Re-writing Nuclear Physics textbooks: 30 years of radioactive ion beam physics

Basic concepts in nuclear reaction theory

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Bibliography

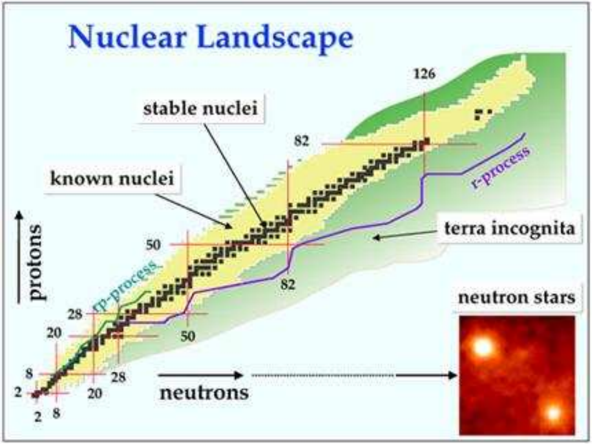
General scattering theory:

- *Quantum collision theory*, C.J. Joachain.

Scattering theory applied to nuclear reactions:

- *Introduction to nuclear reactions*, G.R. Satchler.
- *Direct Reactions*, G.R. Satchler.
- *Direct Nuclear Reactions*, N. Glendenning.
- *Nuclear reactions for astrophysics*, I.J. Thompson and F.M. Nunes.
- *Quantum scattering theory and direct nuclear reactions*, course notes by A.M.M.

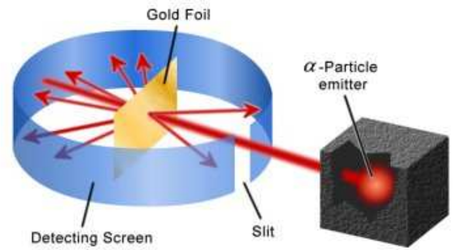
Unstable nuclei and the limits of stability



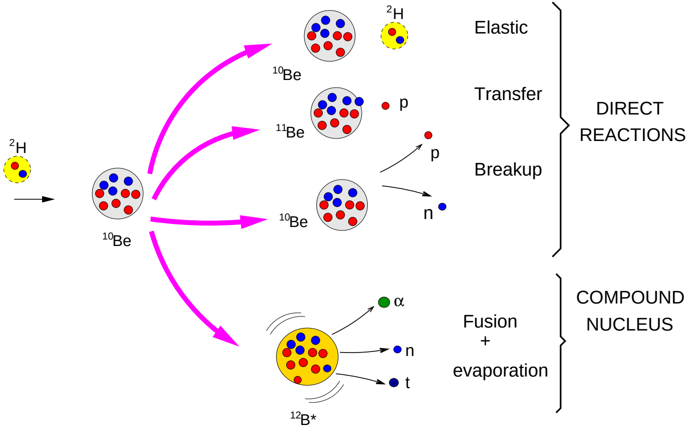
Motivation of reaction theory

➩ The aim of reaction theory is to provide a mathematical description of quantum scattering experiments, in order to extract information on the structure of the colliding nuclei and on their mutual interaction dynamics.

➩ The first experiment of this kind was the α scattering experiment by Rutherford, who lead to the proposal of his celebrated model of the atom and the subsequent formula for the angular dependence of the scattered α particles.



Types of reactions: direct vs. compound nucleus processes



Direct versus compound reactions

DIRECT: elastic, inelastic, transfer,. . .

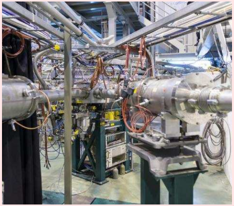
- "fast" collisions (10^{-21} s).
- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about $\pi/2$ (peaked forward)

COMPOUND: complete, incomplete fusion.

- many degrees of freedom involved
- large amount of momentum transfer
- "loss of memory" $\Rightarrow$ almost symmetric distributions forward/backward

