Holographic graphene bilayers

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Gauge/Gravity Duality 2015, April 13-17, 2015

Outline

1. Overview
2. D3/probe D5-$\overline{\text{D5}}$
3. D3/probe D7-$\overline{\text{D7}}$
4. Conclusions

Overview

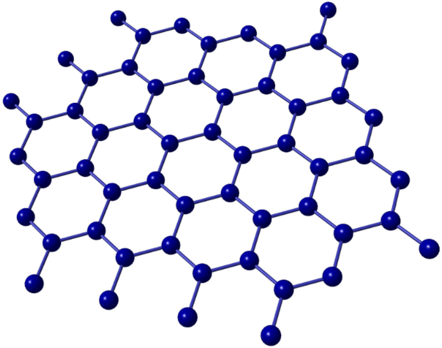
Subject of the talk

Intra-layer and inter-layer exciton condensates in two holographic models of a double monolayer semimetal

Based on: G. G., N. Kim, A.Marini and G. W. Semenoff
arXiv:1410.4911 [hep-th], JHEP 1412 (2014) 091
arXiv:1410.3093 [hep-th]

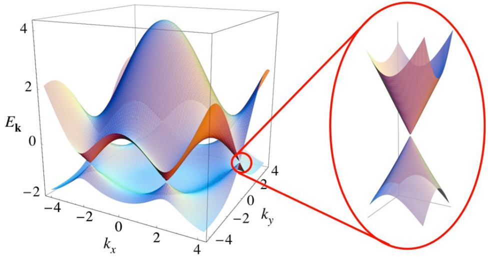
Graphene

- Graphene $\longrightarrow$ two-dimensional material formed by carbon atoms arranged in a honeycomb lattice



Graphene

- Band structure of graphene



- Linearize spectrum near degeneracy points ⟶ relativistic dispersion relation

$$E = \pm\hbar v_F |k| \qquad v_F \simeq \frac{c}{300}$$

- Emergent relativistic Dirac equation for 4 species of massless fermion in 2+1-dim

Semenoff, Phys. Rev. Lett. 53, 2449 (1984)

Holographic graphene

- Graphene $\longrightarrow$ semimetal formed by relativistic massless fermions in 2+1-dim interacting through electromagnetic forces

- Interactions in graphene:

$$\alpha_{\text{graphene}} = \frac{e^2}{4\pi\hbar c}\frac{c}{v_F} \simeq \frac{300}{137} = 2.2$$

- Graphene is a strongly interacting system $\longrightarrow$ AdS/CFT correspondence

- Two top-down holographic models
  - D3/probe D5
  - D3/probe D7

Exciton condensation in double monolayer graphene

- Double monolayer graphene $\longrightarrow$ two monolayers of graphene brought into close proximity but still separated by an insulator
  - no direct transfer of electric charge carriers between the layers
- Exciton $\longrightarrow$ bound state of an electron and a hole
  - intra-layer condensate $\langle\bar{\psi}_1\psi_1\rangle$
  - inter-layer condensate $\langle\bar{\psi}_1\psi_2\rangle$
- Holographic models $\longrightarrow$ D3/probe D5-$\overline{\text{D5}}$ and D3/probe D7-$\overline{\text{D7}}$ systems
  - we use a brane-anti-brane pair so they can partially annihilate $\longrightarrow$ inter-layer condensate

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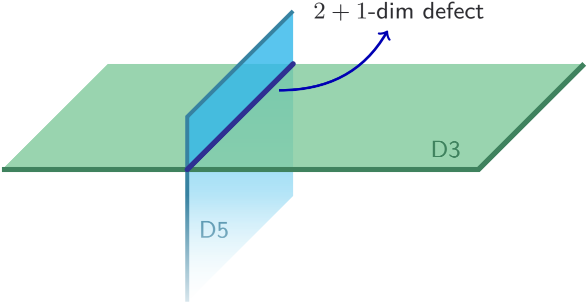
D3/probe D5

- D3/probe D5 system is one of the most studied holographic models

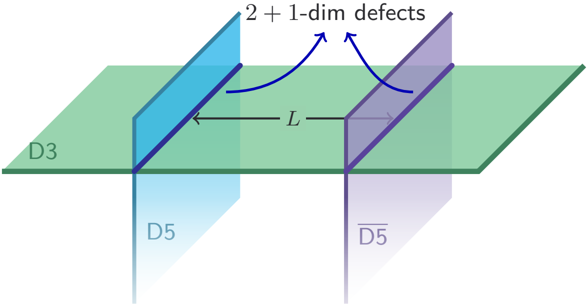


- Dual theory $\longrightarrow$ $\mathcal{N}=4$ SYM at large 't Hooft coupling λ coupled to fundamental hypermultiplets along a 2+1-dim defect

DeWolfe, Freedman, Ooguri [hep-th/0111135]
Karch, Randall [hep-th/0105132]
Erdmenger et al. [hep-th/0203020]

D3/probe D5-$\overline{\text{D5}}$

- We study the D3/probe D5-$\overline{\text{D5}}$ system (at zero temperature)



- Introduce an external magnetic field and a charge density on the D5-branes

Kristjansen, Semenoff [arXiv:1212.5609]

- Zero charge case has been studied by Evans and Kim

Evans, Kim [arXiv:1311.0149]

D3 background

- Stack of N D3-branes $\longrightarrow$ $\mathrm{AdS}_5 \times S^5$ background

$$
\begin{aligned}
ds^2 =& r^2 \left(-dt^2 + dx^2 + dy^2 + dz^2\right) \\
&+ \frac{1}{r^2}\left(dr^2 + r^2 d\psi^2 + r^2 \sin^2\psi d\Omega_2^2 + r^2\cos^2\psi d\tilde{\Omega}_2^2\right)
\end{aligned}
$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $d\tilde{\Omega}_2^2 = d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\phi}^2$

- It is useful to introduce other coordinates

$$
\rho = r\sin\psi \ , \qquad l = r\cos\psi
$$

$$
\begin{aligned}
ds^2 =& (\rho^2 + l^2)\left(-dt^2 + dx^2 + dy^2 + dz^2\right) \\
&+ \frac{1}{\rho^2 + l^2}\left(d\rho^2 + \rho^2 d\Omega_2^2 + dl^2 + l^2 d\tilde{\Omega}_2^2\right)
\end{aligned}
$$

