Present accuracy and future prospects of MC generators for Bhabha and $e^+e^- \to \gamma\gamma$

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with Pavia Theory Group
Montagna, Nicrosini, Piccinini
(and Balossini, Barzè, Bignamini)

Outline

- ⋆ QED scattering processes & radiative corrections for luminometry
- ⋆ Theoretical framework of the event generator **`BabaYaga`**
- ⋆ Independent calculations and tuned comparisons
- ⋆ Theoretical accuracy: comparison to exact NNLO calculations
- ⋆ Conclusions and outlook

Main references:

- Balossini *et al.*, Nucl. Phys. **B758** (2006) 227 & Phys. Lett. **663** (2008) 209
- Actis *et al.* [WG on RC and MCG for Low Energies Collaboration], Eur. Phys. J. C **66** (2010) 585
- Carloni, Czyz, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann *et al.* JHEP **1107** (2011) 126

Typical theory of the MC generators

- ⋆ The most precise MC generators include exact $\mathcal{O}(\alpha)$ (NLO) photonic corrections matched with higher–order leading logarithmic contributions [+ vacuum polarisation, using a data based routine for the calculation of the non–perturbative $\Delta\alpha^{(5)}_{\rm had}(q^2)$ contribution]
- ⋆ The methods used to account for multiple photon corrections are the analytical collinear QED Structure Functions (SF), YFS exponentiation and QED Parton Shower (PS)

- The QED PS [implemented in the generators BabaYaga/BabaYaga@NLO] is a MC solution of the QED DGLAP equation for the electron SF $D(x, Q^2)$

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D(\frac{x}{t}, Q^2)$$

- The PS solution can be cast into the form

$$D(x, Q^2) \;=\; \Pi(Q^2) \sum_{n=0}^{\infty} \int \frac{\delta(x - x_1 \cdots x_n)}{n!} \prod_{i=0}^{n} \left[\frac{\alpha}{2\pi} P(x_i)\; L\; dx_i\right]$$

  - ⋆ $\Pi(Q^2) \equiv e^{-\frac{\alpha}{2\pi} L I_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon} P(x) dx$ $L \equiv \ln Q^2/m^2$ collinear log, ϵ soft–hard separator and Q^2 virtuality scale

- The accuracy is improved by matching exact NLO with higher-order leading log corrections
  - ⋆ theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections, for all QED channels [Bhabha, $\gamma\gamma$ and $\mu^+\mu^-$]

Summary of QED (photonic) radiative corrections

The corrections to the LO cross section can be arranged as (L collinear log)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Blue: LL PS, LL YFS, SF

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NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Red: matched PS, YFS, SF + NLO

Matching NLO and PS in BabaYaga

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma_{LL}^{\infty} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$
- $d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}]\, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{LL}^{SV}(\varepsilon) + d\sigma_{LL}^{H}(\varepsilon)$
- $d\sigma_{\mathrm{NLO}}^{\alpha} = [1 + C_{\alpha}]\, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1 \equiv d\sigma_{\mathrm{NLO}}^{SV}(\varepsilon) + d\sigma_{\mathrm{NLO}}^{H}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,LL}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$

$$d\sigma_{\mathrm{matched}}^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for $n+2$ final-state particles

Matching NLO and PS in **BabaYaga**

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of LL
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\rm NLO}$
- resummation of higher orders LL contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV\ |\ H,i} \times LL$

G. Montagna et al., **PLB** 385 (1996)

- the th. error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \sim 5\times 10^{-4}$$

Results with **BabaYaga@NLO** (Bhabha)

- as examples to show the features of the EG, the following setups and definitions are used (for Bhabha)

  a. $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.408$ GeV, $20^\circ < \theta_\pm < 160^\circ$, $\xi_{max} = 10^\circ$
  b. $\sqrt{s} = 1.02$ GeV, $E_{min} = 0.408$ GeV, $55^\circ < \theta_\pm < 125^\circ$, $\xi_{max} = 10^\circ$
  c. $\sqrt{s} = 10$ GeV, $E_{min} = 4$ GeV, $20^\circ < \theta_\pm < 160^\circ$, $\xi_{max} = 10^\circ$
  d. $\sqrt{s} = 10$ GeV, $E_{min} = 4$ GeV, $55^\circ < \theta_\pm < 125^\circ$, $\xi_{max} = 10^\circ$

