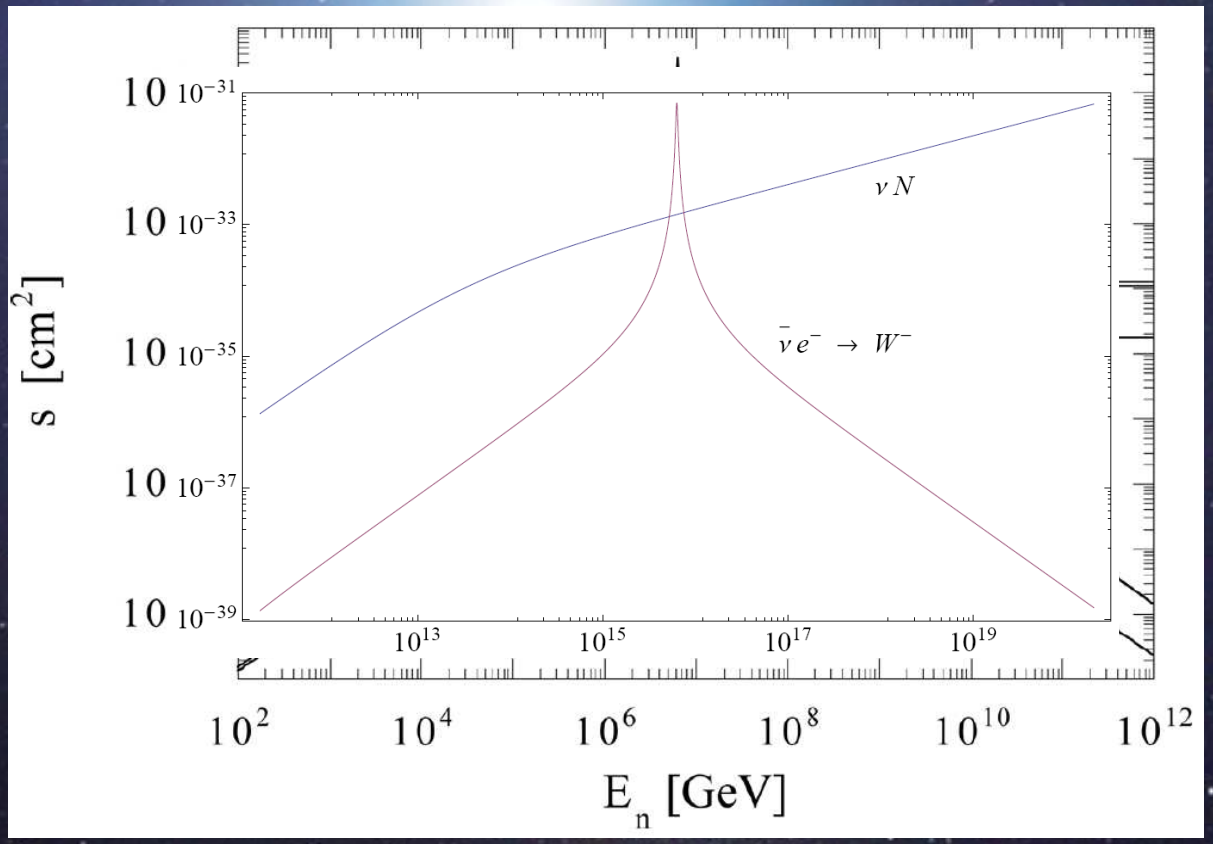
Auger, TA,ASHRA e gli sciami estesi di ν_τ





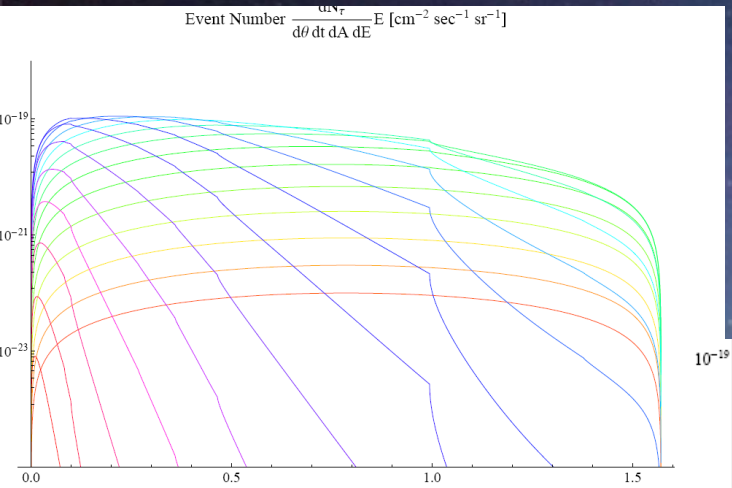


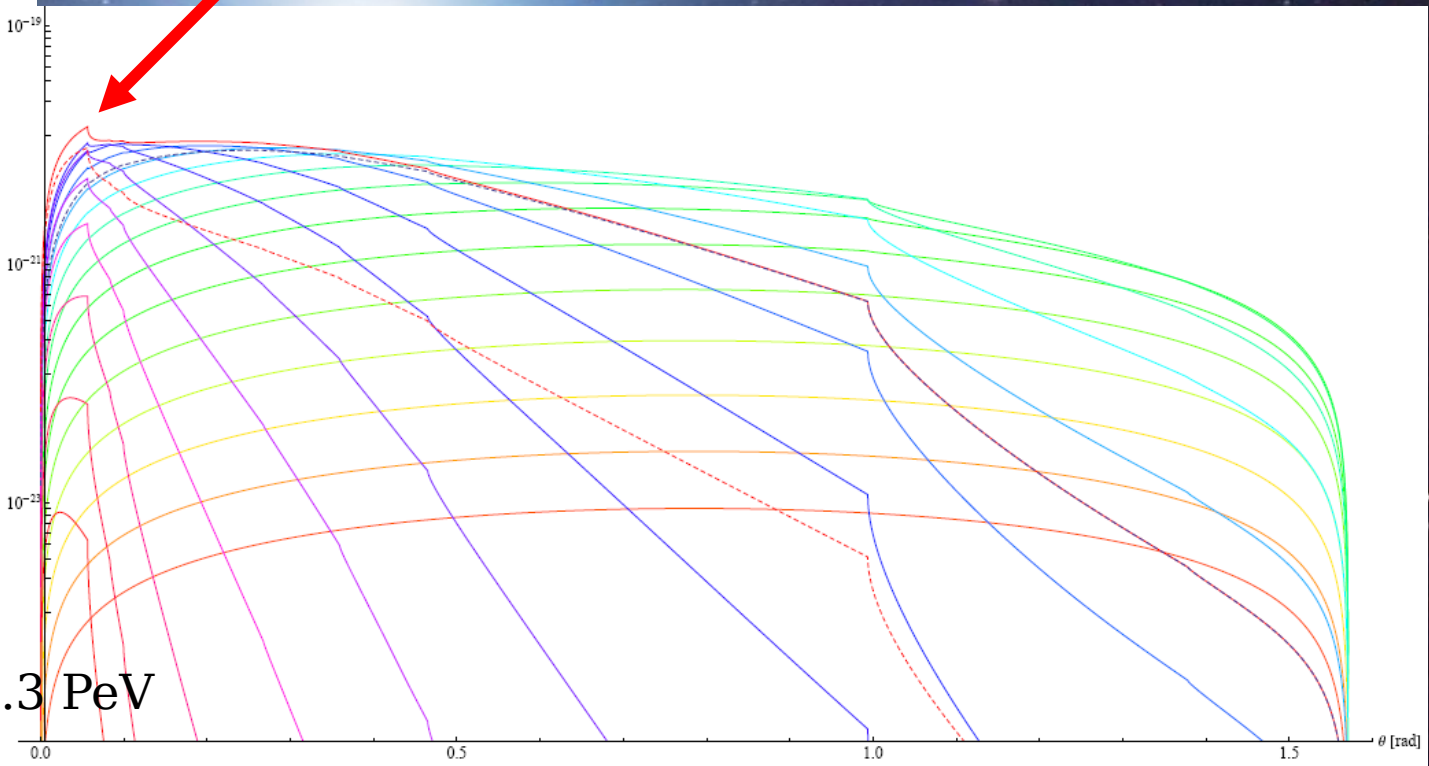
Sezione d'urto risonante W



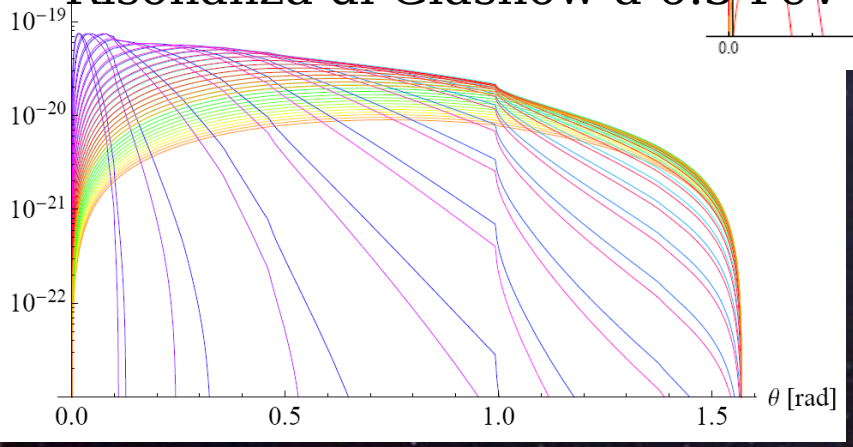
RAPPORTO DEI FLAVORS

Distribuzione angolare dello spettro degli sciami tau da neutrini tau da energie 10 GeV ad ZeV.
Risonanza di Glashow a 6.3 PeV

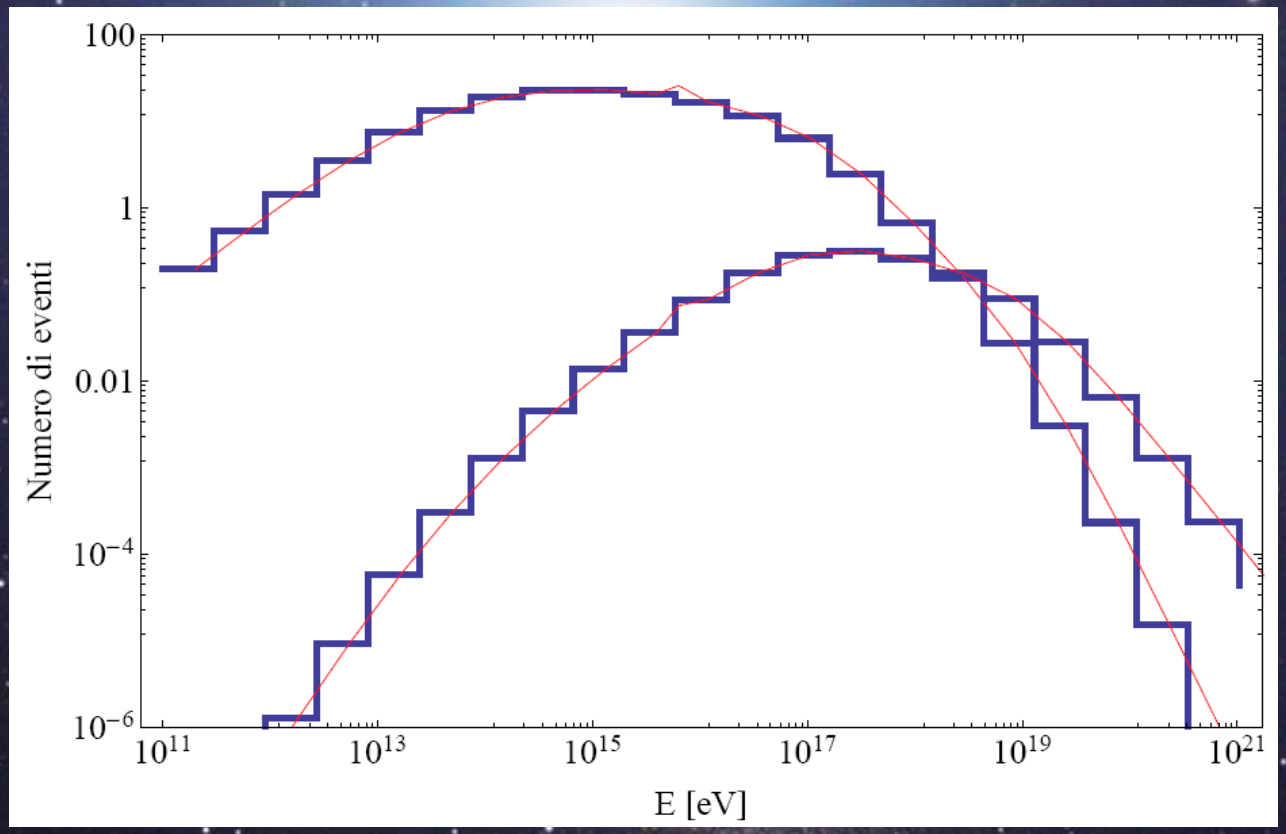




Risonanza di Glashow a 6.3 PeV



Numero di eventi previsti per 3 anni in Auger



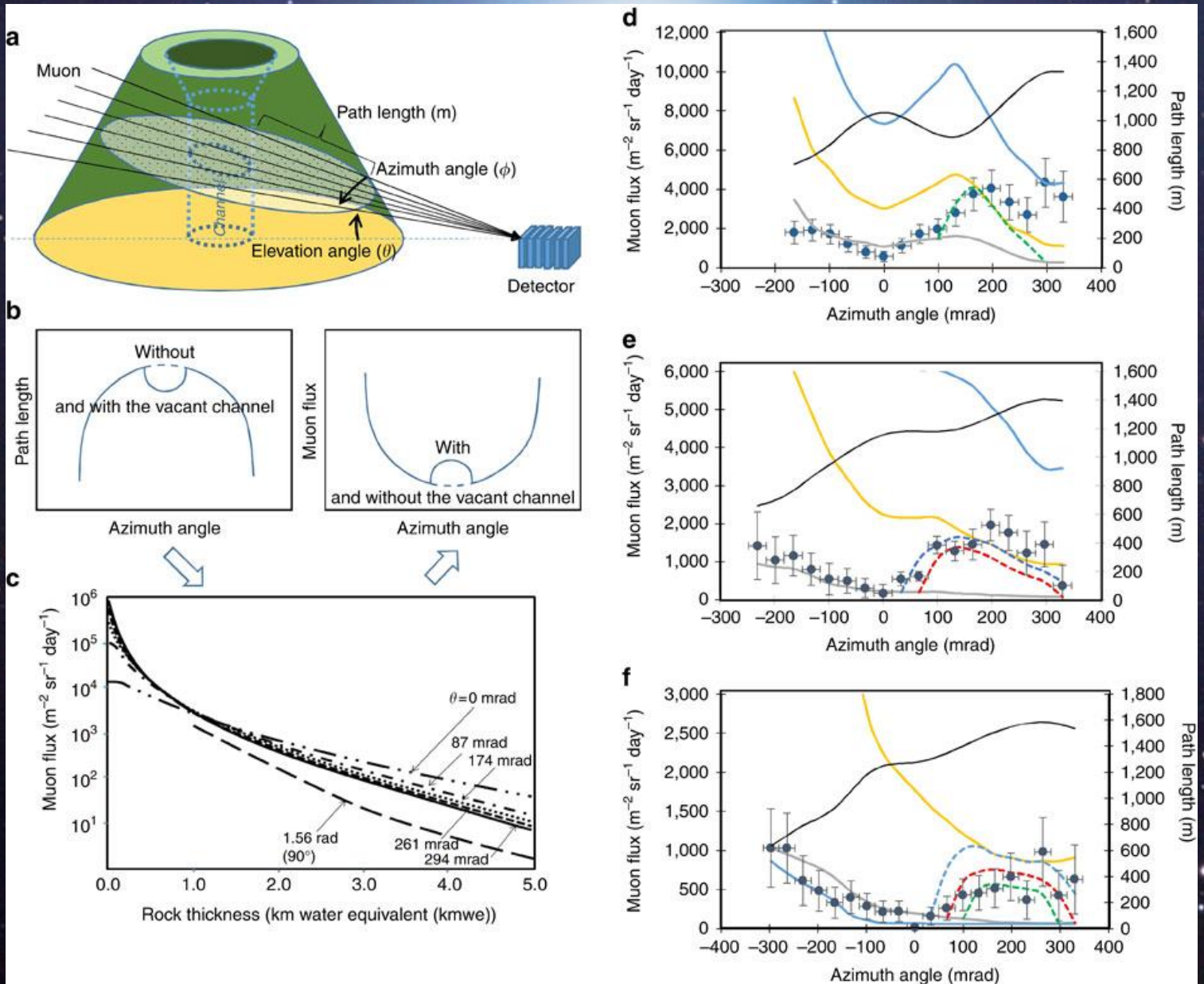
Energia media	$6.3\ 10^{15}$	$1.18\ 10^{16}$	$3.54\ 10^{16}$	$1.06\ 10^{17}$	$3.2\ 10^{17}$	$9.5\ 10^{17}$	$2.87\ 10^{18}$
Eventi	0.07	0.08	0.18	0.28	0.32	0.26	0.17

La somma degli eventi nell'intervallo da $6.3 \cdot 10^{15}$ a $2.9 \cdot 10^{18}$ eV = 1.3 per 3 anni

TAU-MUON ARRAY

Recording horizontal muons across a canyon or across a vulcano, waiting for rare,sudden horizontal airshower originated by a PeVs Horizontal Tau Airshower coming from the opposite side .