Poincaré horizon at $r = 0 \longrightarrow \rho = l = 0$

D5-$\overline{\text{D5}}$ embedding

- Embed N_5 D5 and $\overline{\text{D5}}$ probes in this background ($N_5 \ll N$)
- DBI + WZ actions

$$S = T_5 N_5 \left[- \int d^6\sigma \sqrt{- \det(g + 2\pi\alpha' F)} + 2\pi\alpha' \int C^{(4)} \wedge F \right]$$

- Worldvolume coordinates and ansatz for the embedding of the D5/$\overline{\text{D5}}$

	t	x	y	z	ρ	l	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
D3	×	×	×	×	–	–	–	–	–	–
D5/$\overline{\text{D5}}$	×	×	×	$z(\rho)$	×	$l(\rho)$	×	×	–	–

- l asymptotically gives the distance between the D3 and the D5-brane $\longrightarrow$ the bare fermion mass

D5-$\overline{\text{D5}}$ geometry

Worldvolume geometry of D5/$\overline{\text{D5}}$ is for the most part determined by symmetry

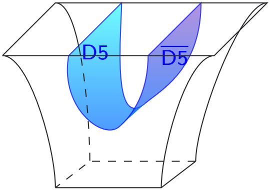
- Poincaré invariance in 2+1-d $\longrightarrow$ branes wrap t, x, y
- SO(3) symmetry $\longrightarrow$ branes wrap S^2 (θ, ϕ)
- Choose ρ as the last worldvolume coordinate
- None of the remaining variables depend on t, x, y, θ, ϕ
- $z(\rho)$ and $l(\rho)$ are the dynamical embedding functions
- The point $l = 0$ $\left(\psi = \frac{\pi}{2}\right)$ $\longrightarrow$ additional SO(3) symm.

Symmetry breaking

The geometry of the D5-$\overline{\text{D5}}$ can break two symmetries

1. $l(\rho) \neq 0 \longrightarrow$ SO(3) $\times$ SO(3) $\rightarrow$ SO(3)
   - intra-layer condensate

2. $z(\rho) \neq$ const $\longrightarrow$ U(N_5) $\times$ U(N_5) $\rightarrow$ U(N_5)
   - partial annihilation of D5 and $\overline{\text{D5}}$
   - inter-layer condensate



D5-$\overline{\text{D5}}$ embedding

- Induced metric on the D-branes worldvolume

$$ds^2 = (\rho^2 + l^2)\left(-dt^2 + dx^2 + dy^2\right) + \frac{\rho^2}{\rho^2 + l^2} d^2\Omega_2 + \frac{d\rho^2}{\rho^2 + l^2}\left(1 + ((\rho^2 + l^2)z')^2 + l'^2\right)$$

  - For $z(\rho) =$const and $l(\rho) =$const $\longrightarrow$ D5/$\overline{\text{D5}}$ wv is $\mathrm{AdS}_4 \times S^2$

- Charge density and external magnetic field $\longrightarrow$ D5 worldvolume gauge fields (in the $a_\rho = 0$ gauge)

$$\frac{2\pi}{\sqrt{\lambda}} F = a_0'(\rho) d\rho \wedge dt + b dx \wedge dy$$

$$b = \frac{2\pi}{\sqrt{\lambda}} B \qquad a_0 = \frac{2\pi}{\sqrt{\lambda}} A_0$$

DBI action

- DBI action for N_5 D5 ($\overline{\text{D5}}$)

$$S = \mathcal{N}_5 \int d\rho \frac{\rho^2}{\rho^2 + l^2} \sqrt{(\rho^2 + l^2)^2 + b^2} \sqrt{1 + l'^2 + ((\rho^2 + l^2)z')^2 - {a_0'}^2}$$

where $\mathcal{N}_5 = \frac{\sqrt{\lambda} N N_5}{2\pi^3} V_{2+1}$

- $a_0(\rho)$ and $z(\rho)$ are cyclic variables $\longrightarrow$ their canonical momenta are constants

$$Q = -\frac{\delta \mathscr{L}}{\delta a_0'} \equiv \frac{2\pi \mathcal{N}_5}{\sqrt{\lambda}} q \qquad q = \frac{\rho^2 a_0' \sqrt{(\rho^2 + l^2)^2 + b^2}}{(\rho^2 + l^2)\sqrt{1 + l'^2 + ((\rho^2 + l^2)z')^2 - (a_0')^2}}$$

$$\Pi_z = \frac{\delta \mathscr{L}}{\delta z'} \equiv \mathcal{N}_5 f \qquad f = \frac{(\rho^2 + l^2)\rho^2 z' \sqrt{(\rho^2 + l^2)^2 + b^2}}{\sqrt{1 + l'^2 + ((\rho^2 + l^2)z')^2 - {a_0'}^2}}$$

  - $q =$ charge density on the D5 ($\overline{\text{D5}}$)

Equations of motion

- Solving for $a_0'(\rho)$ and $z'(\rho)$ in terms of q and f we get

$$a_0' = \frac{q(\rho^2 + l^2)\sqrt{1 + l'^2}}{\sqrt{\rho^4\left(b^2 + (\rho^2 + l^2)^2\right) + q^2(\rho^2 + l^2)^2 - f^2}}$$

$$z' = \frac{f\sqrt{1 + l'^2}}{(\rho^2 + l^2)\sqrt{\rho^4\left(b^2 + (\rho^2 + l^2)^2\right) + q^2(\rho^2 + l^2)^2 - f^2}}$$

- EoM for $l(\rho)$

$$\begin{aligned} &-\left(l^2 + \rho^2\right) l'' \left(-f^2 + l^2\left(l^2 + 2\rho^2\right)\left(\rho^4 + q^2\right) + \rho^4\left(\rho^4 + q^2 + b^2\right)\right) \\ &-2\left(l'^2 + 1\right)\left(\rho\left(f^2 + \rho^2 l^2\left(3\rho^2 l^2 + l^4 + 3\rho^4 + b^2\right) + \rho^8\right) l' + \left(\rho^4 - f^2\right) l\right) = 0 \end{aligned}$$

Asymptotic behaviour

Asymptotic behaviour at $\rho \to \infty$ for the embedding functions $l(\rho)$, $z(\rho)$ and the gauge field $a_0(\rho)$

