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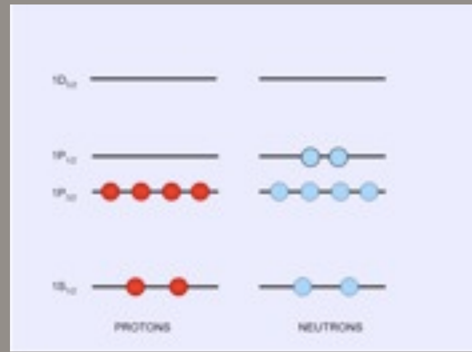
Structure models: from shell models to ab-initio methods

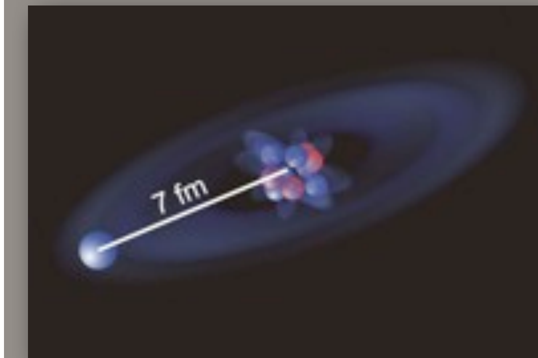
Sonia Bacca | Science Division | TRIUMF

SUMMER SCHOOL UNIVERSITÀ DI PISA

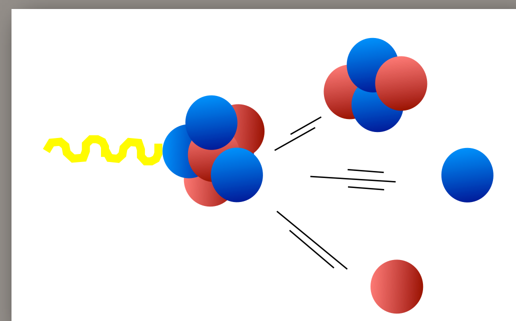
EXOTIC 2015:
Re-writing Nuclear Physics textbooks



Shell Model



Nuclear Halo



Electromagnetic Reactions

Content

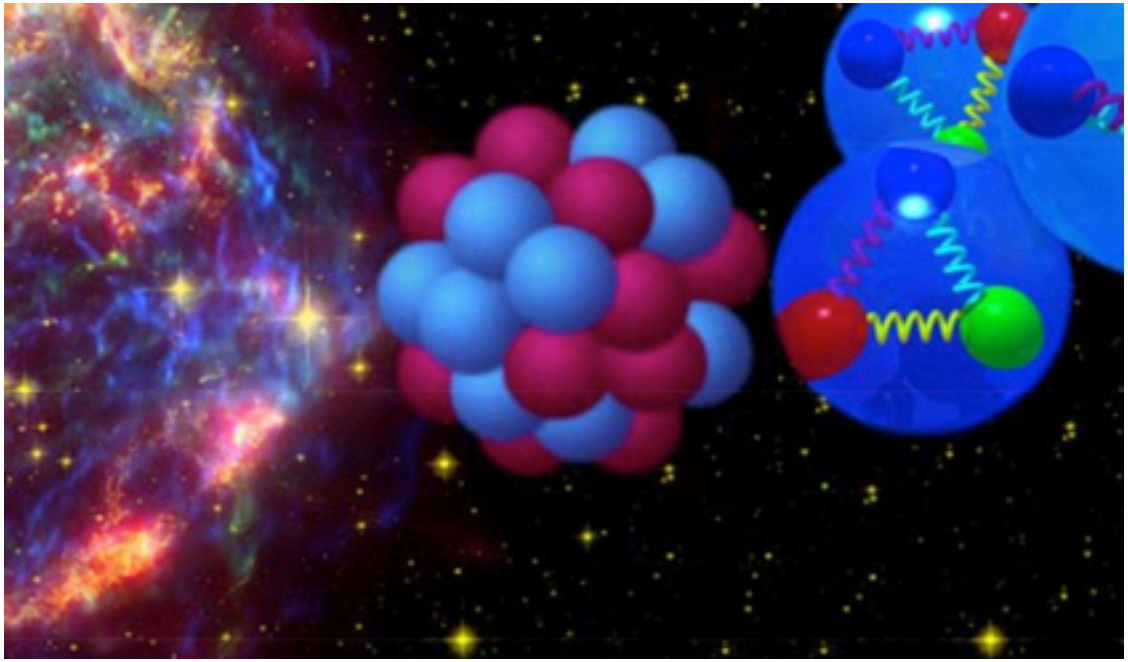
- Evidence of shells in nuclei
- Representation of a many body wave function
- Non interacting shell model
- Interacting shell model
- “Ab-initio” shell model
- Ab-initio methods
  - ➡ Hyperspherical harmonic expansions
  - ➡ Lorentz Integral transform
  - ➡ Coupled-cluster theory
- Selected applications in the physics of exotic nuclei

Nuclear Theory

Our Goal

Develop a unified theory of all nuclei in the nuclear chart

Connecting
to
Astrophysics

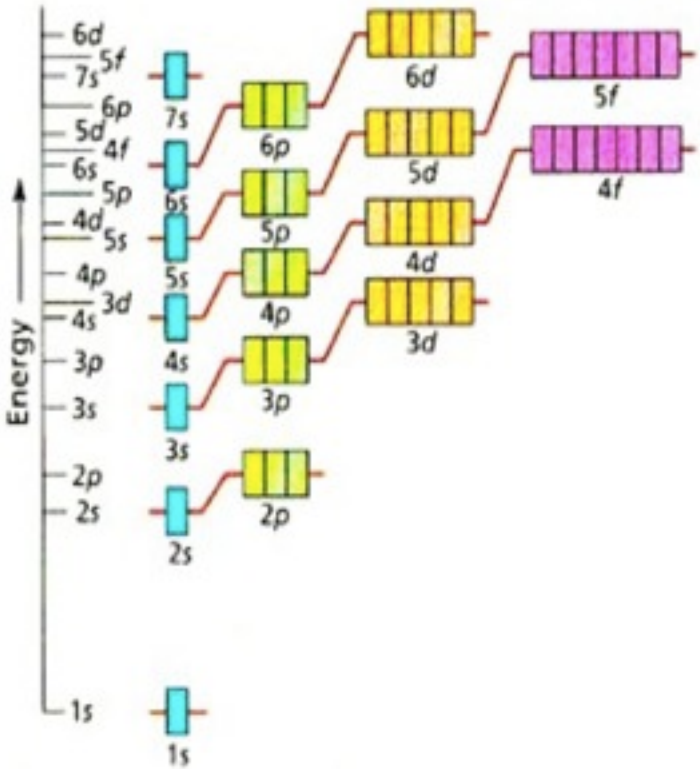


Connecting
to
QCD

$$H|\Psi\rangle = E|\Psi\rangle$$

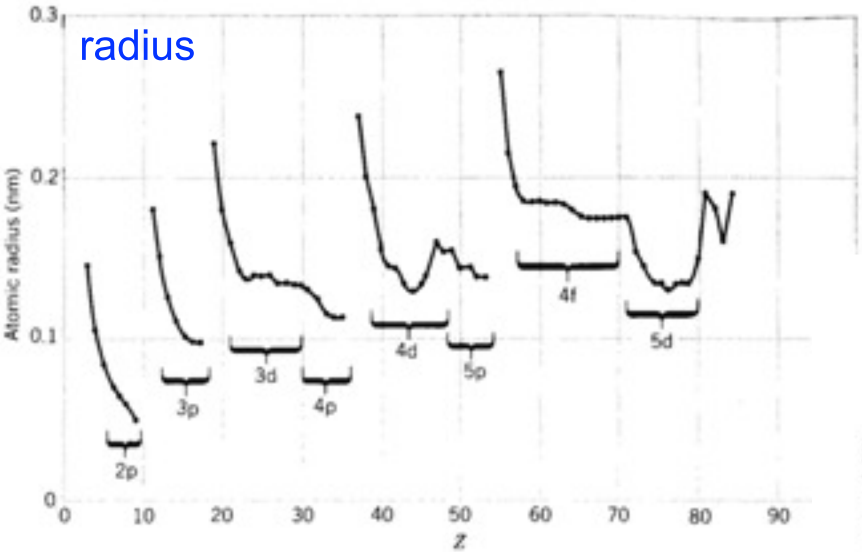
Shell Model in Atoms

Electrons in atoms occupy well defined shells of discrete, well separated energies.

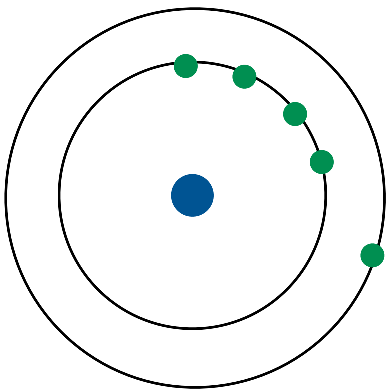


Evidence of electron shells in atoms: sudden jumps in atomic properties as the shell gets filled up, such as atomic radius, ionization energy etc.

Shell Model in Atoms



From Krane
"*Introductory Nuclear Physics*"



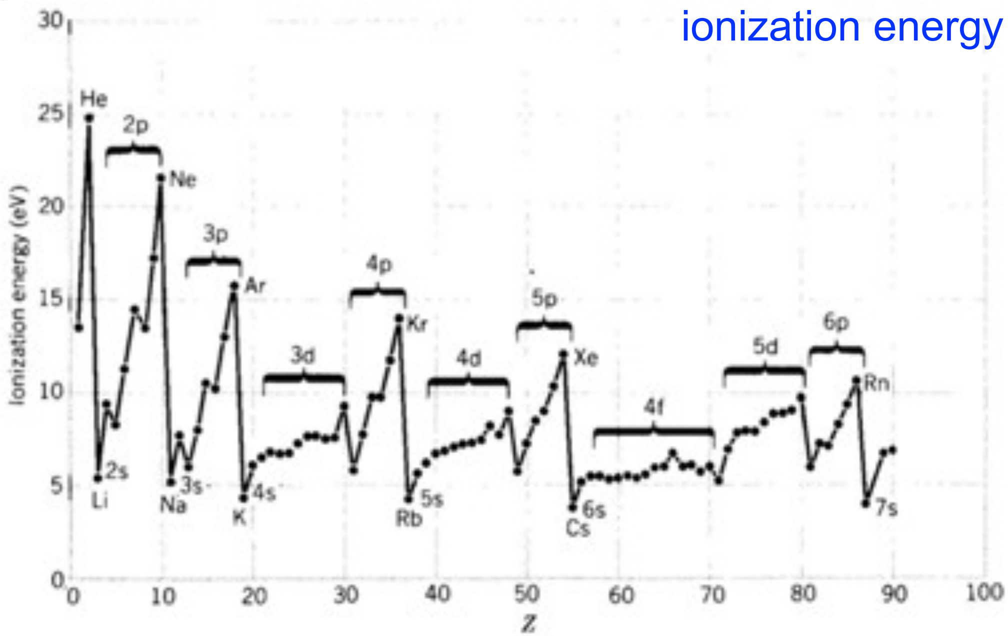


Figure 5.1 Atomic radius (top) and ionization energy (bottom) of the elements. The smooth variations in these properties correspond to the gradual filling of an atomic shell, and the sudden jumps show transitions to the next shell.

Do nucleons inside the nucleus do the same or not?

Friday, 17 July, 15

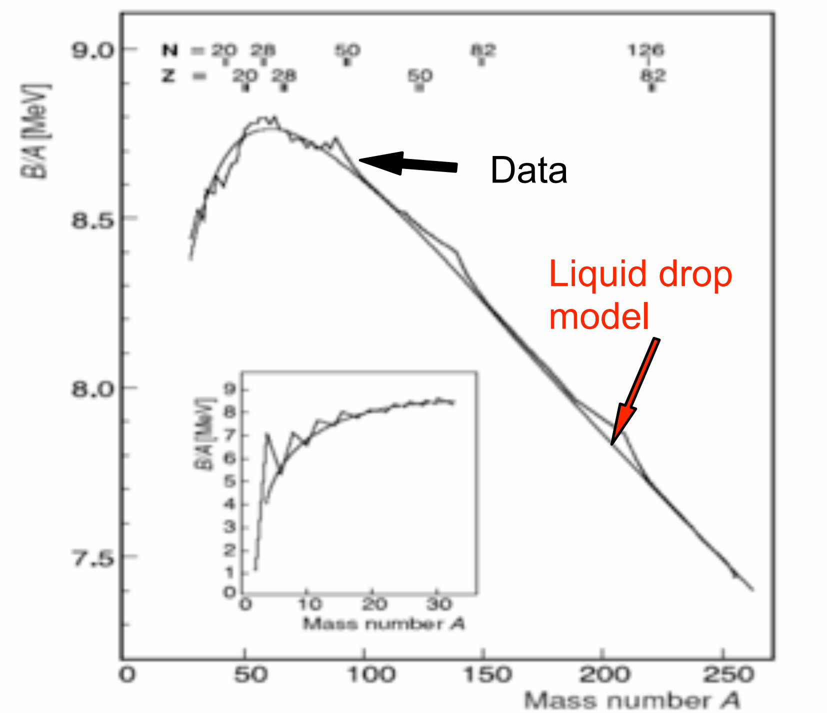
Shell Model in Nuclei?

Shell structure in the nucleus would mean that individual nucleons inhabit orbitals of well defined energy. Not evident a priory why this should be the case. Why?

- The liquid drop model (smooth) is very successful in describing the binging energy.

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{4A} - \frac{\delta}{A^{1/2}}$$

volume term surface term Coulomb term (a)symmetry term pairing term



Von Weizsaecker suggested the liquid drop model in 1935

Shell Model in Nuclei?

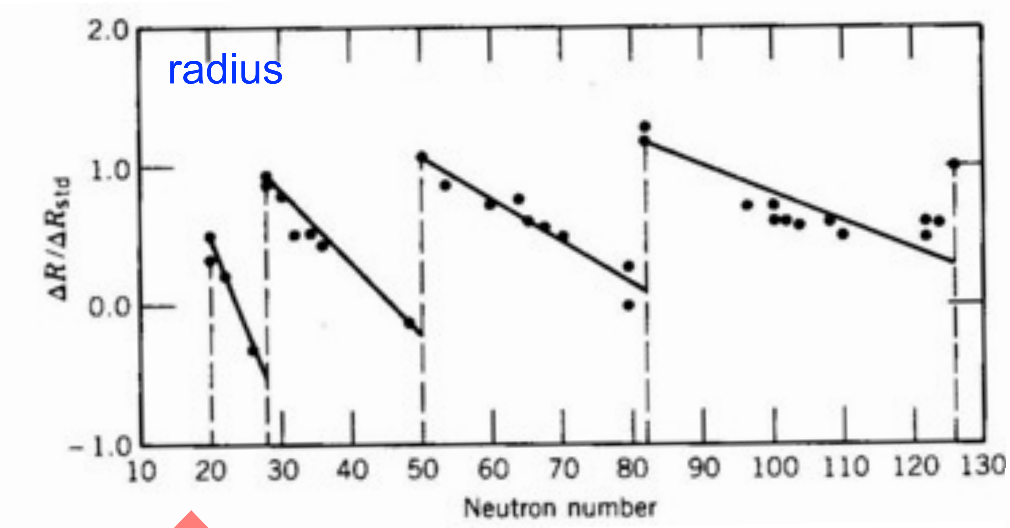
Shell structure in the nucleus would mean that individual nucleons inhabit orbitals of well defined energy. Not evident a priory why this should be the case. Why?

- The liquid drop model (smooth) is very successful in describing the binging energy.
- No obvious centre for nucleons to orbit around.
- No external potential in nuclei, that should be the equivalent of the Coulomb force in atoms.

But the experimental evidence seems to say otherwise!

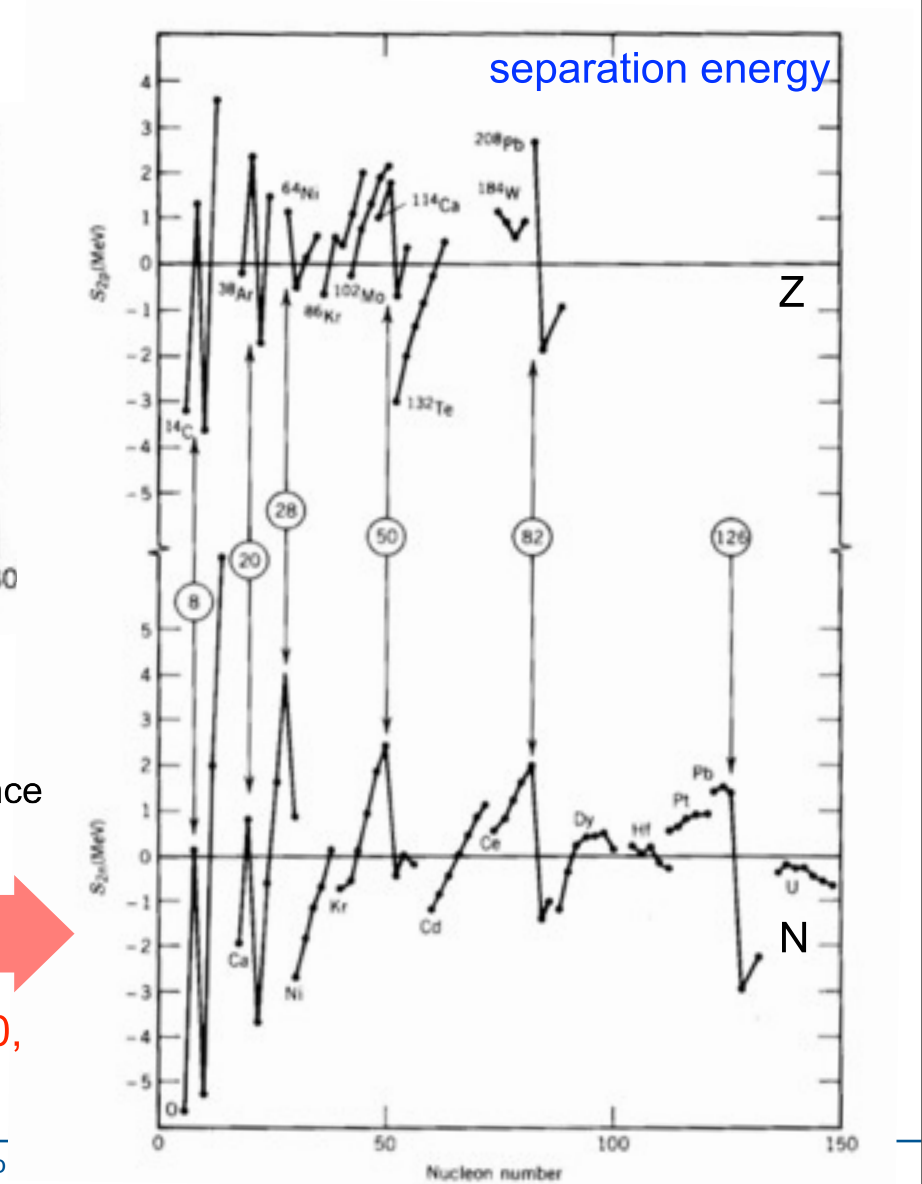
Shell Model in Nuclei?

From Krane "*Introductory Nuclear Physics*"



Here the difference in radius has been divided by the standard ΔR expected from the $A^{-1/3}$ dependence

Here difference between experiment and the prediction of the semi-empirical mass formula.



Jumps/Drops at neutron number: 2, 8, 20, 28, 50, 82, 126 ⇒ evidence of shell structure

Friday, 17 July, 15

Shell Model in Nuclei!

Nuclei exhibit a shell structure!
Experimental data indicate local maxima of the binding energy and local minima of radii in proximity of the neutron or proton
"magic numbers" 2, 8, 20,28, 50, 82, 126 ← only for neutrons

We are physicists, so we do not believe in magic!
Where do these magic numbers come from?
They have to be related to the way nucleons interact with each other.

The theory that explains this is called non interacting shell model or nuclear shell model. It is a simplified theory that accounts though for measured properties and can predict others. It is based on the assumption that the motion of the single nucleon is governed by a potential caused by all other nucleons.

In order to understand where the magic numbers come from and to explain the theory of the nuclear shell model, we need to open a parenthesis on:
- how to represent a many-body wave function
- what is an independent particle model

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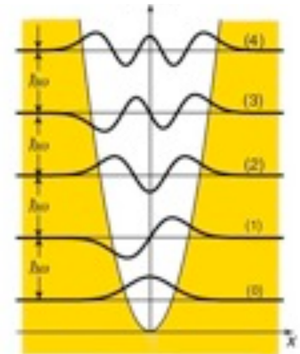
Many-body wave functions

- In order to construct a many-body wave function, one first has to start from a single particle (nucleon) wave function, which is separated in space/spin/isospin components

$$|\varphi_k\rangle = \left[|\varphi_k^{space}\rangle \otimes |\varphi_k^{spin}\rangle\right] \otimes |\varphi_k^{isospin}\rangle$$

This could be the solution of the single nucleon Schrödinger equation

$$h|\varphi_k\rangle = \varepsilon_k|\varphi_k\rangle \qquad (\star)$$



$\{|\varphi_k\rangle\}$ Set of eigenstates of a single nucleon.
Different depending on what Hamiltonian h one uses.
Can assume for now that this is something we can solve analytically or also numerically. E.g., harmonic oscillator

- One can then use these single particle states to construct a many-body wave function. The many-body space is in general the product of many single particle Hilbert spaces

$$H^A = h_1 \otimes h_2 \otimes \cdots \otimes h_A$$

Each single particle state is spanned by $\{|\varphi_k\rangle\}$ as solution of ($\star$)

Many-body wave functions

- We can construct a many-body wave function as the product of single particle wave functions that each live in their own single particle Hilbert space

$$\underbrace{|\psi^A\rangle}_{\text{A-body}} = \underbrace{|\varphi_{k_1}\rangle \otimes |\varphi_{k_2}\rangle \otimes \cdots \otimes |\varphi_{k_A}\rangle}_{\text{1-body}}$$

The symbol $\otimes$ is for an ordered product, which means if you exchange the index 1 with the index 2 you have a different w.f.

$$|\varphi_{k_1}\rangle \otimes |\varphi_{k_2}\rangle \neq |\varphi_{k_2}\rangle \otimes |\varphi_{k_1}\rangle,$$

The first position refers to the first particle, the second position refers to the second particle and so on...

Since we deal with identical particles which are fermions, we need to work with many-body states that are antisymmetrized with respect to the exchange of two particles

Many–body wave functions

- Antisymmetrized many-body wave function

$$|\psi^A\rangle = \mathcal{A}\left\{|\varphi_{k_1}\rangle \otimes |\varphi_{k_2}\rangle \otimes \cdots \otimes |\varphi_{k_A}\rangle\right\}$$

where the antisymmetrizer operator is

$$\mathcal{A} = \frac{1}{A!}\sum_{\text{all perm P}} sign(P)\, P_P$$

mass number

where P_P is the permutation operator

$$sign(P) = (-1)^{n_p}\ ,\ n_p : \text{number of pair exchanges}$$

Example of permutations with A=3 (A!=6)

Permutation	Sign	
1 2 3	+	starting configuration, with assigned sign +
2 3 1	+	$P_{12}(123)=213 \rightarrow P_{13}(213)=231$ — exchange operator - , exchange operator - ; two pair exchanges +
3 1 2	+	
2 1 3	-	
1 3 2	-	$P_{23}(123)=132$ — one pair exchange -
3 2 1	-	

Many-body wave functions

- The Hamiltonian always commute with the antisymmetrization operator $[H, \mathcal{A}] = 0$
  (the Hamiltonian is always written in a symmetric form)
  As a consequence they can have the same eigenvectors

$$|\psi^A\rangle = \mathcal{A}\left\{|\varphi_{k_1}\rangle \otimes |\varphi_{k_2}\rangle \otimes \cdots \otimes |\varphi_{k_A}\rangle\right\}$$

Slater Determinant:
antisymmetrized product of single particle states

Imposing antisymmetrization means respecting Pauli principle $\rightarrow$
If we put two particle in the same state, when we permute, the antisymmetrizer will give zero.

Example A=2
Suppose we neglect spin-isospin now and use a coordinate representation of the single particle states, i.e., $\langle r|\varphi_k\rangle = \varphi_k(r)$

$$\langle r_1| \otimes \langle r_2|\, \mathcal{A}\left\{|\varphi_{k_1}\rangle \otimes |\varphi_{k_2}\rangle\right\} = \langle r_1| \otimes \langle r_2|\; \left(|\varphi_{k_1}\rangle \otimes |\varphi_{k_2}\rangle - |\varphi_{k_2}\rangle \otimes |\varphi_{k_1}\rangle\right)\frac{1}{2}$$

$$= \frac{1}{2}\left(\varphi_{k_1}(r_1)\varphi_{k_2}(r_2) - \varphi_{k_2}(r_1)\varphi_{k_1}(r_2)\right) = \frac{1}{2}\det\begin{pmatrix} \varphi_{k_1}(r_1) & \varphi_{k_1}(r_2) \\ \varphi_{k_2}(r_1) & \varphi_{k_2}(r_2) \end{pmatrix}$$

here particle one is in state k_2

Many-body wave functions

Case of A particles coordinate space representation

Slater Determinant

$$\langle r_1, r_2, ..., r_A | \psi^A \rangle = \frac{1}{A!} \det \begin{pmatrix} \varphi_{k_1}(r_1) & \varphi_{k_1}(r_2) & \cdots & \varphi_{k_1}(r_A) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{k_A}(r_1) & \varphi_{k_A}(r_2) & \cdots & \varphi_{k_A}(r_A) \end{pmatrix}$$

Determinant of an AxA matrix, with same particle in each column and same single particle state in each row.

