Turbulent strings in AdS/CFT

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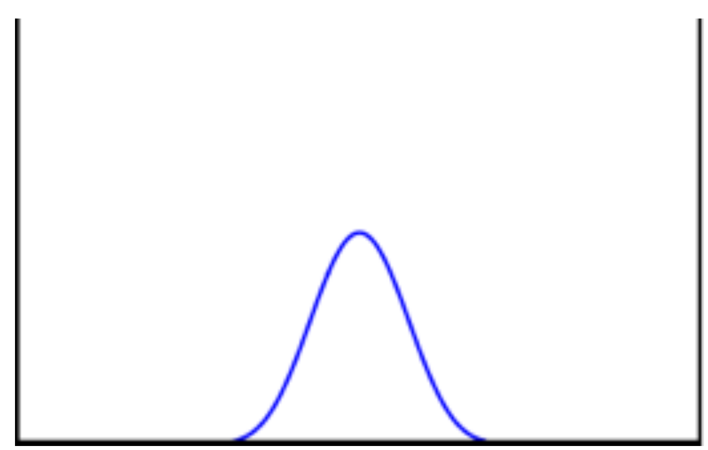
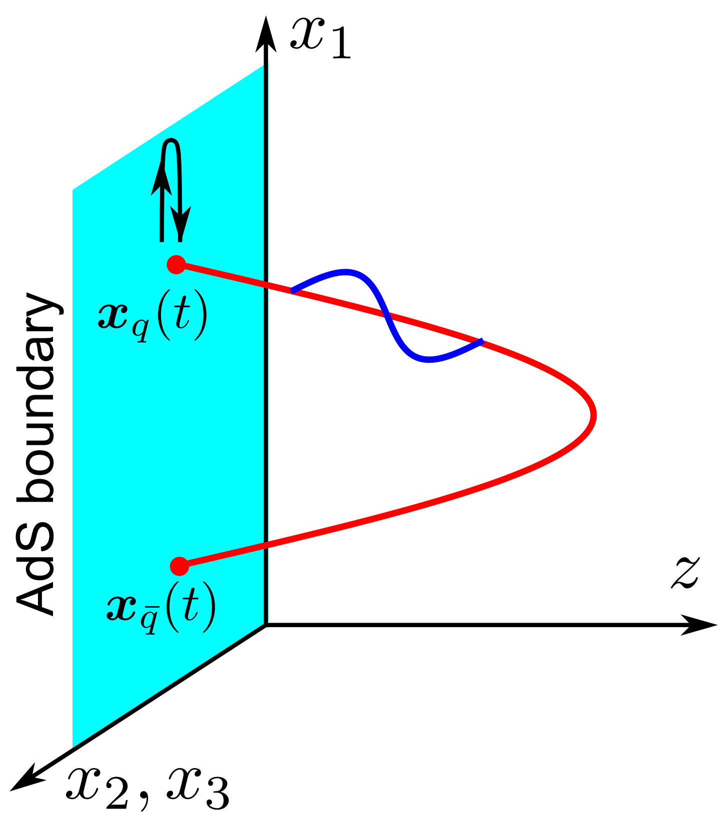
arXiv:1504.02190 with Keiju Murata



14 Apr 2015, HoloGrav 2015@GGI

Plan

Perturb holographic quark-antiquark potential



Motivations

- AdS turbulence
- Turbulent instability on D7

[Hashimoto-Kinoshita-Oka-Murata]

We solve nonlinear time evolution

c.f.) Cosmic strings in flat space

Contents

Time-like holographic Wilson loop

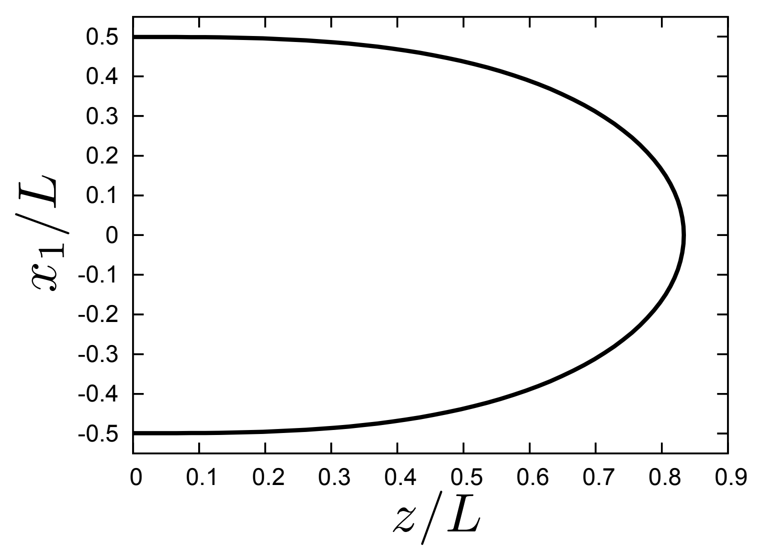
[Maldacena, Rey-Yee]

In $AdS_5 x S^5$ $$ds^2 = \frac{\ell^2}{z^2}\left(-dt^2 + dz^2 + d\boldsymbol{x}^2\right) + \ell^2 d\Omega_5^2$$

Static gauge: (τ,σ)=(t,z)
Target space embedding: $x_1=X_1(z)$

Solution for separation L

$$X_1(z) = \pm z_0 \int_{z/z_0}^{1} dw \frac{w^2}{\sqrt{1-w^4}}$$
$$= \pm z_0 \left[\Gamma_0 + F(z/z_0;i) - E(z/z_0;i)\right]$$