$$\delta_{VP} \equiv \frac{\sigma_{0,VP} - \sigma_0}{\sigma_0} \qquad \delta_\alpha \equiv \frac{\sigma_\alpha^{NLO} - \sigma_0}{\sigma_0}$$

$$\delta_{HO} \equiv \frac{\sigma_{matched}^{PS} - \sigma_\alpha^{NLO}}{\sigma_0} \qquad \delta_{HO}^{PS} \equiv \frac{\sigma^{PS} - \sigma_\alpha^{PS}}{\sigma_0}$$

$$\delta_\alpha^{non\text{-}log} \equiv \frac{\sigma_\alpha^{NLO} - \sigma_\alpha^{PS}}{\sigma_0} \qquad \delta_\infty^{non\text{-}log} \equiv \frac{\sigma_{matched}^{PS} - \sigma^{PS}}{\sigma_0}$$

Results with **BabaYaga@NLO** (Bhabha)

set up	(a)	(b)	(c)	(d)
δ_{VP}	1.76	2.49	4.81	6.41
δ_{α}	-11.61	-14.72	-16.03	-19.57
δ_{HO}	0.39	0.82	0.73	1.44
δ_{HO}^{PS}	0.35	0.74	0.68	1.34
$\delta_{\alpha}^{non\text{-}log}$	-0.34	-0.56	-0.34	-0.56
$\delta_{\infty}^{non\text{-}log}$	-0.30	-0.49	-0.29	-0.46

Table: Relative corrections (in per cent) to the Bhabha cross section for the four setups

- ⋆ in short, the fact that $\delta_{\alpha}^{non\text{-}log} \simeq \delta_{\infty}^{non\text{-}log}$ and $\delta_{HO} \simeq \delta_{HO}^{PS}$ means that the matching algorithm preserves both the advantages of exact NLO calculation and PS approach:
  - $\rightarrow$ it includes the missing NLO RC to the PS
  - $\rightarrow$ it adds the missing higher-order RC to the NLO

Results with **BabaYaga@NLO** for $\gamma\gamma$ final state

- $\gamma\gamma$ final state has a lower x-section, but, at least up to NNLO, it does not depend on VP, which is a source of th. error
- Similar setups and definitions were used to study $\gamma\gamma$ FS

$$\begin{cases} \sqrt{s} = 1. \; - \; 3. \; - \; 10.\mathrm{GeV} \\ E_\gamma^{\min} = 0.3 \; \times \sqrt{s} \\ \vartheta_\gamma^{\min} = 45^\circ, \quad \vartheta_\gamma^{\max} = 135^\circ \\ \xi_{\max} = 10^\circ \end{cases}$$

$$\delta_\alpha = 100 \times \frac{\sigma_\alpha^{\rm NLO} - \sigma}{\sigma} \qquad \delta_\infty = 100 \times \frac{\sigma_{\rm exp} - \sigma}{\sigma}$$

$$\delta_{\rm exp} = 100 \times \frac{\sigma_{\rm exp} - \sigma_\alpha^{\rm NLO}}{\sigma_\alpha^{\rm NLO}} \qquad \delta_\alpha^{\rm NLL} = 100 \times \frac{\sigma_\alpha^{\rm NLO} - \sigma_\alpha^{\rm PS}}{\sigma_\alpha^{\rm PS}}$$

$$\delta_\infty^{\rm NLL} = 100 \times \frac{\sigma_{\rm exp} - \sigma_{\rm exp}^{\rm PS}}{\sigma_{\rm exp}^{\rm PS}}$$

Results with **BabaYaga@NLO** for $\gamma\gamma$

$\sqrt{s}$ (GeV)	1	3	10
σ	137.53	15.281	1.3753
$\sigma_{\alpha}^{\mathrm{PS}}$	128.55	14.111	1.2529
$\sigma_{\alpha}^{\mathrm{NLO}}$	129.45	14.211	1.2620
$\sigma_{\mathrm{exp}}^{\mathrm{PS}}$	128.92	14.169	1.2597
σ_{exp}	129.77	14.263	1.2685
δ_{α}	-5.87	-7.00	-8.24
δ_{∞}	-5.65	-6.66	-7.77
δ_{exp}	0.24	0.37	0.51
$\delta_{\alpha}^{\mathrm{NLL}}$	0.70	0.71	0.73
$\delta_{\infty}^{\mathrm{NLL}}$	0.66	0.66	0.69

Table: Photon pair production cross sections (in nb) to different accuracy levels and relative corrections (in per cent)

Estimating the theoretical accuracy

- It is of utmost importance to compare with independent calculations/implementations, in order to
  - ⋆ asses the technical precision, spot bugs (with the same th. ingredients)
  - ⋆ estimate the theoretical "error" when including partial/incomplete higher-order corrections
- Generators exist on the market, some of them including QED h.o. and NLO corrections according to different approaches (collinear SF + NLO, YFS exponentiation,...)

