Reminder on nuclear structure

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Credits:
Part of this material has been adapted from "slidedshare.net"

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Nuclear Structure: global properties

Nuclear Structure

- The atomic nucleus consists of positively charged protons and neutral neutrons.

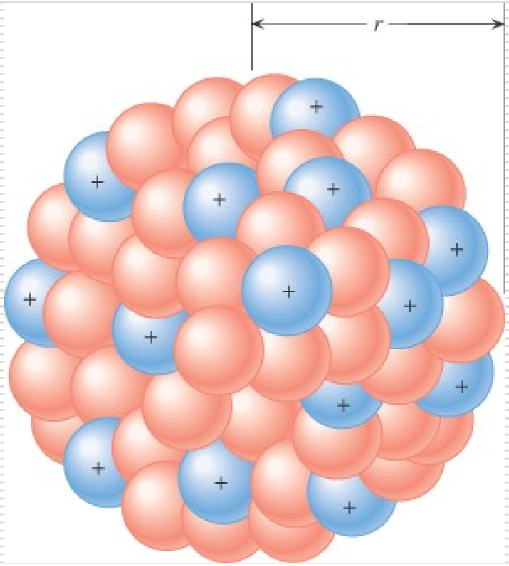


Table 31.1 Properties of Select Particles

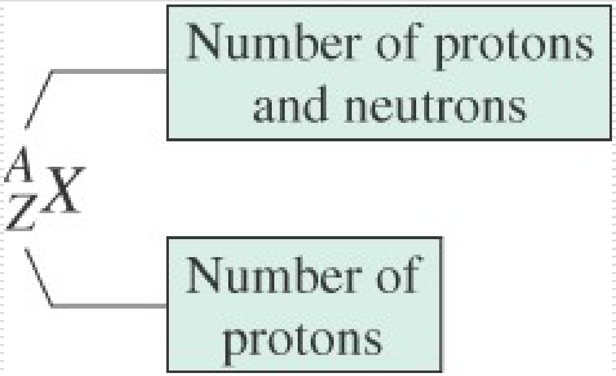
Particle	Electric Charge (C)	Mass: Kilograms (kg)	Mass: Atomic Mass Units (u)
Electron	-1.60×10^{-19}	$9.109\,382 \times 10^{-31}$	$5.485\,799 \times 10^{-4}$
Proton	$+1.60 \times 10^{-19}$	$1.672\,622 \times 10^{-27}$	1.007 276
Neutron	0	$1.674\,927 \times 10^{-27}$	1.008 665
Hydrogen atom	0	$1.673\,534 \times 10^{-27}$	1.007 825

Identifying Variables

mass
number

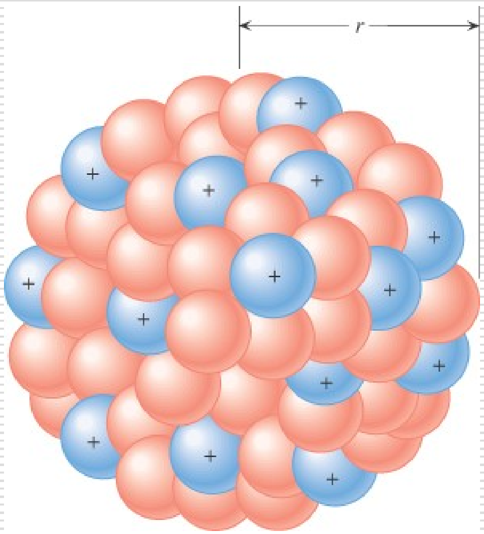
atomic
number

$$A = N + Z$$



- Nuclei can contain the same number of protons but a different number of neutrons (***isotopes)***