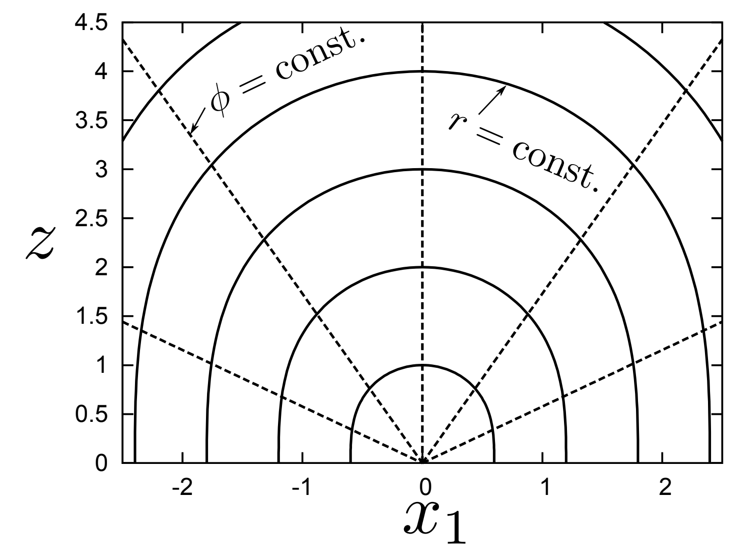


z_0: string tip
Γ_0=0.599

$$\frac{L}{2} = z_0\Gamma_0$$

A convenient parametrization

Polar-like coordinates (r,ϕ) in which the static solution is $r=z_0$

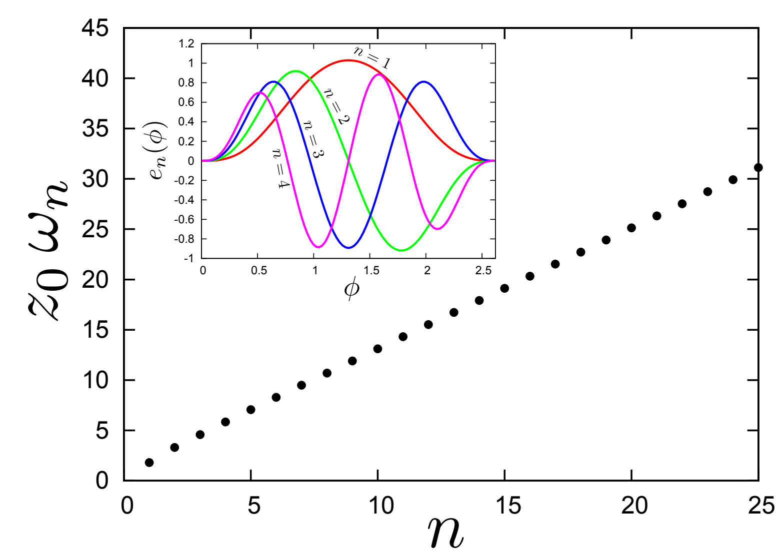


$$z = r f(\phi) = r\,\mathrm{sn}(\phi; i)$$

$$x_1 = r g(\phi) = r \begin{cases} \phi - E(\mathrm{sn}(\phi;i);i) + \Gamma_0 & (\phi \le \beta_0/2) \\ \phi + E(\mathrm{sn}(\phi;i);i) - \Gamma_0 - \beta_0 & (\phi > \beta_0/2) \end{cases}$$

We prepare eigenvalues/functions in linearized perturbations

[Callan-Guijosa, Klebanov-Maldacena-Thorn]



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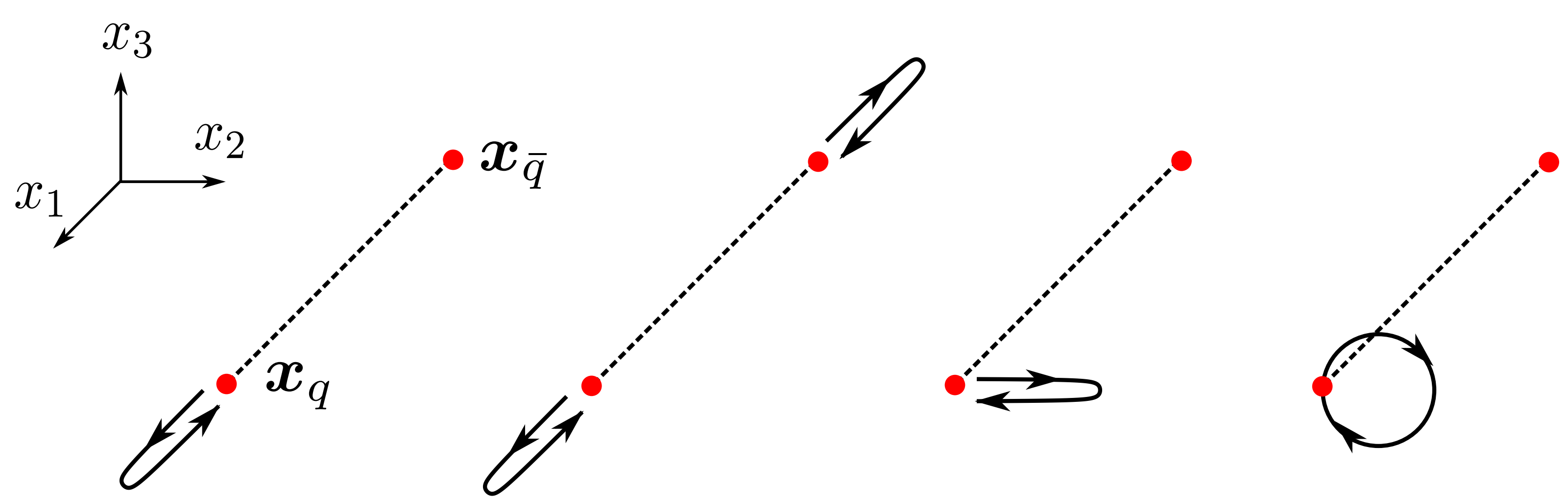
Perturb the string endpoints



Longitudinal one-sided quench

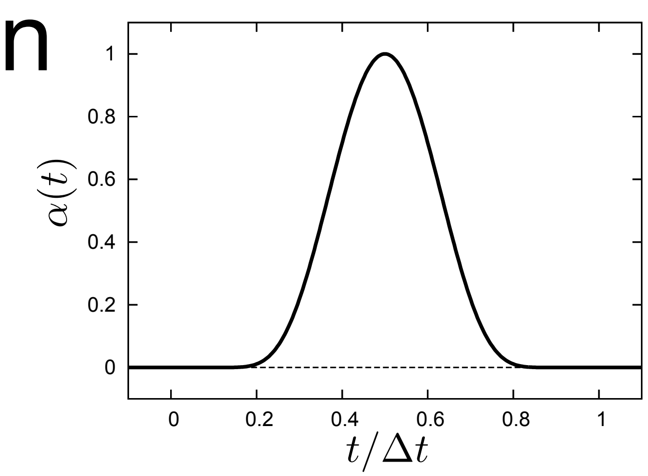
Longitudinal Z_2 - symmetric quench

Transverse linear quench

Transverse circular quench

Quench profile: a compact C$^\infty$ function

$$\alpha(t) = \exp\left[2\left(\frac{\Delta t}{t-\Delta t} - \frac{\Delta t}{t} + 4\right)\right] \quad (0 < t < \Delta t)$$