Generator	Processes	Theory	Accuracy	Web address
BHAGENF/BKQED	$e^+e^-/\gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$	1%	www.lnf.infn.it/~graziano/bhagenf/bhabha.html
BabaYaga v3.5	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	Parton Shower	$\sim\ 0.5\%$	www.pv.infn.it/~hepcomplex/babayaga.html
BabaYaga@NLO	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + PS	$\sim 0.1\%$	www.pv.infn.it/~hepcomplex/babayaga.html
BHWIDE	e^+e^-	$\mathcal{O}(\alpha)$ YFS	0.5%(LEP1)	placzek.home.cern.ch/placzek/bhwide
MCGPJ	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + SF	$< 0.2\%$	cmd.inp.nsk.su/~sibid

Without vacuum polarisation, to compare QED corrections consistently

At the Φ and τ–charm factories (cross sections in nb)

By BabaYaga people, Ping Wang and A. Sibidanov

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \leq \vartheta_\mp \leq 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \leq \vartheta_\mp \leq 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\vert\vartheta_+ + \vartheta_- - \pi\vert \leq 0.25$ rad	35.20(2)	—	35.181(5)	0.050

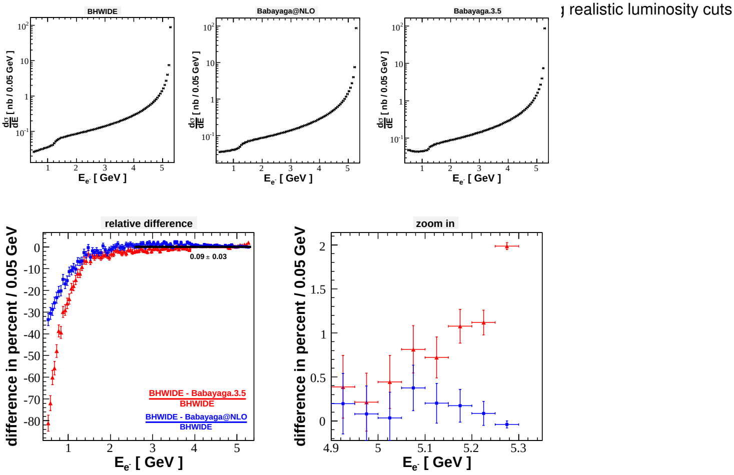
★ Agreement well below 0.1%! ★

At BaBar (cross sections in nb)

By A. Hafner and A. Denig

angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

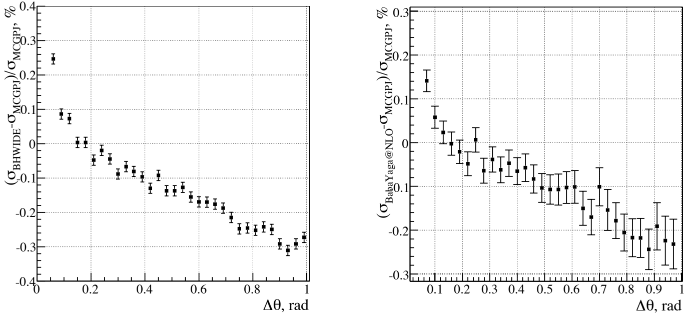
★ Agreement at the $\sim 0.1\%$ level! ★



- BabaYaga@NLO and BHWIDE well agree (at a few per mille level) also for distributions. Larger differences correspond to very hard photon emission and do not influence noticeably the luminosity measurement

MCGPJ, BabaYaga@NLO and BHWIDE at VEPP–2M

From the Luminosity Section of the WG Report
ity @ CMD–2



- The three generators agree within 0.1% for the typical experimental acollinearity cut $\Delta\theta \sim 0.2 \div 0.3$ rad

★ Main conclusion from tuned comparisons: technical precision of the generators well under control, the small remaining differences being due to slightly different details in the calculation of the same theoretical ingredients [additive vs factorized formulations, different scales for higher–order leading log corrections]