- $$l(\rho) \underset{\rho\to\infty}{\simeq} m + \frac{c}{\rho} + \dots$$
  - $m \propto$ mass term for the fermions $\longrightarrow$ we consider solution with $m = 0$
  - $c \propto$ expectation value for the intra-layer condensate
- $$z(\rho) \underset{\rho\to\infty}{\simeq} \pm\frac{L}{2} \mp \frac{f}{5\rho^5} + \dots \qquad \text{(for D5/}\overline{\text{D5}}\text{)}$$
  - $L =$ separation between the D5 and the $\overline{\text{D5}}$
  - $f \propto$ expectation value for the inter-layer condensate
- $$a_0(\rho) \underset{\rho\to\infty}{\simeq} \mu - \frac{q}{\rho} + \dots$$
  - $\mu =$ chemical potential

Rescalings

- The magnetic field b can be rescaled to 1 performing the following rescalings

$$\rho \to \sqrt{b}\,\rho \qquad l \to \sqrt{b}\,l \qquad z \to \frac{z}{\sqrt{b}} \qquad a_0 \to \sqrt{b}\,a_0$$
$$f \to b^2 f \qquad q \to b\,q \qquad m \to \sqrt{b}\,m \qquad c \to b\,c$$
$$L \to \frac{L}{\sqrt{b}} \qquad \mu \to \sqrt{b}\mu \qquad S \to b^{3/2} S$$

- b disappears from all the equations. For instance, the action becomes

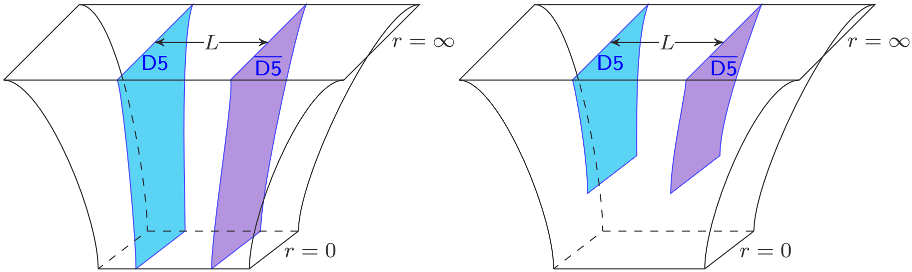
$$S = \mathcal{N}_5 \int d\rho \frac{\rho^2}{\rho^2 + l^2} \sqrt{(\rho^2 + l^2)^2 + 1} \sqrt{1 + l'^2 + ((\rho^2 + l^2) z')^2 - {a_0'}^2}$$

Unconnected solutions

Eq. for $z(\rho) \longrightarrow z' = \dfrac{f\sqrt{1+l'^2}}{(\rho^2+l^2)\sqrt{\rho^4\left(1+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$

- If $f = 0 \longrightarrow$ the solution is trivial $\longrightarrow z = \pm L/2$ (for D5/$\overline{\text{D5}}$)

Unconnected solution



"Black hole" embedding — Minkowski embedding

$r = 0 \longrightarrow l = \rho = 0$

Connected solutions

- If $f \neq 0$ the solution for $z(\rho)$ is

$$z(\rho) = f \int_{\rho_0}^{\rho} d\tilde{\rho} \frac{\sqrt{1+l'^2}}{(\rho^2+l^2)\sqrt{\rho^4\left(1+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$$

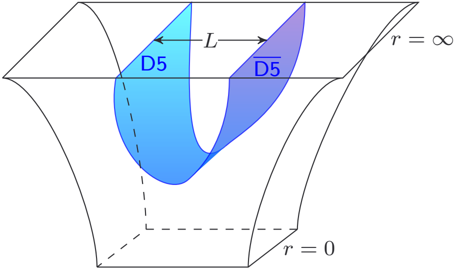
- ρ_0 such that $\rho_0^4 \left(1+(\rho_0^2+l^2(\rho_0))^2\right)+q^2(\rho_0^2+l(\rho_0)^2)-f^2=0$

- $z'(\rho_0)=\infty$

- D-brane worldvolume interrupts at $\rho=\rho_0>0$

Connected solutions

- In order to have a sensible solution we have to glue smoothly the D5/$\overline{\text{D5}}$ solutions at $\rho = \rho_0 \longrightarrow$ connected solution

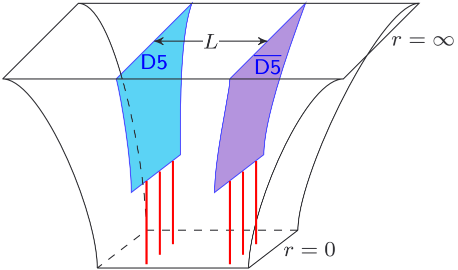


- $f_{\text{D5}} = f_{\overline{\text{D5}}}$ and $q_{\text{D5}} = -q_{\overline{\text{D5}}} \longleftrightarrow$ D5-$\overline{\text{D5}}$ system is neutral
- Inter-layer condensate exists only when the Fermi surfaces in the two layers are perfectly nested

Minkowski vs. BH embeddings

- $(f = 0, c \neq 0)$-solutions can in principle be either BH or Mink. embeddings
- In practice if $q \neq 0$ only BH embeddings are allowed
- Mink. embeddings $\longrightarrow$ D-brane pinches off at $\rho = 0 \longrightarrow l(0) \neq 0$
- If $q \neq 0 \longrightarrow$ there must be charge sources $\longrightarrow$ F-strings suspended between the D5 and the Poincaré horizon $(r = 0)$
- $T_{\mathsf{F1}} > T_{\mathsf{D5}} \longrightarrow$ strings pull the D5 to $r = 0 \longrightarrow$ BH embed.

  Kobayashi et al. [hep-th/0611099]
- For unconnected solutions $(f = 0)$ Mink. embeddings are allowed only if $q = 0$



Classification of the solutions

Scheme of the possible types of solutions

	$f = 0$	$f \neq 0$
$c = 0$	Type 1 unconnected $l = 0$ BH embedding chiral symm.	Type 2 connected $l = 0$ inter
$c \neq 0$	Type 3 unconnected $l(\rho)$ not constant BH ($q \neq 0$)/Mink ($q = 0$) intra	Type 4 connected $l(\rho)$ not constant intra + inter

D-brane separation and chemical potential

- Separation between the D5 and the $\overline{\text{D5}}$ for the connected solution $(f \neq 0)$

$$L = 2\int_{\rho_0}^{\infty} d\rho\, z'(\rho) = \int_{\rho_0}^{\infty} d\rho\, \frac{2f\sqrt{1+l'^2}}{(\rho^2+l^2)\sqrt{\rho^4\left(1+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$$