Linking theory with experiments: the cross section

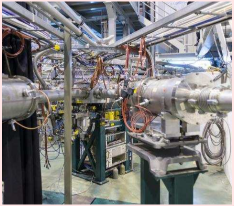
EXPERIMENT



THEORY
$(H\Psi = E\Psi)$

Linking theory with experiments: the cross section

EXPERIMENT

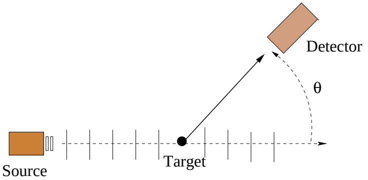


THEORY
$(H\Psi = E\Psi)$

CROSS SECTIONS

$$\frac{d\sigma}{d\Omega}, \frac{d\sigma}{dE}, etc$$

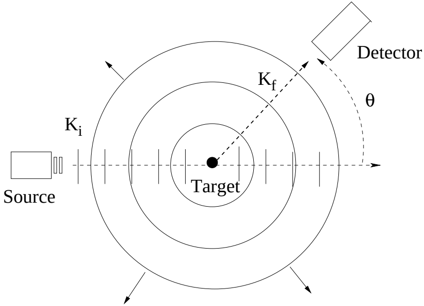
Experimental cross section

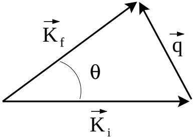


$$\Delta I = I_0 \, n_t \, \frac{d\sigma}{d\Omega} \Delta\Omega$$

- ΔI: detected particles per unit time in $\Delta\Omega$
- I_0: incident particles per unit time
- n_t: number of target nuclei per unit surface
- $\Delta\Omega$: solid angle of detector
- $d\sigma/d\Omega$: differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\text{flux of scattered particles through } dA = r^2 d\Omega}{\text{incident flux}}$$

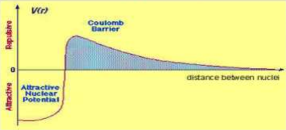




Among the many mathematical solutions of $[H - E]\Psi = 0$ we are interested in those behaving asymptotically as:

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \to \Phi_\alpha(\xi_\alpha) e^{i\mathbf{K}_\alpha \cdot \mathbf{R}_\alpha} + \text{(outgoing spherical waves)}$$

Energy domains



Competition between (attractive) nuclear and (repulsive) Coulomb interactions give rise to different physics depending on incident energies

A) "Very low" (astrophysical) energies (<< 1 MeV)

- Dominated by Coulomb interaction
- Few open channels
- Eg.: radiative capture: $^7Be(p,\gamma)^8B$ (Solar neutrino problem!)

B) "Low" energies (Coulomb barrier) (~ 1-10 MeV/u)

- Interplay between Coulomb and nuclear
- Many open channels (inelastic, transfer, breakup...)
- Strong dynamical effects

C) "Intermediate" energies (~10^2-10^3 MeV)

- Dominated by nuclear forces
- Classical-like trayectories
- More violent processes (eg. knock-out)

E (- → +)

Elastic scattering

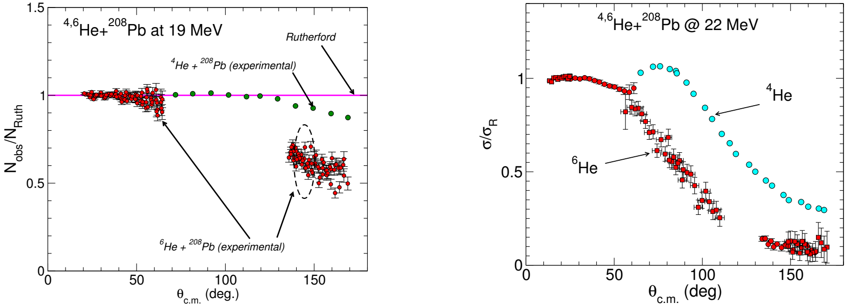
What can we learn by measuring elastic scattering?

- Studying the angular dependence of elastically scattered particles we can infer information on:
  - The interplay between Coulomb and nuclear forces.
  - The presence of non-elastic channels, that will show up as a reduction of the elastic cross section with respect to the case of inert objects (*absorption*).
- From scattering theory, the angular distribution is calculated from the scattering wavefunction as:

$$\Psi^{(+)}(\mathbf{K},\mathbf{R}) \rightarrow e^{i\mathbf{K}\cdot\mathbf{R}} + f(\theta)\frac{e^{iKR}}{R}$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

Rutherford experiment...100 years later



- ^{4}He follows Rutherford formula at 19 MeV but not at 22 MeV.Why?
- ^{6}He drastically departs from Rutherford formula at both energies. Why?

