The role of experimental data as input for precise hadronic calculations: $(g-2)_\mu$, $P\to e^+e^-$, η-η' mixing

Pere Masjuan
Johannes Gutenberg-Universität Mainz
(masjuan@kph.uni-mainz.de)

Work done in collaboration with
Pablo Sanchez-Puertas

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Cluster of Excellence Precision Physics, Fundamental Interactions and Structure of Matter PRISMA

JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

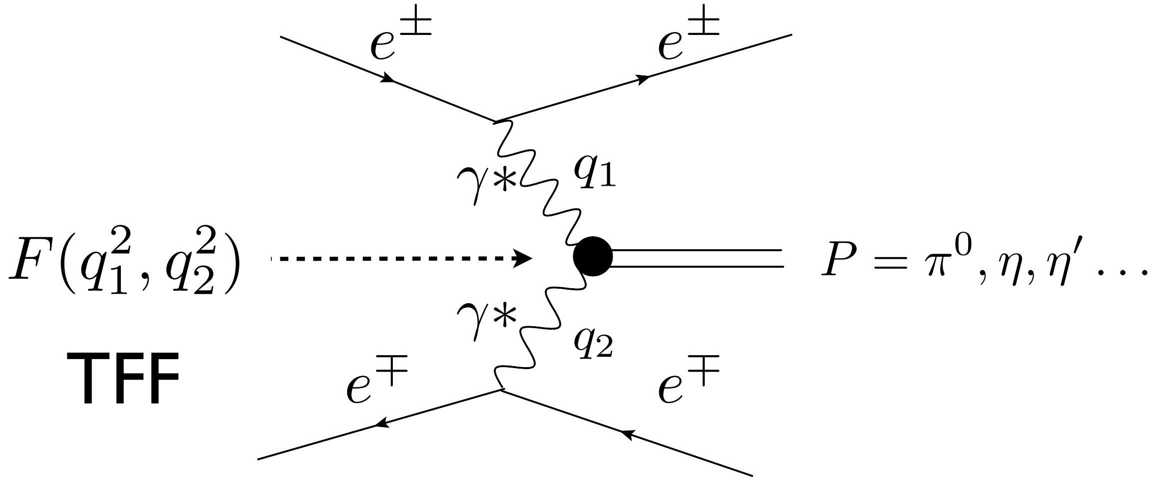
Radio MCLow WG, Frascati, 18 Nov

Outline

- Pseudoscalar Transition Form Factors
- How to use data for dressing the TFFs
- Applications
  - (g-2), $P \to e^+e^-$, η-η' mixing, time-like TFF
- Conclusions

Pseudoscalar Transition Form Factors

- Study of ee→eeγ*γ* with γ*γ*→π,η,η' but also P→eeγ, 4e,2e



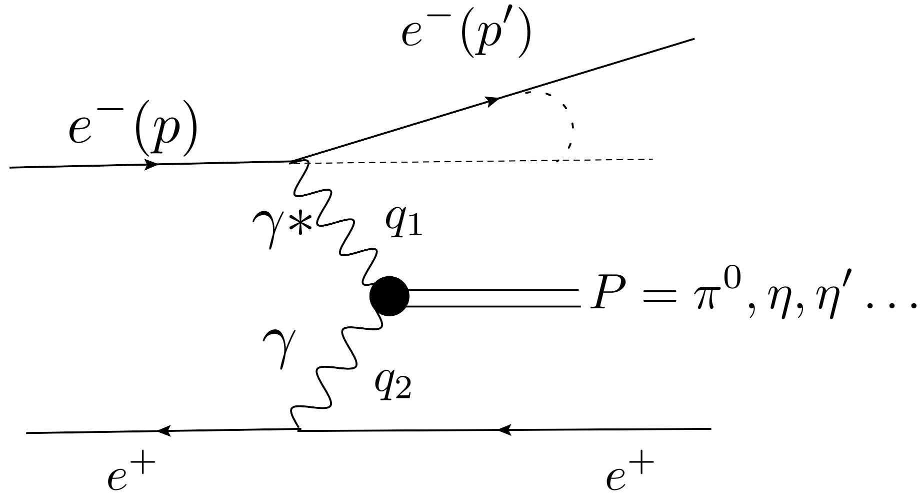
- Meson Structure
  - Transition Form Factors (TFF) give access to Meson Distribution Amplitudes
- Precision Tests of the Standard Model
  - Relation to mixing parameters, rare decays, and muon anomaly $(g-2)_\mu$

How do we do that?

- Single Tag Method can access the Meson Transition Form Factor

Selection criteria

- 1 e^- detected
- 1 e^+ along beam axis
- Meson full reconstructed



Momentum transfer

- tagged: $Q^2 = -q_1^2 = -(p-p')^2$

⇒highly virtual photon

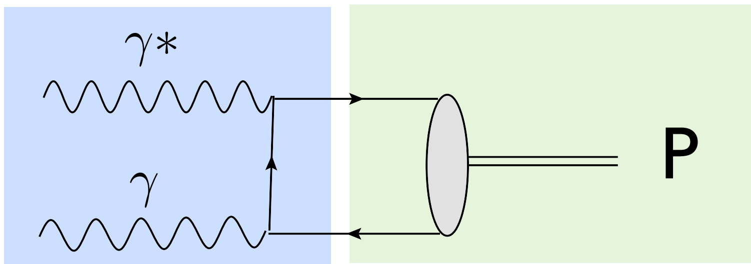
- untagged: $q^2 = -q_2^2 \sim 0\,\mathrm{GeV}^2$

⇒quasi-real photon

How do we do that?

Cross section for P production depends only on $F(q_1^2, q_2^2)$

With the Single Tag Method: $F(q_1^2, q_2^2) \to F(Q^2)$



$$F(Q^2) = \int T_H(x, Q^2)\Phi_P(x, \mu_F)\mathrm{d}x$$

$T_H(\gamma^*\gamma \to q\bar{q})$ $\Phi_P(q\bar{q} \to P)$

- μ_F is scale between soft and hard
- x-dependence of $\Phi_P(x,Q^2)$ not known but models

- Experimental data on $F(Q^2)$ is needed

convolution of perturbative and nor-perturbative regimes

The role of experimental data

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

Use hadronic models constrained with chiral and large-Nc arguments

Use data from the Transition Form Factor for input calculations

The role of experimental data

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

Use hadronic models constrained with chiral and large-Nc arguments

Use data from the Transition Form Factor for input calculations

The role of experimental data

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

- We want a method, not a *model*
- Simple (not black box as disp. rel)
- Approaches yes (improvable), assumptions no
- Systematic:
  - easy to update with new data
  - error from incompleteness of the data set
- Predictive (checkable)

The role of experimental data

Use data from
the Transition Form Factor
for numerical integral

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

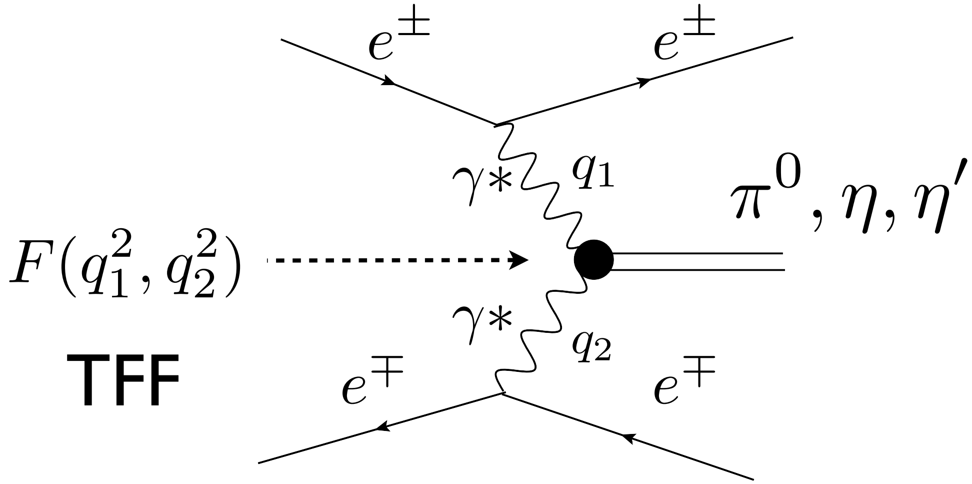
The role of experimental data

~~Use data from the Transition Form Factor for numerical integral~~

~~$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$~~

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

double-tag method



The role of experimental data

~~Use data from the Transition Form Factor for numerical integral~~

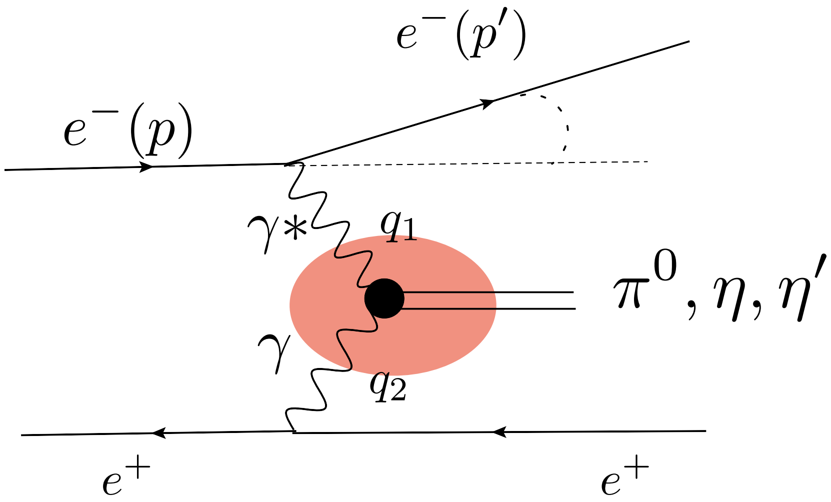
$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

Use data from
the Transition Form Factor
to constrain your
hadronic model

$$F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)$$

single-tag method



The role of experimental data

Use data from
the Transition Form Factor
for numerical integral

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

$$\Downarrow$$

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

$$\Downarrow$$

Use data from
the Transition Form Factor
to constrain your
hadronic model

$$\boxed{F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)}$$

How??

Nice synergy between experiment and theory

Simple, easy, systematic, user friendly method

Our proposal: use Padé Approximants

[P.M.'12; P.M., M.Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

Our proposal: use Padé Approximants

[P.M.'12; P.M., M. Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

$$F_{P\gamma*\gamma}(Q^2,0) = a_0^P\left(1 + b_P\frac{Q^2}{m_P^2} + c_P\frac{Q^4}{m_P^4} + \dots\right)$$

$\Gamma_{P\to\gamma\gamma}$ ↗ a_0^P ; slope ↑ b_P ; curvature ↑ c_P

We have published space-like data for $Q^2F_{P\gamma*\gamma}(Q^2,0)$

$$Q^2F_{P\gamma*\gamma}(Q^2,0) = a_0Q^2 + a_1Q^4 + a_2Q^6 + \dots$$

$$P_M^N(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0Q^2 + a_1Q^4 + a_2Q^6 + \dots + \mathcal{O}((Q^2)^{N+M+1})$$

Our proposal: use Padé Approximants

[P.M.'12; P.M., M. Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

$$F_{P\gamma*\gamma}(Q^2,0) = a_0^P \left(1 + b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \dots\right)$$

a_0^P ↗ $\Gamma_{P\to\gamma\gamma}$; b_P ↑ slope; c_P ↑ curvature

We have published space-like data for $Q^2 F_{P\gamma*\gamma}(Q^2,0)$

$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2} \longrightarrow \begin{array}{l} P_1^N(Q^2) = P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \dots \\ P_N^N(Q^2) = P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \dots \end{array}$$

sequence of approximations, i.e., <u>theoretical error</u>

Our proposal: use Padé Approximants

[P.M.'12; P.M., M.Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2} \longrightarrow \begin{array}{l} P_1^N(Q^2) = P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \dots \\ P_N^N(Q^2) = P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \dots \end{array}$$

<u>Convergence</u> (making use of analytical properties):

$$\lim_{N\to\infty} P_1^N(Q^2) = F_{P\gamma^*\gamma}(Q^2,0)$$ Montessus Theorem

Conv. from pole at -Q^2 to Q^{*2}: good at LE, bad at HE. Fantastic for LEPs and cheap

Our proposal: use Padé Approximants

[P.M.'12; P.M., M.Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \ldots$$

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2} \longrightarrow \begin{array}{l} P_1^N(Q^2) = P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \ldots \\ P_N^N(Q^2) = P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \ldots \end{array}$$

<u>Convergence</u> (making use of analytical properties):

$$\lim_{N\to\infty} P_N^N(Q^2) = F_{P\gamma^*\gamma}(Q^2,0)$$ Pommerenke Theorem

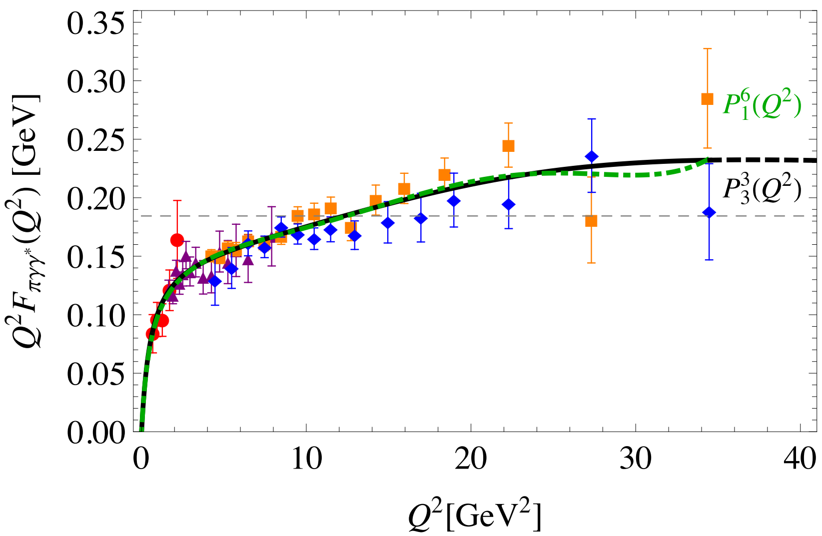
Conv. from cut at -Q^2 to ∞: good at LE and HE. Good for LEPs and no cheap