Approximate size of a Nucleus

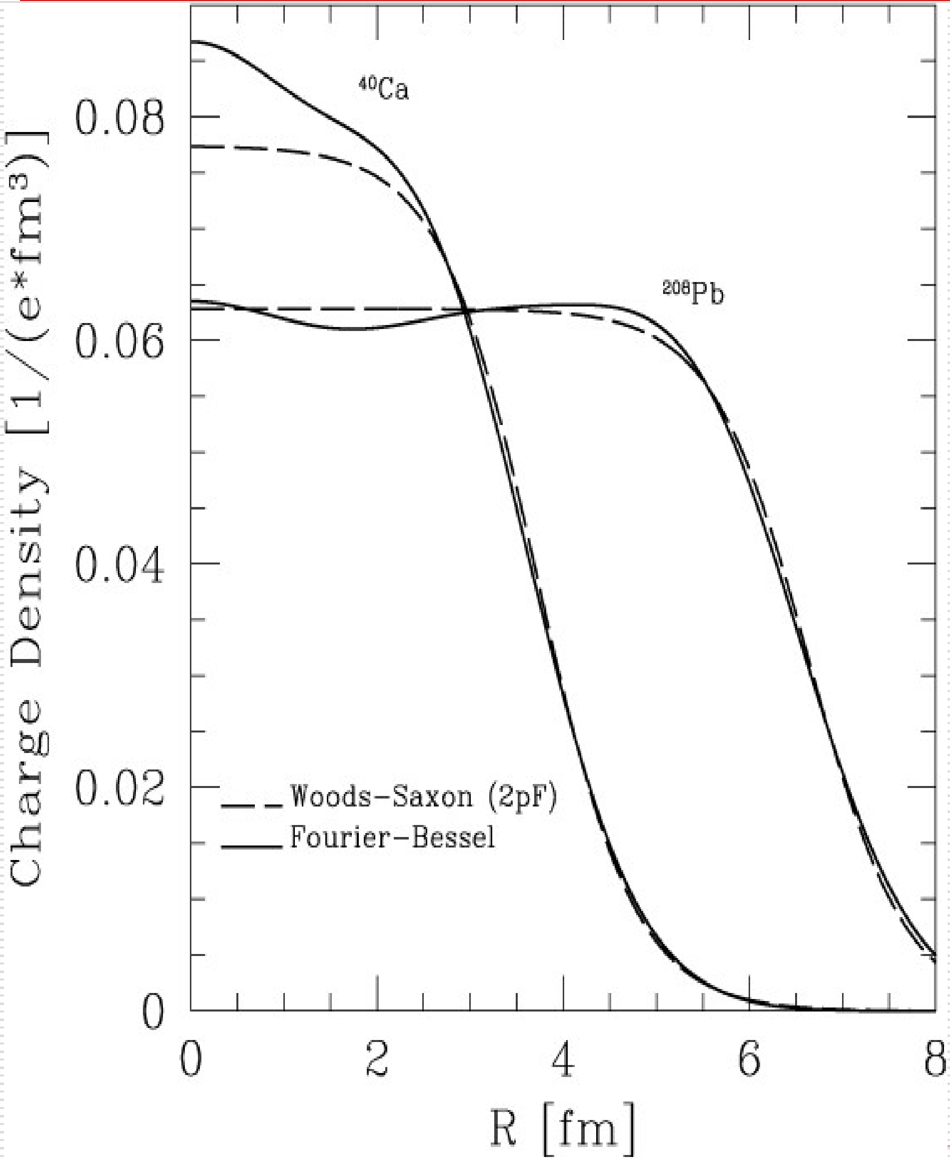


- The mutual repulsion of the protons due to the electric force should is compensated by the ***strong nuclear force.***
- The short-range nature of the nuclear force gives rise to a densely packed structure with a radius of the order:

$$r \approx \left(1.2 \times 10^{-15}\, m\right) A^{1/3}$$

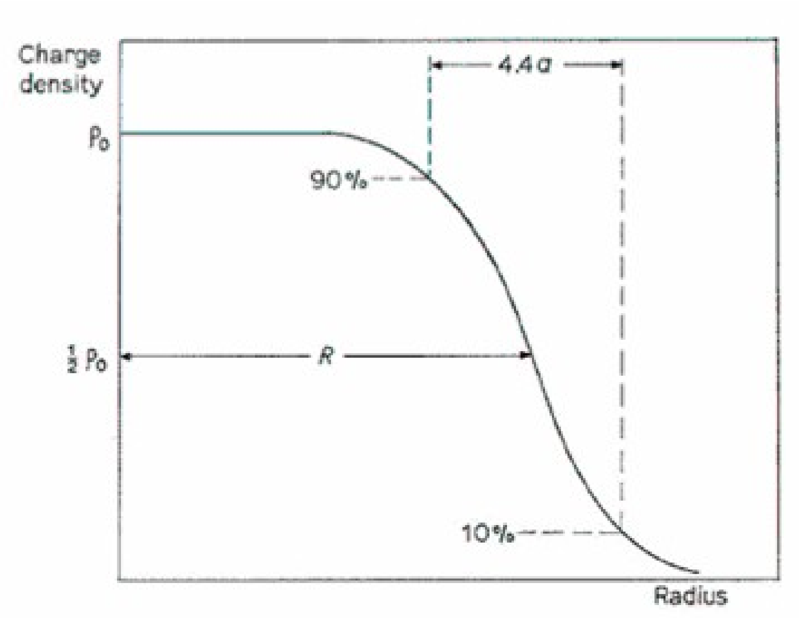
mass number

Density profiles



- In ordinary nuclei, the nuclear density is roughly constant at short distances (*saturation*) and falls abruptly at the surface

Parametrizing the density profile

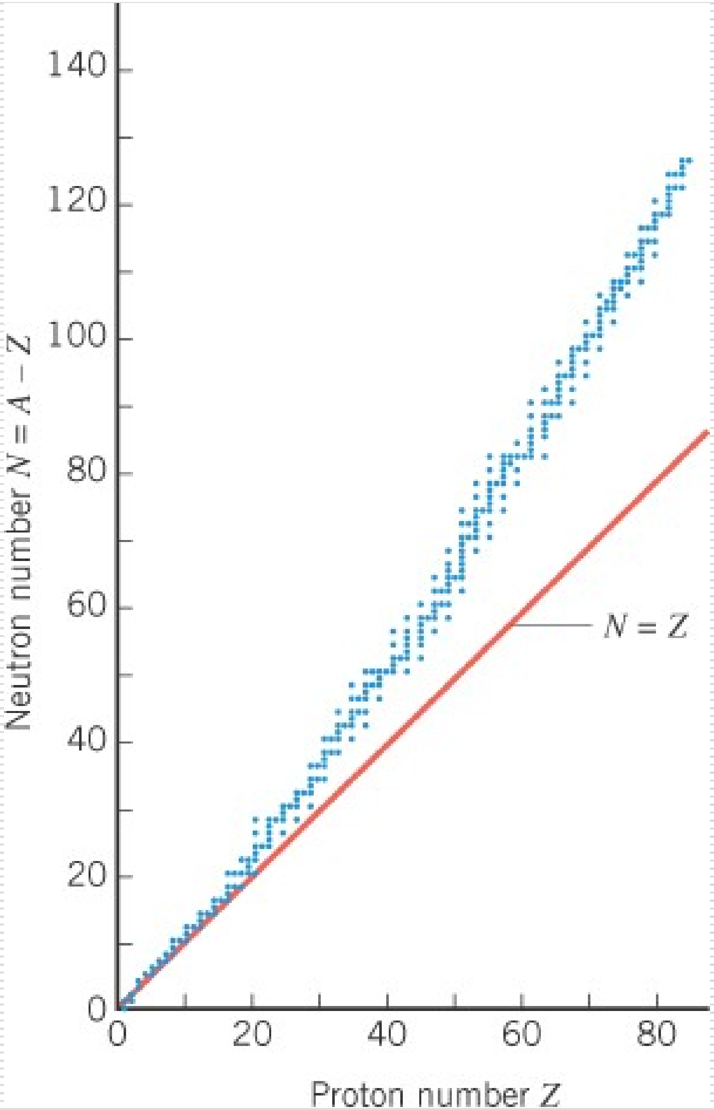


- Fermi parametrization of the nuclear density:

  ρ(r) = ρ0 / [1 + exp(r – R / a)]