Reaction and interaction cross sections

Interaction cross sections

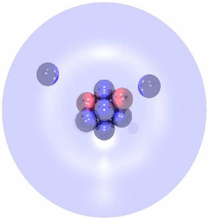
Tanihata et al, Phys. Rev. Lett. 55 (1985) 2676

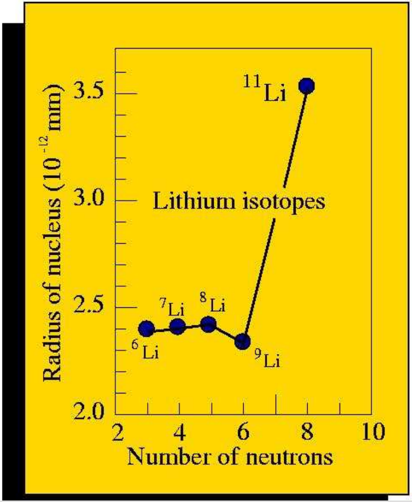
➩ Interaction cross section:

$$\sigma_I = \sigma_{\rm tot} - \sigma_{\rm inel} - \sigma_{\rm el}$$

➩ Interaction radius:

$$\sigma_I = \pi \left(R_I^{proj} + R_I^{targ}\right)^2$$

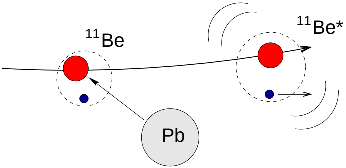


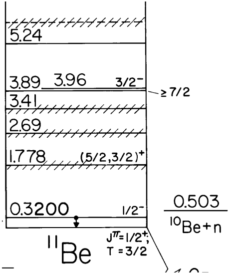


Inelastic scattering

Inelastic scattering

- Nuclei are not inert or *frozen* objects; they do have an internal structure of protons and neutrons that can be modified (excited) during the collision.
- Quantum systems exhibit, in general, an energy spectrum with bound and unbound levels.



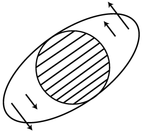


Inelastic scattering

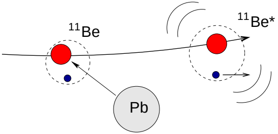
- Direct reactions → nuclei make "glancing" contact and separate immediately.
- Energy/momentum transferred from relative motion to internal motion so the projectile and/or target are left in an excited state.
- Involve small number of degrees of freedom.
- The colliding nuclei preserve their identity: $a + A \to a^* + A^*$
- Typically, they are peripheral (surface) processes.

Models for inelastic excitations

1. COLLECTIVE: Involve a collective motion of several nucleons which can be interpreted macroscopically as rotations or surface vibrations of the nucleus.



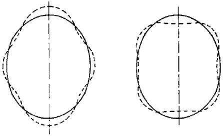
2. FEW-BODY/SIGLE-PARTICLE: Involve the excitation of a nucleon or cluster.



Types of collective excitations

The nucleons can move inside the nucleus in a coherent (collective) way.

1. Vibrations (spherical nuclei): small surface oscillations in shape.

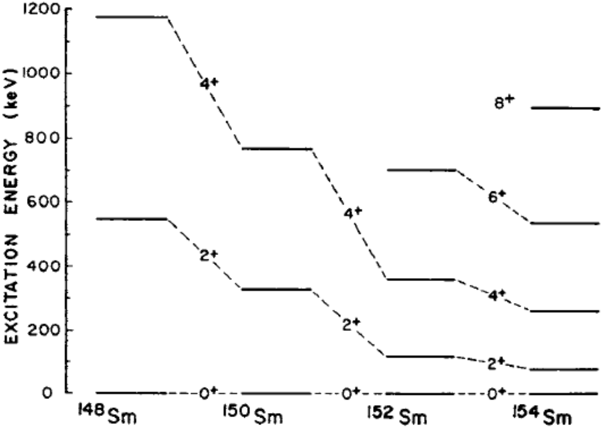


2. Rotations (non-spherical nuclei): permanent deformation.
3. Monopole (*breathing*) mode: oscillations in the size (radius).
4. Isovector excitations (protons and neutrons move out of phase) (eg. giant dipole resonance)

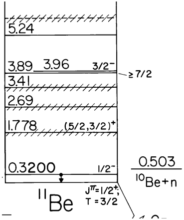
Types of collective excitations

☞ The type of collective motion is closely related to the kind of energy spectrum.