It is a (simple) way to construct antisymmetrized states.

Exercise: Play with the three-particle Slater Determinant.

Independent particle model

In an independent particle model it is assumed that particles do not interact with each other. They are only subject to the Pauli principle.

Formally this means that one can write the Hamiltonian for A particles as

$$H = \sum_i^A h_i, \quad h_i : \text{single particle Hamiltonian}$$

Note: there is nothing that connects particle i with particle j

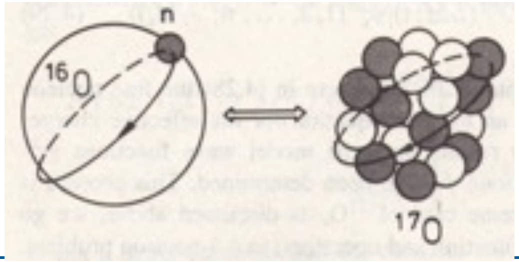
Examples:

$$h_i = \frac{p_i^2}{2m}$$ only kinetic energy (Fermi gas models with SD of plane waves)

$$h_i = \frac{p_i^2}{2m} + U_i$$

U_i is the potential felt by particle i, which could be an external potential like the Coulomb force in atoms or an average potential given to *i* by the presence of all the other A-1 particles.

Assumption: The interaction of a nucleon with ALL the other particles is approximated by a "mean" potential



Independent particle model

$$H = \sum_i^A h_i$$ The solution of such Hamiltonian is obtained by solving the single particle Schrödinger equation

$$h_i|\varphi_k\rangle = \varepsilon_k|\varphi_k\rangle \implies h_i\varphi_k(r_i) = \varepsilon_k\varphi_k(r_i)$$ in coordinate space representation

Then the A-body states are just Slater Determinants of single particle states $\varphi_k(r_i)$

The solution of $H|\psi^A\rangle = E|\psi^A\rangle$ has the following energy

$$E = \sum_k^A \varepsilon_k \boxed{d_k} \longrightarrow$$ degeneracy: measures the occupancy of a single particle state

Exercise: To convince yourself, prove that this is true for A=2

Independent particle model

$$H = \sum_{i}^{A} h_i$$

The solution of such Hamiltonian is obtained by solving the single particle Schrödinger equation

$$h_i|\varphi_k\rangle = \varepsilon_k|\varphi_k\rangle \implies h_i\varphi_k(r_i) = \varepsilon_k\varphi_k(r_i)$$

in coordinate space representation

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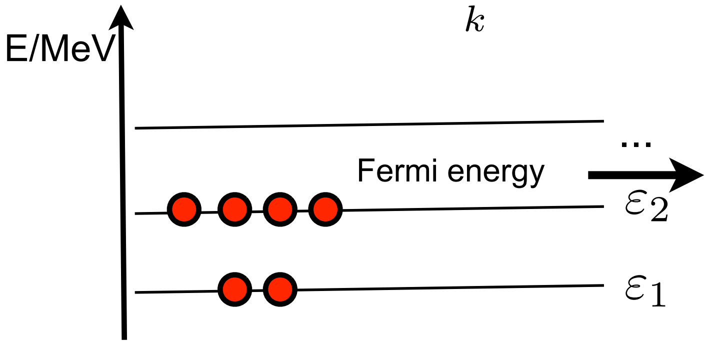
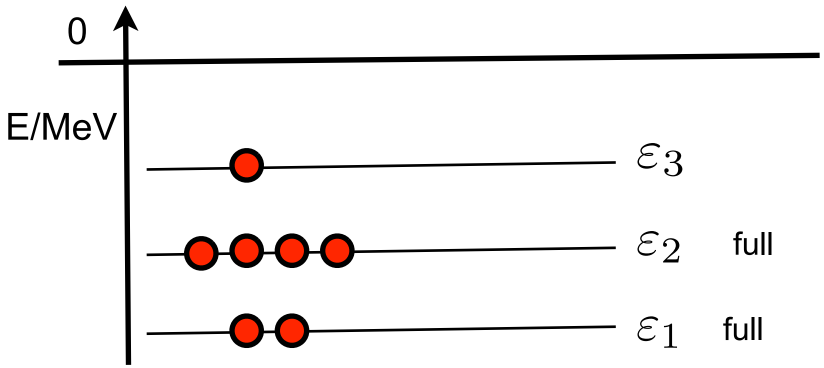


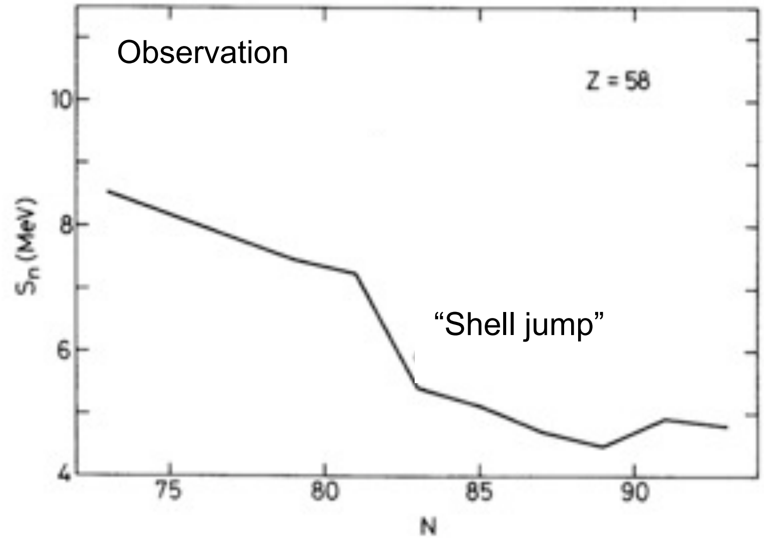
energy of the highest occupied state

Fill in the shells respecting Pauli principle (d_k) up to A particles

Magic numbers arise because the single particle spectrum is not smooth, but is made by discrete levels. Particles are grouped into shells with relatively large gaps between them.

Separation Energies





$$E_A = \sum_k^A \varepsilon_k d_k$$ g.s. energy (neg. number)

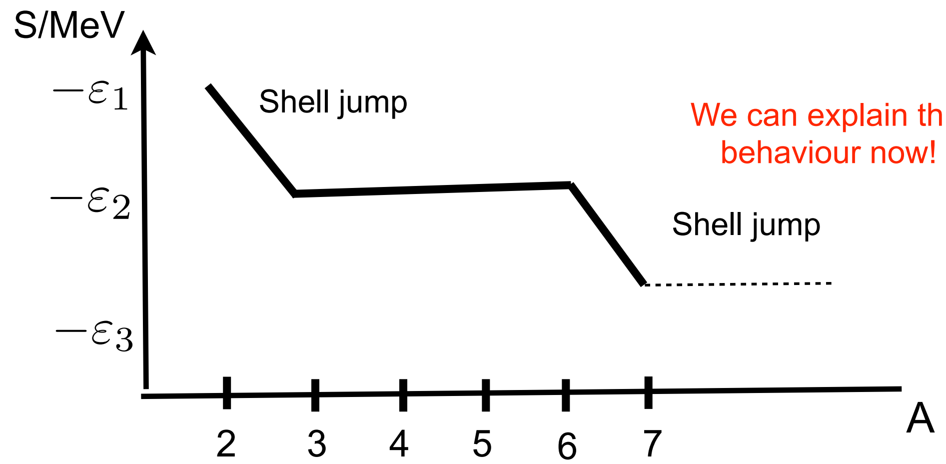
Energy of A-body system

$$E_7 = 2\varepsilon_1 + 4\varepsilon_2 + \varepsilon_3$$
$$E_6 = 2\varepsilon_1 + 4\varepsilon_2$$
$$E_5 = 2\varepsilon_1 + 3\varepsilon_2$$
$$E_4 = 2\varepsilon_1 + 2\varepsilon_2$$
$$E_3 = 2\varepsilon_1 + \varepsilon_2$$
$$E_2 = 2\varepsilon_1$$
$$E_1 = \varepsilon_1$$

Separation energy

$$S^{(A)} = BE(A) - BE(A-1) = E_{A-1} - E_A$$

$$S^{(7)} = -\varepsilon_3$$
$$S^{(6)} = -\varepsilon_2$$
$$S^{(5)} = -\varepsilon_2$$
$$S^{(4)} = -\varepsilon_2$$
$$S^{(3)} = -\varepsilon_2$$
$$S^{(2)} = -\varepsilon_1$$



We can explain this behaviour now!

Friday, 17 July, 15

Nuclear shell model

Case of the spherical HO potential

$$U_i = \frac{1}{2} m\omega^2 r_i^2 \quad \Longrightarrow \quad h_i = \frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 r_i^2$$

$$h_i \; \varphi(\vec{r}_i) = \varepsilon_k \; \varphi(\vec{r}_i)$$

HO in 3 dimensions

k bunch of quantum numbers $k = n\ell m$

For every particle (omit *i* index)

$$\varphi_{n\ell m}(\vec{r}\,) = R_{n\ell}(r) \; Y_{\ell m}(\hat{r})$$

analytical solution of the radial equation

spherical harmonics

n radial quantum number

ℓ, m quantum numbers related to angular momentum and its projection

$$\varepsilon_{n\ell} = \left(N + \frac{3}{2}\right)\hbar\omega = \left(\underbrace{2(n-1)+\ell}_{N} + \frac{3}{2}\right)\hbar\omega = \varepsilon_N$$

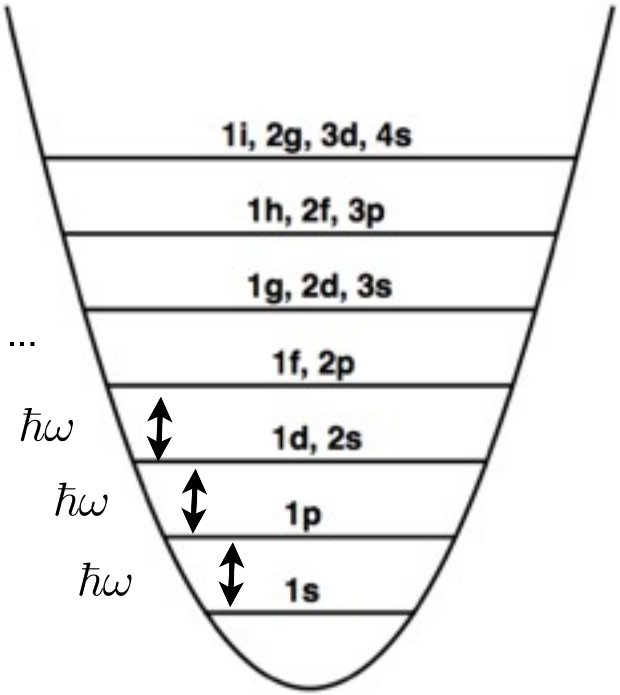
with degeneracy $d_N = 2 \;\; (2\ell + 1)$

two possible spin projections

all possible values of m for a given

Nuclear shell model

Case of the spherical HO potential



The integrated degeneracy is related to the magic numbers

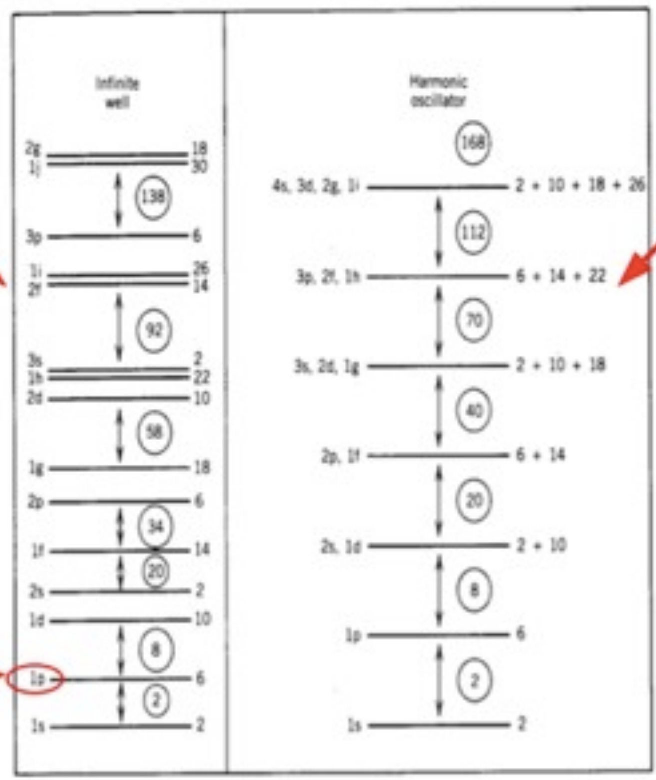
N	E_N	d_N	$\boxed{\sum_N d_N}$	$n(l)$	parity
0	$\frac{3}{2}\hbar\omega$	2	2	1s	+
1	$\frac{5}{2}\hbar\omega$	6	8	1p	-
2	$\frac{7}{2}\hbar\omega$	12	20	1d, 2s	+
3	$\frac{9}{2}\hbar\omega$	20	40	1f, 2p	-
4	$\frac{11}{2}\hbar\omega$	30	70	1g, 2d, 3s	+
5	$\frac{13}{2}\hbar\omega$	42	112	1h, 2f, 3p	-
6	$\frac{15}{2}\hbar\omega$	56	168	1i, 2g, 3d, 4s	+

The magic numbers are wrong after the first three!

Nuclear shell model

Does it depend on the “mean” potential we chose? We can do the same using a different U_i

infinite square well potential



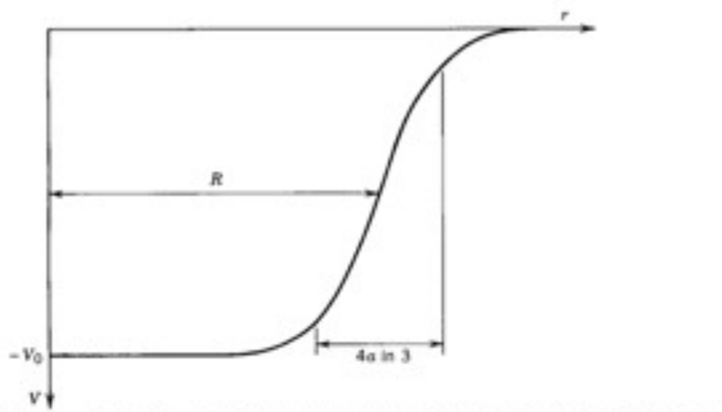
harmonic oscillator potential

Figure 5.4 Shell structure obtained with infinite well and harmonic oscillator potentials. The capacity of each level is indicated to its right. Large gaps occur between the levels, which we associate with closed shells. The circled numbers indicate the total number of nucleons at each shell closure.

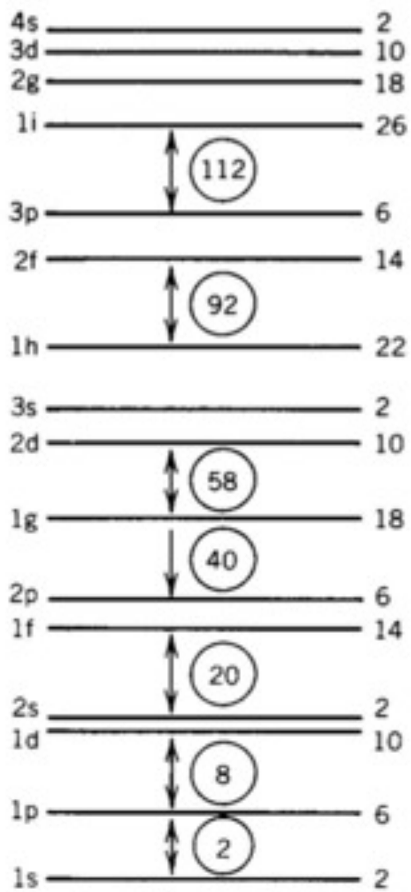
Krane, Introductory Nuclear Physics

Nuclear shell model

One can try to use a Wood-Saxton form for U_i



From Krane "*Introductory Nuclear Physics*"



Still the empirical magic number are not reproduced
2, 8, 20,28, 50, 82, 126

None of these single particle potentials seemed to work properly

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Nuclear shell model

Fermi's suggestion: any evidence for a spin-orbit force?

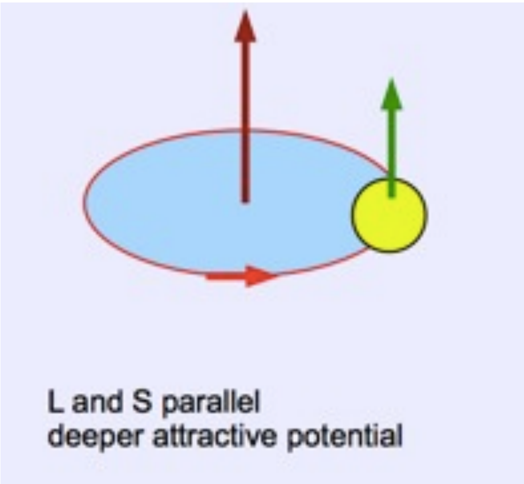
Mean field central potential plus an empirical spin-orbit term like

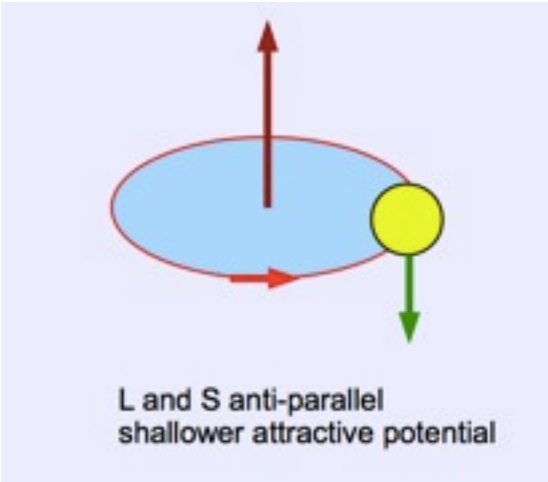
$$U(r) = V_0(r) + V_{\ell s}(r)\ \vec{\ell}\cdot\vec{s}$$

$\vec{\ell}$ orbital angular momentum

$\vec{s}$ spin (intrinsic) angular momentum

with $V_0(r), V_{\ell s}(r)$ being negative (attractive potentials)



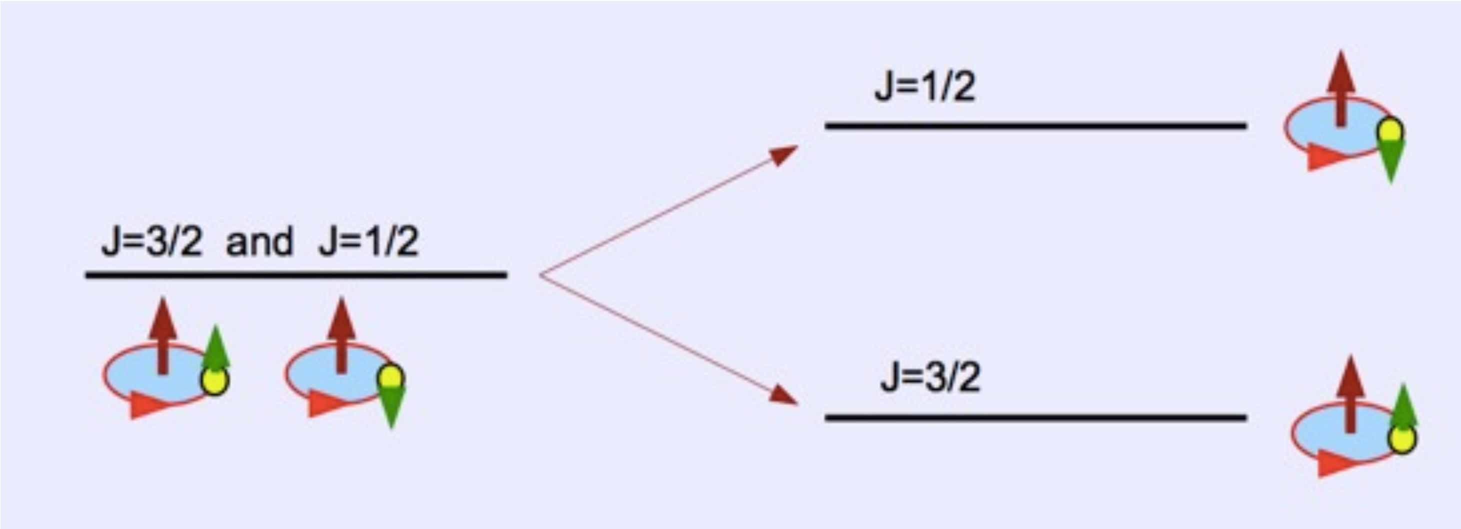


Nuclear shell model

Spin-orbit splitting

Now the good quantum numbers is j, so we have to consider the angular momentum coupling

$$\vec{j} = \vec{\ell} + \vec{s} = \vec{\ell} + \frac{\vec{1}}{2} = \begin{cases} j = \frac{3}{2} \\ j = \frac{1}{2} \end{cases} \qquad \text{for } \ell = 1$$

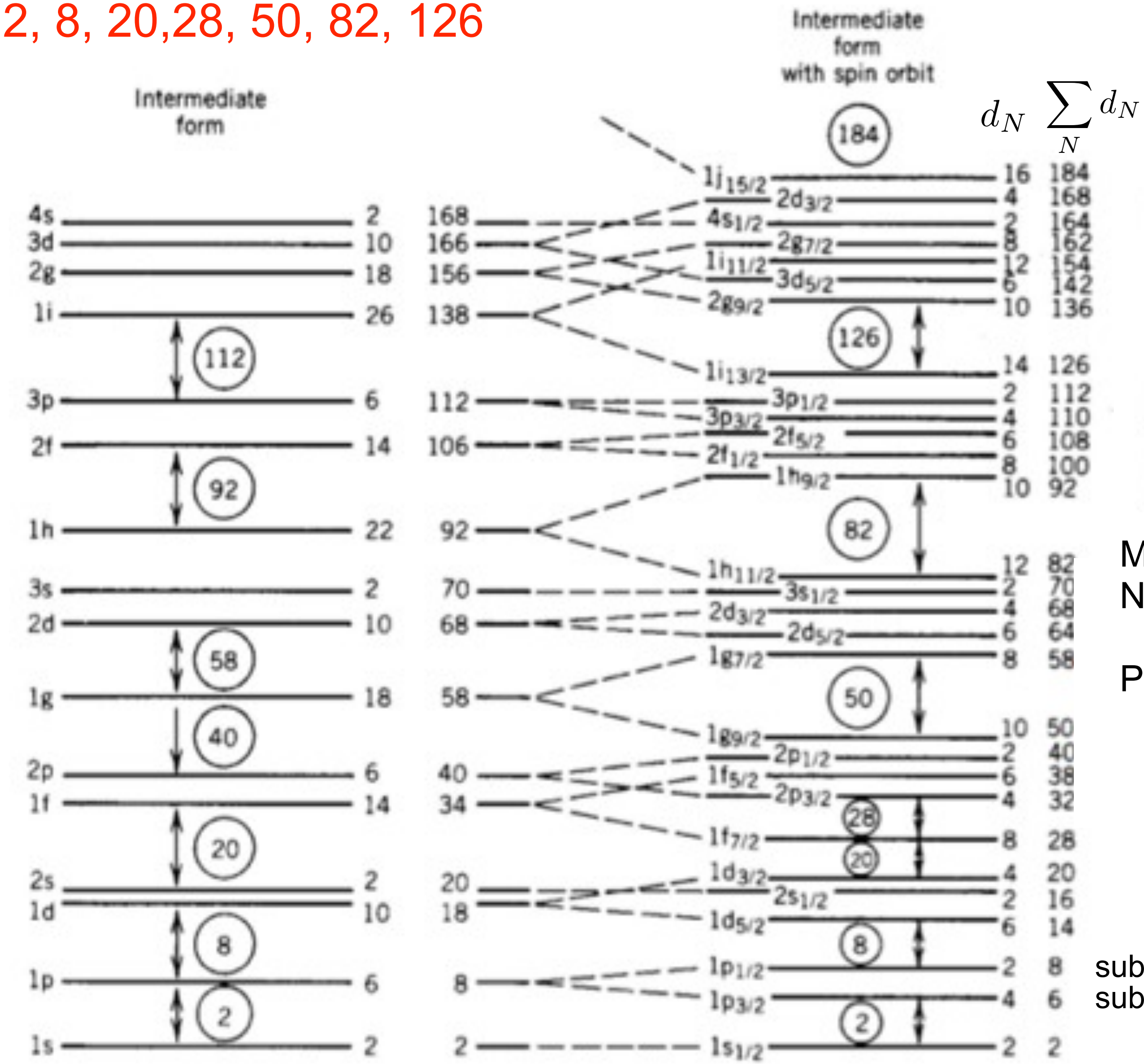


no spin-orbit force $\Longrightarrow$
degenerate levels

with spin-orbit $\Longrightarrow$
splitting of the levels

Nuclear shell model

With the addition of the spin-orbit, the magic numbers are reproduced
2, 8, 20,28, 50, 82, 126



Maria Goppert-Mayer and Hans Jensen
Nobel prize in 1963

Phys. Rev. **75**, 1969 (1949)

Degeneracy with spin-orbit force

$$d_N = (2j+1)$$

sub-shell
sub-shell } P-shell

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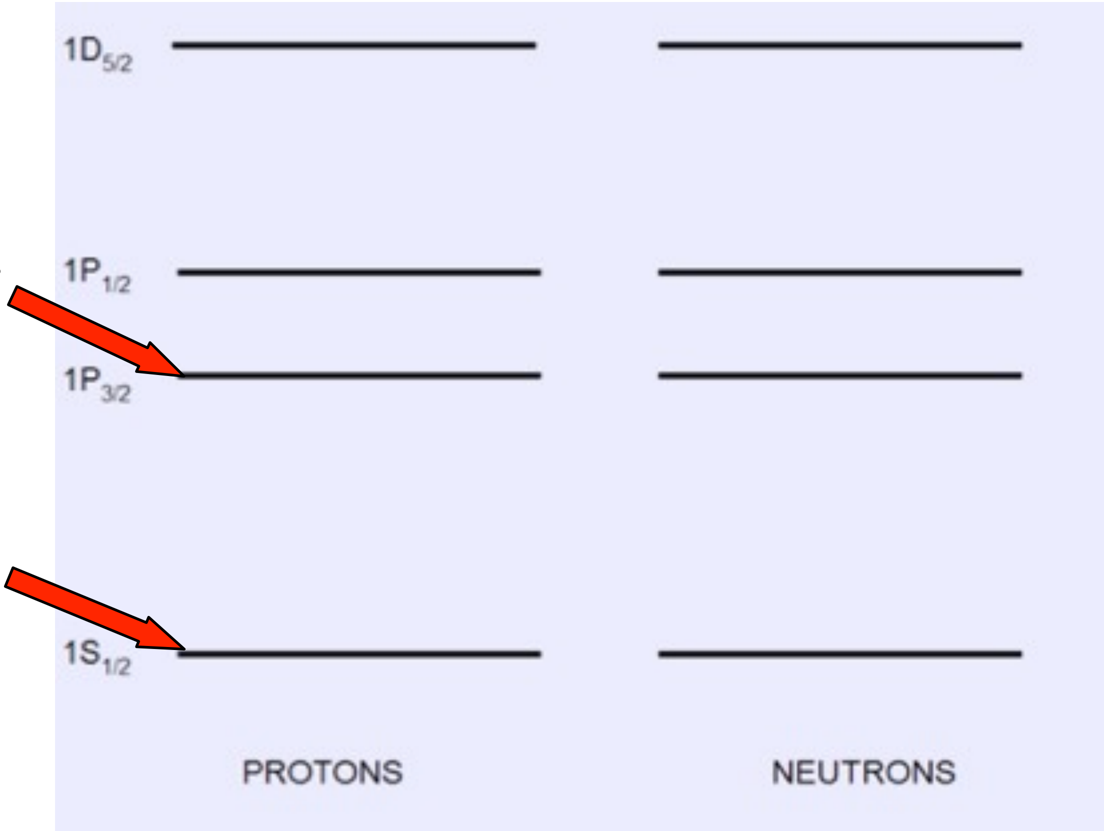
Nuclear shell model

Now we can build up the shell structure of nuclei like, just like we fill up the electron shells in atoms. Only now there are separate proton and neutron shells.

