Resonance phenomena: from compound nucleus decay to proton radioactivity

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Rewriting Nuclear Physics Textbooks 30 years with radioactive Ion Beam Physics

- a) Basic ideas about resonances with simple models.
- b) Two experimental techniques to investigate resonances
- c) Ground-state proton and neutron decay
- d) 3-body resonances
- c) Compound-nucleus decay

1-d independent-particle model

Each nucleon moves in a mean field generated by other nucleons Otherwise interactions between nucleons ignored

- 1) define mean field
- 2) solve schrodinger equation for single-particle orbits
- 3) fill single-particle orbits with **A** nucleons

1-d nucleus Mean-field



Important features

- . 1) well

2) barrier 3)continuum



Bound States



Fill levels with lowest possible energy For ground state





Change depth of well, Single-particle levels move up









Classically it will bounce backwards and forwards. There is probability of barrier penetration at each cycle. Exponential decay. Exponentially decaying solutions (Gamow States)

Use time-dependent Schrodinger Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(r)\right]\Psi(x,t)$$
$$\Psi(x,t) = \exp\left(\frac{-iEt}{\hbar}\right)\psi(x), \text{ where } E = E_r - i\frac{\Gamma}{2}$$
$$\text{thus } |\Psi(x,t)|^2 = \exp\left(\frac{-\Gamma t}{\hbar}\right)|\psi(x)|^2$$

Just need to solve time-independent Schrodinger Equation with complex energy

$$E\psi(x) = \left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(r)\right]\psi(x)$$

Look for solution where out side the barrier we have just a outgoing wave.

$$\Psi(x,t) \propto \exp\left[-i\left(\frac{Et}{\hbar} - kx\right)\right]$$
, for $x > x_0$ where $k = \frac{\sqrt{2mE}}{\hbar}$



Scattering with no "nuclear " potential Time-independent Schrodinger Equation





Look at a) probability of populating the well region b) change in phase in the outside region





































$$\frac{\Gamma^2}{(E-E_r)^2 + (\Gamma/2)^2}$$

Phase shift of π radians



Basic resonance properties

- 1) Populate a resonance state: Exponential decay with lifetime $\frac{\hbar}{\Gamma}$ Decay energy E_r
- 2) Resonance are also seen in scattering: Resonance peak at scattering energy E_r . FWHM of peak is Γ

The value of E_{Γ} and Γ will be the same in the two cases as long as Γ is not too large

Resonances with plane waves (3 dimensions)

Plane wave





Plane wave with scatter

Partial Wave expansion

no potential, plane wave solution $\psi = \exp(ikz)$ In polar coordinates $\psi = \sum_{l=0}^{\infty} (2l+1)i^l j_l(kr) P_l(\cos \theta)$ where $P_l(\cos \theta)$ is Legendre Polynonial, l is orbital angular momentum and j_l are Spherical Bessel Functions where $j_l(kr) \rightarrow \frac{\sin(kr - l\pi/2)}{kr}$ for $r \rightarrow \infty$ thus $\psi = \sum_{l=0}^{\infty} \frac{(2l+1)}{2ikr} [e^{ikr} - (-1)^l e^{-ikr}] P_l(\cos \theta)$ for $r \rightarrow \infty$ Sum of incoming (e^{-ikr}) and outgoing (e^{ikr}) spherical waves of equal amplitude for each l-wave

With Potential

$$\psi = \sum_{l=0}^{\infty} \frac{(2l+1)}{2i \, k \, r} \Big[S_{l,j} e^{ik \, r} - (-1)^l e^{-ik \, r} \Big] P_l(\cos \theta)$$

$$S_{l,j} \text{ is the S-matrix}$$
for elastic processes $S_{l,j} = e^{2i\delta}$ only a phase shift
i.e., $\psi = \sum_{l=0}^{\infty} \frac{(2l+1)}{2i \, k \, r} \frac{\sin (kr - l \, \pi/2 + \delta_{l,j})}{kr} P_l(\cos \theta)$
for non elastic processes $|S_{l,j}| < 1$
 $\psi = e^{ikz} + \sum_{l=0}^{\infty} \frac{(2l+1)}{2i \, k \, r} (S_{l,j} - 1) e^{ikr} P_l(\cos \theta)$ plane wave plus scattered wave


3-dimensions

a)Detector at zero angle can measure loss of flux from beam (total cross sections)(n only) b) Detector at finite angle can detect scattered cross section



non exotic resonances.