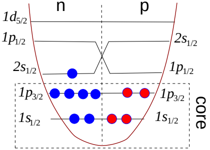
- Rotor: $E_J \propto J(J+1)$
- Vibrator: $E_J \approx n\hbar\omega$



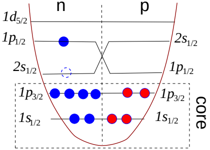
Microscopic description in the IPM: the ^{11}Be case



Ground state ($1/2^{+}$)

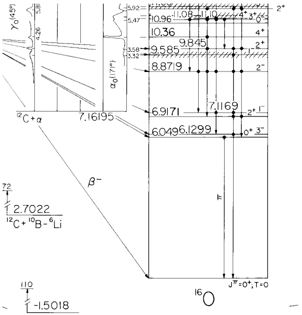


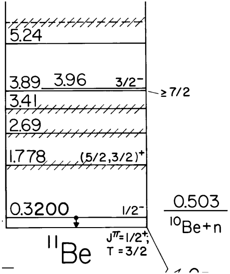
First excited state ($1/2^{-}$)



Models for inelastic excitations

Microscopically, what we describe in both cases are quantum transitions between discrete or continuum states:





☞*Collective excitations can be regarded as a coherent superposition of many single-particle excitations.*

- By doing inelastic scattering experiments we *measure* the *response* of the nucleus to an external field (Coulomb, nuclear). This response is related to some structure property of the nucleus.

  Example: for a Coulomb field:

$$B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f | \mathcal{M}(E\lambda) | \Psi_i \rangle|^2$$

  where $\mathcal{M}(E\lambda, \mu)$ is the electric multipole operator:

$$\mathcal{M}(E\lambda, \mu) \equiv e \sum_i^{Z_p} r_i^\lambda Y^*_{\lambda\mu}(\hat{r}_i)$$

- The structure $\Psi_{i,f}$ can be described in a collective, few-body or microscopic model.

Energy balance for inelastic scattering

- For projectile excitation: $a + A \longrightarrow a^* + A$

$$E^i_{\rm cm} + M_a c^2 + M_A c^2 = E^f_{\rm cm} + M^*_a c^2 + M_A c^2$$

$M_{a^*} = M_a + E_x$ (E_x=excitation energy)

- Q-value:

$$\boxed{Q = M_a c^2 + M_A c^2 - M^*_a c^2 - M^2_A c^2 = -E_x < 0}$$

$$\boxed{E^f_{\rm cm} = E^i_{\rm cm} + Q}$$

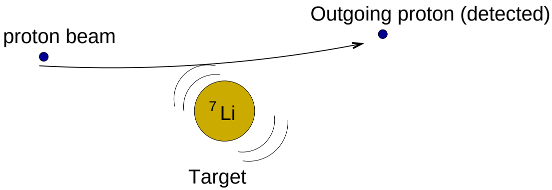
- So

$$\boxed{E_x = E^i_{cm} - E^f_{cm}}$$

What do we measure in an inelastic scattering experiment?

☞ In general, one measures the scattering angle and energy of outgoing particles.

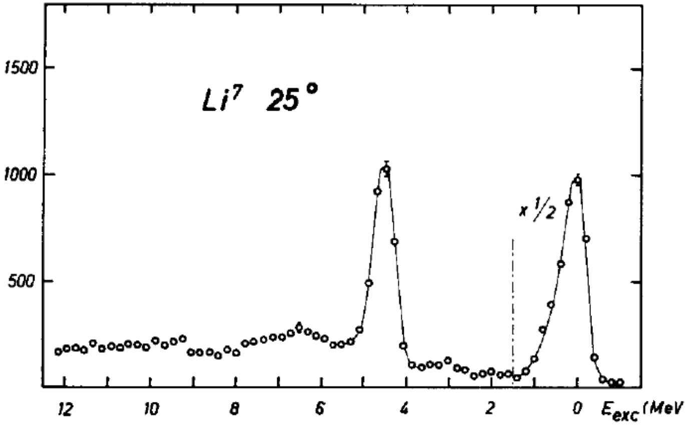
EXAMPLE: p+^{7}Li → p+^{7}Li*



☞ *Eg. energy and angular distribution of the outgoing protons.*

What do we measure in an inelastic scattering experiment?

☞ The proton energy carries information on the ^{7}Li excitation spectrum.



Data from Nuclear Physics 69 (1965) 81-102

What do we measure in an inelastic scattering experiment?

