Ideas and Approaches to Predict the CP Phase(s) in the Leptonic Sector

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XVI International Workshop on Neutrino Telescopes,

02.03.-06.03.2015, Venice, Italy

Outline

- lepton mixing: parametrization and data
- recap: prediction of lepton mixing angles
- ideas for prediction of leptonic CP phase(s)
  - less symmetry
  - corrections
  - involve CP symmetry
- conclusions

Lepton mixing: parametrization

Parametrization

$$U_{PMNS} = \tilde{U}\ \mathrm{diag}(1, e^{i\alpha/2}, e^{i(\beta/2+\delta)})$$

with

$$\tilde{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

and $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Lepton mixing: data

Global fits IH [NH] *(Gonzalez Garcia et al. ('14) [after NOW 2014])*

best fit and $1\,\sigma$ error	$3\,\sigma$ range
$\sin^2\theta_{13} = 0.0219[8]^{+0.0011[0]}_{-0.0010}$	$0.0188[6] \leq \sin^2\theta_{13} \leq 0.0251[0]$
$\sin^2\theta_{12} = 0.304^{+0.013}_{-0.012}$	$0.270 \leq \sin^2\theta_{12} \leq 0.344$
$\sin^2\theta_{23} = \begin{cases} 0.579^{+0.025}_{-0.037} \\ [0.452^{+0.052}_{-0.028}] \end{cases}$	$0.389[2] \leq \sin^2\theta_{23} \leq 0.644[3]$
$\delta = \left(1.41[70]^{+0.35[22]}_{-0.34[9]}\right)\pi$	$0 \leq \delta \leq 2\,\pi$
$\alpha\,,\ \beta$	unconstrained

Lepton mixing angles: ideas

Well-known mixing pattern: tri-bimaximal (TB) mixing

(Harrison et al. ('02), Xing ('02))

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad J_{CP} = 0$$

Lepton mixing angles: ideas

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does not fit data well!

And since θ_{13} vanishes, the CP phase δ is unphysical.

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does not fit data well!

And since θ_{13} vanishes, the CP phase δ is unphysical.

But nice thing about it:

it's derived from symmetries like A_4, S_4

Lepton mixing angles: ideas

Origin of TB mixing ***(Lam ('07,'08))***

$$G_f = S_4$$

$$\swarrow \qquad\qquad\qquad \searrow$$

charged leptons | neutrinos

$$G_e = Z_3 \qquad\qquad G_\nu = Z_2 \times Z_2$$

$$U_e \qquad\qquad U_\nu$$

$$\searrow \qquad\qquad\qquad \swarrow$$

$$U_{PMNS} = U_e^\dagger U_\nu$$

[Masses do not play a role in this approach.]

Lepton mixing angles: ideas

Use different group for mixing with $\theta_{13} \neq 0$, $\theta_{23} \neq \frac{\pi}{4}$

(de Adelhart Toorop/Feruglio/H ('11))

$$G_f = \Delta(384)$$

$$\swarrow \qquad\qquad \searrow$$

charged leptons	neutrinos
$G_e = Z_3$	$G_\nu = Z_2 \times Z_2$
U_e	U_ν

$$\searrow \qquad\qquad \swarrow$$

$$U_{PMNS} = U_e^\dagger U_\nu$$

Lepton mixing angles: ideas

(de Adelhart Toorop/Feruglio/H ('11))

$$||U_{PMNS}|| = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{2}\sqrt{4+\sqrt{2}+\sqrt{6}} & 1 & \frac{1}{2}\sqrt{4-\sqrt{2}-\sqrt{6}} \\ \frac{1}{2}\sqrt{4+\sqrt{2}-\sqrt{6}} & 1 & \frac{1}{2}\sqrt{4-\sqrt{2}+\sqrt{6}} \\ \sqrt{1-\frac{1}{\sqrt{2}}} & 1 & \sqrt{1+\frac{1}{\sqrt{2}}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{4}{8+\sqrt{2}+\sqrt{6}} \approx 0.337\,, \quad \sin^2\theta_{23} = \frac{4-\sqrt{2}+\sqrt{6}}{8+\sqrt{2}+\sqrt{6}} \approx 0.424\,,$$

$$\sin^2\theta_{13} = \frac{4-\sqrt{2}-\sqrt{6}}{12} \approx 0.011\,, \quad \sin\delta = 0$$

"Accidentally" δ is trivial ...

Lepton mixing angles: ideas

- surveys of groups G_f, G_e and G_ν

  (H/Meroni/Vitale ('13), King/Neder/Stuart ('13), Joshipura/Patel ('13), Holthausen et al. ('12), Lam ('12), Fonseca/Grimus ('14))

  have shown that all patterns have trimaximal column ($\sin^2\theta_{12} \gtrsim 1/3$) and Dirac phase $\delta = 0, \pi$, if mixing angles are accommodated well

Leptonic CP phases: ideas

- surveys of groups G_f, G_e and G_ν

  (H/Meroni/Vitale ('13), King/Neder/Stuart ('13), Joshipura/Patel ('13), Holthausen et al. ('12), Lam ('12), Fonseca/Grimus ('14))

  have shown that all patterns have trimaximal column ($\sin^2\theta_{12} \gtrsim 1/3$) and Dirac phase $\delta = 0, \pi$,
  if mixing angles are accommodated well
- Way-outs
  - less symmetry
  - corrections
  - involve CP as symmetry

Leptonic CP phases: less symmetry

Use less symmetry!

(Ge et al. ('11), Hanlon et al. ('13); Hernandez/Smirnov ('12))

some

symmetry

$\swarrow$ $\searrow$

charged leptons	neutrinos
mixing matrix	$G_\nu = Z_2$
U_e	U_ν

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger U_\nu$$

Leptonic CP phases: less symmetry

(Ge et al. ('11), Hanlon et al. ('13))

- diagonal basis of charged leptons: $U_e = \mathbb{1}$
- neutrino sector is invariant under $Z_2^s(k)$ that is generated by

$$G_1(k) = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2\,k & 2\,k \\ 2\,k & k^2 & -2 \\ 2\,k & -2 & k^2 \end{pmatrix}$$

- correlation of Dirac phase δ and mixing angles is derived

$$\cos\delta = -\frac{\left(\sin^2\theta_{12} - \cos^2\theta_{12}\sin^2\theta_{13}\right)\,\cos 2\theta_{23}}{\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}}$$

Leptonic CP phases: less symmetry

(Ge et al. ('11), Hanlon et al. ('13))

- diagonal basis of charged leptons: $U_e = \mathbb{1}$
- neutrino sector is invariant under $\overline{Z}_2^s(k)$ that is generated by

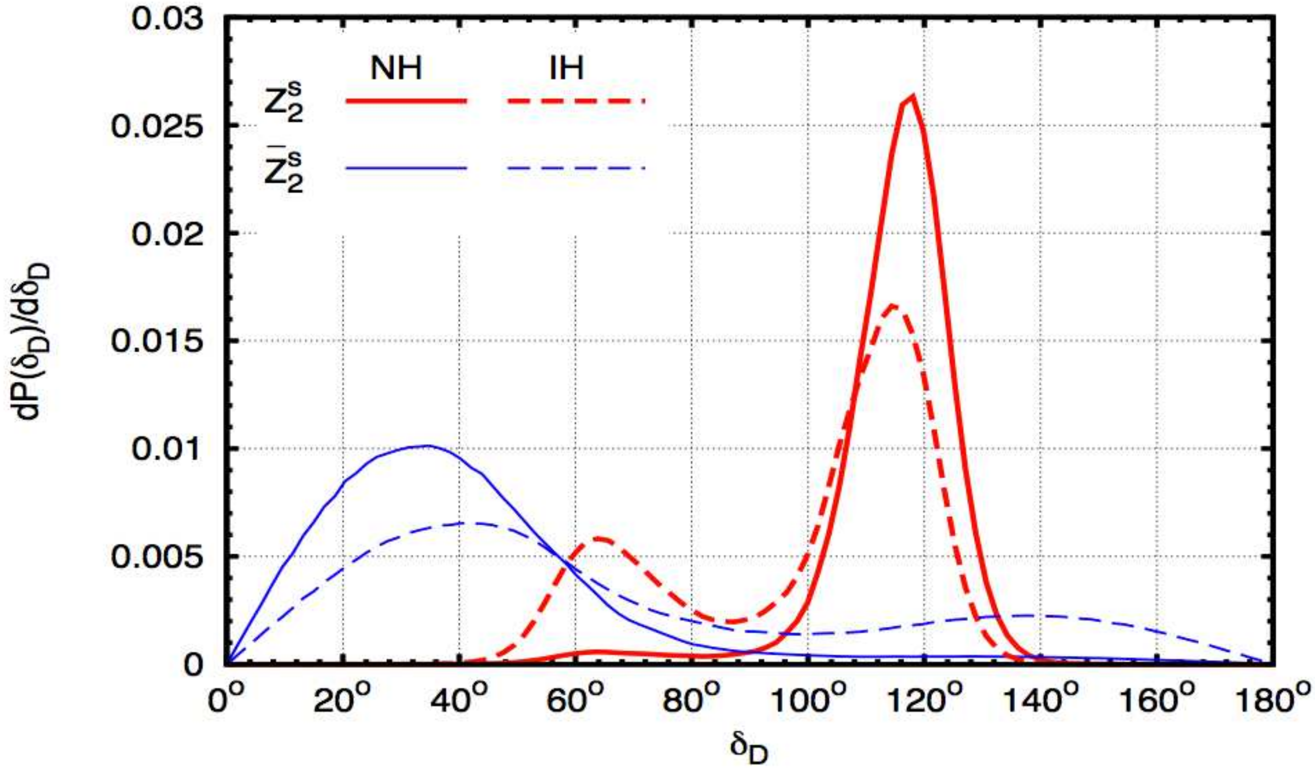
$$G_2(k) = \frac{1}{2+k^2} \begin{pmatrix} 2-k^2 & 2\,k & 2\,k \\ 2\,k & -2 & k^2 \\ 2\,k & k^2 & -2 \end{pmatrix}$$

- correlation of Dirac phase δ and mixing angles is derived

$$\cos\delta = -\frac{\left(\sin^2\theta_{12}\sin^2\theta_{13} - \cos^2\theta_{12}\right)\,\cos 2\theta_{23}}{\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}}$$

Leptonic CP phases: less symmetry

(Hanlon et al. ('13))



Leptonic CP phases: less symmetry

Use less symmetry!

(Hernandez/Smirnov ('12))

$$G_f = \text{PSL}(2,7)$$

$\swarrow$ $\searrow$

charged leptons | neutrinos

$G_e = Z_7$ | $G_\nu = Z_2$

U_e | U_ν

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger U_\nu$$

Leptonic CP phases: less symmetry

(Hernandez/Smirnov ('12))

- one column is fixed

$$|U_{\mu 1}|^2 = \frac{1}{4\left(1+\sin\frac{\pi}{14}\right)} \approx 0.204 \quad , \quad |U_{\tau 1}|^2 = \frac{1}{4\left(1+\cos\frac{\pi}{7}\right)} \approx 0.132$$

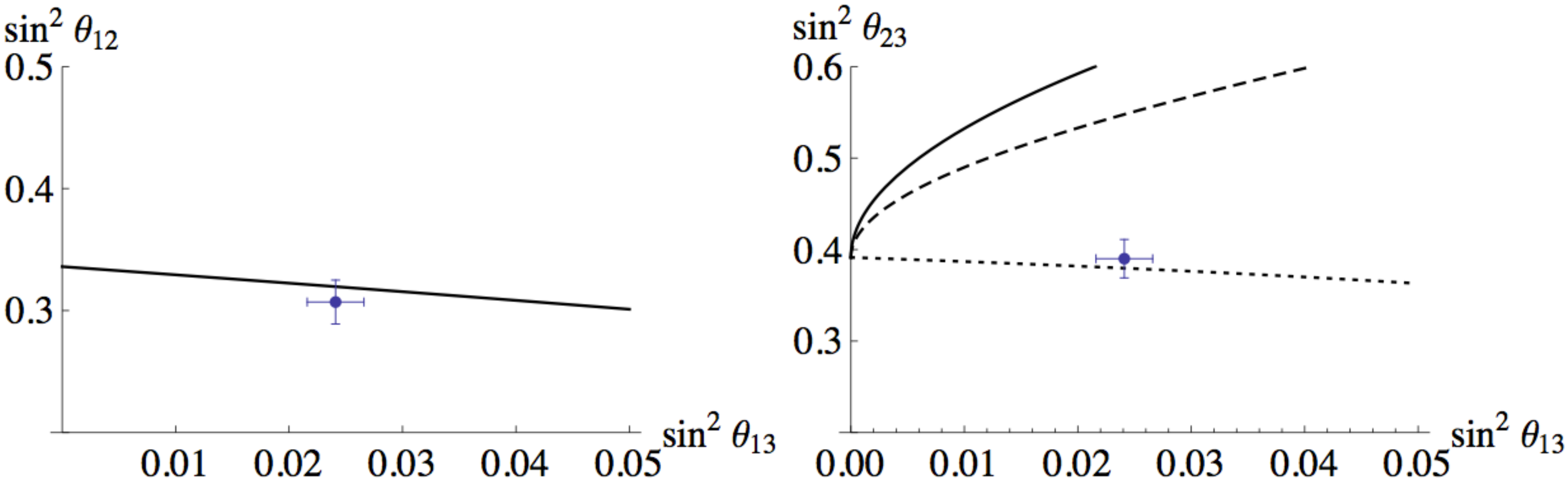
so the first column of the PMNS mixing matrix is of the form

$$\begin{pmatrix} |U_{e1}| \\ |U_{\mu 1}| \\ |U_{\tau 1}| \end{pmatrix} \approx \begin{pmatrix} 0.815 \\ 0.452 \\ 0.363 \end{pmatrix}$$

- in particular notice that $|U_{e1}|^2 = \cos^2\theta_{12}\cos^2\theta_{13} \approx 0.664$

Leptonic CP phases: less symmetry

(Hernandez/Smirnov ('12))



thick line: $\delta = 0$

dashed line: $\delta = \pi/4$

dotted line: $\delta = \pi/2$

Leptonic CP phases: corrections

(Marzocca et al. ('11))