J=3/2, there are four possible projections J_z=3/2,1/2,-1/2,-3/2;
therefore a maximum of 4 nucleons can stay in this sub-shell

J=1/2, there are two possible projections J_z=+1/2,-1/2; therefore a maximum of 2 nucleons can stay in this shell



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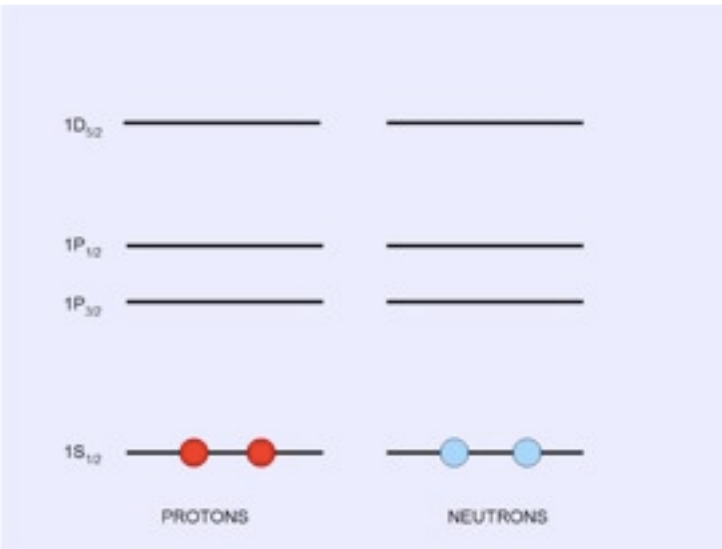
Nuclear shell model

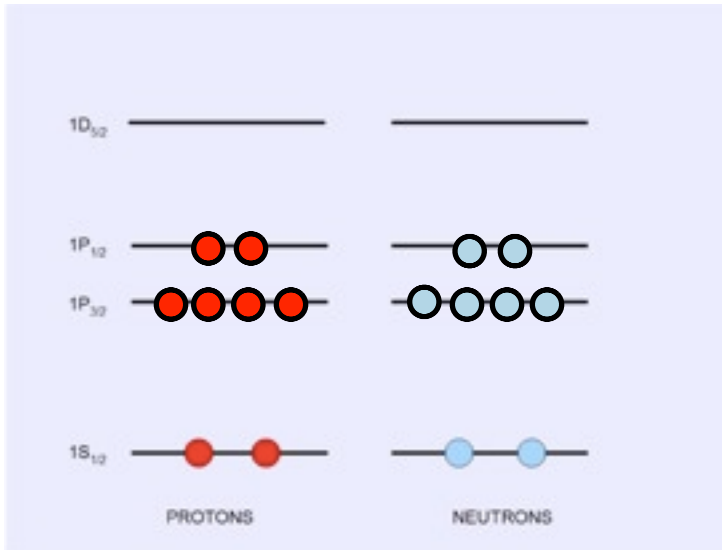
Now we can build up the shell structure of nuclei like, just like we fill up the electron shells in atoms. Only now there are separate proton and neutron shells.

^{4}He



double closed s-shell

^{16}O

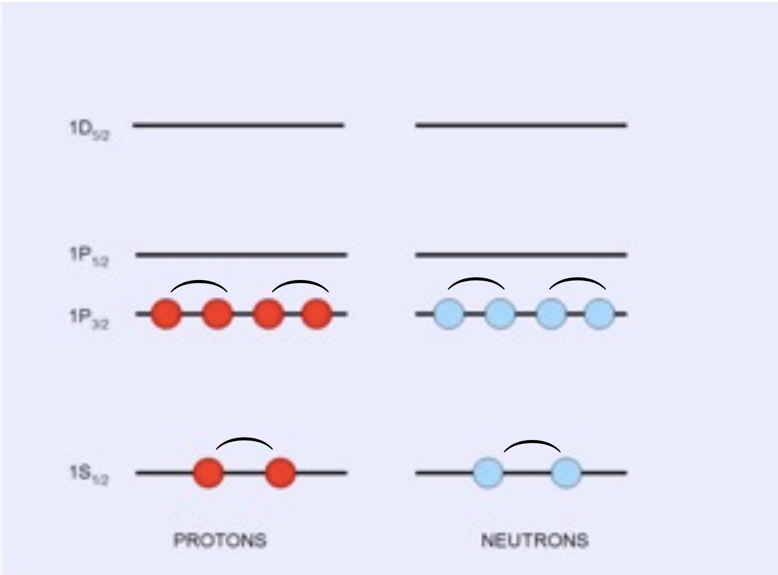


double closed p-shell

double magic nuclei:
extra binding energy, extra small radius, extra low reaction probability

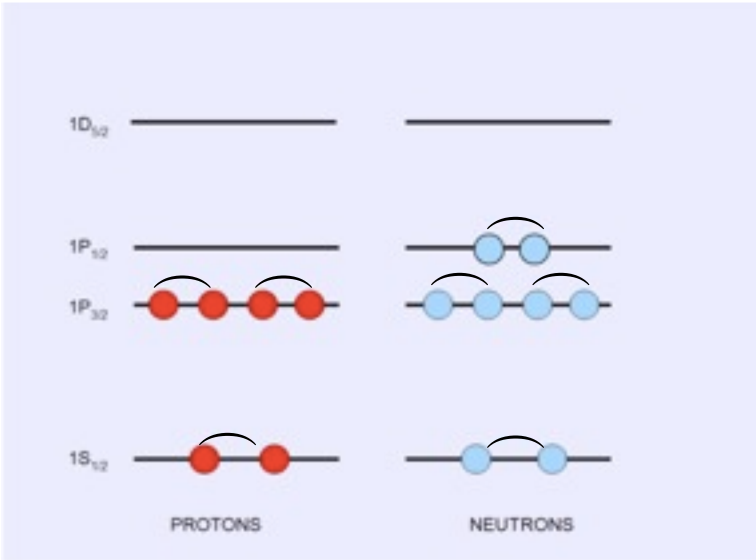
Nuclear shell model

^{12}C



double closed sub-shell
$p_{3/2}$ is full (6 p + 6 n)

^{14}C



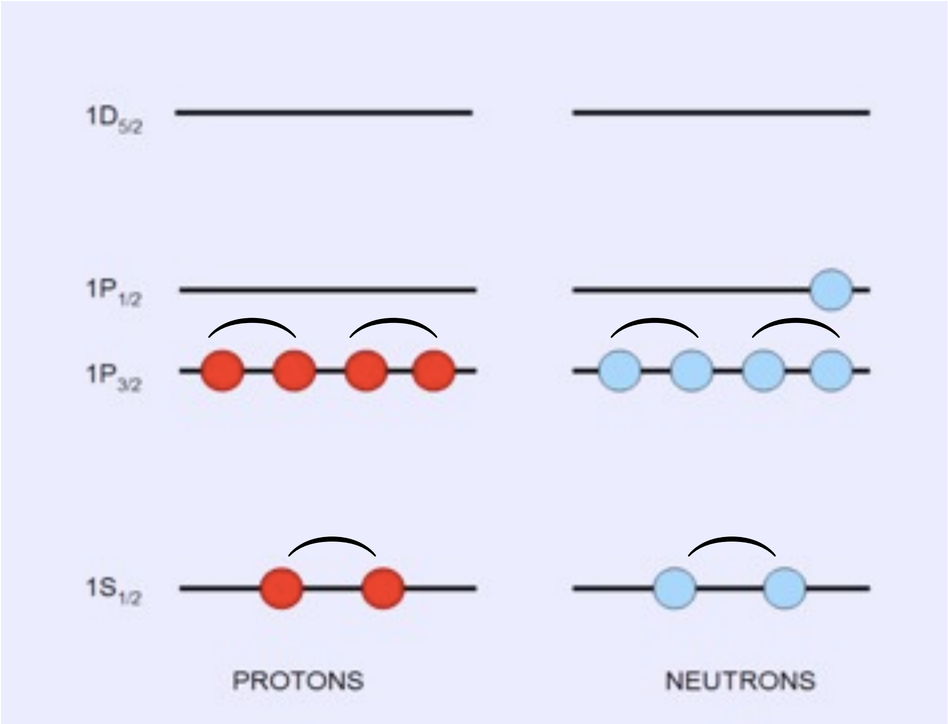
closed p-shell for neutrons (8)
closed $p_{3/2}$ sub-shell for protons (6)

Protons and neutrons pair off, so that if there is an even number of p and n, then the total angular momentum of the nucleus in the g.s. is $J=0^+$

Nuclear shell model

If there is an unpaired p or n, then the total angular momentum J^p of the nucleus in the g.s. is equal to the angular momentum and parity of the single nucleon in the outer shell

^{13}C



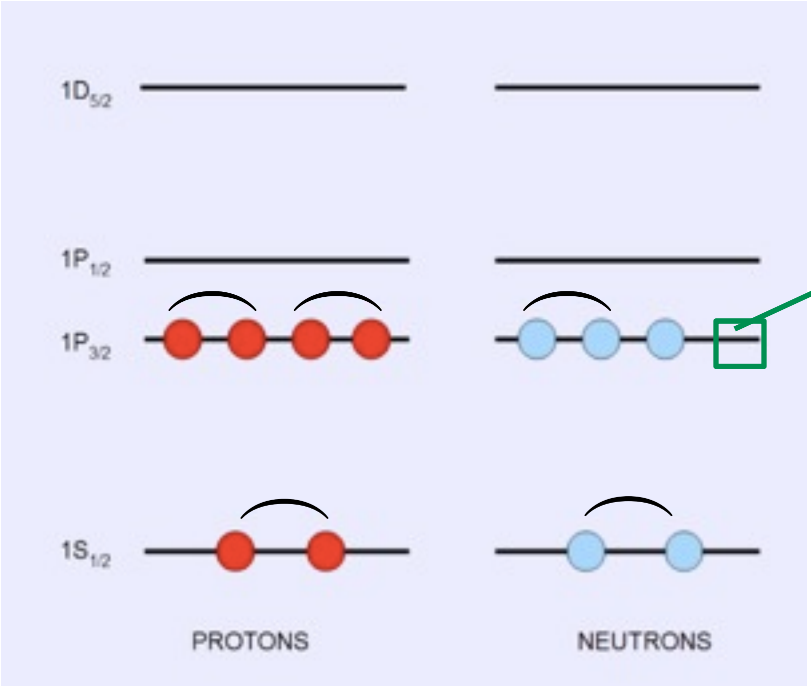
6 protons and 7 neutrons.
One n in the outer p1/2 shell
The nucleus has $J=1/2^-$

Friday, 17 July, 15

Nuclear shell model

If there is an unpaired p or n, then the total angular momentum J^p of the nucleus in the g.s. is equal to the angular momentum and parity of the single nucleon in the outer shell

^{11}C



vacancy in an otherwise filled shell

6 protons and 5 neutrons. One unpaired n in the p3/2 shell, the nucleus has $J=3/2^-$
A vacancy in an otherwise filled shell acts like a lone particle in that same shell in determining the spin parity of the nucleus. Thus, again, $J=3/2^-$

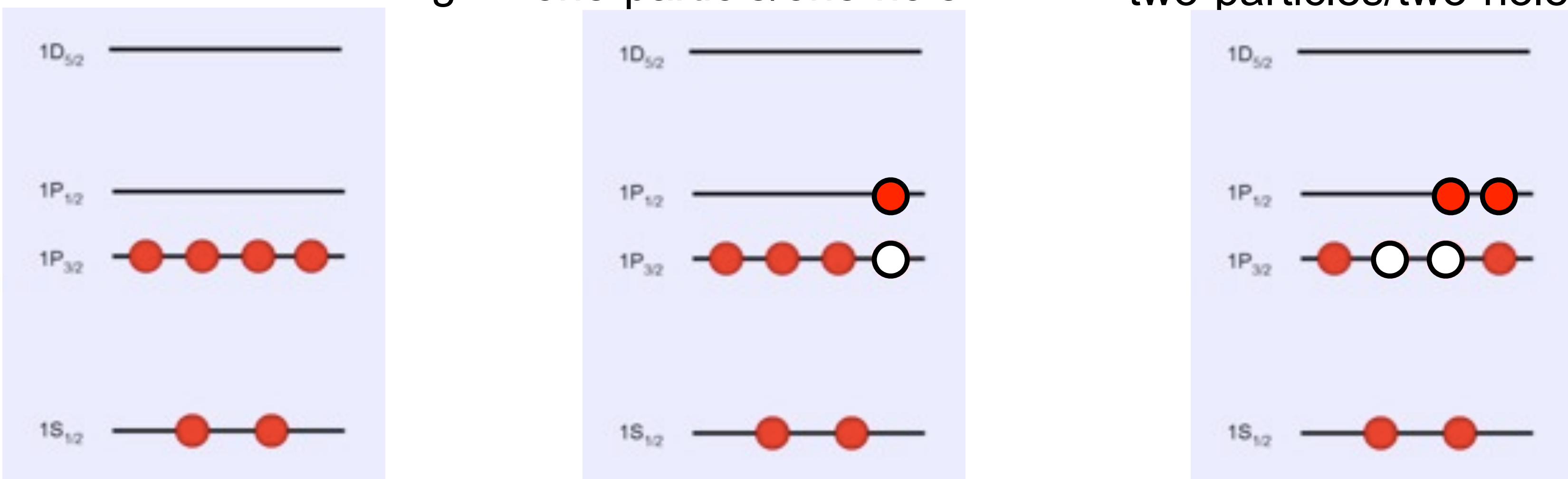
Exercise: Give parity and J assignments of selected nuclei ...

Interacting shell model

What we have described so far is known as non interacting shell model and we have discussed the ground state of nuclei.
However, in modern research what is used is the interacting shell model/shell model.

One can construct excited states or correlated ground states out of particle-hole excitations of the starting Slater determinant.

mean field/non interacting one-particle/one-hole two-particles/two-holes



In this way you construct many Slater determinants, that can form a many-body basis which one can use to expand the many-body wave function. This is also called configuration mixing

$$|\Psi^A\rangle = \sum_i c_i |\psi_i^A\rangle$$

Interacting shell model

Hamiltonian with a two-body potential (now particles are interacting)

$$H = \sum_{i}^{A} \frac{p_i^2}{2m} + \sum_{i<j}^{A} \boxed{V_{ij}}$$

connects particle i with particle j

$$H = \sum_{i}^{A} \frac{p_i^2}{2m} + \sum_{i<j}^{A} V_{ij} + \underbrace{\sum_{i}^{A} U_i - \sum_{i}^{A} U_i}_{=0}$$

$$= \underbrace{\sum_{i}^{A} \frac{p_i^2}{2m} + \sum_{i}^{A} U_i}_{} + \underbrace{\sum_{i<j}^{A} V_{ij} - \sum_{i}^{A} U_i}_{}$$

non interacting Hamiltonian

$$= H^0 + W^{res}$$

residual interaction: total interaction minus the "mean"potential

So far we have not specified what the potentials are U_i, V_{ij}

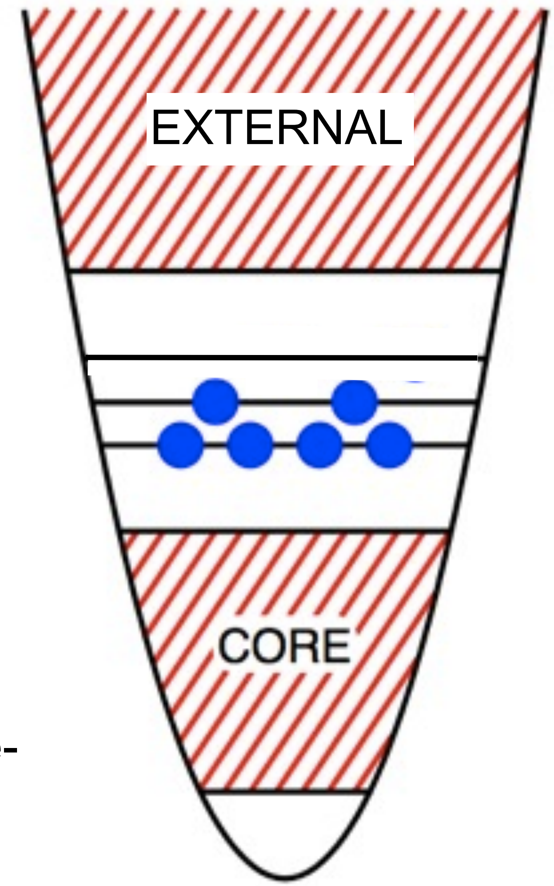
Interacting shell model

Construct orbitals from the HO potential $U_i = \frac{1}{2} m\omega^2 r_i^2$ $\hbar\omega \simeq 41 A^{-\frac{1}{3}}$ MeV

Ansatz:

For a given number on p and n, the mean field orbitals can be grouped in:

- inherent core:
  orbitals that are always full
- valence space:
  orbits that can have particle-hole excitations
- external space:
  all the remaining orbits that are always empty



1. Starting from $\boxed{V_{ij}}$ you construct a V^{eff}_{ij} that lives in the valence space using many body perturbation theory
2. Solve H_0+W_{res} by **diagonalizing** a matrix with particle-hole excitations in your valence space

Diagonalization methods

Solve Schroedinger equation by expanding the w.f. on a set of basis states

$$H|\psi\rangle = E|\psi\rangle \qquad |\psi\rangle = \sum_i^{\cancel{\infty}\,N} c_i |\psi_i\rangle$$

cannot store an infinite vector

→ basis states:
- fast convergence
- large model spaces
- different A

$$\langle\psi_j| \times H \sum_i^N c_i |\psi_i\rangle = E \sum_i^N c_i |\psi_i\rangle$$

$$\sum_i^N \underbrace{\langle\psi_j| H |\psi_i\rangle}_{H_{ji}} c_i = E \sum_i^N c_i \underbrace{\langle\psi_j|\psi_i\rangle}_{\delta_{ji}}$$

$$\boxed{\mathbf{H}\mathbf{c} = E\mathbf{c}}$$

Eigenvalue problem for an Hermitian matrix

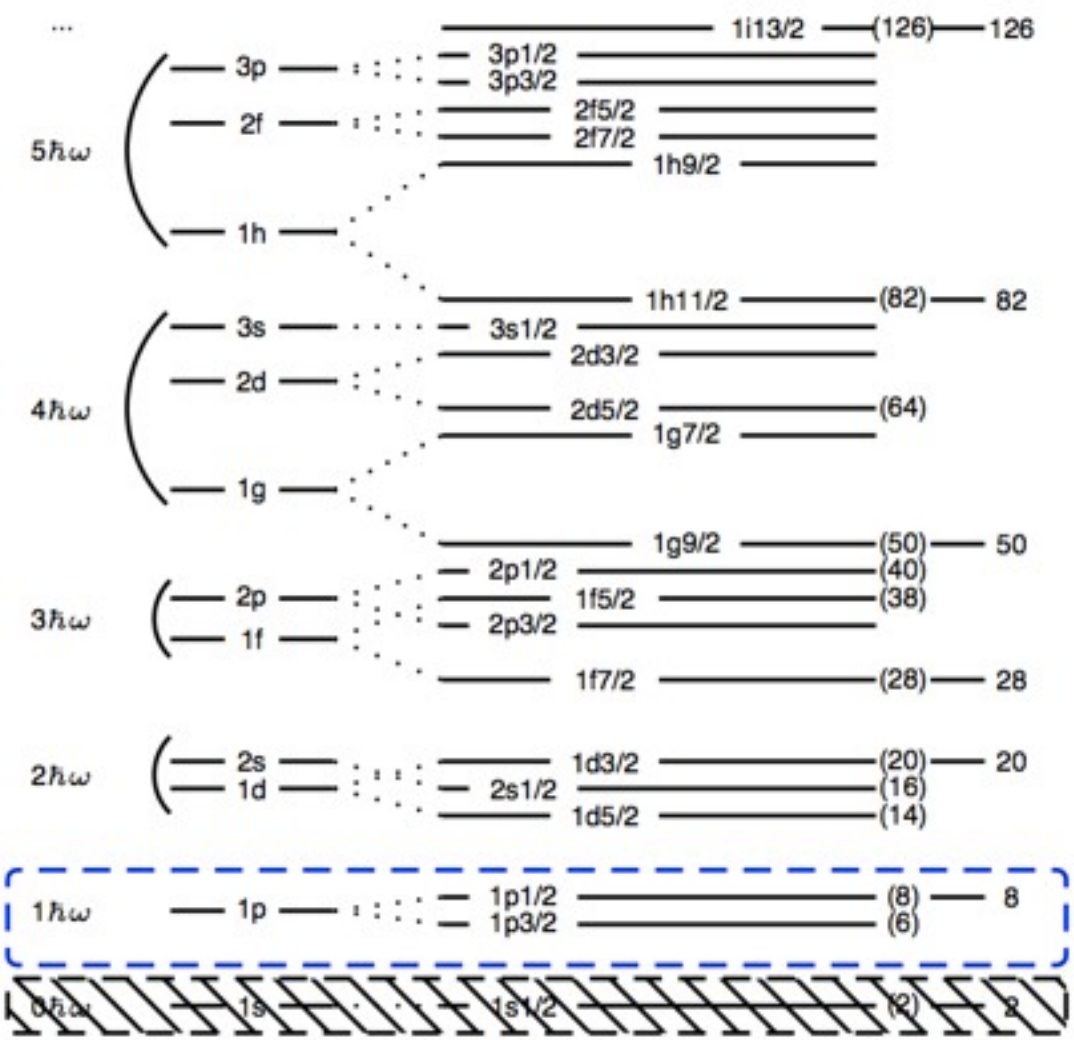
$$\mathbf{H} = \mathbf{H}^\dagger$$

Finding eigenvalues and eigenvectors is equivalent to diagonalize the matrix N^3 operation