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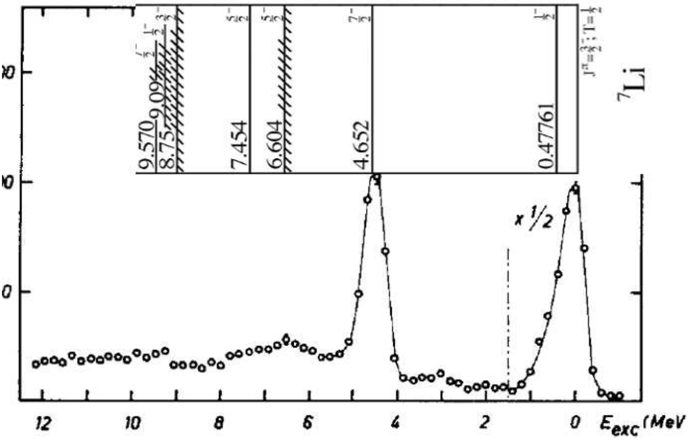
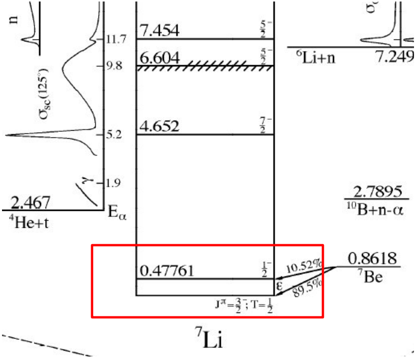


What information do we get from an inelastic scattering experiment?

- The proton energy spectrum shows peaks which correspond to the states of the target (^{7}Li)
- The heights of peak ($\sim$ cross section) are different for each state $\Rightarrow$ not all states are populated with the same probability.
- Some peaks are narrow, other are broad. Why?...
- Above a certain excitation energy, the spectrum becomes continuous and structureless.

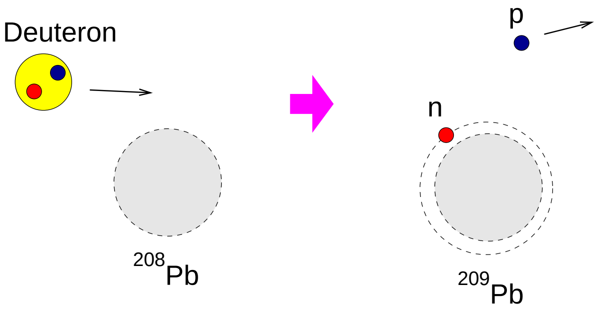
What information do we get from an inelastic scattering experiment?



Transfer reactions

Transfer reactions

Example: $d + {}^{208}Pb \rightarrow p + {}^{209}Pb$



Transfer reactions: Q-value considerations

Consider: $a + A \rightarrow b + B$

- Energy balance (in CM frame):

$$E^{i}_{\rm cm} + M_a c^2 + M_A c^2 = E^{f}_{\rm cm} + M_b c^2 + M_B c^2$$

- Q_0 value:

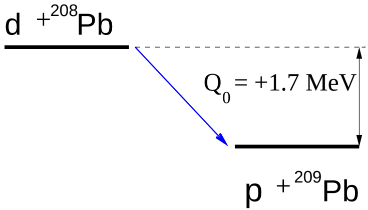
$$Q_0 = M_a c^2 + M_A c^2 - M_b c^2 - M_B c^2$$

$$E^{f}_{\rm cm} = E^{i}_{\rm cm} + Q_0$$

  - $Q_0 > 0$: the system gains kinetic energy (exothermic reaction)
  - $Q_0 < 0$: the system loses kinetic energy (endothermic reaction)

Transfer reactions: Q-value considerations

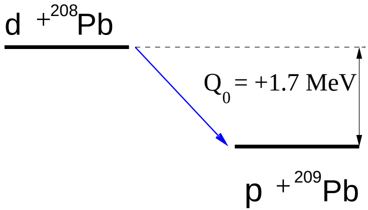
Example: d+^{208}Pb $\rightarrow$ p + ^{209}Pb



$$Q_0 = M_d c^2 + M(^{208}\text{Pb})c^2 - M_p c^2 - M(^{209}\text{Pb})c^2 = +1.7\,\text{MeV}$$

Transfer reactions: Q-value considerations

Example: d+^{208}Pb → p + ^{209}Pb



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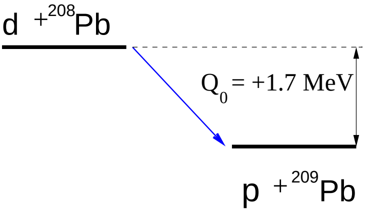
☞ $Q_0 > 0$: the outgoing proton will gain energy with respect to the incident deuteron.