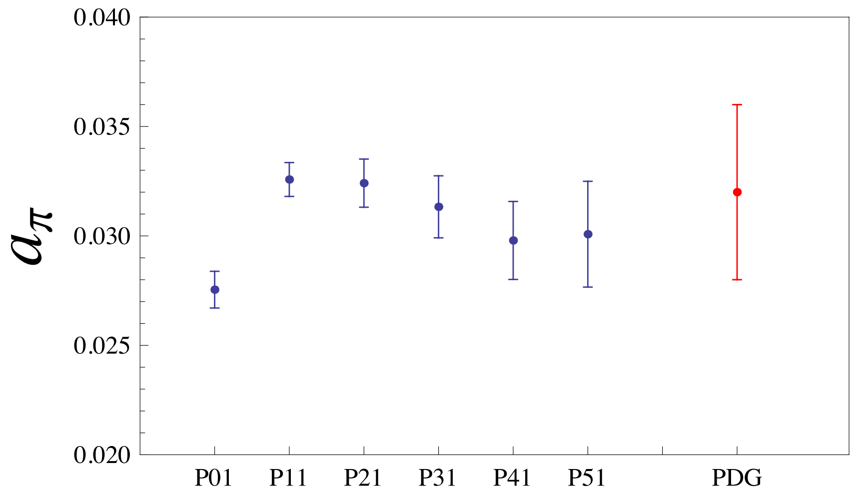
Our proposal: use Padé Approximants

[P.M.'12; P.M., M.Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'09 and Belle'12

$P_1^N(Q^2)$ up to N=5 [P.M, '12]





$P_N^N(Q^2)$ up to N=3

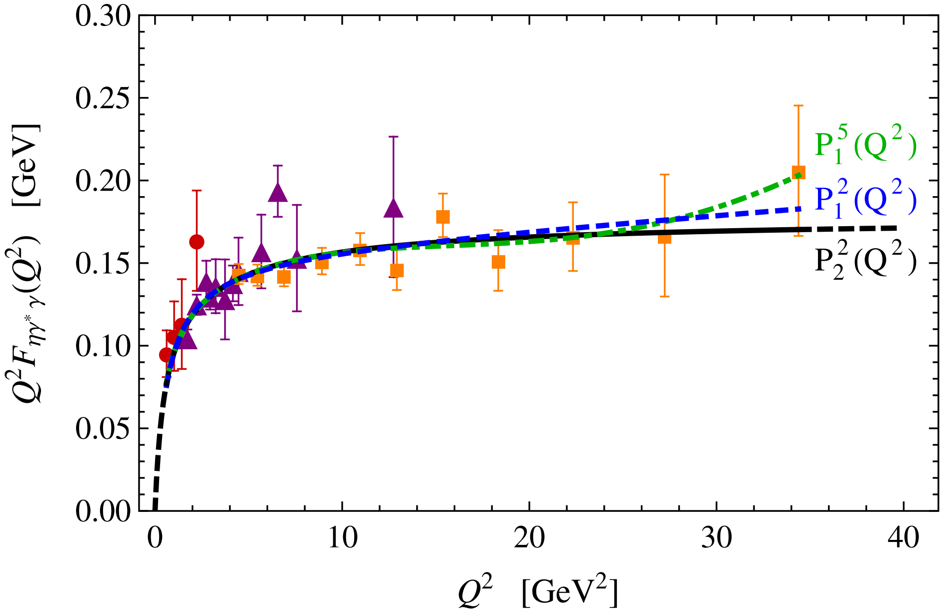
Accurate description of the low-energy region making full use of available experimental data

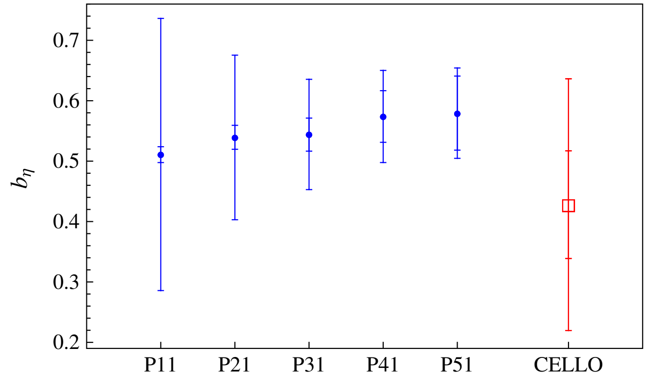
η-TFF

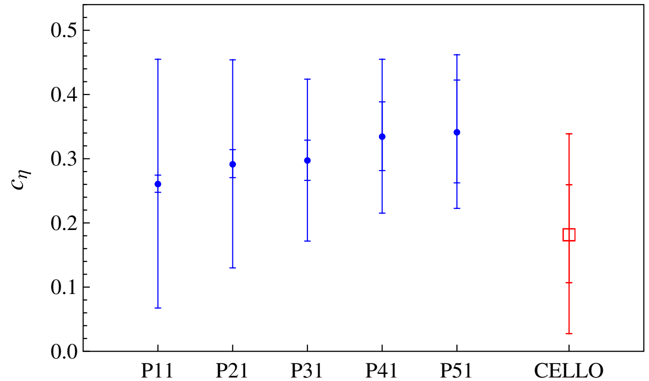
Fit to Space-like data: CELLO'91, CLEO'98, BABAR'11+ $\Gamma_{\eta\to\gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]

$P_1^N(Q^2)$ up to N=4







$P_N^N(Q^2)$ up to N=2

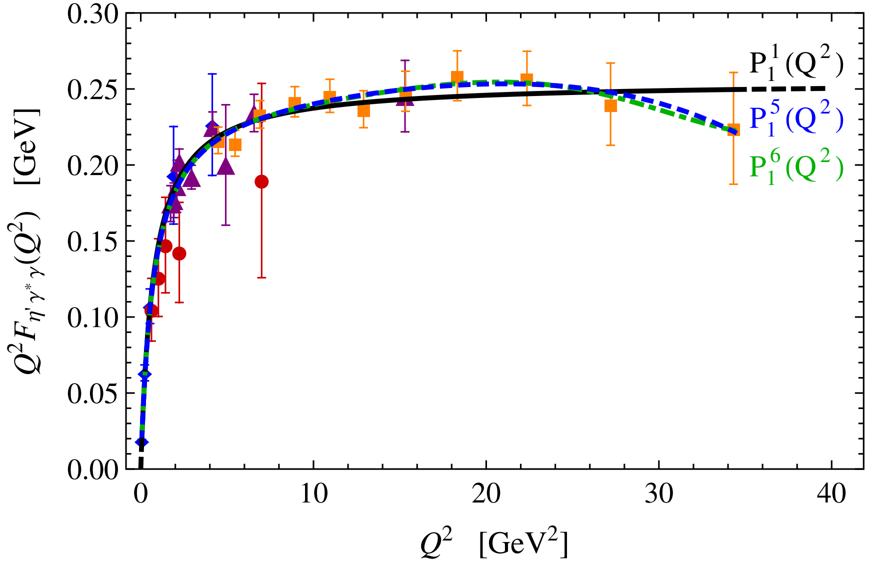
$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.164(2) GeV$$

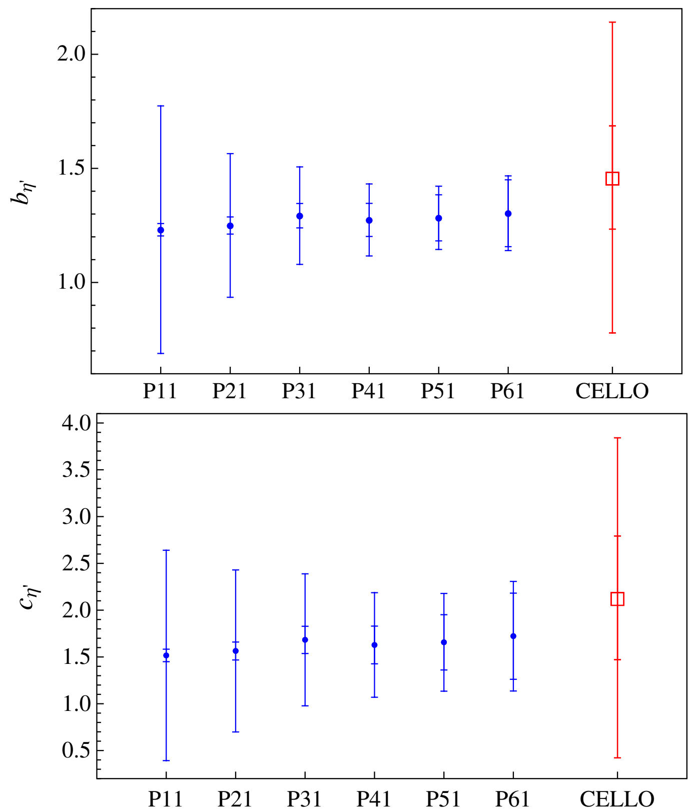
η’-TFF

Fit to Space-like data: CELLO’91, CLEO’98, L3’98, BABAR’11+ $\Gamma_{\eta'\to\gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, ’13]

$P_1^N(Q^2)$ up to N=5





$P_N^N(Q^2)$ up to N=1

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma*\gamma}(Q^2,0) = 0.254(4) GeV$$

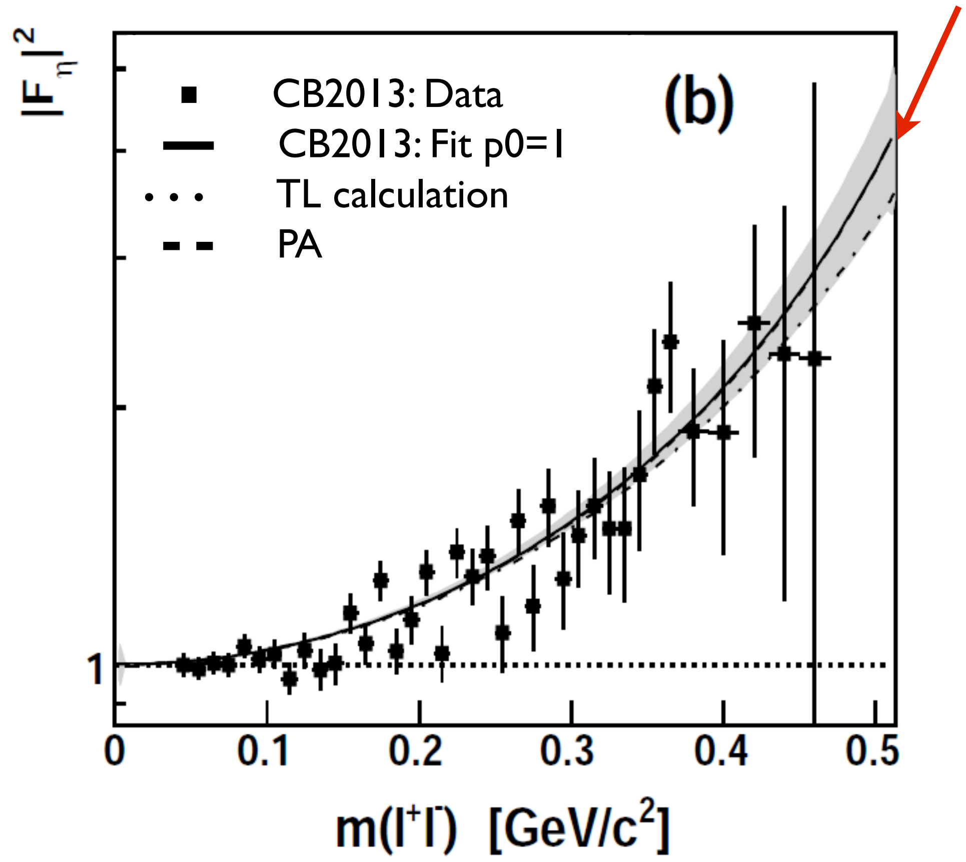
η-TFF

Predictive method!

