Re-writing Nuclear Physics textbooks: 30 years of radioactive ion beam physics
Basic concepts in nuclear reaction theory

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General scattering theory:

- *Quantum collision theory*, C.J. Joachain.

Scattering theory applied to nuclear reactions:

- *Direct Reactions*, G.R. Satchler.
- *Direct Nuclear Reactions*, N. Glendenning.
- *Quantum scattering theory and direct nuclear reactions*, course notes by A.M.M.
Unstable nuclei and the limits of stability
The aim of reaction theory is to provide a mathematical description of quantum scattering experiments, in order to extract information on the structure of the colliding nuclei and on their mutual interaction dynamics. The first experiment of this kind was the $\alpha$ scattering experiment by Rutherford, who lead to the proposal of his celebrated model of the atom and the subsequent formula for the angular dependence of the scattered $\alpha$ particles.
Types of reactions: direct vs. compound nucleus processes
**Direct:** elastic, inelastic, transfer,…

- “fast” collisions ($10^{-21}$ s).
- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about $\pi/2$ (peaked forward)

**Compound:** complete, incomplete fusion.

- many degrees of freedom involved
- large amount of momentum transfer
- “loss of memory” $\Rightarrow$ almost symmetric distributions forward/backward
Linking theory with experiments: the cross section
Linking theory with experiments: the cross section

EXPERIMENT

THEORY

\( H\Psi = E\Psi \)

CROSS SECTIONS

\( \frac{d\sigma}{d\Omega}, \frac{d\sigma}{dE}, etc \)
Experimental cross section

- $\Delta I$: detected particles per unit time in $\Delta \Omega$
- $I_0$: incident particles per unit time
- $n_t$: number of target nuclei per unit surface
- $\Delta \Omega$: solid angle of detector
- $d\sigma/d\Omega$: differential cross section

\[
\Delta I = I_0 \ n_t \ \frac{d\sigma}{d\Omega} \Delta \Omega
\]

\[
\frac{d\sigma}{d\Omega} = \frac{\text{flux of scattered particles through } dA = r^2 d\Omega}{\text{incident flux}}
\]
Among the many mathematical solutions of $[H - E] \Psi = 0$ we are interested in those behaving asymptotically as:

$$\Psi^{(+)}_{K\alpha} \rightarrow \Phi_{\alpha}(\xi_\alpha) e^{iK_\alpha \cdot R_\alpha} + \text{(outgoing spherical waves)}$$
Energy domains

A) “Very low” (astrophysical) energies ($<< 1$ MeV)
   - Dominated by Coulomb interaction
   - Few open channels
   - Eg.: radiative capture: $^7\text{Be}(p,\gamma)^8\text{B}$ (Solar neutrino problem!)

B) “Low” energies (Coulomb barrier) ($\sim 1-10$ MeV/u)
   - Interplay between Coulomb and nuclear
   - Many open channels (inelastic, transfer, breakup...)
   - Strong dynamical effects

C) “Intermediate” energies ($\sim 10^2 - 10^3$ MeV)
   - Dominated by nuclear forces
   - Classical-like trajectories
   - More violent processes (eg. knock-out)
Elastic scattering
What can we learn by measuring elastic scattering?

- Studying the angular dependence of elastically scattered particles we can infer information on:
  - The interplay between Coulomb and nuclear forces.
  - The presence of non-elastic channels, that will show up as a reduction of the elastic cross section with respect to the case of inert objects (absorption).

- From scattering theory, the angular distribution is calculated from the scattering wavefunction as:

\[
\Psi^{(+)}(K, R) \rightarrow e^{iK \cdot R} + f(\theta) e^{iKR} R
\]

\[
\frac{d\sigma}{d\Omega} = |f(\theta)|^2
\]
Rutherford experiment...100 years later

- $^4\text{He} + ^{208}\text{Pb}$ follows Rutherford formula at 19 MeV but not at 22 MeV. Why?
- $^6\text{He}$ drastically departs from Rutherford formula at both energies. Why?
Introduction

Reaction and interaction cross sections

Inelastic scattering  Transfer reactions  Breakup reactions  Knockout reactions  Radiative capture

Re-writing Nuclear Physics textbooks

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Interaction cross sections

⇒ Interaction cross section:

\[ \sigma_I = \sigma_{\text{tot}} - \sigma_{\text{inel}} - \sigma_{\text{el}} \]

⇒ Interaction radius:

\[ \sigma_I = \pi \left( R_{I}^{\text{proj}} + R_{I}^{\text{targ}} \right)^2 \]

Inelastic scattering
Nuclei are not inert or *frozen* objects; they do have an internal structure of protons and neutrons that can be modified (excited) during the collision.

Quantum systems exhibit, in general, an energy spectrum with bound and unbound levels.
Inelastic scattering

- Direct reactions → nuclei make “glancing” contact and separate immediately.

- Energy/momentum transferred from relative motion to internal motion so the projectile and/or target are left in an excited state.

- Involve small number of degrees of freedom.