Worldsheet double null coordinates

Induced metric $ds^2_{F1} = -2\gamma_{uv} du dv$

Worldsheet: u,v

Target space: T(u,v), Z(u,v), $X_{1,2,3}$(u,v)

$$\gamma_{uv} = \frac{\ell^2}{Z^2}(-T_{,u}T_{,v} + Z_{,u}Z_{,v} + \boldsymbol{X}_{,u}\cdot\boldsymbol{X}_{,v})$$

Equations of motion

$$T_{,uv} = \frac{1}{Z}(T_{,u}Z_{,v} + Z_{,u}T_{,v})$$

$$Z_{,uv} = \frac{1}{Z}(T_{,u}T_{,v} + Z_{,u}Z_{,v} - \boldsymbol{X}_{,u}\cdot\boldsymbol{X}_{,v})$$

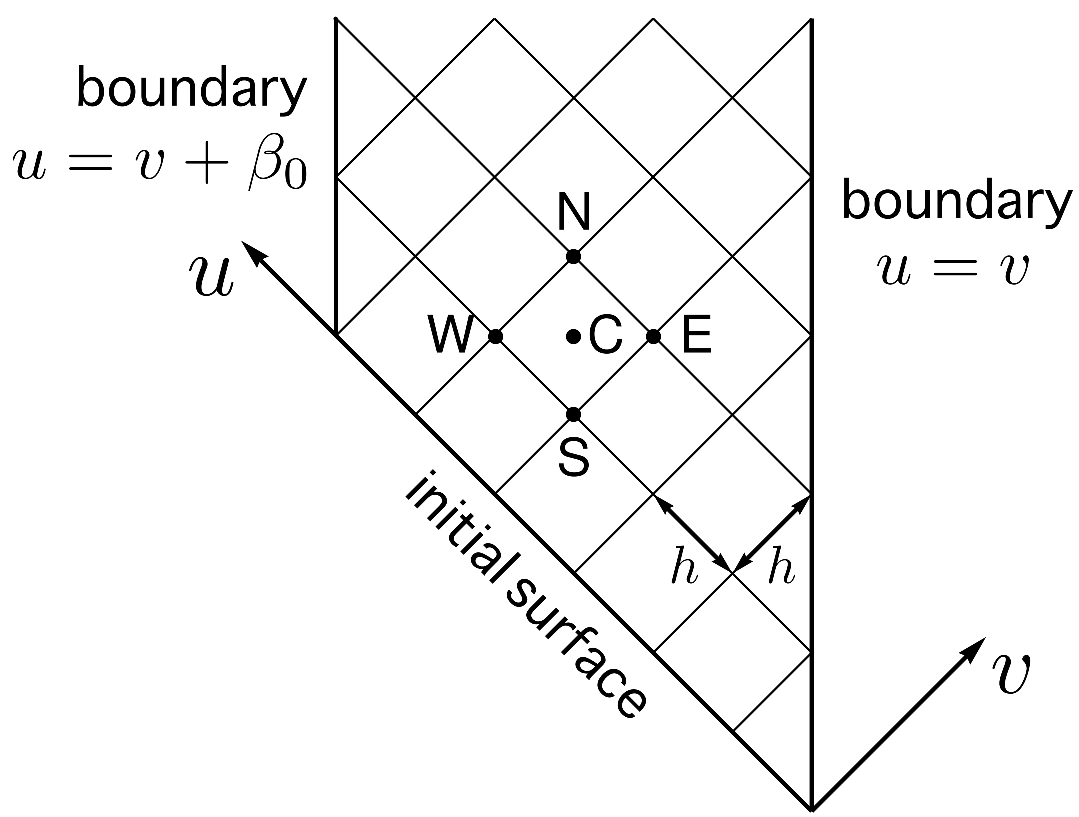
$$\boldsymbol{X}_{,uv} = \frac{1}{Z}(\boldsymbol{X}_{,u}Z_{,v} + Z_{,u}\boldsymbol{X}_{,v})$$

Constraints

$$\gamma_{uu} = \frac{\ell^2}{Z^2}(-T^2_{,u} + Z^2_{,u} + \boldsymbol{X}^2_{,u}) = 0$$

$$\gamma_{vv} = \frac{\ell^2}{Z^2}(-T^2_{,v} + Z^2_{,v} + \boldsymbol{X}^2_{,v}) = 0$$