Ansatz

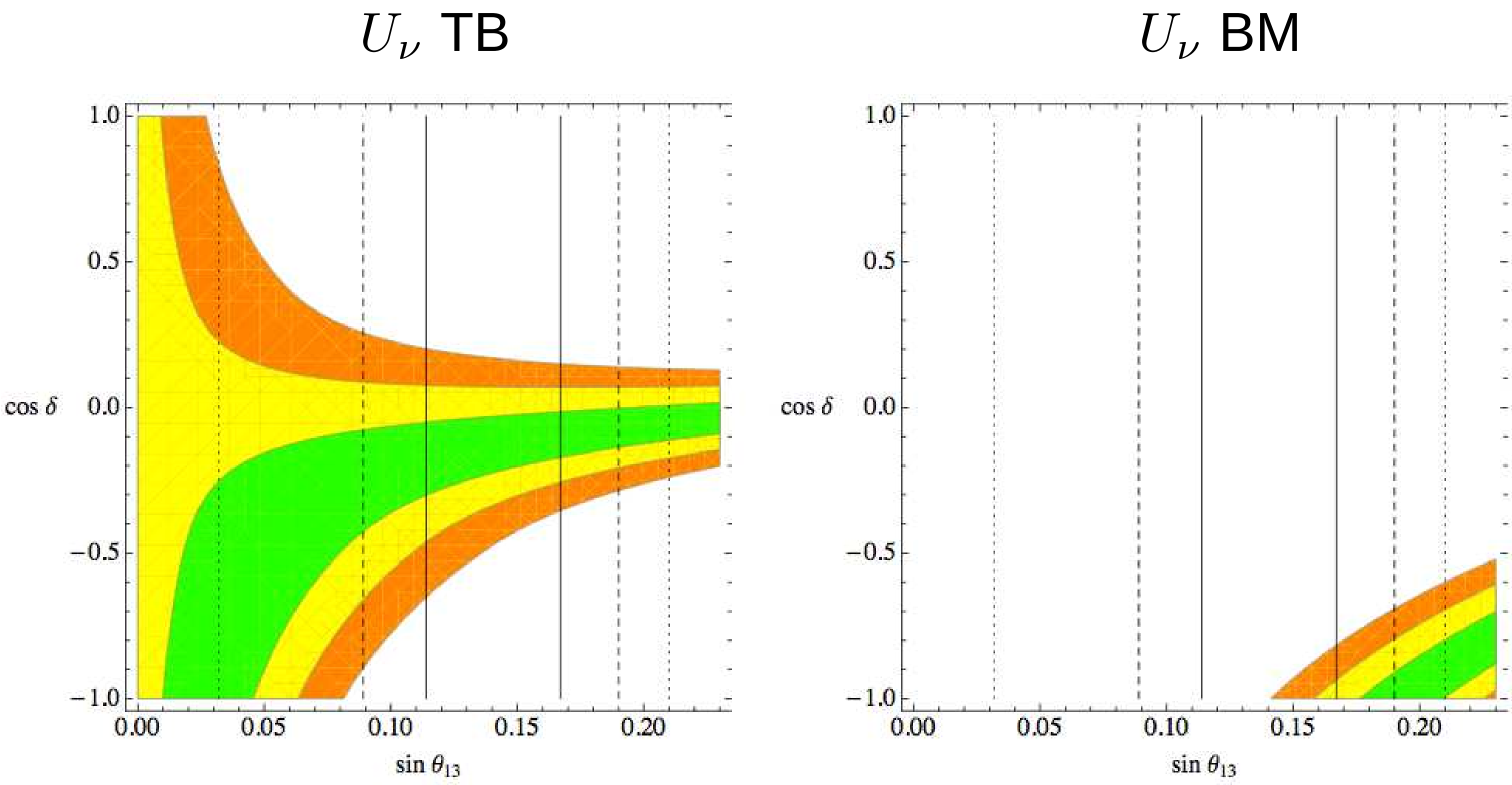
- $U_{PMNS} = U_e^\dagger \, \Psi \, U_\nu$
- U_ν is TB or bimaximal (BM)
- $U_e \neq \mathbb{1}$, but

$$U_e = R_{12}\,(\theta^e_{12})$$

- Ψ diagonal matrix with phase

Leptonic CP phases: corrections

(Marzocca et al. ('11))



N σ regions of $\sin^2\theta_{12}$

Leptonic CP phases: corrections

(Petcov ('14), Ballett et al. ('14), Girardi et al. ('14))

$U_{PMNS} = U_e^\dagger \, \Psi \, U_\nu$ with $U_e = R_{23}\,(\theta_{23}^e)\; R_{12}\,(\theta_{12}^e)$ and Ψ phases

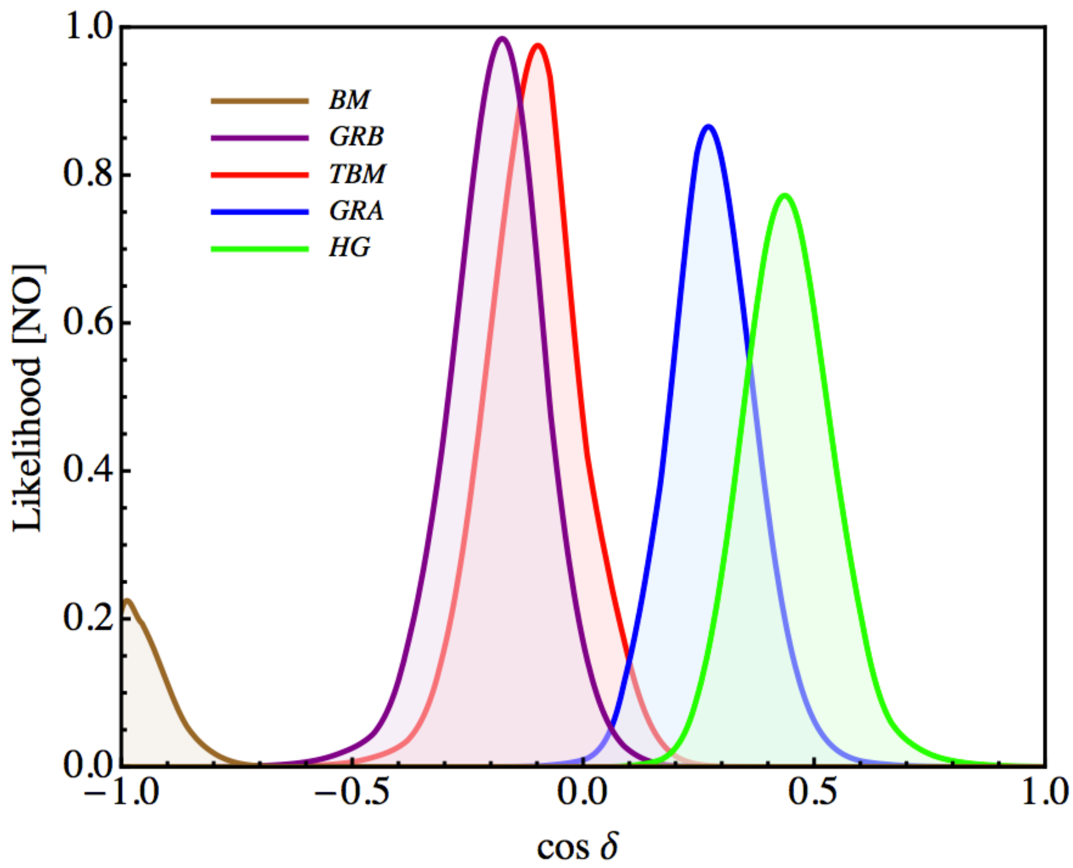
and U_ν is TB, BM, golden ratio (GR) mixings as well as hexagonal mixing
(all have in common $\theta_{13}^\nu = 0$ and $\theta_{23}^\nu = \pi/4$)

Relation of Dirac phase δ, lepton mixing angles θ_{ij} and neutrino mixing angle θ_{12}^ν

$$\cos\delta = \frac{\tan\theta_{23}\sin^2\theta_{12} + \frac{\sin^2\theta_{13}\cos^2\theta_{12}}{\tan\theta_{23}} - \sin^2\theta_{12}^\nu\left(\tan\theta_{23} + \frac{\sin^2\theta_{13}}{\tan\theta_{23}}\right)}{\sin 2\theta_{12}\sin\theta_{13}}$$

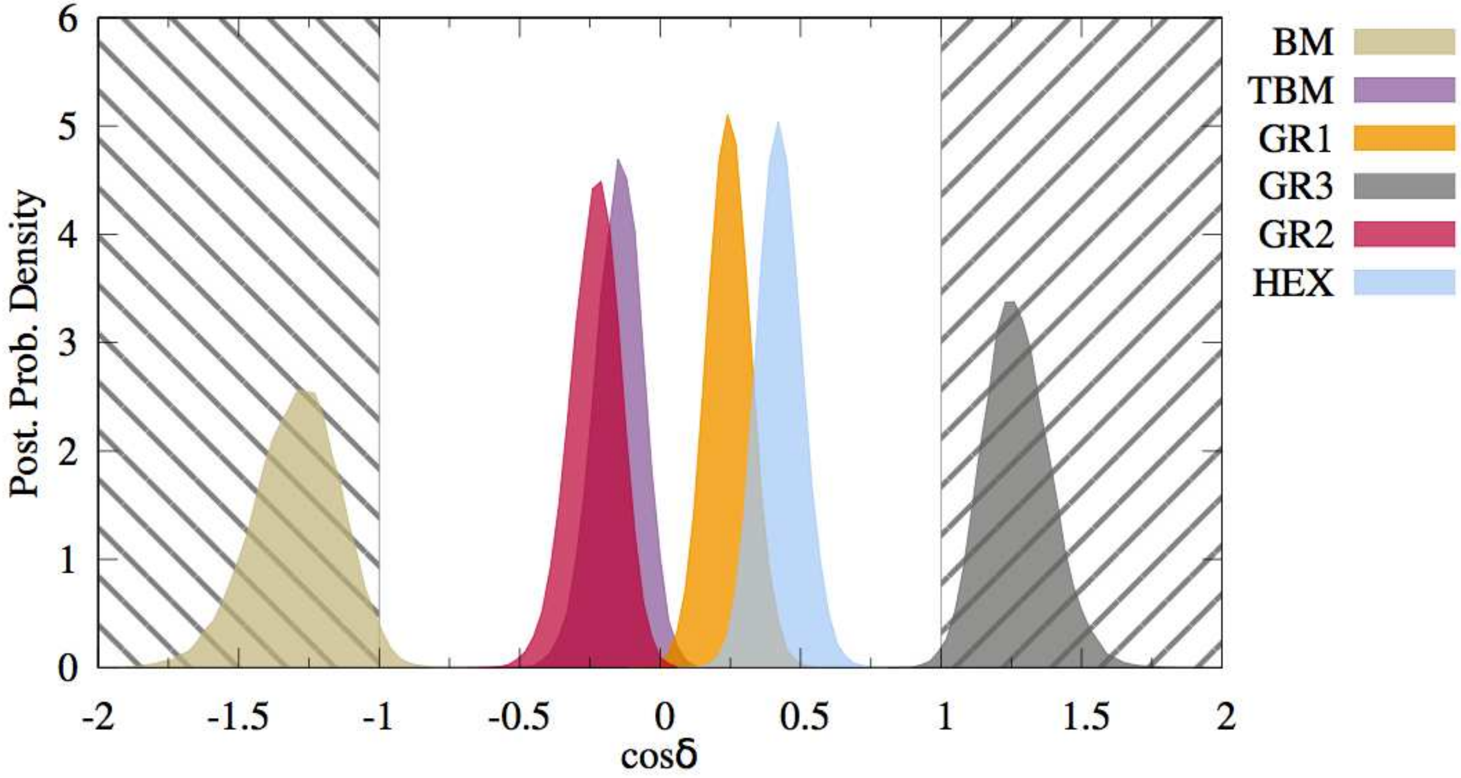
Leptonic CP phases: corrections

(Girardi et al. ('14))



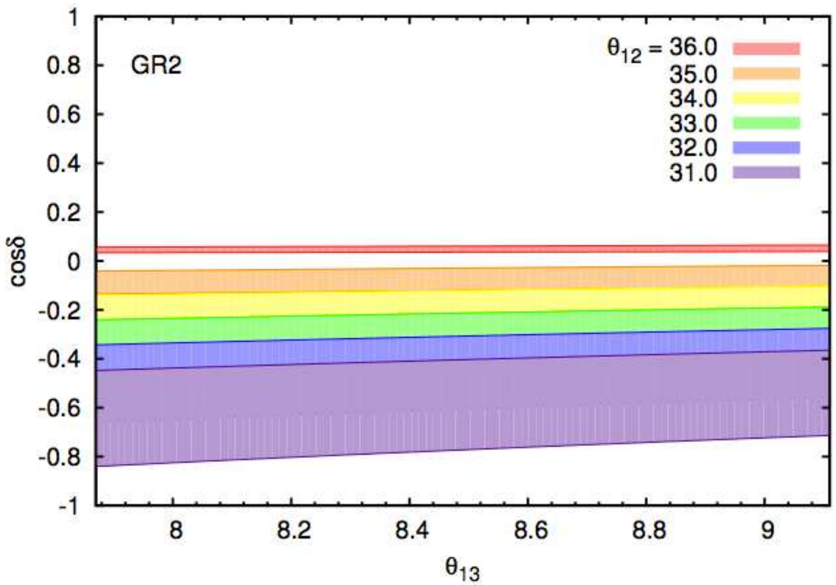
Leptonic CP phases: corrections

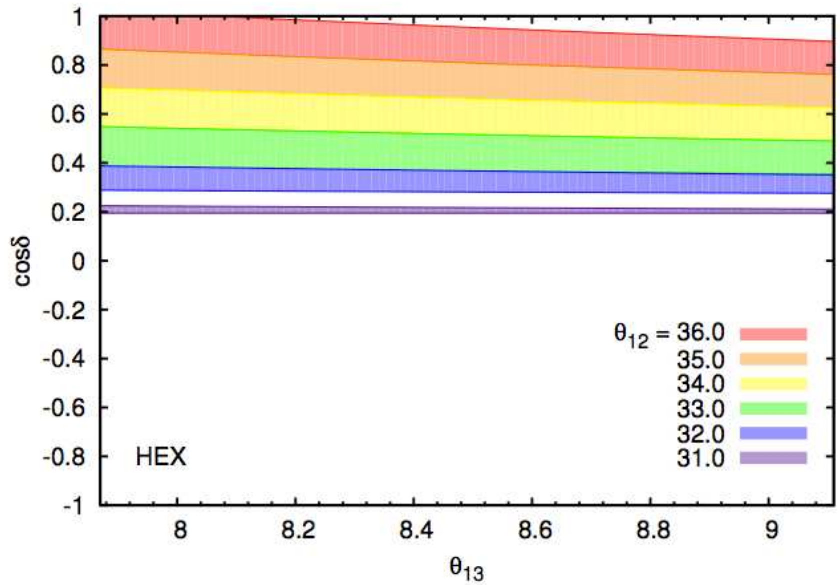
(Ballett et al. ('14))



Leptonic CP phases: corrections

(Ballett et al. ('14))





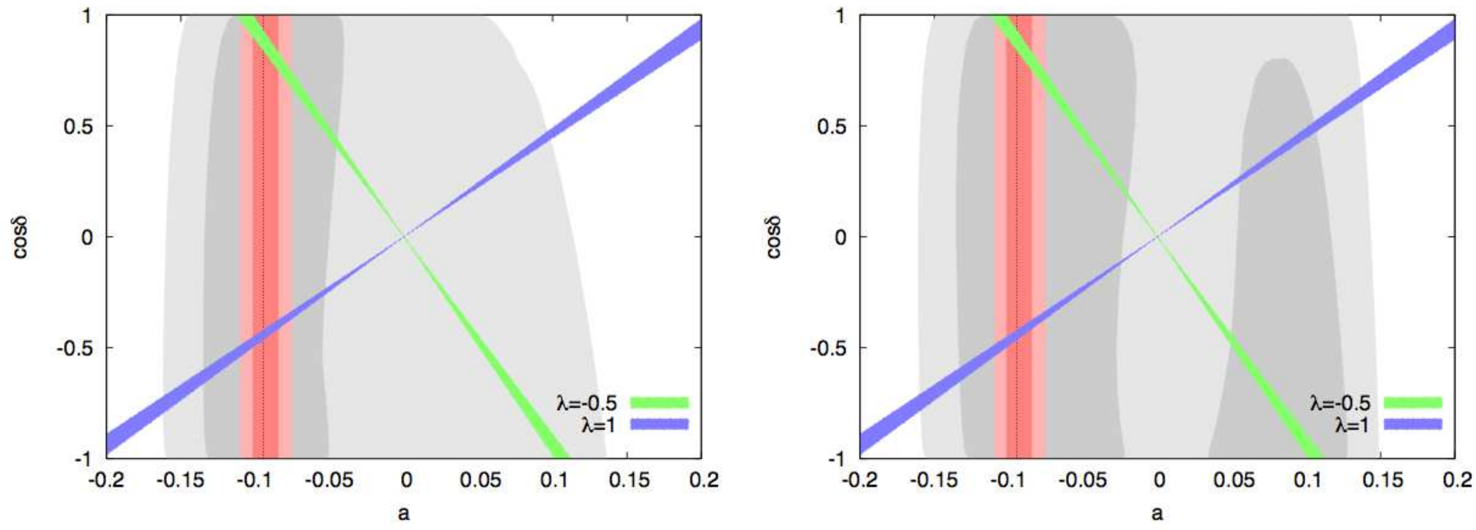
width of bands is generated by varying θ_{23}

Leptonic CP phases: corrections

Sum rule

(Ballett et al. ('13))

$$a \approx a_0 + \lambda\, r\, \cos\delta \quad \text{with} \quad \sin\theta_{13} = \frac{r}{\sqrt{2}} \quad \text{and} \quad \sin\theta_{23} = \frac{1+a}{\sqrt{2}}$$



(a) Normal mass ordering (b) Inverted mass ordering

choice here: $a_0 = 0$

Leptonic CP phases: corrections

(Dasgupta/Smirnov ('14))

Ansatz

$$U_{PMNS} = D(\gamma)\, V_{CKM}^{\dagger}\, U_X\, D(\beta) \quad \text{with} \quad U_X \quad \text{is real}$$

and U_{PMNS} fits experimental data

Note: this reminds of quark-lepton complementarity

Dirac phase

$$\sin\delta \approx -\sin\delta_q \frac{\sin\theta_{13}^q}{\sin\theta_{13}} \cos\theta_{23} \times (1 + 2\,\sin\theta_{13}\,\tan\theta_{23}\,\cot 2\theta_{12})$$

$$\approx -\sin\delta_q\,\lambda^2 \quad \text{and} \quad \lambda \approx 0.22$$

Leptonic CP phases: corrections

(Dasgupta/Smirnov ('14))

Ansatz

$$U_{PMNS} = D(\gamma)\, V_{CKM}^{\dagger}\, U_X\, D(\beta) \quad \text{with} \quad U_X \quad \text{is real}$$

and U_{PMNS} fits experimental data

Note: this reminds of quark-lepton complementarity

Dirac phase $\quad \sin\delta \approx -\sin\delta_q\,\lambda^2 \quad$ and $\quad \lambda \approx 0.22$

Majorana phases $\quad \beta_1 \approx \sin\delta_q\, \sin\theta_{13}^q\, \sin\theta_{23}\, \cot\theta_{12} \approx \sin\delta_q\,\lambda^3$

$\beta_2 \approx -\sin\delta_q\, \sin\theta_{13}^q\, \sin\theta_{23}\, \tan\theta_{12} \approx -\sin\delta_q\,\lambda^3$

All leptonic CP phases are suppressed.

