Current status of Monte Carlo generator Tauola

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18.11.2014

TAUOLA (Monte Carlo generator for tau decay modes)

***CPC* version** R. Decker, S.Jadach, M.Jezabek, J.H.Kuhn, Z. Was,
Comp. Phys. Comm. 76 (1993) 361

***Cleo* version** Alain Weinstein : http://www.cithep.caltech.edu/~ajw/korb_doc.html#files

* ***BaBar* version**

* ***Belle version***

***Aleph* version** B. Bloch, private communications

Features of all versions:

* based on VMD, i.e. 3 scalar modes BW(V1)*BW(V2) , reproduces LO ChPT limit
* wrong normalization for 2 scalar modes, except 2 π, only vector FF , no scalar FF

* *not correct low energy behaviour of the vector part for Kkπ modes*

* *3 scalar mode results are not able to reproduce experimental data*

Belle* (2π, *K* π) *spectra, BaBar* 3 meson *invariant mass spectra

published

Hadronic currents for two and three meson decay modes

$$J_\mu = \langle \text{Hadrons} | (V-A)_\mu \, e^{iS_{QCD}} | 0 \rangle = \Sigma_i (\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

Hadronic form factors are:

• **Model: Resonance Chiral Lagrangian (*Chiral lagrangian with the explicit inclusion of resonances , G.Ecker et al., Nucl. Phys B321(1989)311*)**

* The resonance fields ($V_{\mu\nu}$, $A_{\mu\nu\nu}$ *antisymmetric tensor field*) is added by explicit way
* Reproduces NLO prediction of ChPT (at least)
* Correct high energy behaviour of form factors → relation between model parameters

Finite numbers of parameters *(one octet: f_π , F_V, G_V, F_A)*

Modes: 2π, $K\pi$, KK, $KK\pi$ → 88% of tau hadronic width

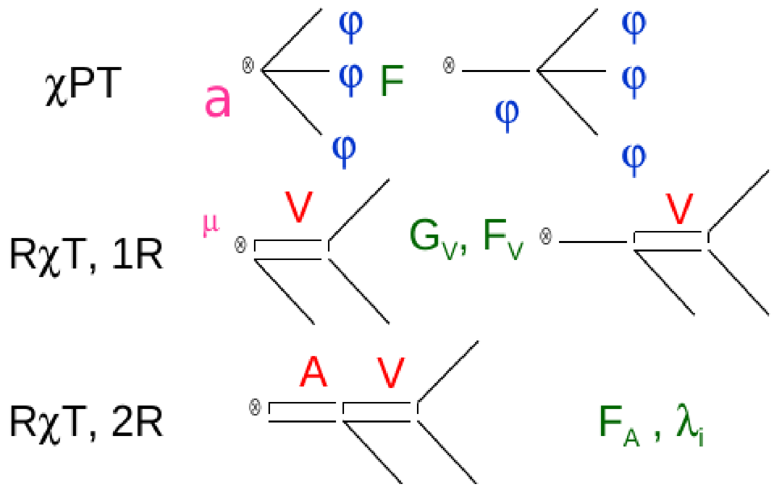
self consistent results within RChL for TAUOLA

We will start with $\tau \to \pi^- \pi^- \pi^+ \nu_\tau$

$Br(\tau^- \to \nu_\tau)/Br(\tau \to \text{hadrons}\ \nu_\tau) = 14.1\%$

Three pion modes: $\tau \to \pi^+ \pi^- \pi^+ \nu_\tau$

$$J^{\mu} = N \left\{ T^{\mu}_{\nu} \left[c_1 (p_2 - p_3)^{\nu} F_1 + c_2 (p_3 - p_1)^{\nu} F_2 + c_3 (p_1 - p_2)^{\nu} F_3 \right] + c_4 q^{\nu} F_4 - \frac{i}{4\pi^2 F^2} c_5 \varepsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} F_5 \right\}$$



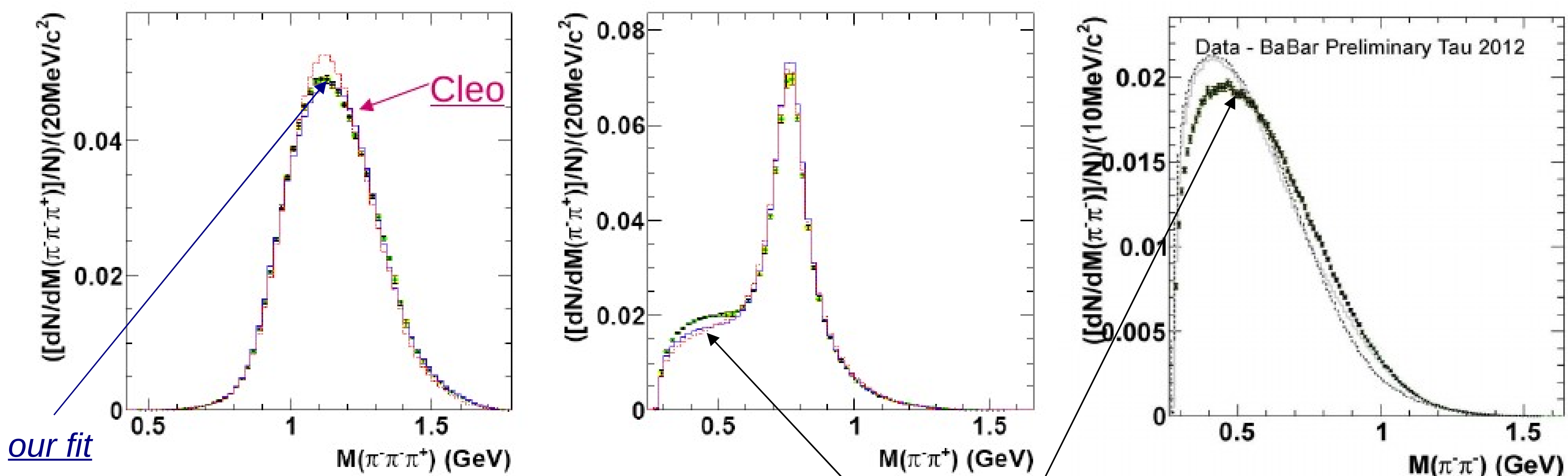
For 3 pion modes $F_5 = 0$; $F4 \sim m_\pi^2/q^2$;