Discretization



To solve EoMs, we use O(h^2) central finite differential

$$\Psi_{,uv}|_C = \frac{\Psi_N - \Psi_E - \Psi_W + \Psi_S}{h^2}$$

$$\Psi_{,u}|_C = \frac{\Psi_N - \Psi_E + \Psi_W - \Psi_S}{2h}$$

$$\Psi_{,v}|_C = \frac{\Psi_N + \Psi_E - \Psi_W - \Psi_S}{2h}$$

$$\Psi|_C = \frac{\Psi_E + \Psi_W}{2}$$

Compute N by using EWS data

Initial data (v=0): static solution

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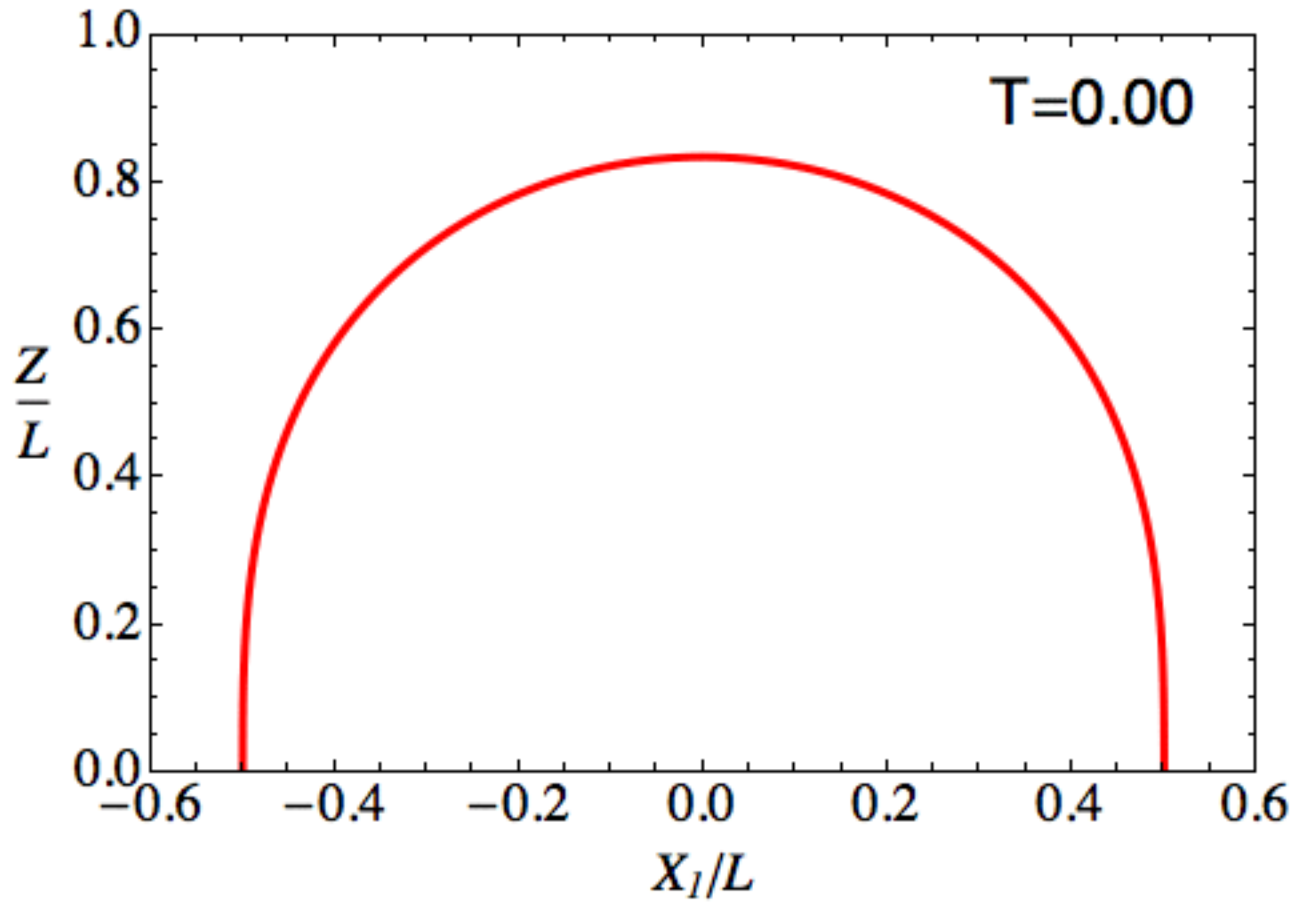
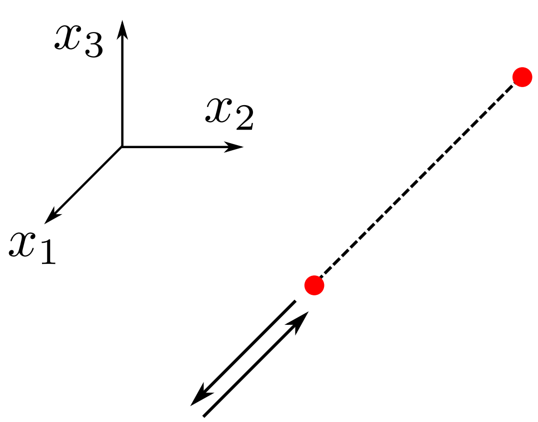
Longitudinal one-sided quench