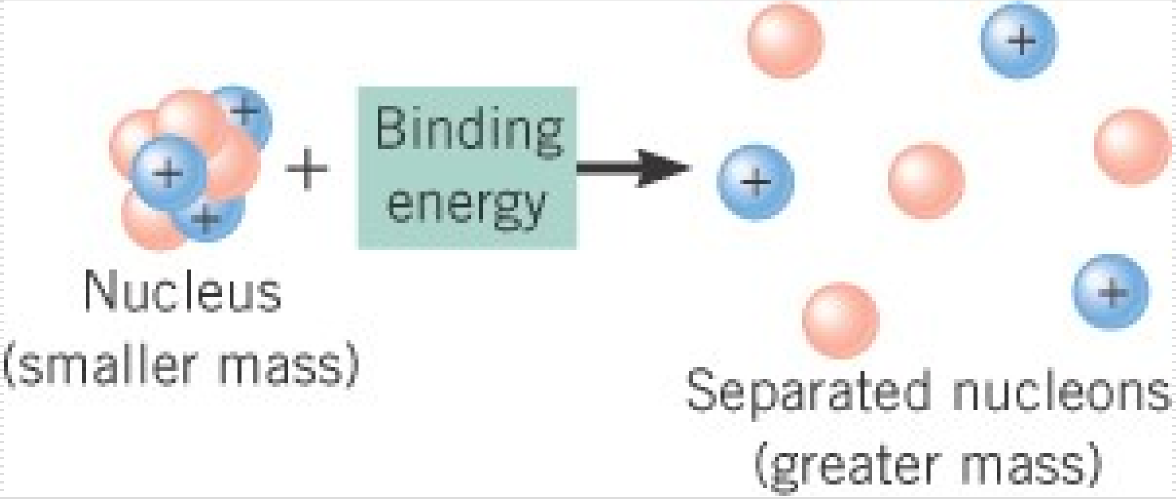
- ρ0=density in the core
- R= matter radius
- a= diffuseness (~0.5-0.6 fm)

Stability of the Nucleus



- As nuclei get larger, more neutrons are required for stability.
- The neutrons act like glue without adding more repulsive force.
- For “small” elements
  - Ratio N/P ~ 1
- For “large” element
  - Ratio of N/P ~ 2

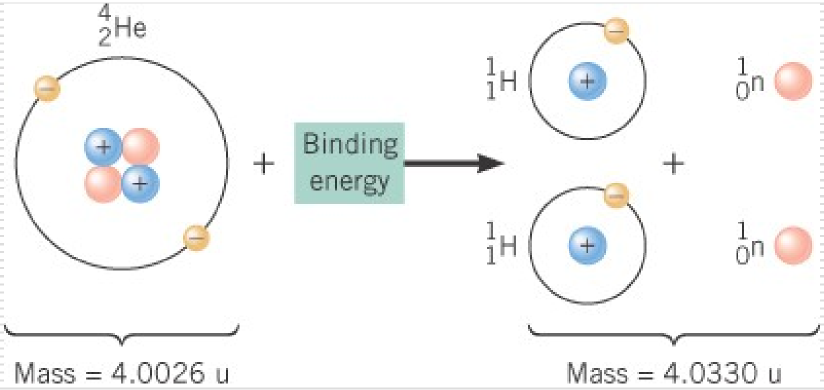
Mass Deficit



$$\text{Binding energy} = (\text{Mass deficit})\,c^2 = (\Delta m)\,c^2$$

Example The Binding Energy of the Helium Nucleus

The atomic mass of helium is 4.0026u and the atomic mass of hydrogen is 1.0078u. Using atomic mass units obtain the binding energy of the helium nucleus.