$F_2(q^2,s_1,s_2) = F_1(q^2,s_2,s_1)$

$A = a_1$; $V = \rho$; ρ'

D. Gomez Dumm et al, 0911.4436

Tauola 2012: Implementation + technical tests



missing contribution at low energy of 2 pion

Modification of RChL → inclusion of σ meson

* *σ meson is not in RChL scheme*
* *BW approach, the RChL current structure*

$$F_1^{\mathrm{R}} \rightarrow F_1^{\mathrm{R}} + \frac{\sqrt{2}F_V G_V}{3F^2}\left[\alpha_\sigma BW_\sigma(s_1)F_\sigma(q^2,s_1) + \beta_\sigma BW_\sigma(s_2)F_\sigma(q^2,s_2)\right]$$

$$F_1^{\mathrm{RR}} \rightarrow F_1^{\mathrm{RR}} + \frac{4F_A G_V}{3F^2}\frac{q^2}{q^2 - M_{a_1}^2 - iM_{a_1}\Gamma_{a_1}(q^2)}\left[\gamma_\sigma BW_\sigma(s_1)F_\sigma(q^2,s_1) + \delta_\sigma BW_\sigma(s_2)F_\sigma(q^2,s_2)\right]$$

$$BW_\sigma(x) = \frac{m_\sigma^2}{m_\sigma^2 - x - im_\sigma\Gamma_\sigma(x)} \quad \Gamma_\sigma(x) = \Gamma_\sigma\frac{\sigma_\pi(x)}{\sigma_\pi(m_\sigma^2)} \quad F_\sigma(q^2,x) = \exp\left[\frac{-\lambda(q^2,x,m_\pi^2)R_\sigma^2}{8q^2}\right]$$

Fit parameters $M_A,\ M_\rho,\ M_{\rho'},\ F_V,\ F_A,\ \beta_\rho,\ F \quad + \quad \alpha_\sigma,\ \beta_\sigma,\ \gamma_\sigma,\ \delta_\sigma,\ R_\sigma$

d Γ/dq2 ds1 ds2 -> 1d dimensional distributions (s1, s2, q2) to fit to BaBar data

$$\frac{G_F^2|V_{ud}|^2}{128(2\pi)^5 M_\tau F^2}\left(\frac{M_\tau^2}{q^2}-1\right)^2\left[W_{SA} + \frac{1}{3}\left(1+2\frac{q^2}{M_\tau^2}\right)(W_A + W_B)\right]$$

(W_{SA} = 0, W_B = 0)

$$W_A = -(V_1^\mu F_1 + V_2^\mu F_2 + V_3^\mu F_3)(V_{1\mu}F_1 + V_{2\mu}F_2 + V_{3\mu}F_3)^* \longrightarrow \text{resonances}$$