Computationally challenging for growing N

Effective Potentials

The model space and the effective interaction are very much related. Typically, the effective interaction is a set of two-body matrix elements tuned to reproduce experimental data



p-shell nuclei

$$4 \leq A \leq 16$$

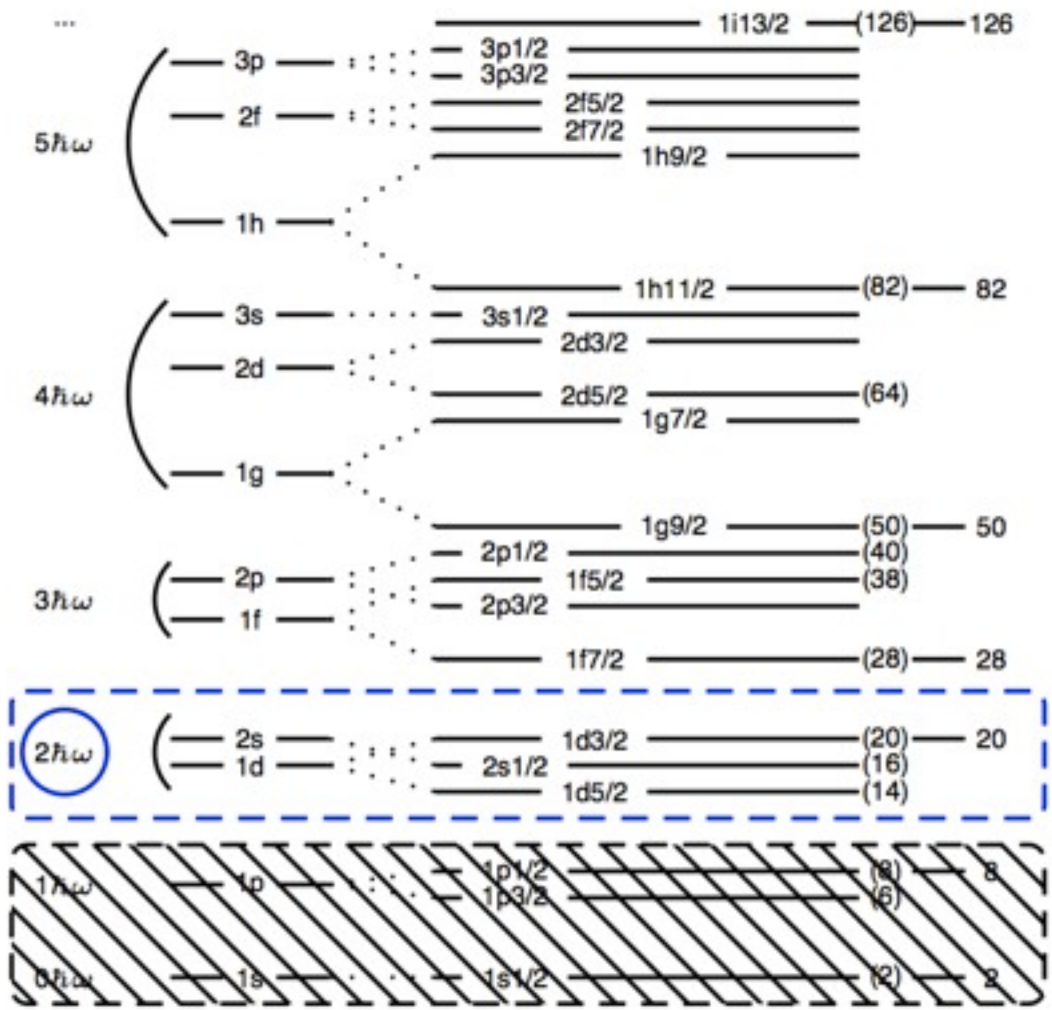
Cohen-Kurath interaction

VALENCE SPACE

CORE

Effective Potentials

The model space and the effective interaction are very much related. Typically, the effective interaction is a set of two-body matrix elements tuned to reproduce experimental data



sd-shell nuclei

$$16 \leq A \leq 40$$

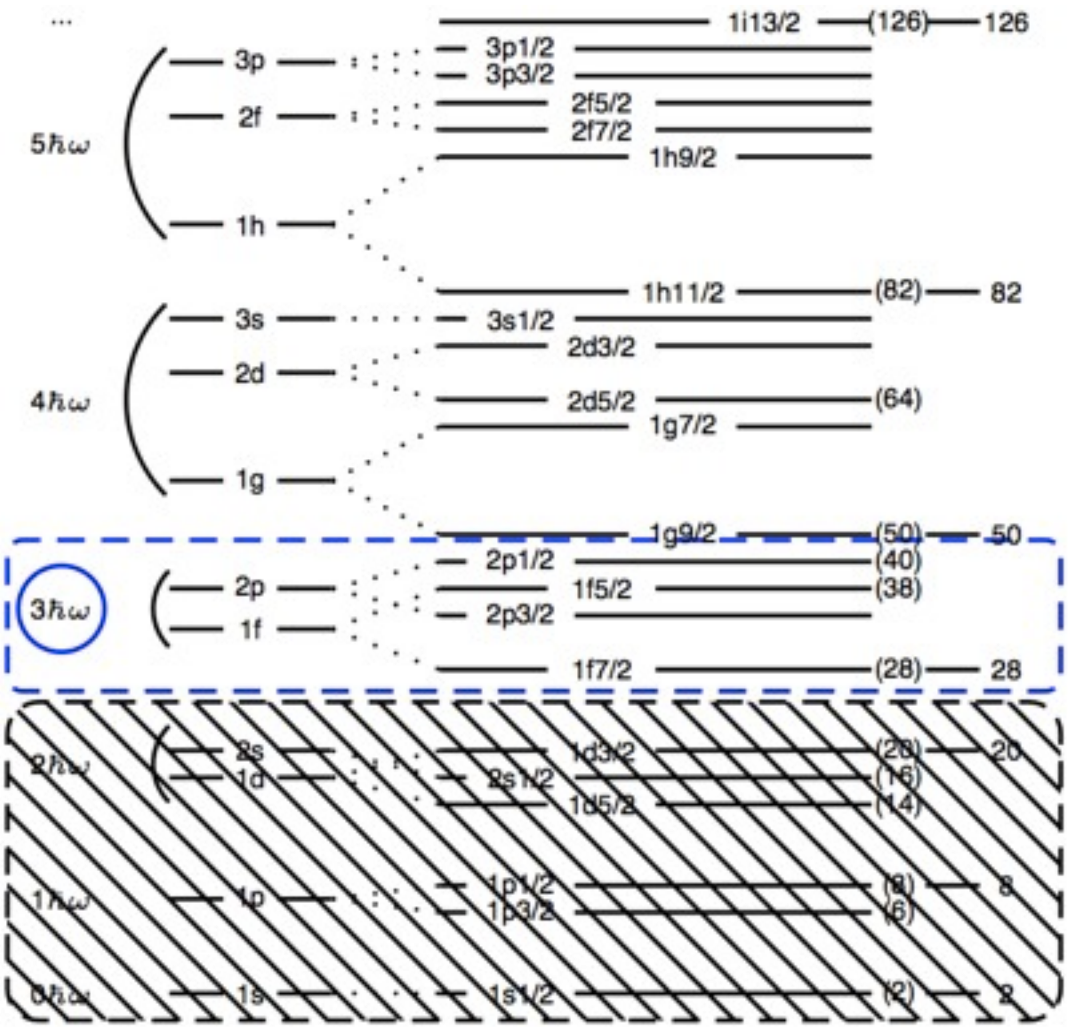
USD interaction

VALENCE SPACE

CORE

Effective Potentials

The model space and the effective interaction are very much related. Typically, the effective interaction is a set of two-body matrix elements tuned to reproduce experimental data



VALENCE SPACE

CORE

pf-shell nuclei

$$40 \leq A \leq 80$$

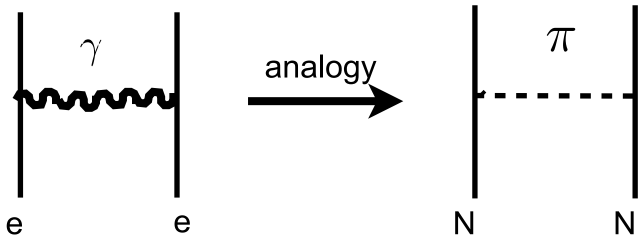
GXPF1 interactions

...

How to go beyond phenomenological potentials: a more fundamental approach to nuclear interactions

A more microscopic view on V_{ij}

Nucleon-Nucleon Forces



$r \sim \frac{1}{m}$

one-pion exchange potential (OPE)

electromagnetic force:
infinite range ⇨ exchange of massless particle

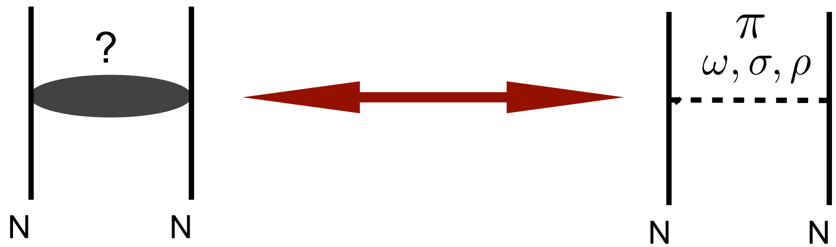
NN force:
finite range ⇨ exchange of massive particle

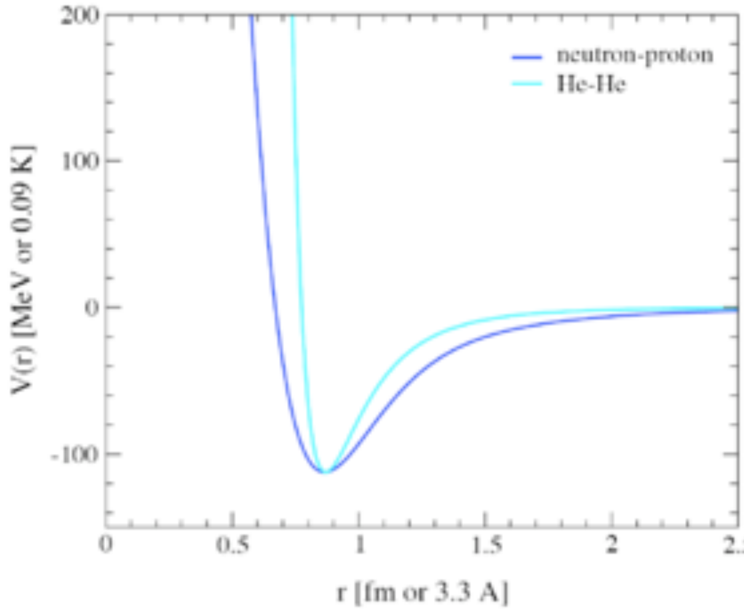
Hideki Yukawa
Nobel prize in 1949

Realistic NN potentials: fit to NN scattering data with $\chi^2 \approx 1$

Phenomenological models

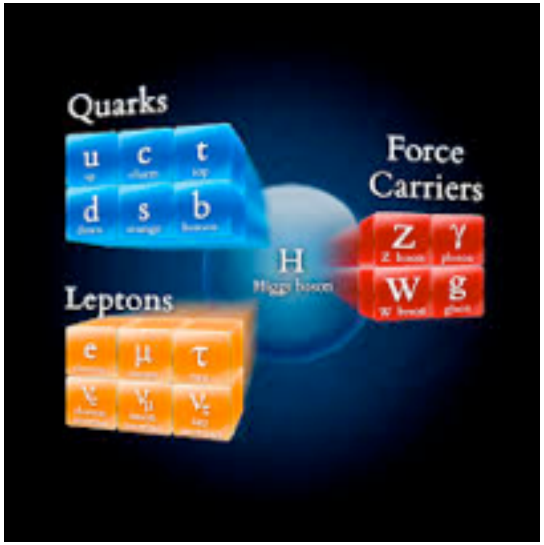
Meson exchange models

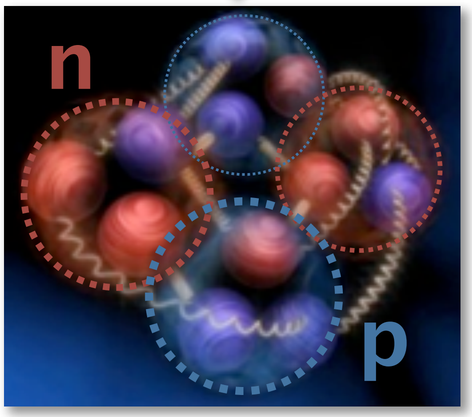




Similar to the potential between two He atoms (liquid Helium) or between two molecules (van der Waals forces)

Chiral Effective Field Theory



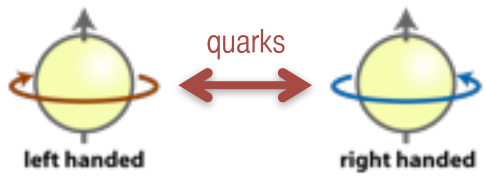


Quark/gluon (high energy) dynamics

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \bar{q}_L i\gamma_\mu D^\mu q_L + \bar{q}_R i\gamma_\mu D^\mu q_R - \bar{q}\mathcal{M}q$$

In the limit of vanishing quark masses the QCD Lagrangian is invariant under chiral symmetry

QCD chiral symmetry



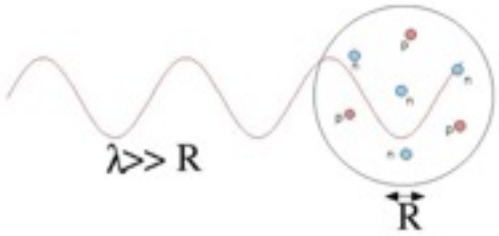
Chiral symmetry is explicit and spontaneous broken

Nucleon/pion (low energy) dynamics

$$\mathcal{L}_{eff} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots$$

Compatible with explicit and spontaneous **chiral symmetry breaking**

Chiral Effective Field Theory



Limited resolution at low energy

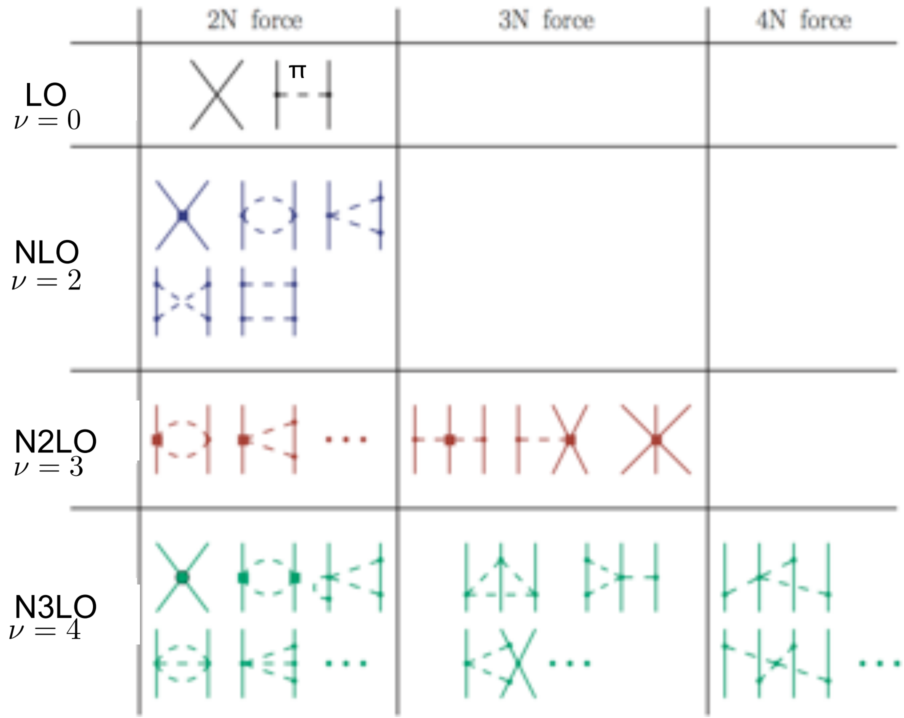
Separation of scales

$$\frac{1}{\lambda} = Q \ll \Lambda_b = \frac{1}{R}$$

Systematic expansion

$$\mathcal{L} = \sum_{\nu} c_{\nu} \left(\frac{Q}{\Lambda_b} \right)^{\nu}$$

Details of short distance physics not resolved, but captured in low energy constants (LEC)

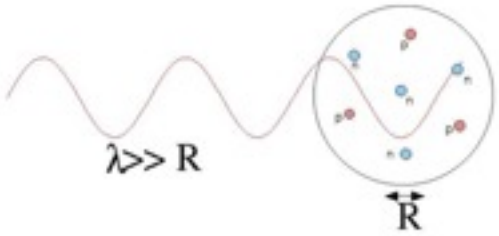


Power counting

$$\nu = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2)$$

Exercise on power counting.

Chiral Effective Field Theory



Separation of scales

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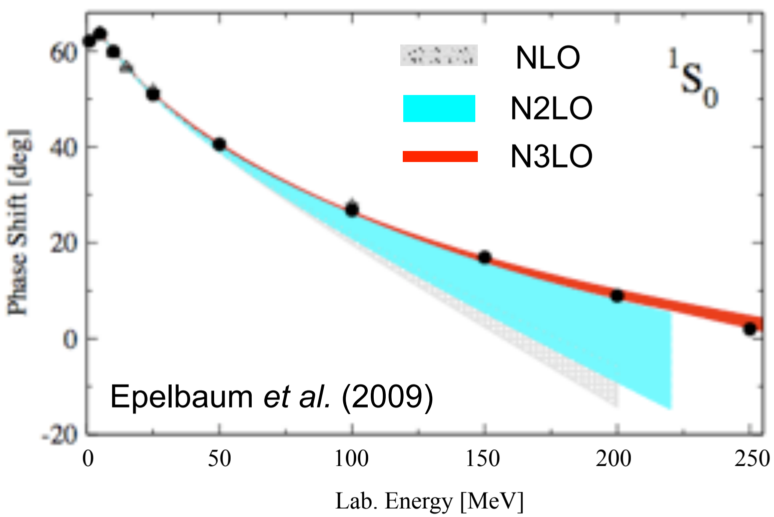
Limited resolution at low energy

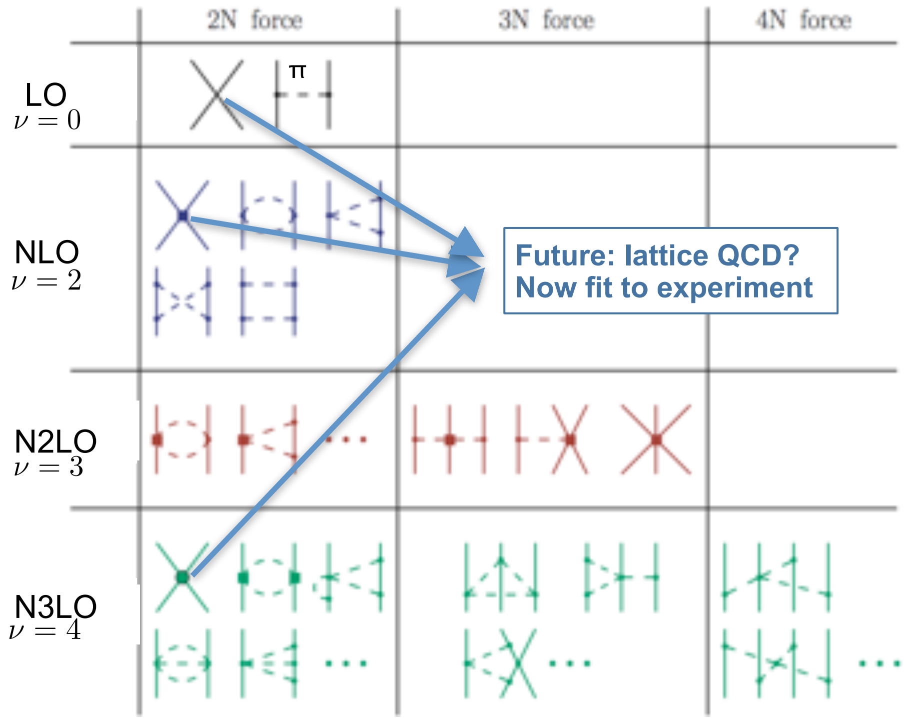
Systematic expansion

$$\mathcal{L} = \sum_{\nu} c_\nu \left(\frac{Q}{\Lambda_b}\right)^{\nu}$$

Details of short distance physics not resolved, but captured in low energy constants (LEC)

LEC fit to experiment - NN sector -





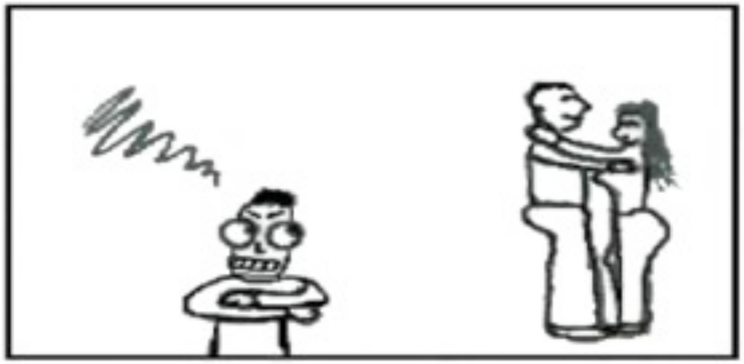
Friday, 17 July, 15

Three-Body Forces

What is the origin of three-body forces?

Nucleons are effective degrees of freedom

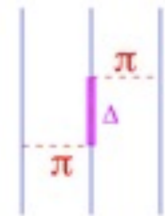
“The three-body force is a force that does not exist in a two-nucleon system, but appears in a system with three objects or more” $A \geq 3$



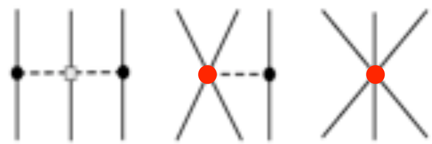
From N. Kalantar, FM50

As an analogy, if we identify nucleons with human beings and forces with emotions, then jealousy is a good example of a three-body force

- The first 3N force was introduced by Fujita and Miyazawa in 1957



- In chiral effective theory there are 3 terms (at N^2LO) with only two new low energy constants

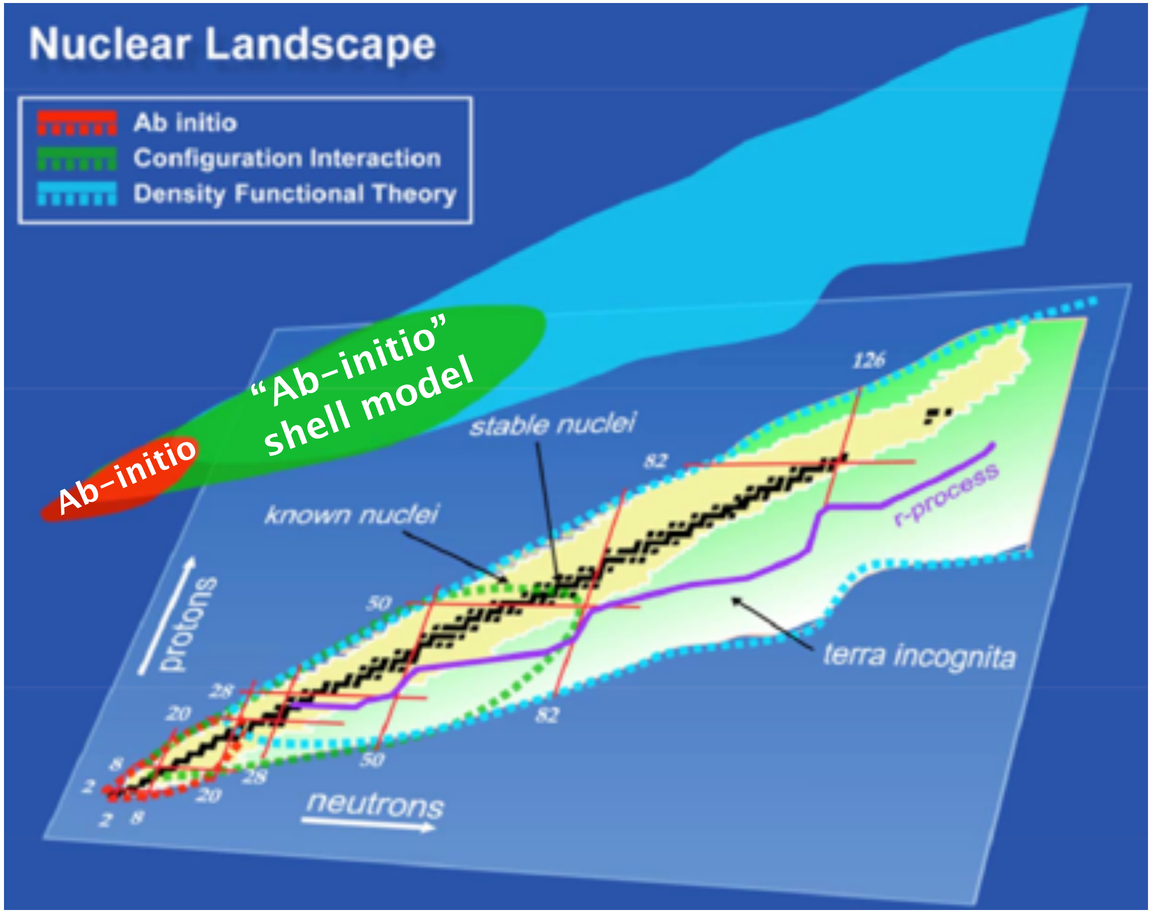


$c_D(\Lambda)$ $c_E(\Lambda)$

Goal: Calibrate Hamiltonian and then predict other observables

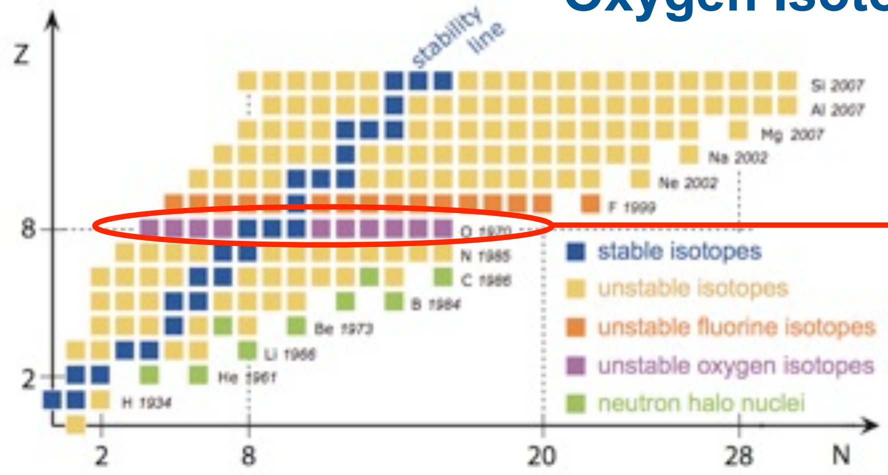
Ab-initio methods

Let's now use more fundamental interactions and solve the many-body problem



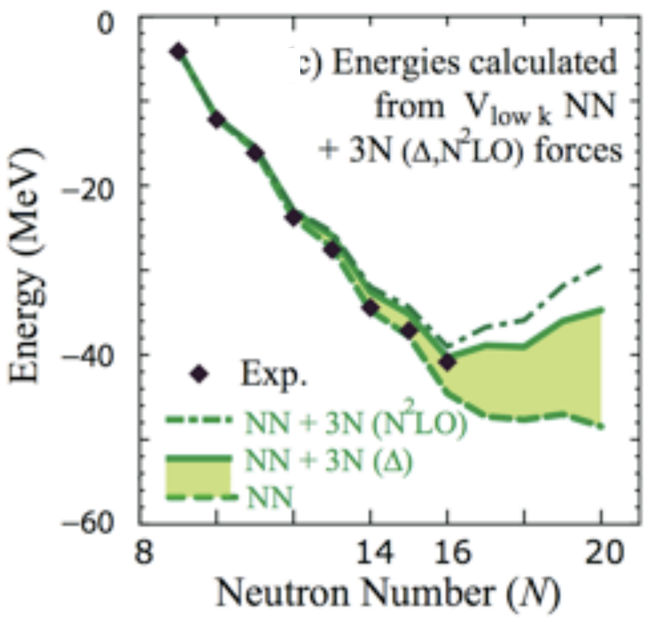
“Ab-initio”shell model:
using realistic interactions in shell model