Transfer reactions: Q-value considerations

Example: d+^{208}Pb → p + ^{209}Pb



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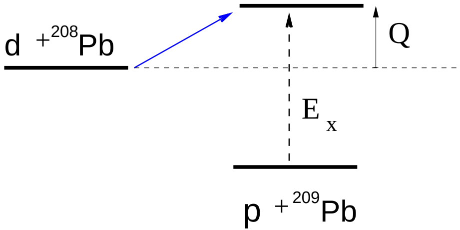
☞ $Q_0 > 0$: the outgoing proton will gain energy with respect to the incident deuteron.

For a transfer reaction, the Q value is just the difference in binding energies of the transferred particle/cluster in the initial and final nuclei:

$$Q_0 = \varepsilon_b(f) - \varepsilon_b(i) = 3.936 - 2.224 = +1.7\,\text{MeV}$$

Transfer reactions: Q-value considerations

If the transfer leads to an excited state, the Q-value will change, and hence the kinetic energy of the outgoing nuclei.



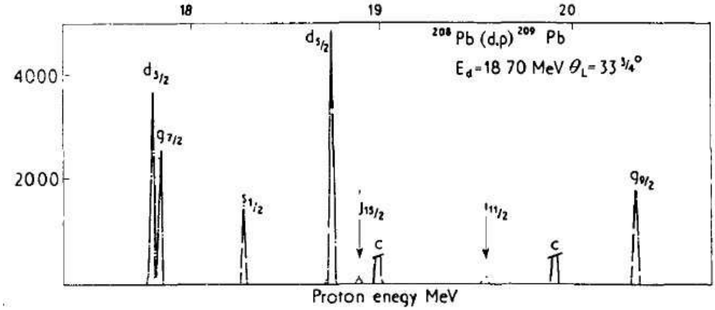
Energy balance:

$$E^f_{\rm cm} = E^i_{\rm cm} + Q = E^i_{\rm cm} + Q_0 - E_x$$

☞*If we know Q_0 we can infer the excitation energies (E_x) measuring the final kinetic energy of outgoing fragments.*

What we do observe in a transfer experiment?

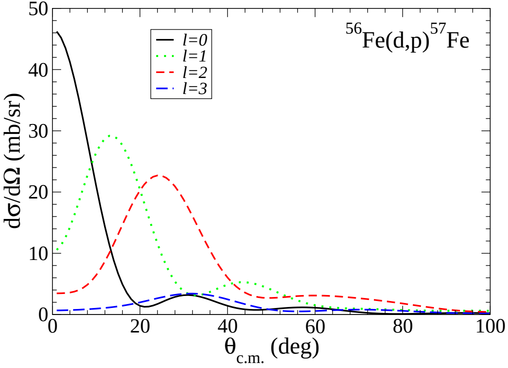
Example: d+^{208}Pb → p + ^{209}Pb



- ☞ The proton energy spectrum shows some peaks which reflect the excitation energy spectrum of the residual nucleus (^{209}Pb).
- ☞ The population probability will depend on the reaction dynamics and on the structure properties of these states.

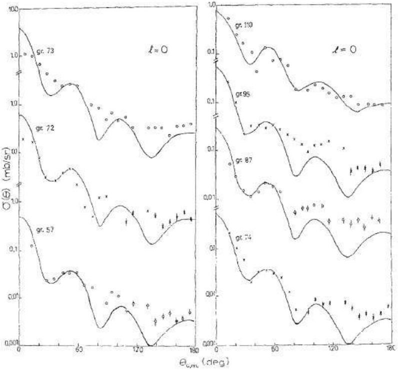
Transfer example: $^{56}Fe(d,p)^{57}Fe$

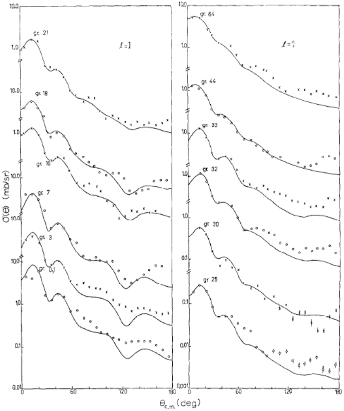
Selectivity of ℓ:



Transfer example: ^{56}Fe(d,p)^{57}Fe

Selectivity of ℓ:



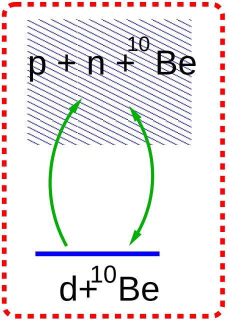


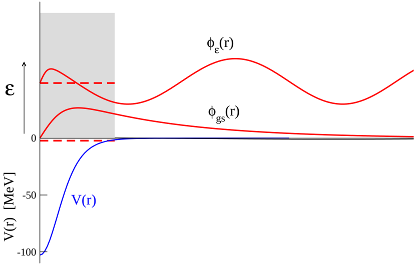
H.M. Sen Gupta et al, Nucl. Phys. A160, 529 (1971)