More stable nuclei have a number of Particle-bound excited state and only have resonances above the neutron or proton separation energy.

Neutron resonances in ¹³C are clearly seen in the total neutron cross section on ¹²C. Not all states above the Neutron separation are "strong" neutron Resonances.

Data: Rapp et al, Nucl. Sci. Eng. 172(2012)268



Can we modify the depth of mean field potential? Yes via the symmetry energy. Semi-Empirical Mass formula

$$E = a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_A \frac{(N-Z)^2}{A} + \dots$$

volume surface Coulomb asymmetry

Protons feel a greater attraction to neutrons than other protons and via versa. Add more neutrons and proton mean field potential becomes deeper Remove neutrons and proton mean field potential becomes shallower.

Depth of mean field potential

$$V = V_0 + v_A \frac{(N-Z)}{A} \text{ protons}$$
$$= V_0 - v_A \frac{(N-Z)}{A} \text{ neutrons}$$



are excited states of ¹⁹F

Ground state is a p+¹⁵O resonance

1**-**d nucleus Mean-field model



Important features

1) well

2) barrier 3)continuum

In the single-particle model, we need a barrier for a resonance state.

Protons – Coulomb + centrifugal

Neutrons – Centrifugal

s-wave neutrons (I=0) no barrier?

If the potential well is almost deep enough to make a bound state, it is called a virtual state. Large scattering cross section at low energy, but not a resonance, no lifetime defined, phase shift < π radians

Examples: "dineutron" n+n J=0 ⁹He "ground state" $n+^{8}$ He J= 1/2⁺ ? ¹⁰Li "ground state" $n+^{9}$ Li J = 1/2⁺ ?

Virtual states are found at just the threshold energy

However neutron s-wave resonances are possible.

Failure of the single-particle picture? Nuclei are more complicated.

An s-wave neutron can couple to another channel which is bound or has a Barrier.

Example E*=6.3 MeV J=1/2^{+ 17}F - n+¹⁶O is the only energy energetically allowed exit-channel.

This channel couples strongly to the α +⁹Be channel which is just bound at this excitation energy.

Scattering
$$n+{}^{16}O(I=0) \rightarrow \alpha + {}^{13}C(I=1) \rightarrow n+{}^{16}O(I=0)$$

unbound bound

So far we have considered in the independent-particle model where a nuclear resonance is achieved by placing a nucleon in a single-particle resonance state. (Some states have strong single-particle character where this model is a good approximation)

In this model barriers are needed for single-particle resonances (Coulomb + Centrifugal)

Some obvious problems.

a) No resonances for s-wave (*I*=0) neutrons.

b) No resonances for when the channels energy is above the height of the barrier.

Yet resonances are observed?

The shell model takes single-particle states and adds mixing between them.

This configuration mixing allows an entrance channel with no barrier to mix with other states with are localized. If the state then couples back to the original channel, then we get elastic scattering.

Configuration mixing allows for multiple exit channels.

Consider a resonance C which couples to two channels: a+b and d+e

In scattering we can observed the following reaction with the indicated cross sections

$$a+b \to C \to a+b \qquad \sigma = \frac{\pi}{k_{\infty}^2} g \frac{\Gamma_{a+b}^2}{(E-E_r^{a+b})^2 + (\Gamma_{total}/2)^2}$$

$$a+b \to C \to d+e \qquad \sigma = \frac{\pi}{k_{\infty}^2} g \frac{\Gamma_{a+b}\Gamma_{d+e}}{(E-E_r^{a+b})^2 + (\Gamma_{total}/2)^2}$$

$$d+e \to C \to d+e \qquad \sigma = \frac{\pi}{k_{\infty}^2} g \frac{\Gamma_{d+e}^2}{(E-E_r^{d+e})^2 + (\Gamma_{total}/2)^2}$$