Oxygen Isotope Chain



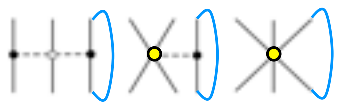
Shell Model: ^{16}O core

any SM calculation with realistic 2NF predicts bound $^{25\text{-}28}$O in contrast with experimental observation

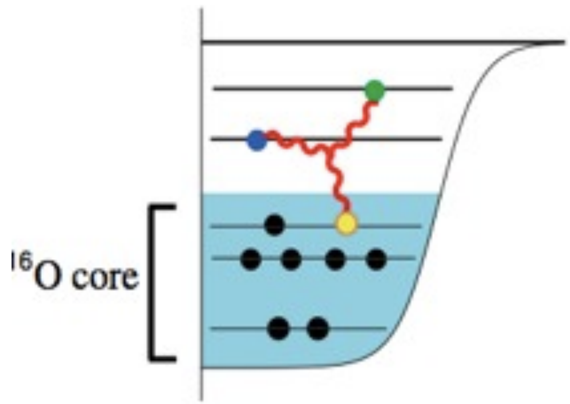


Otsuka et al. PRL 105, 032501 (2010)

First results with 3NF (effective 2NF)



3NF fits to E(^{3}H) and ^{4}He rms



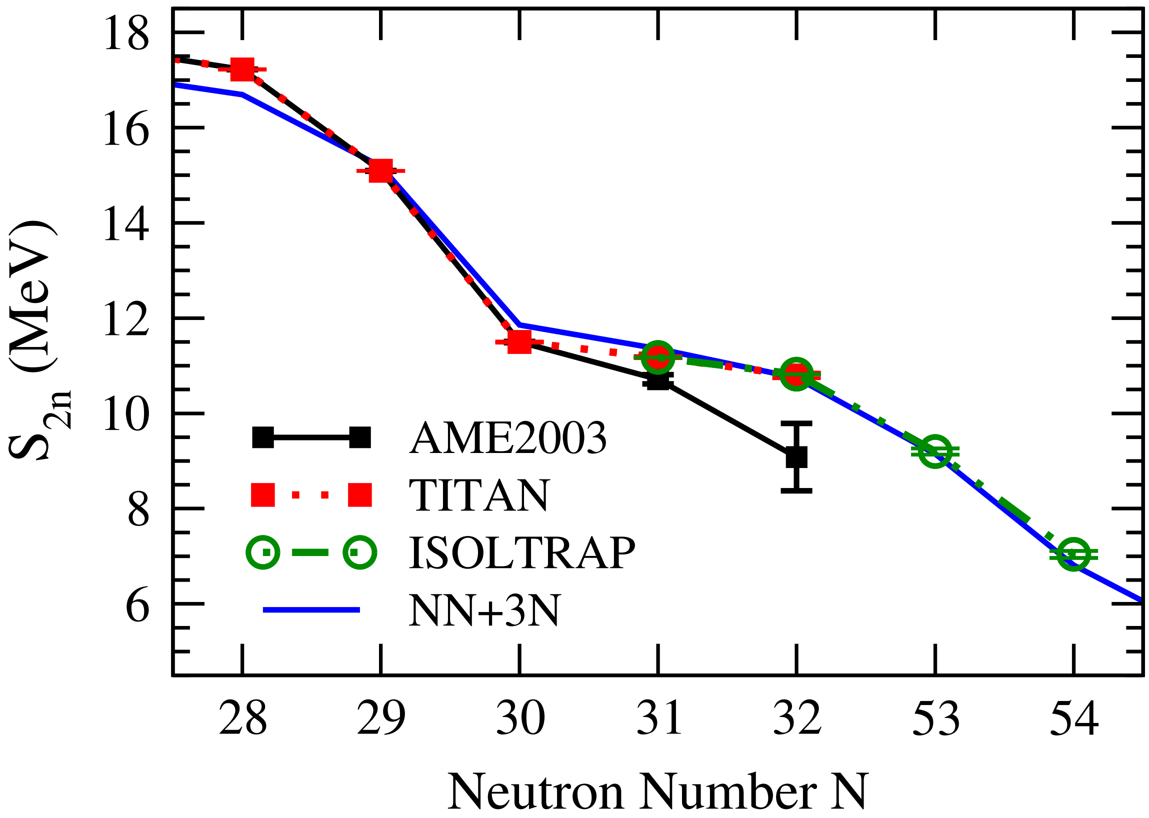
Three-body forces are needed here to explain the drip-line

“Ab-initio”shell model:
using few-body interactions in shell model

Calcium Isotope Chain

J. D. Holt, et.al., J. Phys. G 39, 085111 (2012)

F. Wienholtz, et al., Nature 498, 346 (2013).

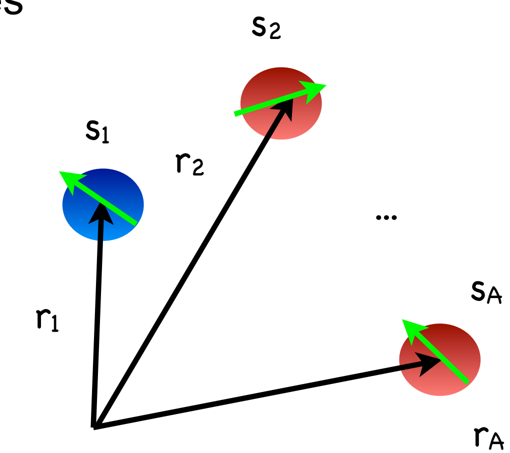


With three-body forces one reproduces precise mass data from the traps

Ab-initio methods

- Start from neutrons and protons interacting with realistic forces

- Solve the non-relativistic quantum mechanical problem of A-interacting nucleons



$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \dots$$

- Find numerical solutions with no approximations or controllable approximations

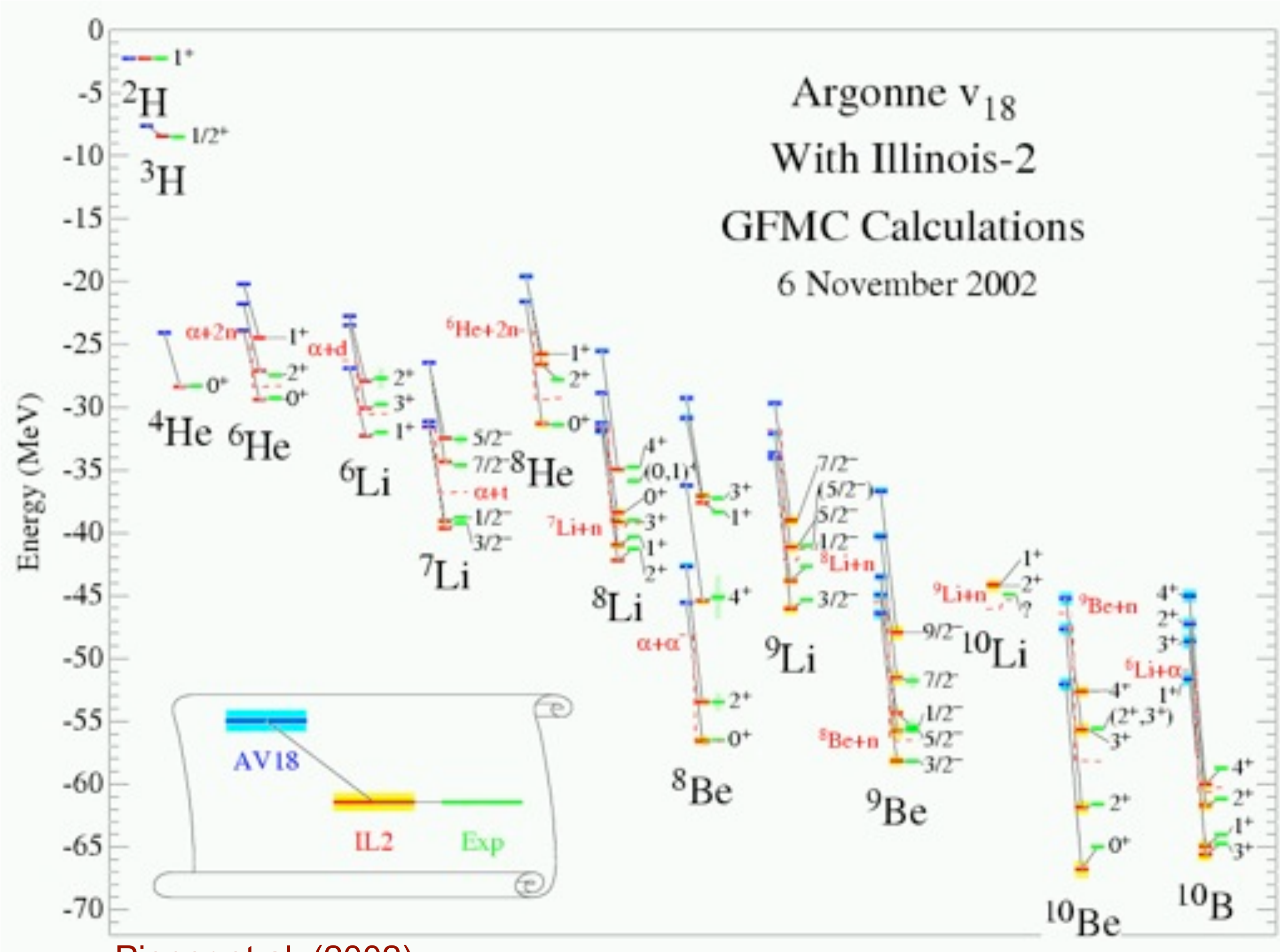
- Calculate low-energy observables for A-body nuclei and compare with experiment to test nuclear forces and provide predictions when experiments are hard or not possible

TRIUMF

Ab-initio methods

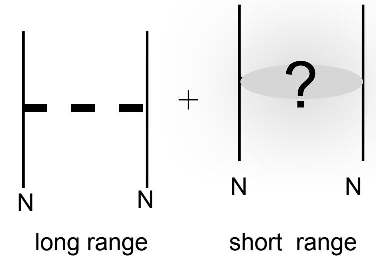
- Monte Carlo Methods, like Green's Function Monte Carlo, Lattice Effective field theory etc. (Carlson, Lovato, Gandolfi, Lee, ...)
- Faddeev and Faddeev-Yakubovsky methods (Deltuva, Lazauskas,...)
- No Core Shell model methods (Vary, Navratil, Qualgioni, Roth, ...)
- Hyperspherical Harmonics expansions (Barnea, Kievsky, Viviani, Marcucci, ...)
- Coupled-cluster theory (Hagen, Papenbrock, Hjorth-Jensen, ...)
- In-medium SRG (Bogner, Hergert, Holt, Schwenk ...)
- Self consistent Green's functions (Barbieri, Soma, ...)
- ...

Quantum Monte Carlo methods



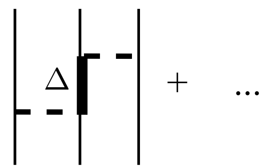
Pieper et al. (2002)

AV18 - NN force



IL2 - 3N force

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi,R} + V_{ijk}^{R}$$



N.B.: parameters of the IL2 force are obtained from a fit of 17 states of A<9 including the binding energy of ^{6}He and ^{8}He

This method can go up to ^{12}C

Friday, 17 July, 15

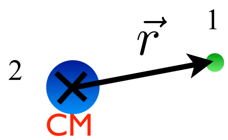
A few-body method

(bound states and reactions)

Hyperspherical Harmonics

A basis set, mostly used for A=3,4. Challenge to go up to A=7,8

Hydrogen atom



- Solve the problem in the CM frame

$$[T + V(r)]\,\psi(\vec{r}) = E\psi(\vec{r})$$

- Use spherical coordinates

$$\vec{r} = (r, \underbrace{\theta, \phi}_{\Omega})$$

$$\psi(\vec{r}) \sim Y_{\ell m}(\Omega) u_\ell(r)$$

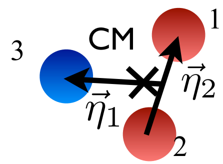
$$T = T_r - \frac{\hat{\ell}^2}{r^2}$$

$$\hat{\ell}^2\; Y_{\ell m}(\Omega) = \ell(\ell+1) Y_{\ell m}(\Omega)$$

- Solve the radial equation

$$\left[T_r - \frac{\ell(\ell+1)}{r^2} + V(r) - E\right] u_\ell(r) = 0$$

Three-body Nucleus



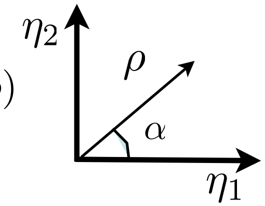
- Solve the problem in the CM frame

$$[T + V(\eta_1, \eta_2)]\,\psi(\vec{\eta}_1, \vec{\eta}_2) = E\psi(\vec{\eta}_1, \vec{\eta}_2)$$

- Use hyperspherical coordinates

$$\rho = \sqrt{\eta_1^2 + \eta_2^2} \qquad \Omega = (\theta_1, \phi_1, \theta_2, \phi_2, \alpha)$$

$$\psi(\vec{\eta}_1, \vec{\eta}_2) \sim \mathcal{Y}_{[K]}(\Omega) R_{[K]}(\rho)$$



$$T = T_\rho - \frac{\hat{K}^2}{\rho^2}$$

$$\hat{K}^2\; \mathcal{Y}_{[K]}(\Omega) = K(K+4) \mathcal{Y}_{[K]}(\Omega)$$

- Solve the hyperradial equation

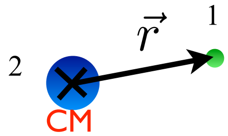
only for hyper-radial potentials

$$\left[T_\rho - \frac{K(K+4)}{\rho^2} + V(\rho) - E\right] R_K(\rho) = 0$$

Hyperspherical Harmonics

A basis set, mostly used for A=3,4. Challenge to go up to A=7,8

Hydrogen atom



- Solve the problem in the CM frame

$$[T + V(r)]\,\psi(\vec{r}) = E\psi(\vec{r})$$

- Use spherical coordinates

$$\vec{r} = (r, \underbrace{\theta, \phi}_{\Omega})$$

$$\psi(\vec{r}) \sim Y_{\ell m}(\Omega) u_\ell(r)$$

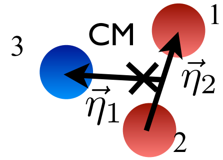
$$T = T_r - \frac{\hat{\ell}^2}{r^2}$$

$$\hat{\ell}^2\; Y_{\ell m}(\Omega) = \ell(\ell+1) Y_{\ell m}(\Omega)$$

- Solve the radial equation

$$\left[T_r - \frac{\ell(\ell+1)}{r^2} + V(r) - E\right] u_\ell(r) = 0$$

Three-body Nucleus



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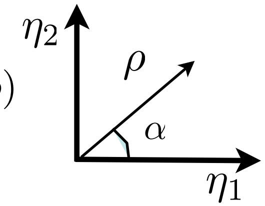
$$\psi(\vec{\eta}_1, \vec{\eta}_2) \sim \mathcal{Y}_{[K]}(\Omega) R_{[K]}(\rho)$$



$$T = T_\rho - \frac{\hat{K}^2}{\rho^2}$$

$$\hat{K}^2\; \mathcal{Y}_{[K]}(\Omega) = K(K+4)\mathcal{Y}_{[K]}(\Omega)$$

- Solve the hyperradial equation

general case

$$\left[\left(T_\rho - \frac{K(K+4)}{\rho^2} - E\right)\delta_{[K],[K']} + \langle \mathcal{Y}_{[K]}|V(\rho,\Omega)|\mathcal{Y}_{[K']}\rangle\right] R_K(\rho) = 0$$

$$\sum_{\nu}^{\nu_{max}} e^{-\rho} L_\nu(\rho)$$

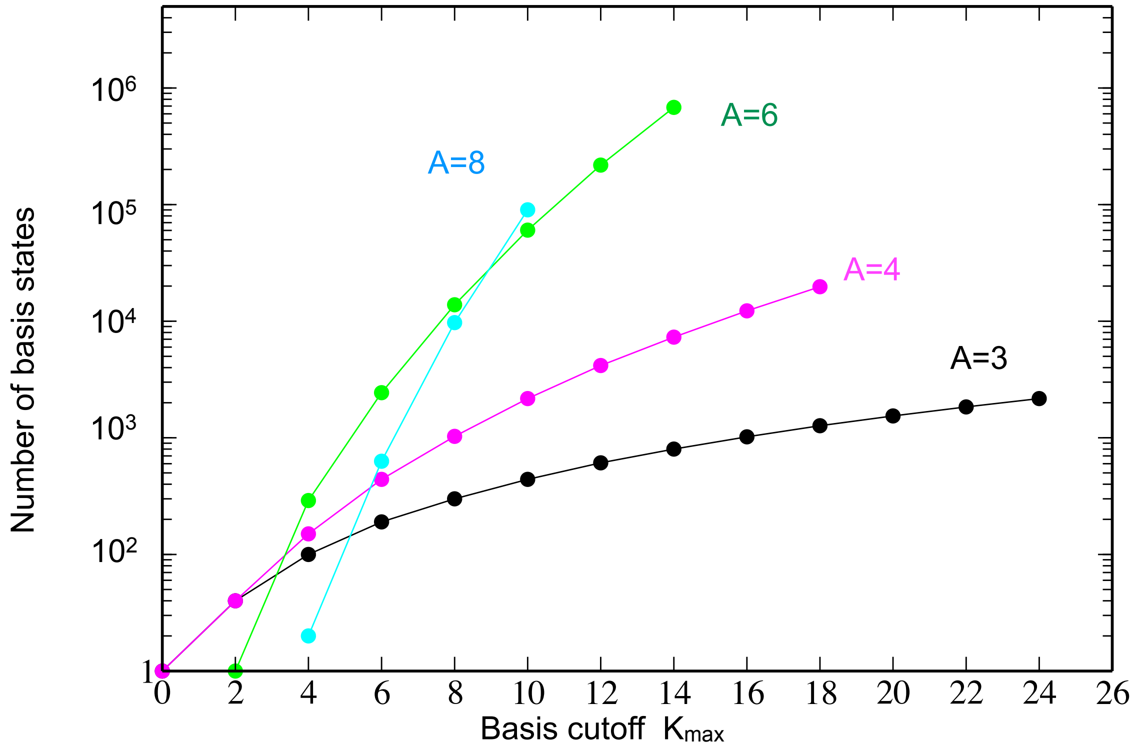
Friday, 17 July, 15

Hypersphercal Harmonics

A basis set, mostly used for A=3,4. Challenge to go up to A=7,8

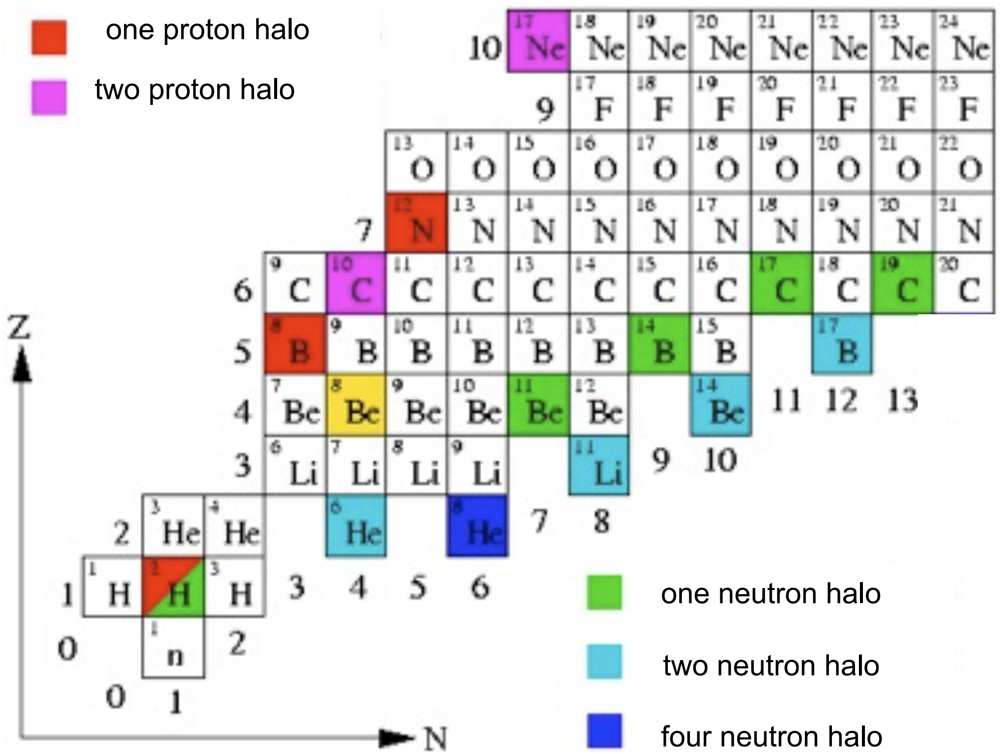
$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2} \rho^{n/2} L_\nu^n(\rho) [\mathcal{Y}_{[K]}^{\mu}(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^{a}$$