- Chemical potential

$$\mu = \int_{\rho_0}^{\infty} a_0'(\rho)\, d\rho = \int_{\rho_0}^{\infty} d\rho\, \frac{q(\rho^2+l^2)\sqrt{1+l'^2}}{\sqrt{\rho^4\left(1+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$$

where, for $f \neq 0$, ρ_0 is the solution of

$$\rho_0^4\left(1+(\rho_0^2+l^2(\rho_0))^2\right)+q^2(\rho_0^2+l(\rho_0)^2)-f^2=0$$

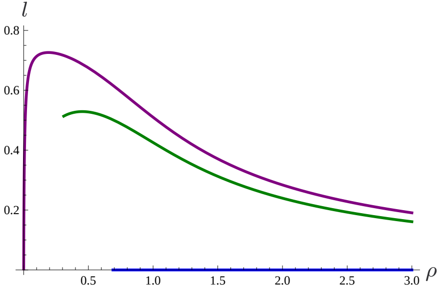
if $f = 0 \longrightarrow \rho_0 = l(\rho_0) = 0$ for $q \neq 0$ and $\rho_0 = 0$ for $q = 0$

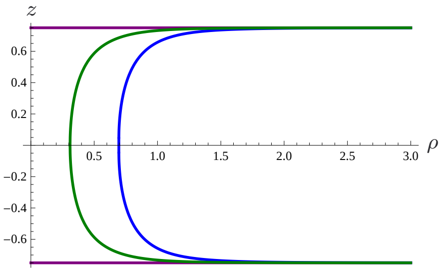
Solutions

- We must look for non-trivial (*i.e.* non-constant) solutions for $l(\rho)$
- EoM for l is a non-linear ODE
- Numerical method to find solutions imposing the suitable asymptotic condition
$$l(\rho) \underset{\rho\to\infty}{\simeq} \frac{c}{\rho} + \dots \quad \text{massless fermions!}$$
- We used a shooting technique
- Four types of solutions are allowed
  1. $f = 0$, $c = 0$ ($z = \pm L/2$, $l = 0$) $\longrightarrow$ chiral symm.
  2. $f \neq 0$, $c = 0$ $\longrightarrow$ inter
  3. $f = 0$, $c \neq 0$ $\longrightarrow$ intra
  4. $f \neq 0$, $c \neq 0$ $\longrightarrow$ intra and inter

Plot of solutions

- Example of plots of non-trivial solutions with $L \simeq 1.5$ and $\mu \simeq 0.77$
  - $f \neq 0$, $c = 0 \longrightarrow$ inter
  - $f = 0$, $c \neq 0 \longrightarrow$ intra
  - $f \neq 0$, $c \neq 0 \longrightarrow$ inter and intra

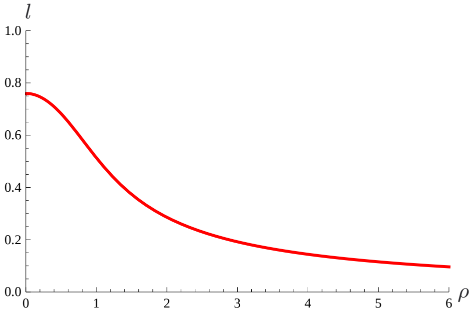




Solutions with zero charge density

- We are interested in solutions at fixed L and μ
- Eq. for a_0 is $\longrightarrow$ $a_0' = \dfrac{q(\rho^2+l^2)\sqrt{1+l'^2}}{\sqrt{\rho^4\left(1+(\rho^2+l^2)^2\right)+q^2(\rho^2+l^2)^2-f^2}}$
- It has a trivial solution $\longrightarrow$ $a_0 = \text{const}$ when $q=0$
- Other solutions with $q=0$ and $a_0=\mu$
- Among these the only relevant one $\longrightarrow$ Minkowski embedding with $f=0$ and $c\neq 0$

Evans,Kim [arXiv:1311.0149]



Free energy

Which configuration is favored?

- Compare the free energies of the different solutions at the same L and μ
- The right quantity to define the free energy is the action evaluated on solutions $\longrightarrow \mathcal{F}_1[L,\mu] \sim S[l,z,a_0]$

$$\delta\mathcal{F}_1 \sim \int_0^\infty d\rho \left(\delta l \frac{\partial\mathcal{L}}{\partial l'} + \delta a_0 \frac{\partial\mathcal{L}}{\partial a_0'} + \delta z \frac{\partial\mathcal{L}}{\partial z'}\right)' = -q\delta\mu + f\delta L$$

$$\mathcal{F}_1[L,\mu] = \int_{\rho_0}^\infty d\rho \, \frac{\rho^4\left(1+\left(l^2+\rho^2\right)^2\right)\sqrt{\frac{1+l'^2}{-f^2+q^2\left(l^2+\rho^2\right)^2+\rho^4\left(1+\left(l^2+\rho^2\right)^2\right)}}}{l^2+\rho^2}$$

- $\mathcal{F}_1 \longleftrightarrow$ implicit function of L and μ

Regularized free energy

- The free energy of each solution is UV divergent
  the integrand in $\mathcal{F}_1$ goes like ρ^2 for $\rho \to \infty$

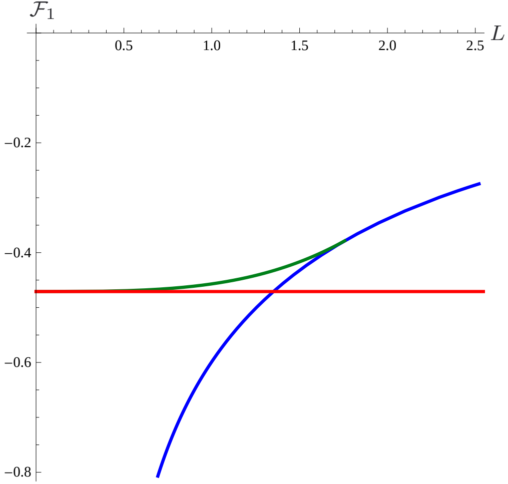
- Regularization $\longrightarrow$ subtracting to the free energy of each solution that of the trivial chirally symmetric solution (with the same μ)

$$\Delta\mathcal{F}_1[L,\mu] \equiv \mathcal{F}_1[L,\mu] - \mathcal{F}_1(l=0; f=0)[\mu]$$

- We use the regularized free energy to study the dominant configuration at fixed values of L and μ