Large CP phases can arise from neutrino sector.

Leptonic CP phases: CP symmetry

Involve CP as symmetry into the game!

(Feruglio/H/Ziegler ('12,'13), Holthausen et al. ('12), Chen et al. ('14), Grimus/Rebelo ('95))

G_f and CP

$\swarrow$ $\searrow$

charged leptons | neutrinos

G_e | $G_\nu = Z_2 \times$ CP

U_e | U_ν

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger U_\nu$$

Leptonic CP phases: CP symmetry

Comments:

- again, masses are not fixed in this approach
- well-known example is "μ-τ reflection symmetry"

  (Harrison/Scott ('02,'04), Grimus/Lavoura ('03))

$$\nu_\mu \quad \to \quad \text{CP}\,(\nu_\tau) \quad \text{and} \quad \nu_\tau \quad \to \quad \text{CP}\,(\nu_\mu)$$

consequences

$$|U_{\mu i}| = |U_{\tau i}| \quad \text{for} \quad i = 1, 2, 3$$

Leptonic CP phases: CP symmetry

Comments:

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- well-known example is "μ-τ reflection symmetry"

(Harrison/Scott ('02,'04), Grimus/Lavoura ('03))

$$\nu_\mu \;\to\; \text{CP}\,(\nu_\tau) \quad \text{and} \quad \nu_\tau \;\to\; \text{CP}\,(\nu_\mu)$$

consequences

$$\sin\theta_{23} = \cos\theta_{23}$$

$$\sin 2\theta_{12} \sin\theta_{13} \cos\delta = 0$$

Leptonic CP phases: CP symmetry

Comments:

- again, masses are not fixed in this approach
- well-known example is "μ-τ reflection symmetry"

  (Harrison/Scott ('02,'04), Grimus/Lavoura ('03))
- all CP phases and mixing angles are strongly correlated, since the PMNS mixing matrix only contains on free real parameter θ

Leptonic CP phases: CP symmetry

Involve CP as symmetry into the game!

(Feruglio/H/Ziegler ('12,'13), Holthausen et al. ('12), Chen et al. ('14), Grimus/Rebelo ('95))

$$G_f = S_4 \text{ and CP}$$

$\swarrow$ $\searrow$

charged leptons	neutrinos
$G_e = Z_3$	$G_\nu = Z_2 \times \text{CP}$
U_e	U_ν

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger U_\nu$$

Leptonic CP phases: CP symmetry

case I

(Feruglio/H/Ziegler ('12,'13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \ , \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} \ , \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1 \ , \quad |J_{CP}| = \frac{|\sin 2\theta|}{6\sqrt{3}} \ , \quad \sin\alpha = 0 \ , \quad \sin\beta = 0$$

Leptonic CP phases: CP symmetry

case I

(Feruglio/H/Ziegler ('12,'13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

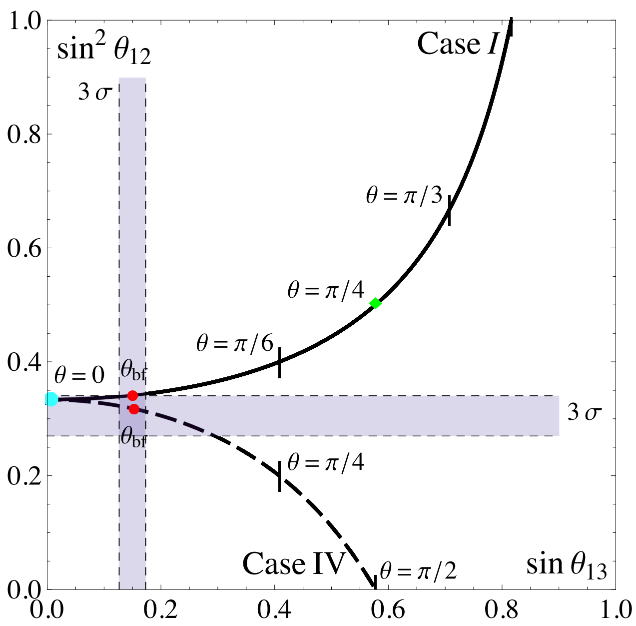
$$\sin^2\theta_{13} \approx 0.023 \;, \quad \sin^2\theta_{12} \approx 0.341 \;, \quad \sin^2\theta_{23} = \frac{1}{2}$$

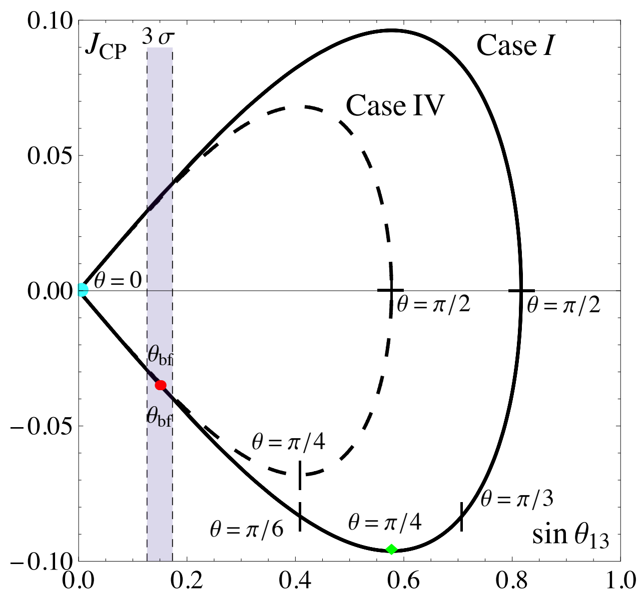
and

$$|\sin\delta| = 1 \;\;, \quad |J_{CP}| \approx 0.0348 \;\;, \quad \sin\alpha = 0 \;\;, \quad \sin\beta = 0 \quad \text{for} \quad \theta \approx 0.185$$

Leptonic CP phases: CP symmetry

(Feruglio/H/Ziegler ('12,'13))





Leptonic CP phases: CP symmetry

Involve CP as symmetry into the game!

(Di Iura/H/Meloni, in preparation)

$$G_f = A_5 \text{ and CP}$$

$\swarrow$ $\searrow$

charged leptons	neutrinos
$G_e = Z_5$	$G_\nu = Z_2 \times \text{CP}$
U_e	U_ν

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger U_\nu$$

Leptonic CP phases: CP symmetry

(Di Iura/H/Meloni, in preparation)

case II $\tan\varphi = 1/\phi$, $\phi = \frac{1}{2}\left(1+\sqrt{5}\right)$ and $\Phi = \frac{2\pi}{5}$

$$U_{PMNS} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2}\,\cos\varphi & -\sqrt{2}\,\sin\varphi & 0 \\ -\,e^{-3\,i\,\Phi}\,\sin\varphi & e^{-3\,i\,\Phi}\,\cos\varphi & -e^{-7\,i\,\Phi/4} \\ -e^{-2\,i\,\Phi}\,\sin\varphi & e^{-2\,i\,\Phi}\,\cos\varphi & e^{-3\,i\,\Phi/4} \end{pmatrix} R_{13}(\theta)\;K_\nu$$

$$\sin^2\theta_{13} = \frac{1}{10}\left(5+\sqrt{5}\right)\sin^2\theta\,, \quad \sin^2\theta_{12} = \frac{2}{2+(3+\sqrt{5})\,\cos^2\theta}, \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1\;, \quad |J_{CP}| = \frac{1}{20\,\sqrt{2}}\sqrt{5+\sqrt{5}}\,|\sin 2\theta|\;, \quad \sin\alpha = 0\;, \quad \sin\beta = 0$$

Leptonic CP phases: CP symmetry

(Di Iura/H/Meloni, in preparation)

case ||

$$U_{PMNS} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2}\,\cos\varphi & -\sqrt{2}\,\sin\varphi & 0 \\ -\,e^{-3\,i\,\Phi}\,\sin\varphi & e^{-3\,i\,\Phi}\,\cos\varphi & -e^{-7\,i\,\Phi/4} \\ -e^{-2\,i\,\Phi}\,\sin\varphi & e^{-2\,i\,\Phi}\,\cos\varphi & e^{-3\,i\,\Phi/4} \end{pmatrix} R_{13}(\theta)\,K_\nu$$

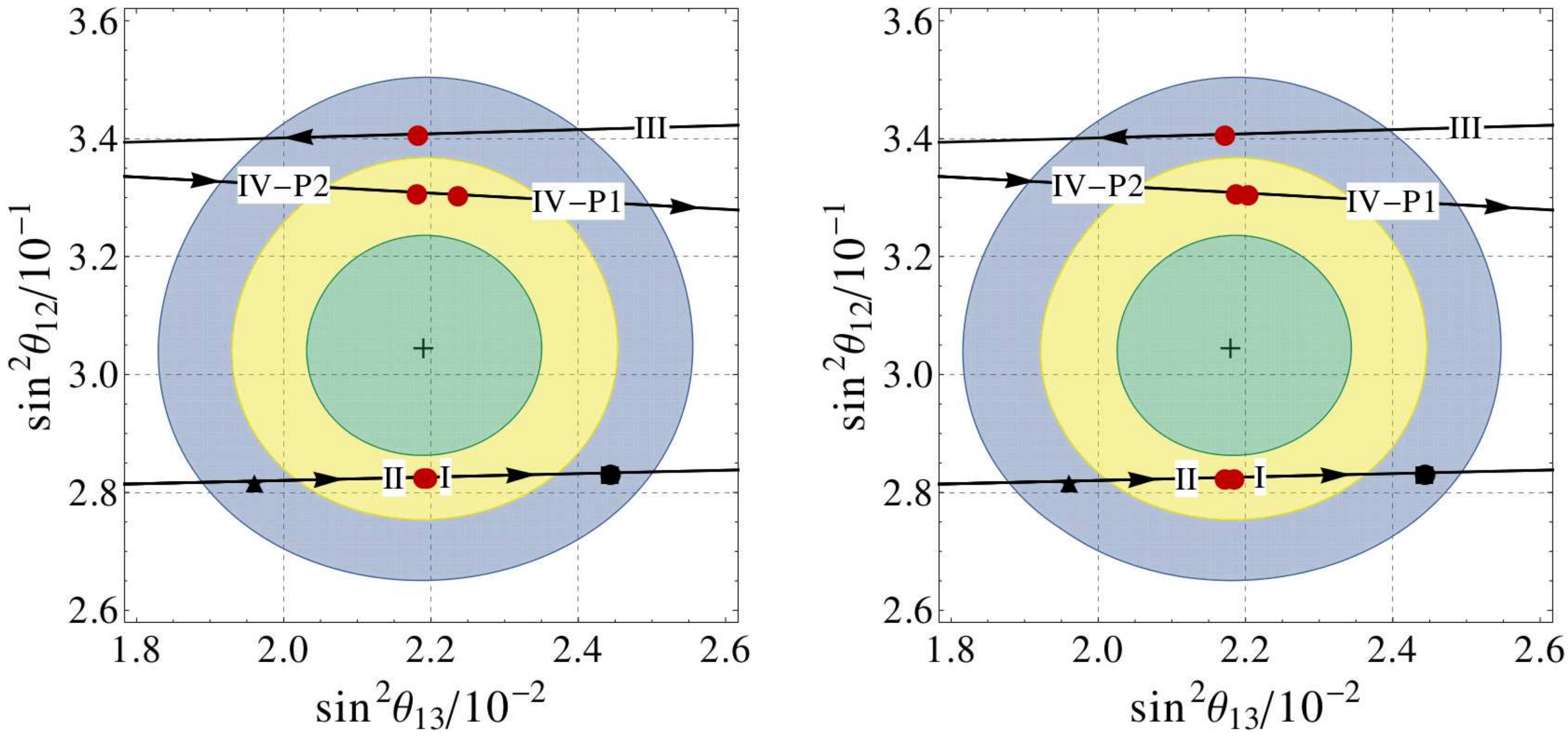
$$\sin^2\theta_{13} \approx 0.0219\;,\quad \sin^2\theta_{12} \approx 0.283,\quad \sin^2\theta_{23} = \tfrac{1}{2}$$

and

$$|\sin\delta| = 1\;,\quad |J_{CP}| \approx 0.0325\;,\quad \sin\alpha = 0\;,\quad \sin\beta = 0 \quad \text{for} \quad \theta \approx 0.175$$

Leptonic CP phases: CP symmetry

(Di Iura/H/Meloni, in preparation)



Leptonic CP phases: CP symmetry

(Everett et al. ('15))

Side remark:

- if $G_\nu = Z_2 \times Z_2 \times$ CP, we recover GR mixing with $\theta_{13} = 0$ and unphysical δ
- if CP is not constrained to correspond to automorphism of flavor group $G_f = A_5$, Majorana phases α and β can be non-trivial (and mixing angles are accommodated well)

Leptonic CP phases: CP symmetry

Involve CP as symmetry into the game!

(H/Meroni/Molinaro ('14); see also Ding et al. ('14))

$G_f = \Delta(3n^2),\ \Delta(6n^2)$ and CP

$\swarrow \qquad \searrow$

charged leptons	neutrinos
$G_e = Z_3$	$G_\nu = Z_2 \times$ CP
U_e	U_ν

$\searrow \qquad \swarrow$

$$U_{PMNS} = U_e^\dagger U_\nu$$

Leptonic CP phases: CP symmetry

(H/Meroni/Molinaro ('14))

Case 1)

$$U_{PMNS} = \Omega_1 \, R_{13}(\theta) \, K_\nu$$

with

$$\Omega_1 = e^{i\,\phi_s} \, U_{TB} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-3\,i\,\phi_s} & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

and

$$\phi_s = \frac{\pi s}{n}$$

Leptonic CP phases: CP symmetry

(H/Meroni/Molinaro ('14))

Case 1)

$$U_{PMNS} = \Omega_1 \, R_{13}(\theta) \, K_\nu$$

This mixing matrix has a trimaximal column. Thus, $\sin^2\theta_{12} \gtrsim 1/3$.

Results for CP phases

$$\sin\delta = 0 \quad \text{and} \quad \sin\beta = 0$$

$$\sin\alpha = (-1)^{k_1+1} \, \sin 6\,\phi_s$$

Very similar results are obtained, if $G_\nu = Z_2 \times Z_2 \times$ CP

(King/Neder ('14))

Leptonic CP phases: CP symmetry

(H/Meroni/Molinaro ('14))

Case 2)

$$U_{PMNS} = \Omega_2 \, R_{13}(\theta) \, K_\nu$$

with

$$\Omega_2 = e^{i\,\phi_v/6} \, U_{TB} \, R_{13}\left(-\frac{\phi_u}{2}\right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\,\phi_v/2} & 0 \\ 0 & 0 & -i \end{pmatrix}$$

and

$$\phi_u = \frac{\pi u}{n} \quad \text{and} \quad \phi_v = \frac{\pi v}{n}$$

Leptonic CP phases: CP symmetry

(H/Meroni/Molinaro ('14))

Case 2)

$$U_{PMNS} = \Omega_2 \, R_{13}(\theta) \, K_\nu$$

This mixing matrix has a trimaximal column. Thus, $\sin^2\theta_{12} \gtrsim 1/3$.