Exact method

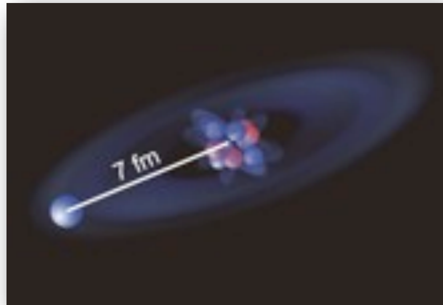


Bad computational scaling

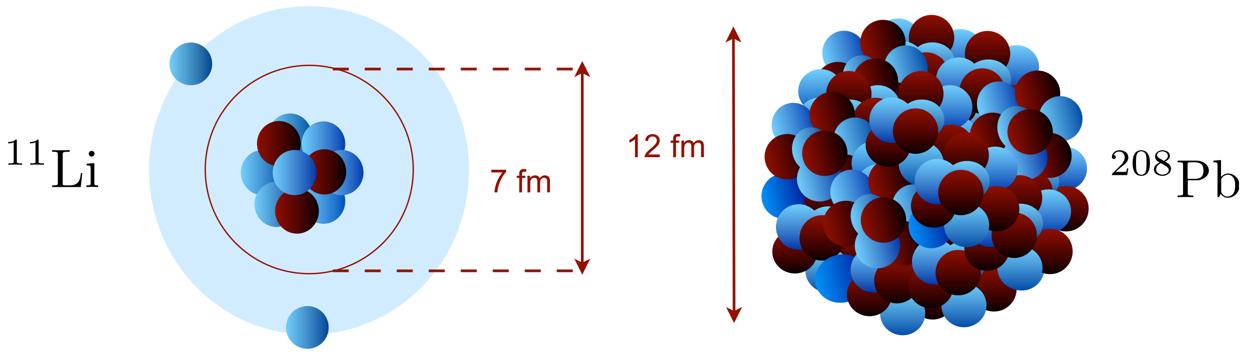
Halo Nuclei



- Exotic nuclei with an interesting structure



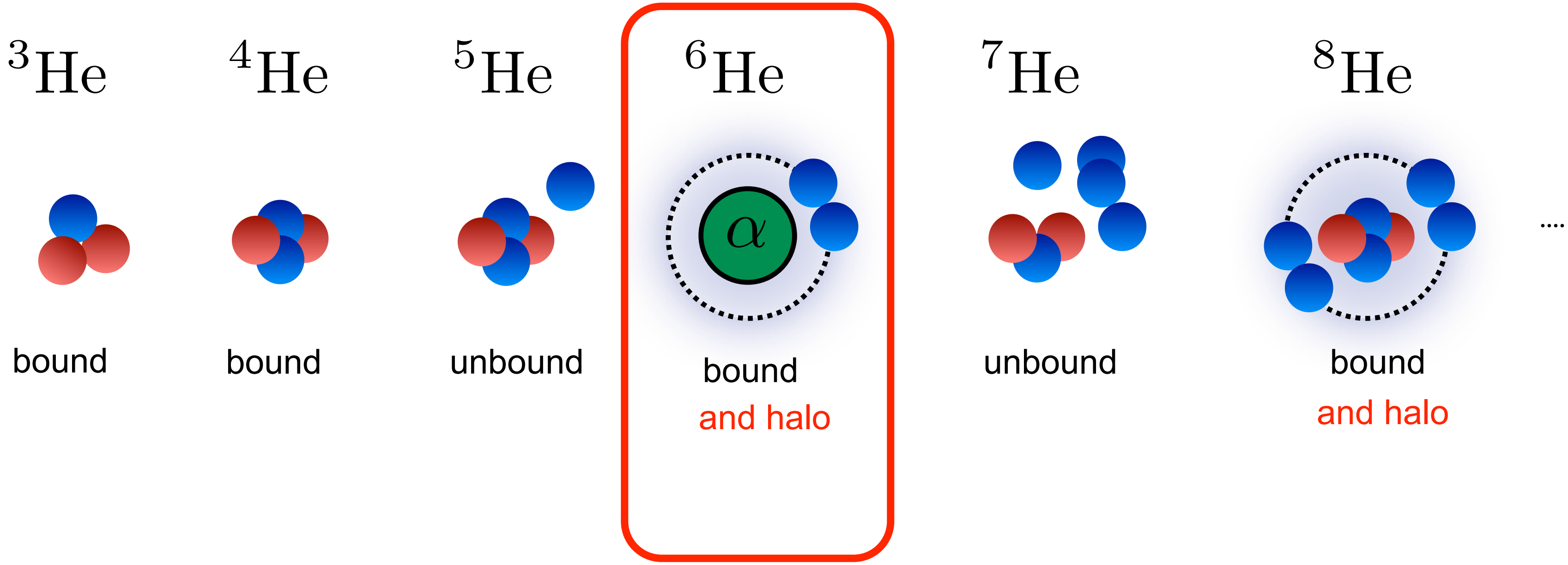
- Neutron halos:
  Large n/p ratio (neutron-rich)



Halo	n/p
^{6}He	2
^{8}He	3
^{11}Li	2.66
^{12}C	1

Halo Nuclei

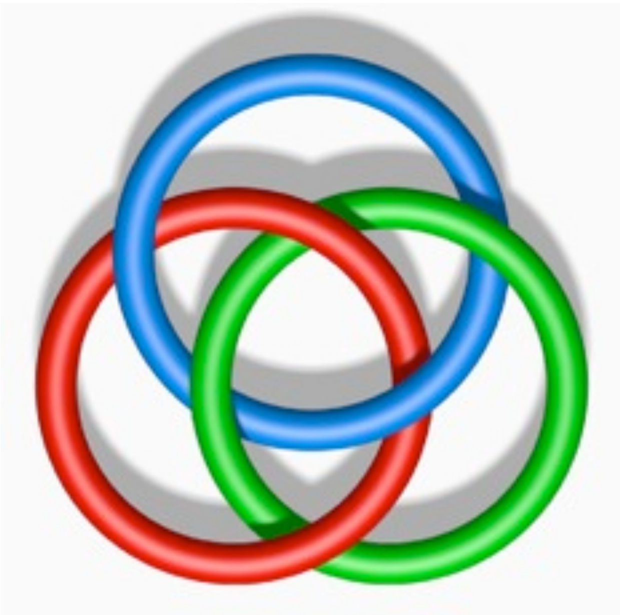
The Helium Isotope Chain



Borromean Nucleus

Halo Nuclei

Named after Borromean rings by M.V. Zhukov *et al*., Phys. Rep **231**, 151 (1993)

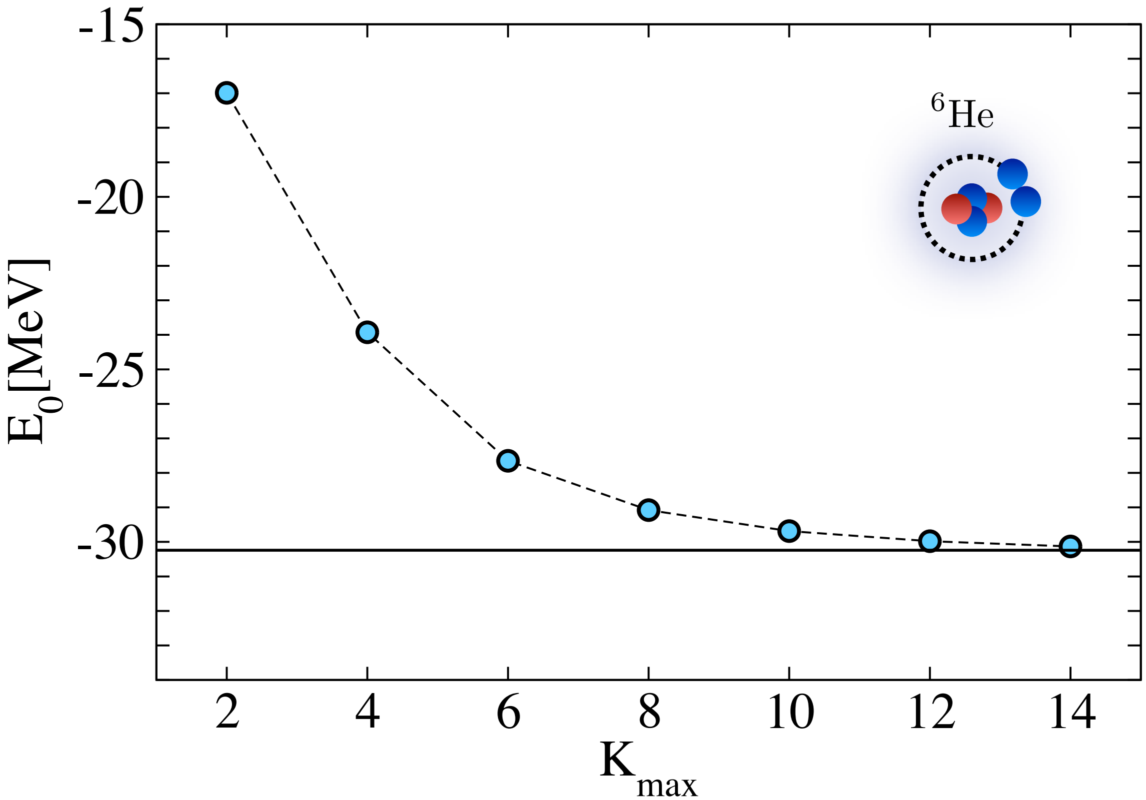


Isola Bella, Lago Maggiore, Italia

pic credit P.Capel

Halo Nuclei

Let's solve it as a six-body problem with hyperspherical harmonics expansions

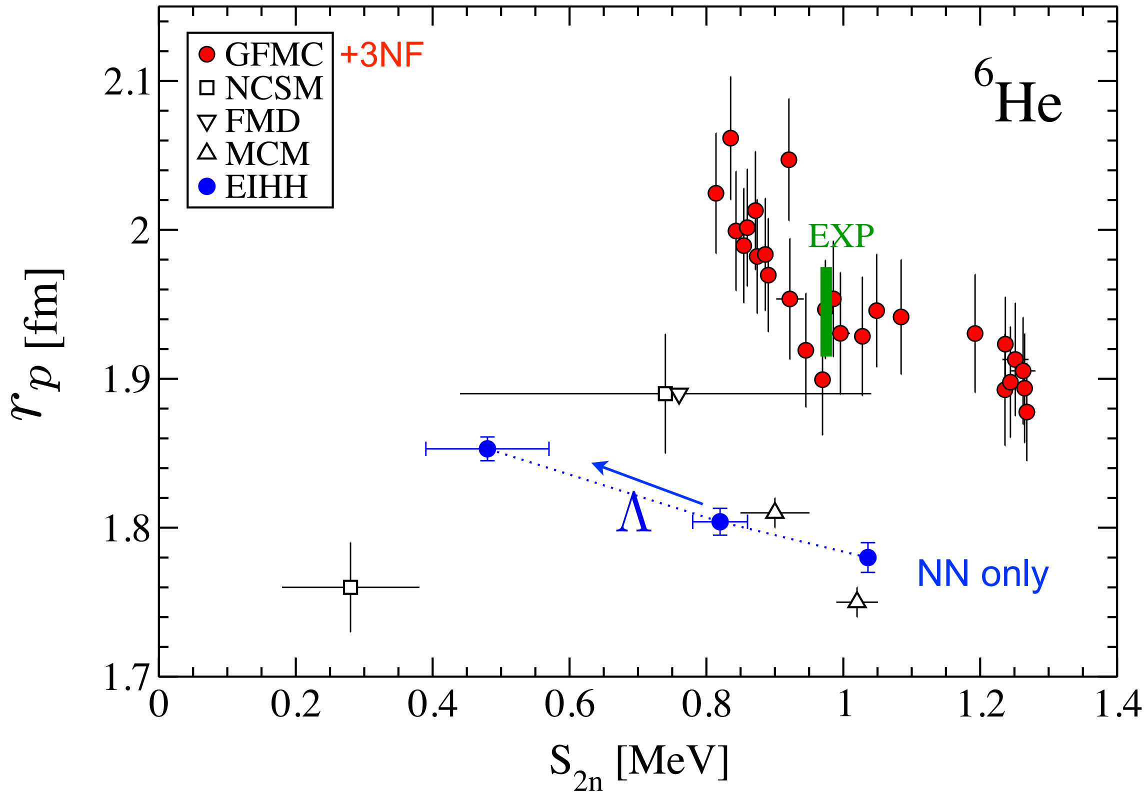


High-precision Penning trap and laser spectroscopy techniques allow accurate measurements of **energies and charge radii** of exotic halo nuclei

Halo Nuclei

Separation energies from TITAN, Penning trap @ TRIUMF

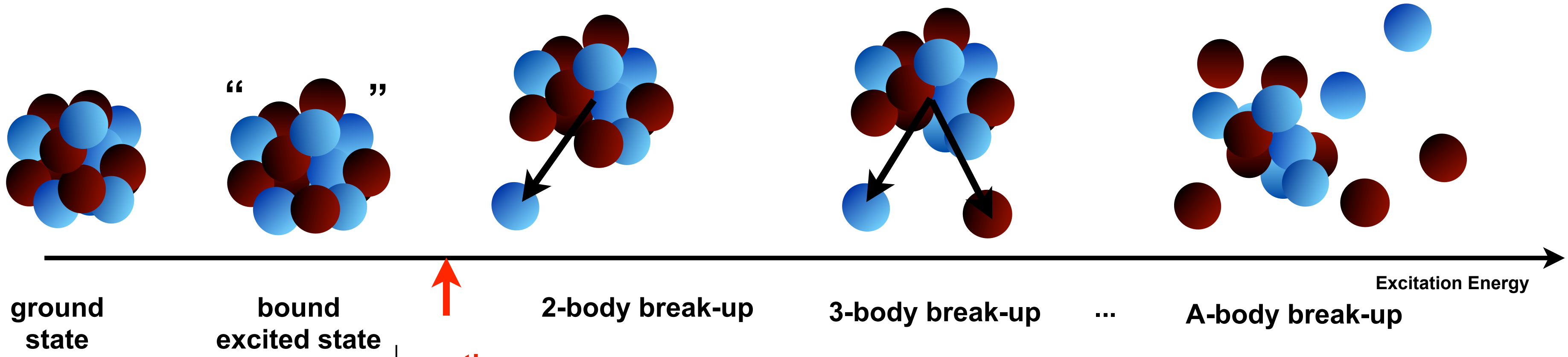
M. Brodeur *et al.*, PRL **108**, 052504 (2012)
S.B. *et al.*, PRC **86**, 034321 (2012)

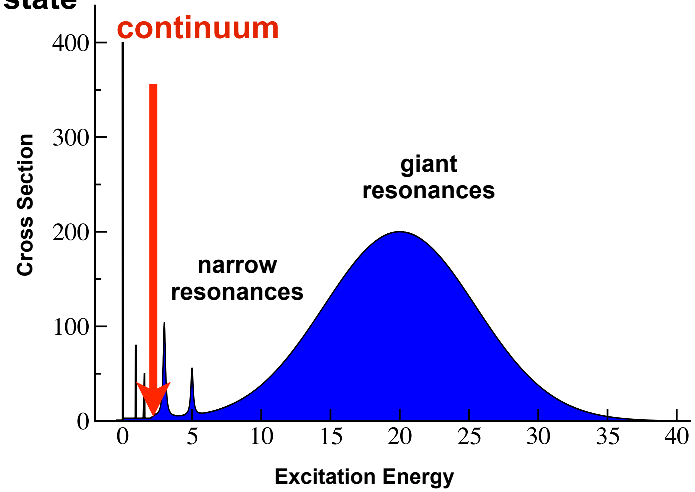


Correlation line r_p-S_{2n} does not go though exp
Need to add three-nucleon forces

Extension to electroweak reactions

The Continuum Problem





$$\sigma \propto \left| \langle \Psi_f | J^\mu | \Psi_0 \rangle \right|^2$$

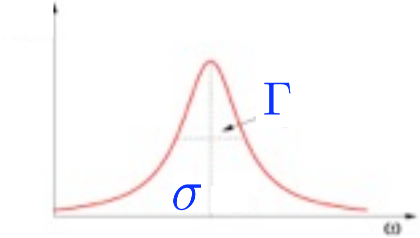
Exact knowledge limited in energy and mass number

Lorentz Integral Transform Method

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

Reduce the continuum problem to a bound-state problem

$$R(\omega) = \sum_f \left| \left\langle \psi_f \middle| J^\mu \middle| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$



$$L(\sigma,\Gamma) = \int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \left\langle \tilde{\psi} \middle| \tilde{\psi} \right\rangle < \infty$$

where $\left|\tilde{\psi}\right\rangle$ is obtained solving

$$\boxed{(H - E_0 - \sigma + i\Gamma)|\tilde{\Psi}\rangle = J^\mu|\Psi_0\rangle}$$

- Due to imaginary part Γ the solution $|\tilde{\psi}\rangle$ is unique
- Since $\langle\tilde{\psi}|\tilde{\psi}\rangle$ is finite, $|\tilde{\psi}\rangle$ has bound state asymptotic behaviour

⇨ Use bound-states techniques to solve the Schrödinger equation

$$L(\sigma,\Gamma) \xrightarrow{\text{inversion}} R(\omega)$$

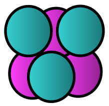
The exact final state interaction is included in the continuum rigorously!

Giant Dipole Resonance in A=6

with Hyperspherical Harmonics

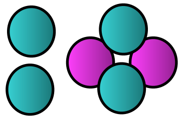
$$\sigma_\gamma = \frac{4\pi^2\alpha}{3}\omega R^{E1}(\omega)$$

$$\mathrm{E1} = \sum_i^Z (z_i - Z_{cm})$$



^{6}Li

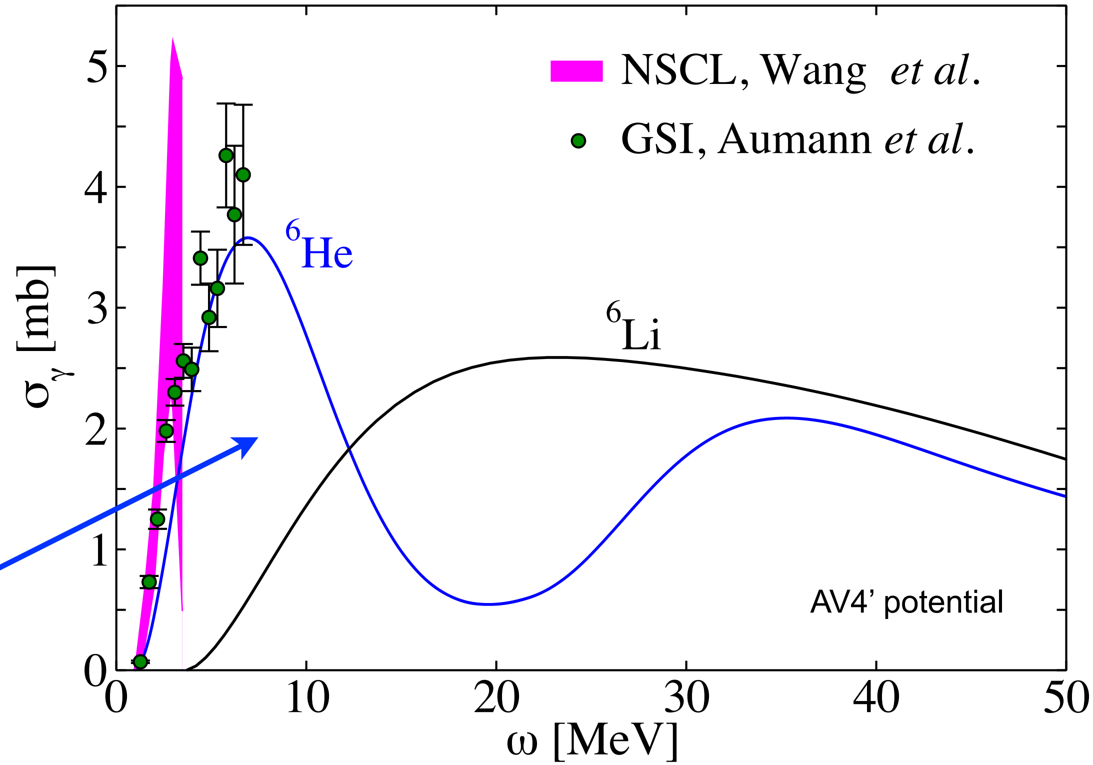
stable



^{6}He

unstable

$T_{1/2} = 806$ ms



Soft Dipole Mode

Future:

Explore soft dipole modes in ^{8}He, ^{22}C and ^{68}Ni which will be measured at RIKEN

S.B. *et al.* PRL **89** 052502 (2002) and PRC **69** 052502 (2004)

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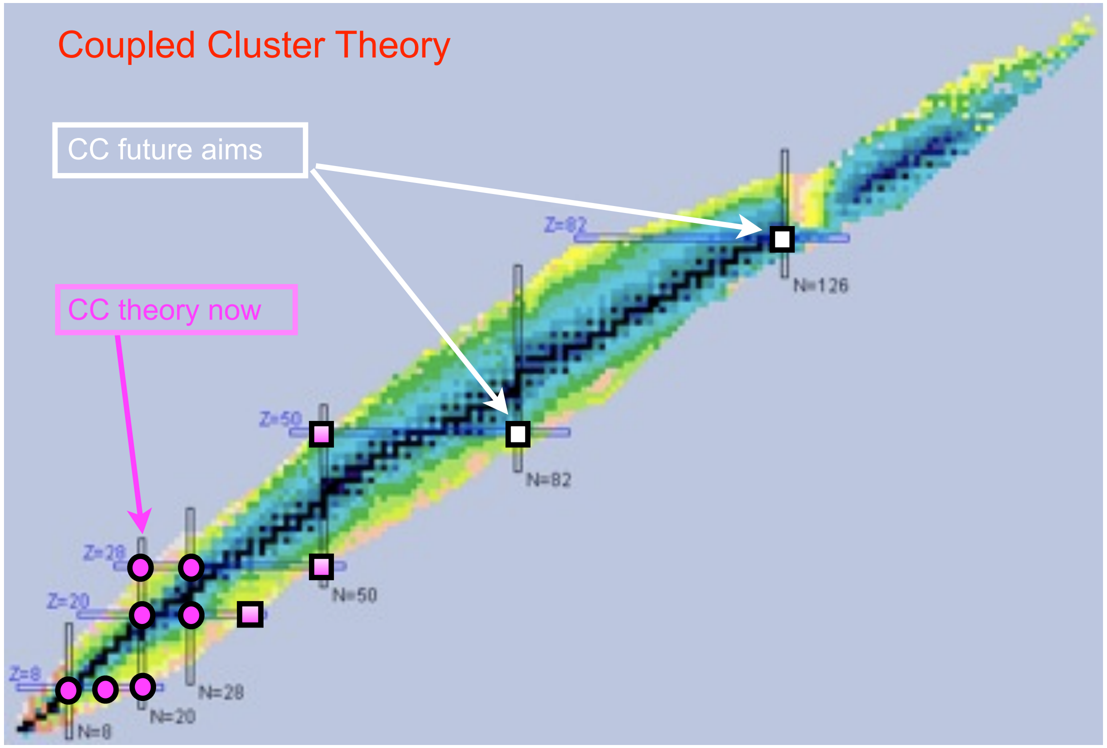
A many-body method (bound states and reactions)

Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei



- CC is optimal for closed shell nuclei (1,±2)±

Uses particle coordinates

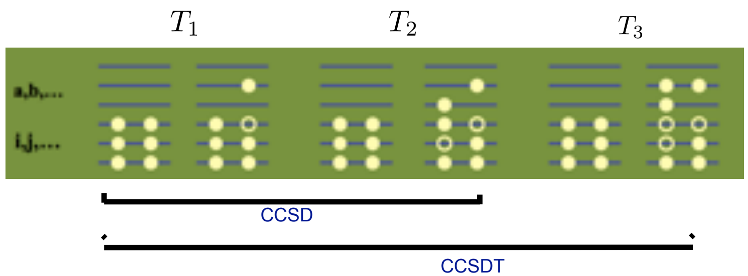
$$|\psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle$$

reference SD with any sp states

$$T = \sum T_{(A)}$$ cluster expansion

$$T_1 = \sum_{ia} t_i^a a_a^\dagger a_i$$

$$T_2 = \frac{1}{4} \sum_{ij,ab} t_{ij}^{ab} \; a_a^\dagger a_b^\dagger a_j a_i \qquad \cdots$$



For the ground state energy

$$E_0 = \langle \phi_0 | e^{-T} H e^T | \phi_0 \rangle \qquad \bar{H} = e^{-T} H e^T$$ similarity transformed Hamiltonian

$$0 = \langle \phi_i^a | e^{-T} H e^T | \phi_0 \rangle$$

$$0 = \langle \phi_{ij}^{ab} | e^{-T} H e^T | \phi_0 \rangle$$

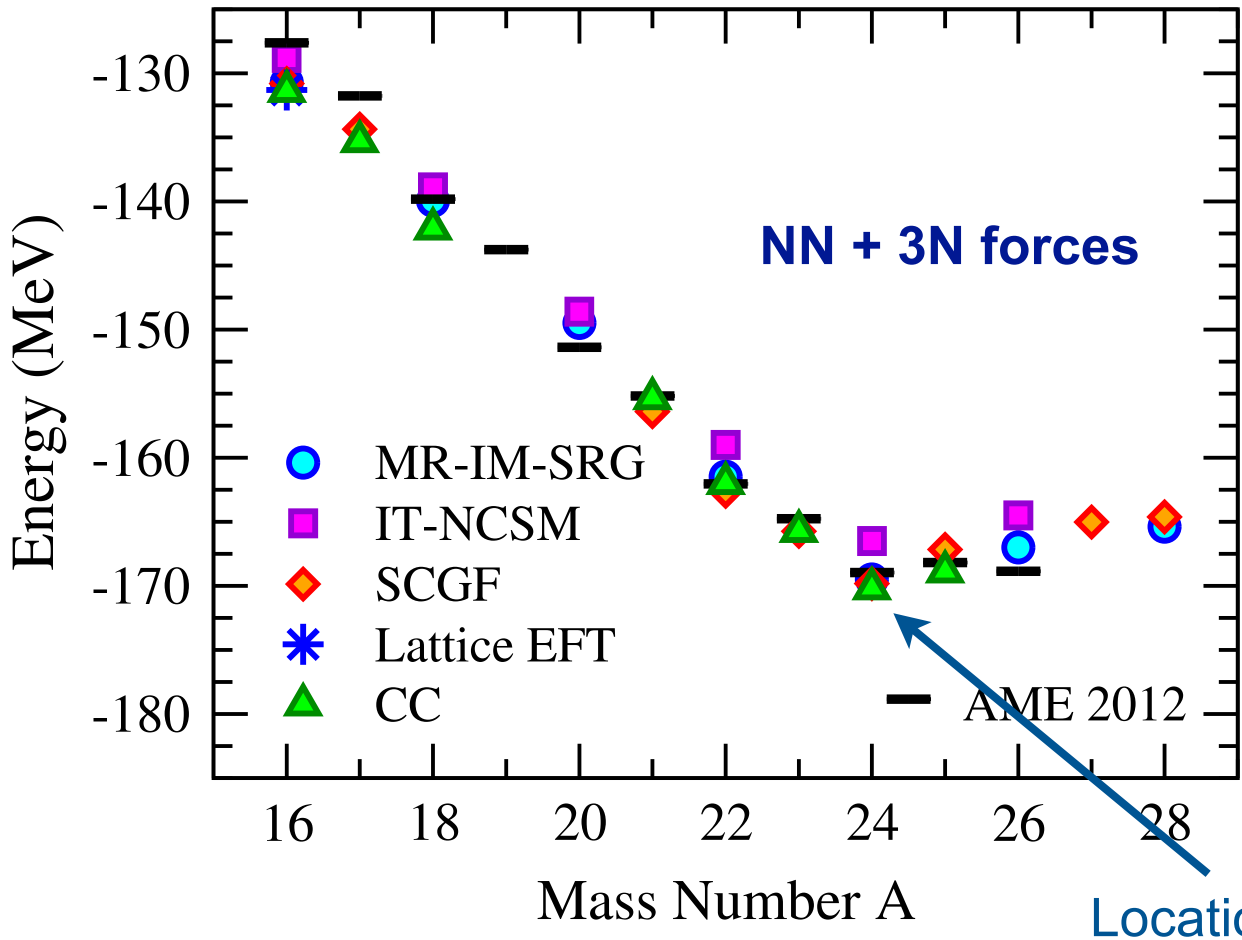
Leads to CCSD equations for the t-amplitudes