$$d+e \to C \to a+b \qquad \sigma = \frac{\pi}{k_{\infty}^2} g \frac{\Gamma_{d+e}\Gamma_{a+b}}{(E-E_r^{d+e})^2 + (\Gamma_{total}/2)^2}$$

$$2J_C+1$$

 $g = \frac{2 \sigma_{C} + 1}{(2J_1 + 1)(2J_2 + 1)}$ where J_1 and J_2 are the spin of the entrance-channel particles $\Gamma_{total} = \Gamma_{a+b} + \Gamma_{d+e}$ Branching ratio to a+b channel $\frac{\Gamma_{a+b}}{\Gamma_{total}}$ Branching ratio tp d+e channel $\frac{\Gamma_{d+e}}{\Gamma_{total}}$ R-matrix theory partial decay widths are

 $\Gamma_{\lambda} = 2 k_{\infty} R P_{l} \Theta_{\lambda}^{2} \gamma_{\lambda}^{2}$ *R* is channel radius $P_{l} \text{ is barrier penetration probability}$ $\gamma_{\lambda}^{2} = \frac{3 \hbar^{2}}{2 M R^{2}} \text{ is reduced single-particle width}$ $\Theta_{\lambda}^{2} \text{ is fractional reduced singple-particle width}$ $\Theta_{\lambda}^{2} = 1 \text{ pure single-particle configuration}$ To obtain small resonance width we need either a small P_{l} (hindered by barrier)
and/or a small Θ_{λ}^{2} (large configuration mixing)

Low energy we can narrow-single particle states due to the barrier or configuration mixing High-excitation energy we get narrow state from configuration mixing.



3-dimensions

a)Detector at zero angle can measure loss of flux from beam (total cross sections)(n only) b) Detector at finite angle can detect scattered cross section Probing beyond the proton-drip line with thick-target inverse-kinematics elastic resonance scattering.



The gas a) slows the beam and proton down

- b) is the target
- c) stops the beam if not nuclear interaction

For elastic scattering there is a unique proton energy for each reaction distance Proton: lower charge and higher velocities -> reduced energy loss in gas. Need to know energy loss very well!

Time measurement allows one to remove other nuclear processes.

Can use He gas for α -particle scattering.



Goldberg et al Physics Letters B 692(2010) 307



¹²O (t_{1/2} ~10^{-21 s}) no beam possible, cannot make
 ¹³F resonances with this technique
 Cannot access all nuclei beyond proton drip line.

Cannot do resonant neutron scattering at neutron drip line. No neutron target.

Invariant-Mass Method

In relativity – the invariant mass of a single object is its rest mass.

For a system of objects, its their total mass-energy in their center-of-mass frame. The invariant mass of the decay products of a resonance is just the rest mass of the resonance.

If we subtract the rest mass of each decay product, then we are left with the decay energy of the resonance.

 $E_T = 1/2 m_p V_p^2 + 1/2 m_{150} V_{150}^2$



Need to measure energy and angle of all decay products to get their velocity vectors.

HiRA array at Michigan State University



Telescope, measure position, energy, and identify particles.



¹⁵O + p invariant-mass excitation spectrum

Brown et al



¹²O (t_{1/2} ~10^{-21 s}) no beam possible, cannot make
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Higgs Boson is a resonance!



Invariant mass distribution of $\gamma - \gamma$ pairs From the ATLAS detector at CERN Note difference in energy scales.

Aad et al. Physics Letters B 716 (2012) 1

3-body resonances

Three-body scattering is impracticable in the laboratory, but occurs in stars, ...

However, three-body resonances can be formed in knockout, transfer, .. reactions.

Classification of 3-body resonances

a) Prompt or "true" 3-body - all three final decay products are created at the same time

 b) sequential – decay occurs in a series of two-body steps.
 Really a two-body resonance but where one of the decay products is also a two-body rsonance.



Sequential 3-body decay – Hoyle state in ¹²C



Decay to ⁸Be ground state resonance.

This is a narrow, long-lived resonance

First-emitted α particle travels 370000 fm during its half life

That is 68000 ¹²C diameters.

No interactions between first α particle and the α particles from the decay of ⁸Be. Hoyle state responsible for most Carbon production is stars.