Approximate results for CP phases

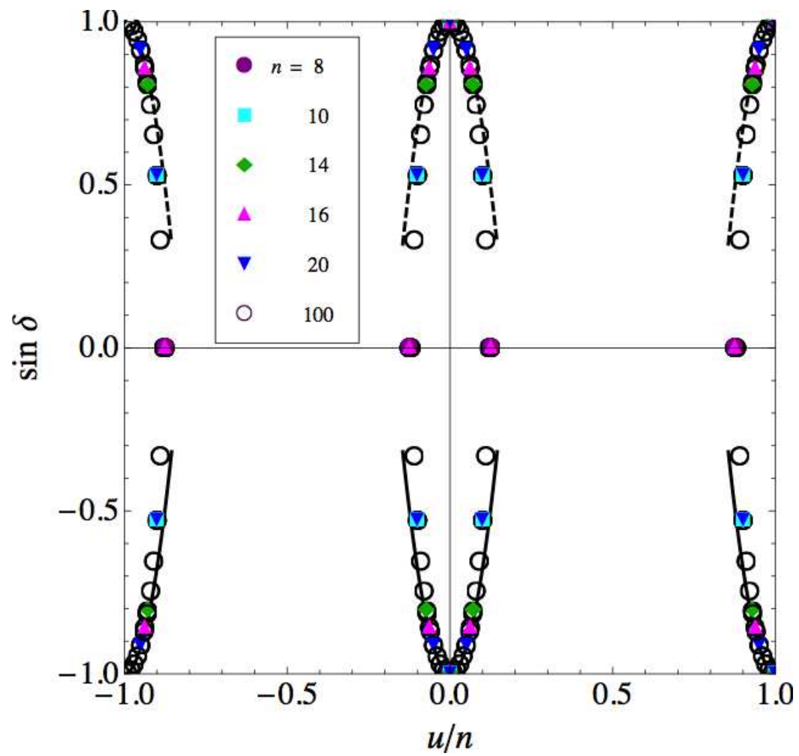
$$\sin\delta \approx \pm 1 \mp 3.3 \left(\phi_u - \bar{\phi}\right)^2$$

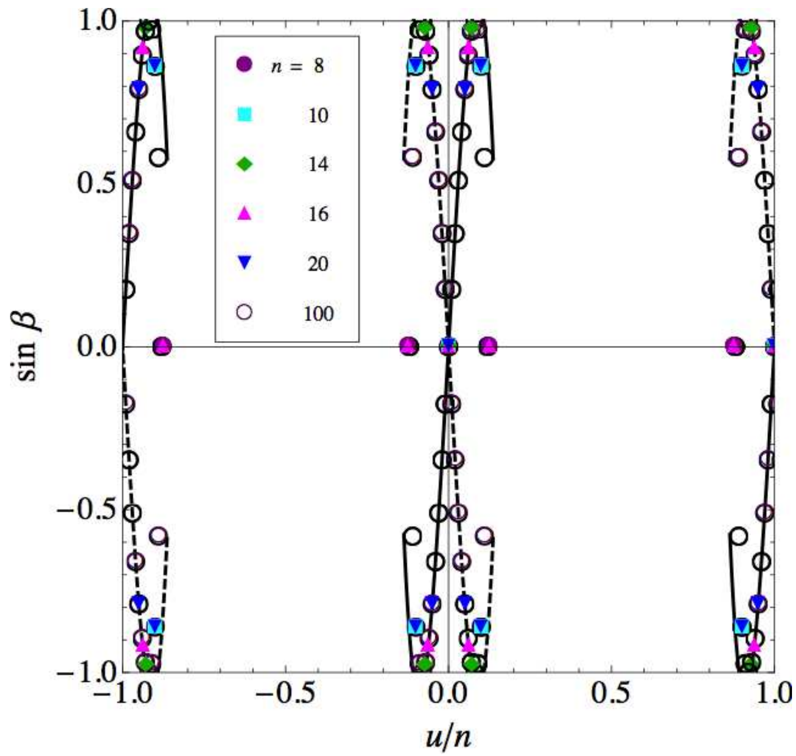
$$\sin\beta \approx \mp 5.6 \left(\phi_u - \bar{\phi}\right) \pm 23 \left(\phi_u - \bar{\phi}\right)^3 \quad \text{for} \quad \bar{\phi} = 0, \pm\pi$$

$$\sin\alpha \approx -\sin\phi_v \quad \text{for} \quad \bar{\phi} = 0$$

Leptonic CP phases: CP symmetry

Results for Dirac phase δ and Majorana phase β
(with θ expressed in u/n)

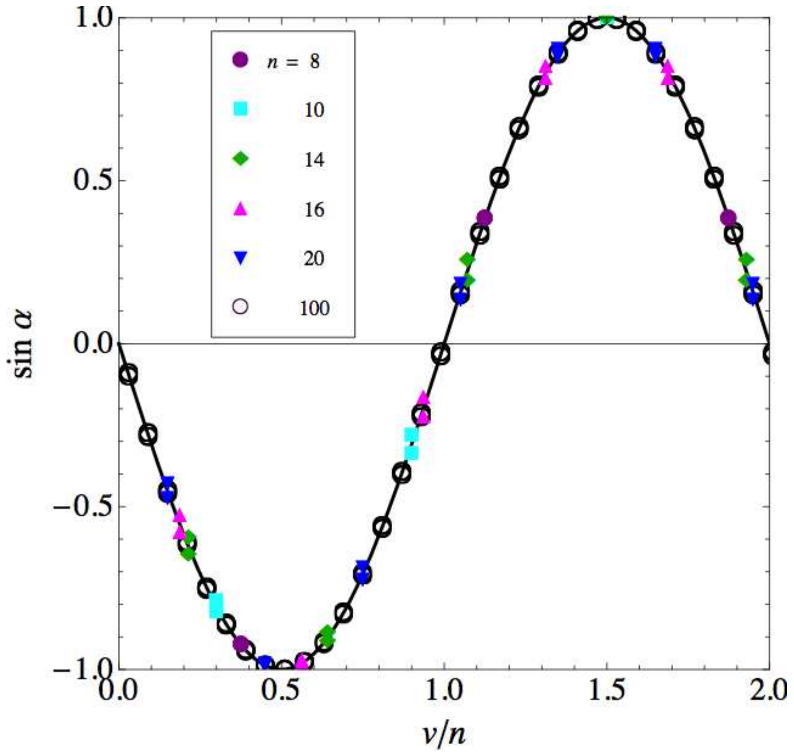




Leptonic CP phases: CP symmetry

Results for Majorana phase α

(with θ and u fixed with $u = 1$)



Leptonic CP phases: CP symmetry

Numerical example: $n = 8$

u	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin\delta$	$\sin\beta$
$u = 0$	0.0218	0.341	0.5	1	0
$u = -1$	0.0254	0.342	0.387	0	0
$u = 1$	0.0254	0.342	0.613	0	0

values of $\sin\alpha$ for $u = 0$

$\sin\alpha = 0$, $\sin\alpha = 1$ and $\sin\alpha = -1/\sqrt{2}$

values of $\sin\alpha$ for $u = \pm 1$

$\sin\alpha \approx -0.924$ and $\sin\alpha \approx 0.383$

Leptonic CP phases: CP symmetry

(H/Meroni/Molinaro ('14))

Case 3a)

$$U_{PMNS} = \Omega_3 \, R_{12}(\theta) \, K_\nu$$

with

$$\Omega_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \Omega_1 \, R_{13}(\phi_m)$$

and

$$\phi_m = \frac{\pi m}{n}$$

We find $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ as functions of ϕ_m.

Leptonic CP phases: CP symmetry

(H/Meroni/Molinaro ('14))

Case 3a)

$$U_{PMNS} = \Omega_3 \, R_{12}(\theta) \, K_\nu$$

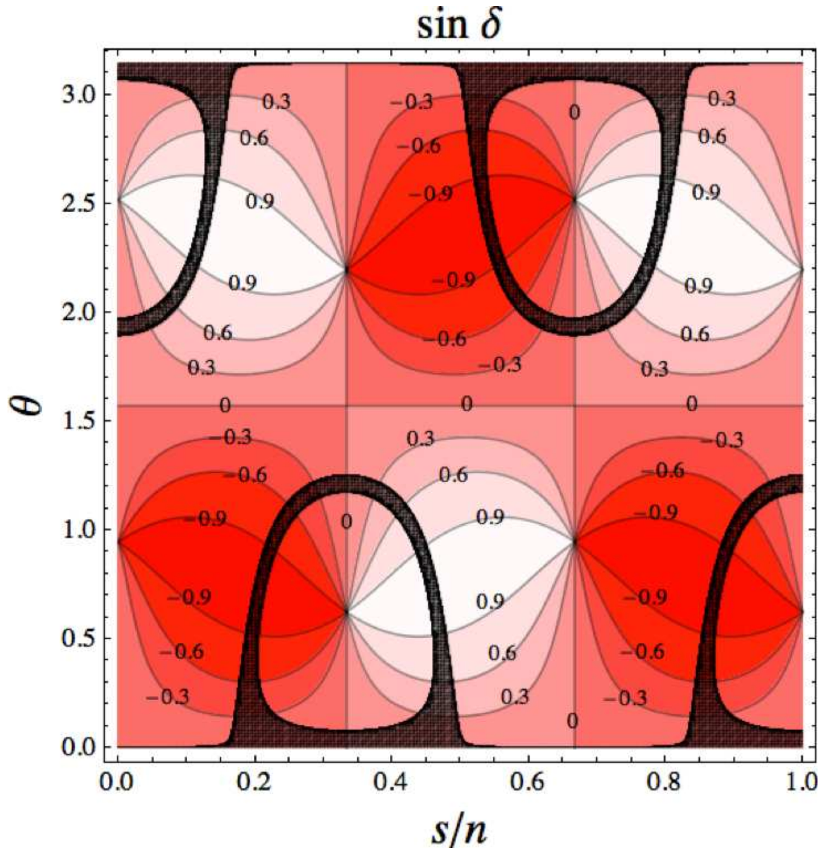
- $m/n = 1/16$ ($m/n = 15/16$) leads to good fit of data:

$$\sin^2\theta_{13} \approx 0.0254 \quad \text{and} \quad \sin^2\theta_{23} \approx \begin{cases} 0.613 \\ 0.387 \end{cases}$$

- solar mixing angle depends on additional parameters ϕ_s and θ

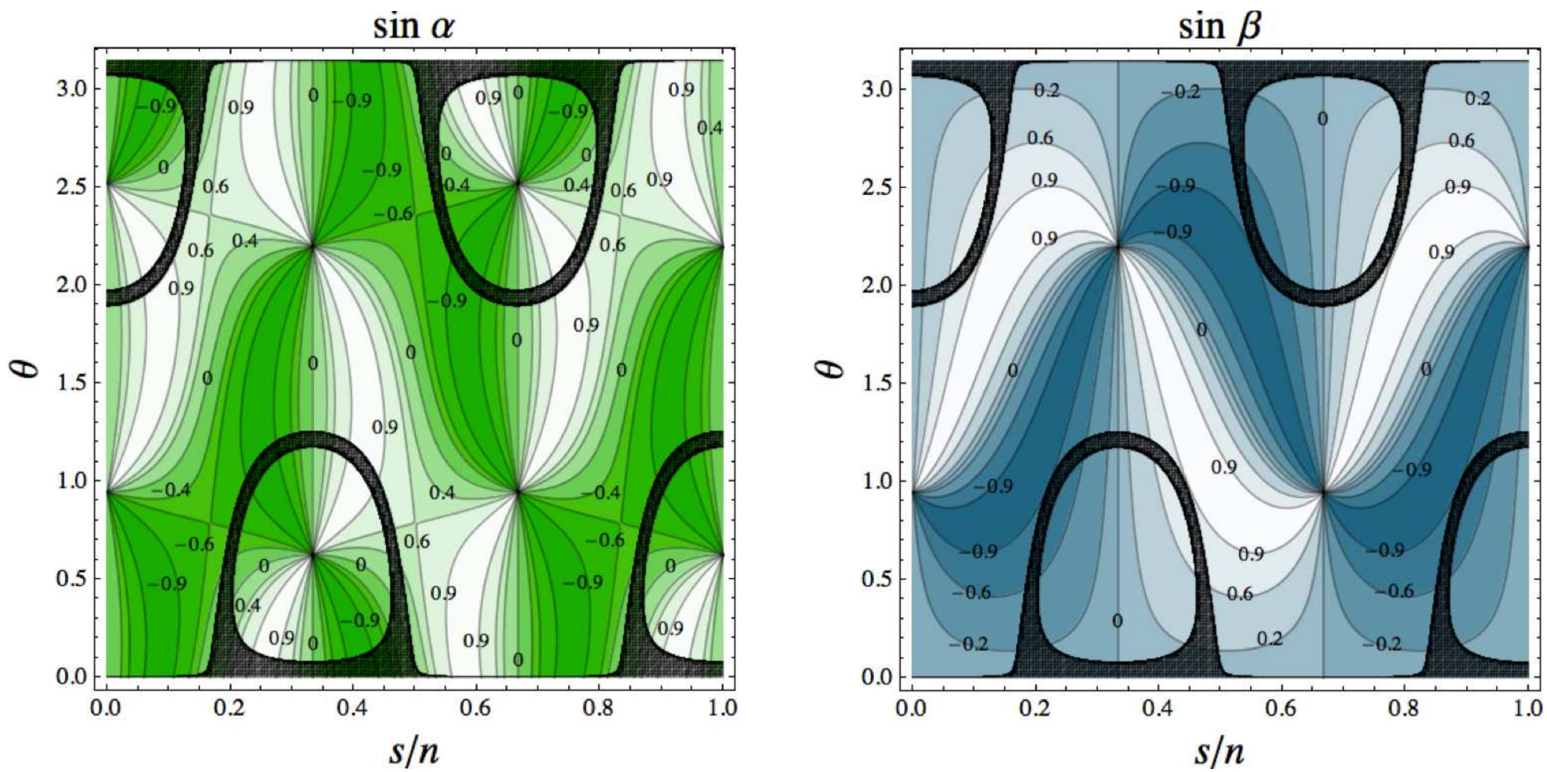
Leptonic CP phases: CP symmetry

Results for Dirac phase δ for $m/n = 1/16$



Leptonic CP phases: CP symmetry

Results for Majorana phases α and β for $m/n = 1/16$



Leptonic CP phases: CP symmetry

Numerical example: $n = 16$, $m = 1$

$\sin^2\theta_{13} \approx 0.0254$ and $\sin^2\theta_{23} \approx 0.613$

some viable choices of s

s	$\sin^2\theta_{12}$	$\sin\delta$	$\sin\alpha$	$\sin\beta$
$s = 0$	0.304	0	0	0
$s = 1$	0.304	0.458	0.939	0.662
	0.304	0.0594	-0.939	0.0383
$s = 3$	0.317	-0.533	0	-0.357

Leptonic CP phases: CP symmetry

(H/Meroni/Molinaro ('14))

Case 3b.1)

$$U_{PMNS} = \Omega_3 \, R_{12}(\theta) \, K_\nu \, P$$

- P permutation that changes ordering of columns
- for $m = n/2$ the first column is like in TB mixing
- thus the solar mixing angle is constrained

$$\sin^2 \theta_{12} \lesssim \frac{1}{3}$$

Leptonic CP phases: CP symmetry

(H/Meroni/Molinaro ('14))

Case 3b.1)

- for $m/n = 1/2$ also the expressions of the Majorana phases become very simple

$$\sin\alpha \propto \sin 6\,\phi_s \quad \text{and} \quad \sin\beta \propto \sin 6\,\phi_s$$