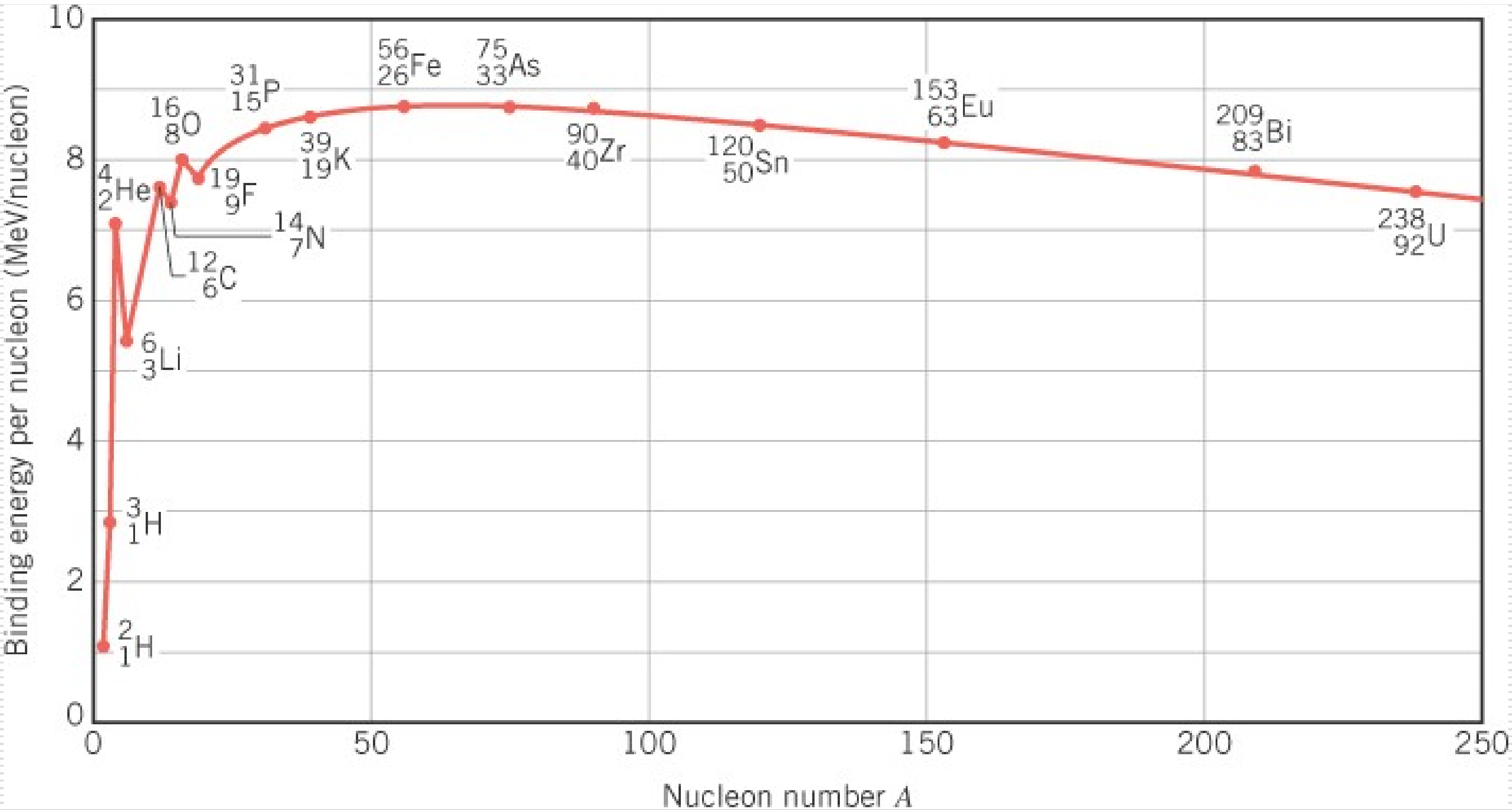


$$\Delta m = 4.0330 \text{ u} - 4.0026 \text{ u} = 0.0304 \text{ u}$$

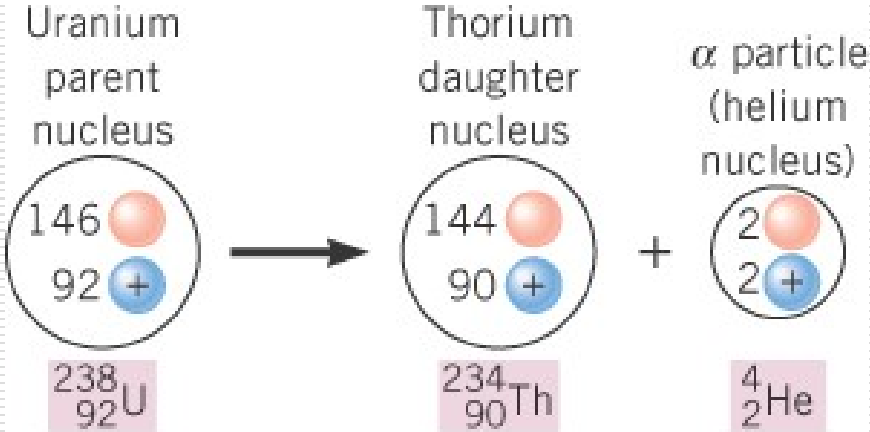
$$1 \text{ u} \leftrightarrow 931.5 \text{ MeV}$$

$$\text{Binding energy} = 28.3 \text{ MeV}$$

Binding Energy



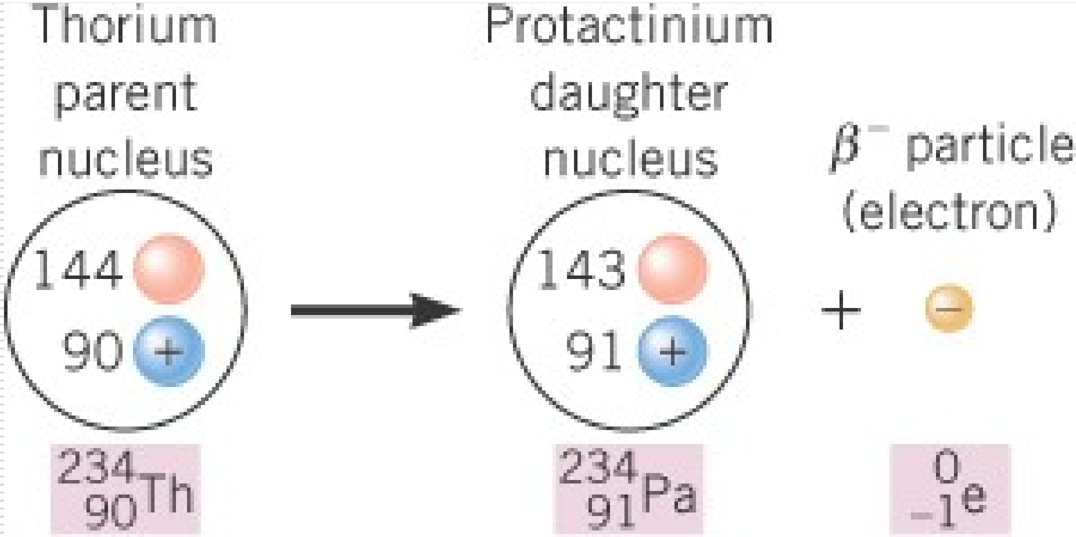
α Decay



$$^{A}_{Z}\mathrm{P} \rightarrow \ ^{A-4}_{Z-2}\mathrm{D} + \ ^{2}_{2}\mathrm{He}$$

β Decay

- Neutron "switches" into a proton
  - Electron and associated neutrino is released
- Other similar reactions can occur
  - Positron emission
  - Electron capture
  - Positron capture



$$^{A}_{Z}\mathrm{P} \rightarrow {}^{A}_{Z+1}\mathrm{D} + {}^{0}_{-1}e$$

γ decay

$${}^{A}_{Z}P^{¿} \rightarrow {}^{A}_{Z}\mathrm{P} + \gamma$$

excited energy state

lower energy state

The shell model

- Proposed by Maria Goeppert-Mayer (1940s)

- Assume nucleons move in some effective potential created by the other nucleons.

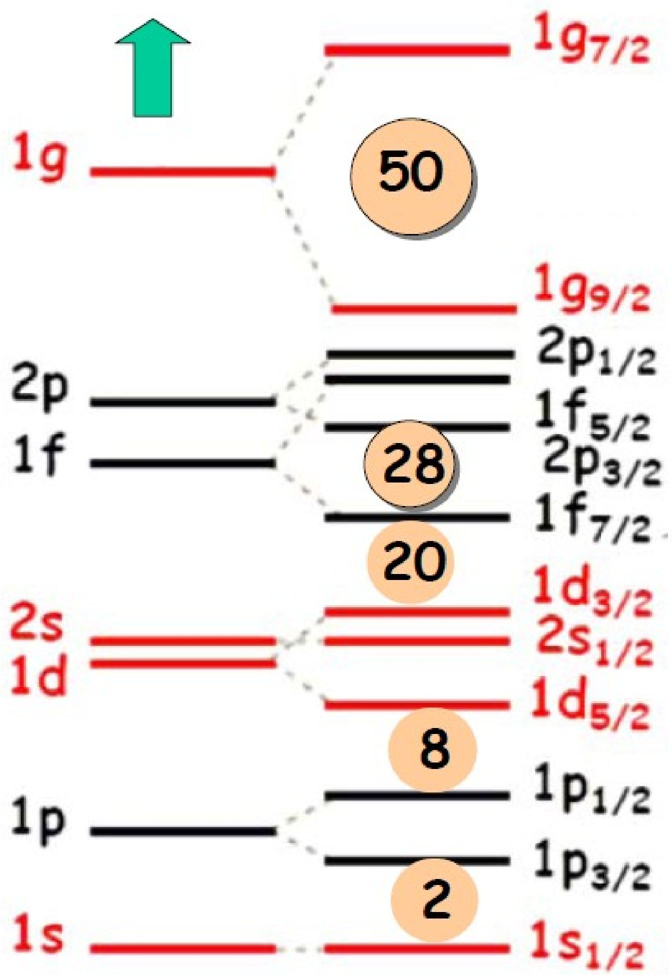
- This gives rise to an energy quantization and to the appearance of the so-called **magic numbers** (analog of noble gases): 2, 8, 20, 28, 50, 82, 126