Sequential decay requires a love-lived intermediate resonance

Prompt decay a) Goldansky's (1960) two-proton decay: ⁴⁵Fe, ⁴⁸Ni, ⁵⁴Zn

One-proton decay not possible



Democratic 2p decay (Bochkarev et al. 1989)



Wide (short-lived) intermediate state-

"first" proton cannot travel any significance distance before "second" proton is emitted.

Odd-Z ground states have larger widths than even-Z ground states





In two-proton decay, the two protons should heave similar energies (Goldansky)

 $prob \propto P(E_1) P(E_2)$ where $E_1 + E_2 = E_T$

Product of barrier penetration factors peaks at equal proton energies.

Distributions of proton energies becomes narrower as the more subbarrier one is.

Higher Z_{core} , or lower E_{T}



Increasing Coulomb Barrier gives narrower proton energy distributions and longer lifetimes.

Two-neutron emitters – no strong A dependence?

Lundenberg et al Physics Review Letter (2012) 142503



Invariant-mass spectra (peak dominated by experimental resolution)



²⁶O – prompt two-neutron emitter

 $t_{1/2} \sim 4.5 \text{ ps}$ (Kohley et al 2013) Usually long half-life given there is only a centrifugal barrier! Need extremely small decay energy to get such a life time $E_{T} < 1 \text{ keV}$ to get experimental half-life from 3-body model Grigorenko et al (2013)

Ground-state proton decay in heavier Nuclei

Large Coulomb barrier gives large half-lives for proton emission outside the drip line. β -decay can complete favorably so not all nuclei beyond drip line have been observed the proton decay.



 ^{147}Tm has a 15% proton decay branch, $t_{_{1/2}}$ =0.9 s β -delayed proton emission also common in this region

2*p* predictions from Olsen *et al* PRL 110 (2013) 222501 Compound Nucleus decay and the Nuclear level density

Density of nuclear levels increase with nucleon number A and excitation energy.

Width of the levels increase with increasing excitation energy.

Level Spacing becomes much smaller than the level widths. Overlapping resonances.

Configuration mixing is very strong, entrance channel couples with an extremely large number of other configurations. Unlikely to exit with same entrance channel.

Lifetime of compound nucleus comes from the wait until the mixing between channels concentrates strength in a channel which can decay.

Decay of Compound Nuclei is independent of how it was formed (Bohr)

Statistical concept can be used to give the average decay modes

Level density is a simple single-particle model. One particle type, no spin, equal spaced levels.



Constructing nuclear level density from the combinations of single-particle excitations.



Number of ways of achieving a given excitation energy increases rapidly with excitation energy.

Strong configuration mixing.





At E*=8 MeV there are 10⁹ levels per MeV (neutron separation energy for stable isotopes)



Note difference between Heavy and light nuclei

Data: Rapp et al, Nucl. Sci. Eng. 172(2012)268

s-wave resonances in first 210 keV of ²³³Th above neutron separation energy. Probing density of J=1/2+ states in ²²³Th



Can determine "a"


Can only be measured for stable isotopes.



Compound-Nucleus regime

$$\bar{D}\!\ll\!\bar{\Gamma}$$

Average Level spacing much less than average level width

Levels overlap and interfere with there decay.

Ericson Fluctuation – interference between overlapping levels. Variations with energy on a scale consistent with the average level width.

Fourier transformed used to used average level width.

These fluctuations fade out at higher energies and cross sections become smooth functions of energy.

The statistical model of compound Nucleus decay aims to calculate the cross section after the fluctuations have been averaged out.



Nuclear Physics **A109** (1968) 329





s=1/2 spin of neutron V = volume of box p = momentum of neutron $S_n =$ neutron separation energy $\sigma_{inv} =$ neutron capture cross section for inverse process v = neutron velocity m = neutron mass

Neutron evaporation in statistical model using detail balance (Weisskopf-Ewing) Put compound nucleus in a reflecting box.

Assumed equilibrium between original state and daughter nucleus + neutron at kinetic energy from E_k to $E_k + dE_k$.

