The electron g-2: recent developments

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Outline

- The SM prediction of the electron g-2
- Testing the SM with the electron g-2
- Positronium contribution to the electron g-2

1. The SM prediction of the electron g-2

The QED prediction of the electron g-2

$$a_e^{QED} = + (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,55(33)\,(\alpha/\pi)^2$$

Schwinger 1948   Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\ldots$$

$O(10^{-18})$ in a_e

$$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,68\,(26) \times 10^{-7}$$

$$A_2^{(4)}(m_e/m_\tau) = 1.837\,98\,(33) \times 10^{-9}$$

$$+\ 1.181\,234\,016\,816\,(11)\,(\alpha/\pi)^3$$

$O(10^{-19})$ in a_e

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587\ldots$$

$$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,62\,(27) \times 10^{-6}$$

$$A_2^{(6)}(m_e/m_\tau) = -6.5830\,(11) \times 10^{-8}$$

$$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,82\,(34) \times 10^{-13}$$

$$-\ 1.9097\,(20)\,(\alpha/\pi)^4$$

$0.6\ 10^{-13}$ in a_e

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012;
Kurz, Liu, Marquard & Steinhauser 2014: analytic mass dependent part.

$$+\ 9.16\,(58)\,(\alpha/\pi)^5$$

Complete Result! (12672 mass indep. diagrams!)

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807; work in progress to reduce the error.

$0.4\ 10^{-13}$ in a_e

NB: $(\alpha/\pi)^6 \sim O(10^{-16})$

The SM prediction of the electron g-2

- The SM prediction is:

$$a_e^{SM}(\alpha) = a_e^{QED}(\alpha) + a_e^{EW} + a_e^{HAD}$$

- The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [value from Codata10]

$$a_e^{EW} = 0.2973\,(52) \times 10^{-13}$$

- The Hadronic contribution (LO+NLO+NNLO) is:

Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{HAD} = 17.10\,(17) \times 10^{-13}$$

$$a_e^{HLO} = +\,18.66\,(11) \times 10^{-13}$$

$$a_e^{HNLO} = [-2.234(14)_{vac} + 0.39(13)_{lbl}] \times 10^{-13}$$

$$a_e^{HNNLO} = +\,0.28\,(1) \times 10^{-13}$$

- Which value of α should we use to compute a_e^{SM}?

The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $a_e^{SM}(\alpha) = a_e^{EXP}$ → best determination of alpha:

$$\alpha^{-1} = 137.035\,999\,177\,(34) \qquad [0.25\ \text{ppb}]$$

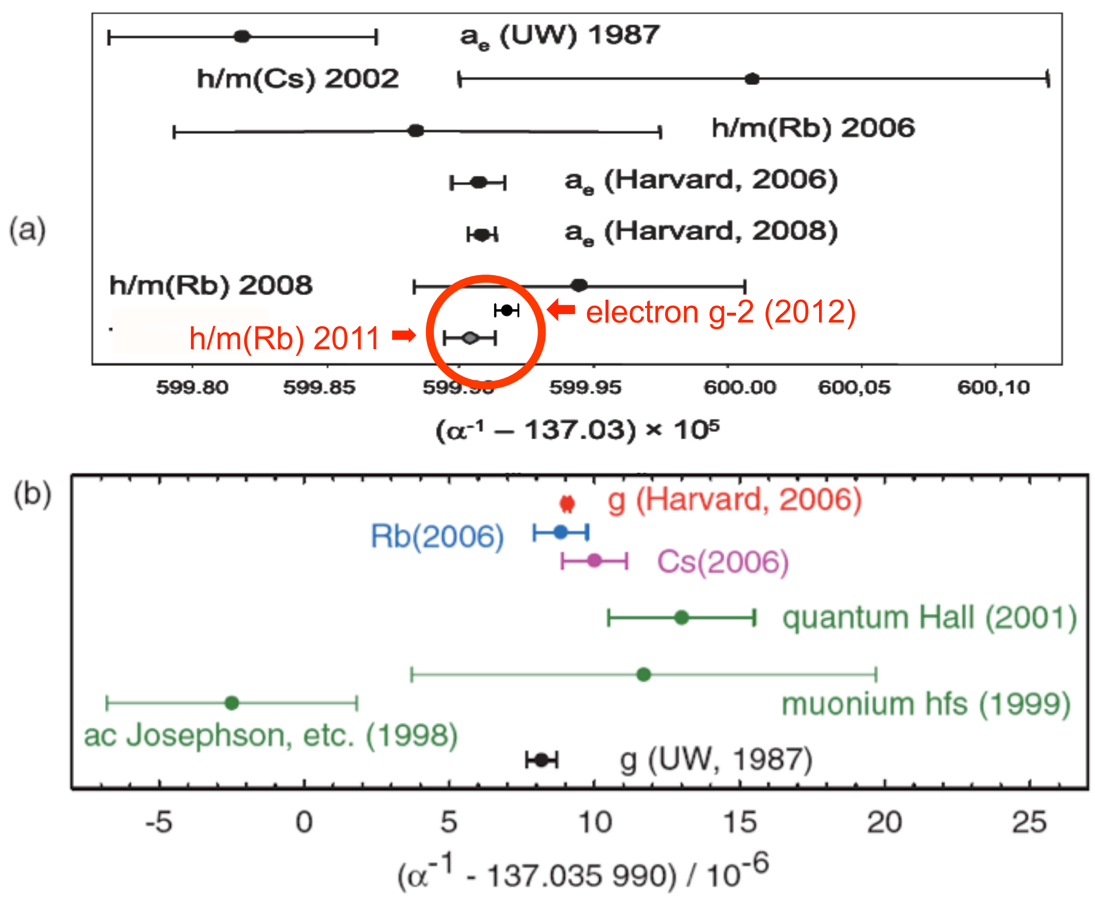
- Compare it with other determinations (independent of a_e):

$\alpha^{-1} = 137.036\,000\,0\,(11)$ [7.7 ppb] PRA73 (2006) 032504 (Cs)

$\alpha^{-1} = 137.035\,999\,049\,(90)$ [0.66 ppb] PRL106 (2011) 080801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801
Bouchendira et al, PRL106 (2011) 080801

2. Testing the SM with the electron g-2

G.F. Giudice, P. Paradisi & MP, arXiv:1208.6583 (JHEP 2012)

The electron g-2: SM vs Experiment

- Using $\alpha = 1/137.035\,999\,049\ (90)$ [^{87}Rb, 2011], the SM prediction for the electron g-2 is

$$a_e^{SM} = 115\,965\,218\,18.1\ (0.6)\ (0.4)\ (0.2)\ (7.6) \times 10^{-13}$$

δC_4^{qed} δC_5^{qed} δa_e^{had} from $\delta\alpha$

- The EXP-SM difference is (note the negative sign):

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -10.8\ (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1.3σ).
NB: The 4-loop contrib. to a_e^{QED} is $-556 \times 10^{-13} \sim 70\ \delta\Delta a_e$!
(the 5-loop one is 6.2×10^{-13})

The electron g-2 sensitivity and NP tests

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.6)_{\mathrm{QED4}}, \quad (0.4)_{\mathrm{QED5}}, \quad (0.2)_{\mathrm{HAD}},}_{(0.7)_{\mathrm{TH}}} \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\mathrm{EXP}}}$$

⬅ may drop to 0.2 or 0.3

- The $(g\text{-}2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.