Theoretical accuracy, comparisons with NNLO

- NLO RC being included, the theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) (although large NNLO RC already included by h.o. exponentiation and by $\mathcal{O}(\alpha)$ LL $\times$ finite-NLO)

⋆ The NNLO QED corrections to Bhabha scattering have been calculated in the last years

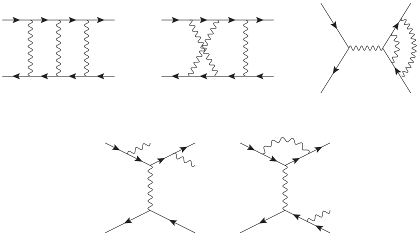
- e.g., **BabaYaga** formulae can be truncated at $\mathcal{O}(\alpha^2)$ to be consistently compared with all the classes of NNLO corrections

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}$$

  - $\sigma^{\alpha^2}_{\rm SV}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2)$ $\longrightarrow$ compared with the corresponding available NNLO QED calculation
  - $\sigma^{\alpha^2}_{\rm SV,H}$: one–loop soft+virtual corrections to single hard bremsstrahlung $\longrightarrow$ presently estimated relying on existing (partial) results
  - $\sigma^{\alpha^2}_{\rm HH}$: double hard bremsstrahlung $\longrightarrow$ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section, to register really negligible differences (at the 1×10^{-5} level)

NNLO Bhabha calculations

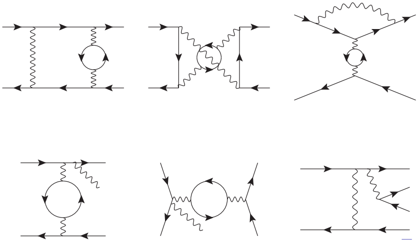
- Photonic corrections A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- Electron loop corrections

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280

S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26

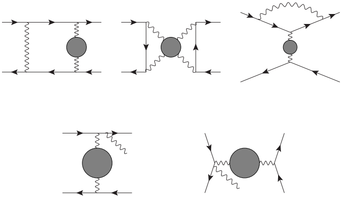


NNLO Bhabha calculations

- Heavy fermion and hadronic loops

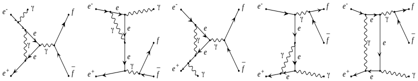
R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601
S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419, `arXiv:0909.1750`

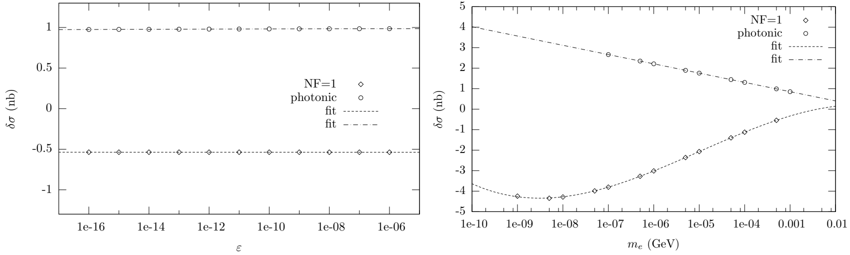


Comparison with NNLO calculation for $\sigma_{\rm SV}^{\alpha^2}$

Using realistic cuts for luminosity @ KLOE

Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of BabaYaga@NLO with

- Penin (photonic): switching off the vacuum polarisation contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut–off (left plot) and of a fictitious electron mass (right plot)



★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\text{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

Lepton and hadron loops & pairs at NNLO

- The exact NNLO soft+virtual corrections and $2 \to 4$ matrix elements $e^+e^- \to e^+e^-(l^+l^-, l = e, \mu, \tau), e^+e^-(\pi^+\pi^-)$ are now available
- In comparison with the *approximation* in BabaYaga@NLO and using realistic luminosity cuts ($S_i \equiv \sigma_i^{\text{NNLO}}/\sigma_{BY}$)

	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$ [‰]	S_{lep} [‰]	S_{had} [‰]	S_{tot} [‰]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BB@NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BB@NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BB@NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BB@NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

⋆ The uncertainty due to lepton and hadron pair corrections is at the level of a few units in 10^{-4}