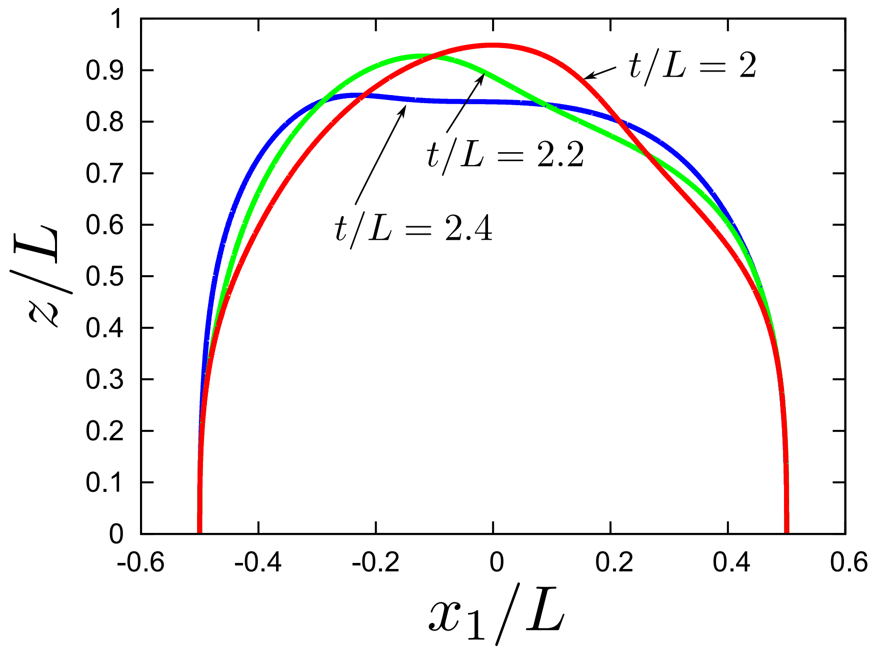
ε=0.03, Δt/L=2

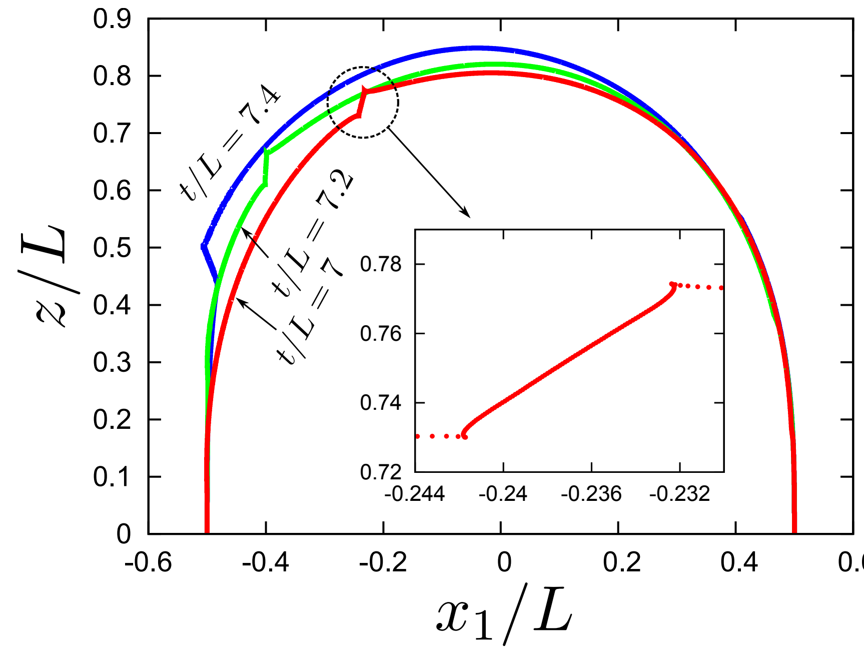


Amplitude: ε=Δx/L

Duration: Δt/L

Cusp formation





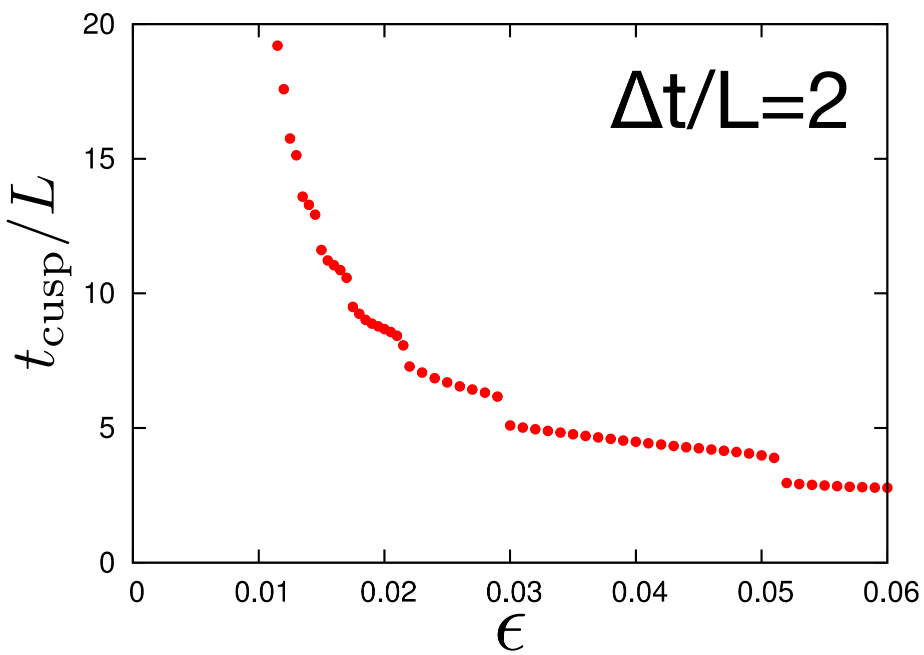
- Cusps are seen in target space (x,z)-coordinates
- Fields on worldsheet (u,v)-coordinates are regular
- Cusps are created in a pair (around t/L~5)

Cusp detection

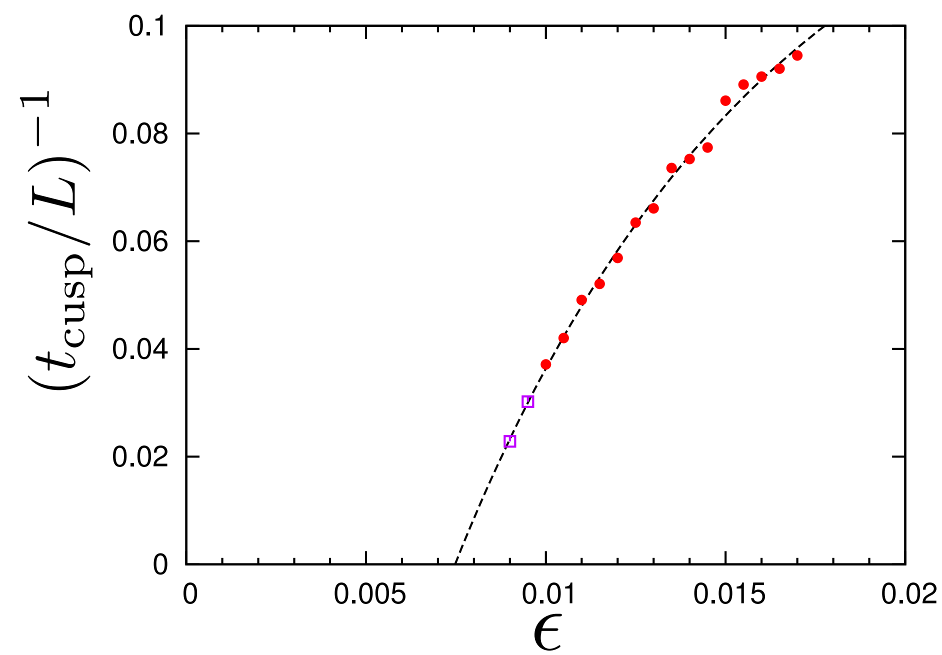
The conditions satisfied at a cusp:

$$J_z \equiv T_{,u} Z_{,v} - T_{,v} Z_{,u} = 0$$

$$J_i \equiv T_{,u} X_{i,v} - T_{,v} X_{i,u} = 0$$

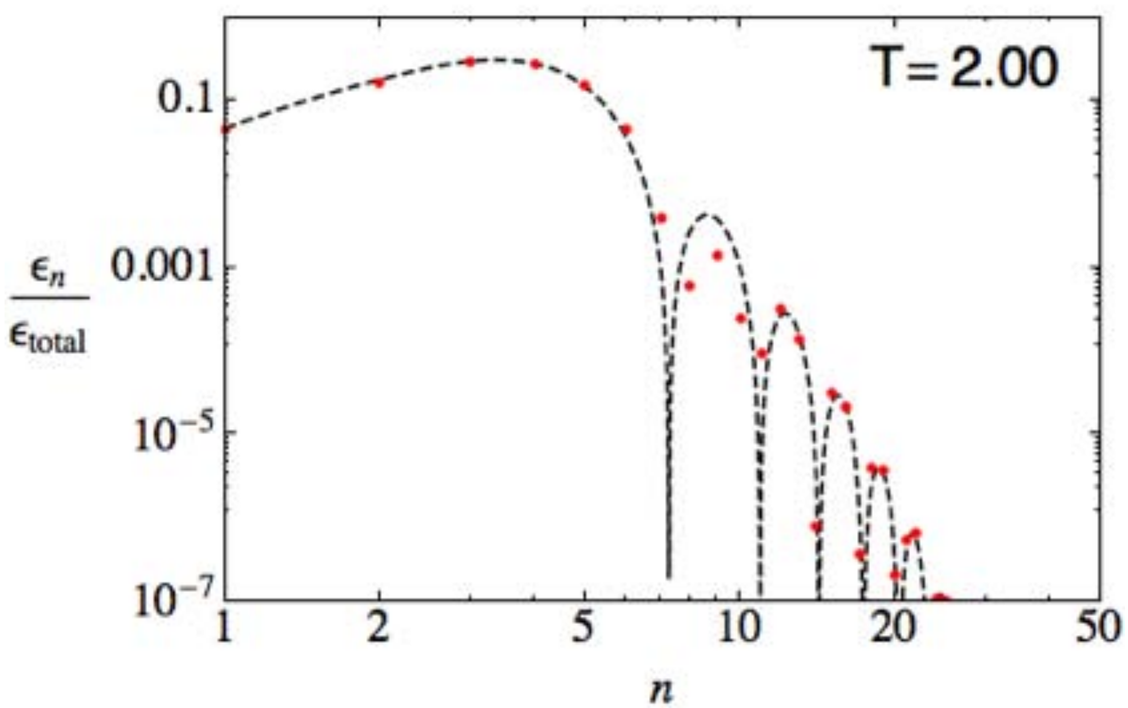


Longer cusp formation time as ε is decreased

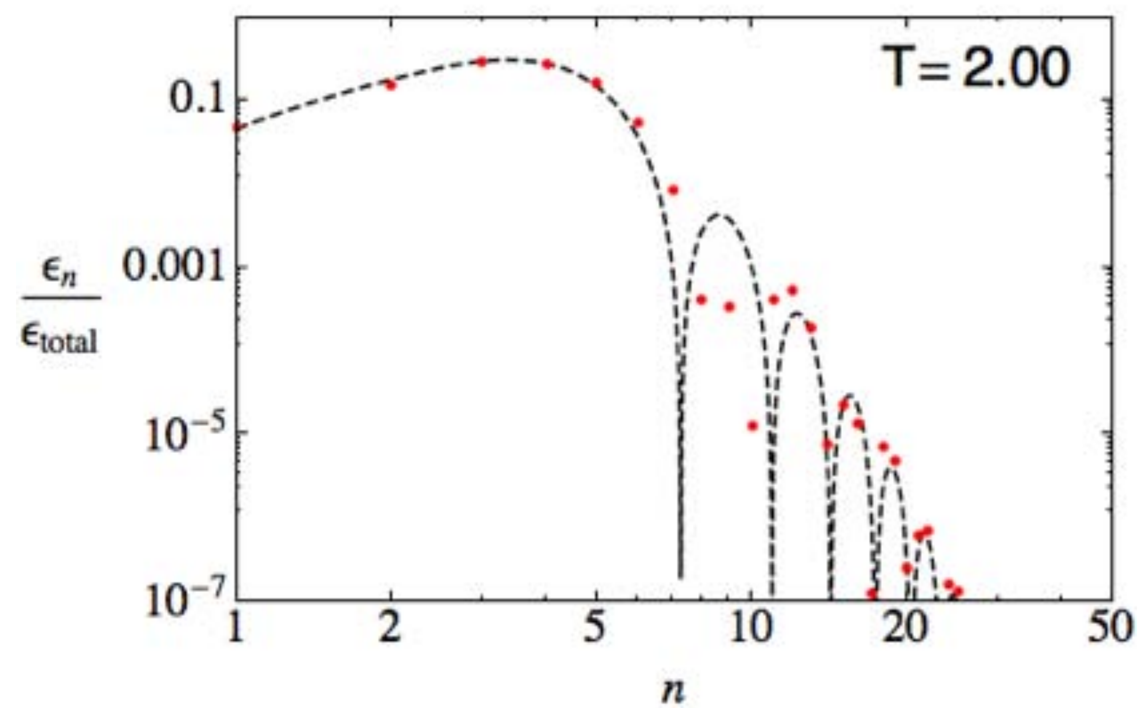


Minimal amplitude for cusps: ε_{crit}~0.075

Energy spectrum (Log-log plot)



ε=0.005, Δt/L=2 (no cusp)