Muon array Detector



A COMPLEMENTARY ARRAY

To test Vulcano or hazard geological sites

The proposal is to combine array of horizontal Muon track detectors (around mountain vulcano) that inspect crossing muons across, while at same time may discover rare up-going electromagnetic airshowers

Horizontal ARRAY

The detector array maybe as usual scintillators and as simple as Hawk like cilindrical water Cherenkov rooms, possibly dig inside the Vulcano inclined walls yo screen them from noisy vertical showers.

Hawk

Conclusioni

Stiamo incontrando l'astronomia dei UHECR?

Connessione con le sorgenti?

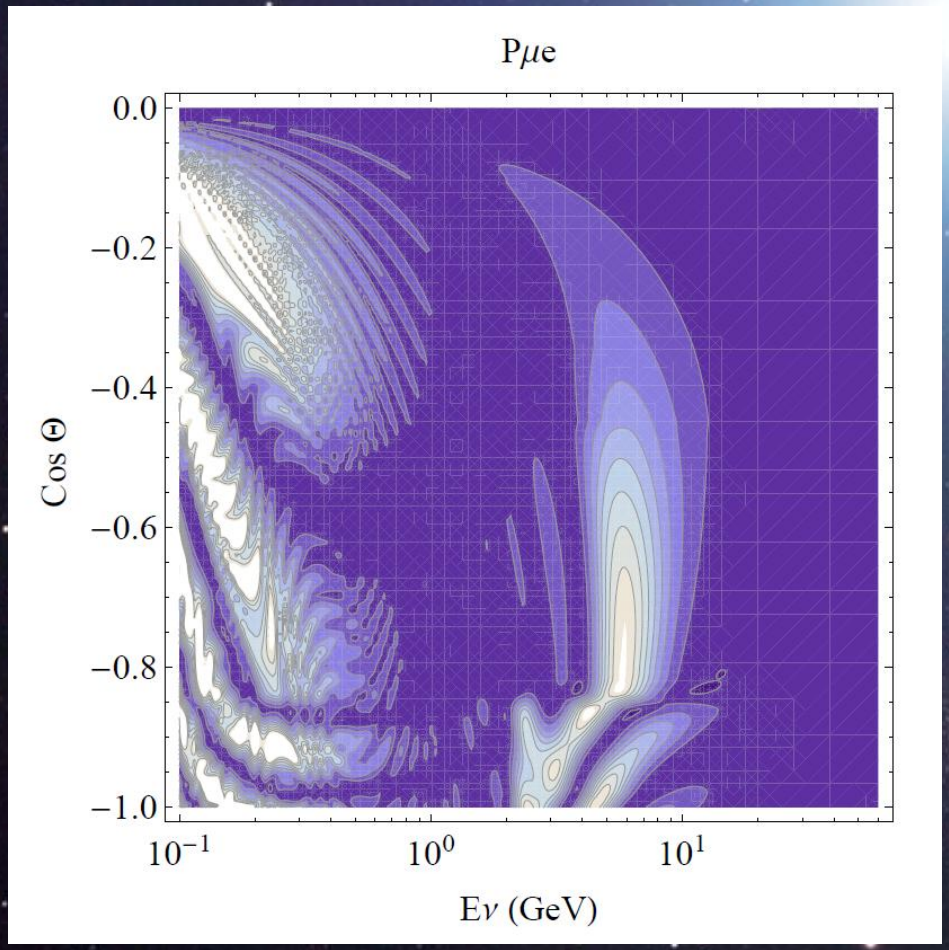
Astronomia dei PeV neutrini?

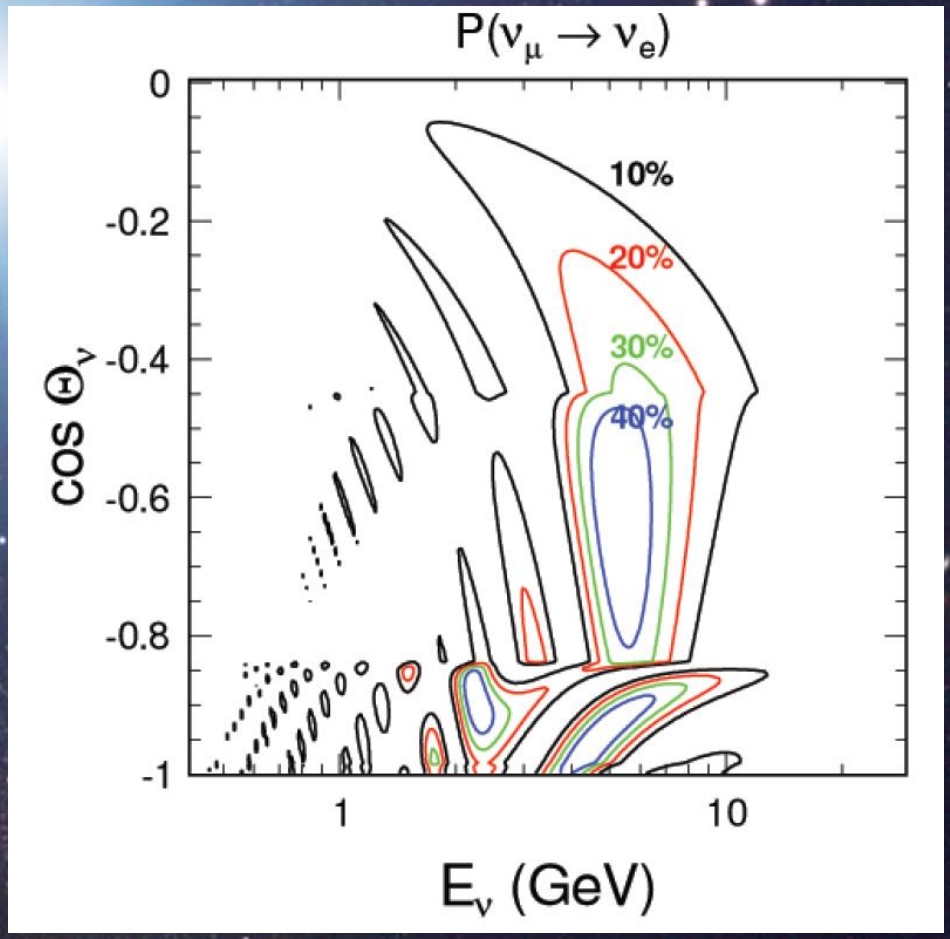
Astronomia dei tau neutrini?

Conosciamo i flavor?

Beaming neutrinos and Tau Arrays may help

$P_{\mu e}$

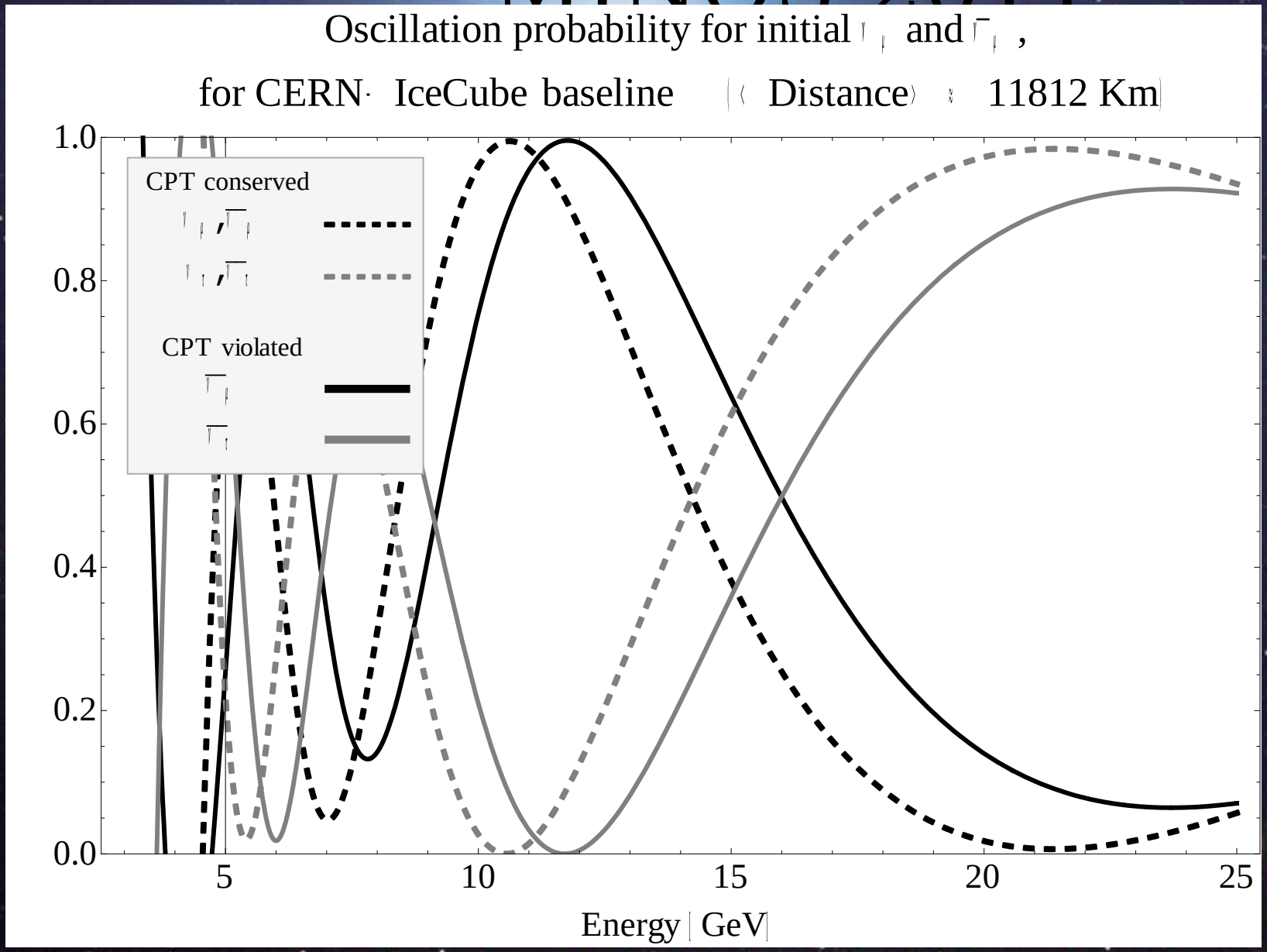

Pμe
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
Cos Θ
10^{-1}
10^{0}
10^{1}
Eν (GeV)


$P(\nu_\mu \rightarrow \nu_e)$
0
-0.2
-0.4
-0.6
-0.8
-1
$\cos\Theta_\nu$
10%
20%
30%
40%
1
10
E_ν (GeV)

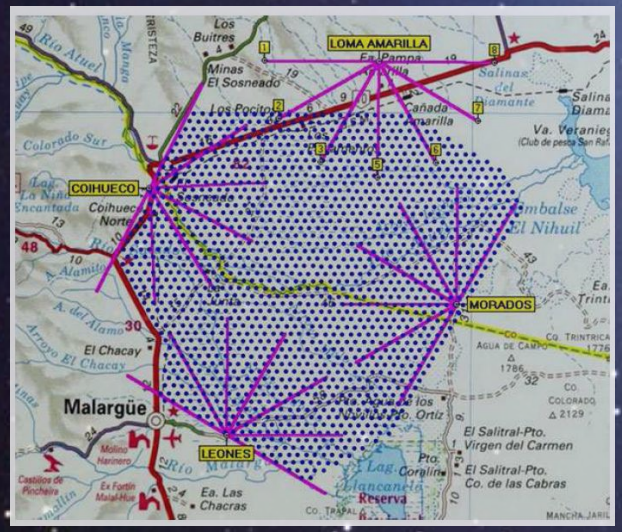
Eventi di ν_μ e $\bar{\nu}_\mu$ in assenza di oscillazione

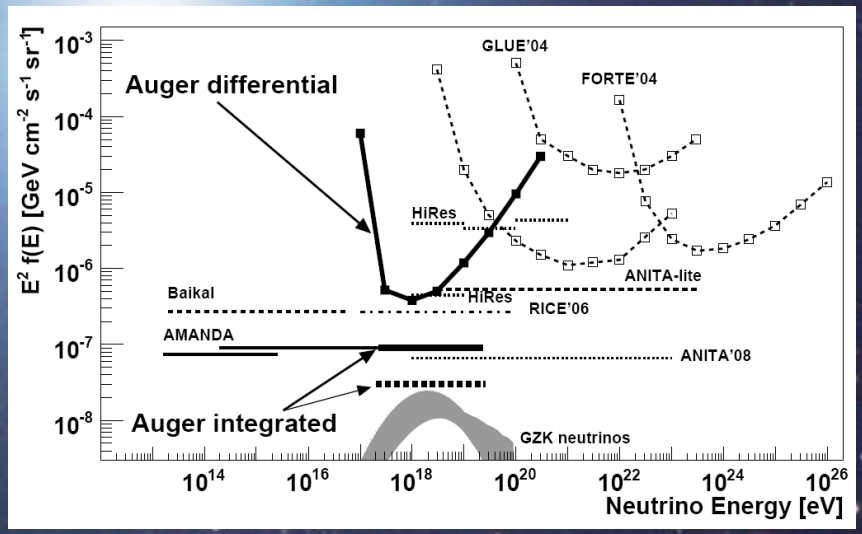
Baseline	E_ν GeV	N_{ν_μ} no osc. born in detector year^{-1}	N_{ν_μ} no osc. born outside year^{-1}	N_{ν_μ} no osc. total year^{-1}	$N_{\bar{\nu}_\mu}$ no osc. total year^{-1}
CERN–OPERA	17	2847	18278	21125	10562
CERN–SK	15.8	324	1621	1945	972
Fermilab–SK	16.5	342	1708	2050	1025
CERN–IceCube	21.8	93343	23336	116679	58340
Fermilab–IceCube	21.4	93951	23488	117439	58720

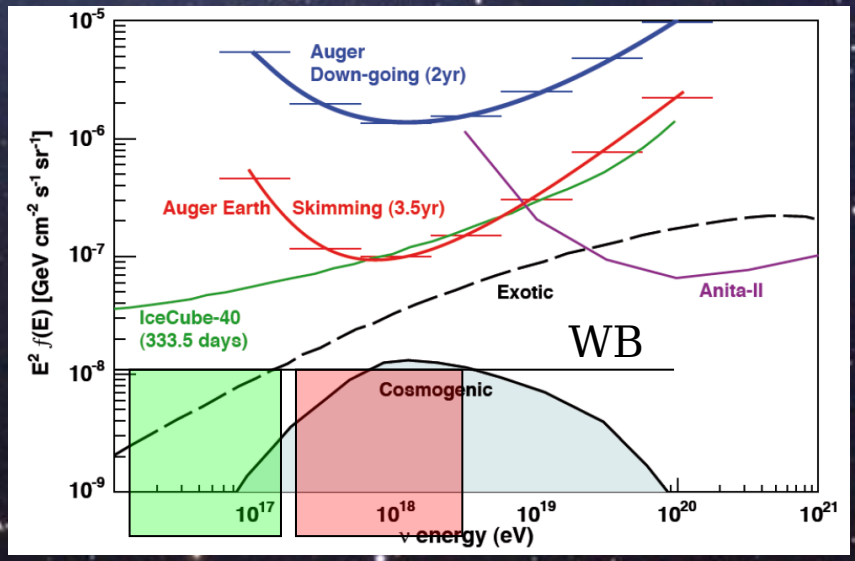
Oscillazioni con i parametri di MINOS 2011