Breakup reactions

Breakup reactions

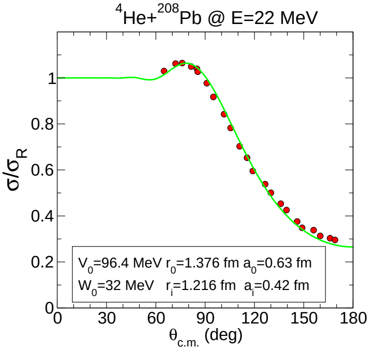
- Direct processes of the from: $a + A \rightarrow b + x + A$
- Can be interpreted (and modelled) as an inelastic excitation to the continuum spectrum.

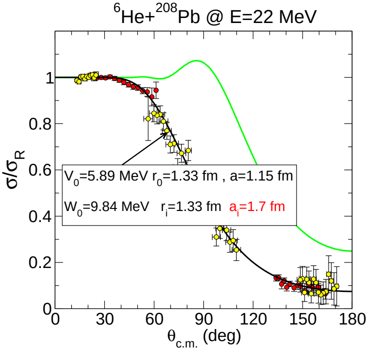




- Important for weakly-bound nuclei (eg. halo nuclei)

Influence of breakup on elastic scattering



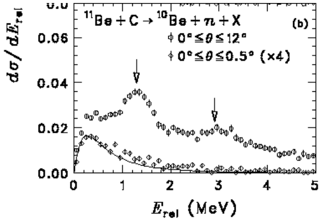


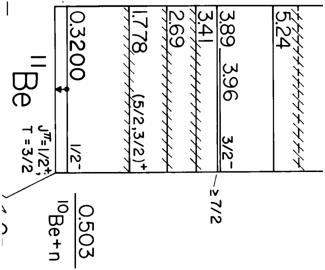
- ^{4}He+^{208}Pb shows typical Fresnel pattern → *strong absorption*
- ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section due to the flux going to other channels (mainly break-up)
- ^{6}He+^{208}Pb requires a large imaginary diffuseness → *long-range absorption*

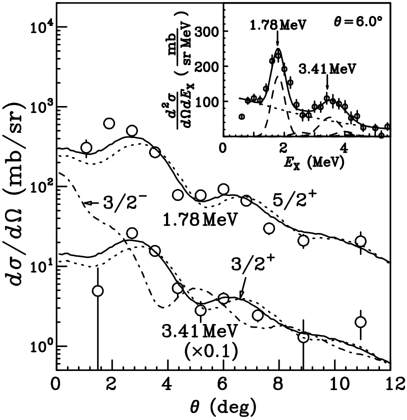
Extracting information from the continuum with breakup reactions

Example: Populating resonances by "inelastic scattering" in ^{11}Be+^{12}C

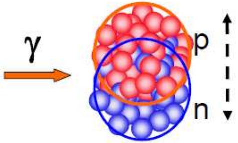
Fukuda et al, Phys. Rev. C70 (2004) 054606)



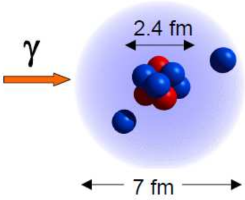




Coulomb response of halo nuclei

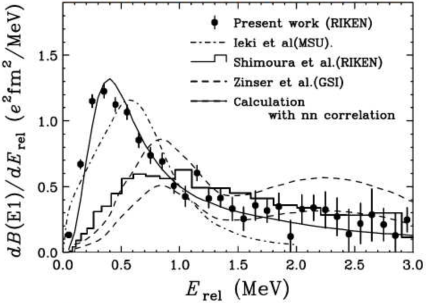


(Giant) electric Dipole excitation → 10-20 MeV



^{6}He (2n,1 MeV) ^{11}Be (1n,0.5 MeV)
^{11}Li (2n,0.5 MeV) ^{14}Be (2n,~1 MeV)