→ sensitivity of 10^{-13} may be reached with ongoing exp. work

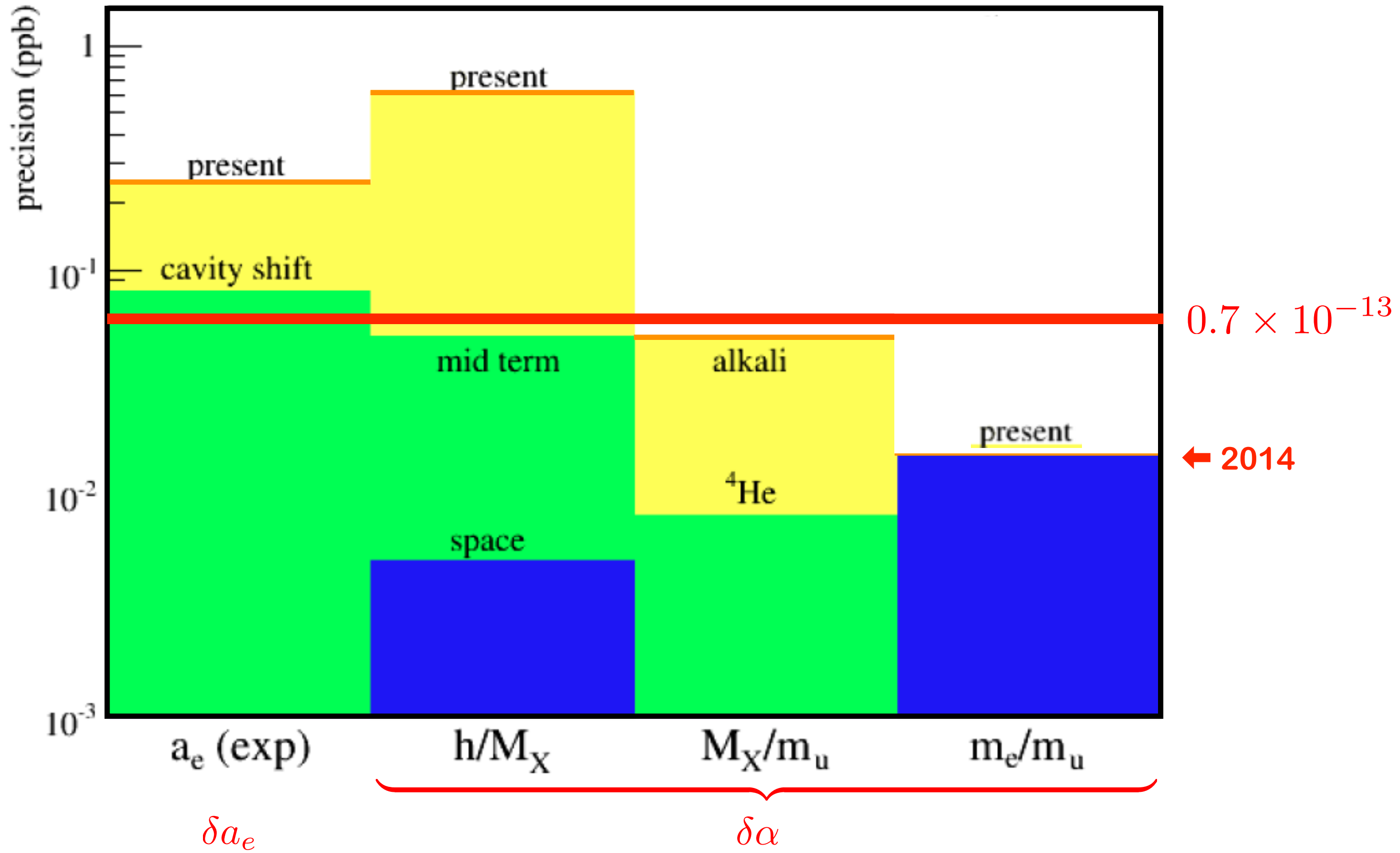
- In a broad class of BSM theories, contributions to a_l scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

The electron g-2 sensitivity and NP tests (2)



Summary of the exp. contributions to the relative uncertainty of Δa_e (in ppb)