- Study Dalitz decays
  $\eta \to \gamma^*\gamma \to e^+e^-\gamma$

- Prediction of the time-like from space-like data

A2@MAMI

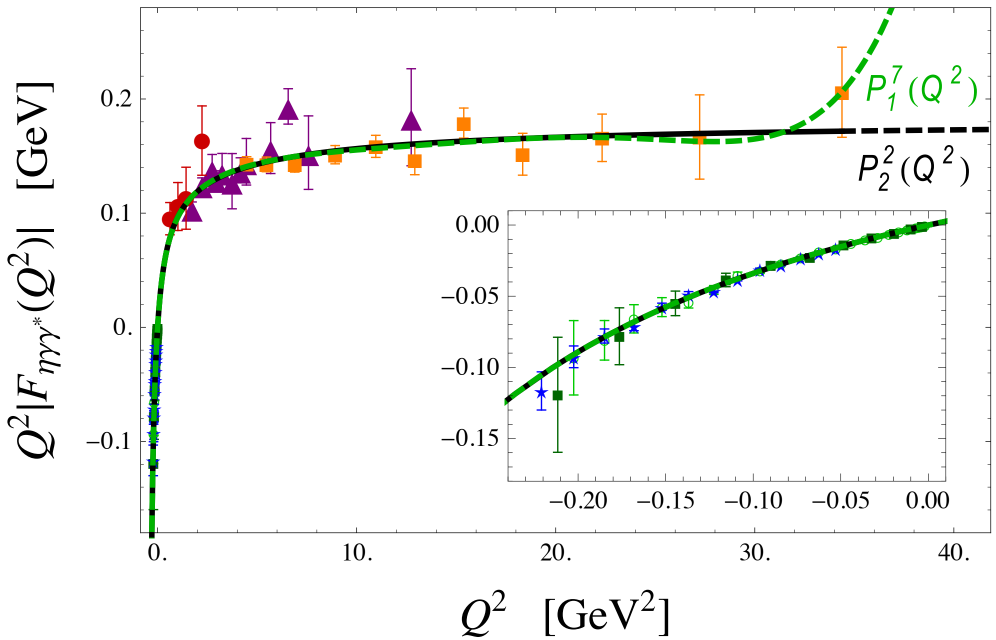


[A2 Coll. PRC89 2014]

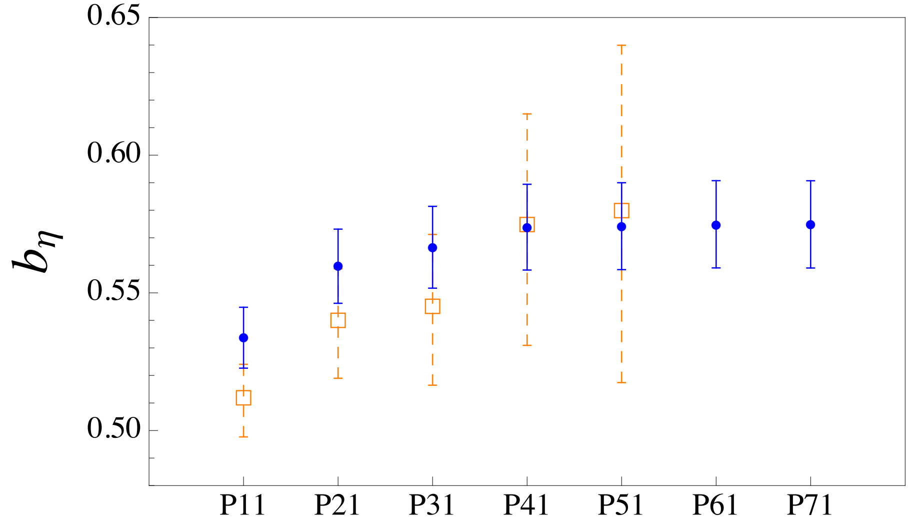
η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+ $\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '14]



$P_1^N(Q^2)$ up to N=7



$P_N^N(Q^2)$ up to N=2

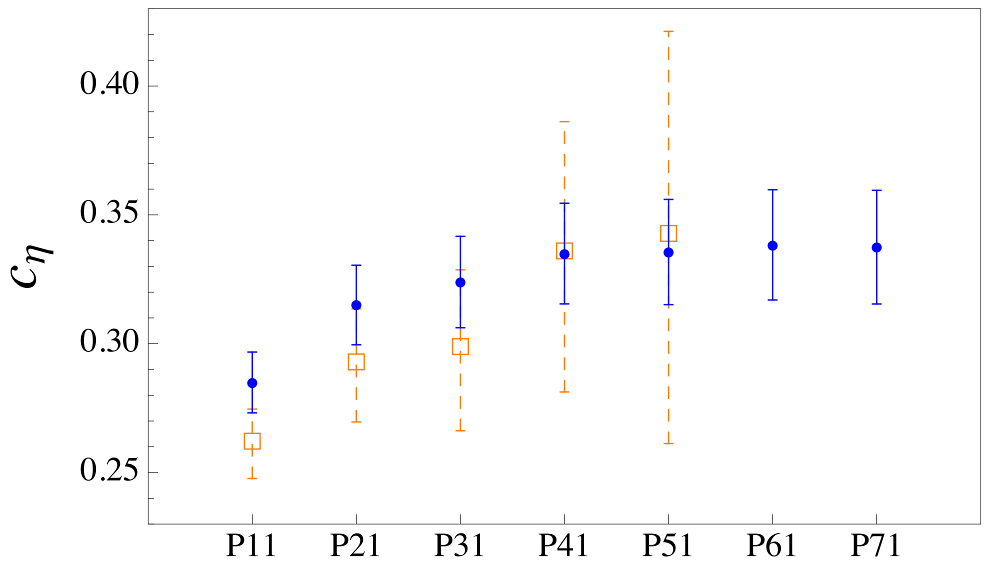
$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.177(15)GeV$$

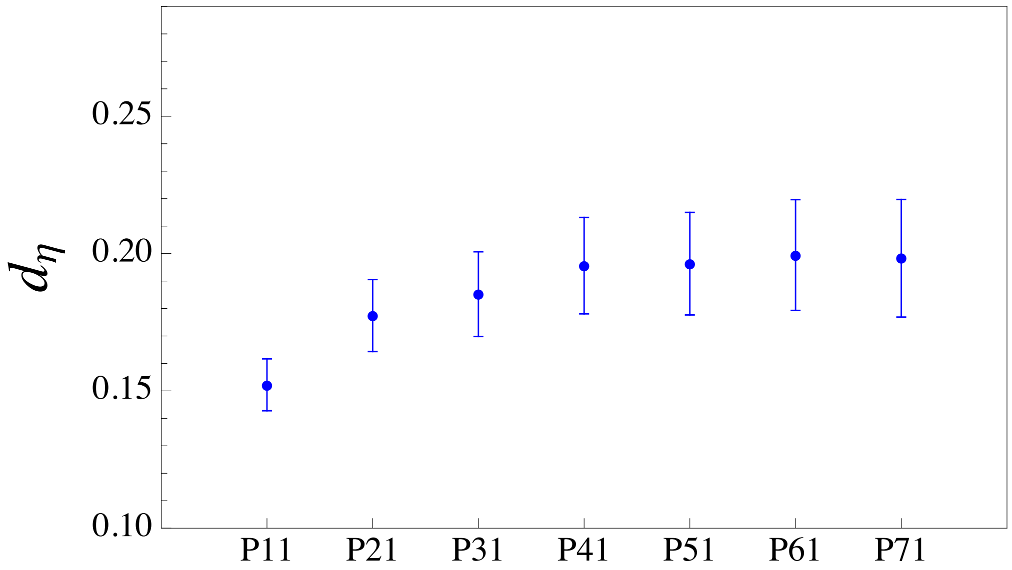
η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+ $\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

$P_1^N(Q^2)$ up to N=7

[R.Escribano, P.M., P. Sanchez-Puertas, '14]

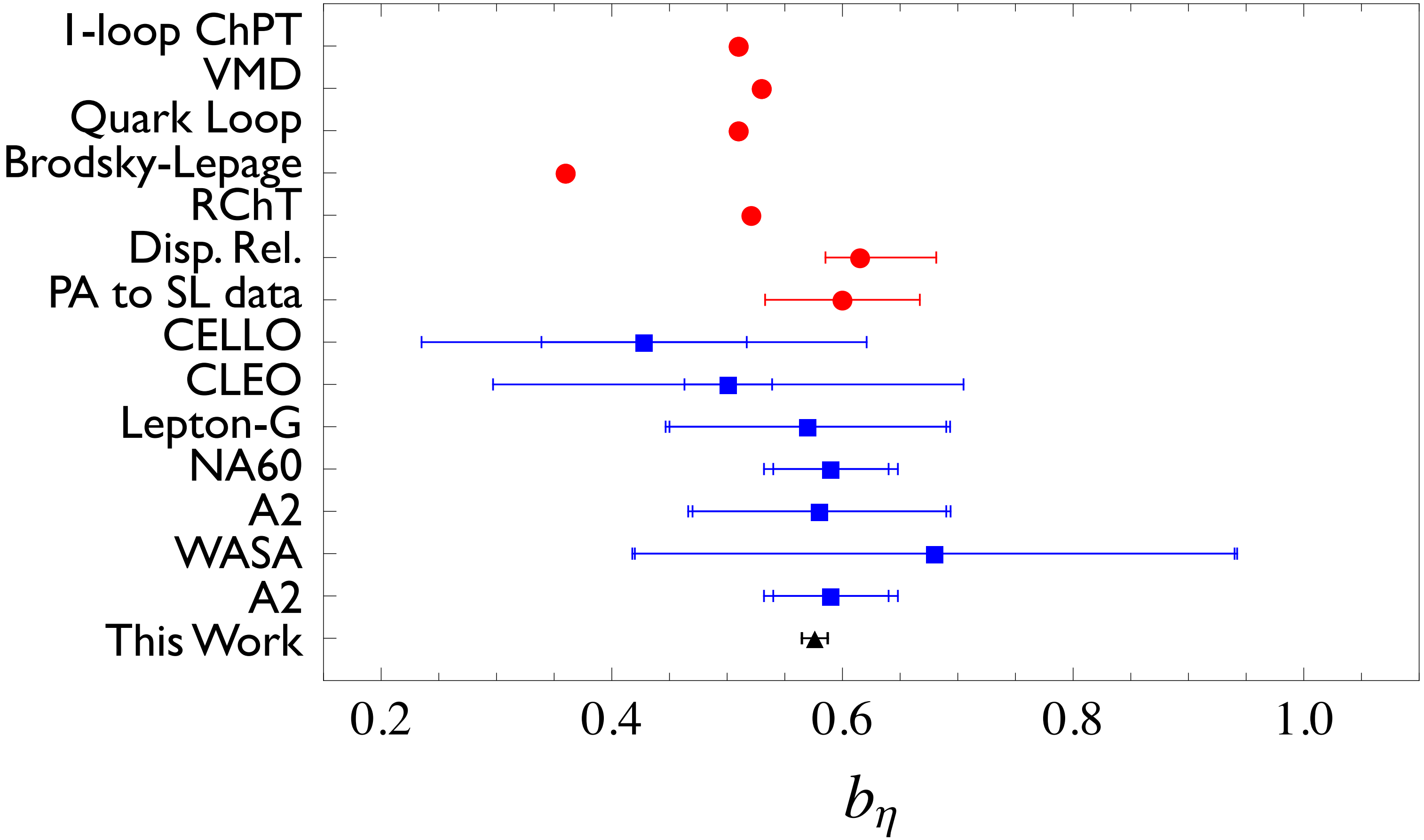




η-TFF

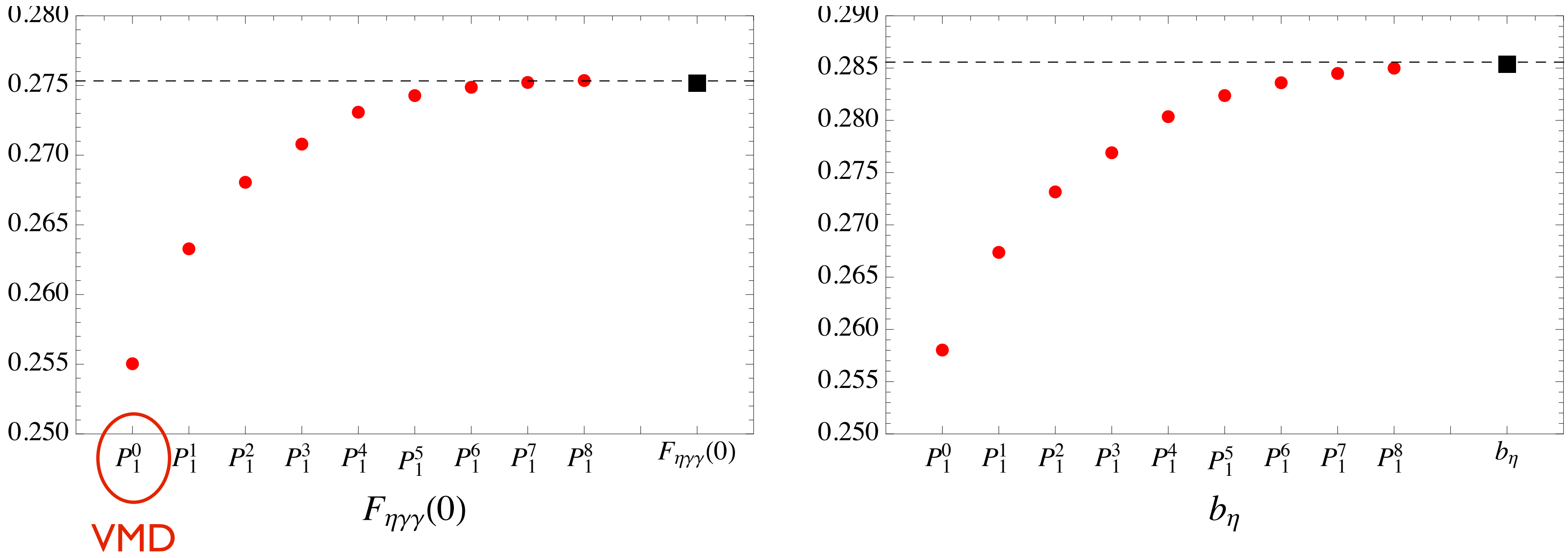
Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+ $\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '14]