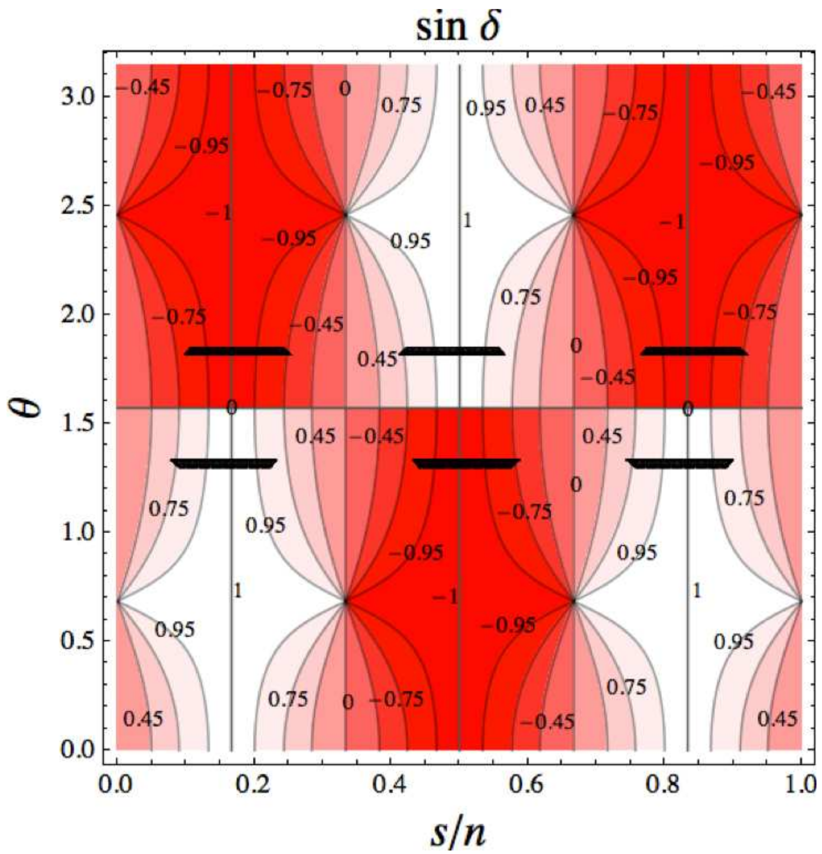
- only the expression for $\sin\delta$ is still complicated; however, lower limit of $|\sin\delta|$ is found

$$|\sin\delta| \gtrsim 0.71$$

if mixing angles are accommodated well

Leptonic CP phases: CP symmetry

Results for Dirac phase δ for $m/n = 1/2$ and $n = 20$



Leptonic CP phases: CP symmetry

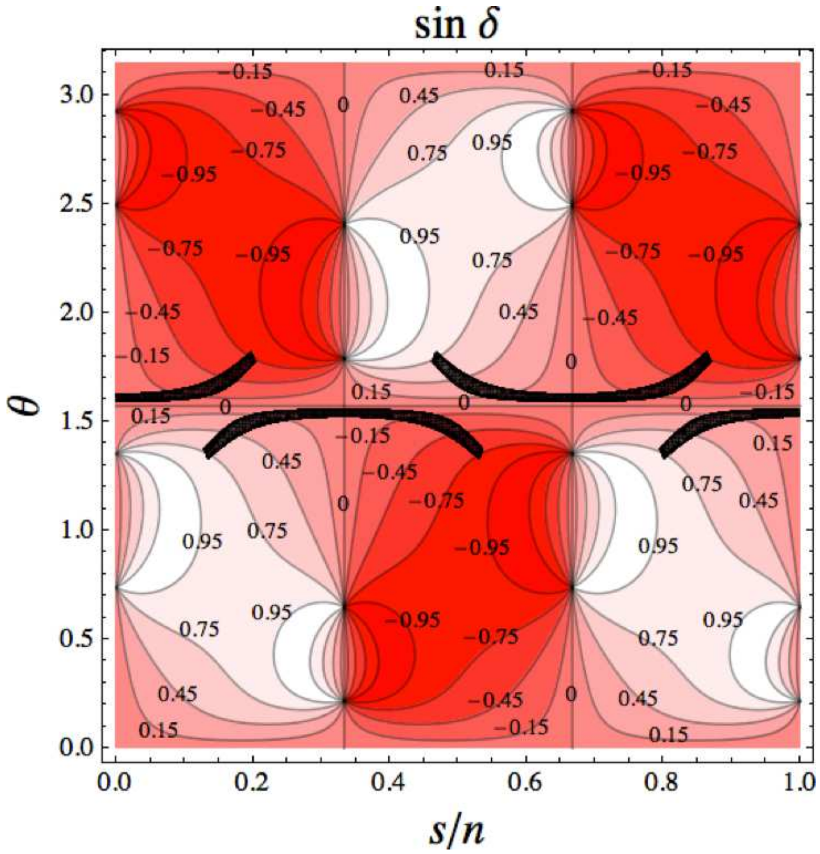
Numerical example: $n = 8$, $m = 4$

some viable choices of s

s	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin\delta$	$\sin\alpha = \sin\beta$
$s = 1$	0.0220	0.318	0.579	0.936	$-1/\sqrt{2}$
	0.0220	0.318	0.421	-0.936	$-1/\sqrt{2}$
$s = 2$	0.0216	0.319	0.645	-0.739	1
$s = 4$	0.0220	0.318	0.5	∓ 1	0

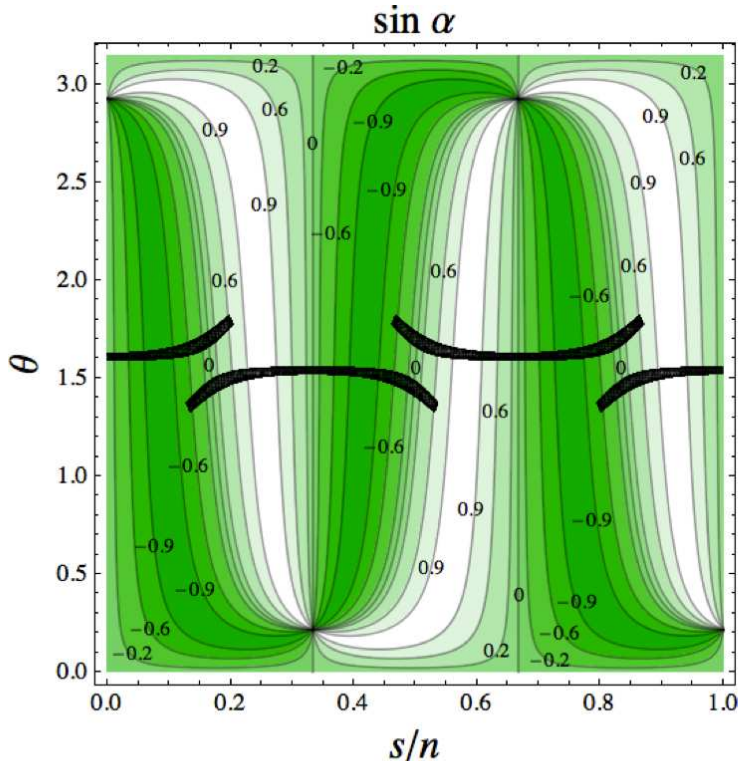
Leptonic CP phases: CP symmetry

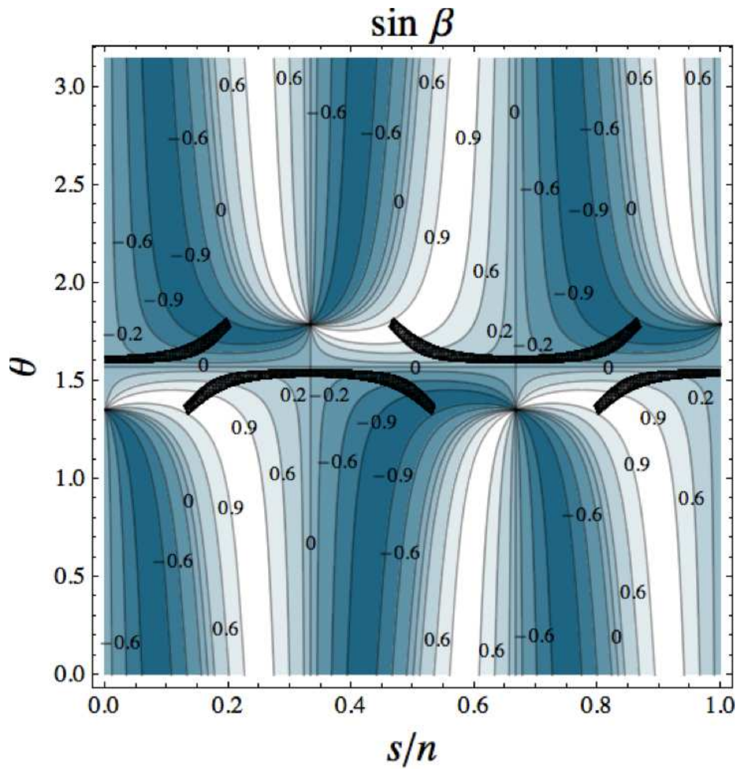
Results for Dirac phase δ for $m = 11$ and $n = 20$



Leptonic CP phases: CP symmetry

Results for Majorana phases α and β for $n = 20$ and $m = 1$





Leptonic CP phases: CP symmetry

Two CPs in neutrino sector

(Chen et al. ('14))

CP symmetries

$\swarrow$ $\searrow$

charged leptons
diagonal mass
matrix

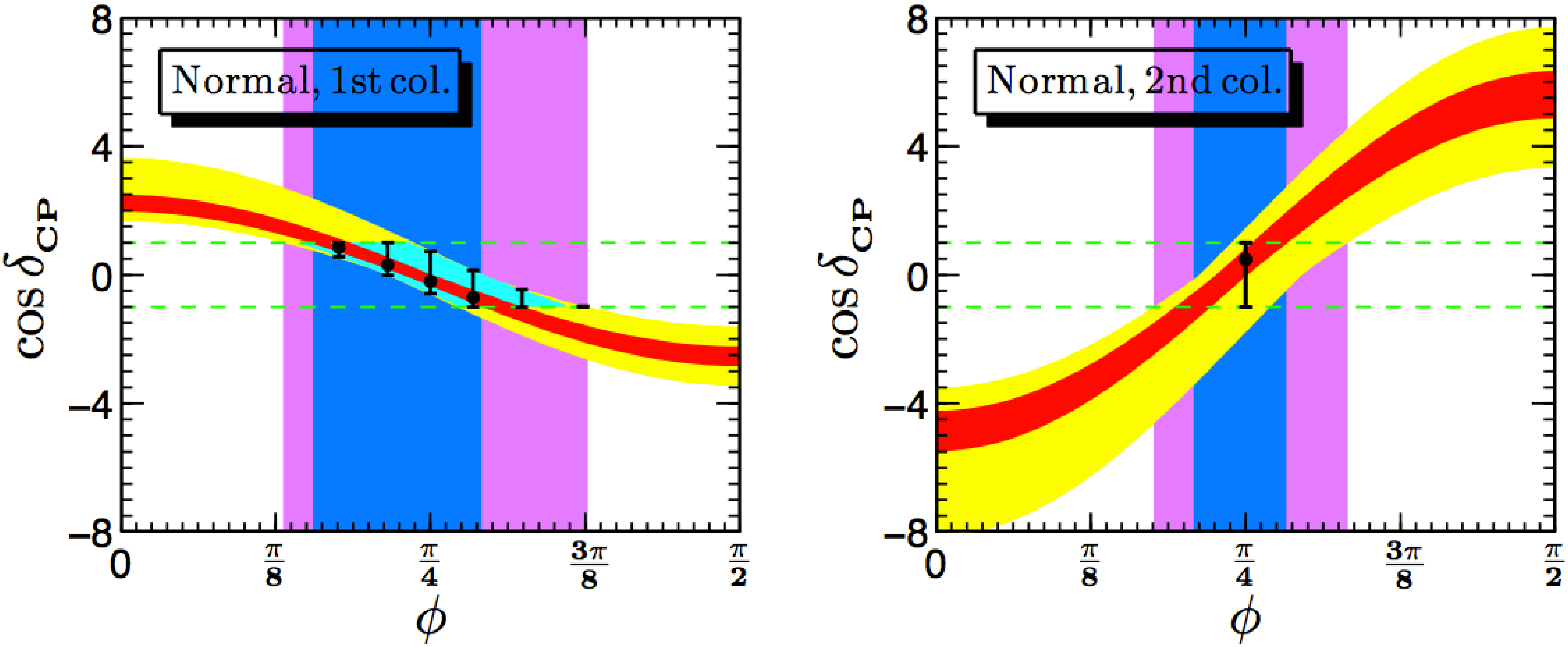
neutrinos
two CPs

$\searrow$ $\swarrow$

$$U_{PMNS} = U_e^\dagger U_\nu$$

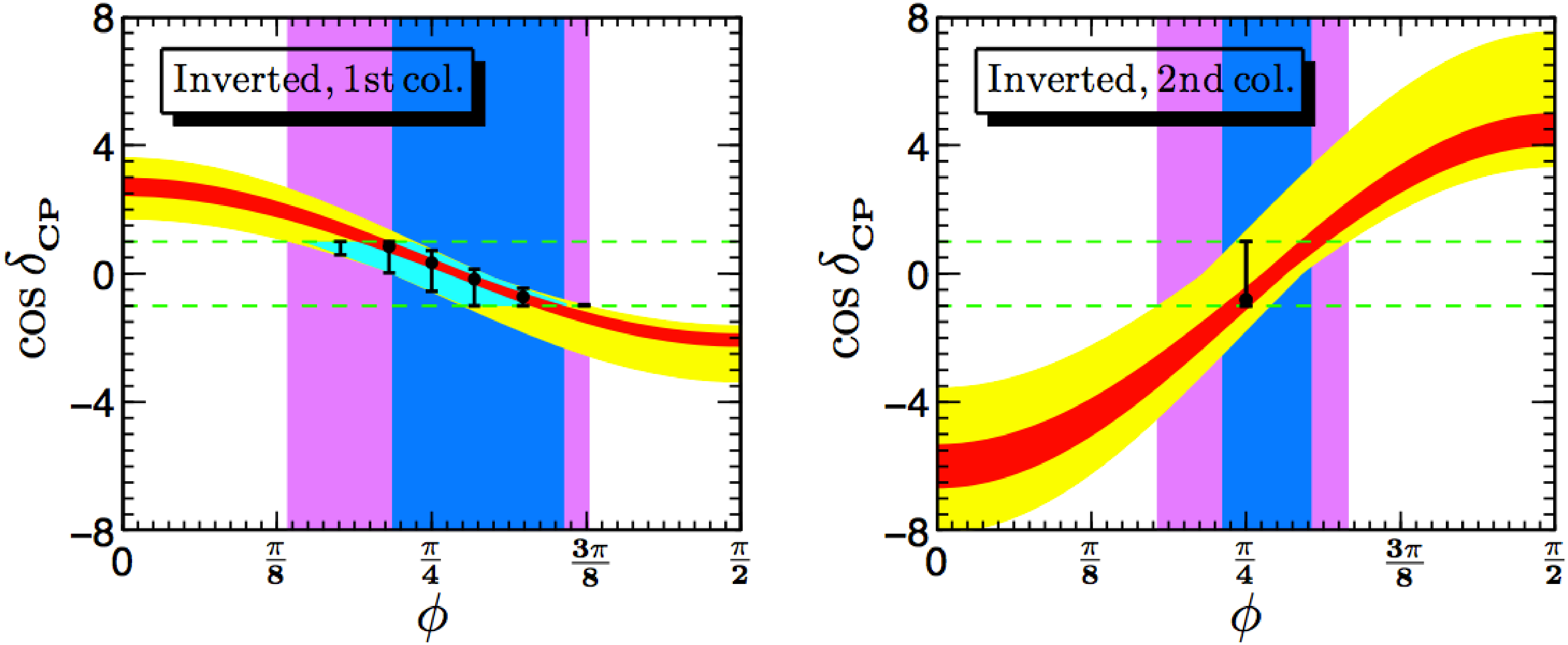
Leptonic CP phases: CP symmetry

(Chen et al. ('14))