To smooth integrand

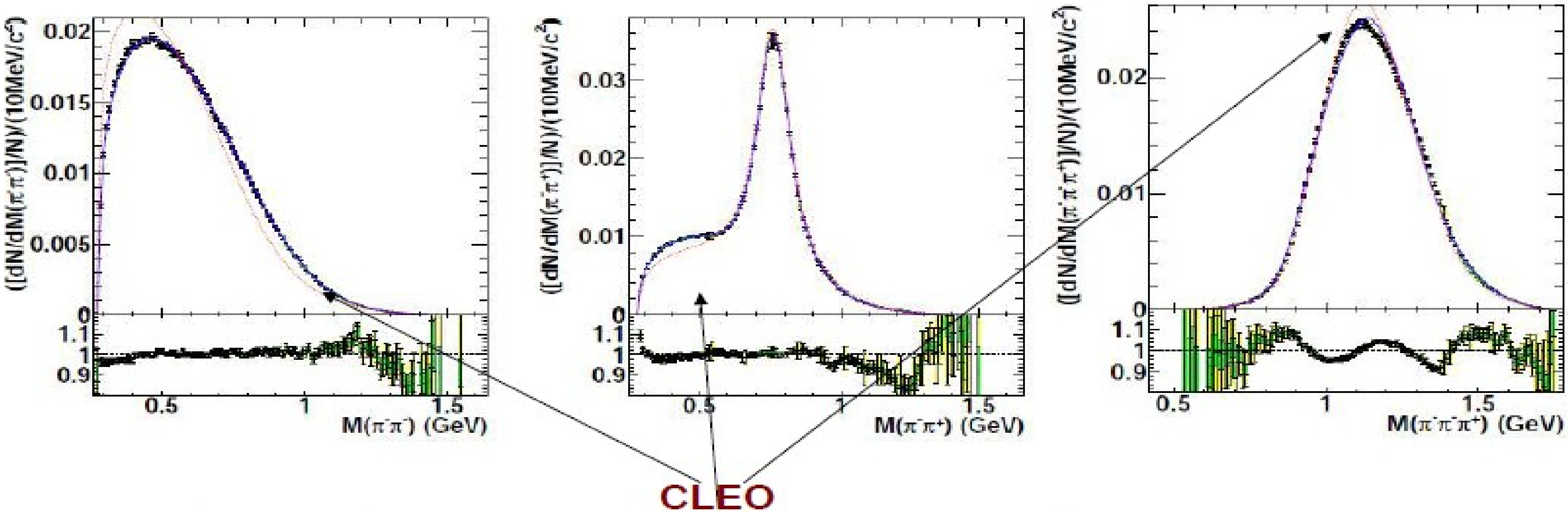
$$\int_{x_1}^{x_2} f(x)dx = \int_0^1 g'(t)f(g(t))dt \qquad x = g(t) = \mathrm{A}^2 + \mathrm{AB}\,\mathrm{tg}(y_1 + t(y_2 - y_1)) \qquad y_1 = \mathrm{arctg}\left(\frac{x_1 - A^2}{AB}\right)$$

$A = 0.77,\ B = 1.8$

Fit results

BaBar data *** 10 MeV/bin (twice decreased)**

*** separated statistical and systematical errors**



	M_ρ [8]	$M_{\rho'}$	$\Gamma_{\rho'}$	M_{a_1}	M_σ	Γ_σ	F	F_V
Min	0.767	1.35	0.30	0.99	0.400	0.400	0.088	0.11
Max	0.780	1.50	0.50	1.25	0.550	0.700	0.094	0.25
Fit	0.771849	1.350000	0.448379	1.091865	0.487512	0.700000	0.091337	0.168652

	F_A	$\beta_{\rho'}$	α_σ	β_σ	γ_σ	δ_σ	R_σ
Min	0.1	-0.37	-10.	-10.	-10.	-10.	-10.
Max	0.2	-0.17	10.	10.	10.	10.	10.
Fit	0.131425	-0.318551	-8.795938	9.763701	1.264263	0.656762	1.866913

$\chi^2/\text{ndf} = 6658/401$ stat

$\chi^2/\text{ndf} = 889/401$ stat+syst ← 2262 / 132

$$\Gamma_{\tau^- \to \pi^-\pi^-\pi^+\nu_\tau} = 2.0001 \cdot 10^{-13}\ \text{GeV}$$ (Tauola2012)

Validation of results

* Statistical errors and correlations between model parameters

* Convergence of the fitting procedure

* Toy MC studies to check of behaviour near the minimum

* Estimation of systematic uncertainties

Validation of results

* Statistical errors and correlations between model parameters
- Hesse algorithm of Minuit package

	α_σ	β_σ	γ_σ	δ_σ	R_σ	M_ρ	$M_{\rho'}$	$\Gamma_{\rho'}$	M_{a_1}	M_σ	Γ_σ	F_π	F_V	F_A	$\beta_{\rho'}$
α_σ	1	0.60	0.36	-0.29	-0.41	-0.69	0.46	0.68	-0.77	-0.09	0.02	0.78	0.76	0.52	-0.78
β_σ	0.60	1	0.44	-0.39	-0.42	-0.75	0.55	0.79	-0.89	-0.16	0.04	0.89	0.88	0.58	-0.88
γ_σ	0.36	0.44	1	-0.56	-0.22	-0.59	0.16	0.37	-0.47	-0.28	0.00	0.49	0.45	0.30	-0.45
δ_σ	-0.29	-0.39	-0.56	1	0.46	0.46	-0.24	-0.42	0.49	0.01	0.01	-0.49	-0.47	-0.31	0.47
R_σ	-0.41	-0.42	-0.22	0.46	1	0.42	-0.33	-0.56	0.62	0.34	0.02	-0.53	-0.56	-0.42	0.48
M_ρ	-0.69	-0.75	-0.59	0.46	0.42	1	-0.27	-0.64	0.79	0.29	-0.02	-0.83	-0.74	-0.48	0.75
$M_{\rho'}$	0.46	0.55	0.16	-0.24	-0.33	-0.27	1	0.67	-0.61	-0.13	0.03	0.61	0.66	0.37	-0.65
$\Gamma_{\rho'}$	0.68	0.79	0.37	-0.42	-0.56	-0.64	0.67	1	-0.88	-0.24	0.03	0.86	0.88	0.57	-0.88
M_{a_1}	-0.77	-0.89	-0.47	0.49	0.62	0.79	-0.61	-0.88	1	0.28	-0.03	-0.96	-0.97	-0.62	0.95
M_σ	-0.09	-0.16	-0.28	0.01	0.34	0.29	-0.13	-0.24	0.28	1	-0.02	-0.30	-0.29	-0.20	0.30
Γ_σ	0.02	0.04	0.00	0.01	0.02	-0.02	0.03	0.03	-0.03	-0.02	1	0.03	0.03	0.03	-0.04
F_π	0.78	0.89	0.49	-0.49	-0.53	-0.83	0.61	0.86	-0.96	-0.30	0.03	1	0.95	0.55	-0.97
F_V	0.76	0.88	0.45	-0.47	-0.56	-0.74	0.66	0.88	-0.97	-0.29	0.03	0.95	1	0.63	-0.96
F_A	0.52	0.58	0.30	-0.31	-0.42	-0.48	0.37	0.57	-0.62	-0.20	0.03	0.55	0.63	1	-0.56
$\beta_{\rho'}$	-0.78	-0.88	-0.45	0.47	0.48	0.75	-0.65	-0.88	0.95	0.30	-0.04	-0.97	-0.96	-0.56	1