- We construct the phase diagram working on a series of constant L slices

Free energy as a function of the separation: no charge



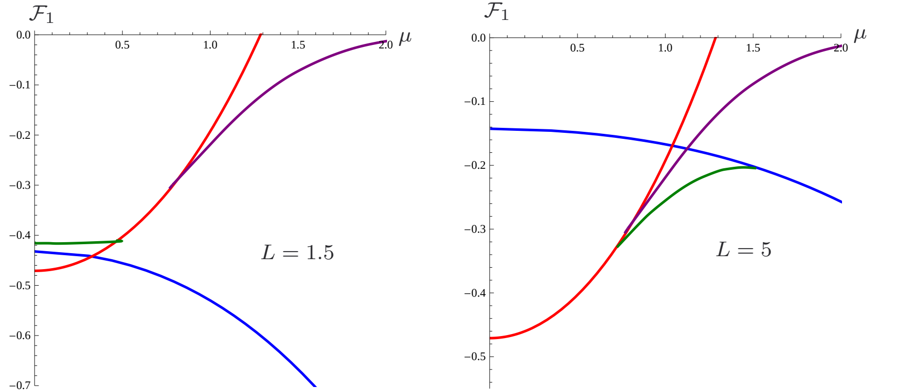
Evans,Kim [arXiv:1311.0149]

—— Minkowski embedding unconnected, only intra-layer condensate

—— connected ρ-independent, only inter-layer condensate

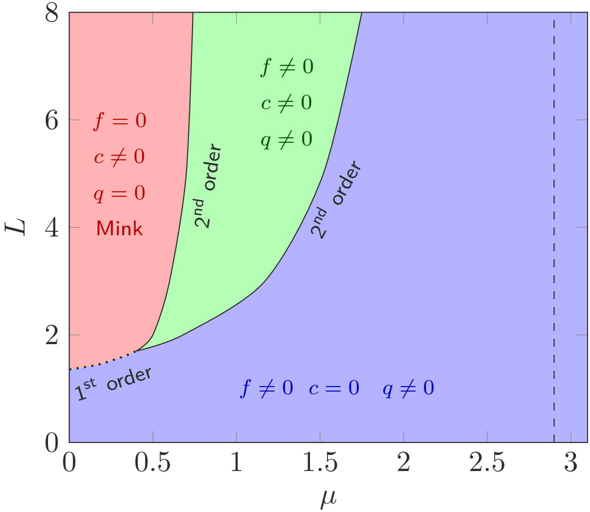
—— connected ρ-dependent, both inter- and intra-layer condensate

Free energy as a function of the chemical potential



- Minkowski embedding unconnected, only intra-layer condensate
- Black-hole embedding unconnected, only intra-layer condensate
- connected ρ-independent, only inter-layer condensate
- connected ρ-dependent, both inter- and intra-layer condensate

(μ, L)–phase diagram for D3/D5-$\overline{\text{D5}}$



Free energy as a function of q and L

- Compare different configurations at fixed charge density q and separation L
- Different definition for the free energy $\longrightarrow$ Legendre transform of $\mathcal{F}_1$

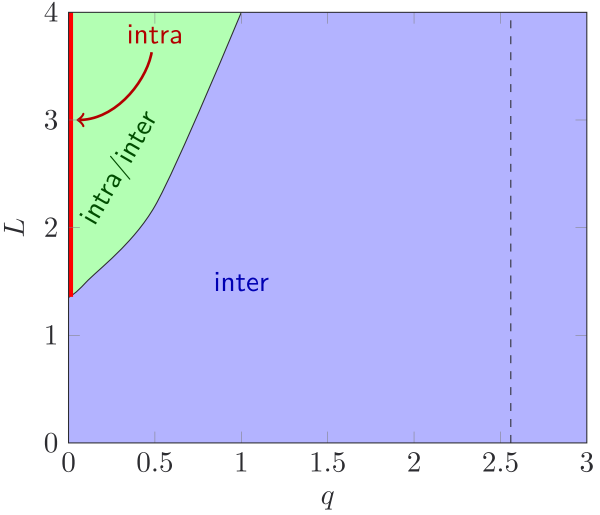
$$\mathcal{F}_2[L,q] = \mathcal{F}_1 + q\,\mu =$$

$$\int_{\rho_0}^{\infty} d\rho \, \frac{q^2\left(l^2+\rho^2\right)^2 + \rho^4\left(1+\left(l^2+\rho^2\right)^2\right)\sqrt{\frac{1+l'^2}{-f^2+q^2(l^2+\rho^2)^2+\rho^4\left(1+(l^2+\rho^2)^2\right)}}}{l^2+\rho^2}$$

- $\mathcal{F}_2$ is divergent $\longrightarrow$ regularization

$$\Delta\mathcal{F}_2[L,q] \equiv \mathcal{F}_2[L,q] - \mathcal{F}_2(l=0; f=0)[q]$$

(q, L)–phase diagram for D3/D5-$\overline{\text{D5}}$



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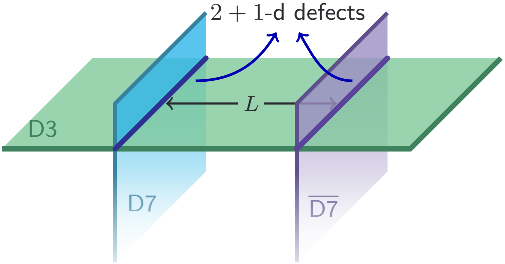
4. Conclusions

D3/probe D7-$\overline{\text{D7}}$

- D3/probe D7 system as an holographic model for graphene

S.Y. Rey [arXiv:0911.5295]

- Dual theory $\longrightarrow$ $\mathcal{N} = 4$ SYM at large 't Hooft coupling λ coupled to massless fermions along a 2+1-dim defect
- We study the D3/probe D7-$\overline{\text{D7}}$ system



- D7-branes with the appropriate boundary conditions are unstable in $\mathrm{AdS}_5 \times S^5$ background
- Embed the D7 in the extremal black D3-brane geometry

Davis, Kraus, Shah [arXiv:0809.1876]

D3 background

- Extremal black D3 brane geometry

$$ds^2 = \frac{1}{\sqrt{1+\frac{R^4}{r^4}}}\left(-dt^2+dx^2+dy^2+dz^2\right)+\sqrt{1+\frac{R^4}{r^4}}\left(dr^2+r^2 d\Omega_5^2\right)$$

where $d\Omega_5^2 = d\psi^2 + \sin^2\psi d\Omega_4^2$ and $R^4 = \lambda\alpha'^2$