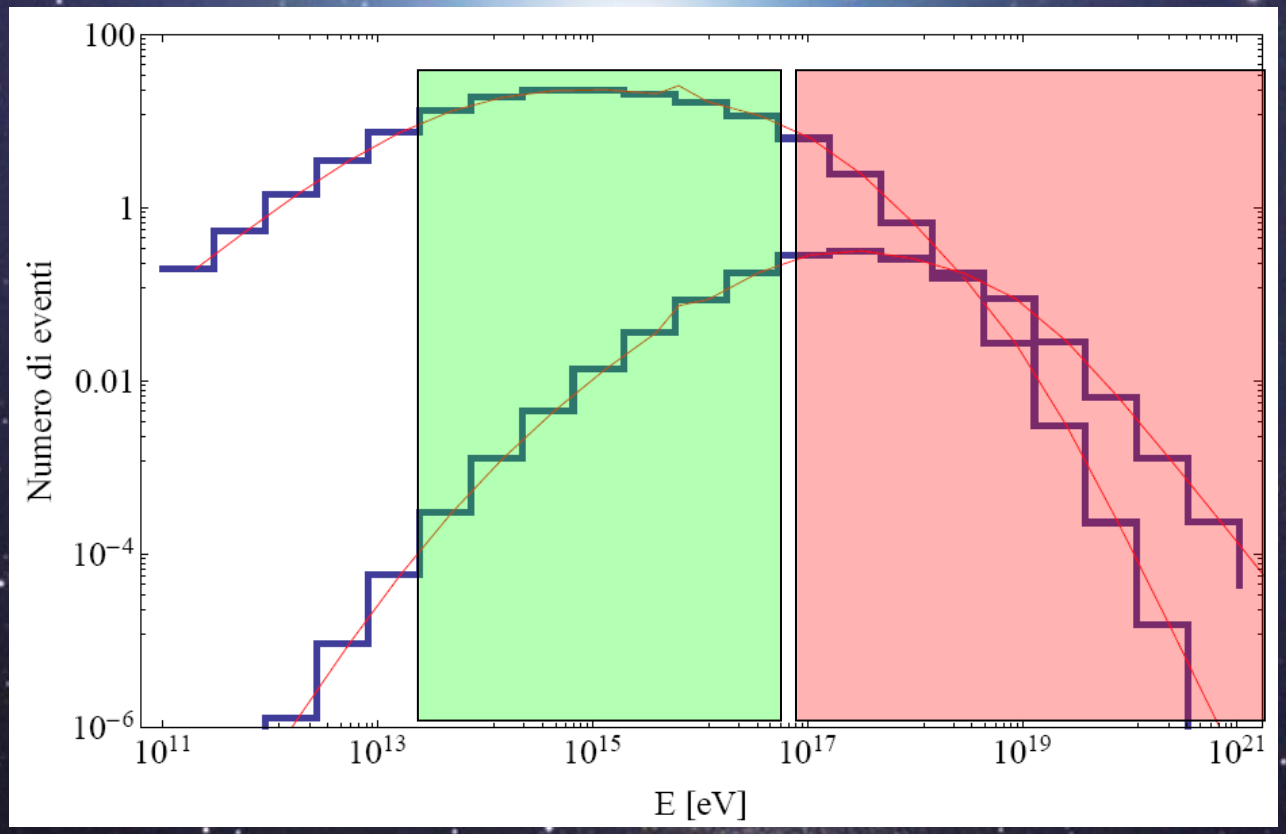
Auger e gli sciami estesi di ν_τ







Numero di eventi previsti per 3 anni in Auger



Energia media	$6.3\ 10^{15}$	$1.18\ 10^{16}$	$3.54\ 10^{16}$	$1.06\ 10^{17}$	$3.2\ 10^{17}$	$9.5\ 10^{17}$	$2.87\ 10^{18}$
Eventi	0.07	0.08	0.18	0.28	0.32	0.26	0.17

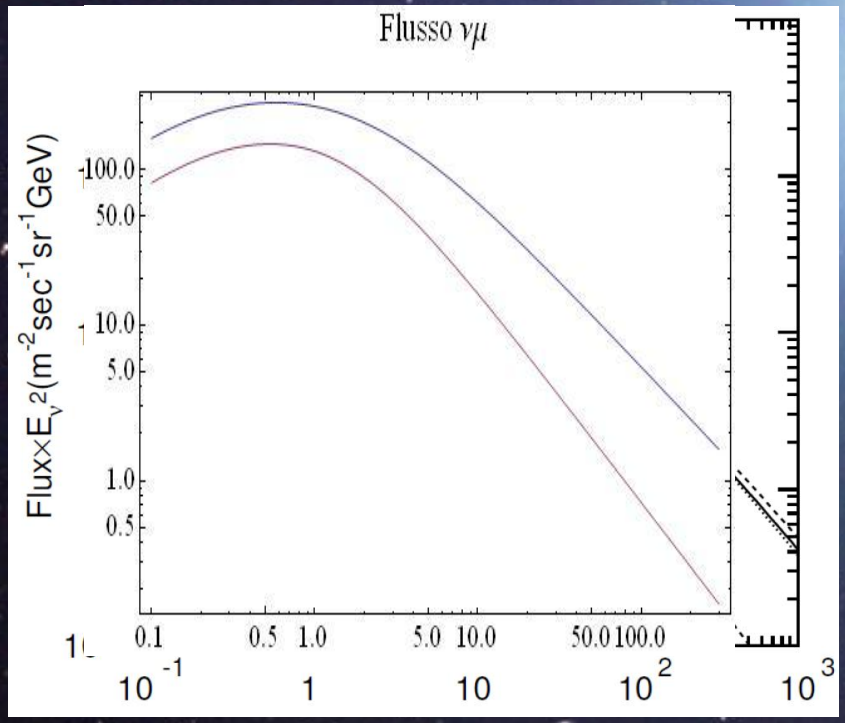
La somma degli eventi nell'intervallo da $6.3 \cdot 10^{15}$ a $2.9 \cdot 10^{18}$ eV

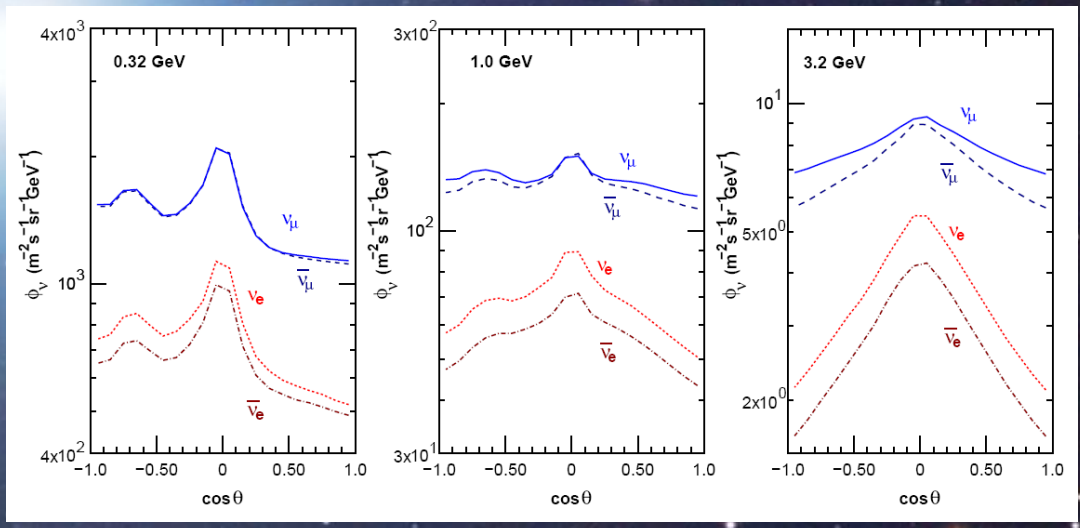
Prevedere i flussi dei Neutrini atmosferici

- Analisi dei dati atmosferici in **DeepCore** per discriminare tra gli scenari di violazione CPT
- Calcolo degli eventi di ν atmosferici a partire dal flusso
- Calibrazione del calcolo sui dati di **SuperKamiokande**

- Estensione a DeepCore e previsioni

Flusso dei ν atmosferici





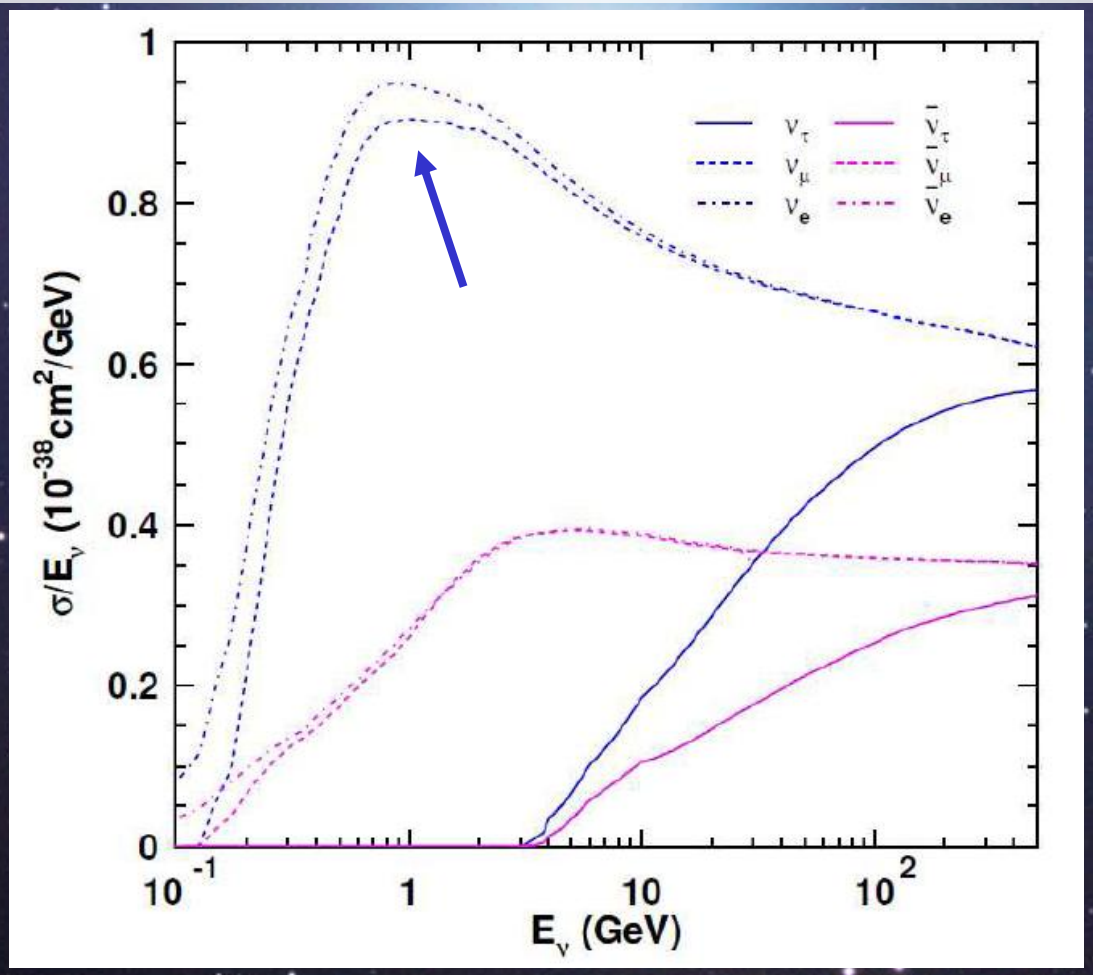
$$\phi_\nu(E_\nu) = 10^2 \left(\frac{1}{9.5 \cdot E_\nu^{-1.12}} + \frac{1}{4} + \frac{1}{27 \cdot E_\nu} \right)^{-1} m^{-2} s^{-1} sr^{-1} GeV$$

$$= f(E_\nu, \theta)$$

$$\left[1 + \left(\frac{E_\nu}{E_0} \right)^{\alpha} \cdot (\sin\theta)^{\beta} \right]^{-1} + \left(1 - \frac{\int_0^{\pi/2} d\theta f(E_\nu, \theta)}{\pi/2} \right)$$

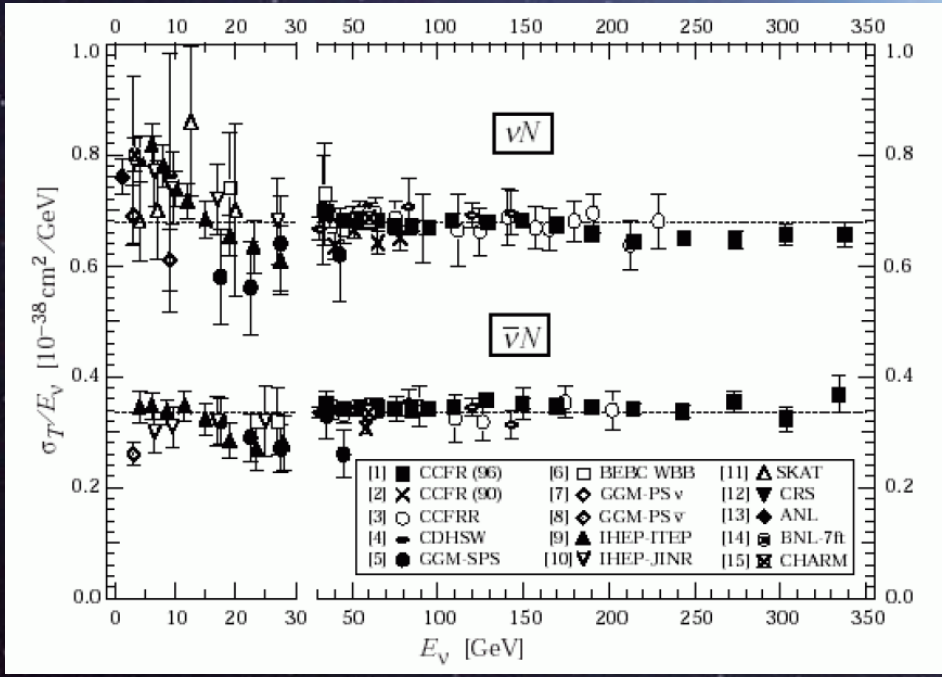
$$F(E_\nu, \theta)$$

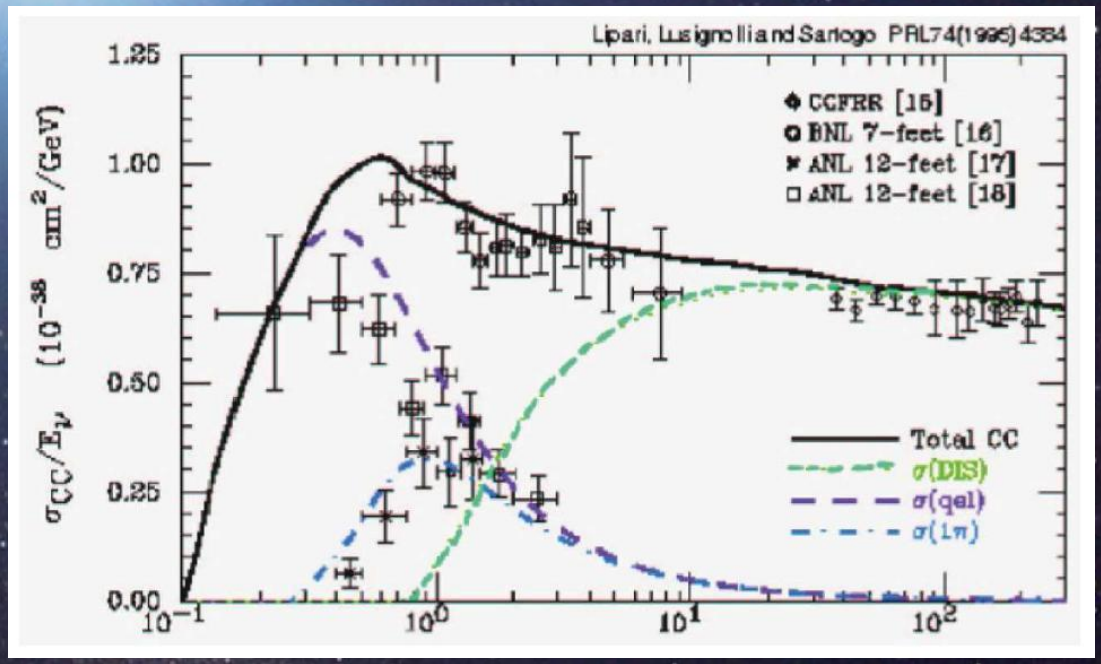
Sezione d'urto ν N



$$\sigma_{CC}(E_\nu) = \left(\frac{1}{\frac{15}{E_\nu} + \frac{5}{E_\nu(2+.8\log E_\nu)^3}} + \frac{0.9}{8 + \frac{7}{E_\nu^2} + \frac{120}{(2.1+0.75\log E_\nu)^3}} \right) \cdot 10^{-37} cm^2$$