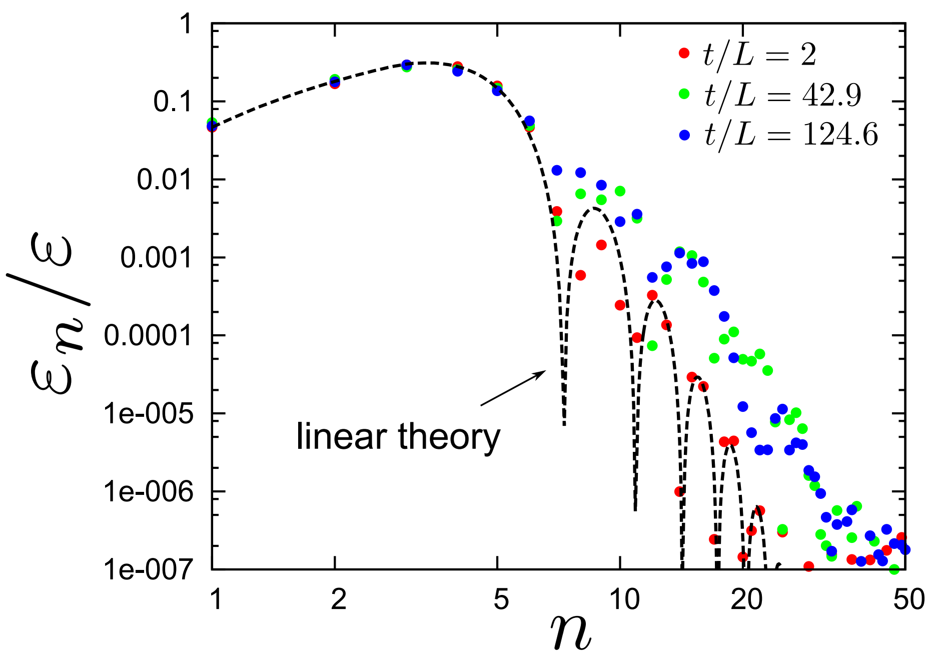
ε=0.01 (cusps T~27)

Decompose nonlinear solutions in linear eigenmodes e_n

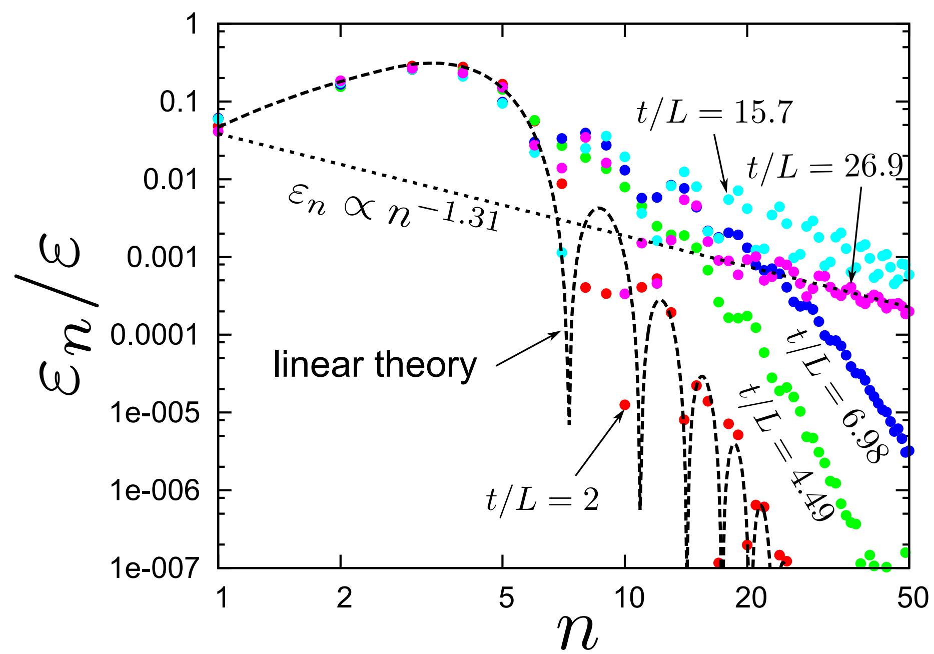
$$\chi_1 = \sum_{n=1}^{\infty} c_n(t) e_n(\phi) \qquad \varepsilon_n(t) = \frac{\sqrt{\lambda} z_0}{4\pi} \left(\dot{c}_n{}^2 + \omega_n^2 c_n{}^2 \right)$$

***Dashed lines are in the linearized theory

Energy cascade



ε=0.005, Δt/L=2 (no cusp)



ε=0.01 (cusps T~27)

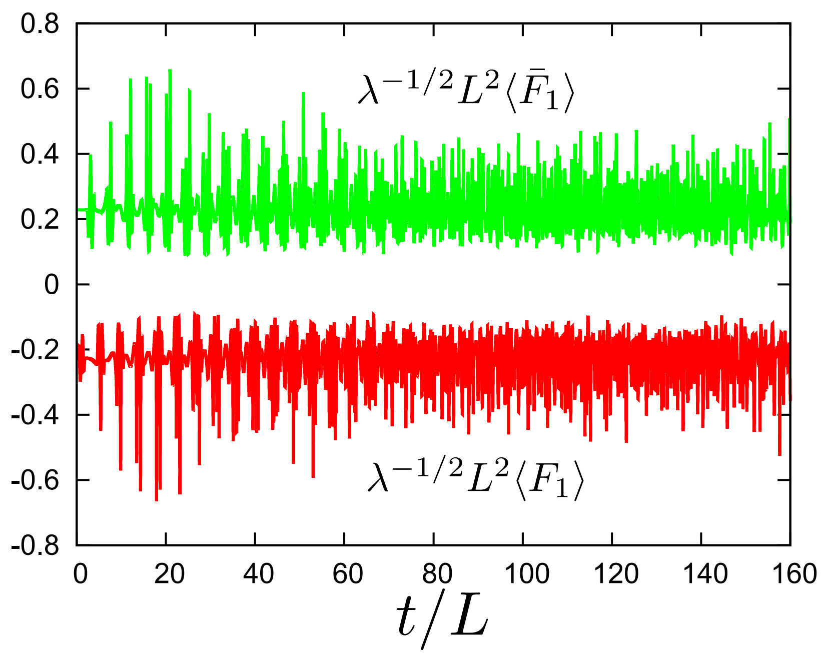
Cusp formation: direct energy cascade → power law