F. Terranova & G.M. Tino, PRA89 (2014) 052118

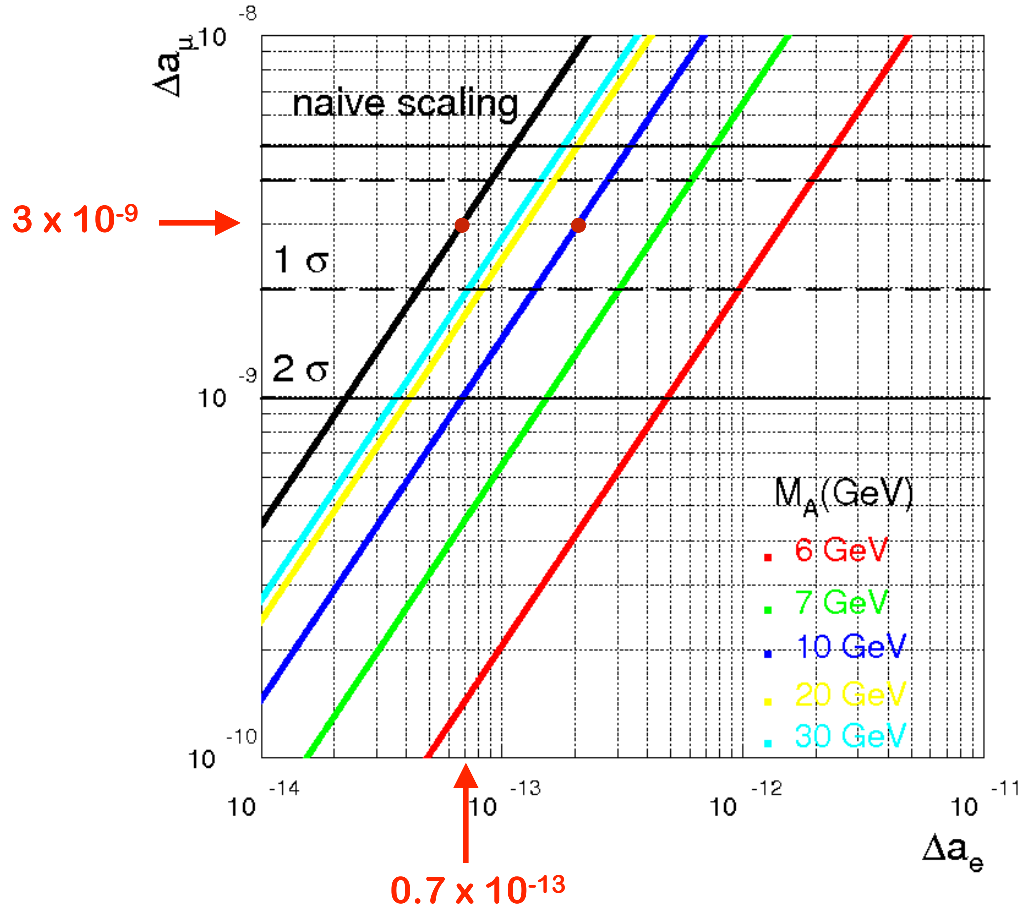
The electron g-2 sensitivity and NP tests (3)

- The experimental sensitivity in Δa_e is not very far from what is needed to test if the discrepancy in $(g\text{-}2)_\mu$ also manifests itself in $(g\text{-}2)_e$ under the naive scaling hypothesis.

- NP scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e and contributions to EDMs, LFV or lepton universality breaking observables.

- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach 10^{-12} (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

The electron g-2 sensitivity and NP tests (4)



- **Example: light pseudoscalars. Interplay between 1-loop and 2-loop contributions. NS systematically violated, Δa_e always lies above its naive expectation.**

The electron g-2 sensitivity and NP tests (5)