Sezione d'urto ν N



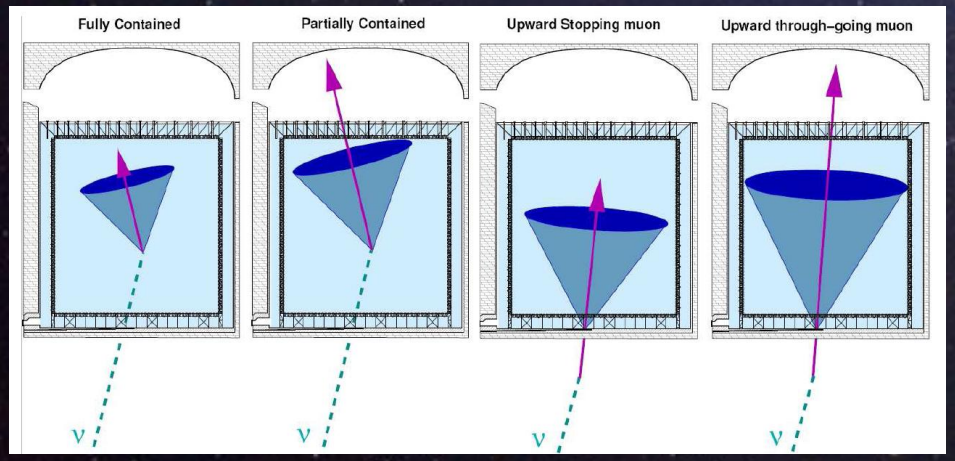


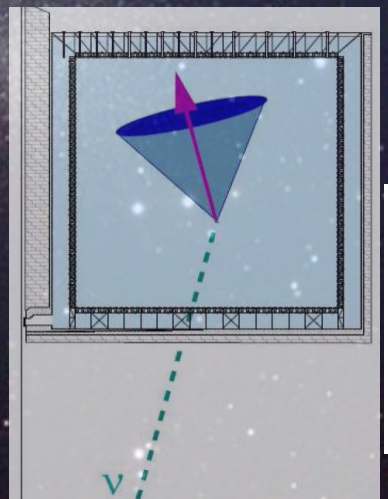
Calibrazione degli Eventi differenziali in SK

$$\frac{dN}{dE_\nu\, dt\, d\Omega}(E_\nu,\theta) = 2\pi \frac{\phi_\nu(E_\nu)}{E_\nu^2} F(E_\nu,\theta)\, \sigma_{CC}(E_\nu)\, N_A\, V(E_\nu,\theta)\, P_{\nu_\mu\to\nu_\mu}(E_\nu,\theta).$$

SuperKamiokande
V_{tot} = 22’458 m^3

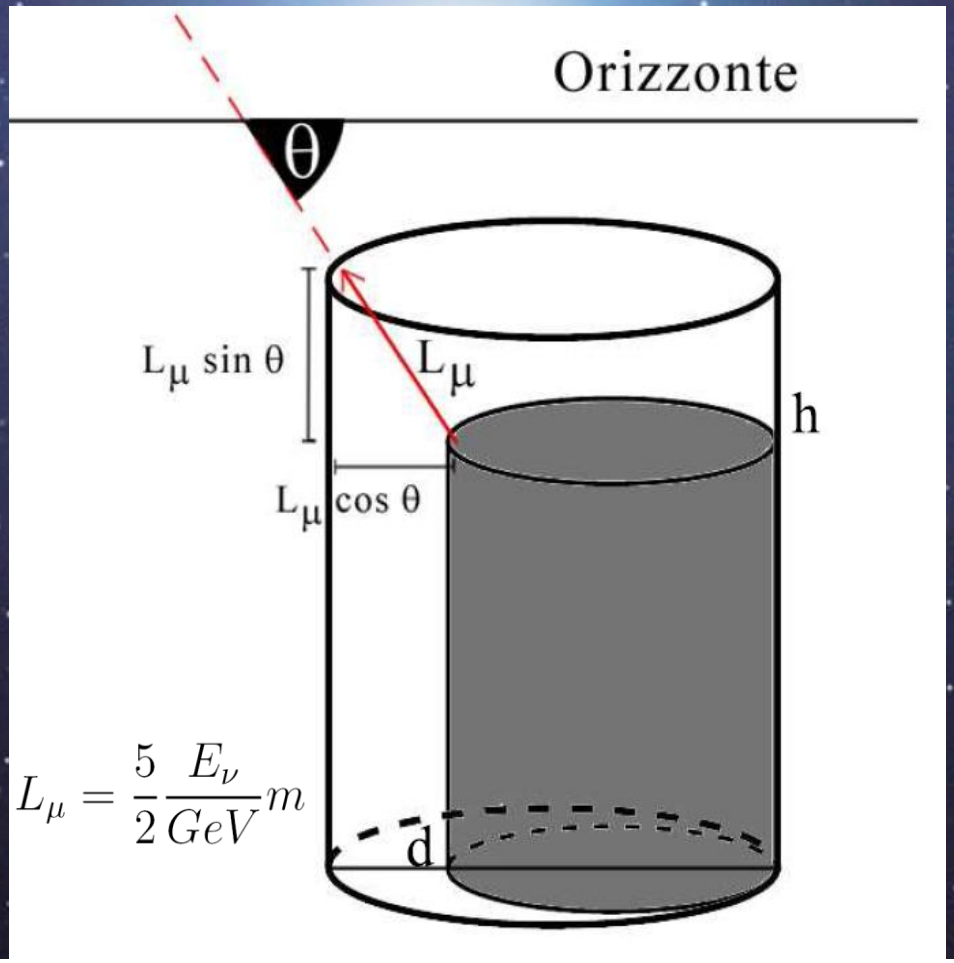
Non dipende dall’energia
Né dalla direzione di arrivo





V_{FC}
Fully Contained

SK Volume (E,θ) per eventi ν Fully Contained



$$L_\mu = \frac{5}{2}\frac{E_\nu}{GeV}m$$

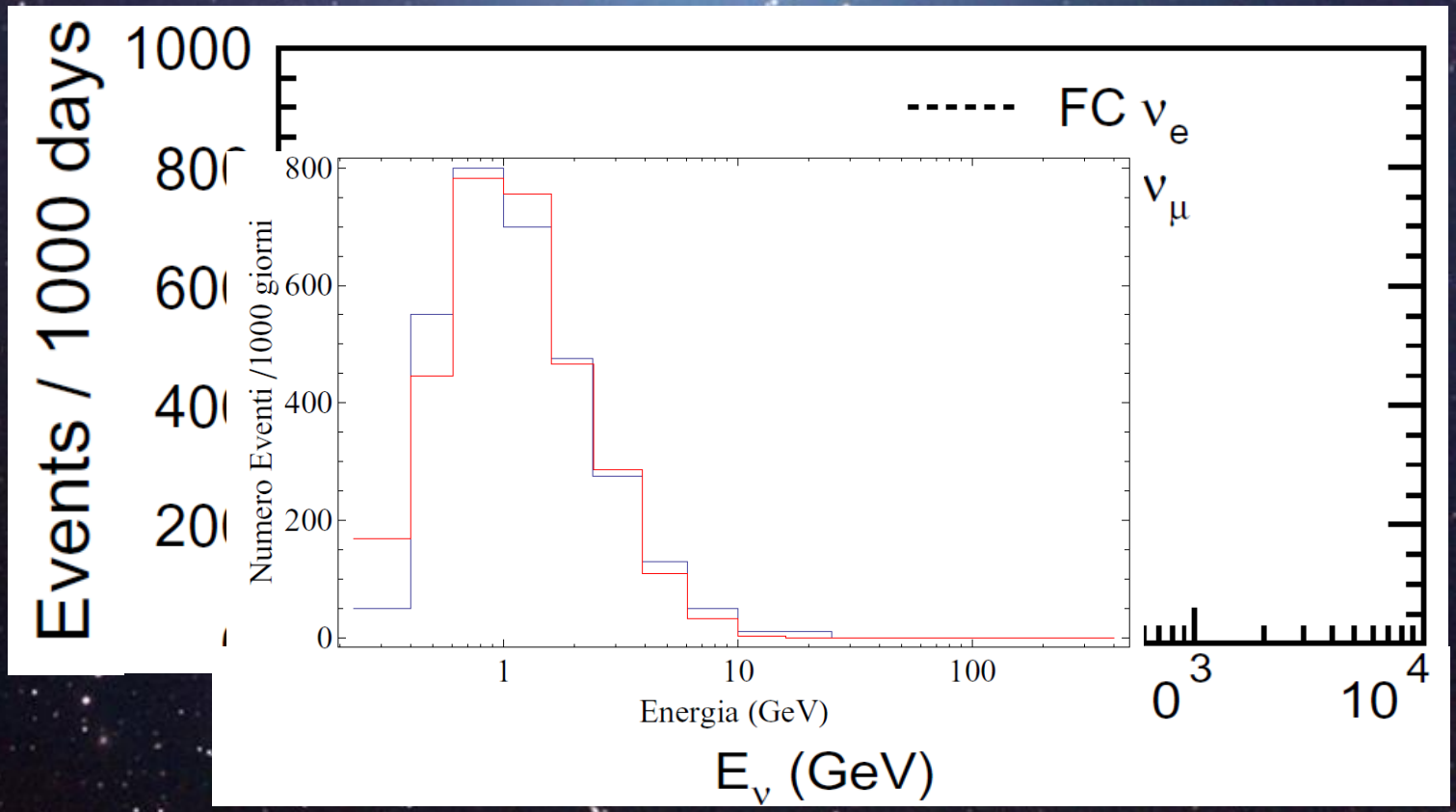
$$V_{FC}(E_\nu,\theta) = (h - L_\mu(E_\nu)\cdot|\sin\theta|)\cdot\tfrac{\pi}{4}(d - L_\mu(E_\nu)\cdot|\cos\theta|)^2$$

$$h > L_\mu\cdot|\sin\theta|$$

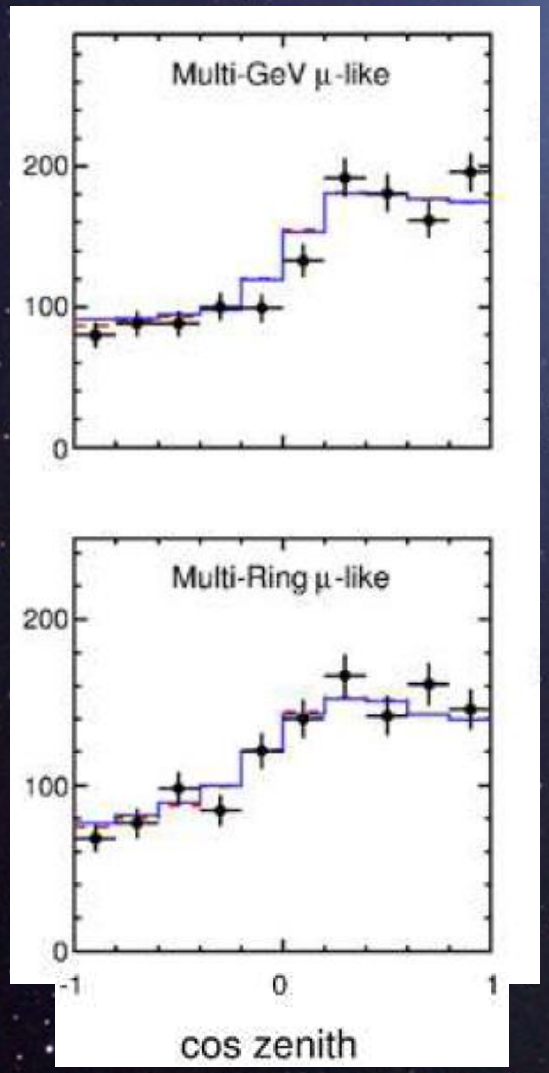
$$d > L_\mu\cdot|\cos\theta|$$

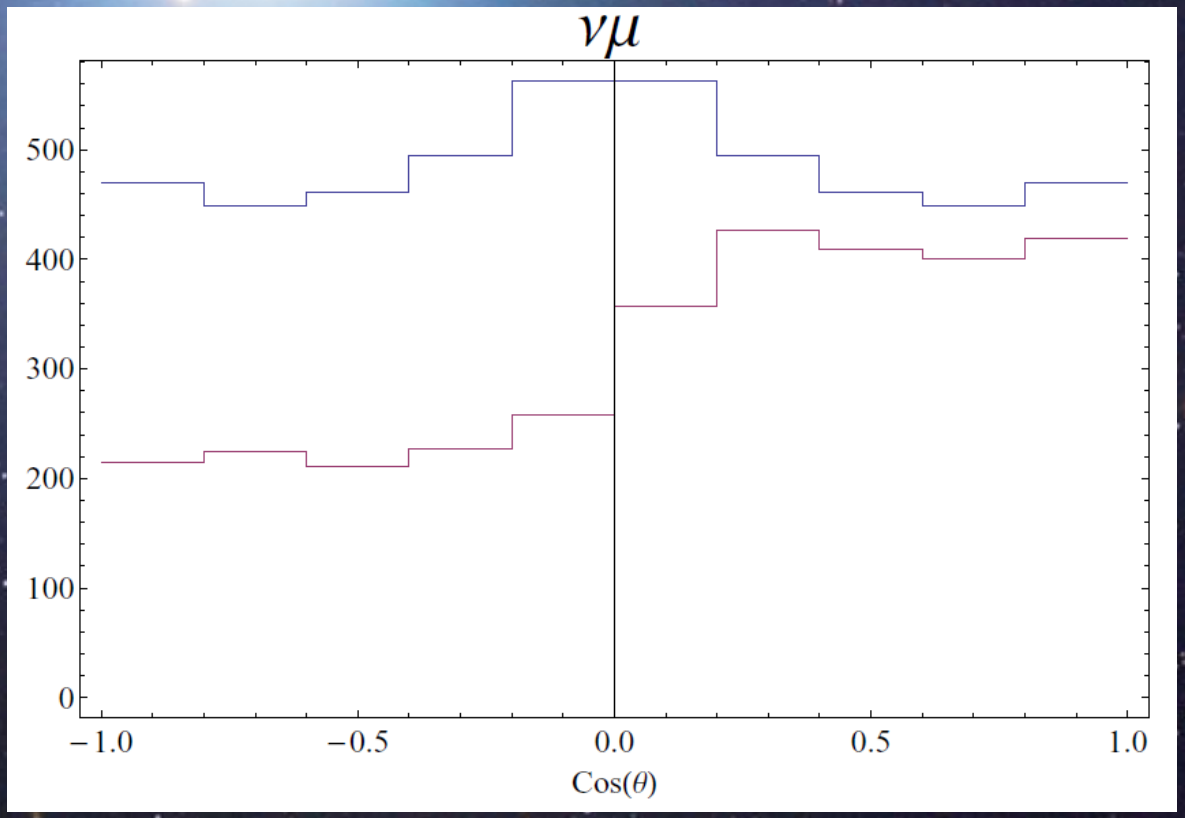
Per verificare il mio modello analitico, confronto le previsioni con gli Eventi ν_μ Fully Contained in SK in funzione dell'energia: appurata la capacita' di previsione

$$\Delta N(E_\nu) = \int_{E_i}^{E_i+\Delta_i E} dE_\nu \int_{-1}^{0} d(\cos\theta) \frac{dN}{dE_\nu\, dt\, d(\cos\theta)}(E_\nu, \theta) \cdot \Delta t$$