Strong correlation > 0.95 $M_{a_1}, F_\pi, F_V, \beta_{\rho'}$

Validation of results

*

*   Convergence of the fitting procedure

to verify that the found minimum is a global minimum

- start with random scan of 210 K points
- select 1K with the best chi2
- from them select 20 points with maximum distance
- use them as a start point for the full fit and apply the full fit procedure

> 50% converge to the minimum

(*others falls with number of parameters at their limits,*
converge to local minimum with higher chi2)

Indicates that the found minimum point is a global minimum
and the fitting procedure does not depend on an initial point

Validation of results

*

*

* Toy MC studies to check of behaviour near the minimum
8 MC samples (different seeds) of 20 million generated with
(I) the fit parameter values ('global minimum'), i.e. difference is “statistical error”, a set “Toy”
(II) the set “Toy” is fitted
(a) the starting point is the 'global' minimum
(b) the starting point is the initial parameter values

The results of fit are consistent, i.e. the fitting procedure is stable

Validation of results

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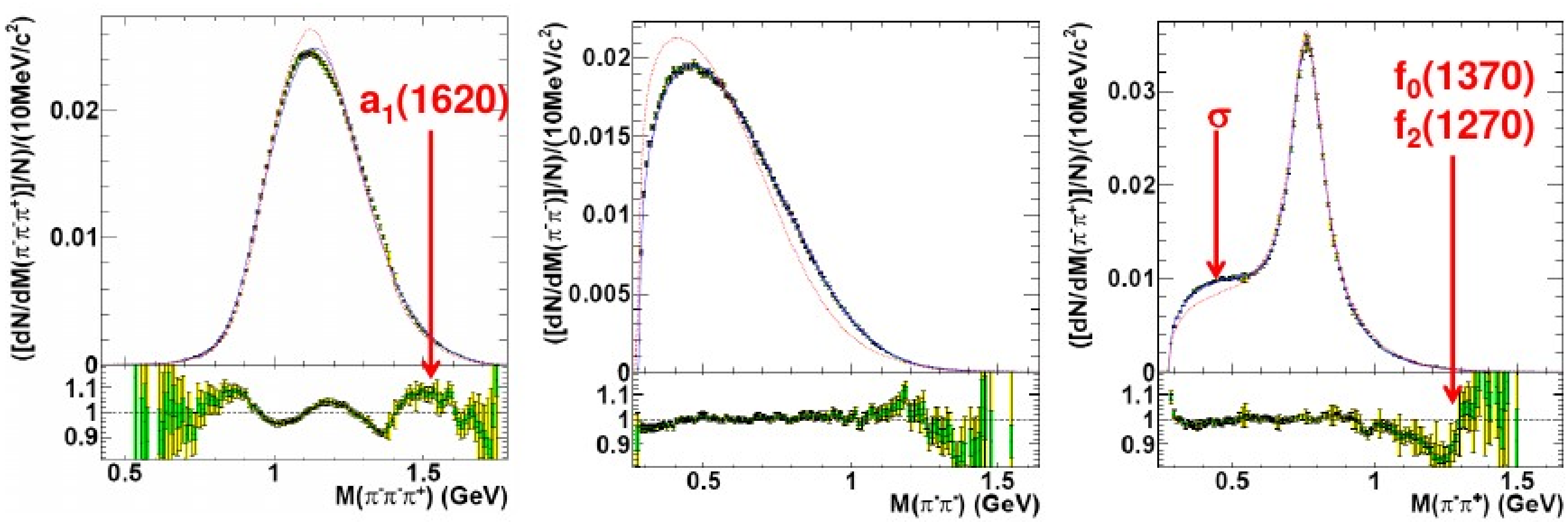
* Estimation of systematic uncertainties

Used systematical covariance matrix from BaBar experiment to include the correlations between bins

Limitations of the model

TAUOLA 2014



TAUOLA 2012

No data for π0 π0 π- !!!!

… and will be not available in near future.