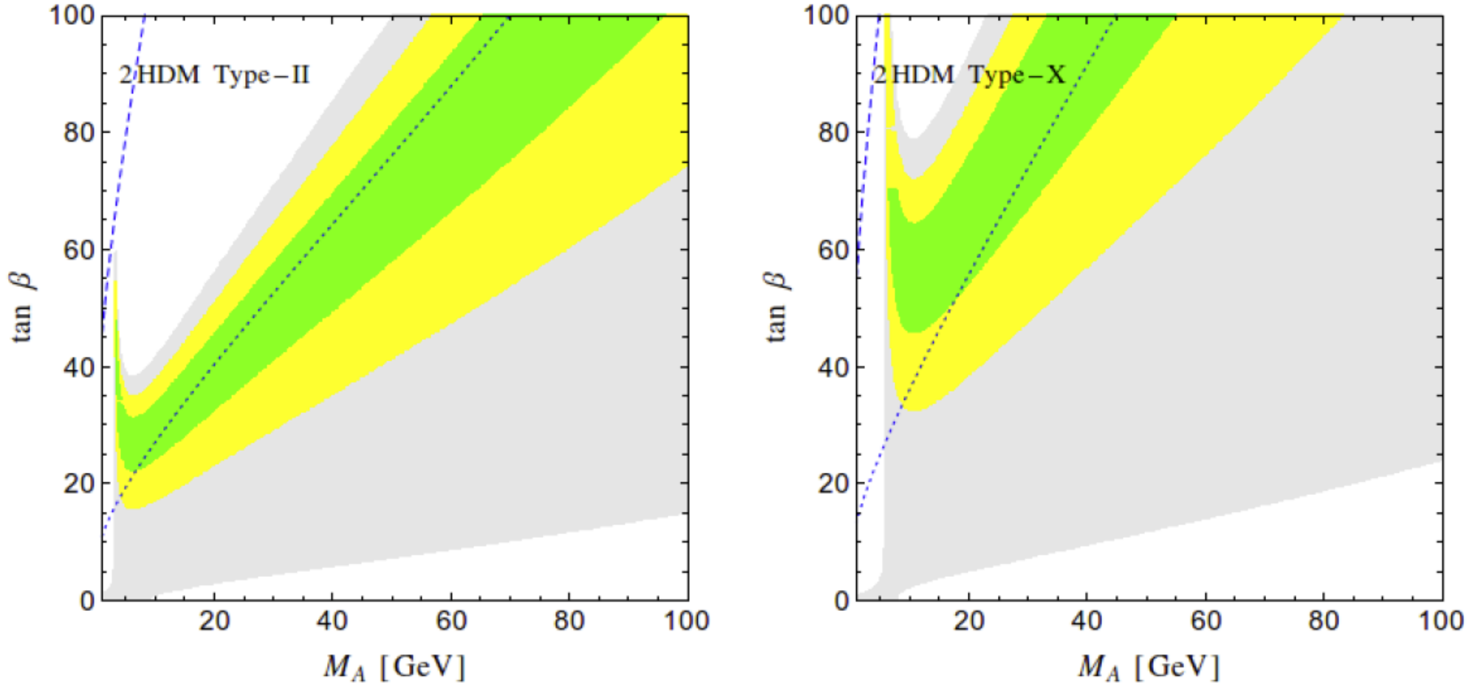


FIG. 4. The 1σ, 2σ and 3σ regions allowed by Δa_μ in the M_A-$\tan\beta$ plane taking the limit of $\beta - \alpha = \pi/2$ and $M_{h(H)} = 125$ (200) GeV in type II (left panel) and type X (right panel) 2HDMs. The regions below the dashed (dotted) lines are allowed at 3σ (1.4σ) by Δa_e.

- **Example: 2HDMs** **Broggio, Chun, MP, Patel, Vempati, arXiv:1409.3199 (JHEP 2014)**

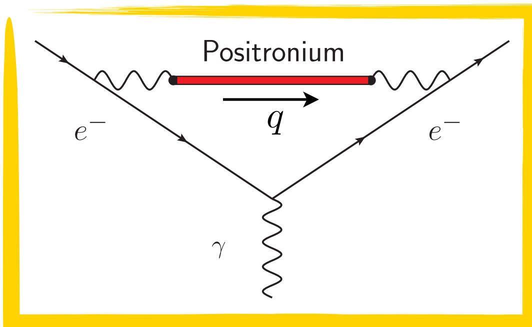
3. Positronium contribution to the electron g-2

M. Fael & MP, arXiv:1402.1575 (PRD 2014)

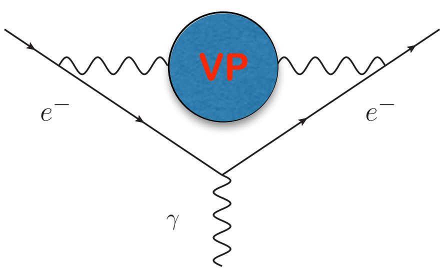
Positronium contribution to the electron g-2

- **The leading contribution of positronium to a_e comes from:**

Mishima 1311.7109; Fael & MP 1402.1575; Melnikov et al. 1402.5690; Eides 1402.5860; Hayakawa 1403.0416



- **The e^+e^- bound states appear as poles in the vac. pol. $\Pi(q^2)$ right below the branch-point $q^2 = (2m)^2$. Their contribution is:**



$$a_e(\mathrm{vp}) = \frac{\alpha}{\pi^2} \int_0^\infty \frac{ds}{s} \, \mathrm{Im}\,\Pi(s + i\epsilon)\, K(s)$$

$$a_e^{\mathrm{P}} = \frac{\alpha^5}{4\pi}\zeta(3) \underbrace{\left(8 \ln 2 - \frac{11}{2}\right)}_{K(4m^2)} = 0.9 \times 10^{-13} = 1.3 \left(\frac{\alpha}{\pi}\right)^5$$