- The colliding nuclei preserve their identity: $a + A \rightarrow a^* + A^*$

- Typically, they are peripheral (surface) processes.
Models for inelastic excitations

1. **COLLECTIVE**: Involve a collective motion of several nucleons which can be interpreted macroscopically as rotations or surface vibrations of the nucleus.

2. **FEW-BODY/SINGLE-PARTICLE**: Involve the excitation of a nucleon or cluster.
Types of collective excitations

The nucleons can move inside the nucleus in a coherent (collective) way.

1. **Vibrations** (spherical nuclei): small surface oscillations in shape.

   ![Vibrations](image)

2. **Rotations** (non-spherical nuclei): permanent deformation.

3. **Monopole** (*breathing*) mode: oscillations in the size (radius).

4. **Isovector** excitations (protons and neutrons move out of phase) (eg. giant dipole resonance)
The type of collective motion is closely related to the kind of energy spectrum.

- **Rotor**: $E_J \propto J(J + 1)$
- **Vibrator**: $E_J \approx n\hbar \omega$
Microscopic description in the IPM: the $^{11}$Be case

Ground state ($1/2^+$)

First excited state ($1/2^-$)
Models for inelastic excitations

**Microscopically**, what we describe in both cases are quantum transitions between discrete or continuum states:

Collective excitations can be regarded as a coherent superposition of many single-particle excitations.
By doing inelastic scattering experiments we measure the response of the nucleus to an external field (Coulomb, nuclear). This response is related to some structure property of the nucleus.

Example: for a Coulomb field:

\[
B(E\lambda; i \rightarrow f) = \frac{1}{2I_i + 1} |\langle \Psi_f | M(E\lambda) | \Psi_i \rangle|^2
\]

where \( M(E\lambda, \mu) \) is the electric multipole operator:

\[
M(E\lambda, \mu) \equiv e \sum_i Z_p r_i^\lambda Y_{\lambda\mu}^*(\hat{r}_i)
\]

The structure \( \Psi_{i,f} \) can be described in a collective, few-body or microscopic model.
Energy balance for inelastic scattering

- For projectile excitation: \( a + A \rightarrow a^* + A \)

\[
E^i_{\text{cm}} + M_a c^2 + M_A c^2 = E^f_{\text{cm}} + M^* a c^2 + M_A c^2
\]

\[M^* a = M_a + E_x \quad (E_x=\text{excitation energy})\]

- \( Q \)-value:

\[
Q = M_a c^2 + M_A c^2 - M^* a c^2 - M_A^2 c^2 = -E_x < 0
\]

\[
E^f_{\text{cm}} = E^i_{\text{cm}} + Q
\]

- So

\[
E_x = E^i_{\text{cm}} - E^f_{\text{cm}}
\]
In general, one measures the **scattering angle** and **energy** of outgoing particles.

**Example:** $p + ^7\text{Li} \rightarrow p + ^7\text{Li}^*$

*Eg. energy and angular distribution of the outgoing protons.*
What do we measure in an inelastic scattering experiment?

☞ The proton energy carries information on the $^7\text{Li}$ excitation spectrum.

Data from Nuclear Physics 69 (1965) 81-102
What do we measure in an inelastic scattering experiment?

The proton energy carries information on the $^7\text{Li}$ excitation spectrum.
What information do we get from an inelastic scattering experiment?

- The proton energy spectrum shows peaks which correspond to the states of the target ($^7\text{Li}$).

- The heights of peak (~ cross section) are different for each state ⇒ not all states are populated with the same probability.

- Some peaks are narrow, other are broad. Why?

- Above a certain excitation energy, the spectrum becomes continuous and structureless.
What information do we get from an inelastic scattering experiment?
Transfer reactions
Transfer reactions

Example: $d + ^{208}\text{Pb} \rightarrow p + ^{209}\text{Pb}$
Transfer reactions: \( Q \)-value considerations

Consider: \( a + A \rightarrow b + B \)

- Energy balance (in CM frame):

\[
E_{\text{cm}}^i + M_a c^2 + M_A c^2 = E_{\text{cm}}^f + M_b c^2 + M_B c^2
\]

- \( Q_0 \) value:

\[
Q_0 = M_a c^2 + M_A c^2 - M_b c^2 - M_B c^2
\]

\[
E_{\text{cm}}^f = E_{\text{cm}}^i + Q_0
\]

- \( Q_0 > 0 \): the system gains kinetic energy (exothermic reaction)
- \( Q_0 < 0 \): the system loses kinetic energy (endothermic reaction)
Transfer reactions: $Q$-value considerations

Example: $d + ^{208}\text{Pb} \rightarrow p + ^{209}\text{Pb}$

\[
Q_0 = M_d c^2 + M(^{208}\text{Pb}) c^2 - M_p c^2 - M(^{209}\text{Pb}) c^2 = +1.7 \text{ MeV}
\]
Transfer reactions: \(Q\)-value considerations

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\(Q_0 > 0\): the outgoing proton will gain energy with respect to the incident deuteron.
Transfer reactions: \( Q \)-value considerations

Example: \( d + {}^{208}\text{Pb} \rightarrow p + {}^{209}\text{Pb} \)

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Q_0 = M_d c^2 + M(208\text{Pb})c^2 - M_p c^2 - M(209\text{Pb})c^2 = +1.7 \text{ MeV}
\]

\( Q_0 > 0 \): the outgoing proton will gain energy with respect to the incident deuteron.