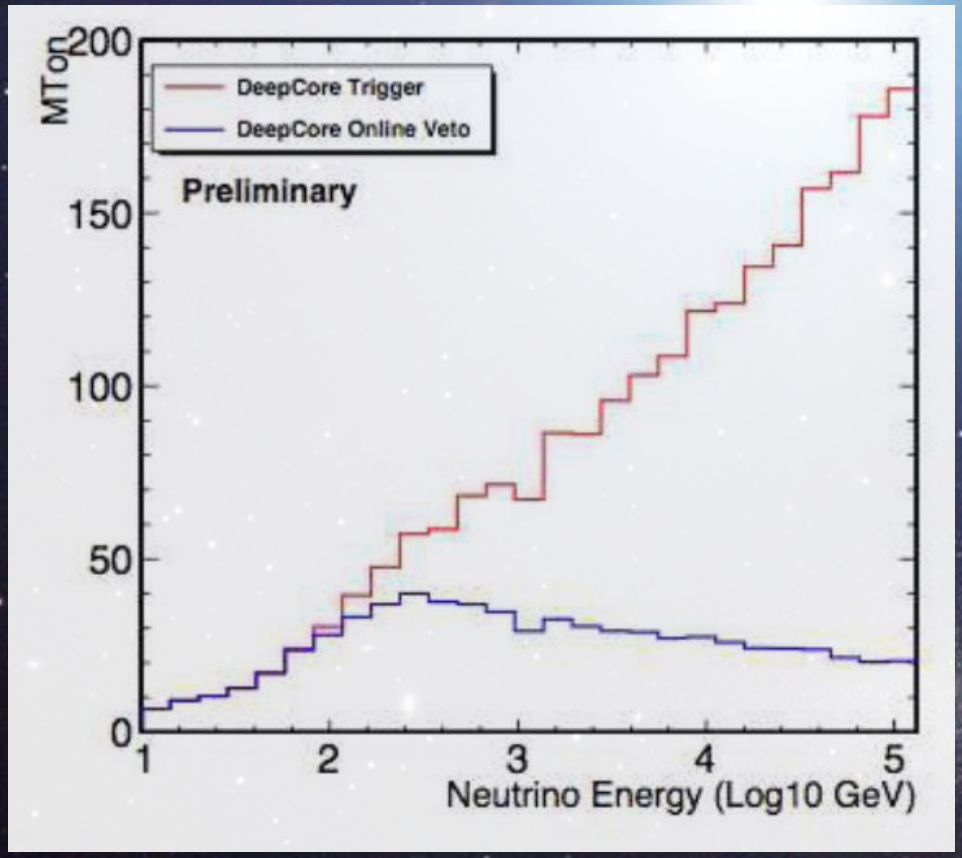
Verifica degli Eventi ν_μ Fully Contained in funzione dell'angolo per finestra energetica: 1.33-10 GeV





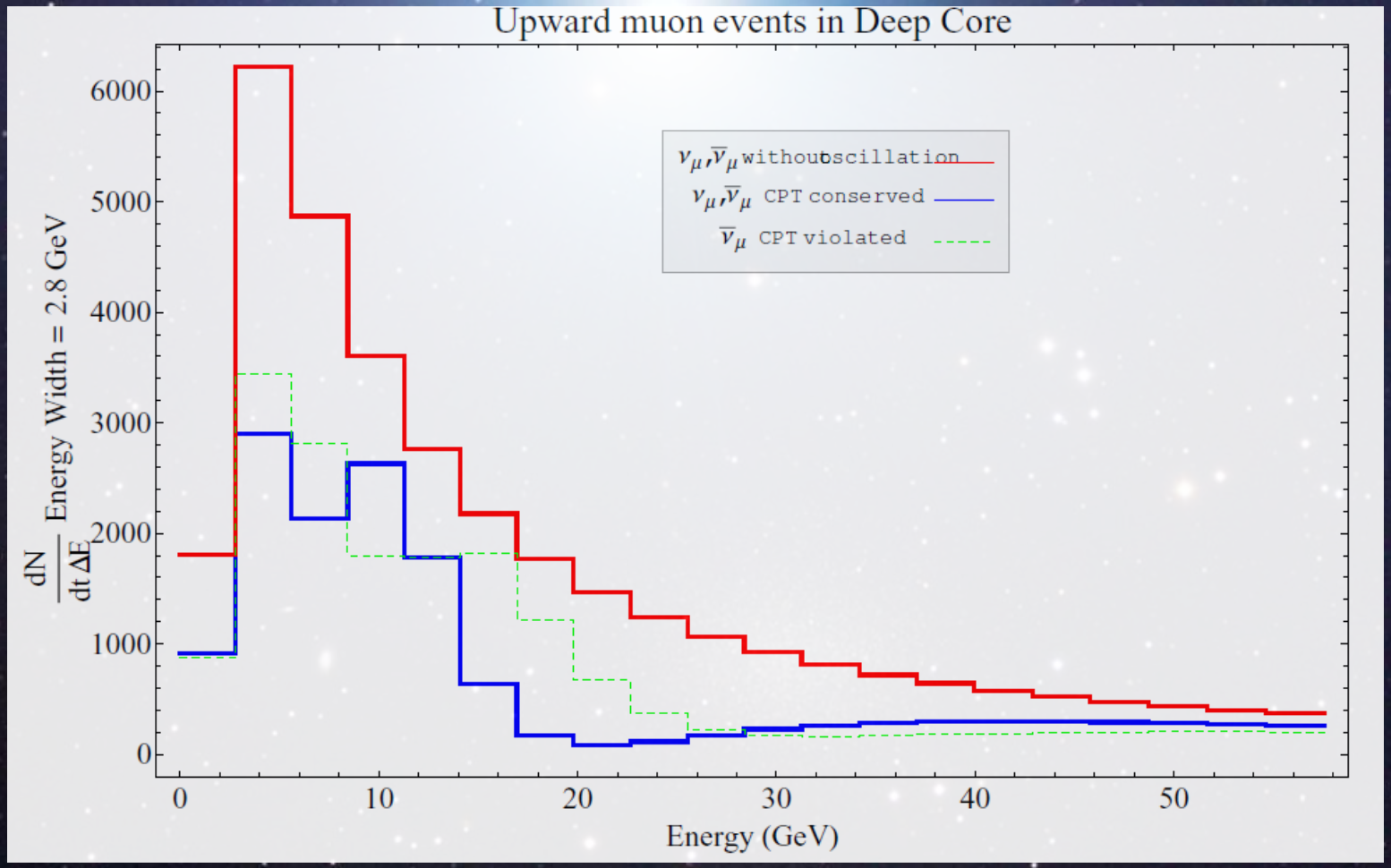
SKI + SKII + SKIII =
1489 + 799+518 giorni