> $\rho_a W_{ab} = \rho_b W_{ba}$ ρ_a = density of states for configuration (a) $=\rho_{CN}(E^*)$ compound nucleus level density ρ_b = density of states for configuration (b) $= (2s+1)V \frac{4\pi p^2}{h^3} \frac{dp}{dE_k} dE_k \times \rho_d (E^* - S_n - E_k)$ w_{ba} = transition rate from (b) to (a) $=\frac{v\,\sigma_{inv}(E_k)}{V}$ Thus $w_{ab} = \frac{\Gamma_n(E_k) dE_k}{\hbar}$ transition from (a) to (b) $=\frac{(2s+1)m}{(\pi\hbar)^2}E_k\sigma_{inv}(E_k)\frac{\rho_d(E^*-S_n-E_k)}{\rho_{CN}(E^*)}$

Total neutron decay width is:

$$\Gamma_{n}(E^{*}) = \frac{(2s+1)m}{(\pi\hbar)^{2}\rho_{CN}(E_{K})} \int_{0}^{E^{*}-S_{n}} E_{k}\sigma_{inv}(E_{k})\rho_{d}(E^{*}-Sn-E_{k})dE_{k}$$

Can derive similar expressions for proton, alpha, gamma-rays, etc.

$$\Gamma_{total} = \Gamma_n + \Gamma_p + \Gamma_\alpha + \Gamma_\gamma$$

Hauser-Feshback developed more complicated formula with explicit Angular momentum coupling.



Making ¹⁴⁷Tm (ground-state proton emitter $t_{1/2} = .9$ s)

Fusion Evaporation reaction

261 MeV ⁵⁸Ni + ${}^{92}Mo \rightarrow {}^{150}Yb^*$

¹⁵⁰Yb compound nucleus is excited and decays by the evaporation of protons, neutrons, alpha particles.

Many possible decay paths

 ${}^{150}\text{Yb}^* \rightarrow {}^{149}\text{Tm}^* + p \rightarrow {}^{148}\text{Er}^* + p + p \rightarrow {}^{147}\text{Ho}^* + p + p + p + p - > {}^{147}\text{Ho}_{g.s}^* + p + p + p + \gamma's$ ${}^{150}\text{Yb}^* \rightarrow {}^{149}\text{Tm}^* + p \rightarrow {}^{148}\text{Er}^* + p + p \rightarrow {}^{147}\text{Er}^* + n + p + p \rightarrow {}^{147}\text{Er}_{g.s.}^* + n + p + p + \gamma's$ ${}^{150}\text{Yb}^* \rightarrow {}^{149}\text{Yb}^* + n \rightarrow {}^{148}\text{Tm}^* + p + n \rightarrow {}^{147}\text{Tm}^* + n + p + n \rightarrow {}^{147}\text{Tm}_{g.s.}^* + n + p + n + \gamma's$

 $^{58}\text{Ni} + {}^{92}\text{Mo} \rightarrow {}^{150}\text{Yb}^*$

Only a small fraction of the evaporation cascade stops at ¹⁴⁷Tm (<1%) via the p2n channel. Sellein et al Physical Review C **47** (1993) 1933



¹⁴⁶Tm made with same fusion reaction at slightly higher bombarding energy. Allows for the evaporation of more particles. *p3n* channel this time. ¹⁴⁵Tm was observed with *p4n* channel. Compound Nucleus decay on the neutron-rich side of the chart of nuclides

Neutron capture reactions are important in the R-process nuclosynthesis. One may not be able to measure these cross sections in the Lab, but Compound-nucleus theory can be used to predict the cross sections in some cases.



Is there an asymmetry dependence as well as an A dependence?

Conclusions

Resonances are important in all nuclei, especially as one goes near and past the driplines where eventually even the ground states are resonances.

We can populate resonances via a variety of reactions mechanisms (knockout, pickup,transfer) after which the probability of survival decays Exponentially. Alternatively we can probe resonances by elastic scattering.

Both barrier penetration and/or configuration mixing are important ingredients to produce narrow resonances.

Beyond the driplines, ground states are either single and two nucleon emitters. Odd-Even dependence.

In region of overlapping resonances there is strong configuration mixing. Statistical concepts come into place to determine the average decay widths. Need information of the level density for proton rich and neutron rich systems