Automation of the leading order calculations for $e^+e^- ightarrow$ hadrons

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The effective Lagrangian of the *Wtb* interaction containing operators of dimension four and five has the following form:

$$\begin{split} L_{Wtb} &= \frac{g}{\sqrt{2}} V_{tb} \left[W_{\mu}^{-} \bar{b} \gamma^{\mu} \left(f_{1}^{L} P_{L} + f_{1}^{R} P_{R} \right) t \right. \\ &\left. - \frac{1}{m_{W}} \partial_{\nu} W_{\mu}^{-} \bar{b} \sigma^{\mu\nu} \left(f_{2}^{L} P_{L} + f_{2}^{R} P_{R} \right) t \right] + \text{h.c.} \end{split}$$

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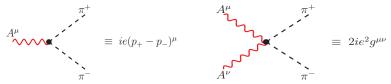
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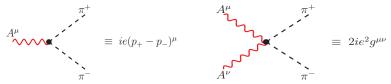


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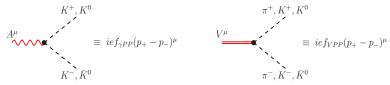


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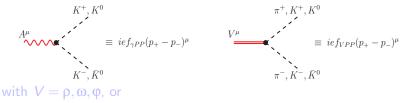
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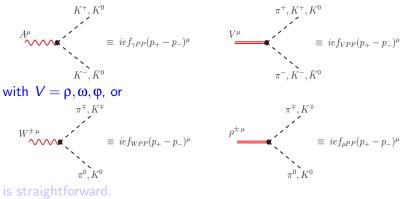
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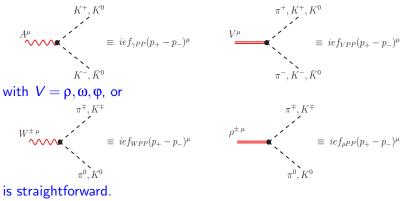
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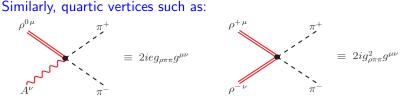


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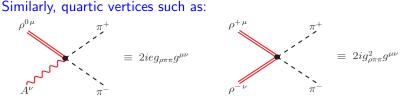
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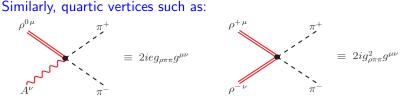
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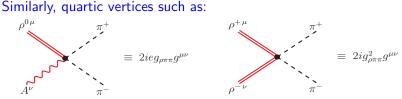
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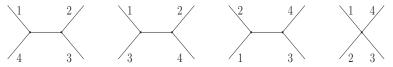


Line 4 is attached to each line and to the vertex \Rightarrow 4 topologies of a 4 particle process.

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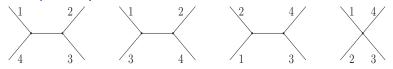
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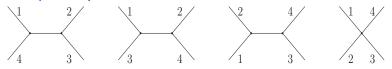


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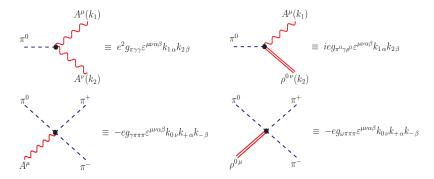
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Vertices of RChPT

Vertices of the Resonance Chiral Perturbation Theory (RChPT):



Their implementation in the program required just a few new subroutines for computation of the corresponding helicity amplitudes.

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