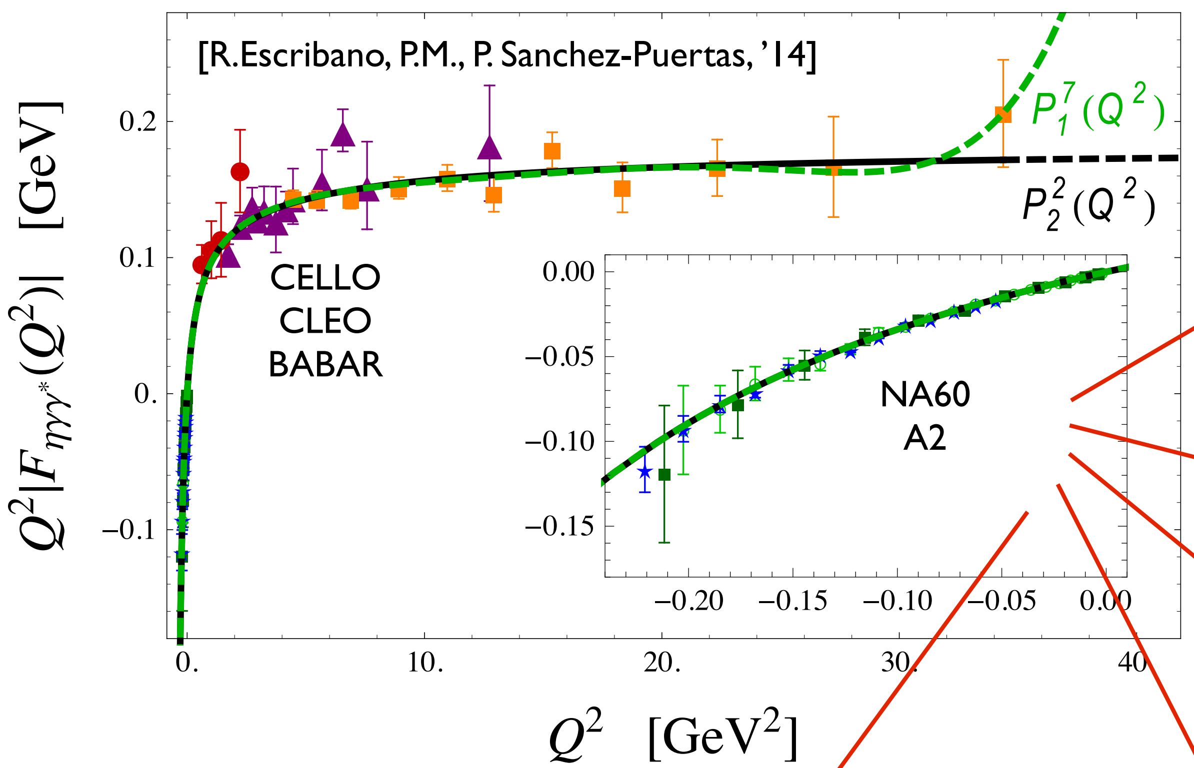
A word on systematics

- Consider a model for η TFF
- Generate a pseudodata set emulating the physical situation (SL+TL)
- Build up your PA sequence
- Fit and compare



PS-TFF

space-like and time-like data



Low-energy parameters up to the third derivative!

$(g-2)_\mu$

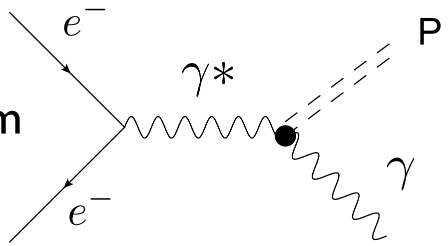
mixing parameters of the η-η' system

f_q, f_s, ϕ

Rare decays

$\Gamma_{P\to l^+l^-}$ $\Gamma_{P\to 4l}$

PS continuum on charmonium region



beautiful synergy experiment - theory

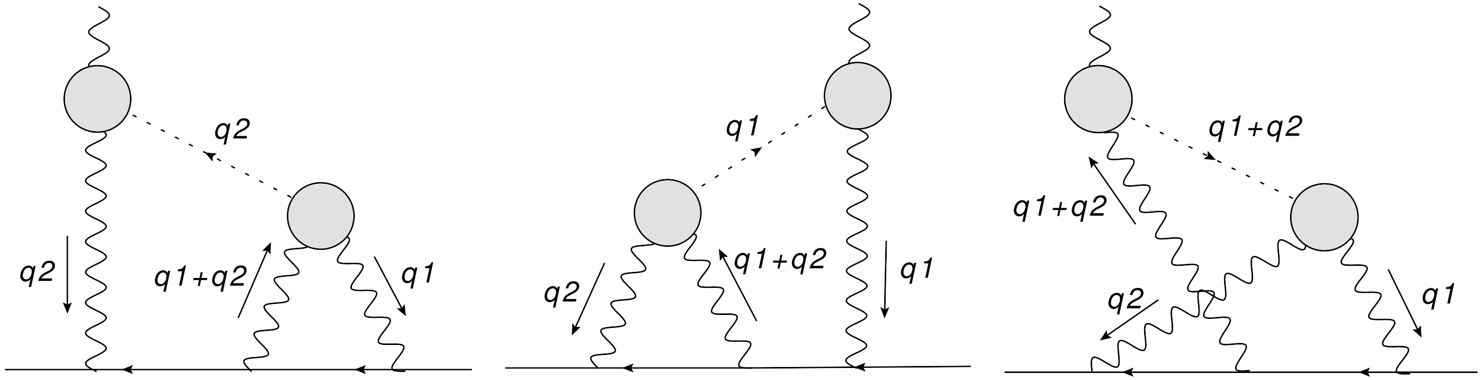
Applications

1. Hadronic Light-by-Light contribution to muon (g-2)
2. PS decays into lepton pairs ($\pi^0 \to e^+e^-$)
3. η-η' mixing
4. Time-like TFF prediction (charmonium backgrounds)

Applications

1. Hadronic Light-by-Light contribution to muon (g-2)
2. PS decays into lepton pairs ($\pi^0 \to e^+e^-$)
3. η-η' mixing
4. Time-like TFF prediction (charmonium backgrounds)

Dissection of the HLBL contribution



$$a_\mu^{LbL;P} = -e^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

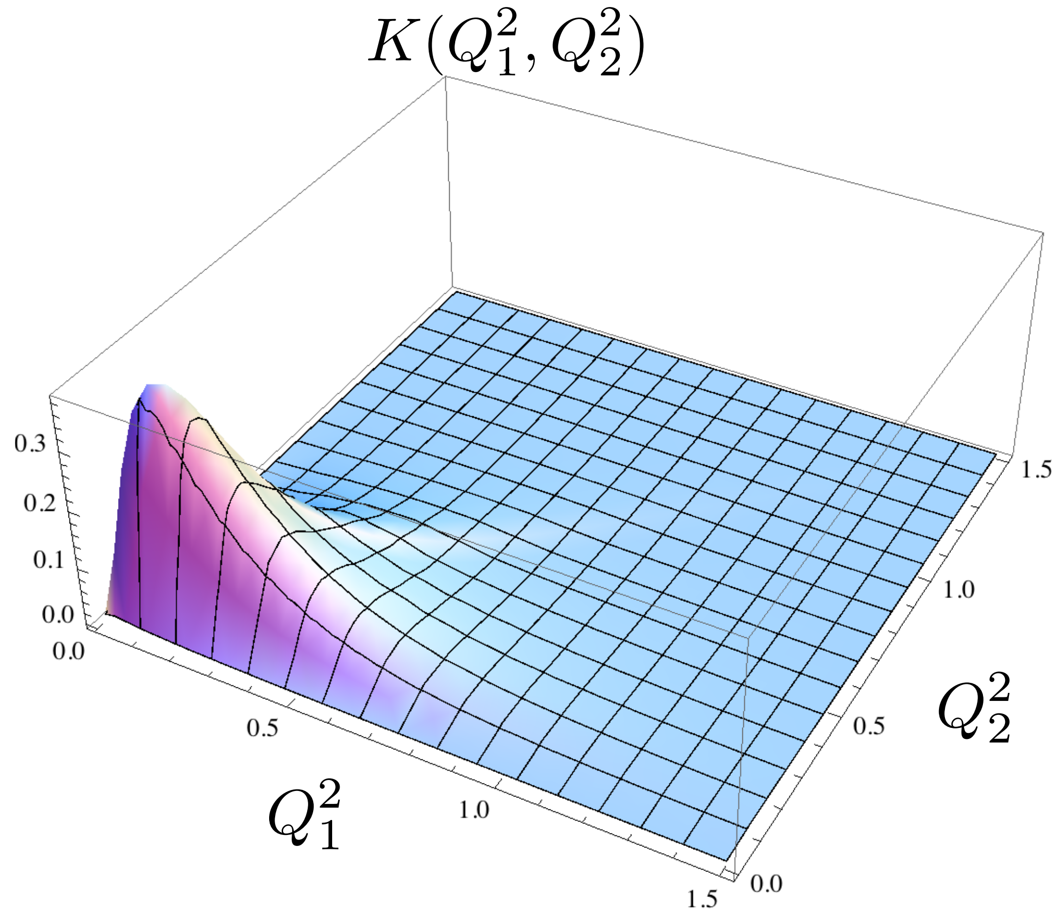
$$\times \left(\frac{F_{P^*\gamma^*\gamma^*}(q_2^2, q_1^2, (q_1+q_2)^2) F_{P^*\gamma^*\gamma^*}(q_2^2, q_2^2, 0)}{q_2^2 - M_P^2} T_1(q_1, q_2; p) \right.$$

$$\left. + \frac{F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, (q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_P^2} T_2(q_1, q_2; p) \right)$$

Dissection of the HLBL contribution

$$a_\mu^{LbyL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2)$$

Using $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim VMD(Q_1^2, Q_2^2)$



(main energy range from 0 to 1 GeV^2)

Dissection of the HLBL contribution

a la Knecht-Nyffeler

Central value:

$$F_{\pi^0\gamma^*\gamma^*}^{LMD+V}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2 q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2 q_1^2 q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:

$$F_\pi = 92.4 \text{ MeV}$$
$$m_\rho = 769 \text{ MeV}$$
$$m_{\rho'} = 1465 \text{ MeV}$$
$$h_1 = 0 \,(\text{ BL limit})$$
$$h_5 = 6.93 \text{ GeV}^4$$
$$h_2 = -10 \text{ GeV}^2$$

$$a_\mu^{\text{HLBL},\pi} = 6.3 \times 10^{-10}$$

Dissection of the HLBL contribution

a la Knecht-Nyffeler

Central value:

$$F_{\pi^0\gamma^*\gamma^*}^{LMD+V}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:		Preliminary, using exp data:
$F_\pi = 92.4$ MeV	⟶	$\Gamma_{\pi^0\to\gamma\gamma}$
$m_\rho = 769$ MeV		$m_\rho = 775$ MeV
$m_{\rho'} = 1465$ MeV	⟶	curvature
$h_1 = 0$ (BL limit)		$h_1 = 0$ (BL limit)
$h_5 = 6.93$ GeV^4	⟶	slope
$h_2 = -10$ GeV^2		$h_2 = -10$ GeV^2
$a_\mu^{\mathrm{HLBL},\pi} = 6.3\times 10^{-10}$	⟶	$a_\mu^{\mathrm{HLBL},\pi} = 7.5\times 10^{-10}$

Dissection of the HLBL contribution

a la Knecht-Nyffeler

Error budget:

$$F^{VMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\frac{M_V^2}{(q_1^2-M_V^2)}\frac{M_V^2}{(q_2^2-M_V^2)}$$

$$F^{LMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{(q_1^2+q_2^2)-c_V}{(q_1^2-M_V^2)(q_2^2-M_V^2)}$$

$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

$$\Delta F_\pi \Rightarrow 2\Delta a_\mu^{\text{HLBL, P}}$$
$$\Delta\ \text{slope} \Rightarrow 0.75\Delta a_\mu^{\text{HLBL, P}}$$
$$\Delta\ \text{curv.} \Rightarrow 0.5\Delta a_\mu^{\text{HLBL, P}}$$
$$\Delta m_\rho = \Gamma/2 \Rightarrow 1.3\Delta a_\mu^{\text{HLBL, P}}$$

Current exp. precision:

$$\Delta F_\pi \sim 1.1\%$$
$$\Delta\ \text{slope} \sim 13\%$$
$$\Delta\ \text{curvature} \sim 25\%$$