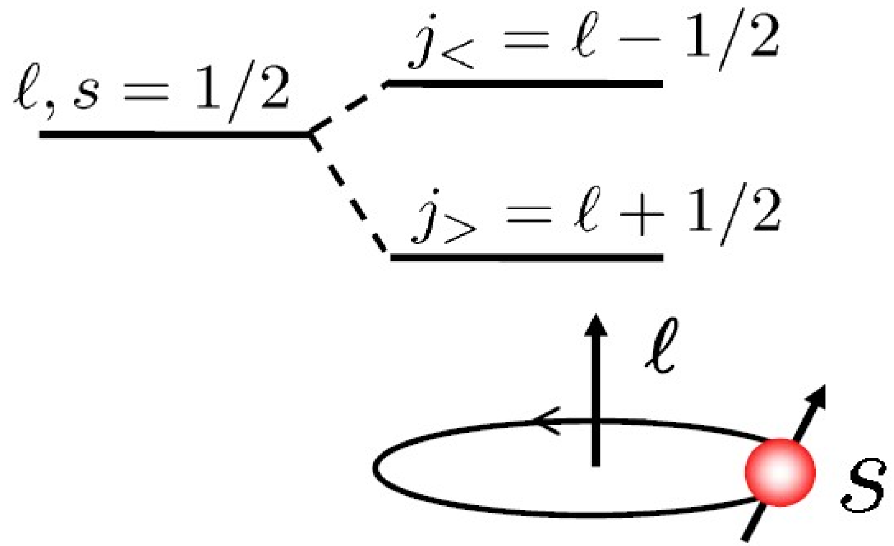
- Pauli principle prevents nucleons from occupying the same quantum states.

- The main features can be described with a simple central + spin-orbit potential:

$$\boxed{\begin{gathered} V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s} \\ V_{so}(r) < 0 \end{gathered}}$$

Particle orbits in mean-field potential

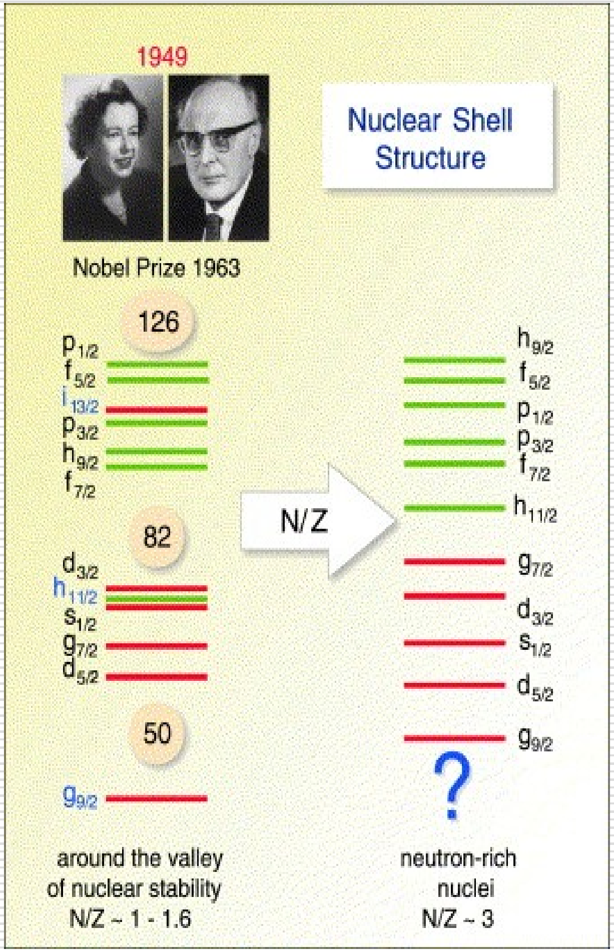




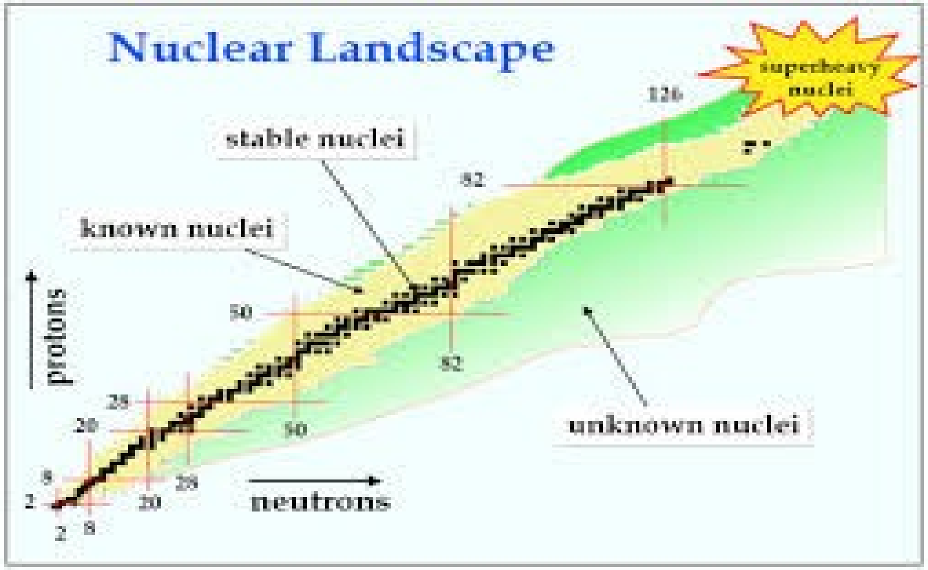
$$V_{\ell s}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$
$$V_{so}(r) < 0$$