Difference is with the sigma meson contribution

fit SIGMA parameters to **π0 π0 π-** BaBar data

$$F_1^{R} \rightarrow F_1^{R} + \frac{\sqrt{2}F_V G_V}{3F^2}\alpha_\sigma^0 BW_\sigma(s_3)F_\sigma(q^2,s_3),$$

$$F_1^{RR} \rightarrow F_1^{RR} + \frac{4F_A G_V}{3F^2}\frac{q^2}{q^2 - M_{a_1}^2 - iM_{a_1}\Gamma_{a_1}(q^2)}\gamma_\sigma^0 BW_\sigma(s_3)F_\sigma(q^2,s_3).$$

π+π− π− $\alpha_\sigma = \beta_\sigma,\ \gamma_\sigma = \delta_\sigma$

$$\alpha_\sigma = 1.139486,\ \gamma_\sigma = 0.889769,\ R_\sigma = 0.000013,\ M_\sigma = 0.550\quad \Gamma_\sigma = 0.700.$$

$$\alpha_\sigma^0 = \alpha_\sigma \cdot Scaling_{factor}^{\gamma}$$

↑ **CLEO**

$$Scaling_{factor}^{\gamma} = 2.1/3.35 = 0.63$$

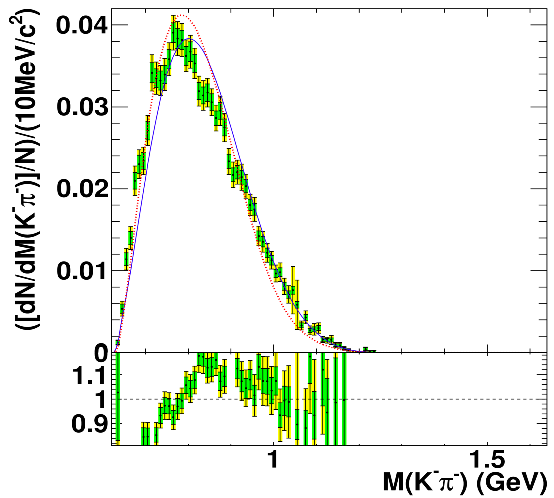
$$\Gamma = (2.1440 \pm 0.02\%) \cdot 10^{-13}$$

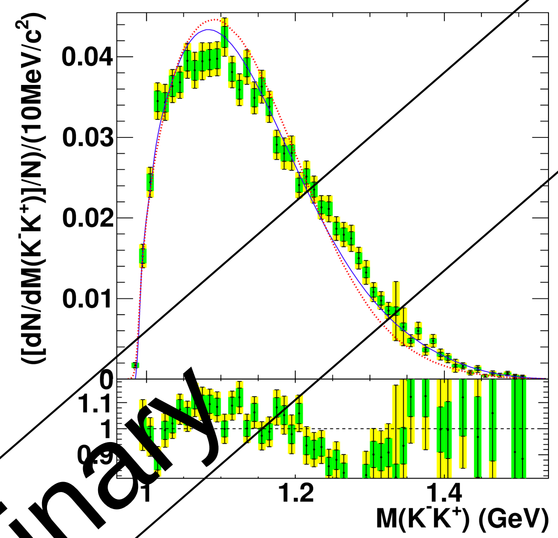
2.1% higher than the PDG value

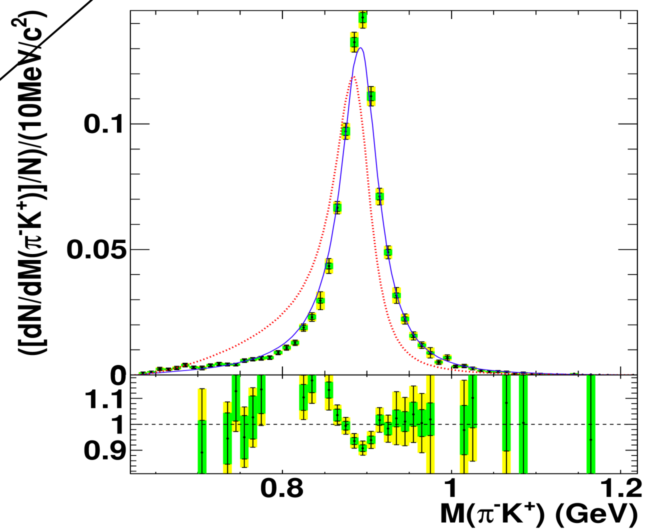
$\tau^- \to K^+ K^- \pi^- \nu$

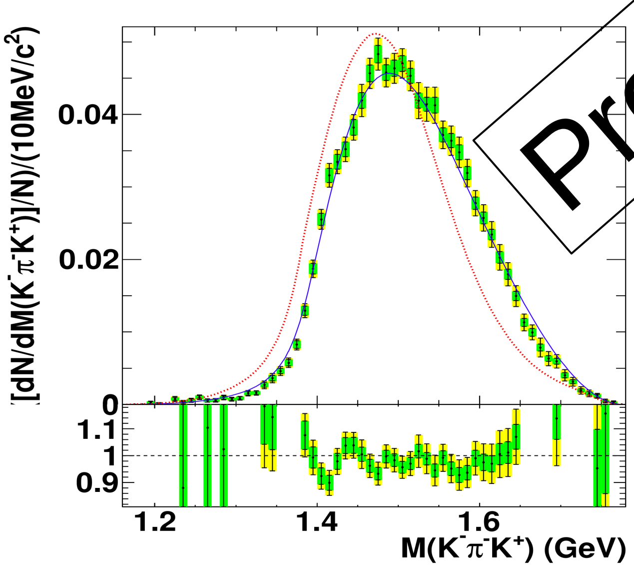
First fitting results to BaBar data , fixed table for the a1 width