Chiral limit $$F_0 \to F_\pi \sim 5\%$$

1/Nc $$\Delta m_\rho \sim 10\%$$

$$\Delta a_\mu^{\text{HLBL},\pi} \sim 15\%$$

Dissection of the HLBL contribution

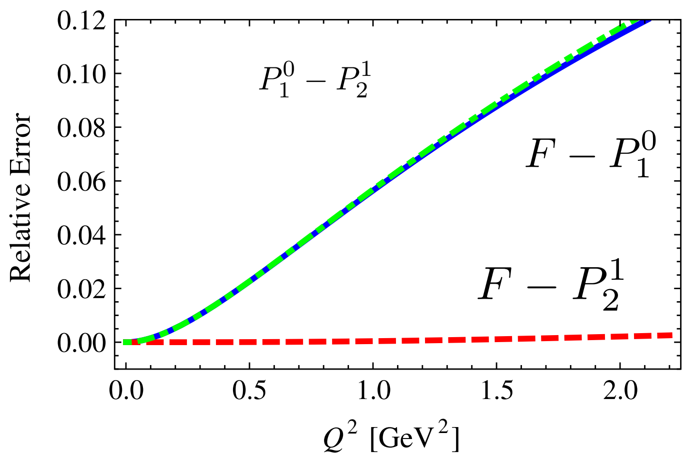
a la Padé

P.M., S. Peris, 07 P.M. '12
P.M., Vanderhaeghen'12
R. Escribano, P.M., P. Sanchez-Puertas, 13

$$F^{P01}_{P^*\gamma^*\gamma^*}(P_P^2, Q_1^2, Q_2^2) = a\frac{b}{Q_1^2+b}\frac{b}{Q_2^2+b}(1+cP_P^2)$$

$$F^{P12}_{P^*\gamma^*\gamma^*}(P_P^2, Q_1^2, Q_2^2) = \frac{a+bQ_1^2}{(Q_1^2+c)(Q_1^2+d)}\frac{a+bQ_2^2}{(Q_2^2+c)(Q_2^2+d)}(1+cP_P^2)$$

	b_P	c_P	$\lim_{Q^2\to\infty} Q^2 F_{P\gamma^*\gamma}(Q^2)$	$a_\mu^{\text{HLBL;P}}$
π^0	0.0324(22)	$1.06(27)\cdot 10^{-3}$	$2f_\pi$	$6.49(56)\cdot 10^{-10}$
η	0.60(7)	0.37(12)	0.160(24)GeV	$1.25(15)\cdot 10^{-10}$
η'	1.30(17)	1.72(58)	0.255(4)GeV	$1.27(19)\cdot 10^{-10}$



Systematic error from approach:

$P_1^0(Q_1^2, Q_2^2)$ vs $P_2^1(Q_1^2, Q_2^2)$ → 5%

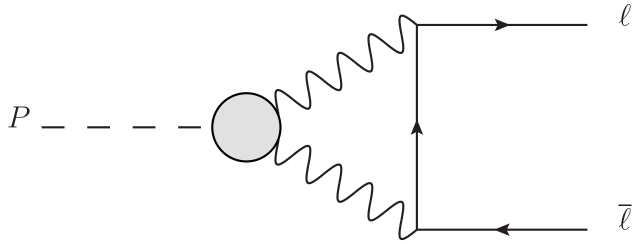
[P.M.,Peris,'07]

Applications

1. Hadronic Light-by-Light contribution to muon (g-2)
2. PS decays into lepton pairs ($\pi^0 \rightarrow e^+e^-$)
3. η-η' mixing
4. Time-like TFF prediction (charmonium backgrounds)

Introduction and Motivation

Experiment



$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2$$

$$\sim 1.5 \cdot 10^{-10}$$

KTeV '07:

$$BR(\pi^0 \to e^+e^-(\gamma), x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

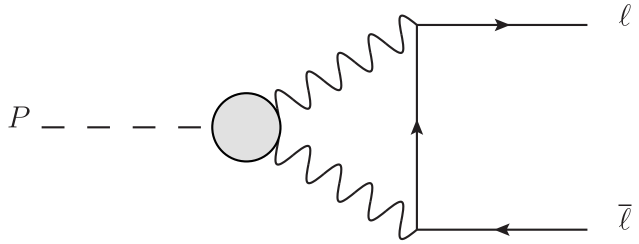
Extrapolation to x=1 + radiative correction + Dalitz decay background

$$BR_{\mathrm{KTeV}}^{w/o\,rad}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

(dominates de PDG)

Introduction and Motivation

Theory



$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2$$

The only unknown $\mathcal{A}(m_P^2)$ from loop calculation where the TFF enters.

$$\mathcal{A}(q^2) = \frac{2i}{\pi^2}\int d^4k \; \frac{q^2k^2 - (k \cdot q)^2}{k^2(k-q)^2((p-k) - m_\ell^2)} \frac{F_{P\gamma^*\gamma^*}(k^2,(q-k)^2)}{F_{P\gamma\gamma}(0,0)}$$

Dissection of $\pi^0 \to e^+e^-$

As model independent as possible:

Cutcosky rules provides the imaginary part

$$Im\mathcal{A}(q^2) = \frac{\pi}{2\beta_l(q^2)} ln\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right) \; ; \quad \beta_l(q^2) = \sqrt{1-\frac{4m_l^2}{q^2}}$$

$$q^2 = m_P^2$$

Assuming $|\mathcal{A}|^2 \geq (\mathrm{Im}\mathcal{A})^2$

$$B(\pi^0 \to e^+e^-) \geq B^{\mathrm{unitary}}(\pi^0 \to e^+e^-) = 4.69 \cdot 10^{-8}.$$

(doesn't depend on TFF)

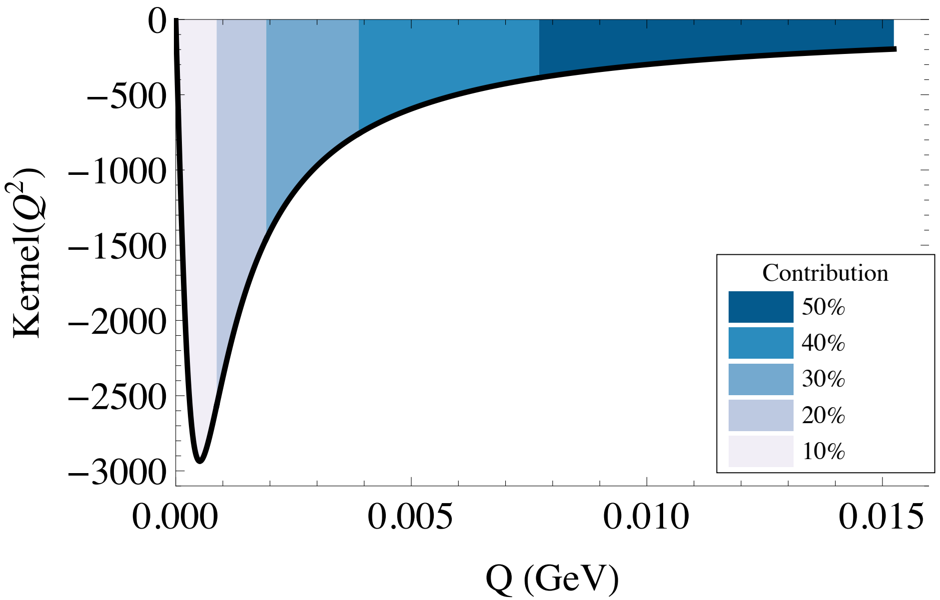
Dissection of $\pi^0 \rightarrow e^+e^-$

$$Re(\mathcal{A}(m_P^2)) = \left(-\frac{5}{4} + \int_0^\infty dQ^2 \; Kernel(Q^2) \right) + \frac{\pi^2}{12} + ln^2 \left(\frac{m_l}{m_P} \right)$$

$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$

Dissection of $\pi^0 \to e^+e^-$

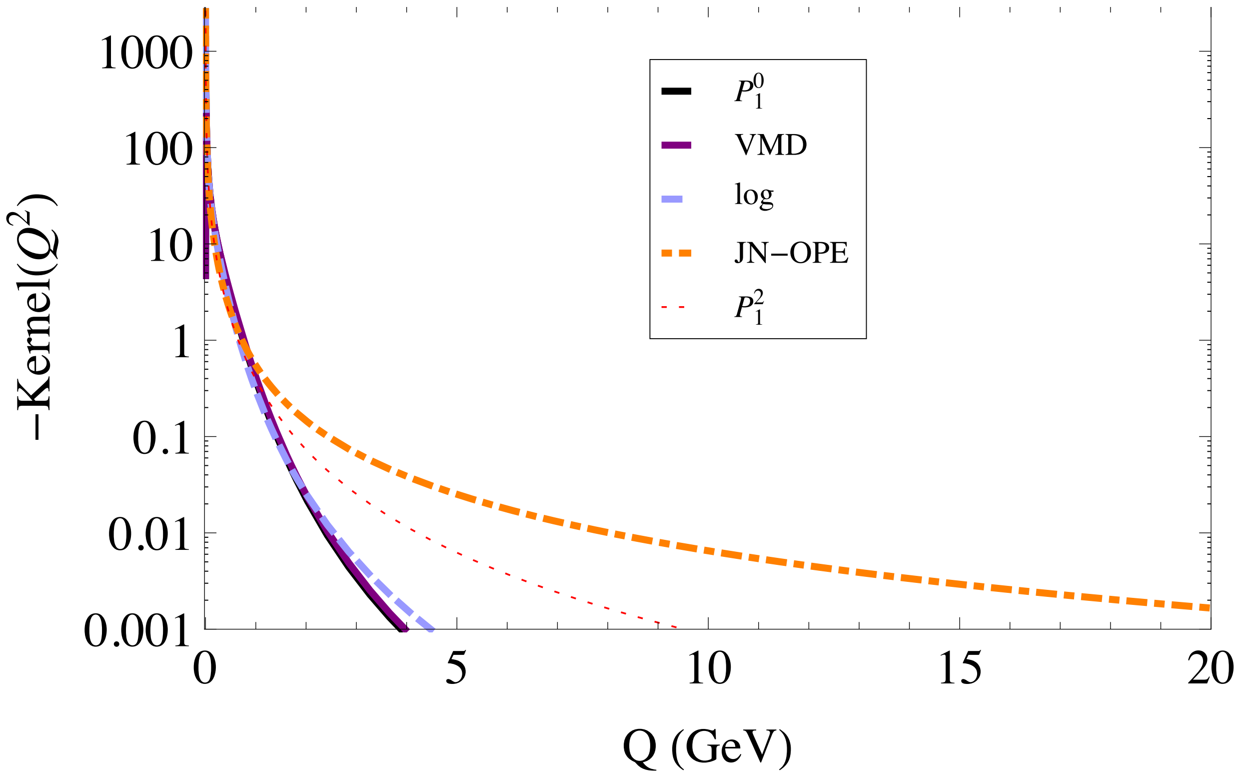
$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$



- Its contribution is negative: lowers the BR.
- Peaks at $\sim\ 2m_e$ and $\langle Q \rangle = 0.09$ GeV.
- Low energies relevant only: slope is enough.

Dissection of $\pi^0 \to e^+e^-$

$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$



Dubna contribution: corrections m_e/m_π, m_e/Λ

Dorokhov and Ivanov, '08

$$\mathcal{O}\left(\frac{m_e}{\Lambda}\right)^2 \qquad \mathcal{O}\left(\frac{m_e}{\Lambda}\log\frac{m_e}{\Lambda}\right)^2$$

Dorokhov, Ivanov and Kovalenko '09

$$\mathcal{O}\left(\frac{m_\pi}{\Lambda}\right)^2 \qquad \mathcal{O}\left(\frac{m_e}{m_\pi}\right)^2$$

Λ
the cut-off
or
VMD "mass"

Resummation of power corrections using Mellin-Barnes techniques.
Conclusion: corrections negligible!

$$BR_{\rm SM}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8} \quad \sim 3\sigma$$

Prague contribution: Radiative corrections

Vasko, Novotny '11 + Husek, Kampf, Novotny'14

$$\frac{\mathrm{BR}(\pi^0 \to e^+e^-(\gamma), x > 0.95)}{\mathrm{BR}(\pi^0 \to \gamma\gamma)} =$$

$$\frac{\Gamma(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} [1 + \delta^{(2)}(0.95) + \Delta^{BS}(0.95) + \delta^{D}(0.95)]$$

$$\delta^{(2)}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}_{\text{soft}}(0.95) = (-5.8 \pm 0.2)\,\% \qquad \text{vs} \quad \sim -13\%$$

$$\Delta^{\text{BS}}(0.95) = (0.30 \pm 0.01)\,\% \qquad \delta^{D}(0.95) = \frac{1.75 \times 10^{-15}}{[\Gamma^{\text{LO}}(\pi^0 \to e^+e^-)/\text{MeV}]}$$

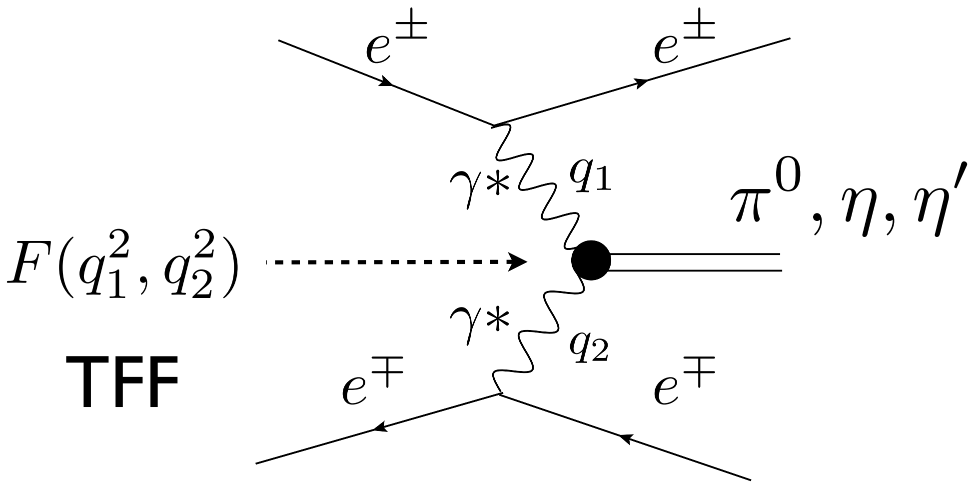
$$BR^{w/o\,rad}_{\text{"KTeV"}}(\pi^0 \to e^+e^-) = (6.87 \pm 0.36) \times 10^{-8}$$