Model space truncation	$N \leq N_{max}$
Computational load	$n_o^2 n_u^4$

Coupled-cluster theory

Compared to other methods on the Oxygen isotope chain

Hebeler, Holt, Menendez, Schwenk, Ann. Rev. Nucl. Part. Sci. (2015)

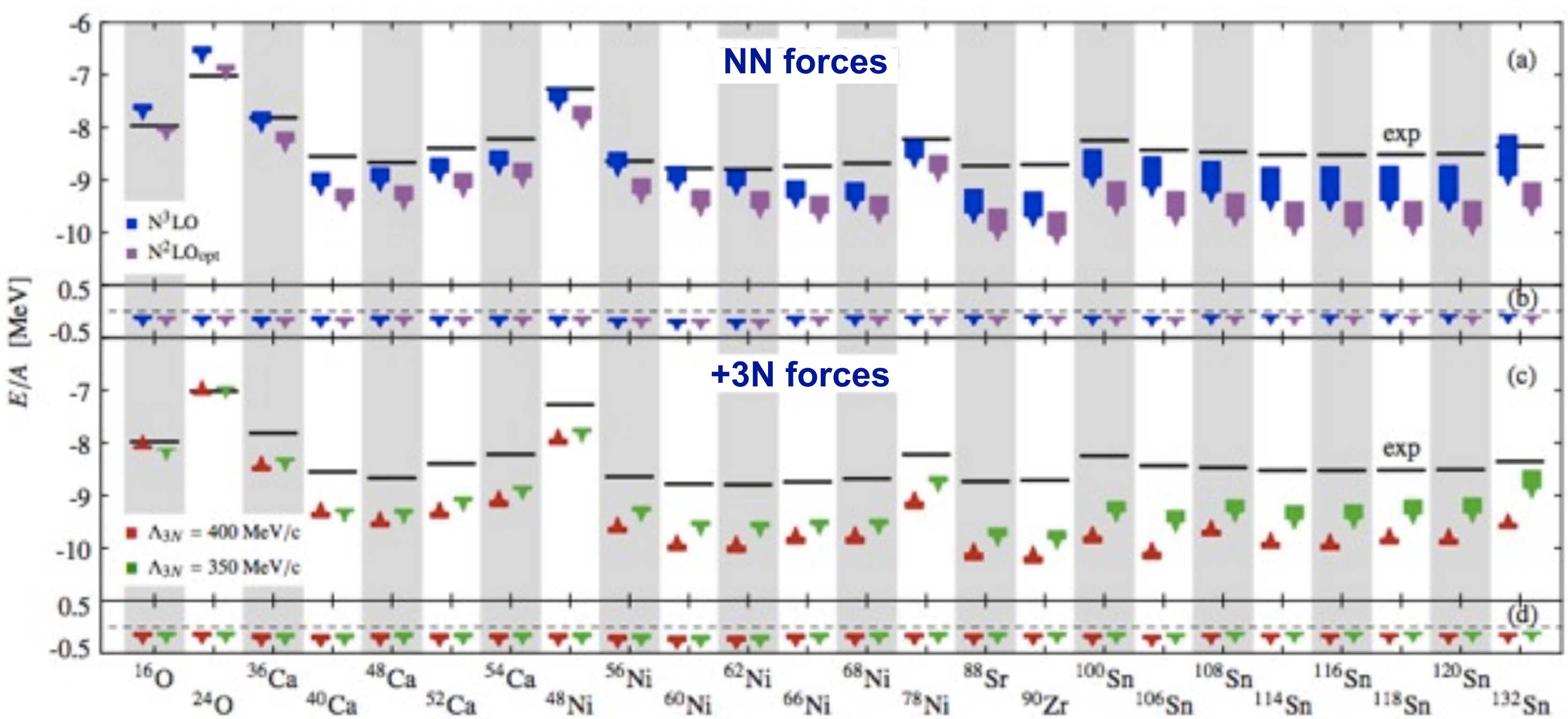


Location of the drip line

Coupled-cluster theory

Pushing the limits in mass number ...

S.Binder *et al.*, Physics Letters B 736 (2014) 119–123



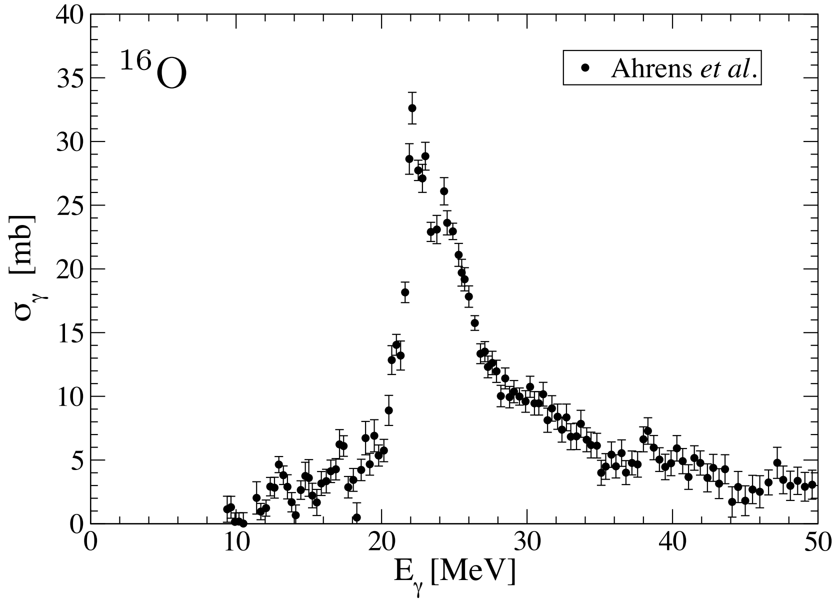
But what about reactions?

Electromagnetic Reactions

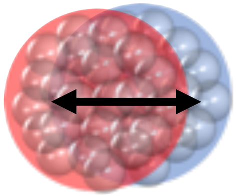
Photo-nuclear Reactions

Reactions resulting from the interaction of a photon with the nucleus

For photon energy 15-25 MeV stable nuclei across the periodic table show wide and large peak



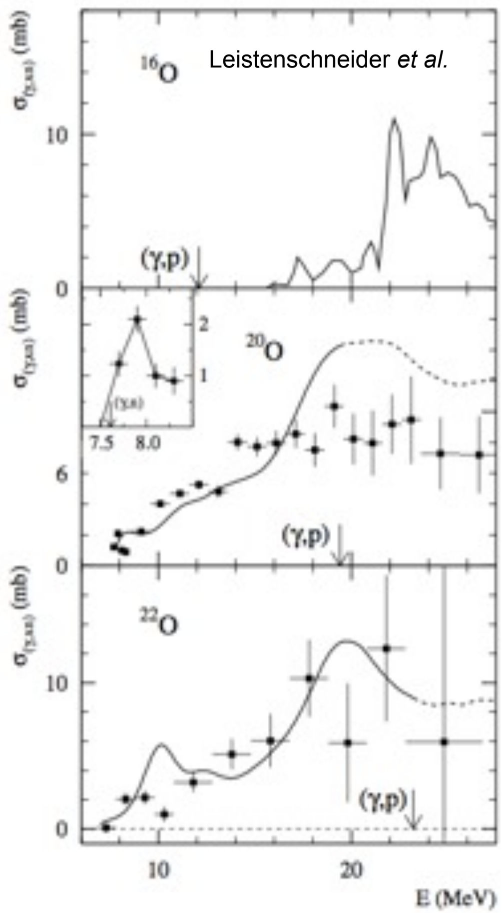
Giant Dipole Resonance



Coulomb excitations

Inelastic scattering between two charged particles. Can use unstable nuclei as projectiles.

Neutron-rich nuclei show fragmented low-lying strength



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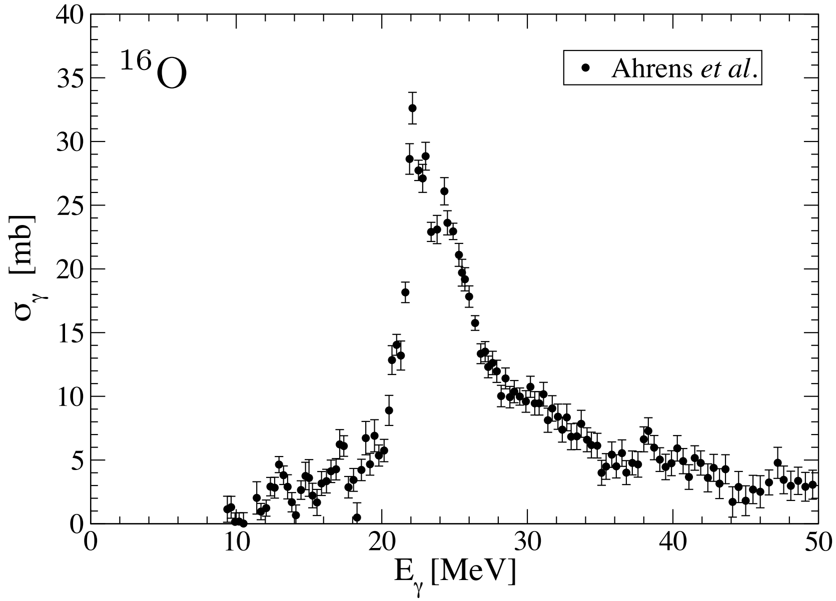
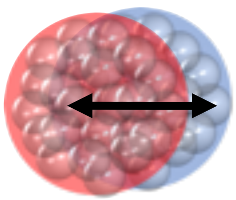


Giant Dipole Resonance



Coulomb excitations

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Can use unstable nuclei as projectiles.

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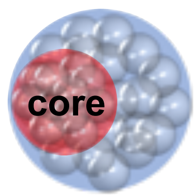
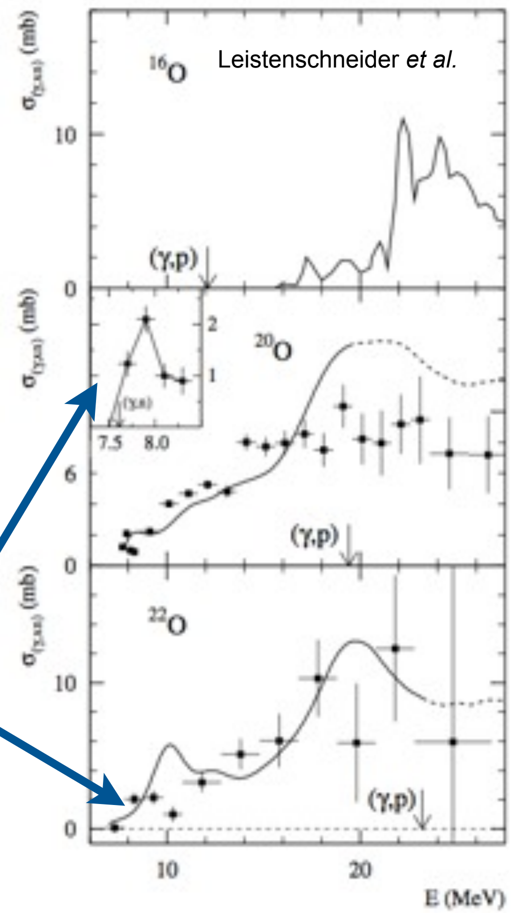


Pigmy Dipole Resonance



- Can we give a microscopic explanation of these observations?

LIT with Coupled Cluster Theory

S.B. *et al.*, PRL **111**, 122502 (2013)

$$\boxed{(H - z^*)|\tilde\Psi\rangle = J^\mu|\psi_0\rangle} \longrightarrow \boxed{(\bar H - z^*)|\tilde\Psi_R(z^*)\rangle = \bar\Theta|\Phi_0\rangle}$$

with $z = E_0 + \sigma + i\Gamma$

$$\bar H = e^{-T} H e^T$$

$$\bar\Theta = e^{-T}\Theta e^T$$

$$L(\sigma,\Gamma) = \left\langle \tilde\Psi | \tilde\Psi \right\rangle \longrightarrow L(\sigma,\Gamma) = \left\langle \tilde\Psi_L | \tilde\Psi_R \right\rangle =$$

$$-\frac{1}{2\pi}\Im\left\{\langle \bar 0_L|\bar\Theta^\dagger\left[|\tilde\Psi_R(z^*)\rangle - |\tilde\Psi_R(z)\rangle\right]\right\}$$

with $|\tilde\Psi_R(z^*)\rangle = \hat R(z^*)|\Phi_0\rangle$

Formulation based on the solution of an equation of motion with a source

No approximations done so far!

Present implementation in the CCSD scheme $T = T_1 + T_2$ $\hat R = \hat R_0 + \hat R_1 + \hat R_2$

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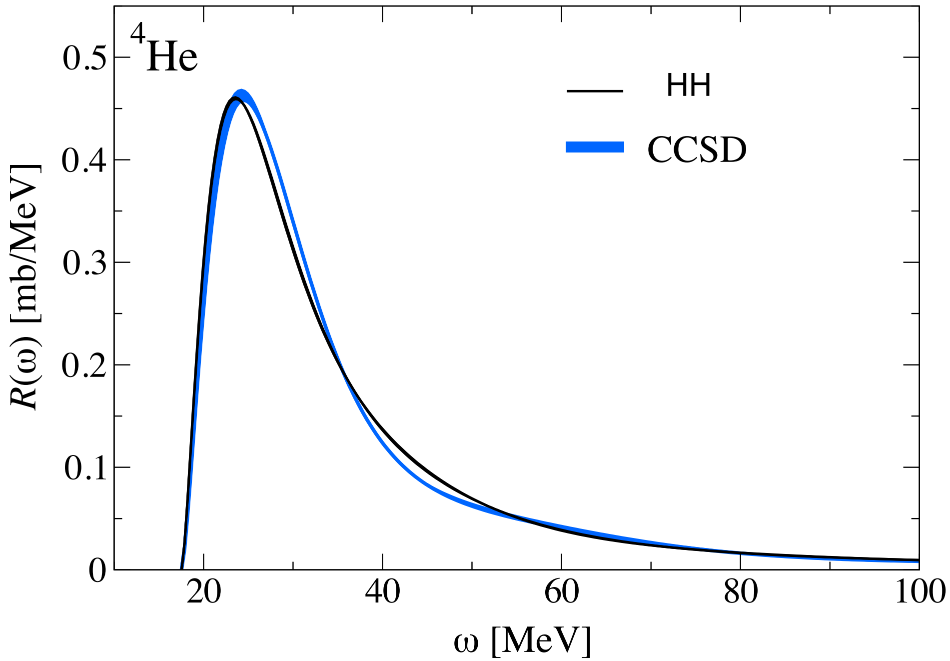
LIT with Coupled Cluster Theory

Validation for ^{4}He

Comparison of CCSD with exact hyperspherical harmonics (HH) with NN forces at N^3LO



The comparison with exact theory is very good!

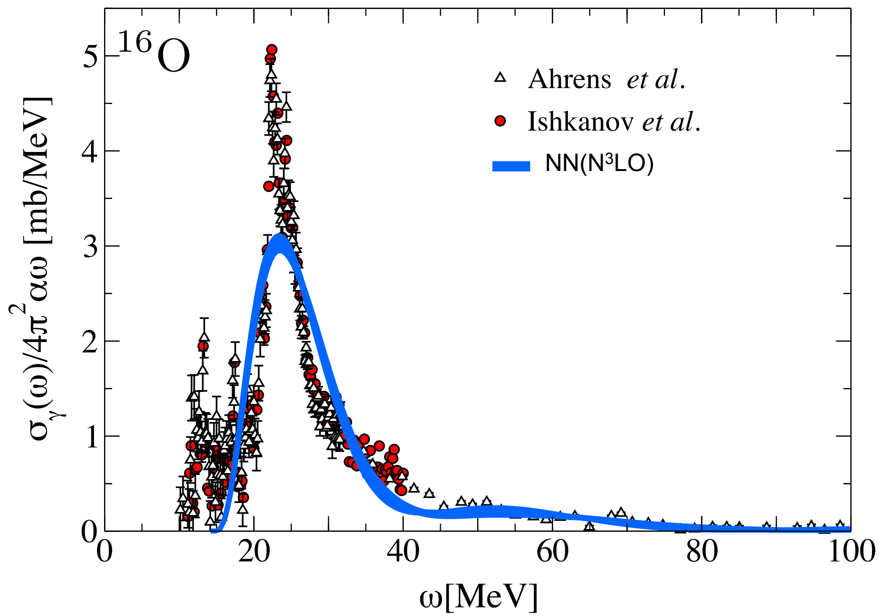
Small difference due to missing triples and quadrupoles

LIT with Coupled Cluster Theory

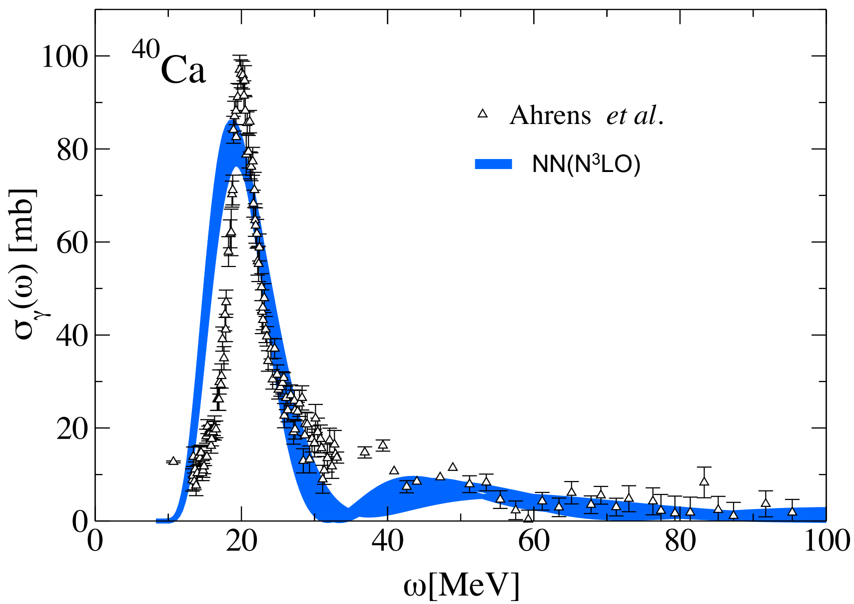
New theoretical method aimed at extending *ab-initio* calculations towards medium mass

Extension to Dipole Response Function to $^{16}O/^{40}Ca$ with NN forces derived from χEFT (N^3LO)

S.B. *et al.*, PRL **111**, 122502 (2013)



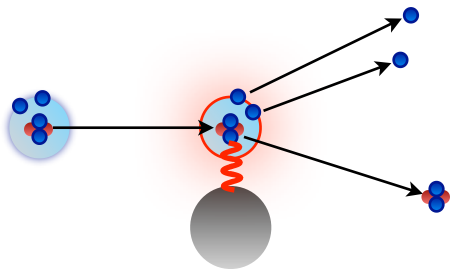
S.B. *et al.*, PRC **90**, 064619 (2014)



First time description of GDR for these nuclei

Pigmy Resonances in Nuclei

Observed e.g. in coulomb excitation reactions



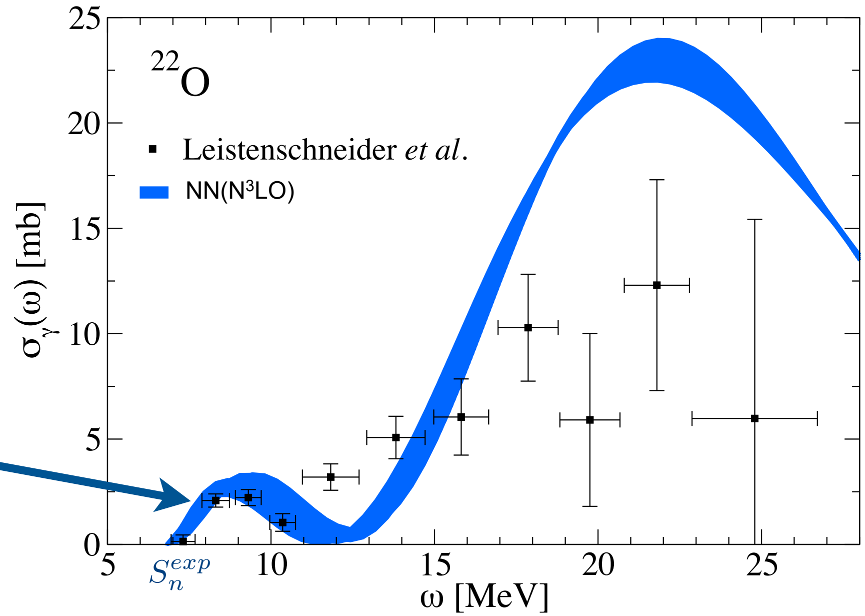
★ Calculation for ^{22}O and ^{22}C

With Mirko Miorelli, PhD student UBC/TRIUMF

PRC **90**, 064619 (2014)

^{22}O data from GSI

^{22}C being measured at RIKEN



Soft dipole mode well described by theory

To do:

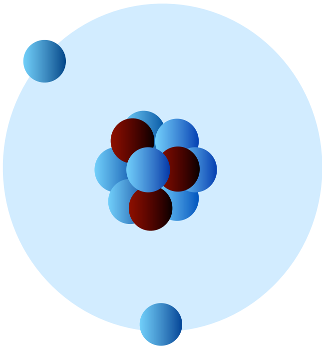
- Include 3NF
- Improve on the approximation scheme CCSD

Pigmy Resonances in Nuclei

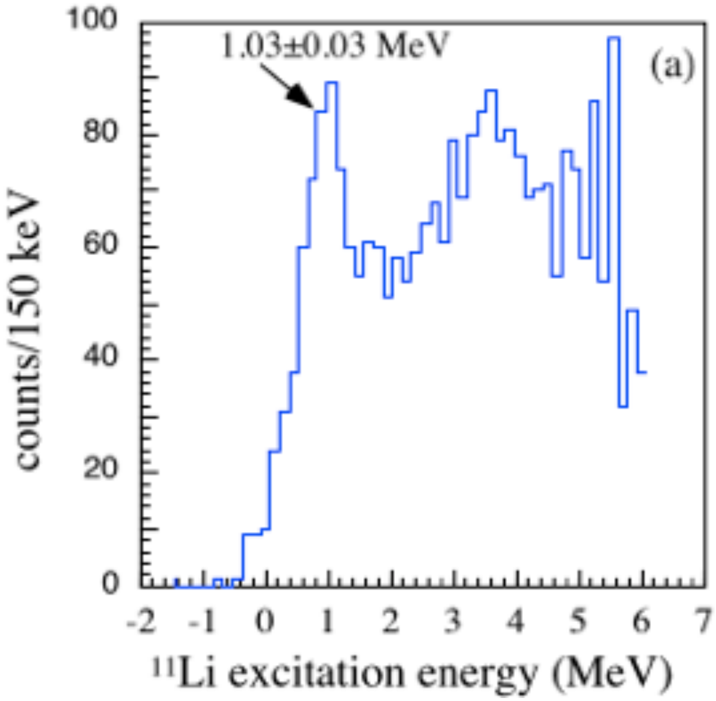
Other resonances in exotic nuclei which await a microscopic explanation ...

With IRIS@TRIUMF

^{11}Li

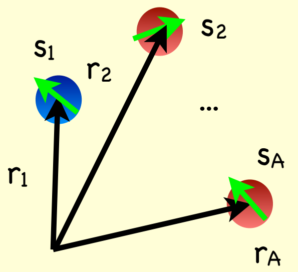


Kanungo *et al.*, Phys. Rev. Lett. **114**, 192502 (2015)



Long sought for isoscalar dipole resonance has been observed

Ab-initio Theory with external probes



$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN} + V_{3N} +$$

High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

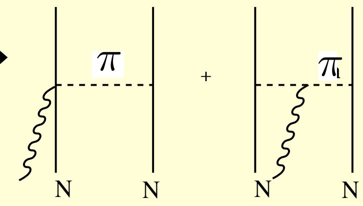
Traditional Nuclear Physics
AV18+UIX, ..., J2

Pastore. Koelling,

Park, Marcucci et.