Preliminary





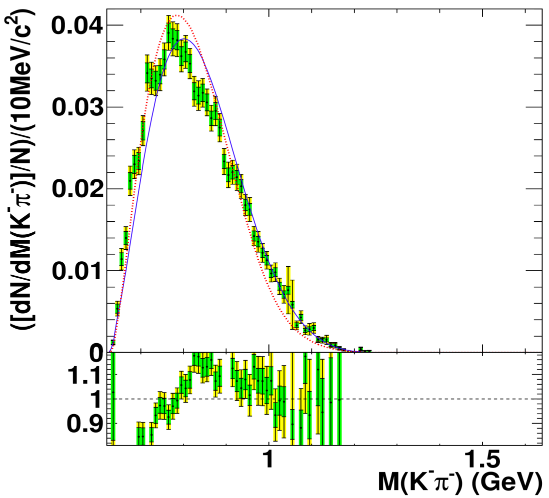


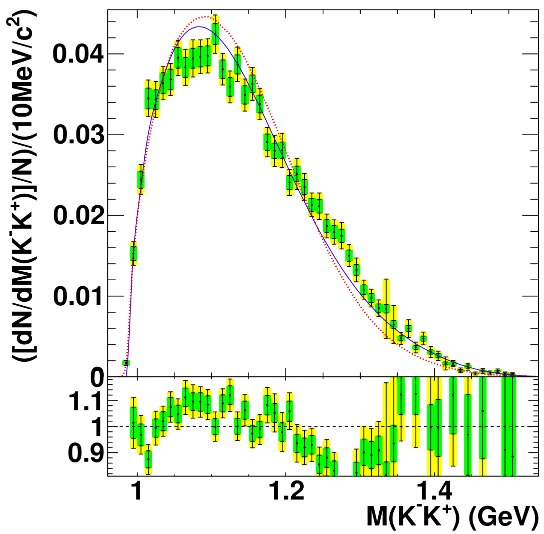


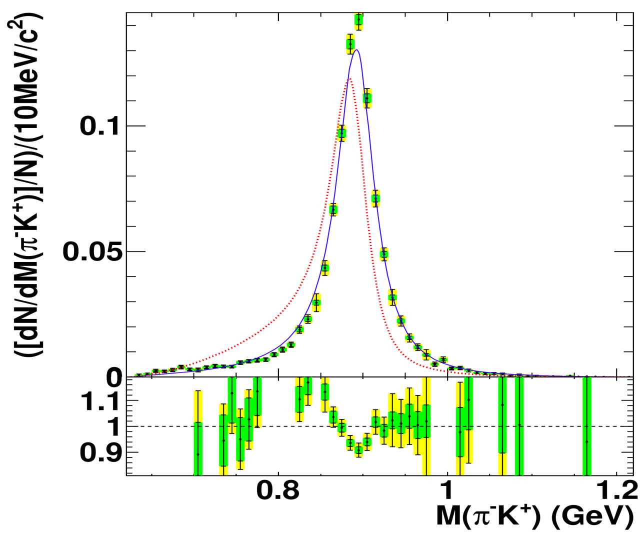
Blue – RChL   Red – Cleo

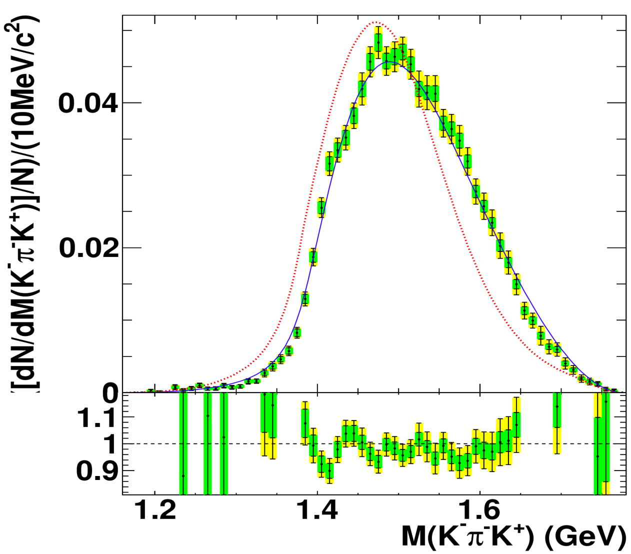
$\tau^- \to K^+ K^- \pi^- \nu$

First fitting results to BaBar data , fixed table for the a1 width









Blue – RChL   Red – Cleo

some parameters on their limits …

* generalization of 3 pion fit strategy

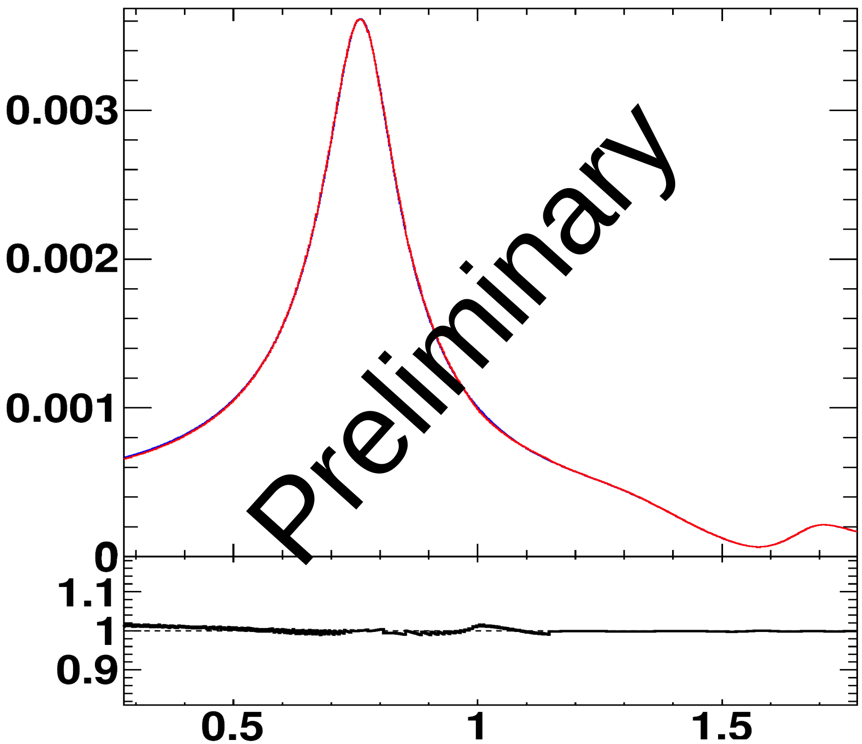
… common fit to $\pi^+ \pi\ \ \pi^-$ and $K^+K^-\pi^-$

$\tau^- \to \pi^0 \pi^- \nu$

Two meson modes:

$$J^\mu = N\left[(p_1 - p_2)^\mu F^V(s) + (p_1 + p_2)^\mu F^S(s)\right]$$

= 0 (for $\pi^0 \pi^-$)



Vector FF: D.Gomez Dumm & P. Roig

s> 1.35 GeV [2]

$$F_V^\pi(s) = \frac{M_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''})\, s}{M_\rho^2 \left[1 + \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right] - s} - \frac{\alpha' e^{i\phi'}\, s}{M_{\rho'}^2\left[1 + s\, C_{\rho'} A_\pi(s)\right] - s} - \frac{\alpha'' e^{i\phi''}\, s}{M_{\rho''}^2\left[1 + s\, C_{\rho''} A_\pi(s)\right] - s}$$