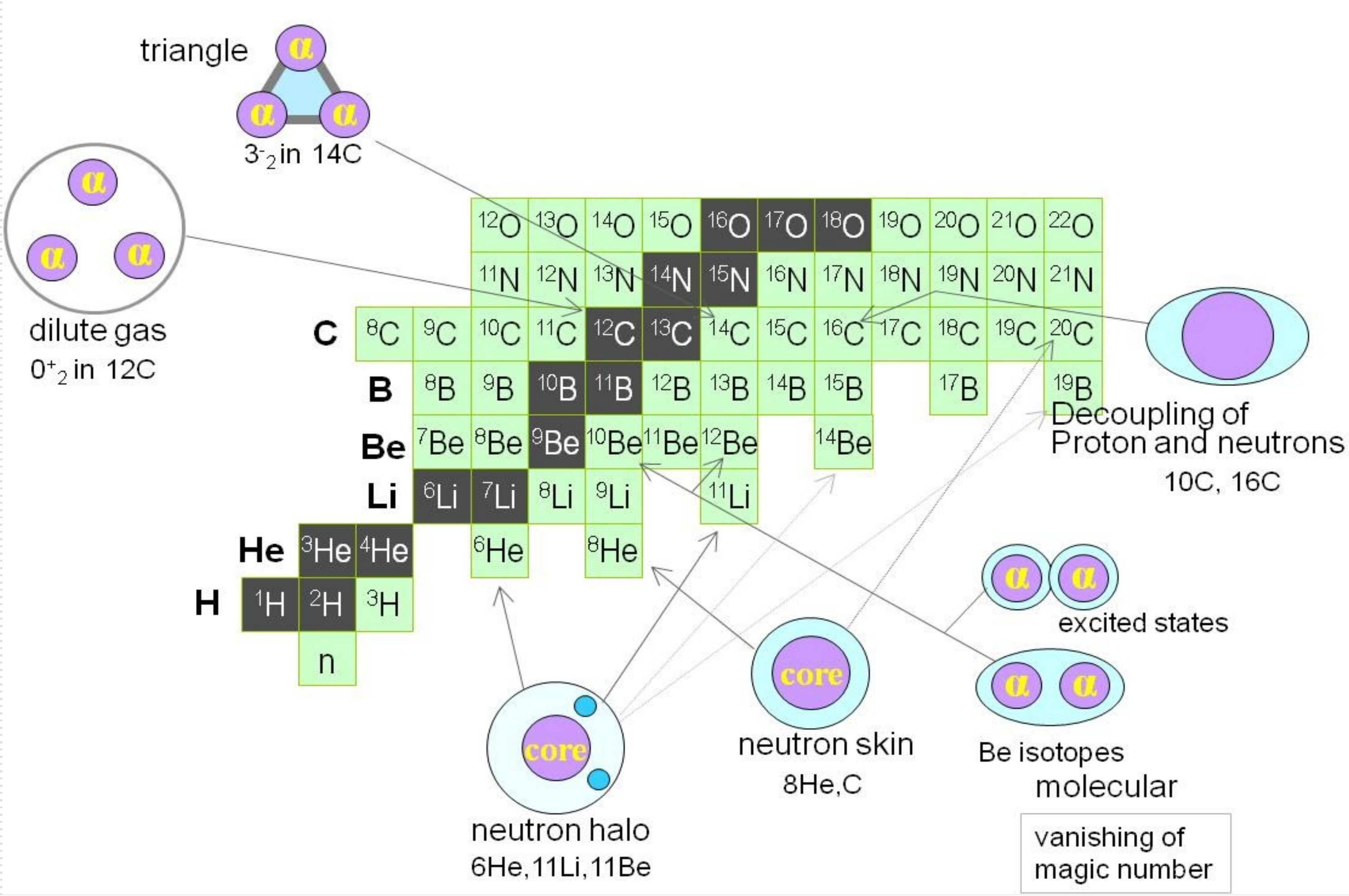
Approaching the driplines



- Stable nuclei constitute a small fraction of "existing" nuclei.
- Unstable nuclei are short-lived, decaying usually by **beta** emission, but they are stable against **particle** emission.
- Where are the limits of the driplines?
- How do the properties of these nuclei differ from those of ordinary nuclei?



Light exotic nuclei



(courtesy of Alessia di Pietro)

Halos, Borromean systems, etc

- Radioactive nuclei: they typically decay by β emission.

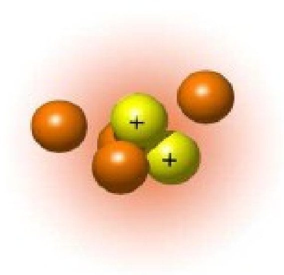
  E.g.: $^6\mathrm{He} \xrightarrow{\beta^-} {}^6\mathrm{Li} \quad (\tau_{1/2} \simeq 807\ \mathrm{ms})$

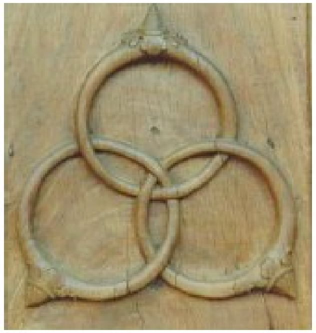
- Weakly bound: typical separation energies are around 1 MeV or less.

- Spatially extended

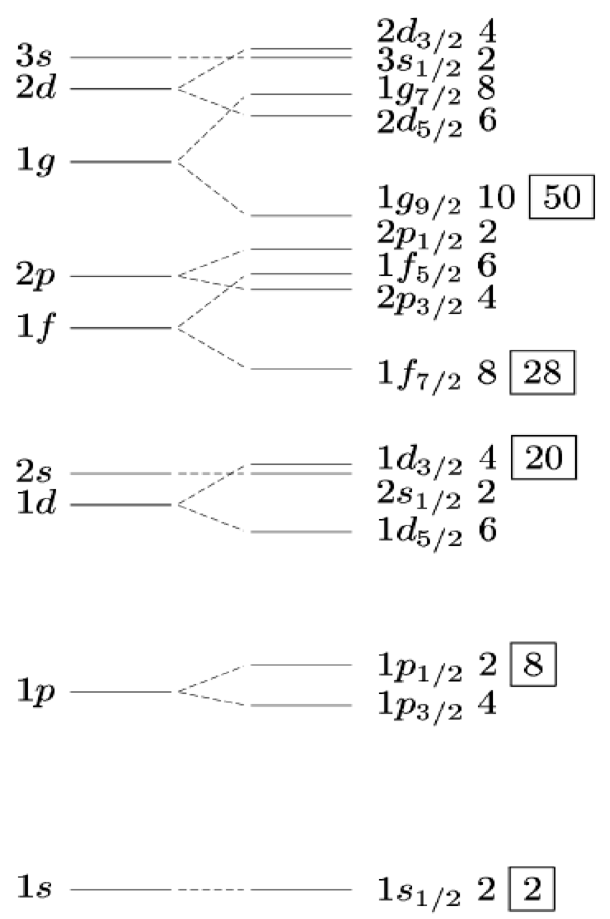
- Halo structure: one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.