For a transfer reaction, the \( Q \) value is just the difference in binding energies of the transferred particle/cluster in the initial and final nuclei:

\[
Q_0 = \varepsilon_b(f) - \varepsilon_b(i) = 3.936 - 2.224 = +1.7 \text{ MeV}
\]
Transfer reactions: $Q$-value considerations

If the transfer leads to an excited state, the $Q$-value will change, and hence the kinetic energy of the outgoing nuclei.

![Diagram](image)

**Energy balance:**

$$E_{cm}^f = E_{cm}^i + Q = E_{cm}^i + Q_0 - E_x$$

If we know $Q_0$ we can infer the excitation energies ($E_x$) measuring the final kinetic energy of outgoing fragments.
What we do observe in a transfer experiment?

**Example:** $d + ^{208}\text{Pb} \rightarrow p + ^{209}\text{Pb}$

- The proton energy spectrum shows some peaks which reflect the excitation energy spectrum of the residual nucleus ($^{209}\text{Pb}$).
- The population probability will depend on the reaction dynamics and on the structure properties of these states.
Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

Selectivity of $\ell$: 

![Graph showing selectivity of $\ell$ for $^{56}\text{Fe}(d,p)^{57}\text{Fe}$ reaction]
Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

Selectivity of $\ell$:

Breakup reactions
Breakup reactions

- Direct processes of the from: $a + A \rightarrow b + x + A$
- Can be interpreted (and modelled) as an inelastic excitation to the continuum spectrum.

- Important for weakly-bound nuclei (eg. halo nuclei)
Influence of breakup on elastic scattering

- $^4\text{He} + ^{208}\text{Pb}$ shows typical Fresnel pattern → *strong absorption*
- $^6\text{He} + ^{208}\text{Pb}$ shows a prominent reduction in the elastic cross section due to the flux going to other channels (mainly break-up)
- $^6\text{He} + ^{208}\text{Pb}$ requires a large imaginary diffuseness → *long-range absorption*
Extracting information from the continuum with breakup reactions

**Example:** Populating resonances by “inelastic scattering” in $^{11}\text{Be}+^{12}\text{C}$

Coulomb response of halo nuclei

(1) Electric Dipole excitation $\rightarrow$ 10-20 MeV

$\gamma$

(Giant) electric Dipole excitation $\rightarrow$ 10-20 MeV

$\gamma$

$\gamma$

$^6\text{He} (2n,1 \text{ MeV})$ $^1\text{Be} (1n,0.5 \text{ MeV})$

$^1\text{Li} (2n,0.5 \text{ MeV})$ $^{14}\text{Be} (2n,\sim 1 \text{ MeV})$

Electric dipole response $- ^{11}\text{Li}$

$\frac{d\sigma(E1)}{dE_{\text{rel}}}$ (e$^2$ fm$^2$/MeV)

$\gamma$

Knockout reactions

- Fast-moving projectile on a (typically) light target.
- One nucleon suddenly removed (absorbed) due to its interaction with the target.
- The remaining nucleons remain unchanged and is detected.
- The momentum of the core is traced back to that of the removed nucleon because in the rest frame of the projectile $\vec{P} = 0$

\[ P = p_c + p_1 = 0 \]
Knockout reactions

- The shape is determined by the orbital angular momentum $\ell$.
- The magnitude is determined by the amount of $s_{1/2}$ (spectroscopic factor)
Knockout reactions

\[ \sigma_{\text{strip}} = \int db \, \langle \phi_0 | S_c |^2 (1 - |S_1|^2) | \phi_0 \rangle \]

- \[ |S_c(b_c)|^2 \] = probability of survival of the core.
- \[ 1 - |S_1(b_1)|^2 \] = probability of absorption of the neutron.
Radiative capture
Radiative capture

Radiative capture: $b + c \rightarrow a + \gamma$

Photo-absorption: $a + \gamma \rightarrow b + c$

Related by detailed balance:

$$\sigma^{(rc)}_{E\lambda} = \frac{2(2J_a + 1)}{(2J_b + 1)(2J_c + 1)} \frac{k^2}{k^2} \sigma^{(phot)}_{E\lambda}$$

($\hbar k_\gamma = E_\gamma/c$)

Astrophysical S-factor:

$$S(E_{c.m.}) = E_{c.m.} \sigma^{(rc)}_{E\lambda} \exp[2\pi\eta(E_{c.m.})]$$
Example: $p + ^{16}O \rightarrow ^{17}F + \gamma$

Morlock, PRL79, 3837 (1997)
Implications in astrophysics: the r-process

Most neutron-rich isotopes of elements heavier than nickel are produced, by the beta decay of very radioactive matter synthesized during the so-called r process.