Volume efficace V(E) per Eventi ν_μ in IceCube - DeepCore

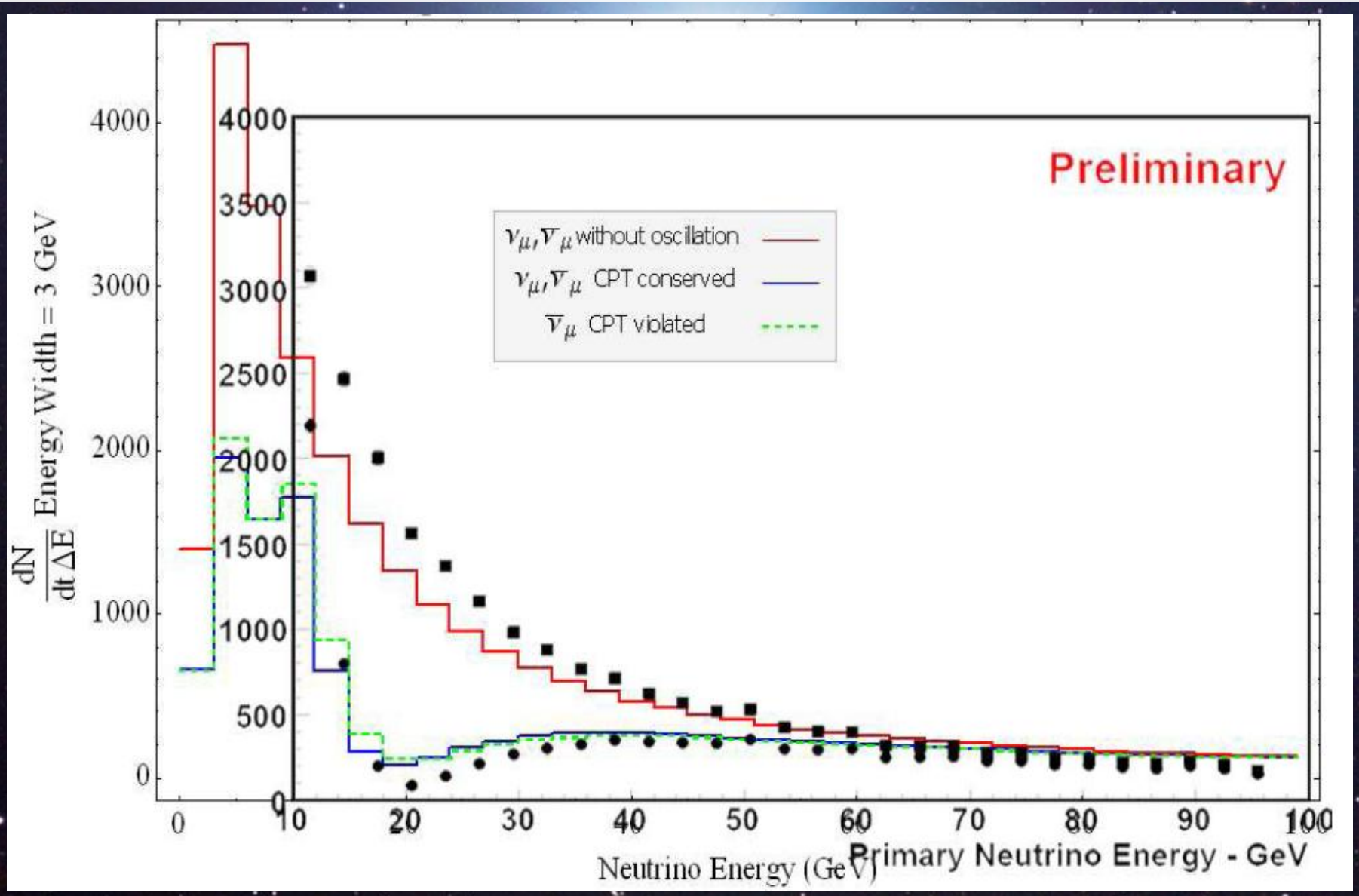


Volume in funzione di E_ν

Da quanto sopra derivo le Previsioni di Eventi ν_μ in IceCube - DeepCore

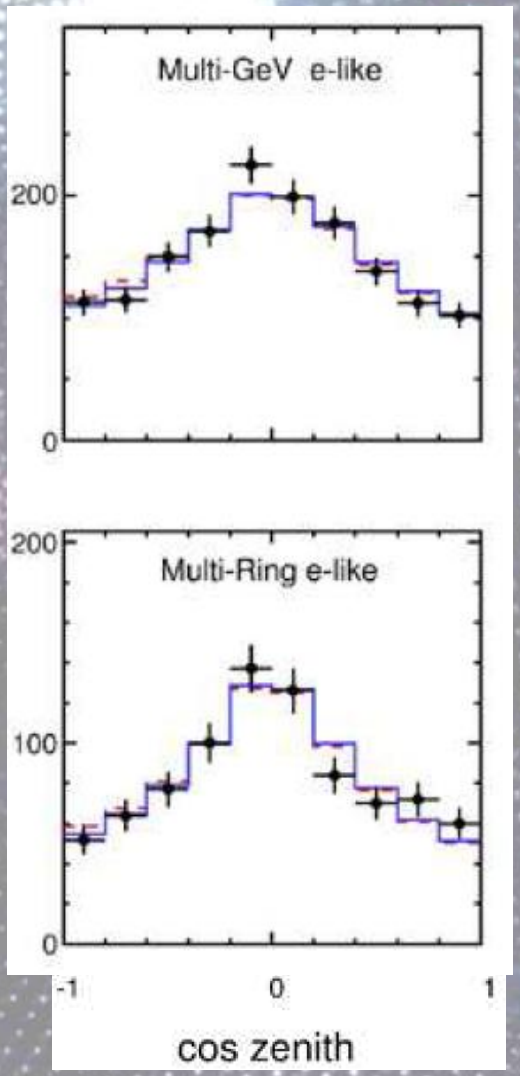


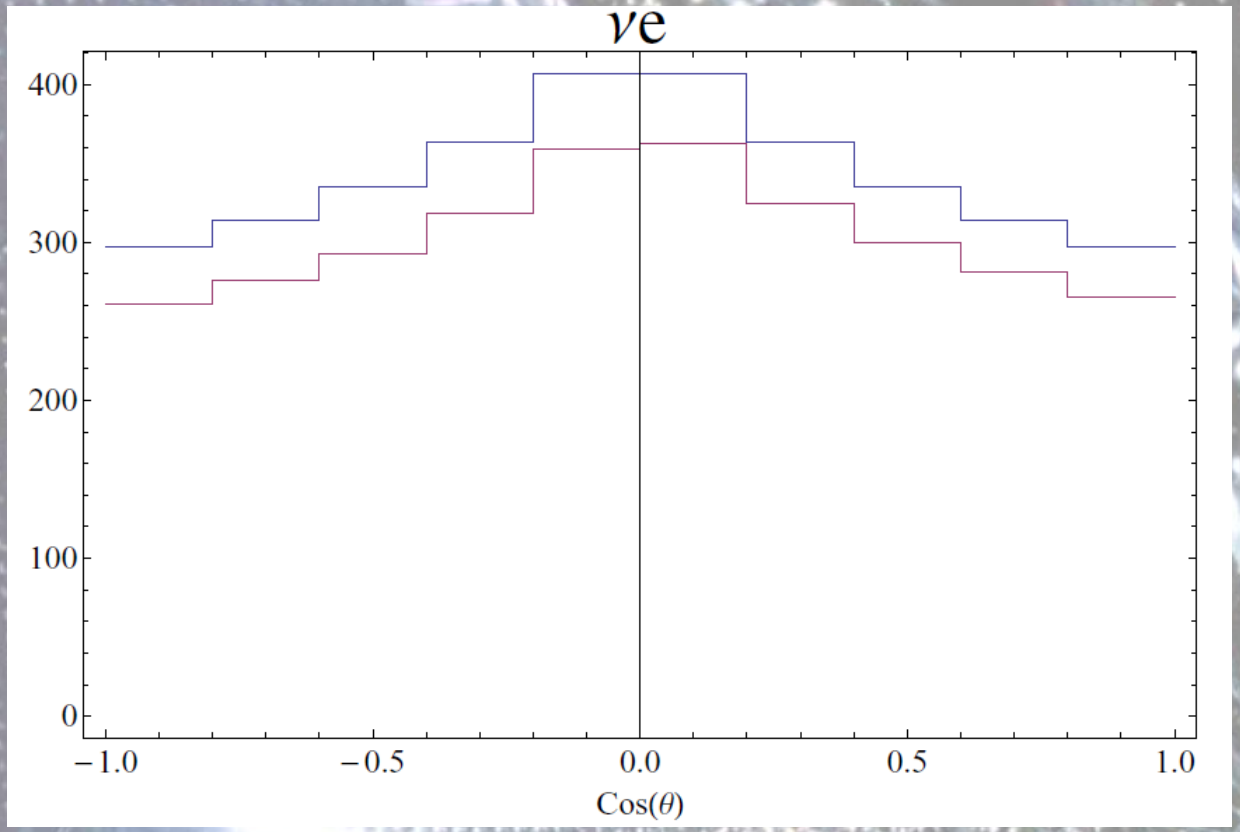
Eventi ν_μ in IceCube - DeepCore



Backup slide

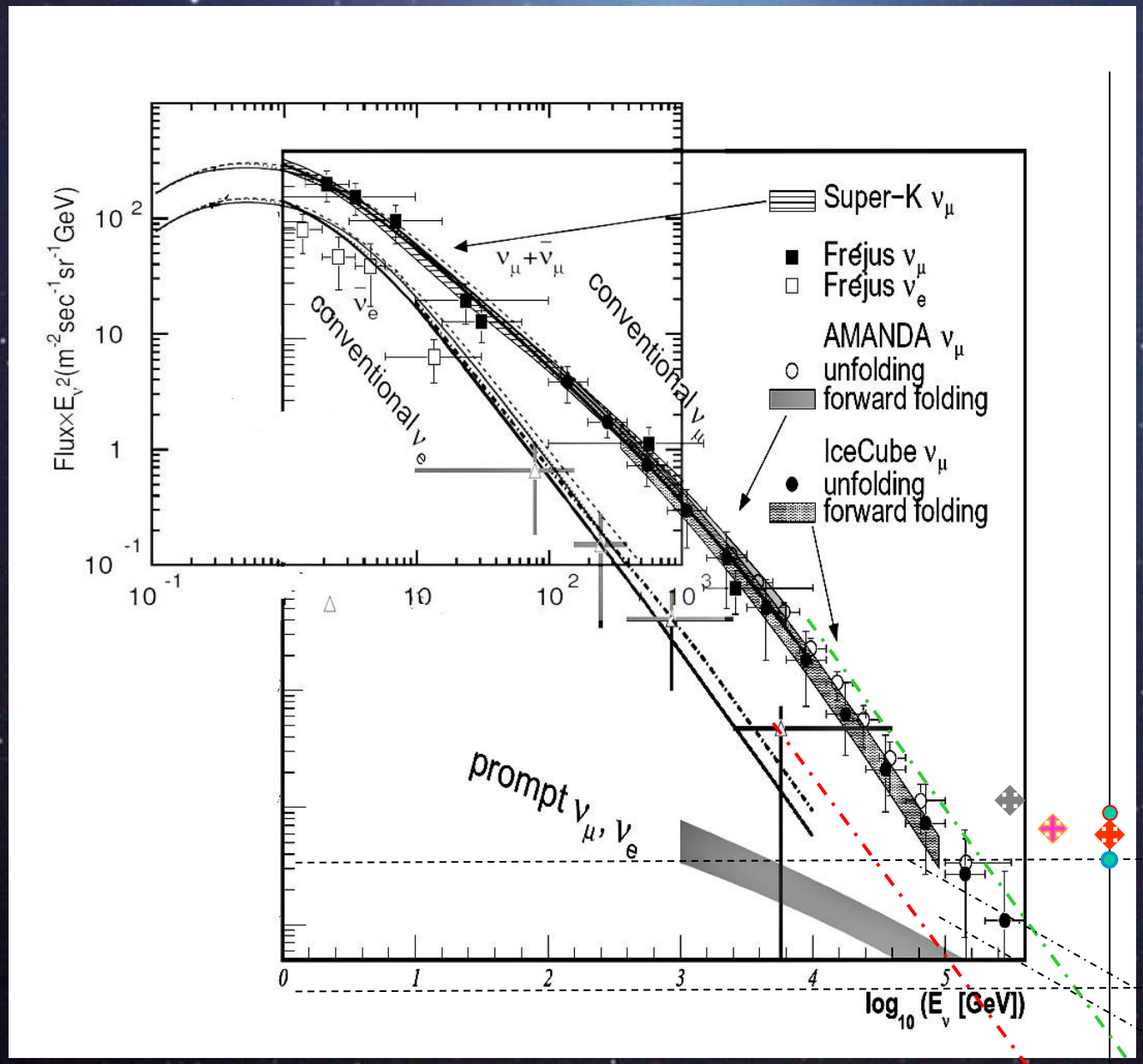
Confronto degli Eventi ν_e Fully Contained in funzione dell'energia





Tesi triennale teorica on: ***Unexpected Icecube Neutrino Revolution***

Rivolgersi presso Prof. D.Fargion st: 113--0649914287

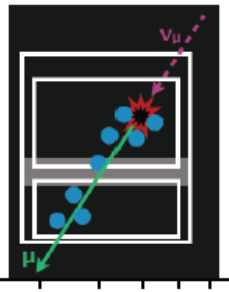


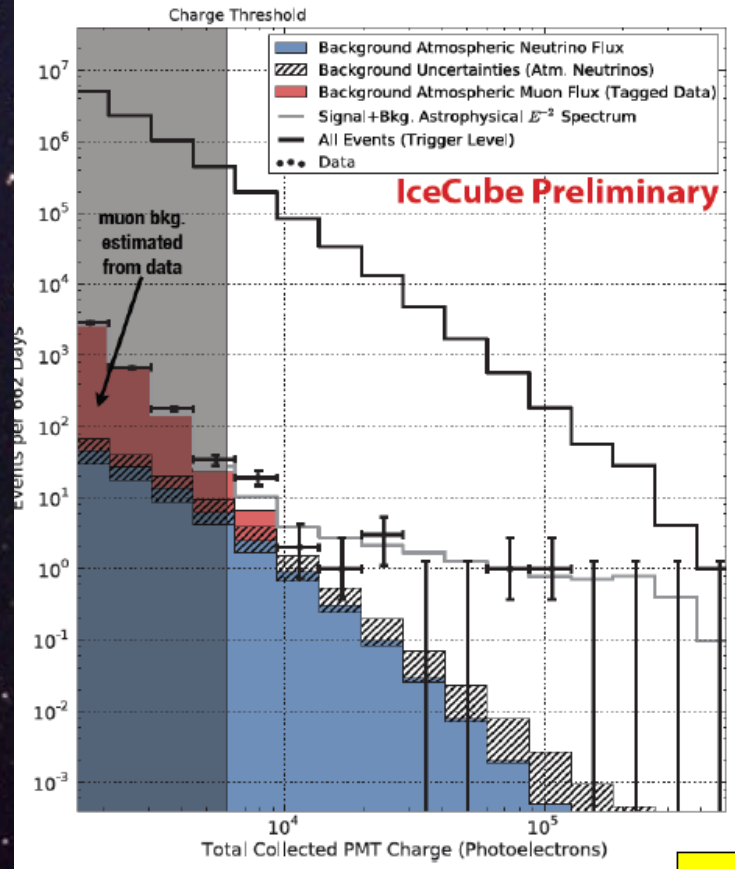
Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector

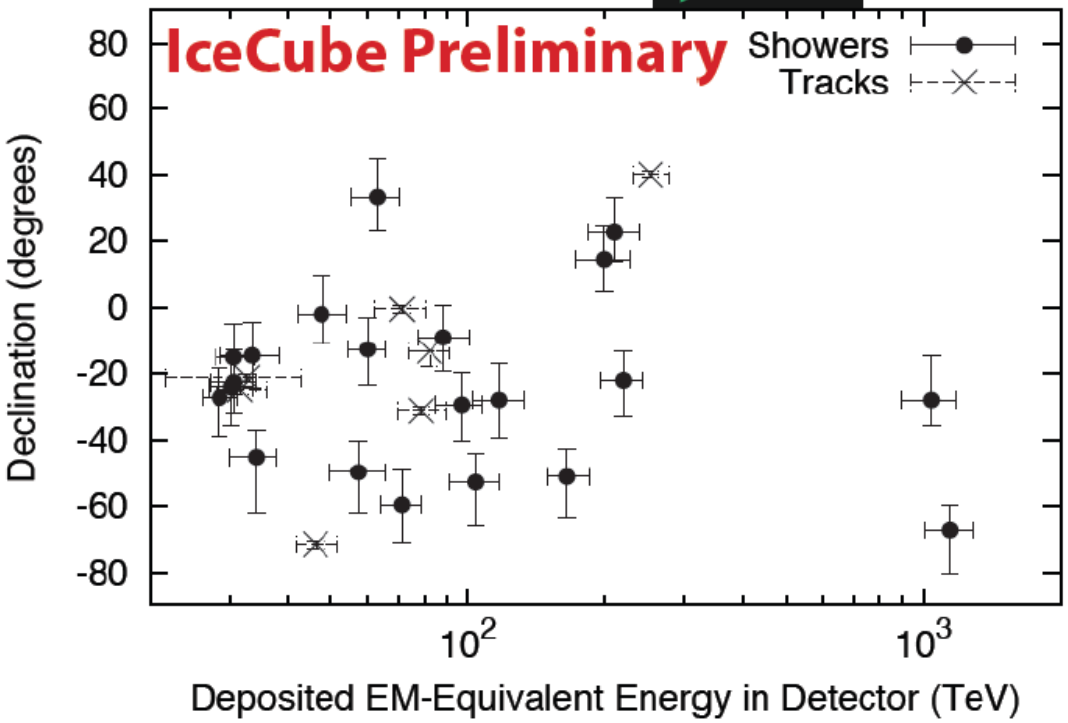
C. Kopper (ID:650)

NU-EX03

Data set: May 2010 – May 2012 (662 days)
Period: 79 and 86 strings
Event selection: Vertex contained in fiducial volume.
(no early timing hits in veto region)







Congratulations!

28 events in 30-1200 TeV

~4σ excess

Background: 10.6 $^{+5.0}_{-3.6}$

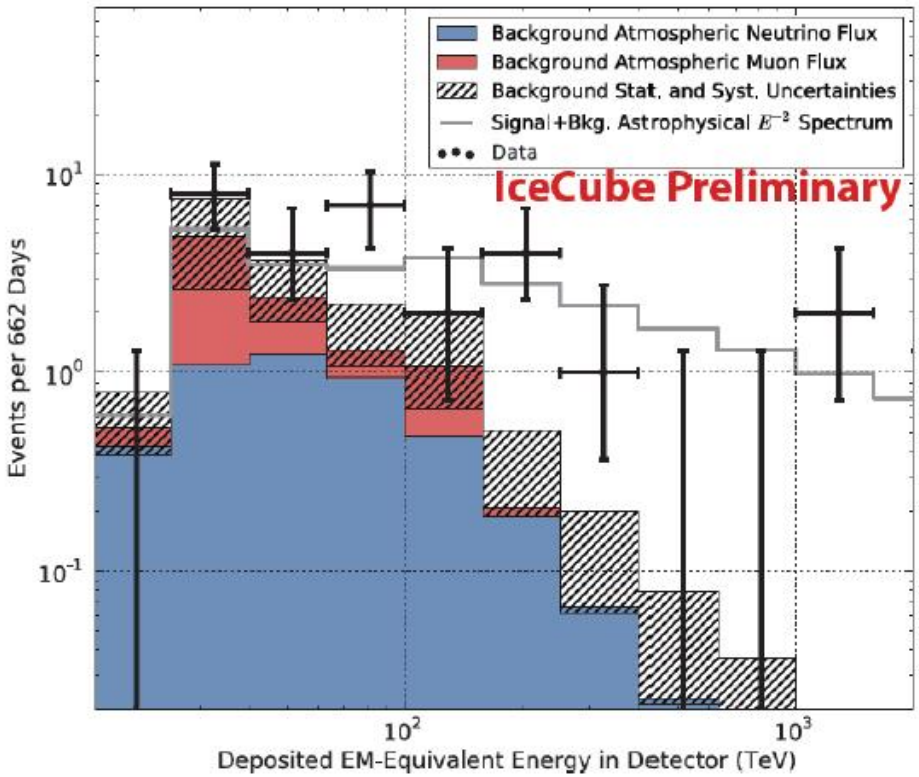
(uncertainty of the prompt component is based on the upper limit of IC59 ν_μ data)

Evidence for High-Energy Extraterrestrial Neutrinos at the IceCube Detector

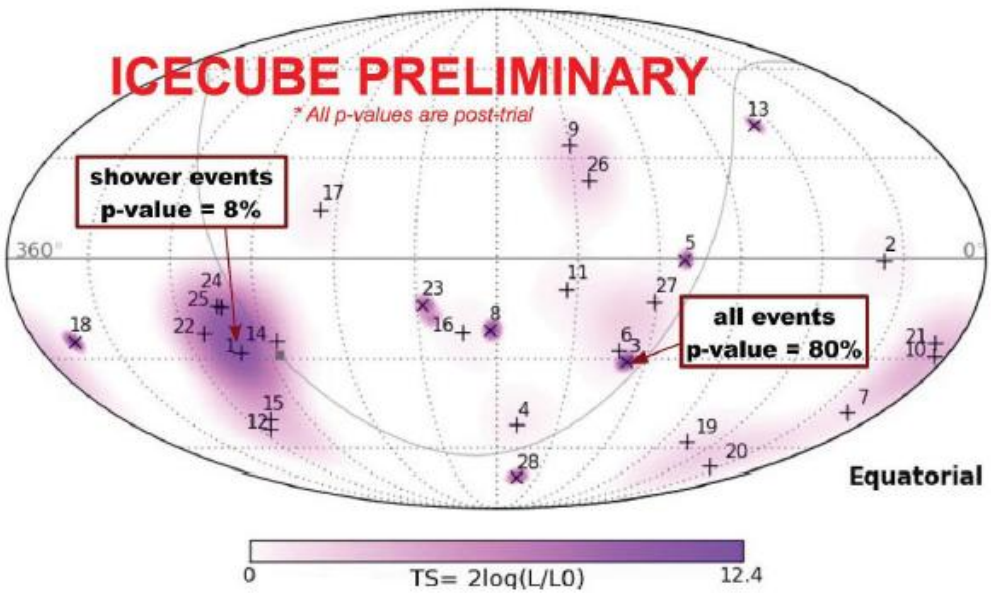
C. Kopper (ID:650)

NU-EX03

Energy spectrum



Skymap



No significant clustering observed?

Potential cutoff at about 2-5 PeV

Best fit: $E^2\phi_\nu = (1.2 \pm 0.4) \times 10^{-8}$ GeV cm^{-2} sr^{-1} s^{-1}

(per ν flavor)

Possible interesting signals in future

Tau neutrinos with double pulses at IceCube

D. Xu(ID:643)

Poster(NU-IN)

A high energy ν_τ charged current (CC) interaction in the ice would produce two showers.