Mainz contribution: TFF parameterization

Use data from
the Transition Form Factor
for numerical integral

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

double-tag method



Remember: only low-energy region is needed

Doubly virtual π⁰-TFF

[P.M., P. Sanchez-Puertas, in preparation]

For $BR_{SM}(\pi^0 \to e^+e^-)$ **we need** $F_{\pi^0\gamma^*\gamma^*}(Q^2, Q^2)$

Proposal: bivariate PA

Chisholm '73

$$P^N_M(Q_1^2, Q_2^2) = \frac{T_N(Q_1^2, Q_2^2)}{R_M(Q_1^2, Q_2^2)} = a_0 + a_1(Q_1^2 + Q_2^2) + a_{1,1}Q_1^2Q_2^2 + a_2(Q_1^4 + Q_2^4) + \cdots$$

$$P^0_1(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2Q_2^2}$$

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data

$a_{1,1}$ from a systematic fit to doubly virtual SL data

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data

$a_{1,1}$ from a systematic fit to doubly virtual SL data

OPE indicates: $\lim_{Q^2\to\infty} P_1^0(Q^2, Q^2) \sim Q^{-2}$ i.e., $a_{1,1} = 2a_1^2$

Doubly virtual π^0-TFF

Chisholm '73

Proposal: bivariate PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data

$$0 \leq a_{1,1} \leq 2a_1^2$$

$$BR_{SM}^{PA}(\pi^0 \to e^+e^-) = (6.22 - 6.36)(4) \times 10^{-8}$$

statistics+theoretical error

method checked for different models
+ to shrink the window: data (data-driven approach)

Doubly virtual π^0-TFF

$$BR^{w/o\,rad}_{\text{"KTeV"}}(\pi^0 \to e^+e^-) = (6.87 \pm 0.36) \times 10^{-8}$$

$$\sim (2.6 - 1.4)\sigma$$

$$BR^{PA}_{SM}(\pi^0 \to e^+e^-) = (6.22 - 6.36)(4) \times 10^{-8}$$

Impact of $\pi^0 \to e^+e^-$ on HLBL

	Model	Published model		Modified model	
		$\pi^0 \to e^+e^-$ $(\times 10^8)$	$HLBL$ $(\times 10^{10})$	$\pi^0 \to e^+e^-$ $(\times 10^8)$	$HLBL$ $(\times 10^{10})$
Jegerlehner and Nyffeler '09	LMD+V	6.33	6.29	6.47	5.22
Dorokhov et al '09	VMD	6.34	5.64	6.87	2.44
Our proposal '14	PA	6.36	5.53	6.87	2.85

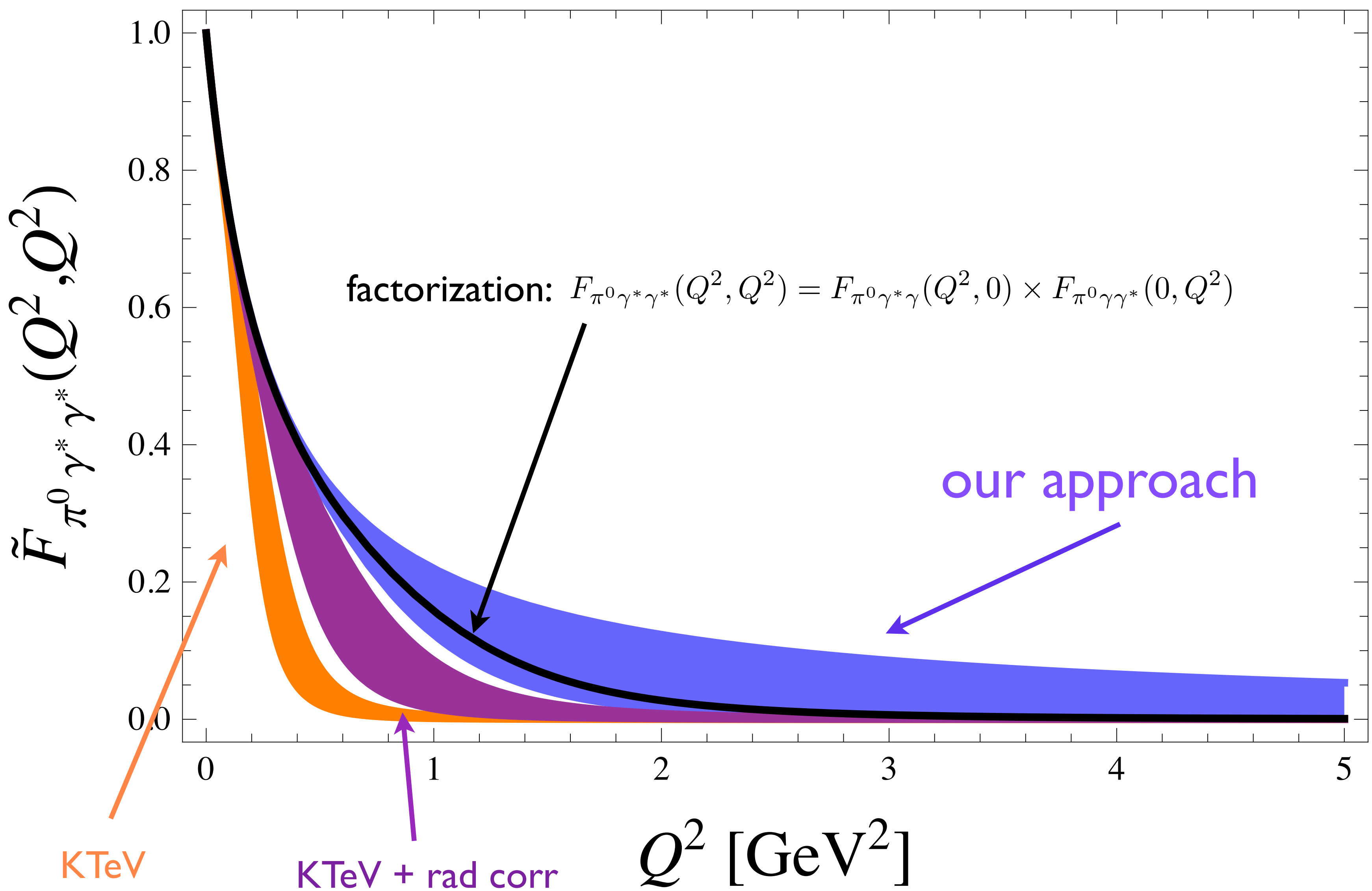
$$\Delta a_\mu^{SM} \sim 6 \times 10^{-10}$$

$$\Delta a_\mu^{HLBL} \sim 4 \times 10^{-10}$$

$$\Delta a_\mu^{HLBL;\pi^0 \to e^+e^-} \sim (2-3) \times 10^{-10}$$

+ similar effect for the η decay!

The role of doubly virtual TFF data



Dissection of η→l⁺l⁻

PDG value dominated by the KTeV measurement

$$\frac{BR(P\to\bar{\ell}\ell)}{BR(P\to\gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2 \begin{array}{ll} = 5.8(8)\cdot 10^{-6} & (\mu^+\mu^-) \\ \leq 5.6\cdot 10^{-6} & (e^+e^-) \end{array}$$

Unitary Bound for the μμ case $= 4.37\cdot 10^{-6}$

SM calculations with $m_\eta^2/\Lambda^2 \sim 0$ $= 4.99\cdot 10^{-6}$

Our result from SL+TL (full result) $= 4.51(2)\cdot 10^{-6}$

Applications

1. Hadronic Light-by-Light contribution to muon (g-2)
2. PS decays into lepton pairs ($\pi^0 \rightarrow e^+e^-$)
3. η-η’ mixing
4. Time-like TFF prediction (charmonium backgrounds)

η-η' mixing

η-η' mixing in the flavor basis

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

From the TFFs we can determine f_q, f_s, ϕ

$$\Gamma_{\eta\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_\eta^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_\eta^q \frac{10}{3} + f_\eta^s \frac{2\sqrt{2}}{3} ,$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3} .$$

η-η' mixing

η-η' mixing in the flavor basis

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

From the TFFs we can determine f_q, f_s, ϕ

$$\Gamma_{\eta\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_\eta^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_\eta^q \frac{10}{3} + f_\eta^s \frac{2\sqrt{2}}{3} ,$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3} .$$

[R.Escribano, P.M., P. Sanchez-Puertas, '14]

$$f_q = 1.07(1) f_\pi , \quad f_s = 1.39(14) f_\pi , \quad \phi = 39.3(1.3)^\circ$$

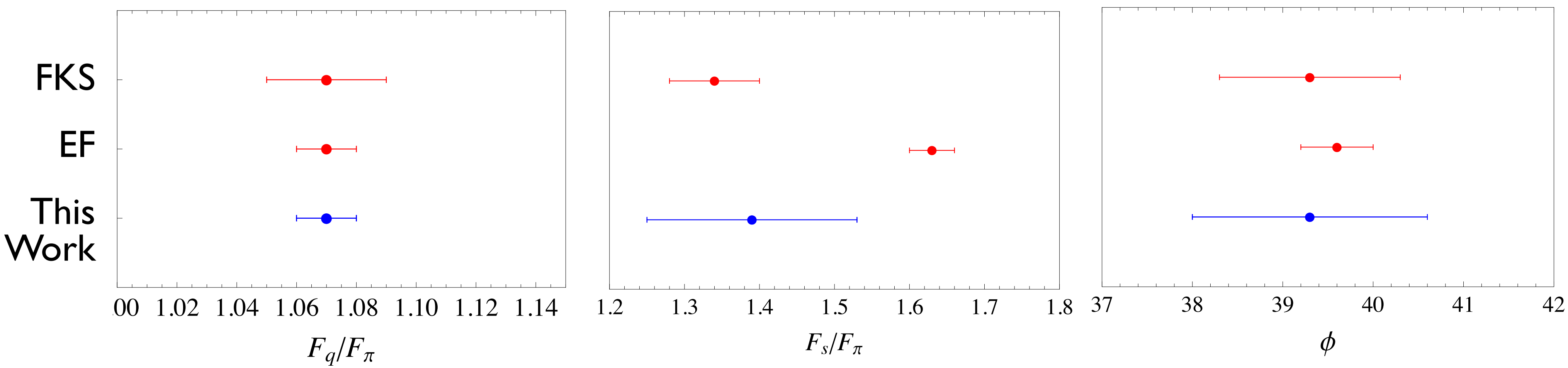
Update of Frere-Escribano '05 with PDG12 using 9 inputs

$$f_q = 1.07(1) f_\pi , \quad f_s = 1.63(2) f_\pi , \quad \phi = 40.4(0.3)^\circ$$

η-η' mixing

η-η' mixing in the flavor basis

From the TFFs we can determine F_q, F_s, ϕ



FKS: Feldmann, Kroll, Stech, PLB 449, 339, (1999)
EF: Escribano, Frere, JHEP 0506, 029 (2005) updated in Escribano, P.M, Sanchez-Puertas, 2013.