Mishima 1311.7109

- **Of the same magnitude of the exp. unc. of a_e & the "naively rescaled" muon Δa_μ. Of the same order of α as the 5-loop term!**

Positronium contribution to the electron g-2 (II)

- Melnikov, Vainshtein & Voloshin (MVV) 1402.5690 determined a nonpert. contrib. of the e^+e^- continuum right above threshold that cancels one-half of a_e^P:

$$a_e(\mathrm{vp})^{\mathrm{cont,np}} = -\frac{|\alpha|^5}{8\pi}\zeta(3)\left(8\ln 2 - \frac{11}{2}\right)$$

- In fact the total positronium poles + continuum nonperturbative contribution to a_e arising from the threshold region at LO in α is:

$$a_e^{\mathrm{thr}}(\mathrm{vp}) = -\frac{\alpha}{\pi}K(4m^2)\,\mathrm{Re}\,A(1)$$

with

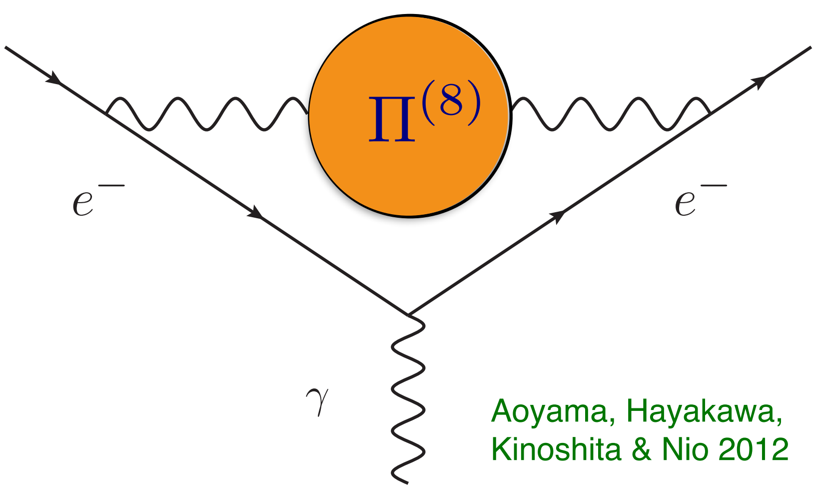
$$A(\beta) = -\frac{\alpha^2}{2}\left[\gamma + \psi\left(1 - \frac{i\alpha}{2\beta}\right)\right] = \frac{\alpha^2}{2}\sum_{k=1}^{\infty}\zeta(k+1)\left(\frac{i\alpha}{2\beta}\right)^k$$

so that

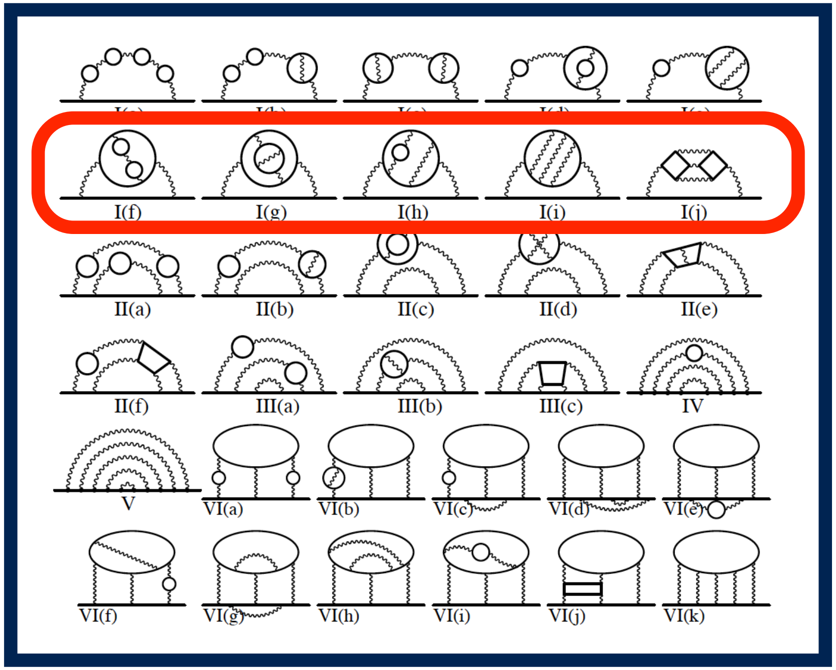
$$a_e^{\mathrm{thr}}(\mathrm{vp}) = \frac{\alpha^5}{8\pi}\zeta(3)\,K(4m^2) = \frac{a_e^{\mathrm{P}}}{2}$$