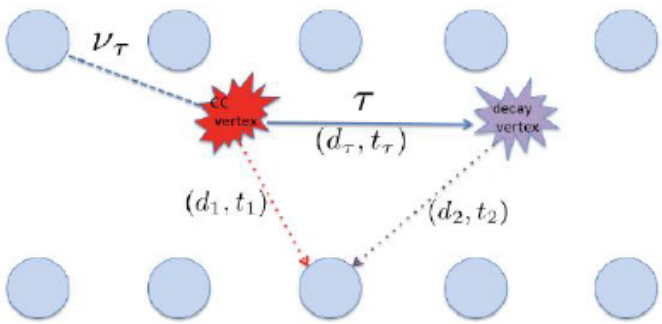


Fig. 2. Sketch of a ν_τ CC event in the IceCube PMT array

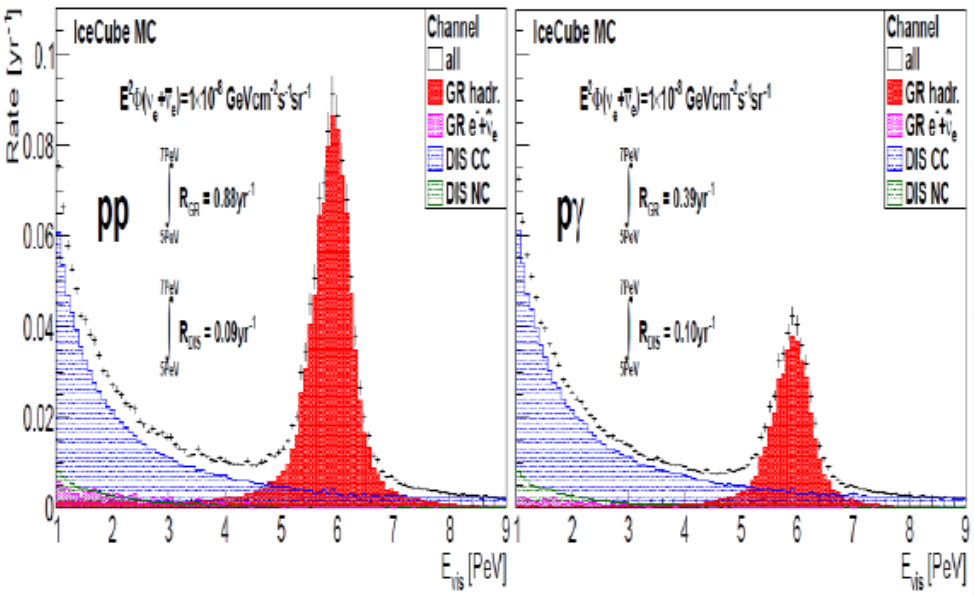
About 0.32±0.04 events/year ν_τ CC events are expected assuming a flux of all neutrino flavors $E^2\phi_\nu$= 3.6x10^{-8} GeVsr^{-1}s^{-1}cm^{-2}.

Glashow resonance at IceCube

J. Kiryluk(ID:494)

Poster(NU-IN)

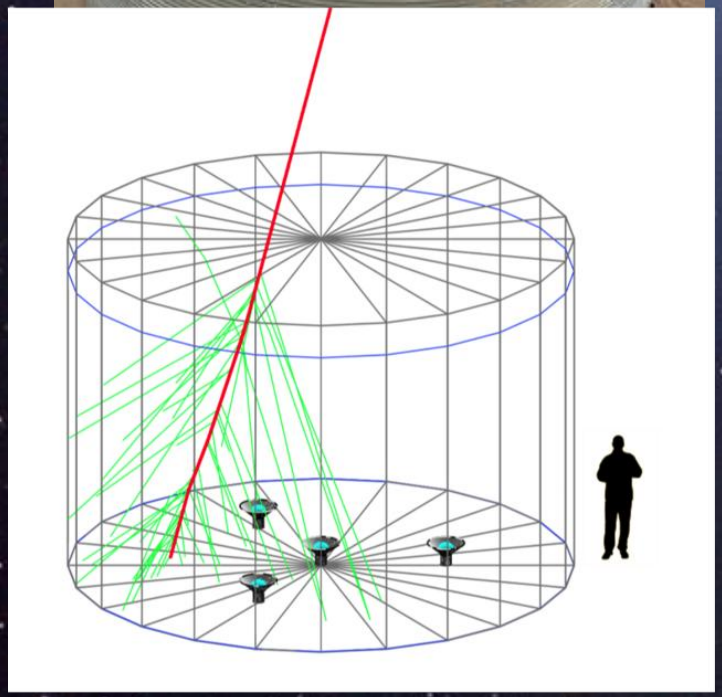
Due to Glashow resonance($\overline{\nu}_e$+e$^-$→W$^-$), event excess is expected at E_ν=~6.3 PeV.

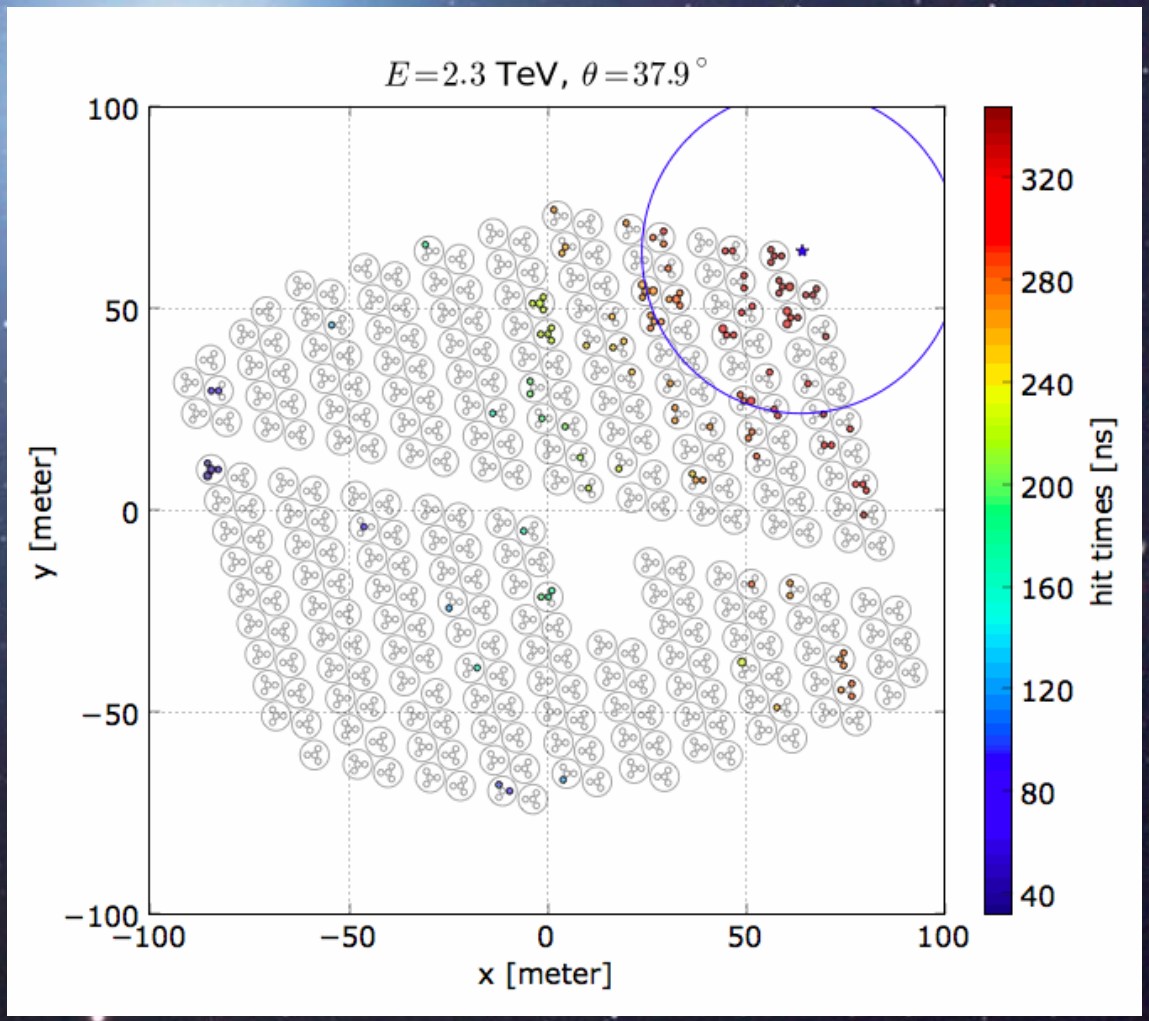


Expect 0.9 (0.4) resonance events/year from pp(pγ) sources for 5PeV< E_{vis}< 7PeV assuming electron type flux of $E^2\,\phi(\nu_e+\overline{\nu}_e)$= 1x10^{-8} GeVsr^{-1}s^{-1}cm^{-2}.