s< 1.35 GeV [2]

$$F_V^\pi(s) = \exp\left[\alpha_1\, s + \frac{\alpha_2}{2}\, s^2 + \frac{s^3}{\pi}\int_{s_{\rm thr}}^{\infty} ds' \, \frac{\delta_1^1(s')}{(s')^3 (s' - s - i\epsilon)}\right]$$

Fit to Belle analytical FF

$\tau^- \to \pi^0 \pi^- \nu$

Two meson modes:

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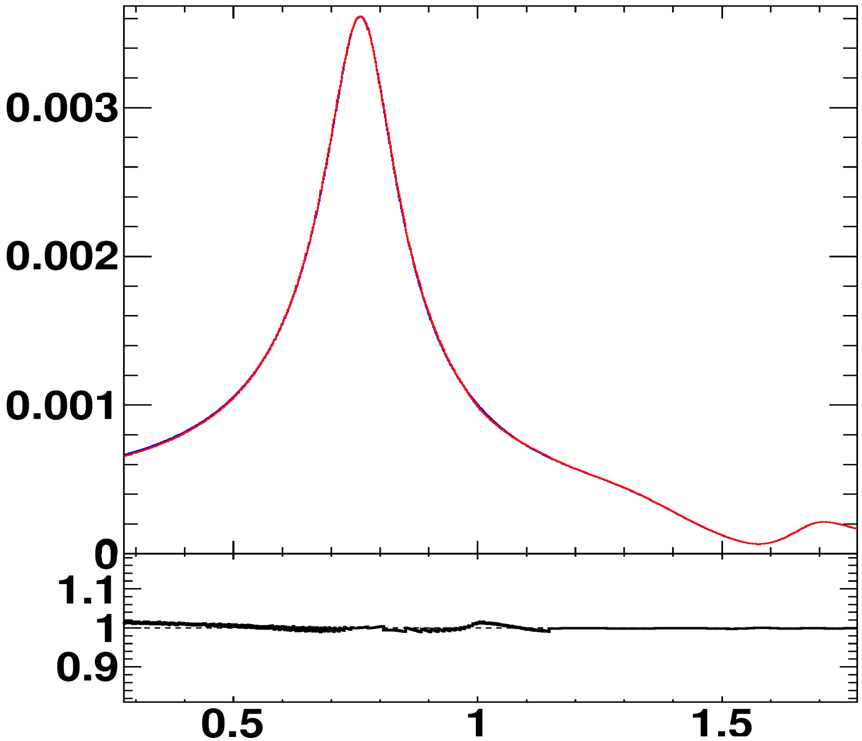

Fit to Belle analytical function

Vector FF: D.Gomez Dumm & P. Roig

s> 1.35 GeV [2]

$$F_V^\pi(s) = \frac{M_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''})\, s}{M_\rho^2\left[1 + \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right] - s} - \frac{\alpha' e^{i\phi'}\, s}{M_{\rho'}^2\left[1 + s\, C_{\rho'} A_\pi(s)\right] - s} - \frac{\alpha'' e^{i\phi''}\, s}{M_{\rho''}^2\left[1 + s\, C_{\rho''} A_\pi(s)\right] - s}$$