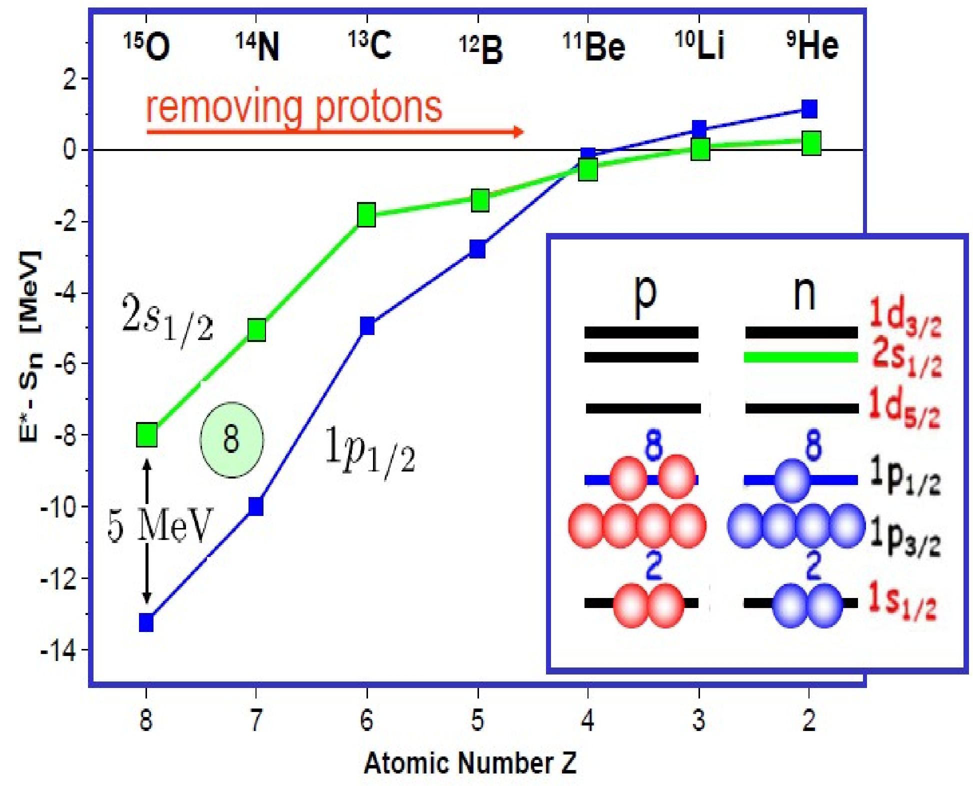
- Borromean nuclei: Three-body systems with no bound binary sub-systems.





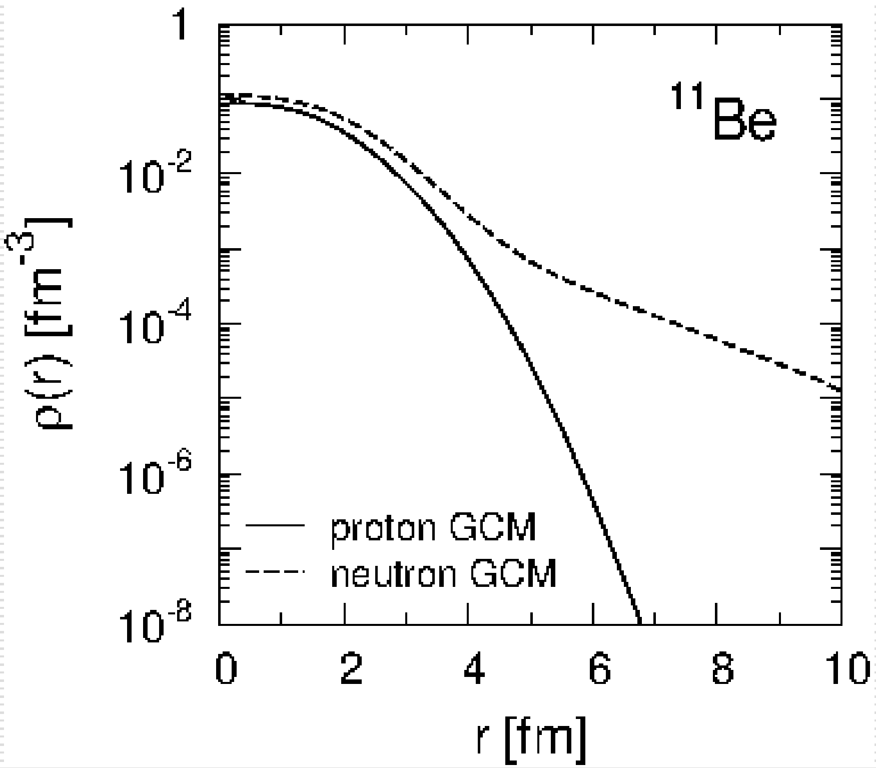
Example: parity inversion in N=7 isotones





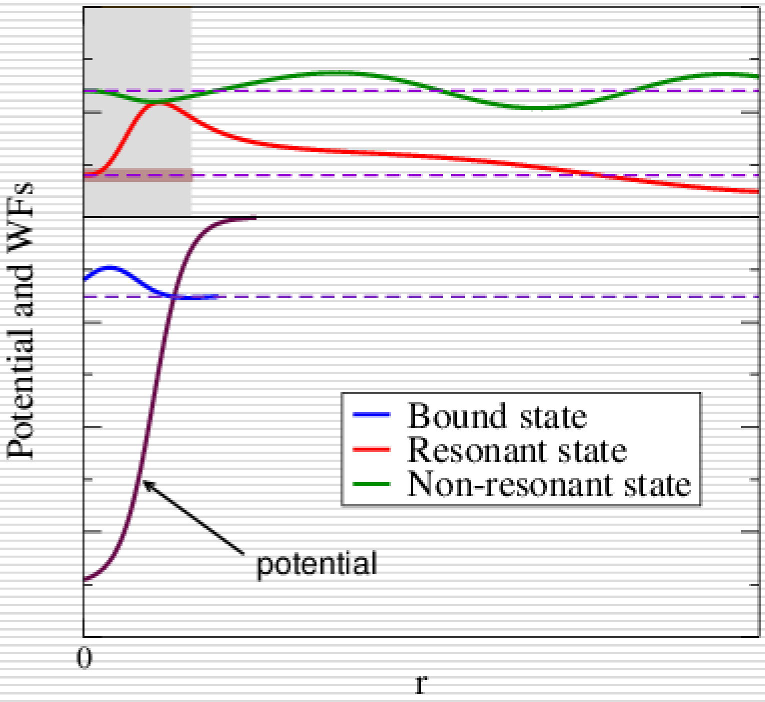
From: P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