- $R^{-1} =$ UV cutoff
- We then introduce the coordinates $\rho = r\sin\psi\,, \quad l = r\cos\psi$

$$ds^2 = \left(1+\frac{R^4}{(\rho^2+l^2)^2}\right)^{-1/2}\left(-dt^2+dx^2+dy^2+dz^2\right)$$
$$+\left(1+\frac{R^4}{(\rho^2+l^2)^2}\right)^{1/2}\left(d\rho^2+dl^2+\rho^2 d\Omega_4^2\right)$$

D3/probe D7-$\overline{\text{D7}}$ embedding

- Embed N_7 D7 and $\overline{\text{D7}}$ probes in this background ($N_7 \ll N$)

- DBI + WZ actions

$$S = T_7 N_7 \left[-\int d^8\sigma \sqrt{-\det(g + 2\pi\alpha' F)} + 2\pi\alpha' \int C^{(4)} \wedge F \wedge F \right]$$

- Ansatz for D7 ($\overline{\text{D7}}$) embedding

	t	x	y	z	ρ	l	θ_1	θ_2	θ_3	θ_4
D3	$\times$	$\times$	$\times$	$\times$	—	—	—	—	—	—
D7/$\overline{\text{D7}}$	$\times$	$\times$	$\times$	$z(\rho)$	$\times$	$l(\rho)$	$\times$	$\times$	$\times$	$\times$

D7-$\overline{\text{D7}}$ embedding

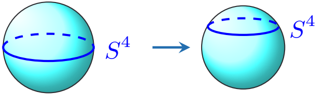
Worldvolume geometry of D7/$\overline{\text{D7}}$ is for the most part determined by symmetry

- Poincaré invariance in 2+1-d $\longrightarrow$ branes wrap t, x, y
- SO(5) symmetry $\longrightarrow$ branes wrap $S^4 \subset S^5$
- Choose ρ as the last worldvolume coordinate
- None of the remaining variables depend on $t, x, y, \theta_1, \theta_2, \theta_3, \theta_4$
- $z(\rho)$ and $l(\rho)$ are the dynamical embedding functions
- $l = 0$ $\left(\psi = \frac{\pi}{2}\right)$ is a point of higher symmetry $\longrightarrow$ parity in the dual defect theory

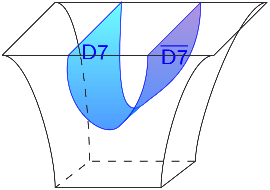
Symmetry breaking

The geometry of the D7-$\overline{\text{D7}}$ can break two symmetries

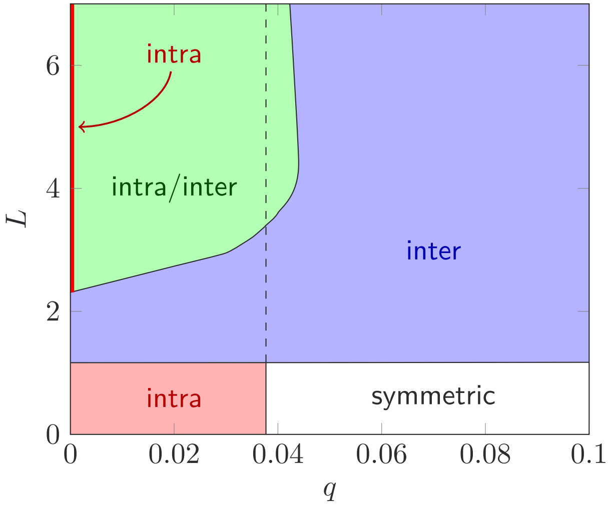
1. $l(\rho) \neq 0 \longrightarrow$ parity breaking
   - intra-layer condensate



2. $z(\rho) \neq$ const $\longrightarrow \mathrm{U}(N_7) \times \mathrm{U}(N_7) \to \mathrm{U}(N_7)$
   - partial annihilation of D7 and $\overline{\text{D7}}$
   - inter-layer condensate



(q, L)–phase diagram for D3/D7-$\overline{\text{D7}}$



Outline

1 Overview

2 D3/probe D5-$\overline{\text{D5}}$

3 D3/probe D7-$\overline{\text{D7}}$

4 Conclusions

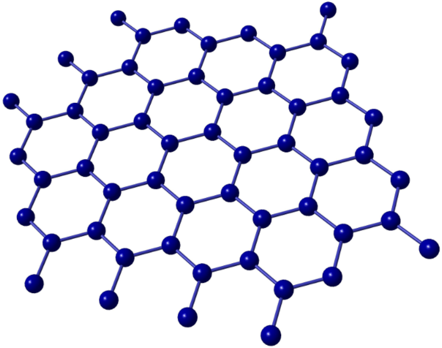
Conclusions

- Two holographic models of a duoble monolayer graphene
  - D3/probe D5-$\overline{\text{D5}}$
  - D3/probe D7-$\overline{\text{D7}}$
- Holographic mechanism for exciton condensation $\longrightarrow$ two channels
  - intra-layer condensate
  - inter-layer condensate
- Inter-layer condensate possible overall neutral system $\longrightarrow$ Fermi surfaces perfectly nested
- When $q \neq 0$ $\longrightarrow$ phase with both inter- and intra-layer condensates
- Study of the phase diagrams
- Perfect fine-tuning of Fermi surfaces is not absolutely necessary for inter-layer condensation if we have more the one fermion species
- Outlook: Turn on the temperature in the models

Extra slides

Graphene

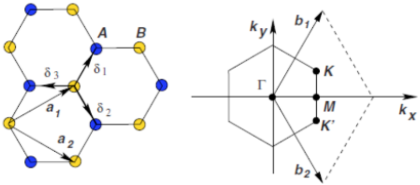
- Graphene $\longrightarrow$ two-dimensional material formed by carbon atoms arranged in a honeycomb lattice



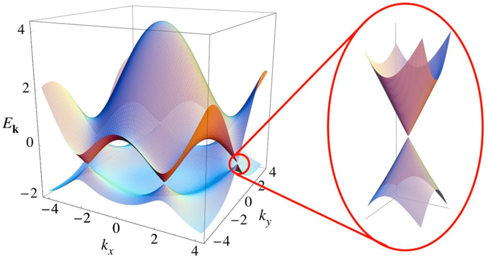
- Carbon atom has four valence electrons
  - Three form strong covalent σ-bonds with neighboring atoms
  - The fourth in the π orbital is unpaired