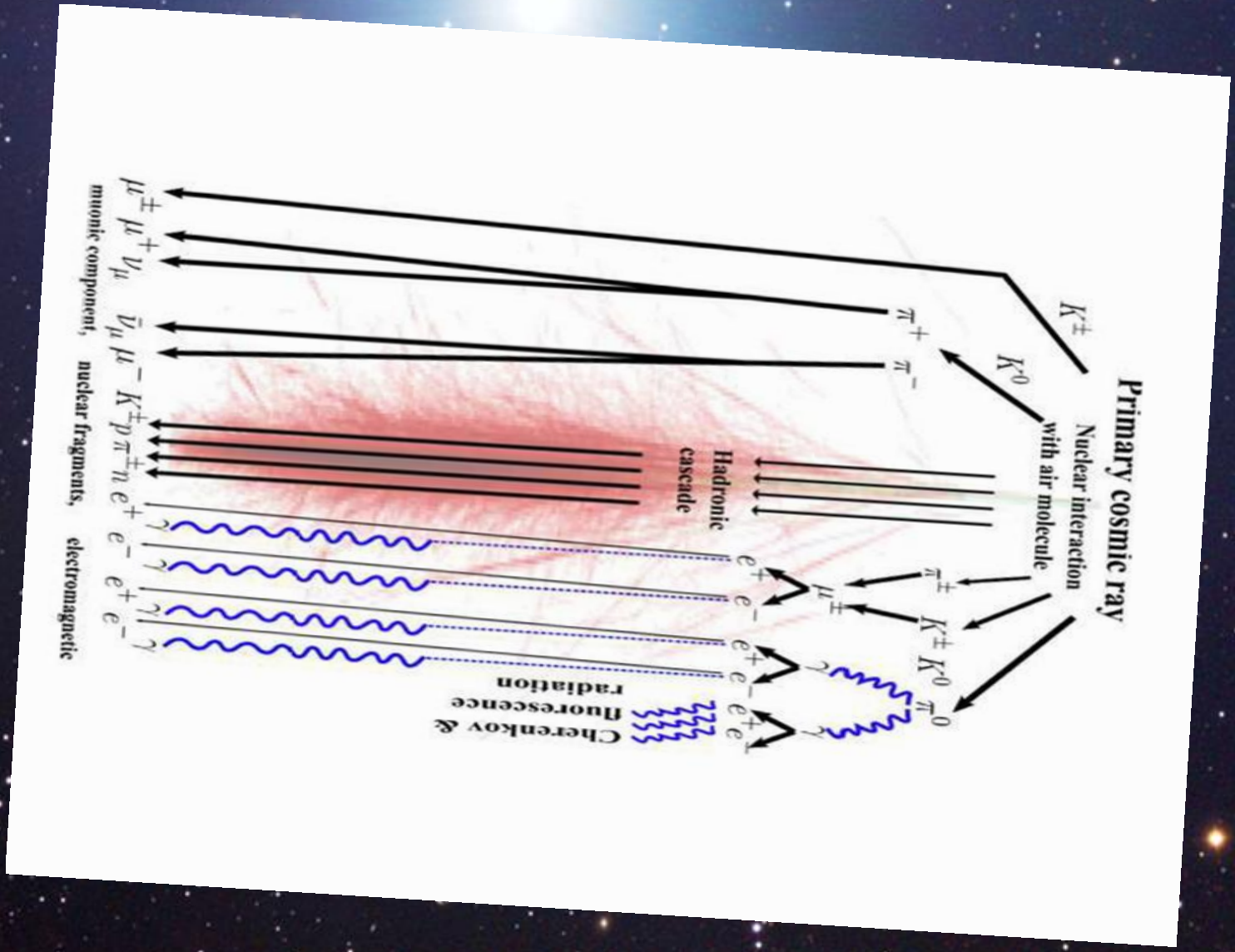
The Simplest- The Best

Array as large as Hawk
But much much spread in few cluster each

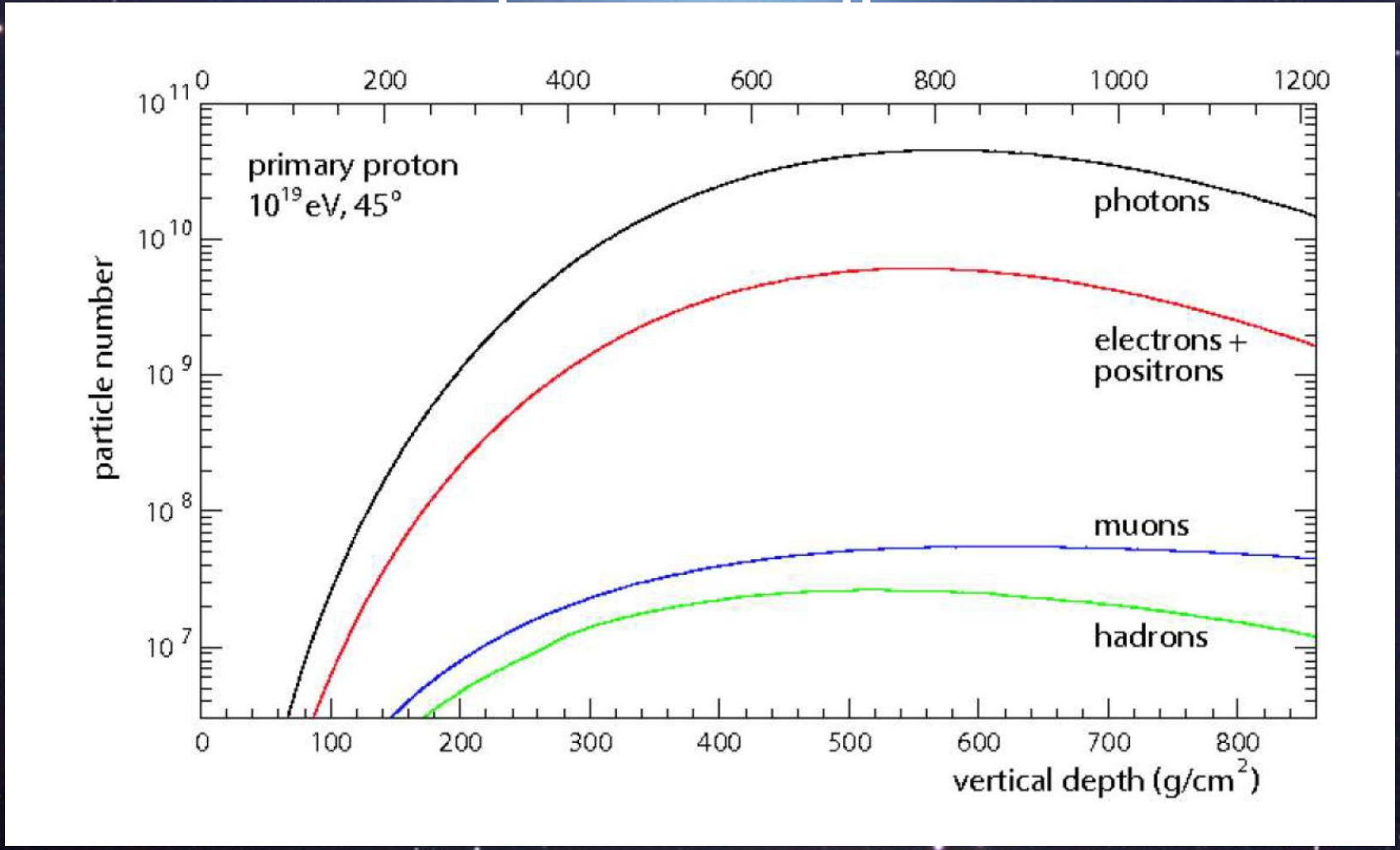




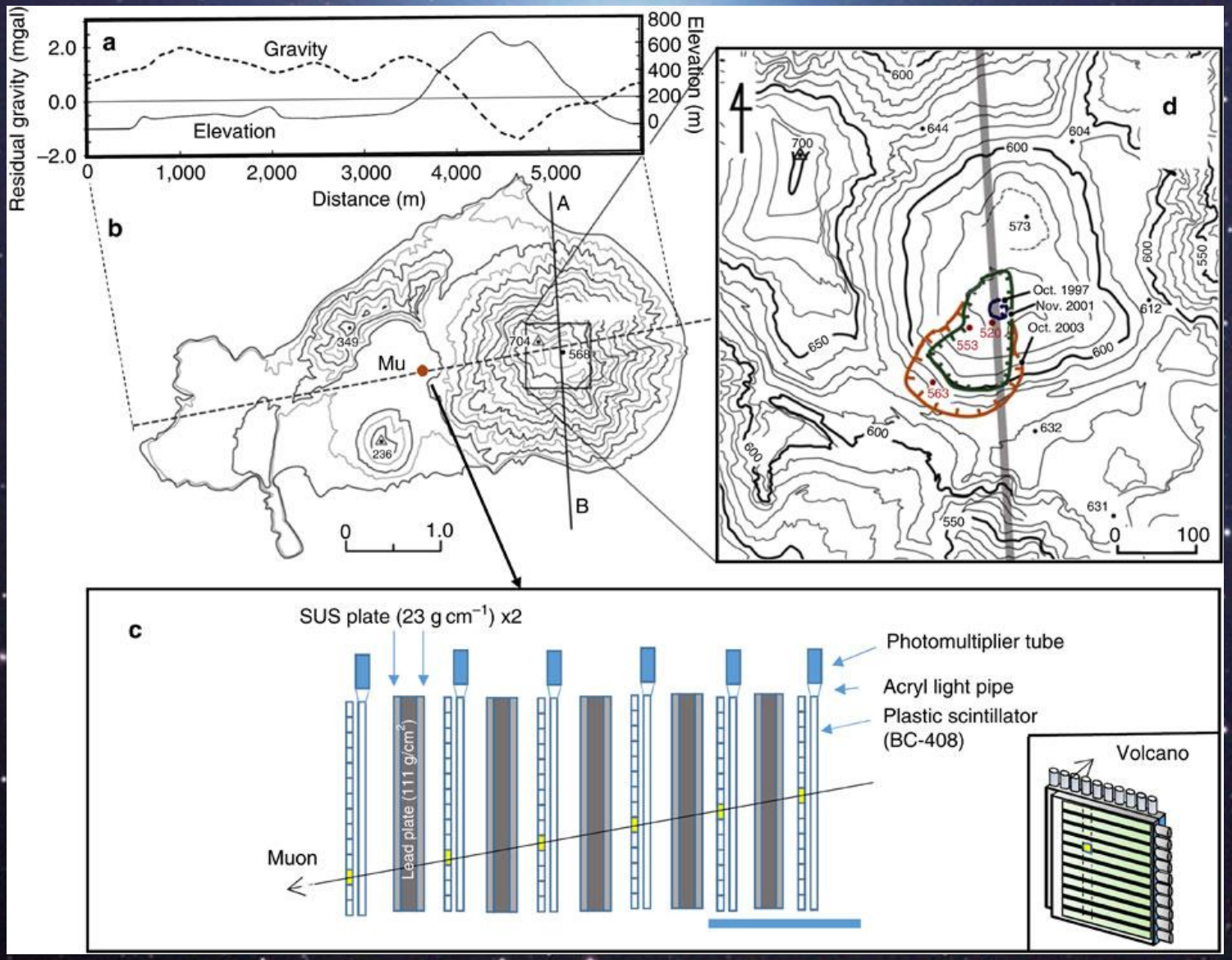
Flux of a 10 EeV Tau Airshower at 100 meter from its axis



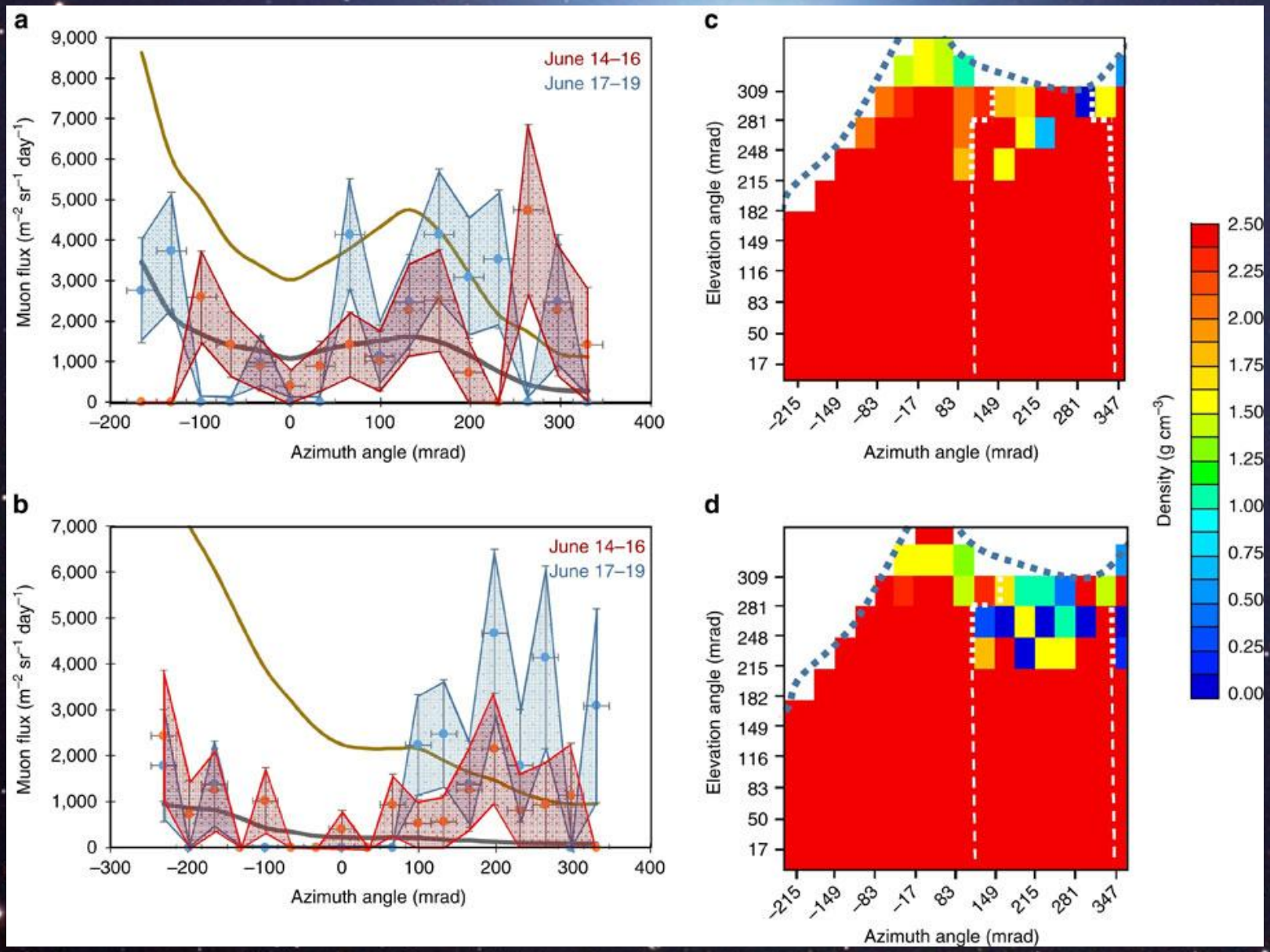
A PeV tau airshower shine a tenth part of a thousand



Muon Radiography



Vulcan Radiography
Comparison of muographic measurement results.



Sistema di 9 equazioni differenziali $P_{\mu\tau}$

$$
\begin{aligned}
& i\ \psi'_{\mu\tau}(x) = \\
& \frac{1}{E_\nu}\,(2.534\ \mathrm{Cos}\,[\theta_{13}]\,(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\Delta_{31} - \mathrm{Sin}\,[\theta_{12}]\,(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \\
& \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\mu e}(x) + 2.534\ \ (\mathrm{Cos}\,[\theta_{13}]^{\,2}\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31} - \\
& (\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}]) \\
& (\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] - \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\mu\mu}(x) + \\
& 2.534\,(\mathrm{Cos}\,[\theta_{13}]^{\,2}\Delta_{31}\mathrm{Cos}\,[\theta_{23}]^{\,2} + (\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \\
& \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])^{\,2}\Delta_{21})\,\psi_{\mu\tau}(x))\,,
\end{aligned}
$$

Sistema di 9 equazioni differenziali

$$i\ \psi'_{\mathrm{ee}}(x) =$$
$$\frac{1}{E_\nu}\{(2.534\ \mathrm{Cos}\,[\theta_{13}]^{\,2}\Delta_{21}\mathrm{Sin}\,[\theta_{12}]^{\,2} + 0.000192\ E_\nu\ \rho(x) + 2.534\ \mathrm{Sin}\,[\theta_{13}]^{\,2}\Delta_{31})\,\psi_{\mathrm{ee}}(x) +$$
$$[\mathrm{Sin}\,[\theta_{12}]\,(2.534\ \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{13}]\,\mathrm{Cos}\,[\theta_{23}] - 1.267\ \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[2\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} +$$
$$1.267\ \mathrm{Sin}\,[2\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31}]\,\psi_{\mathrm{e}\mu}(x) + \mathrm{Cos}\,[\theta_{13}]\,[(-2.534\ \mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{12}]^{\,2} -$$
$$1.267\ \mathrm{Sin}\,[2\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} + 2.534\ \mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\Delta_{31}]\,\psi_{\mathrm{e}\tau}(x)]\,,$$

$$i\ \psi'_{\mu\mathrm{e}}(x) =$$
$$\frac{1}{E_\nu}((2.534\ \mathrm{Cos}\,[\theta_{13}]^{\,2}\Delta_{21}\mathrm{Sin}\,[\theta_{12}]^{\,2} + 0.000192\ E_\nu\ \rho(x) + 2.534\ \mathrm{Sin}\,[\theta_{13}]^{\,2}\Delta_{31})\,\psi_{\mu\mathrm{e}}(x) +$$
$$(\mathrm{Sin}\,[\theta_{12}]\,(2.534\ \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{13}]\,\mathrm{Cos}\,[\theta_{23}] - 1.267\ \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[2\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} +$$
$$1.267\ \mathrm{Sin}\,[2\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31})\,\psi_{\mu\mu}(x) + \mathrm{Cos}\,[\theta_{13}]\,((-2.534\ \mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{12}]^{\,2} -$$
$$1.267\ \mathrm{Sin}\,[2\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} + 2.534\ \mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\Delta_{31})\,\psi_{\mu\tau}(x)\}\,,$$

$$i\ \psi'_{\tau\mathrm{e}}(x) =$$
$$\frac{1}{E_\nu}((2.534\ \mathrm{Cos}\,[\theta_{13}]^{\,2}\Delta_{21}\mathrm{Sin}\,[\theta_{12}]^{\,2} + 0.000192E_\nu\ \rho(x) + 2.534\mathrm{Sin}\,[\theta_{13}]^{\,2}\Delta_{31})\,\psi_{\tau\mathrm{e}}(x) +$$
$$(\mathrm{Sin}\,[\theta_{12}]\,(2.534\ \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{13}]\,\mathrm{Cos}\,[\theta_{23}] - 1.267\ \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[2\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} +$$
$$1.267\ \mathrm{Sin}\,[2\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31})\,\psi_{\tau\mu}(x) + \mathrm{Cos}\,[\theta_{13}]\,((-2.534\ \mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{12}]^{\,2} -$$
$$1.267\ \mathrm{Sin}\,[2\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} + 2.534\ \mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\Delta_{31})\,\psi_{\tau\tau}(x))\,,$$

$$i\ \psi'_{\mathrm{e}\mu}(x) =$$
$$\frac{1}{E_\nu}(2.534\ \mathrm{Cos}\,[\theta_{13}]\,(\mathrm{Sin}\,[\theta_{12}]\,(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] - \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} +$$

Sistema di 9 equazioni differenziali

$$\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31})\,\psi_{\mathrm{ee}}(x) + 2.534\ \ (\mathrm{Cos}\,[\theta_{13}]^{\,2}\Delta_{31}\mathrm{Sin}\,[\theta_{23}]^{\,2} + (\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] -$$
$$\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])^{\,2}\Delta_{21})\,\psi_{\mathrm{e}\mu}(x) + 2.534\,(\mathrm{Cos}\,[\theta_{13}]^{\,2}\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31} -$$
$$(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] -$$
$$\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{e\tau}(x))\,,$$

$$i\ \psi'_{\mu\mu}(x) =$$
$$\frac{1}{E_\nu}\,(2.534\ \mathrm{Cos}\,[\theta_{13}]\,(\mathrm{Sin}\,[\theta_{12}]\,(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] - \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} +$$
$$\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31})\,\psi_{\mu\mathrm{e}}(x) + 2.534\ \ (\mathrm{Cos}\,[\theta_{13}]^{\,2}\Delta_{31}\mathrm{Sin}\,[\theta_{23}]^{\,2} + (\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] -$$
$$\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])^{\,2}\Delta_{21})\,\psi_{\mu\mu}(x) + 2.534\ \ (\mathrm{Cos}\,[\theta_{13}]^{\,2}\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31} -$$
$$(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] -$$
$$\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\mu\tau}(x))\,,$$

$$i\ \psi'_{\tau\mu}(x) =$$
$$\frac{1}{E_\nu}\,(2.534\ \mathrm{Cos}\,[\theta_{13}]\,(\mathrm{Sin}\,[\theta_{12}]\,(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] - \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21} +$$
$$\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31})\,\psi_{\tau e}(x) + 2.534\ \ (\mathrm{Cos}\,[\theta_{13}]^{\,2}\Delta_{31}\mathrm{Sin}\,[\theta_{23}]^{\,2} + (\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] -$$
$$\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])^{\,2}\Delta_{21})\,\psi_{\tau\mu}(x) + 2.534\ \ (\mathrm{Cos}\,[\theta_{13}]^{\,2}\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31} -$$
$$(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] -$$
$$\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\tau\tau}(x))\,,$$

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$$i\ \psi'_{e\tau}(x) =$$
$$\frac{1}{E_\nu}\,(2.534\mathrm{Cos}\,[\theta_{13}]\,(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\Delta_{31} - \mathrm{Sin}\,[\theta_{12}]\,(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] +$$
$$\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\mathrm{ee}}(x) + 2.534\ \ (\mathrm{Cos}\,[\theta_{13}]\,^2\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31} -$$
$$(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])$$
$$(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] - \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\mathrm{e}\mu}(x) +$$
$$2.534\,(\mathrm{Cos}\,[\theta_{13}]\,^2\Delta_{31}\mathrm{Cos}\,[\theta_{23}]\,^2 + (\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] +$$
$$\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,^2\Delta_{21})\,\psi_{e\tau}(x))\,,$$

$$i\ \psi'_{\mu\tau}(x) =$$
$$\frac{1}{E_\nu}\,(2.534\ \mathrm{Cos}\,[\theta_{13}]\,(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\Delta_{31} - \mathrm{Sin}\,[\theta_{12}]\,(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] +$$
$$\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\mu\mathrm{e}}(x) + 2.534\ \ (\mathrm{Cos}\,[\theta_{13}]\,^2\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31} -$$
$$(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])$$
$$(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] - \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\mu\mu}(x) +$$
$$2.534\,(\mathrm{Cos}\,[\theta_{13}]\,^2\Delta_{31}\mathrm{Cos}\,[\theta_{23}]\,^2 + (\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] +$$
$$\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,^2\Delta_{21})\,\psi_{\mu\tau}(x))\,,$$

$$i\ \psi'_{\tau\tau}(x) =$$
$$\frac{1}{E_\nu}\,(2.534\mathrm{Cos}\,[\theta_{13}]\,(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{13}]\,\Delta_{31} - \mathrm{Sin}\,[\theta_{12}]\,(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] +$$
$$\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\tau e}(x) + 2.534\,(\mathrm{Cos}\,[\theta_{13}]\,^2\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{23}]\,\Delta_{31} -$$
$$(\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] + \mathrm{Cos}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{23}])$$
$$(\mathrm{Cos}\,[\theta_{12}]\,\mathrm{Cos}\,[\theta_{23}] - \mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}]\,\mathrm{Sin}\,[\theta_{23}])\,\Delta_{21})\,\psi_{\tau\mu}(x) +$$
$$2.534\,(\mathrm{Cos}\,[\theta_{13}]\,^2\Delta_{31}\mathrm{Cos}\,[\theta_{23}]\,^2 + (\mathrm{Cos}\,[\theta_{23}]\,\mathrm{Sin}\,[\theta_{12}]\,\mathrm{Sin}\,[\theta_{13}] +$$