η-η' mixing

From the TFFs we can determine F_q, F_s, ϕ

and the VPγ and J/Ψ decays used in FKS and EF as inputs

(using $F_{\pi^0} = 131.5 \pm 1.4$ MeV instead of $F_{\pi^-} = 92.21 \pm 0.14$ MeV)

	Our predictions	Experimental determinations
$g_{\rho\eta\gamma}$	1.55(4)	1.58(5)
$g_{\rho\eta'\gamma}$	1.19(5)	1.32(3)
$g_{\omega\eta\gamma}$	0.56(2)	0.45(2)
$g_{\omega\eta'\gamma}$	0.54(2)	0.43(2)
$g_{\phi\eta\gamma}$	–0.83(11)	–0.69(1)
$g_{\phi\eta'\gamma}$	0.98(14)	0.72(1)
$\frac{J/\Psi\to\eta'\gamma}{J/\Psi\to\eta\gamma}$	4.74(60)	4.67(20)

Applications

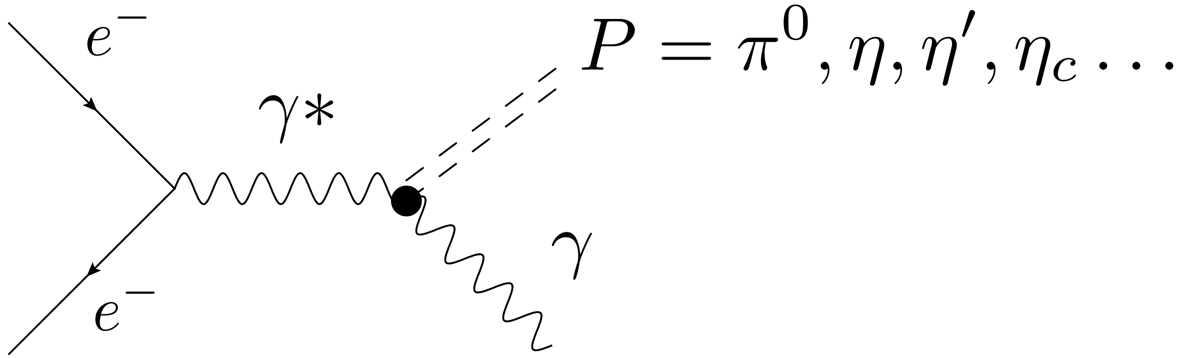
1. Hadronic Light-by-Light contribution to muon (g-2)

2. PS decays into lepton pairs ($\pi^0 \to e^+e^-$)

3. η-η' mixing

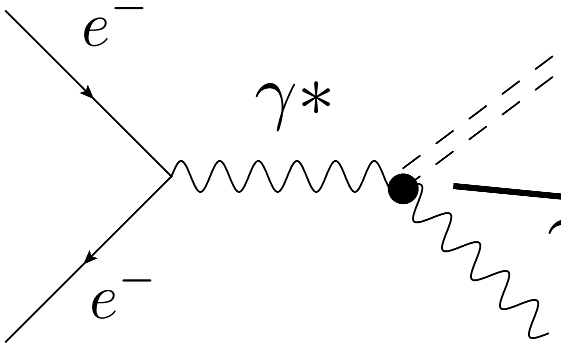
4. Time-like TFF prediction (charmonium backgrounds)

Time-like TFF: prediction



- Asymptotic limits in time-like and space-like FFs are expected to be close, is important to measure this time-like FF because:
  - the charmonium region is between the perturbative and non-perturbative regimes of the π-, η-, and η'-TFF
  - background for charmonium decays: charm quark mass determination

Time-like TFF: prediction



$P = \pi^0, \eta, \eta', \eta_c \ldots$

The vertex of interest is:

$$\Gamma_\mu = -ie^2 F_P(Q^2)\epsilon_{\mu\nu\rho\sigma}p^\nu \epsilon^\rho q^\sigma$$

Differential cross section:

$$\frac{d\sigma(e^+e^- \to \gamma^* \to \gamma P)}{d(\cos\theta)} = \frac{\pi^2\alpha^3}{4}\left(F_{P\gamma^*\gamma}(s,0)\right)^2\left(1-\frac{M_P^2}{s}\right)^3(1+\cos^2\theta)$$

Integrating with respect to cosθ

$$\sigma(e^+e^- \to \gamma^* \to \gamma P) = \frac{2\pi^2\alpha^3}{3}\left(F_{P\gamma^*\gamma}(s,0)\right)^2\left(1-\frac{M_P^2}{s}\right)^3$$

Conclusions

- Transition Form Factors are a good laboratory to study meson properties (one and two virtualities)
- Need for a model independent approach: we use Padé App.
- Padé Approximants' method is *easy*, *systematic* and can be *improved* upon by including new data
- Considering space- and time-like data
  - provides very accurate LECs and asymptotic limits
  - provides insight in mixing scheme and meson structure
  - predicts VPγ, J/Ψ, rare decays, continuum...
  - <u>beautiful synergy experiment - theory</u>

back-up

Dissection of the HLBL contribution

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

Use hadronic models constrained with chiral and large-Nc arguments

Use data from the Transition Form Factor for numerical integral

Dissection of the HLBL contribution

Use hadronic models constrained with chiral and large-Nc arguments

$$F(0) = \frac{1}{4\pi^2 f_\pi}, \qquad F(Q^2) \to \frac{6 f_\pi}{N_c Q^2} + \cdots$$

ABJ and BL

$$\boxed{F(Q^2) = \frac{1}{4\pi^2 f_\pi} \frac{m_\rho^2}{m_\rho^2 + Q^2}}$$

$$f_\pi = ? \quad m_\rho = ?$$

Dissection of the HLBL contribution

Use hadronic models constrained with chiral and large-Nc arguments

$$F(0) = \frac{1}{4\pi^2 f_\pi}, \qquad F(Q^2) \to \frac{6 f_\pi}{N_c Q^2} + \cdots$$ ABJ and BL

$$F(Q^2) = \frac{1}{4\pi^2 f_\pi} \frac{m_\rho^2}{m_\rho^2 + Q^2}$$ $f_\pi = ? \quad m_\rho = ?$

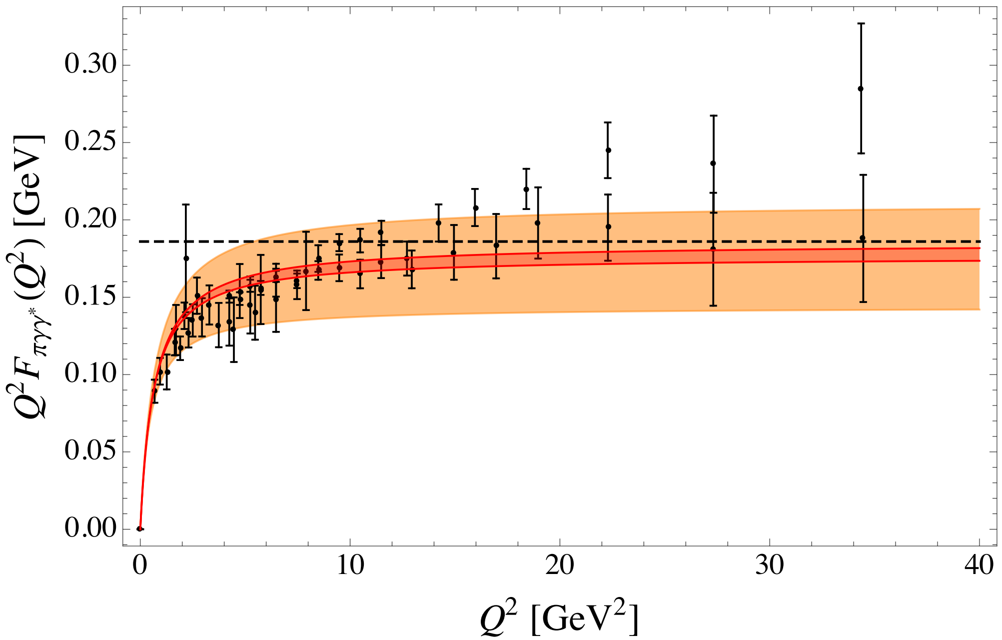
$f_\pi = 92.21(14)$ MeV from PDG

$f_\pi = f_0 = 88.1(4.1)$ MeV from lattice [Ecker et al '14]

$f_\pi = 93(1)$ MeV from $\Gamma_{\pi^0\gamma\gamma}$

$m_\rho^2 = \frac{24\pi^2 f_\pi^2}{N_c}$, imposing BL

$m_\rho \sim 780 - 820$ MeV



Large-Nc and the *half-width* rule

- Mass shift and Width are of the same order in 1/Nc

- Example: 2 point GF $D(s) = \dfrac{1}{s - m_0^2 - \Sigma(s)}$

- The resonance pole $s = s_R = m_R^2 - i m_R \Gamma_R$ is give by:

$$s_R - m_0^2 - \Sigma(s_R) = 0$$

- In large-Nc (perturbative) expansion:

$$s_R = m_0^2 + \mathcal{O}(N_c^{-1})$$

$$s_R = m_0^2 + 2m_0 \Delta m_R - i\Gamma_R m_0 + \mathcal{O}(N_c^{-2})$$

$$\Delta m_R = \frac{1}{2m_0}\mathrm{Re}\Sigma(m_0^2),$$

$$\Gamma_R = -\frac{1}{m_0}\mathrm{Im}\Sigma(m_0^2)$$

Dissection of the HLBL contribution

Use hadronic models constrained with chiral and large-Nc arguments

$$F(0) = \frac{1}{4\pi^2 f_\pi}, \qquad F(Q^2) \to \frac{6 f_\pi}{N_c Q^2} + \cdots$$ ABJ and BL

$$F(Q^2) = \frac{1}{4\pi^2 f_\pi} \frac{m_\rho^2}{m_\rho^2 + Q^2}$$ $f_\pi = ? \quad m_\rho = ?$

$f_\pi = 92.21(14)$ MeV from PDG

$f_\pi = f_0 = 88.1(4.1)$ MeV from lattice [Ecker et al '14]

$f_\pi = 93(1)$ MeV from $\Gamma_{\pi^0\gamma\gamma}$

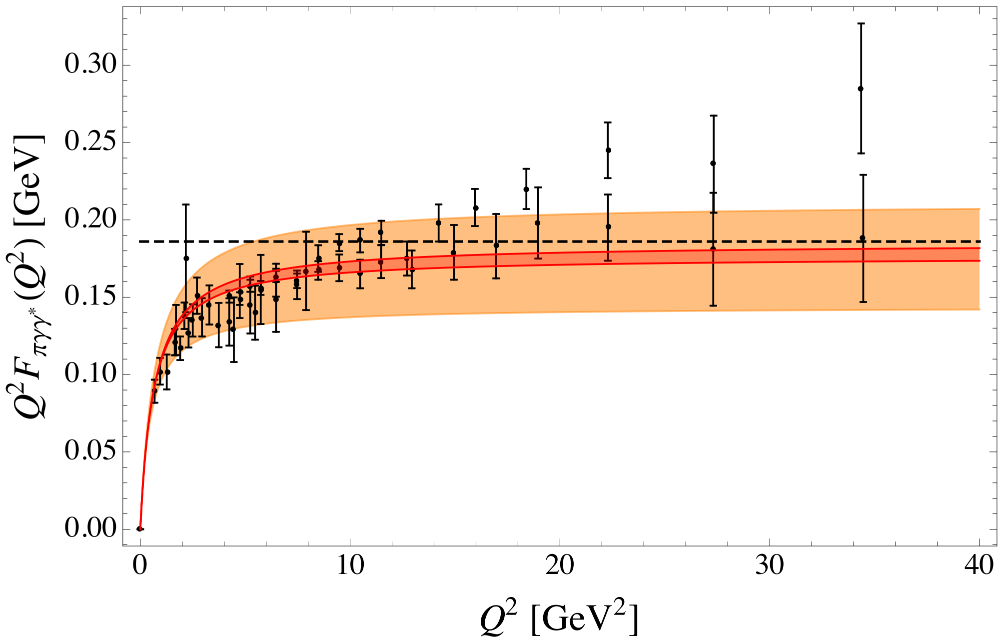
$$m_\rho^2 = \frac{24\pi^2 f_\pi^2}{N_c},$$ imposing BL

$$m_\rho \sim 780 - 820 \text{ MeV}$$

$$m_\rho = (775 \pm \Delta m_{\rho, N_c \to \infty}) \text{ MeV}$$

$$m_\rho = (775 \pm \Gamma/2) \text{ MeV}$$

half-width rule [P.M., Ruiz-Arriola, Broniowski,'13]



Dissection of the HLBL contribution

Use hadronic models constrained with chiral and large-Nc arguments

$$F(0) = \frac{1}{4\pi^2 f_\pi}, \qquad F(Q^2) \to \frac{6 f_\pi}{N_c Q^2} + \cdots$$ ABJ and BL

$$F(Q^2) = \frac{1}{4\pi^2 f_\pi} \frac{m_\rho^2 m_{\rho'}^2 + 24 f_\pi^2 \pi^2 Q^2/N_c}{(m_\rho^2 + Q^2)(m_{\rho'}^2 + Q^2)}$$

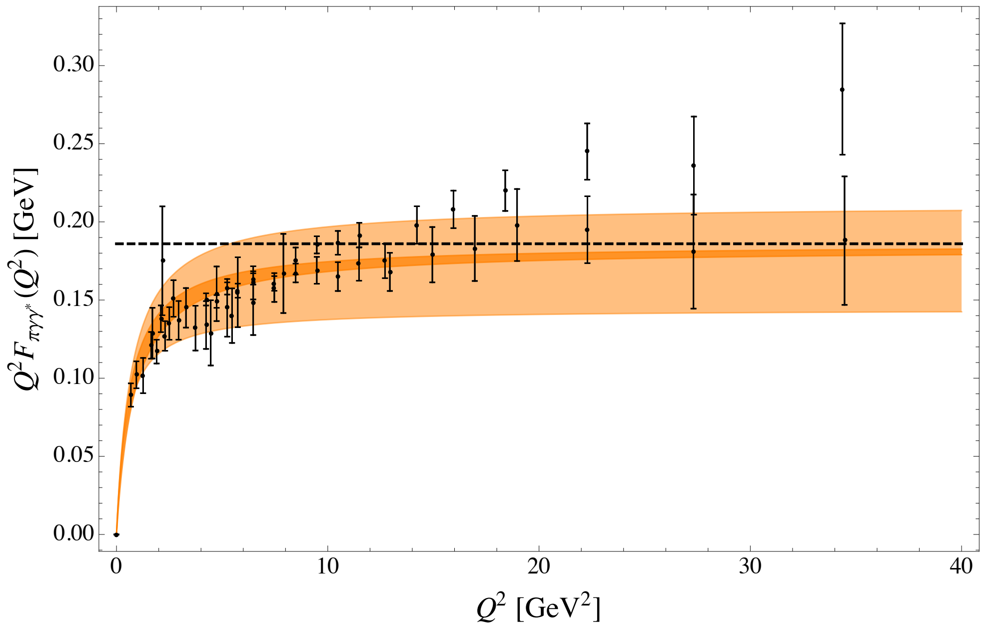
$f_\pi = 92.21(14)$ MeV from PDG

$f_\pi = f_0 = 88.1(4.1)$ MeV from lattice [Ecker et al '14]