Positronium contribution to the electron g-2 (III)

- So, should we add this total threshold contribution $a_e^P/2$ to the perturbative QED 5-loop result of Kinoshita and collaborators?
- Using the Coulomb Green's function, MVV 1402.5690 argued that it is already contained in the contribution of $O(\alpha^5)$.
- Hayakawa 1403.0416 claimed that positronium contributes to a_e only through a specific class of diagrams of $O(\alpha^7)$.
- To address this question: study the 5-loop QED contribution to a_e arising from the insertion of the 4-loop VP in the photon line. This has been computed via:



Aoyama, Hayakawa, Kinoshita & Nio 2012



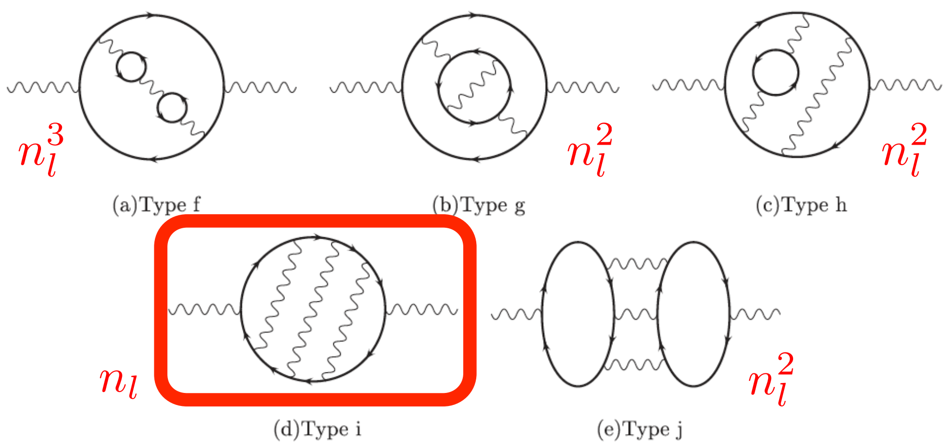
Positronium contribution to the electron g-2 (IV)

- Using explicit expressions for $\Pi^{(8)}(q^2)$ (Baikov, Maier, Marquard '13) in

$$a_e^{(10)}(\text{vp}) = -\frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Pi^{(8)}\Big(-\frac{m^2x^2}{1-x}\Big)$$

we obtain:

$$a_e^{(10)}(\text{vp}) \;=\; n_e\,\frac{\alpha^5}{8\pi}\zeta(3)\,K(4m^2) + \cdots \;=\; \frac{a_e^{\text{P}}}{2} + \cdots$$



- $a_e^P/2$ is already included in the 5-loop contrib. of class I(i).
- **There is no additional contrib of QED bound states beyond PT!**

M.A. Braun 1968; Barbieri, Christillin, Remiddi 1973

Conclusions

- The present sensitivity to NP effects in a_e is 8 x 10^{-13}. It is limited by the experimental uncertainties (7.6 from α, 2.8 from a_e), but a very strong exp program is under way to improve both α & a_e.

- An improvement by roughly a factor of 10 would allow to test NP with a_e. In particular, whether in NS the muon g-2 discrepancy manifests itself also in the electron g-2!

- Many NP scenarios violate Naive Scaling. They can lead to larger effects in Δa_e, well above its naive expectation.

- The positronium contribution to a_e should not be added to that of perturbative QED. There is no additional contrib. of QED bound states beyond perturbation theory.

The End