Leptonic CP phases: CP symmetry

(Chen et al. ('14))



Leptonic CP phases: CP symmetry

(Chen et al. ('14))

Remarks:

- if the two CPs are combined, this transformation acts on flavor space
  (certain correspondence to approach with $G_\nu = Z_2 \times$ CP)
- PMNS mixing matrix contains one real free parameter θ
- if only one CP is preserved in neutrino sector, PMNS mixing matrix contains three real free parameters

Conclusions

- approach with flavor symmetry only leads to trivial Dirac phase
- if we assume less symmetry or corrections, values of δ different from 0 or π can be achieved
- approach with flavor and CP symmetry leads to highly constrained scenario (only one free real parameter):
  all mixing parameters are strongly correlated, in particular predictions for all CP phases can be made
  (variations of approach are possible)

Conclusions

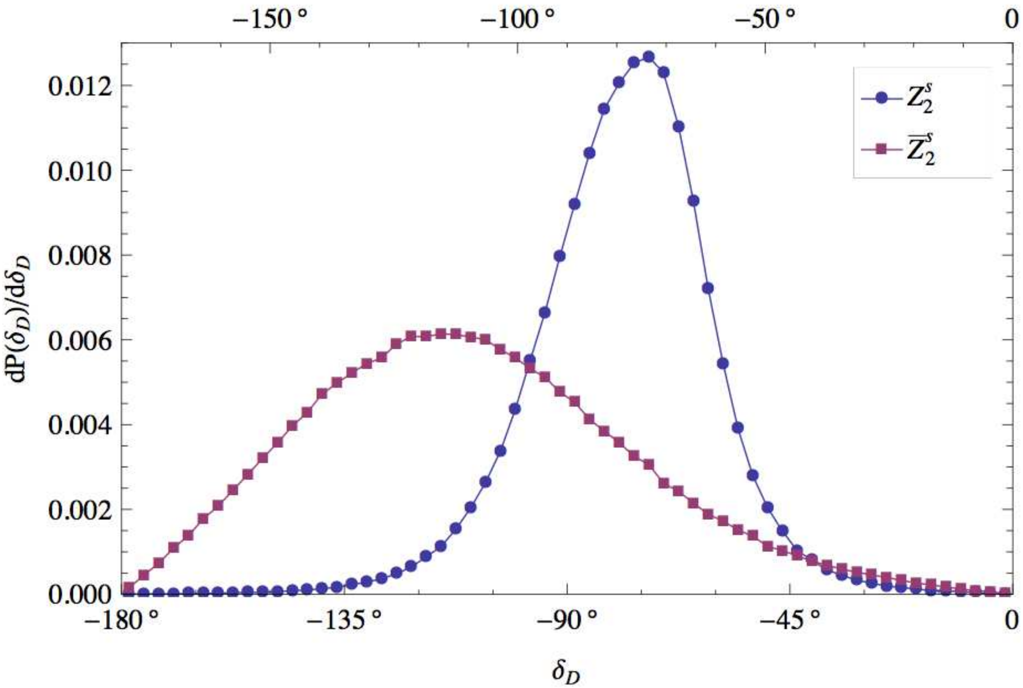
- phenomenology not limited to prediction of Dirac phase measured in neutrino oscillations, but Majorana phases in neutrinoless double beta decay and high energy phases relevant for leptogenesis can be also constrained

Thank you for your attention.

Back up Slides

Leptonic CP phases: less symmetry

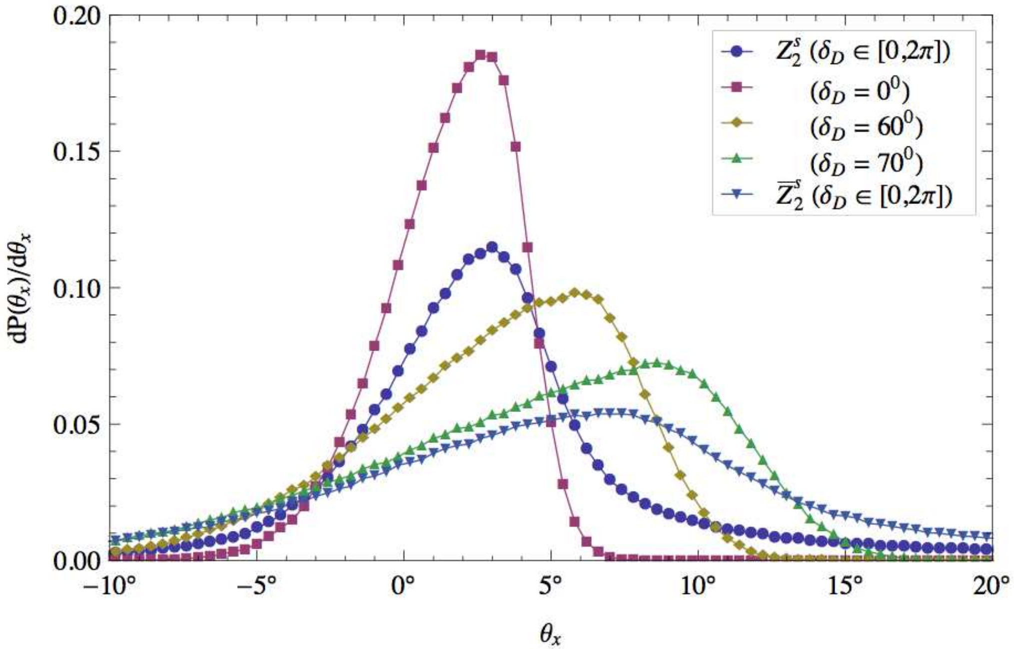
(Ge et al. ('11))



(a) Dirac CP Phase δ_D

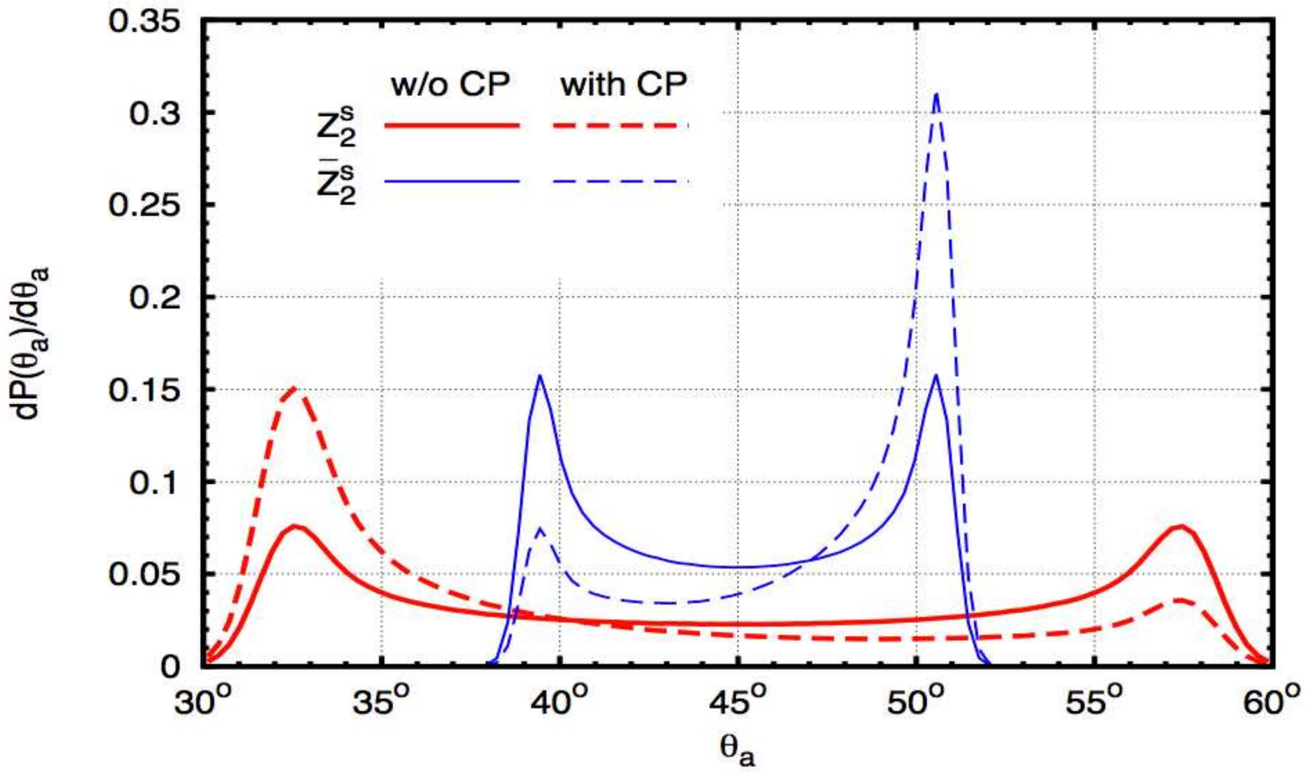
Leptonic CP phases: less symmetry

(Ge et al. ('11))



Leptonic CP phases: less symmetry

(Hanlon et al. ('13))



Leptonic CP phases: corrections

(Marzocca et al. ('13))

Ansatz

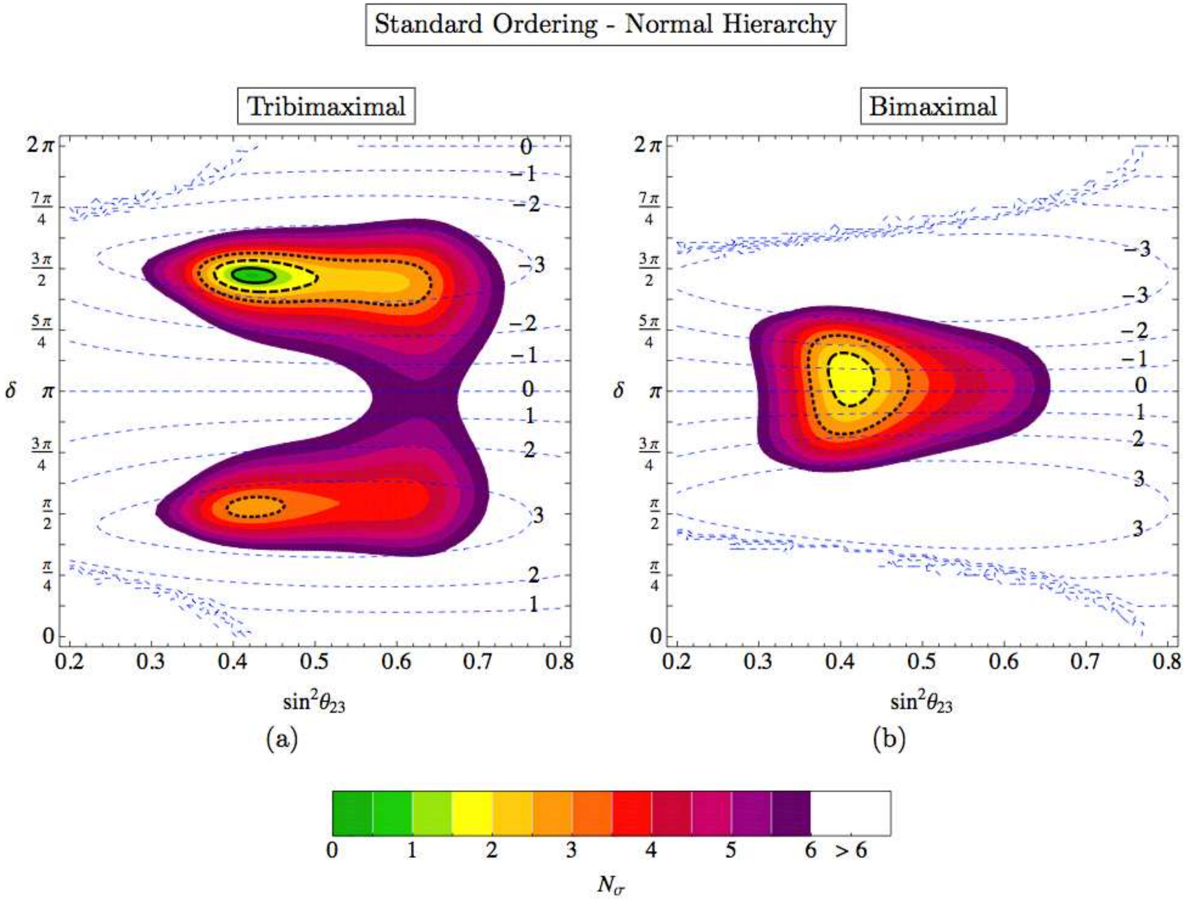
- $U_{PMNS} = U_e^\dagger \, \Psi \, U_\nu$
- U_ν is TB or BM
- U_e now contains two rotations:

$$R_{12}\left(\theta^e_{12}\right) \text{ and } R_{23}\left(\theta^e_{23}\right)$$

- Ψ diagonal matrix with phase

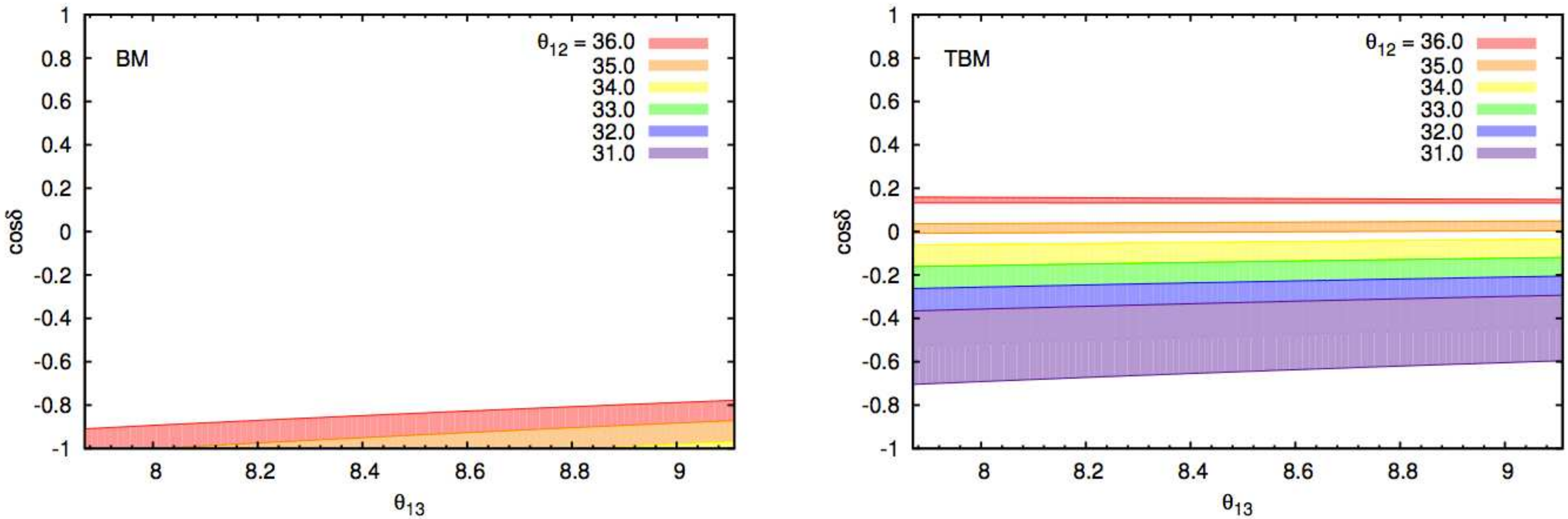
Leptonic CP phases: corrections

(Marzocca et al. ('13))