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$$F_V^\pi(s) = \exp\left[\alpha_1\, s + \frac{\alpha_2}{2}\, s^2 + \frac{s^3}{\pi}\int_{s_{\rm thr}}^{\infty} ds'\, \frac{\delta_1^1(s')}{(s')^3(s' - s - i\epsilon)}\right]$$

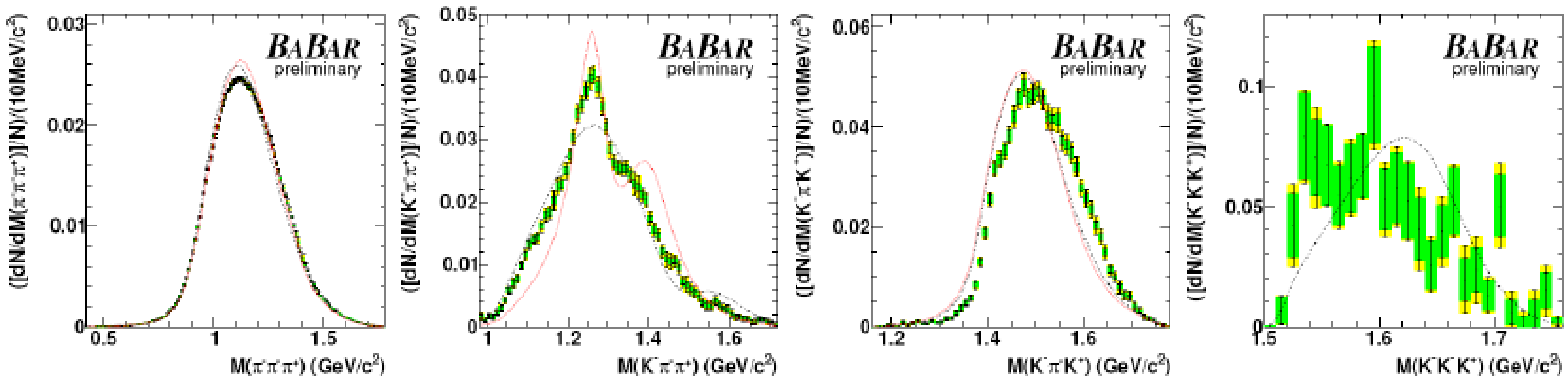
Conclusion and plans

Currently there are (preliminary) 1d invariant mass spectra available for:

I. Nugent

$\tau \to \pi^- \pi^- \pi^+ \nu$ $\tau \to K^- \pi^- \pi^+ \nu$ $\tau \to K^- \pi^- K^+ \nu$ $\tau \to K^- K^- K^+ \nu$

BaBar data



Fitting 1d distributions in $\tau \to \pi^- \pi^- \pi^+ \nu$ has already given us insight into fitting models of low energy QCD (RCHL):

- Information on missing resonances
- Problems and with multi-dimensional fitting – data provided by collaborations

- 1d projection → multi-dimentional fit for 3 pion mode
  K K π + 3 π mode fit

- 2 pion RChL current fit to Belle data
- 4 pion RChL current in Tauola and fit to BaBar data

Belle data analysis ??

tauola -bbb project ⇒

tauola-bbb project

- **Achieved:**
- TAUOLA MC with 200 decay channels, solution similar as presented on TAU04 and used by BaBar. Neutrinoless channels available.
- **Default BaBar Tauola initialization.**
- Alternatively, for 2 and 3 π's, new currents with comparison with experimental data prepared.
- Theoretically motivated currents, 4 and 5 π's decay modes, also as alternative.
- No fits to global properties such as average charged energy. For alternatives, no experimental quality stamps.
- User can re-initialize TAUOLA with own (C++ coded) currents (or matrix elements).

- **Non complete tasks:**
- Results for 3-scalar modes with K's are not incorporated, need quality fits. See e.g. Olga talk.
- Many alternative parametrizations, eg. for 2K 2π modes (BaBar) are not incorporated, even though these are missing channels, at present only flat phase space.
- Environments for fits are not well structured for model independent use.

tauola-bbb project

ChannelForTauola class


```
// get information about existing decay channel
ChannelForTauola *demo_modify = GetChannel(87);

demo_modify->setName( demo_modify->getName() + " modified" );
demo_modify->setBr( demo_modify->getBr() * 1234 );

// redefine decay products
vector<int> products = demo_modify->getProducts();
products[0] = -3; //K-
products[1] =  4; //K0
demo_modify->setProducts(products);

// register modified channel
Tauolapp::RegisterChannel( 87, demo_modify );
demo_modify->print();

// set ME type to flat phase space
demo_modify->setMeType(1);

// register into first available free slot
Tauolapp::RegisterChannel( -1, demo_modify );
demo_modify->print();
```

- Use `tauola-bbb/tauola-c/ChannelForTauola.h` to define user channels. No need to link Tauola library.
- New matrix element or current provided by a pointer to user function. Arguments of the function checked at compile time.
- Use `RegisterChannel` for `*demo_modify` object.
- Can be also used to modify existing channels (change name, BR, decay products, etc.)
- New channel can substitute existing one or be added at the end of the list
- All, except ponters to user provided functions of hadronic currents (ME's) re-initialize content of F77 common blocks: minimal changes in old F77 code.