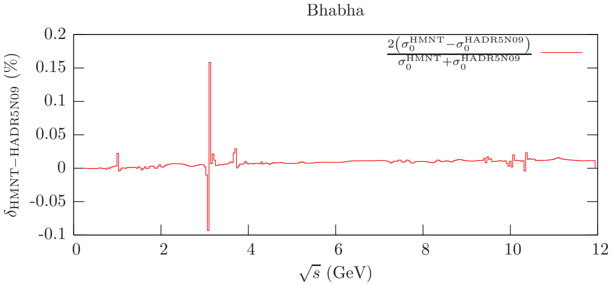
Carloni, Czyz, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann *et al.*, JHEP **1107** (2011) 126

Vacuum polarisation: HADR5N09 vs. HMNT for Bhabha

For a discussion see the Vacuum polarisation Section of the WG Report

HADR5N09, F. Jegerlehner, `http://www-com.physik.hu-berlin.de/~fjeger/hadr5n09.f`
Nucl. Phys. Proc. Suppl. **135** (2008) 181

HMNT: K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, Phys. Lett. **B649** (2007) 173
T. Teubner, K. Hagiwara, R. Liao, A.D. Martin and D. Nomura
Chinese Phys. **C34** (2010) 728, `arXiv:1001.5401`



- Bhabha largely dominated by t-channel (space-like) scattering
- The two parameterisations agree within 0.5 $\times 10^{-3}$ for all c.m. energies, at $\sim 0.1 - 0.2$ % around the J/ψ

Status of the MC theoretical accuracy

Main conclusion of the Luminosity Section of the WG Report

Putting the various sources of uncertainties (for large–angle Bhabha) all together...

Source of error (%)	$\Phi-$factories	$\sqrt{s}$ = 3.5 GeV	$B-$factories
$\lvert\delta_{\rm VP}^{\rm err}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta_{\rm VP}^{\rm err}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta_{\rm SV,\alpha^2}^{\rm err}\rvert$	0.02	0.02	0.02
$\lvert\delta_{\rm HH,\alpha^2}^{\rm err}\rvert$	0.00	0.00	0.00
$\lvert\delta_{\rm SV,H,\alpha^2}^{\rm err}\rvert$ [in progress]	0.05	0.05	0.05
$\lvert\delta_{\rm pairs}^{\rm err}\rvert$	0.03	0.016	0.03
$\lvert\delta_{\rm total}^{\rm err}\rvert$ linearly	0.12	0.1	0.13
$\lvert\delta_{\rm total}^{\rm err}\rvert$ in quadrature	0.07	0.06	0.06

- For the experiments on top of and closely around the J/ψ resonance, the accuracy slightly deteriorates, because of the differences between the predictions of independent $\Delta\alpha^{(5)}_{\rm had}(q^2)$ routines

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements. It is comparable with that achieved for small–angle Bhabha luminosity monitoring at LEP/SLC

What next? (from MC side) & conclusions

- The present MC estimated accuracy is at the level of 0.1% (at least off the narrow hadronic resonances)
- Is a better accuracy needed for future measurements at flavour factories? If yes,
- what can be done in the next future to improve it, from a MC point of view?
- Bhabha (and $e^+e^- \to \mu^+\mu^-$)
  - implementation in all MC codes of *complex* $\alpha(Q^2) \to$ more reliable also on top of narrow hadronic resonances
  - generalisation of NLO matching to NNLO: it can be done (I think)
- $e^+e^- \to \gamma\gamma$
  - perform a more detailed and tuned comparison among existing codes
  - how much of the NNLO Bhabha calculations can be transferred to $\gamma\gamma$?

BACKUP SLIDES

Resummation beyond α^2

- with a complete 2-loop generator at hand, (leading-log) resummation beyond α^2 can be neglected?

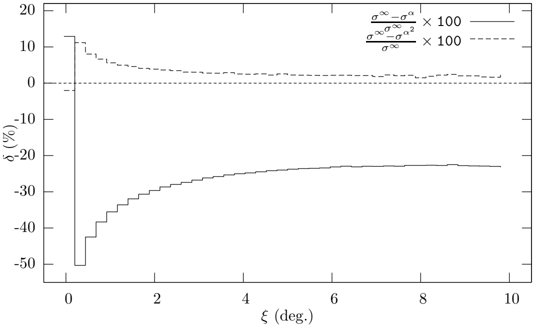


Figure: Impact of α^2 (solid line) and resummation of higher order ($\geq \alpha^3$) (dotted) corrections on the acollinearity distribution

- resummation beyond α^2 still important!