$f_\pi = 93(1)$ MeV from $\Gamma_{\pi^0\gamma\gamma}$

$$m_\rho = (775 \pm \Delta m_{\rho, N_c\to\infty})\ \text{MeV}$$
$$m_\rho = (775 \pm \Gamma/2)\ \text{MeV}$$
$$m_{\rho'} = (1465 \pm 400/2)\ \text{MeV}$$

half-width rule [P.M., Ruiz-Arriola, Broniowski,'13]



Naive New Physics contributions

$$\frac{\mathrm{BR}(\pi^0 \to e^+e^-)}{\mathrm{BR}(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 \beta_e \left| \mathcal{A}(q^2) + \frac{\sqrt{2} F_\pi G_F}{4\alpha^2 F_{\pi\gamma\gamma}} \left(\frac{4 m_W}{m_{A(P)}}\right)^2 \times f^{A(P)} \right|^2$$

$$f^A = c_e^A (c_u^A - c_d^A) \qquad f^P = \frac{1}{4} c_e^P (c_u^P - c_d^P) \frac{m_\pi^2}{m_\pi^2 - m_P^2} \qquad c \sim \mathcal{O}\left(\frac{g}{g_{SU(2)_L}}\right)$$

$$\frac{\mathrm{BR}(\pi^0 \to e^+e^-)}{\mathrm{BR}(\pi^0 \to \gamma\gamma)} = \mathrm{SM}\,(1 + \epsilon_{Z,NP} \times 5\%)$$

Z contribution (Arnellos, Marciano, Parsa '82) $\epsilon_Z \sim 0.3\%$

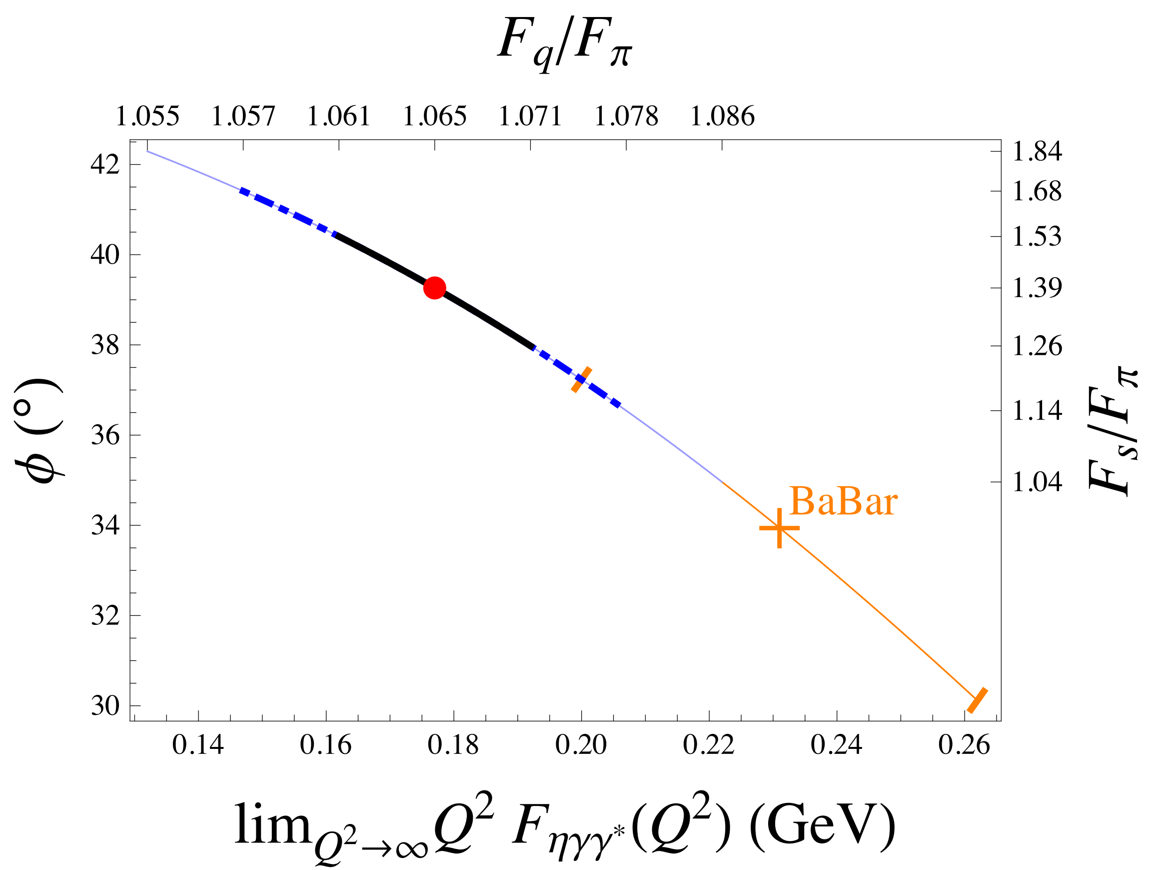
Our estimate based on existing exp. constrains: $\epsilon_{NP} \sim 0.3\%$

negligible!

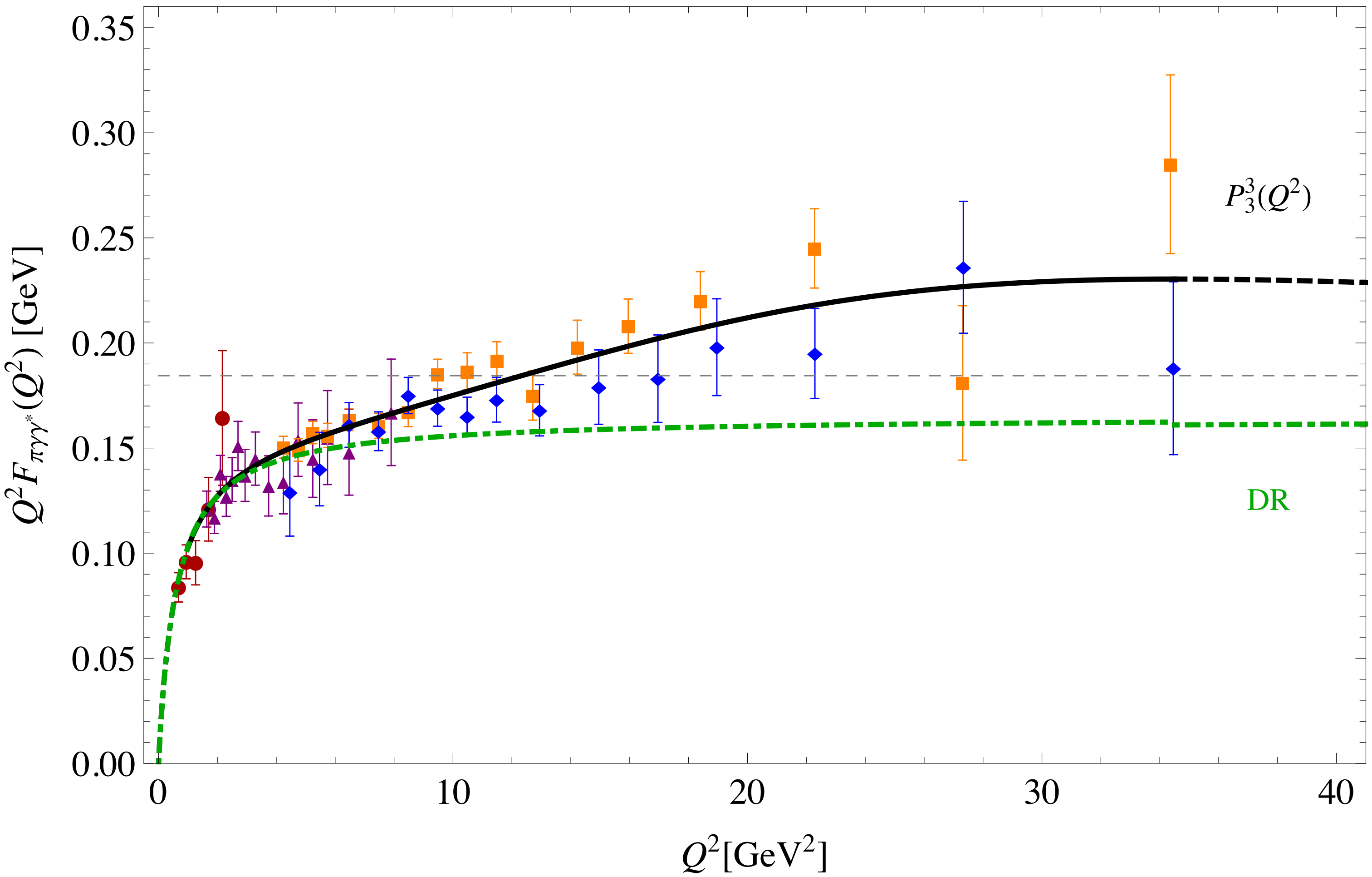
η-η' mixing

η-η' mixing in the flavor basis

From the TFFs we can determine F_q, F_s, ϕ

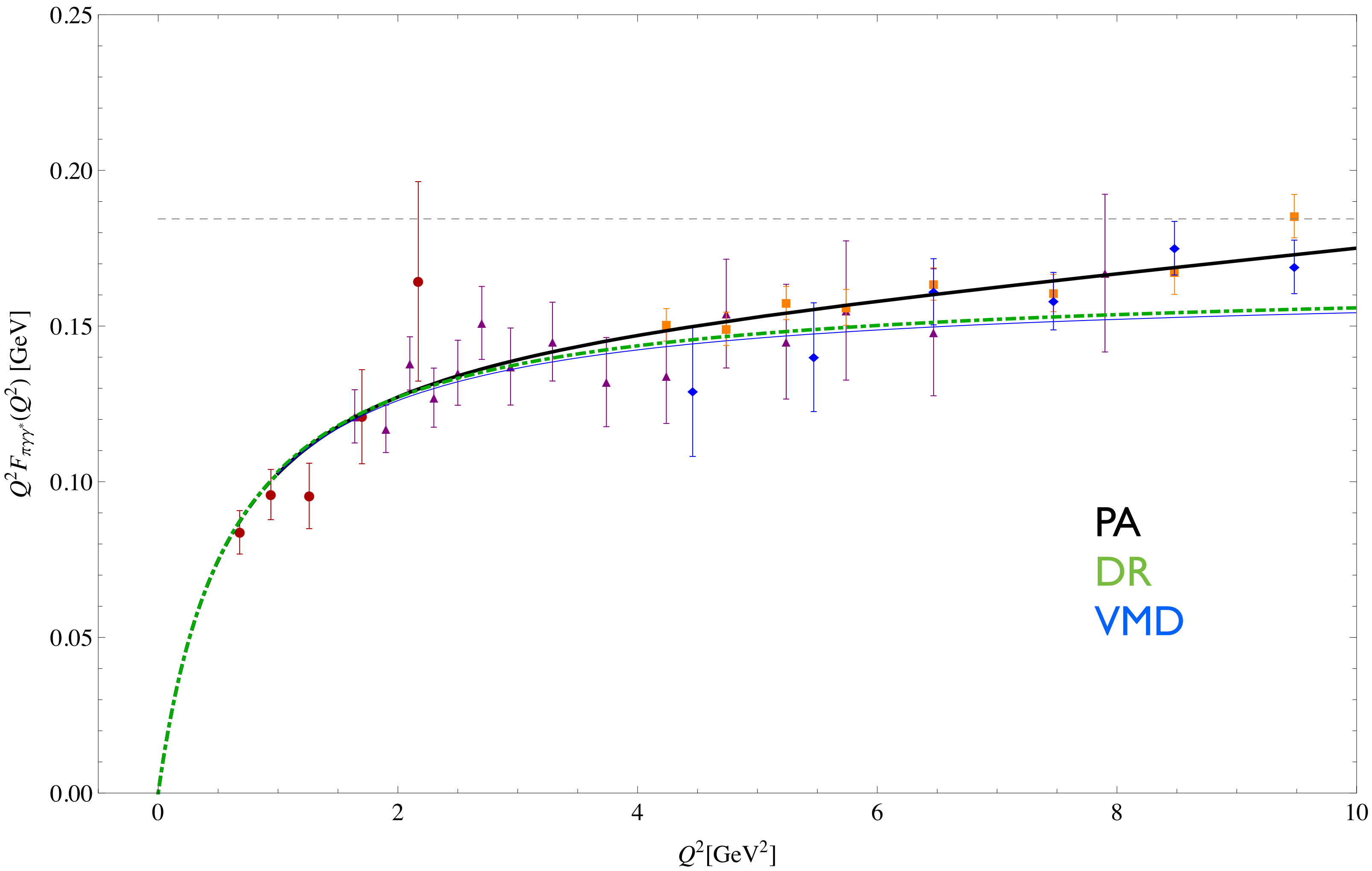


PA vs DR



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$Q^2 F_{\pi\gamma\gamma^*}(Q^2)$ [GeV]
0
10
20
30
40
Q^2[GeV^2]
$P^3_3(Q^2)$
DR

PA vs DR



0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10
$Q^2 F_{\pi\gamma\gamma^*}(Q^2)$ [GeV]
Q^2[GeV^2]
PA
DR
VMD