Electric dipole response – ^{11}Li

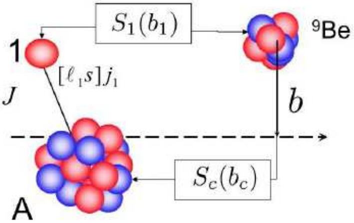


T. Nakamura et al., PRL (2006) in press

Knockout reactions

Knockout reactions

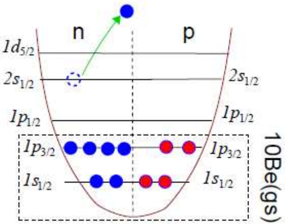
- Fast-moving projectile on a (typically) light target.
- One nucleon suddently removed (absorbed) due to its interaction with the target.
- The remaining nucleons remain unchanged and is detected.
- The momentum of the core is traced back to that of the removed nucleon because in the rest frame of the projectile $\vec{P} = 0$

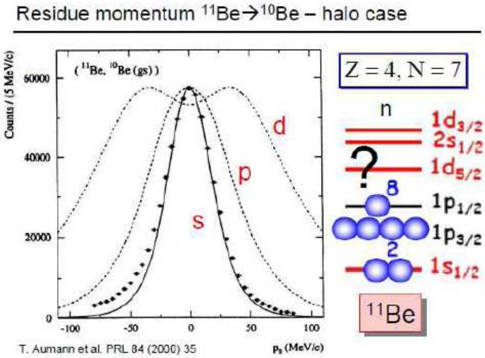


$$\vec{P} = \vec{p}_c + \vec{p}_1 = 0$$

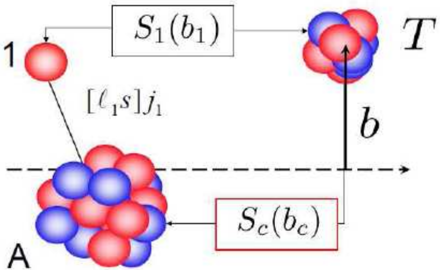
Knockout reactions

- The shape is determined by the orbital angular momentum ℓ.
- The magnitude is determined by the amount of $s_{1/2}$ (spectroscopic factor)





Knockout reactions



$$\sigma_{\text{strip}} = \int d\mathbf{b}\ \langle \phi_0 \,||\, S_c|^2 (1 - |S_1|^2) |\, \phi_0 \rangle$$

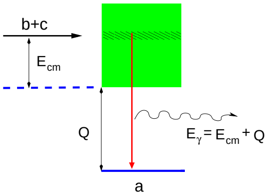
- $|S_c(b_c)|^2$=probability of survival of the core.
- $1 - |S_1(b_1)|^2$=probability of absorption of the neutron.

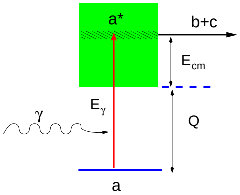
Radiative capture

Radiative capture

Radiative capture: $b + c \rightarrow a + \gamma$

Photo-absorption: $a + \gamma \rightarrow b + c$





➪ Related by detailed balance:

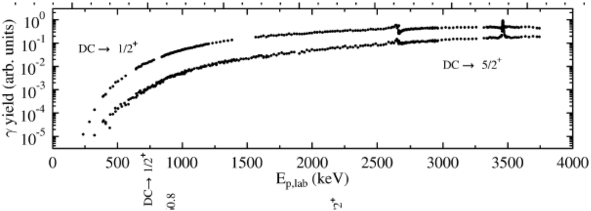
$$\sigma^{(rc)}_{E\lambda} = \frac{2(2J_a+1)}{(2J_b+1)(2J_c+1)} \frac{k_\gamma^2}{k^2} \sigma^{(phot)}_{E\lambda} \qquad (\hbar k_\gamma = E_\gamma / c)$$

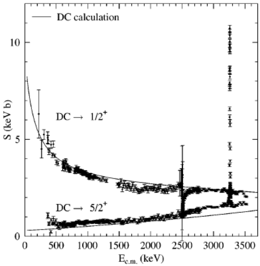
➪ Astrophysical S-factor:

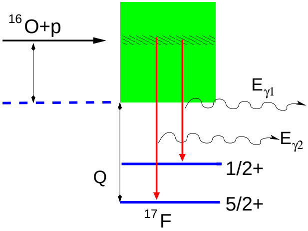
$$S(E_{\mathrm{c.m.}}) = E_{\mathrm{c.m.}} \sigma^{(rc)}_{E\lambda} \exp[2\pi\eta(E_{\mathrm{c.m.}})]$$

Example: p+^{16}O $\rightarrow$ ^{17}F + γ

Morlock, PRL79, 3837 (1997)







Implications in astrophysics: the r-process

⇨ Most neutron-rich isotopes of elements heavier than nickel are produced, by the beta decay of very radioactive matter synthesized during the so-called r process.