Chiral EFT
N^2LO, N^3LO ...

$$J^\mu = J^\mu_N + J^\mu_{NN} + \ldots$$



two-body currents (or MEC) subnuclear d.o.f.

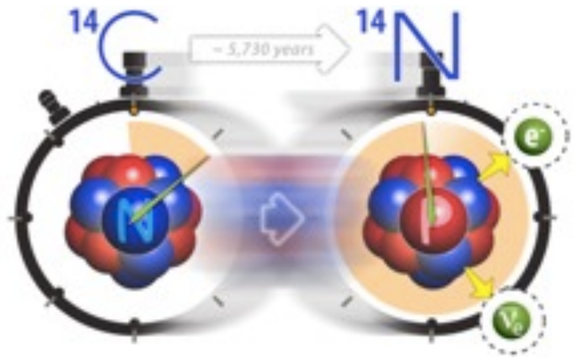
J^μ consistent with V

$$\nabla \cdot J = -i[V, \rho]$$

$$\left|\langle \Psi_f | J^\mu | \Psi_0 \rangle\right|^2$$

N.B.: Two-body currents replace the old shell model language of "effective charges" or "quenching factors"

Beta Decays

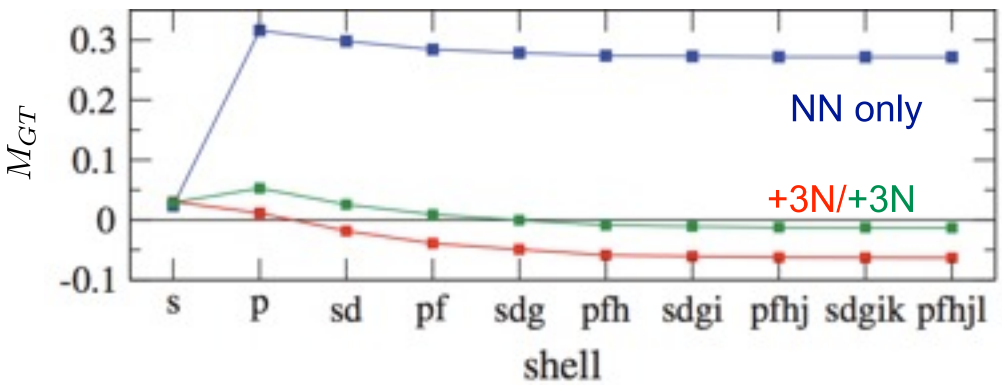


Why is the ^{14}C half life so anomalously long? 5730ys

$$T_{1/2} = \frac{1}{f(Z,E_0)} \frac{2\pi^3\hbar^7 \ln 2}{m_e^5 c^4 G_V^2} \frac{1}{g_A^2 |M_{GT}|^2} \longrightarrow T_{1/2} = \frac{1}{f(Z,E_0)} \frac{2\pi^3\hbar^7 \ln 2}{m_e^5 c^4 G_V^2} \frac{1}{g_A^2 |\ E_A^1\ |^2}$$

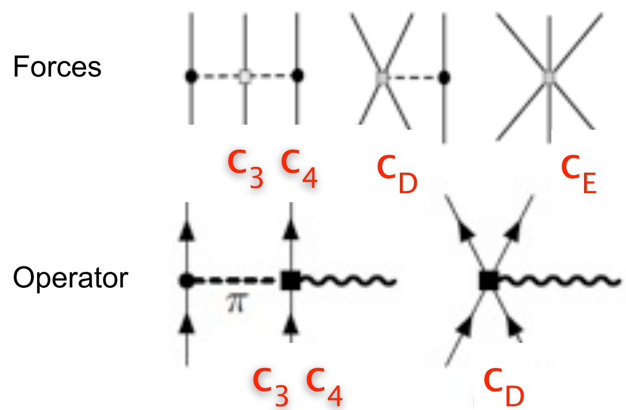
Axial dipole from one- and two-body operators

Maris *et al.*, Phys. Rev. Lett. **106**, 202502 (2011)



3NF needed to explain the long half life of ^{14}C

Three-body forces and two-body operators are deeply connected



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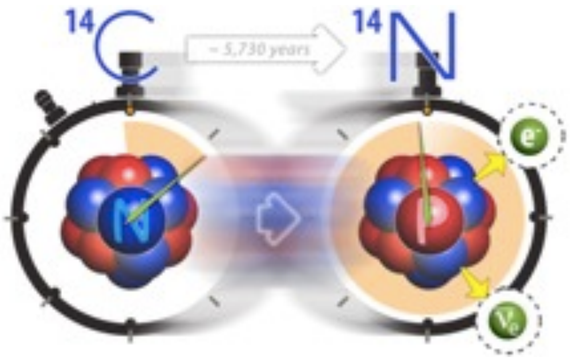
Beta Decays



$$T_{1/2} = \frac{1}{f(Z, E_0)} \frac{2\pi^3\hbar^7 \ln 2}{m_e^5 c^4 G_V^2} \frac{1}{g_A^2 |M_{GT}|^2}$$

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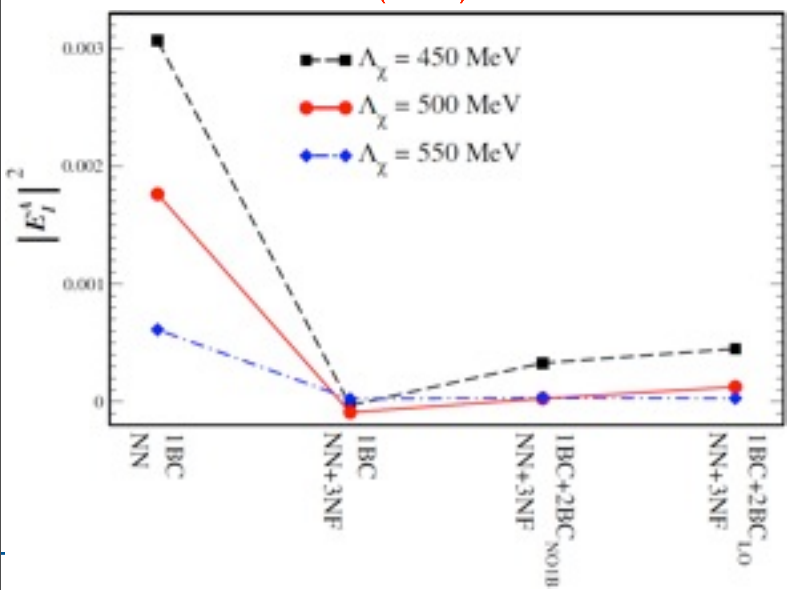
Axial dipole from one- and two-body operators

Revisited with coupled-cluster theory

PRL 113 262504 (2014)

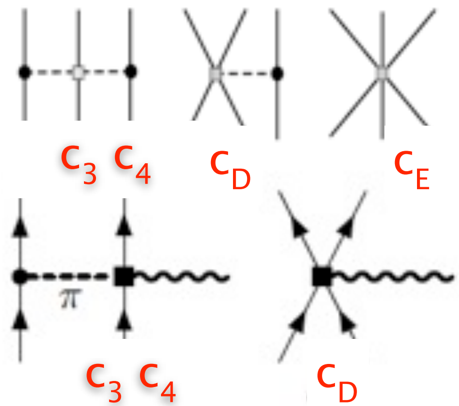


Fit c_D on triton BE and beta decay

Three-body forces and two-body operators are deeply connected

Forces

Operator



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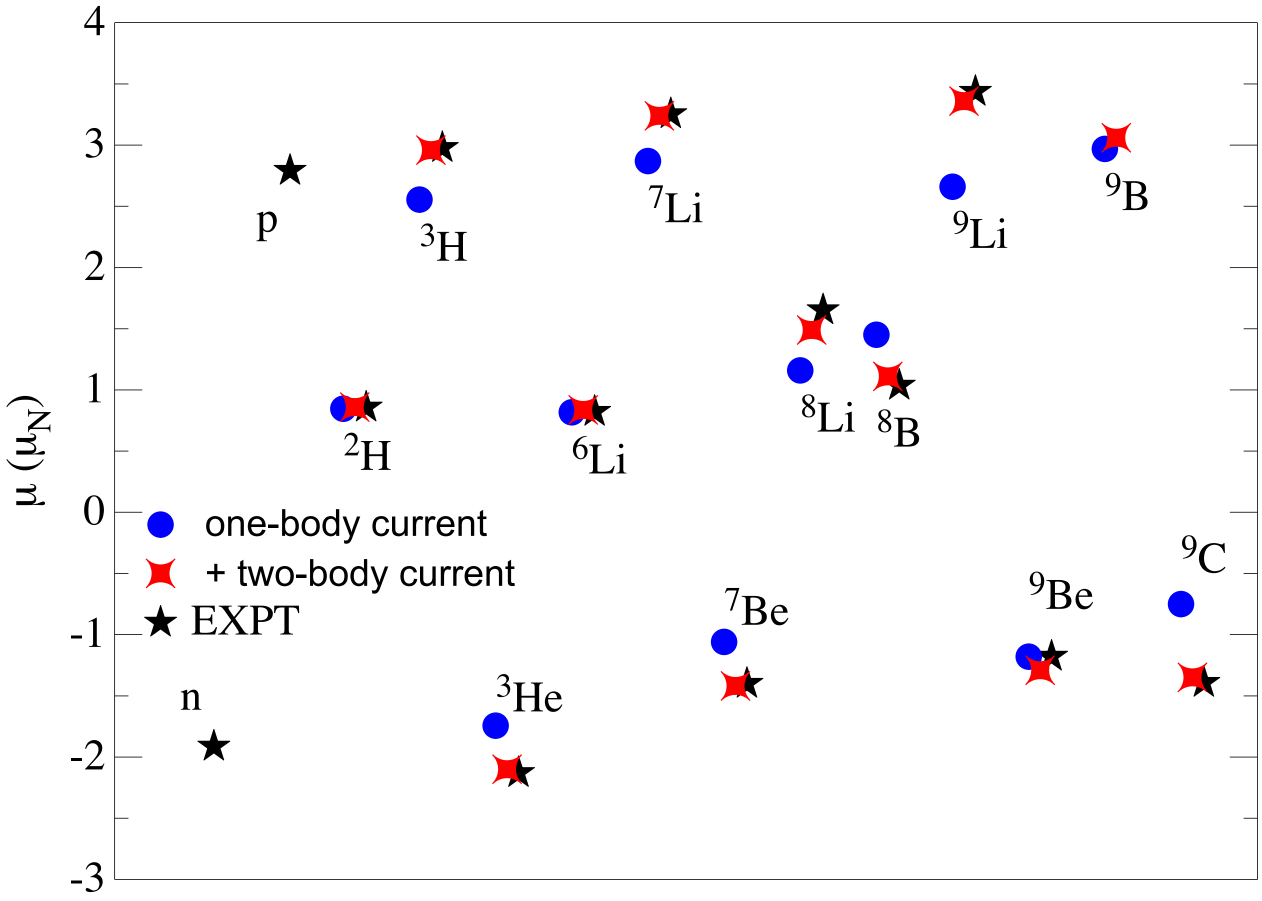
Magnetic Moments

Green's Function Monte Carlo Calculations

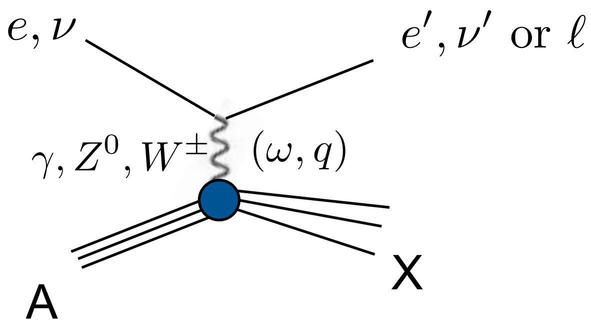
S. Pastore *et al.*, Phys. Rev. C**87**, 035503 (2013).



Two-body currents have a large effect in exotic nuclei

Neutrino-nucleus interactions

Neutrino long baseline experiments (T2K, Miniboon, LBNE, etc.) require theoretical input to simulate the interaction of neutrinos with the detector material (^{12}C, ^{16}O)



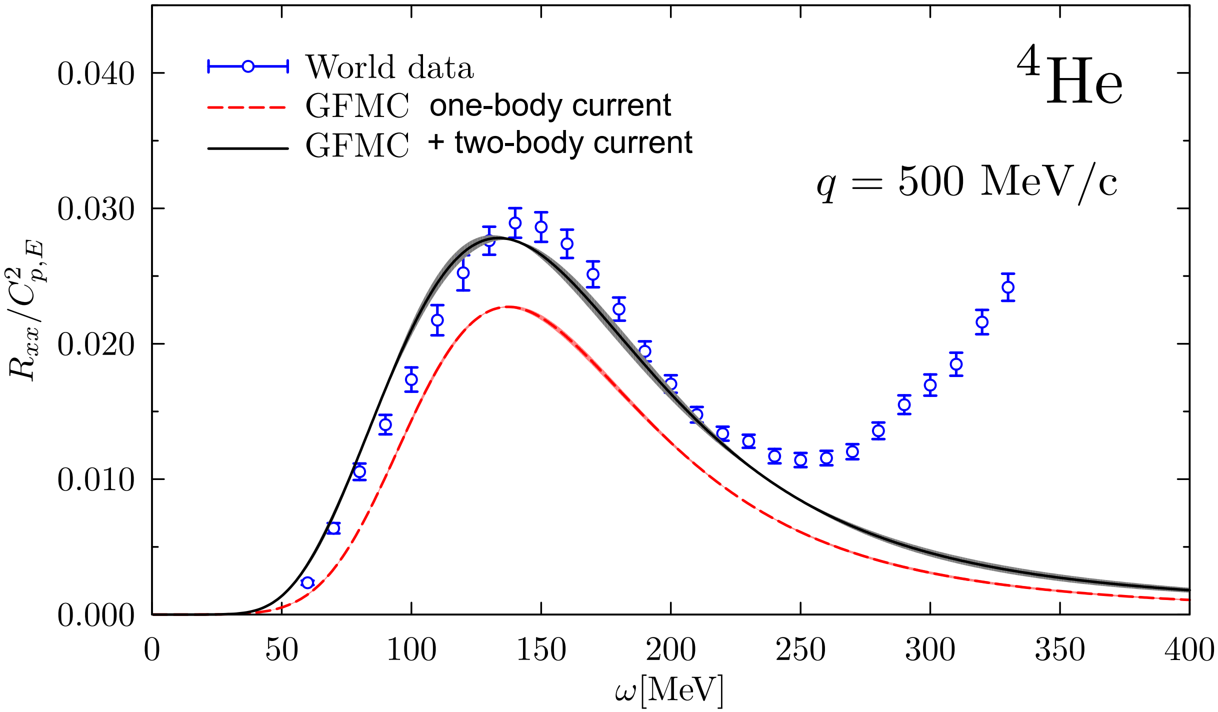
Need response functions

$$R(\omega) = \sum_f \left| \left\langle \psi_f \left| J^\mu \right| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

Lovato et al.,
Phys. Rev. C 91, 062501 (2015)

Two-body currents have a large effect!

Future: address ^{12}C



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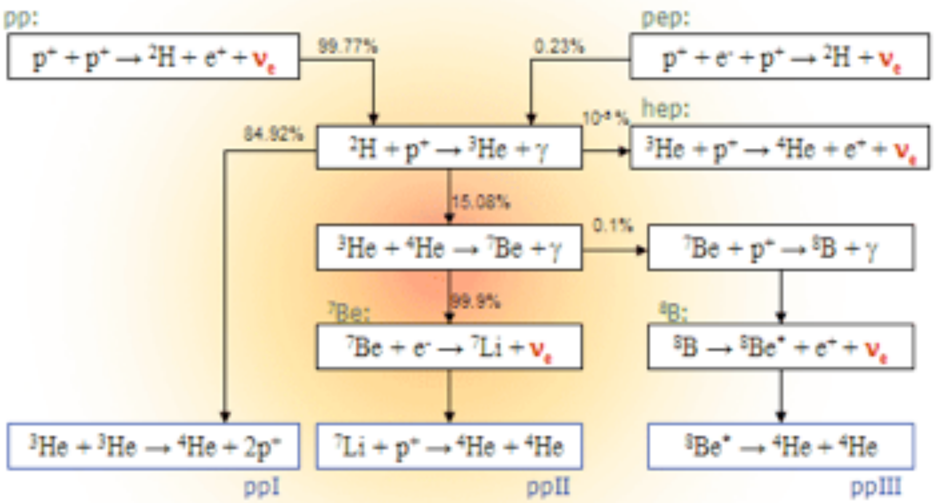
Reactions for Astrophysics



Solar neutrinos measured by SNO and Super-Kamiokande

The predicted solar neutrino flux from the ^{8}B decay is proportional to the thermal average rate of the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ radiative capture reaction

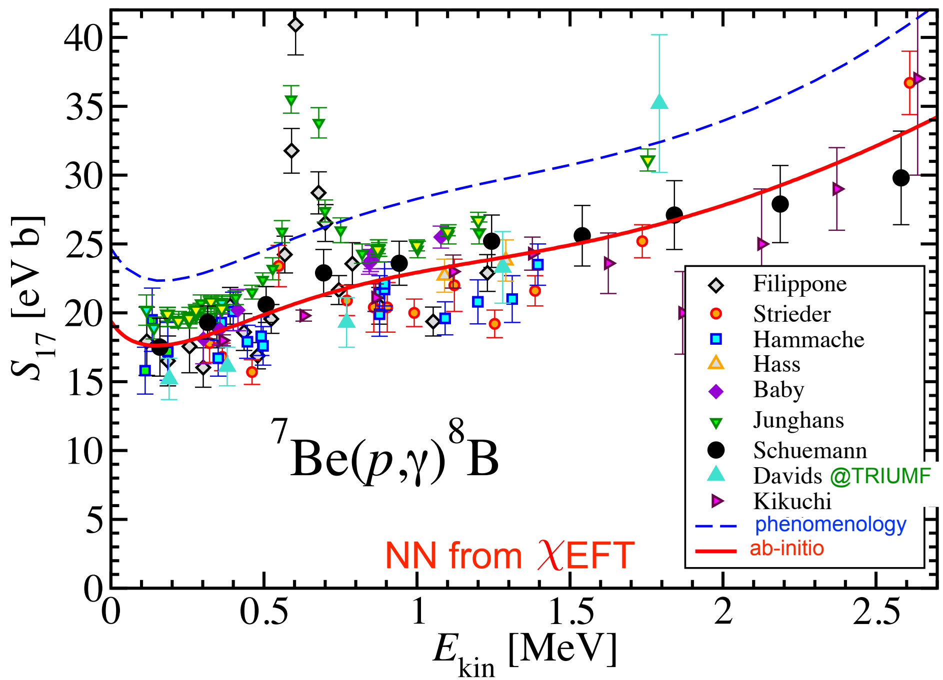
Solar conditions cannot be reproduced in the Lab

Reaction cross sections are measured at higher energy than needed

Need a reliable ab-initio theory (NCSM/RGM) because extrapolations are dangerous

Synergy between experiment and predictive theory is essential

P.Navratil *et al.* Phys.Lett.B **704** 379 (2011)

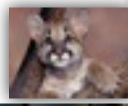


Exciting era in ab-initio nuclear theory with advances on many fronts

The ab-initio approach allows to assess solid theoretical error bars and develop the strong predictive power necessary to tackle exotic nuclei

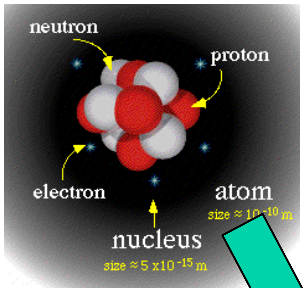
A strong connection of theory with experiment is fundamental in the physics of radioactive ion beams

Dedicated
Theory Cluster
@TRIUMF



Nuclear Structure and Reactions

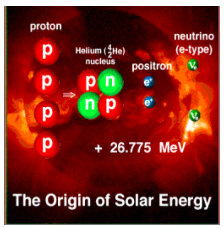
low-energy experiments

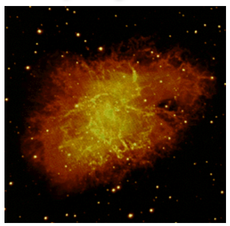


Nuclear Theory

forces
methods
extrapolations

Nuclear Astrophysics





Thank you!

Back-up slides

Nuclear shell model

Calculate the degeneracy in the nuclear shell model

Case of the spherical HO potential

$N = 2(n-1) + \ell$ defines the shell

$n - 1$ number of nodes in $R_{n\ell}(r)$

For fixed N, the angular momentum can vary because n can vary

$$\ell = N - 2(n-1) \implies \ell = N, N-2, \ldots, 2, 0.$$

N	0	1	2	...
ℓ	0	1	0 or 2	...

Because the energy does not depend on the projection m, for a given N, and ℓ we have a degeneracy

$$d_N = 2 \;\; (2\ell + 1)$$

two possible projections of spin +1/2 or -1/2

all possible values of m for a given ℓ

Nuclear shell model

Recap of spin-orbit splitting

Spin-orbit splitting

We have to couple the angular momenta also in the wave functions:

$$\varphi_{n\ell jm}(\vec{r}\,) = R_{n\ell}(r)\;[Y_\ell(\hat{r})\otimes\chi_{1/2}(\sigma)]^j_m$$

To calculate the single particle energy with the new hamiltonian we have to work out the spin-orbit operator

$$j^2 = \vec{j}\cdot\vec{j} = (\vec{\ell}+\vec{s}\,)\cdot(\vec{\ell}+\vec{s}\,) = \ell^2 + s^2 + 2\vec{\ell}\cdot\vec{s}$$

$$\vec{\ell}\cdot\vec{s} = \frac{1}{2}(j^2 - \ell^2 - s^2)$$

So that

$$\vec{\ell}\cdot\vec{s}\,\varphi_{n\ell jm}(\vec{r}\,) = \frac{1}{2}(j(j+1) - \ell(\ell+1) - s(s+1))\varphi_{n\ell jm}(\vec{r}\,)$$

So that, the correction to the single particle energy given by the spin-orbit, leads to the following splitting

$$\langle\vec{\ell}\cdot\vec{s}\,\rangle_{j=\ell+1/2} - \langle\vec{\ell}\cdot\vec{s}\,\rangle_{j=\ell-1/2} = \frac{1}{2}(2\ell+1)$$

bigger with increasing orbital angular momentum

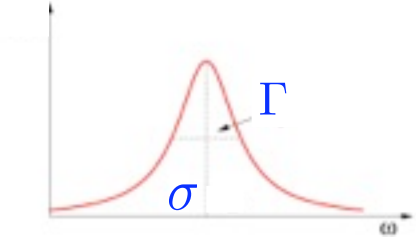
Lorentz Integral Transform Method

Derivation of the bound-state-like equation

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

Reduce the continuum problem to a bound-state problem

$$R(\omega) = \sum\!\!\!\!\!\!\!\int_f \left| \left\langle \psi_f \middle| J^\mu \middle| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$



$$L(\sigma,\Gamma) = \int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \left\langle \tilde{\psi} | \tilde{\psi} \right\rangle < \infty$$

Friday, 17 July, 15

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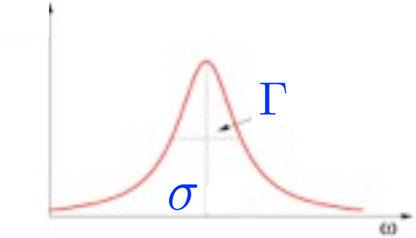

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$$(\omega - \sigma - i\Gamma)(\omega - \sigma + i\Gamma)$$

$$= \sum_f \left\langle \psi_0 | J^\mu \frac{1}{E_f - E_0 - \sigma - i\Gamma} | \psi_f \right\rangle \left\langle \psi_f | \frac{1}{E_f - E_0 - \sigma + i\Gamma} J^\mu | \psi_0 \right\rangle$$

$$= \sum_f \left\langle \psi_0 | J^\mu \frac{1}{H - E_0 - \sigma - i\Gamma} | \psi_f \right\rangle \left\langle \psi_f | \frac{1}{H - E_0 - \sigma + i\Gamma} J^\mu | \psi_0 \right\rangle$$

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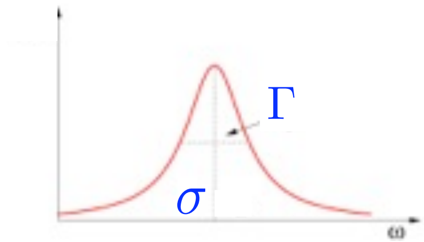

$$L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \left\langle \tilde{\psi} \middle| \tilde{\psi} \right\rangle < \infty$$

where $\left|\tilde{\psi}\right\rangle$ is obtained solving

$$\boxed{(H - E_0 - \sigma + i\Gamma)|\tilde{\Psi}\rangle = J^\mu |\Psi_0\rangle}$$

- Due to imaginary part Γ the solution $|\tilde{\psi}\rangle$ is unique
- Since $\langle\tilde{\psi}|\tilde{\psi}\rangle$ is finite, $|\tilde{\psi}\rangle$ has bound state asymptotic behaviour

⇨ Use bound-states techniques to solve the Schrödinger equation

$$L(\sigma, \Gamma) \xrightarrow{\text{inversion}} R(\omega)$$

The exact final state interaction is included in the continuum rigorously!