Graphene

- Hexagonal lattice ⟶ two triangular sub-lattices

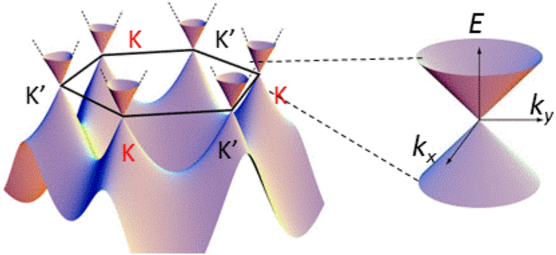


- Band structure of graphene



Graphene

- Linearize spectrum near degeneracy points



- Relativistic dispersion relation

$$E = \pm \hbar v_F |k| \qquad v_F \simeq \frac{c}{300}$$

- Emergent Dirac equation for 4 species of massless fermion in 2+1-dim

Semenoff, Phys. Rev. Lett. 53, 2449 (1984)

D7-$\overline{\text{D7}}$ embedding

- Induced metric on th D-branes worldvolume

$$ds^2 = \left(1 + \frac{R^4}{(\rho^2 + l^2)^2}\right)^{-1/2} \left(-dt^2 + dx^2 + dy^2\right)$$
$$+ \left(1 + \frac{R^4}{(\rho^2 + l^2)^2}\right)^{1/2} \left(d\rho^2 \left(1 + l'(\rho)^2 + \frac{z'(\rho)^2}{1 + \frac{R^4}{(\rho^2 + l^2)^2}}\right) + \rho^2 d^2\Omega_4\right)$$

- Charge density $\longrightarrow$ D7 world-volume gauge fields ($a_\rho = 0$ gauge)

$$2\pi l_s^2 F = a_0'(\rho) d\rho \wedge dt \qquad a_0 = 2\pi l_s^2 A_0$$

DBI action

- DBI action for the D7 ($\overline{\text{D7}}$)

$$S \sim \int d\rho\rho^4 \sqrt{\left(1 + \frac{R^4}{(l^2+\rho^2)^2}\right)(1 - {a_0}'^2 + l'^2) + z'^2}$$

- Perform the rescalings $(\rho, l, z, a_0) \to R(\rho, l, z, a_0) \longleftrightarrow R \to 1$
- $a_0(\rho)$ and $z(\rho)$ are cyclic variables $\longrightarrow$ their canonical momenta are constants

$$Q = -\frac{\delta S}{\delta a_0'} \equiv 2\pi l_s^2 q \ , \qquad q = \frac{\rho^4 {a_0}' \left(2\rho^2 l^2 + l^4 + \rho^4 + 1\right)}{\left(l^2+\rho^2\right)^2 \sqrt{\frac{(2\rho^2 l^2 + l^4 + \rho^4 + 1)(1 - {a_0}'^2 + l'^2)}{(l^2+\rho^2)^2} + z'^2}}$$

$$\Pi_z = \frac{\delta S}{\delta z'} \equiv f \ , \qquad f = \frac{\rho^4 z'}{\sqrt{\frac{(2\rho^2 l^2 + l^4 + \rho^4 + 1)(1 - {a_0}'^2 + l'^2)}{(l^2+\rho^2)^2} + z'^2}} \ ,$$

  - $q =$ charge density on the D7 ($\overline{\text{D7}}$)

Equations of motion

- Solving for $a_0'(\rho)$ and $z'(\rho)$ in terms of q and f we get

$$a_0' = -\frac{q\left(l^2+\rho^2\right)\sqrt{1+l'^2}}{\sqrt{l^2\left(l^2+2\rho^2\right)\left(\rho^8+q^2-f^2\right)+q^2\rho^4+\left(\rho^8-f^2\right)\left(\rho^4+1\right)}}$$

$$z' = \frac{f\left(l^4+2l^2\rho^2+\rho^4+1\right)\sqrt{1+l'^2}}{\left(l^2+\rho^2\right)\sqrt{l^2\left(l^2+2\rho^2\right)\left(\rho^8+q^2-f^2\right)+q^2\rho^4+\left(\rho^8-f^2\right)\left(\rho^4+1\right)}}$$

- EoM for $l(\rho)$

$$2\left(1+l'^2\right)\left\{\rho\, l'\left[2l^2\rho^6\left(l^4+3l^2\rho^2+3\rho^4+1\right)+2\rho^{12}+\rho^8+f^2\right]+l\left(\rho^8-f^2\right)\right\}$$
$$+\left(l^2+\rho^2\right)\left[l^2\left(l^2+2\rho^2\right)\left(\rho^8+q^2-f^2\right)+q^2\rho^4+\left(\rho^4+1\right)\left(\rho^8-f^2\right)\right]l''=0$$

Asymptotic behaviour

Asymptotic behaviour at $\rho \to \infty$ for the embedding functions $l(\rho)$, $z(\rho)$ and the gauge field $a_0(\rho)$

- $$l(\rho) \underset{\rho\to\infty}{\simeq} m + \frac{c}{\rho^3} + \dots$$
  - $m \propto$ mass term for the fermions $\longrightarrow$ we consider solution with $m = 0$
  - $c \propto$ expectation value for the intra-layer condensate

- $$z(\rho) \underset{\rho\to\infty}{\simeq} \pm\frac{L}{2} \mp \frac{f}{\rho^4} + \dots \qquad \text{(for D7/}\overline{\text{D7}}\text{)}$$
  - $L =$ separation between the D7 and the $\overline{\text{D7}}$
  - $f \propto$ expectation value for the inter-layer condensate

- $$a_0(\rho) \underset{\rho\to\infty}{\simeq} \mu - \frac{q}{\rho^4} + \dots$$
  - $\mu =$ chemical potential

Classification of the solutions

Scheme of the possible types of solutions

	$f = 0$	$f \neq 0$
$c = 0$	Type 1 unconnected $l = 0$ BH embedding chiral symm.	Type 2 connected $l = 0$ inter
$c \neq 0$	Type 3 unconnected $l(\rho)$ not constant BH $(q \neq 0)$/Mink $(q = 0)$ intra	Type 4 connected $l(\rho)$ not constant intra + inter

Free energies

- Free energy at fixed separation L and chemical potential $\mu \longrightarrow$ on-shell action
$$\mathcal{F}_1[L,\mu] = S$$
- Free energy at fixed separation L and charge density $q \longrightarrow$ Legendre transform of $\mathcal{F}_1$
$$\mathcal{F}_2[L,q] = \mathcal{F}_1 + q\,\mu$$
- These free energy are divergent $\longrightarrow$ regularization

$$\Delta\mathcal{F}_1[L,\mu] \equiv \mathcal{F}_1[L,\mu] - \mathcal{F}_1(l=0; f=0)[\mu]$$
$$\Delta\mathcal{F}_2[L,q] \equiv \mathcal{F}_2[L,q] - \mathcal{F}_2(l=0; f=0)[q]$$