Leptonic CP phases: corrections

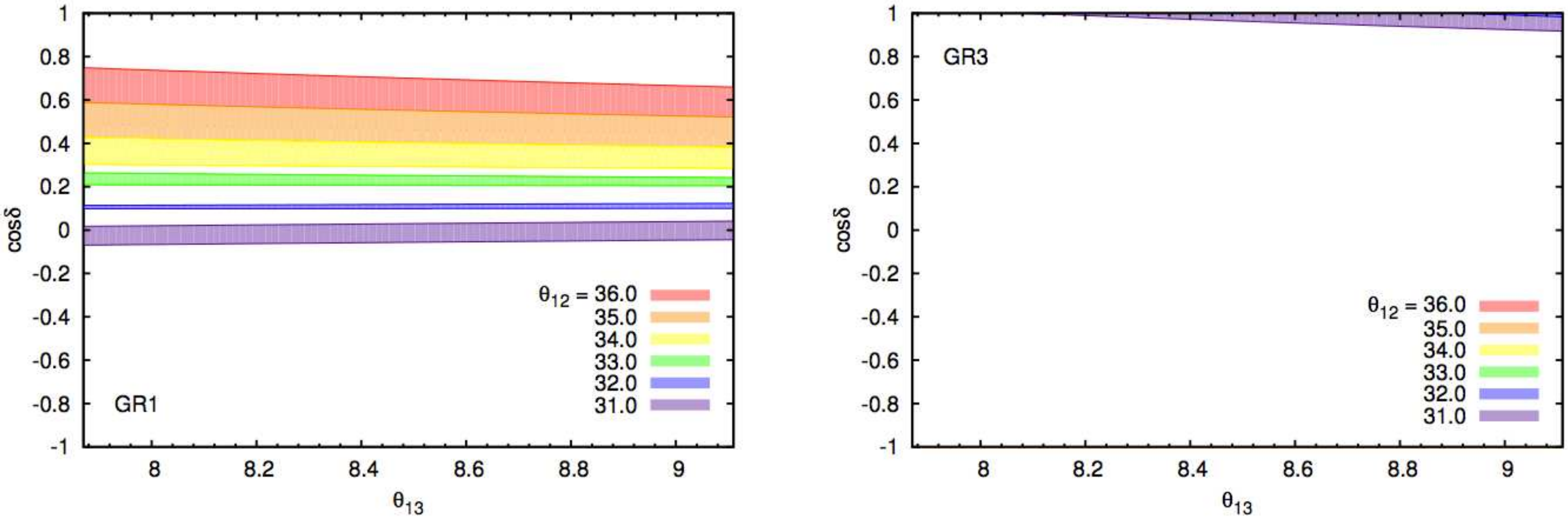
(Ballett et al. ('14))



width of bands is generated by varying θ_{23}

Leptonic CP phases: corrections

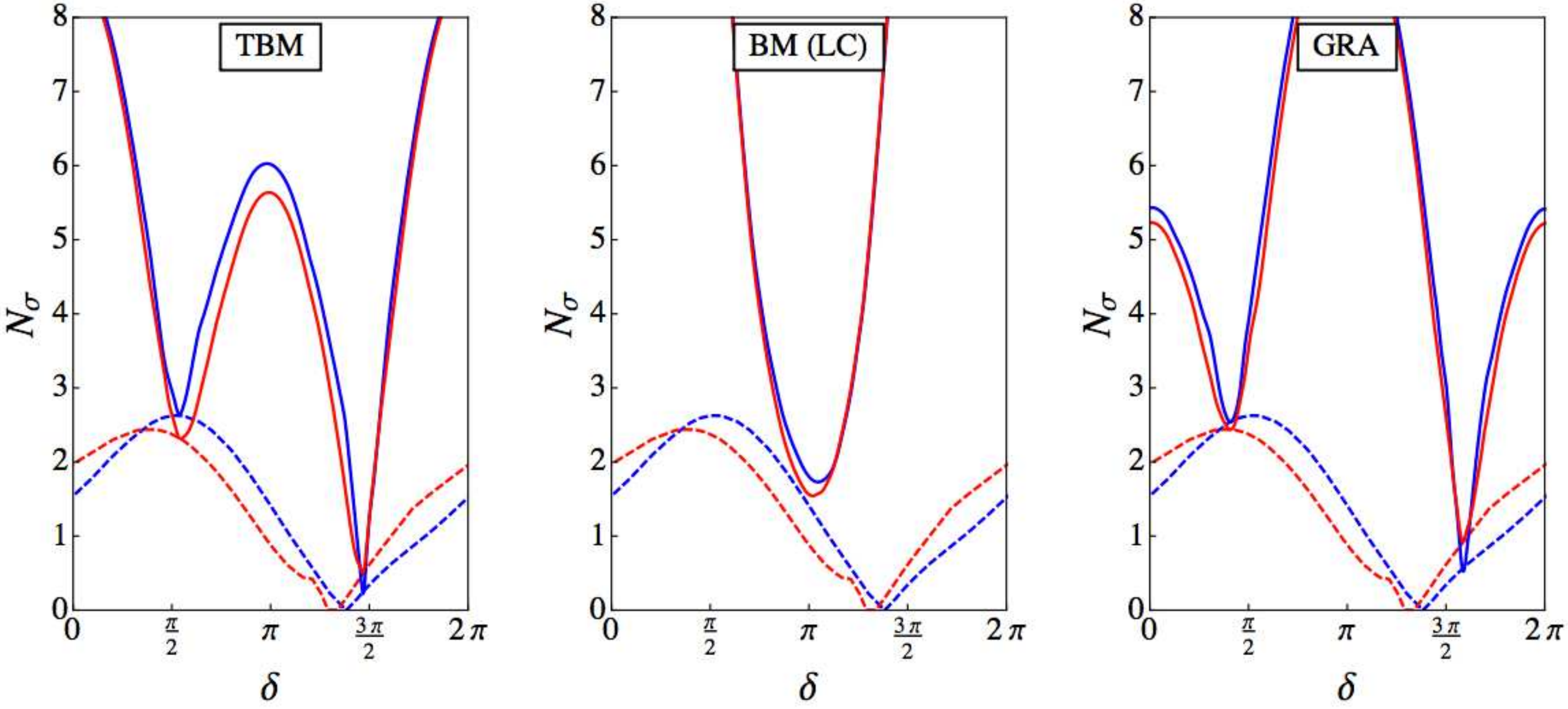
(Ballett et al. ('14))



width of bands is generated by varying θ_{23}

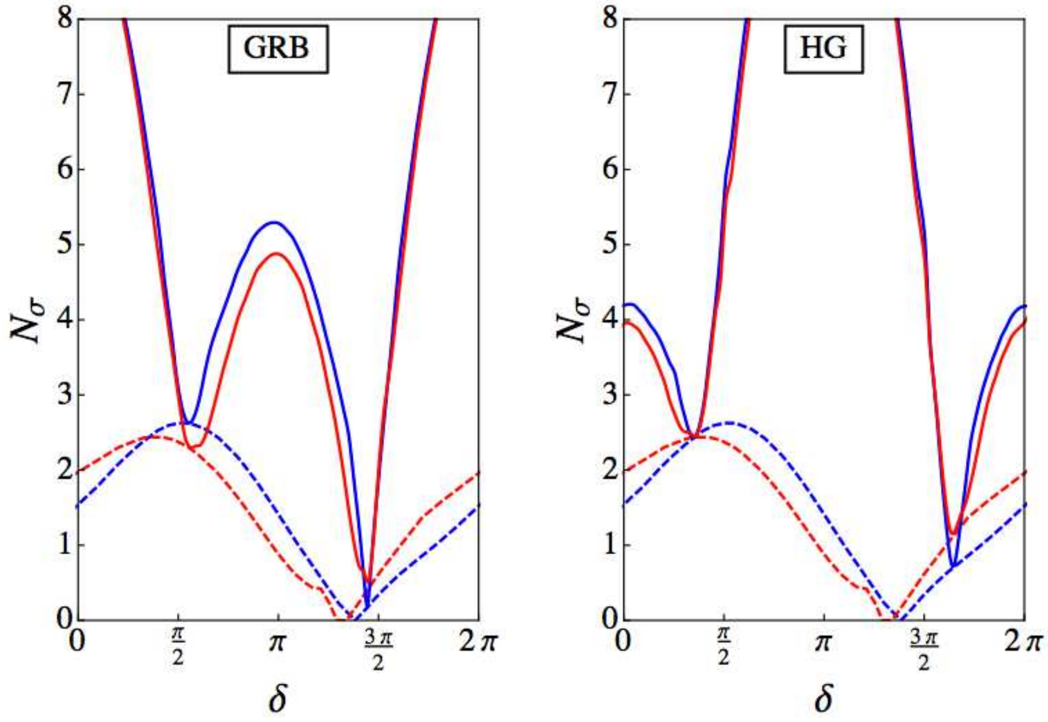
Leptonic CP phases: corrections

(Girardi et al. ('14))



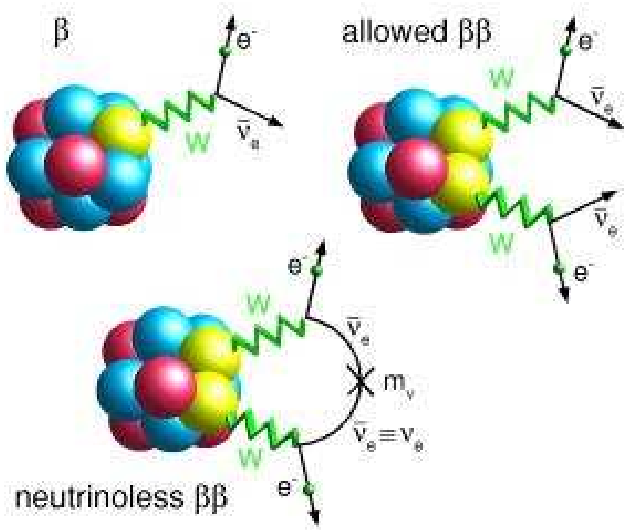
Leptonic CP phases: corrections

(Girardi et al. ('14))



Neutrinoless double beta decay

- neutrinos can be their own antiparticles
- if true, a process called $0\nu\beta\beta$ decay is allowed



Neutrinoless double beta decay

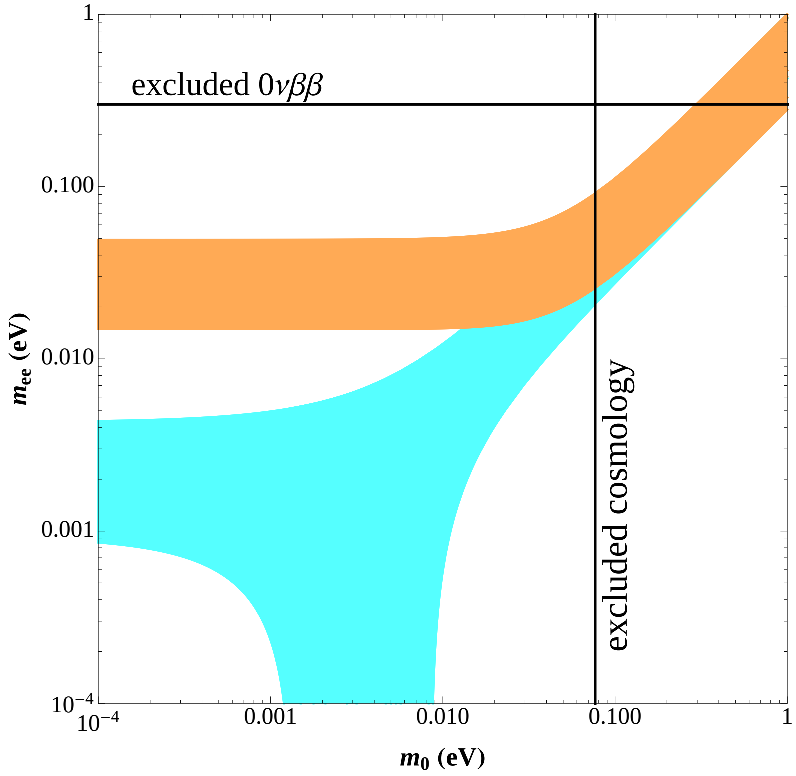
- neutrinos can be their own antiparticles
- if true, a process called $0\nu\beta\beta$ decay is allowed

$$m_{ee} = \left| \cos^2\theta_{12}\,\cos^2\theta_{13}\,m_1 + \sin^2\theta_{12}\,\cos^2\theta_{13}\,e^{i\alpha}\,m_2 + \sin^2\theta_{13}\,e^{i\beta}\,m_3 \right|$$

using the experimentally preferred 3 σ ranges of $\sin^2\theta_{13}$, $\sin^2\theta_{12}$ and of the mass splittings and
using both possible mass orderings (normal and inverted) and
varying the unknown Majorana phases α and β and the lightest neutrino mass m_0 we get ...

Neutrinoless double beta decay

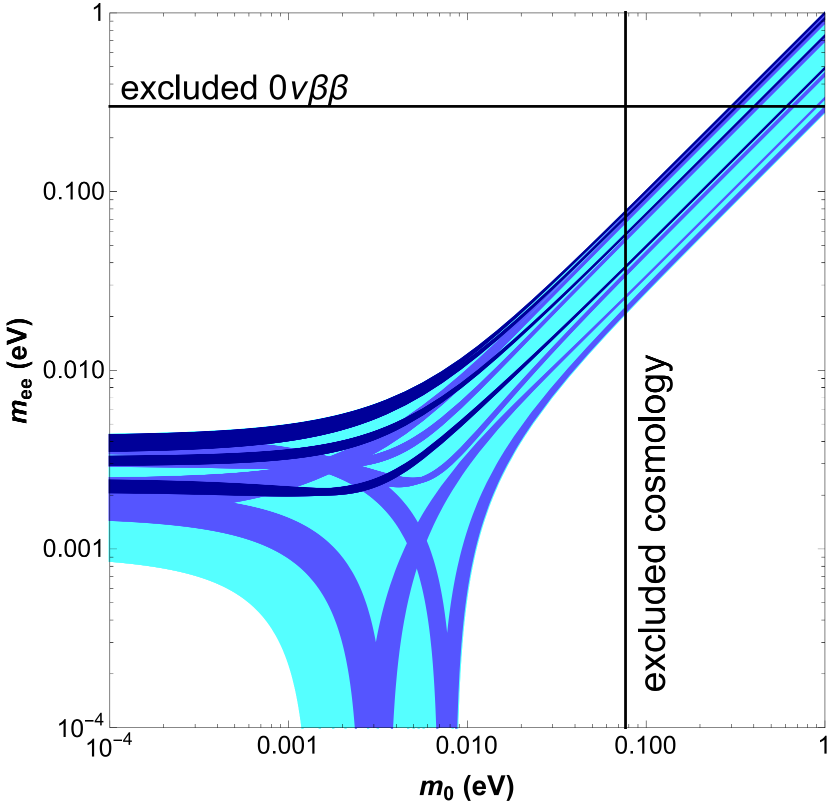
- neutrinos can be their own antiparticles
- if true, a process called $0\nu\beta\beta$ decay is allowed



Neutrinoless double beta decay

(H/Molinaro, in preparation)

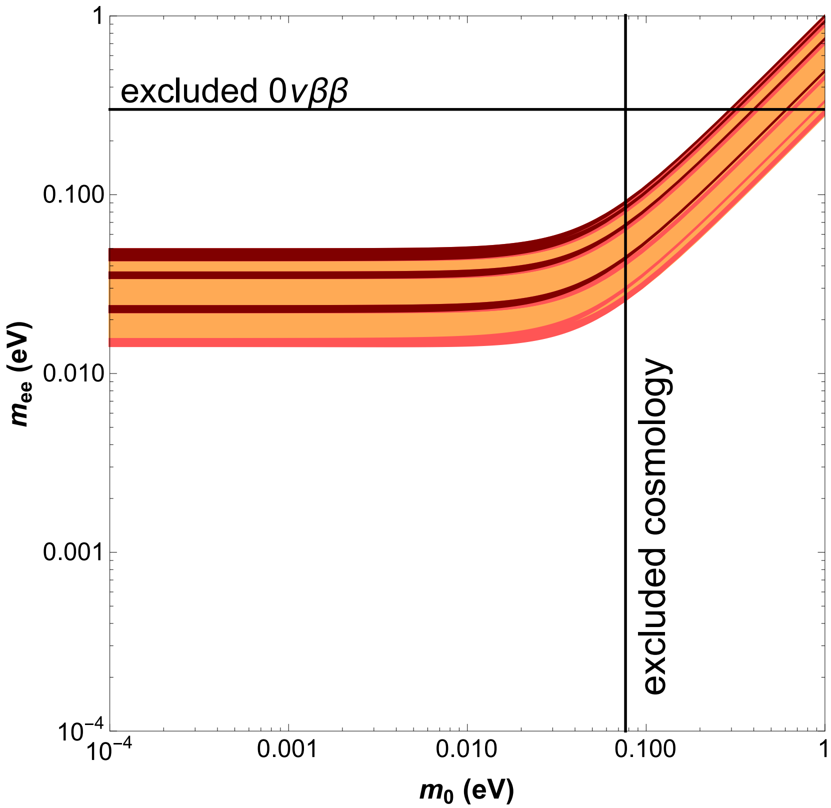
Case 2) with $n = 8$, $u = 0$ and normal ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