Nuclear density of halo nuclei



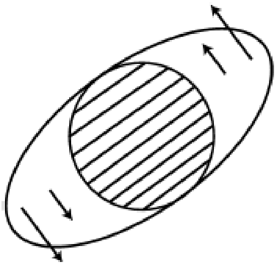
Resonances

- It is a structure on the continuum which may, or may not, produce a maximum in the cross section, depending on the reaction mechanism and the phase space available.
- The resonance occurs in the range of energies for which the phase shift is close to π/ 2.
- In this range of energies, the continuum wavefunctions have a large probability of being in the radial range of the potential.
- The continuum wavefunctions are not square normalizable (oscillatory behaviour)

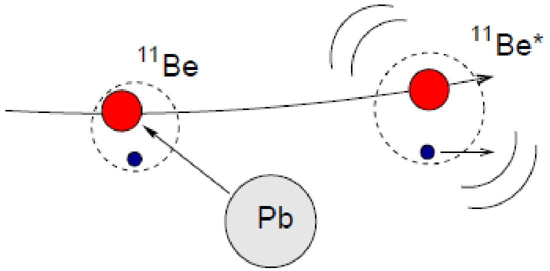


Excited states:
single-particle vs. collective excitations

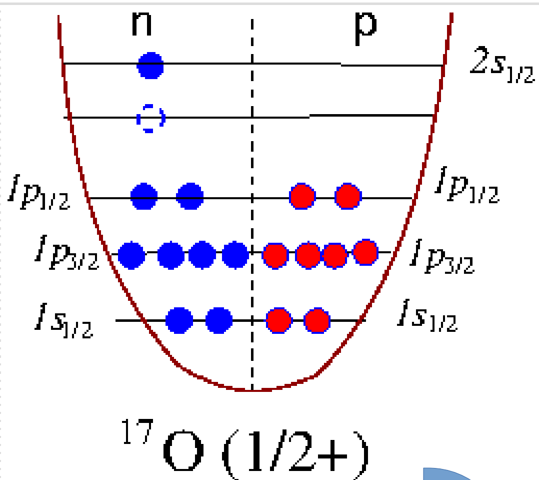
1. COLLECTIVE: Involve a collective motion of several nucleons which can be interpreted macroscopically as rotations or surface vibrations of the nucleus.

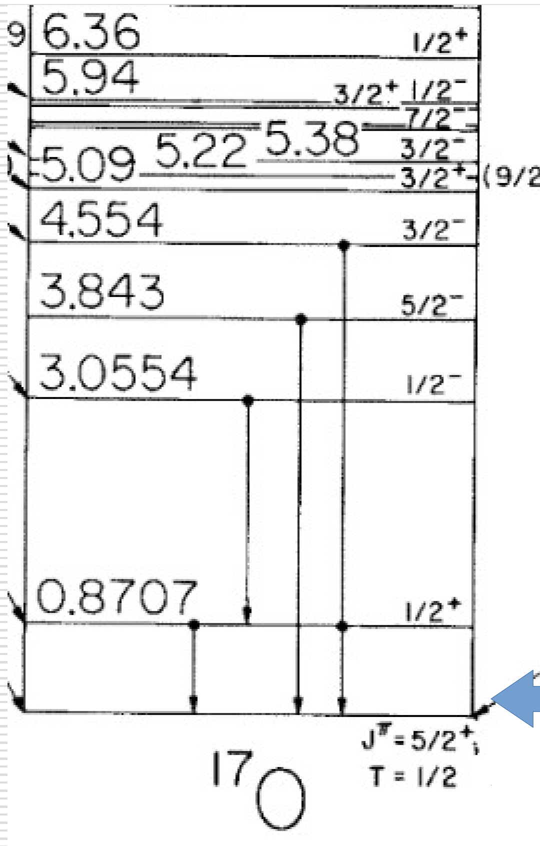


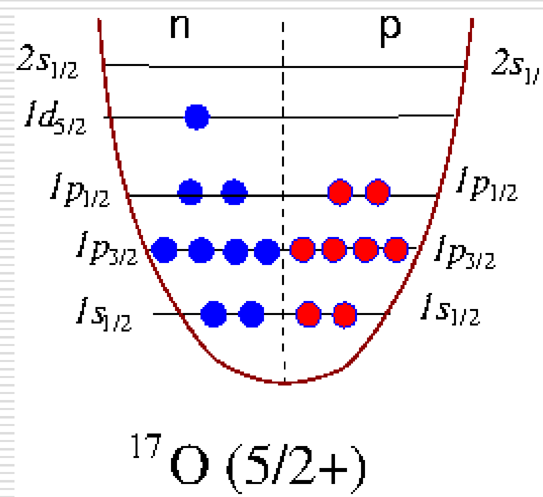
2. FEW-BODY/SIGLE-PARTICLE: Involve the excitation of a nucleon or cluster.



Example of single-particle excitation



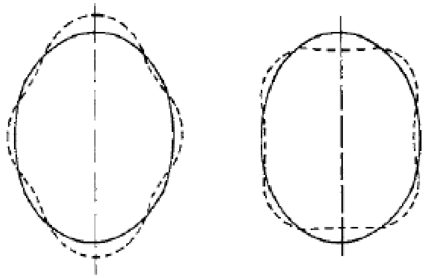




Collective excitations

The nucleons can move inside the nucleus in a coherent (collective) way.

1. Vibrations (spherical nuclei): small surface oscillations in shape.



2. Rotations (non-spherical nuclei): permanent deformation.
3. Monopole (*breathing*) mode: oscillations in the size (radius).
4. Isovector excitations (protons and neutrons move out of phase) (eg. giant dipole resonance)

Collective excitations

The energy spectrum gives information on the kind of excitations

Rotor:

$E_J \sim J(J+1)$

Vibrator:

$E_J \sim n\,hw$