Balanced charge densities

When $Q = \bar{Q} \longrightarrow$ four possible configurations:

1. Two pairs of connected branes
2. All disconnected branes
3. A connected pair and a disconnected pair with BH embeddings
4. A connected pair that absorb all the charge Q and a disconnected pair with zero charge (Mink embedding)

Balanced charge densities

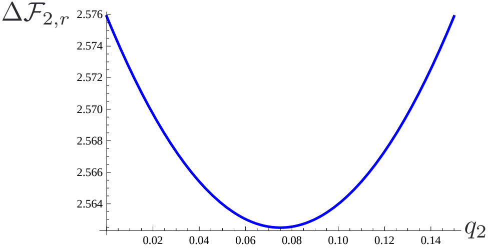
When $Q = \bar{Q} \longrightarrow$ four possible configurations:

1. Two pairs of connected branes $\longrightarrow$ charge evenly distributed
2. All disconnected branes
3. A connected pair and a disconnected pair with BH embeddings
4. A connected pair that absorb all the charge Q and a disconnected pair with zero charge (Mink embedding)

Balanced charge densities

1. All disconnected branes $\longrightarrow$ charge evenly distributed (same as before)

2. All connected branes $\longrightarrow$ $q_1 = Q - q_2 = q_3\,,\ q_4 = q_2$ $\longrightarrow$ charge evenly distributed



For $Q = \bar{Q}$ the energetically favored solution is the one with two connected pairs and all the charges are evenly distributed $q_1 = q_2 = q_3 = q_4 = Q/2$.

Outline

1. Double monolayer with un-balanced charge densities

Double monolayer with two species of fermions

- Consider a double monolayer system with two species of massless fermions with charges Q and $-\bar{Q}$ on the two layers
- Holographic dual $\longrightarrow$ two pairs of D5-$\overline{\text{D5}}$ branes (or D7-$\overline{\text{D7}}$) with total charges Q and $-\bar{Q}$

$$Q = q_1 + q_2 > 0 \qquad \bar{Q} = q_3 + q_4 > 0$$

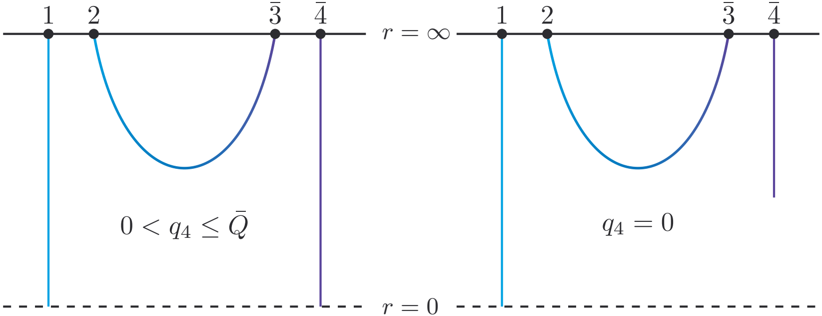
- We fix Q and $\bar{Q}$ and let the q_i vary
- How do the charges Q and $\bar{Q}$ distribute among the branes?
- Which types of solutions give rise to the least free energy?

Un-balnced charges - case 1

- Consider the case of unpaired charges $Q \neq \bar{Q}$
- Suppose $Q > \bar{Q}$

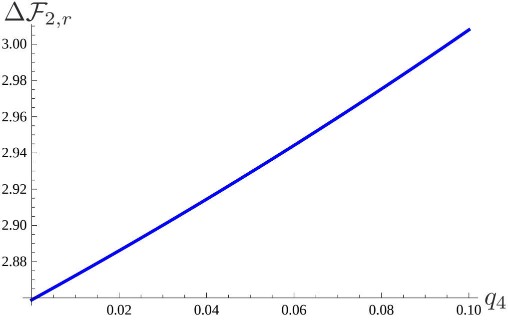
1. A D5 and a $\overline{\text{D5}}$ are connected and the others are unconnected

$$q_2 = q_3 = \bar{Q} - q_4 \ , \qquad q_1 = Q - q_2 = Q - \bar{Q} + q_4$$



Un-balanced charges - case 1

Total free energy as a function of q_4 for $Q = 0.15$, $\bar{Q} = 0.1$ and $L = 1$



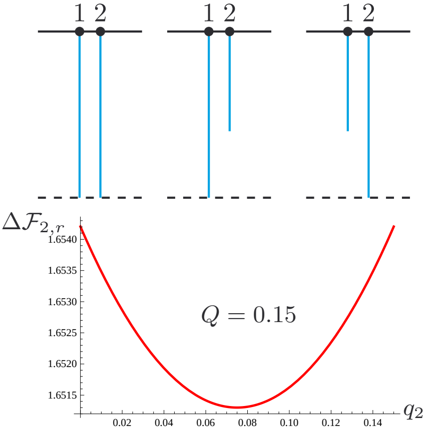
- Least free energy when $q_4 = 0 \longrightarrow$ one $\overline{\text{D5}}$ brane has a Minkowski embedding

Un-balanced charges - case 2

2. The D5 and the $\overline{\text{D5}}$ are all unconnected

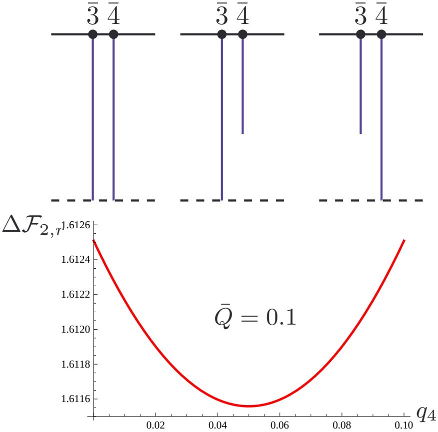
D5-branes

$$q_1 = Q - q_2$$



$\overline{\text{D5}}$-branes

$$q_3 = \bar{Q} - q_4$$



Unbalanced charges

- Comparing the free energies $\longrightarrow$ Configuration 1 with $q_4 = 0$ is the most favored

Recap:

- When there are more the one fermion species $\longrightarrow$ new possible channel for inter-layer condensation when the charges are not balanced $(Q > \bar{Q})$
- Charge can redistribute itself among the species to spontaneously nest one or more pairs of Fermi surfaces
- Perfect fine-tuning of Fermi surfaces is not absolutely necessary for inter-layer condensation