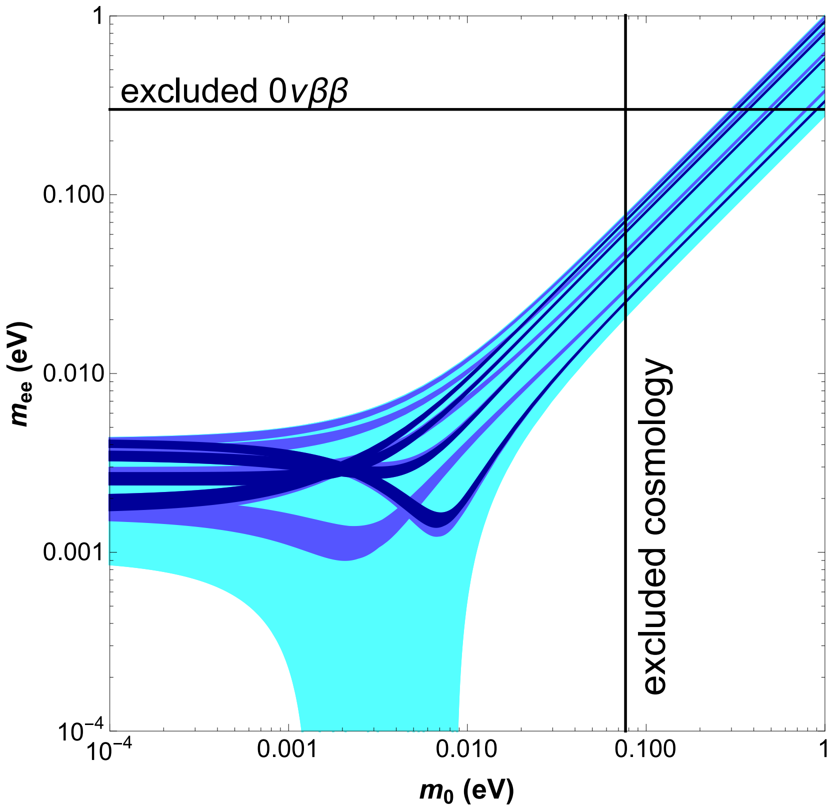
Case 2) with $n = 8$, $u = 0$ and inverted ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

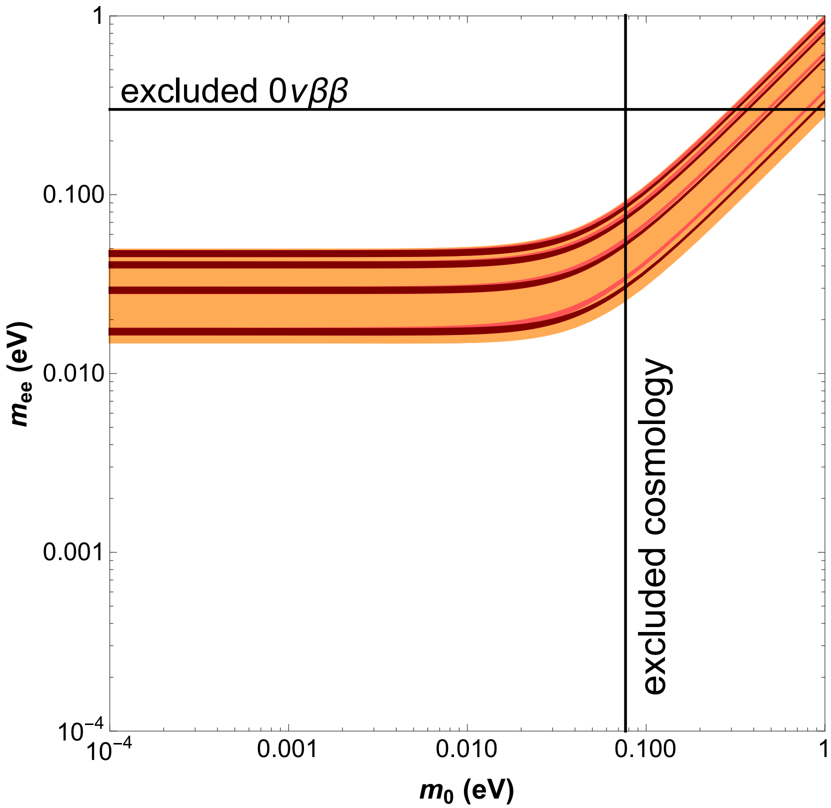
Case 2) with $n = 8$, $u = 1$ and normal ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

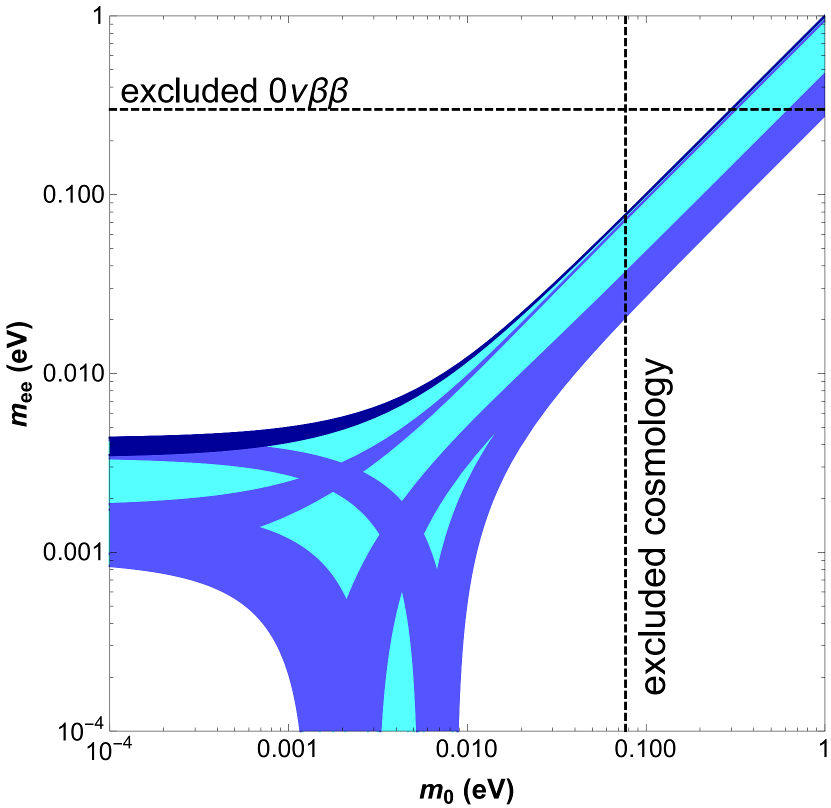
Case 2) with $n = 8$, $u = 1$ and inverted ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

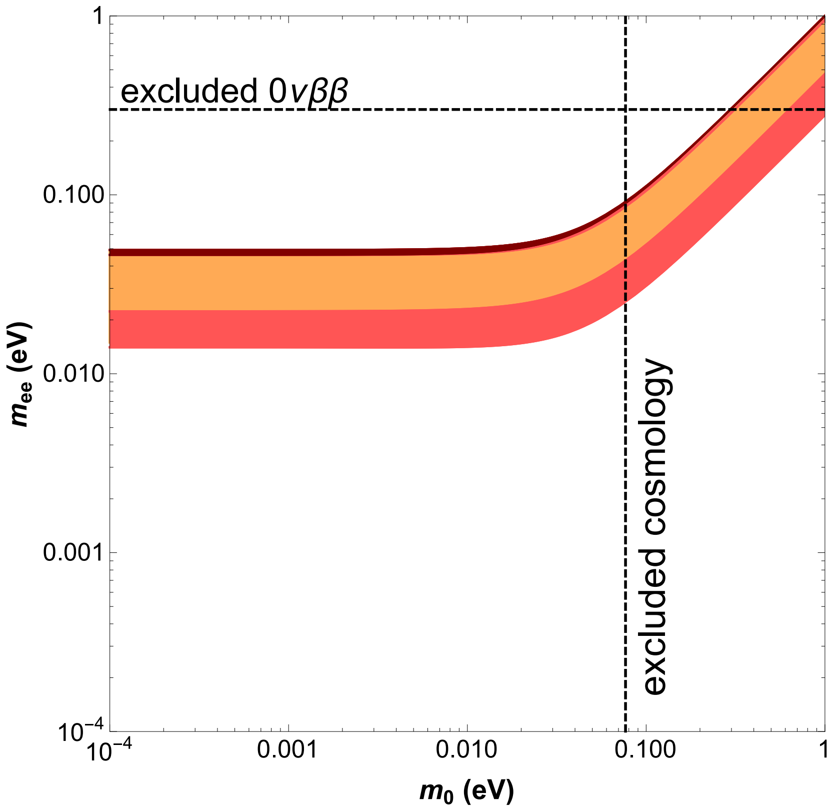
Case 3a) with $n = 16$, $m = 1$, $s = 0$ and normal ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

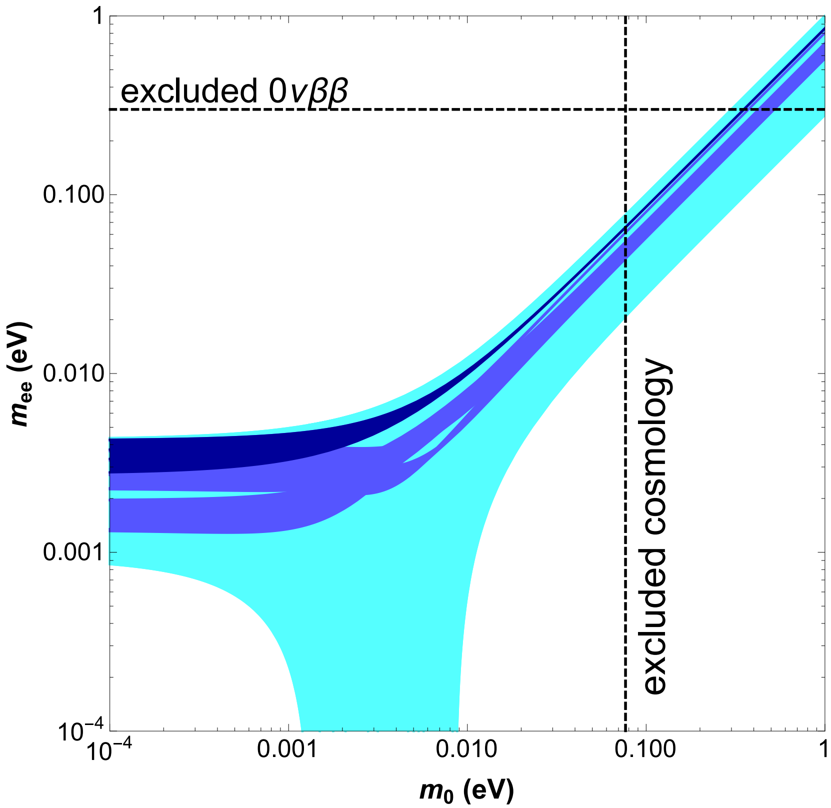
Case 3a) with $n = 16$, $m = 1$, $s = 0$ and inverted ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

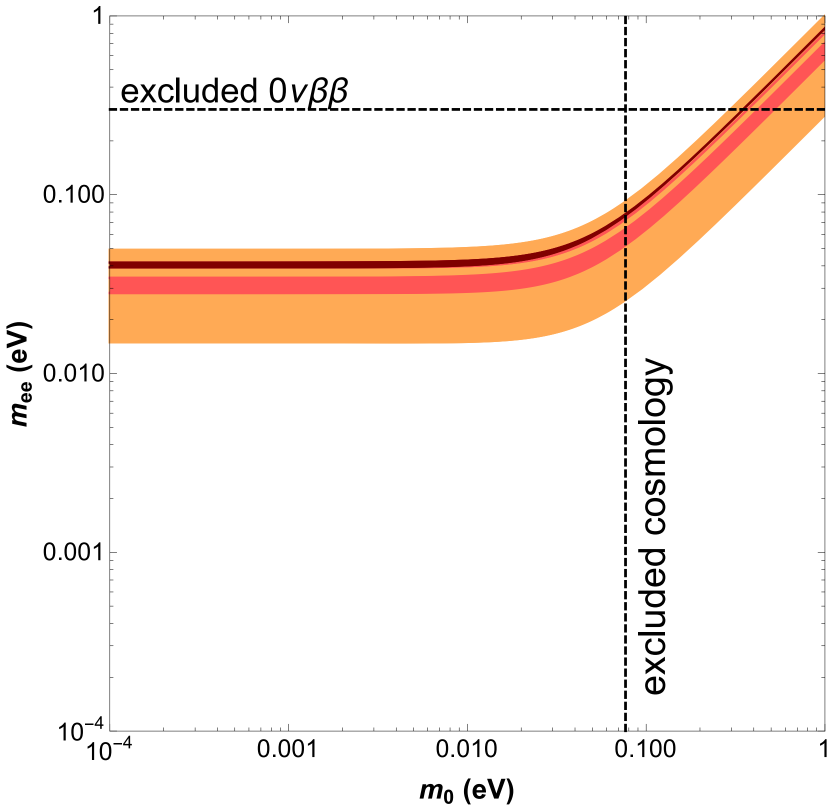
Case 3a) with $n = 16$, $m = 1$, $s = 1$ and normal ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

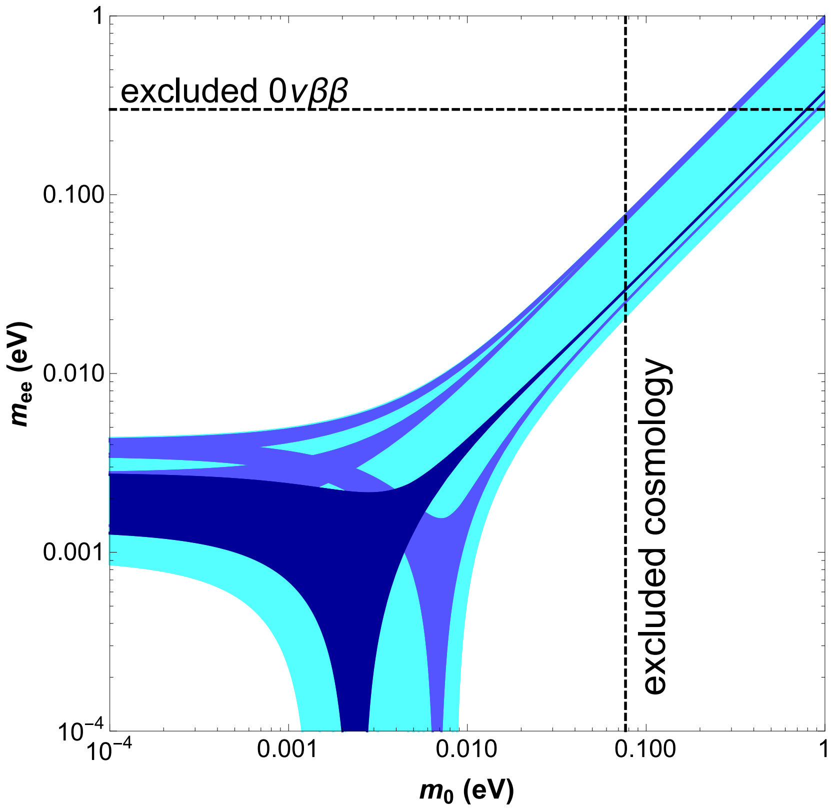
Case 3a) with $n = 16$, $m = 1$, $s = 1$ and inverted ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

Case 3a) with $n = 16$, $m = 1$, $s = 3$ and normal ordering



Neutrinoless double beta decay

(H/Molinaro, in preparation)

Case 3a) with $n = 16$, $m = 1$, $s = 3$ and inverted ordering