No cusp: no power law

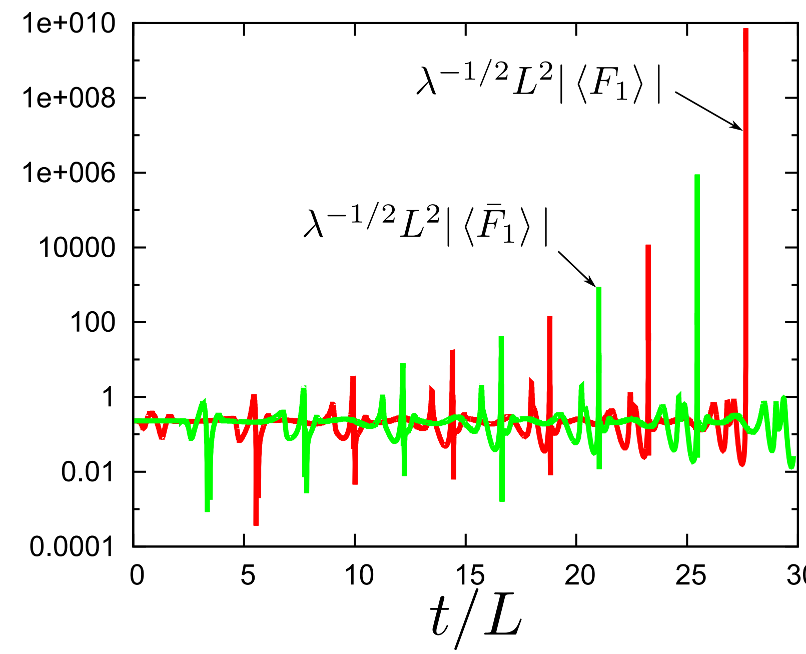
Forces on the endpoints

$$\langle \boldsymbol{F}(t) \rangle = \frac{\delta S_{\text{on-shell}}}{\delta \boldsymbol{x}_q}$$

Force diverges when a cusp reaches the boundary



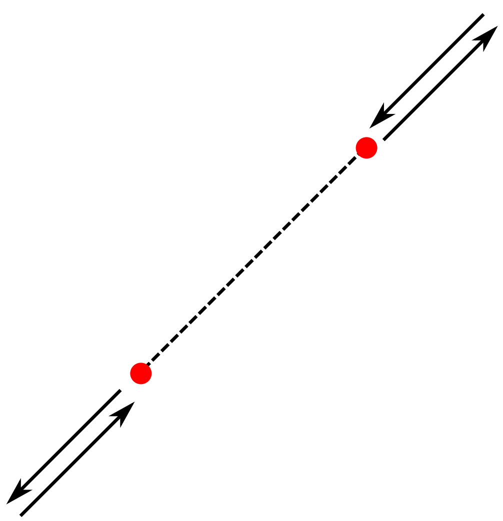
ε=0.005, Δt/L=2 (no cusp)

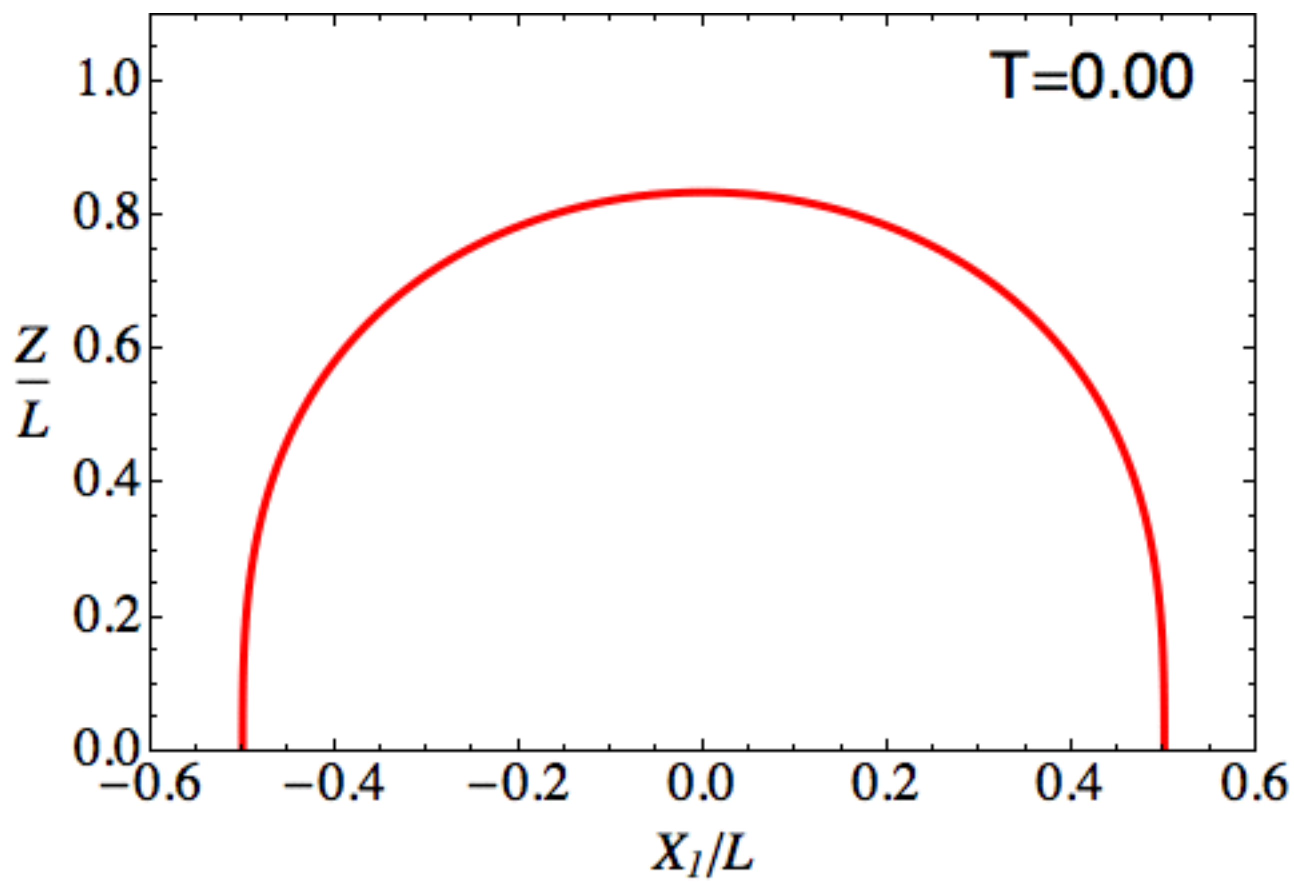


ε=0.01 (cusps T~27)

***Red: x=L/2, green: x=-L/2

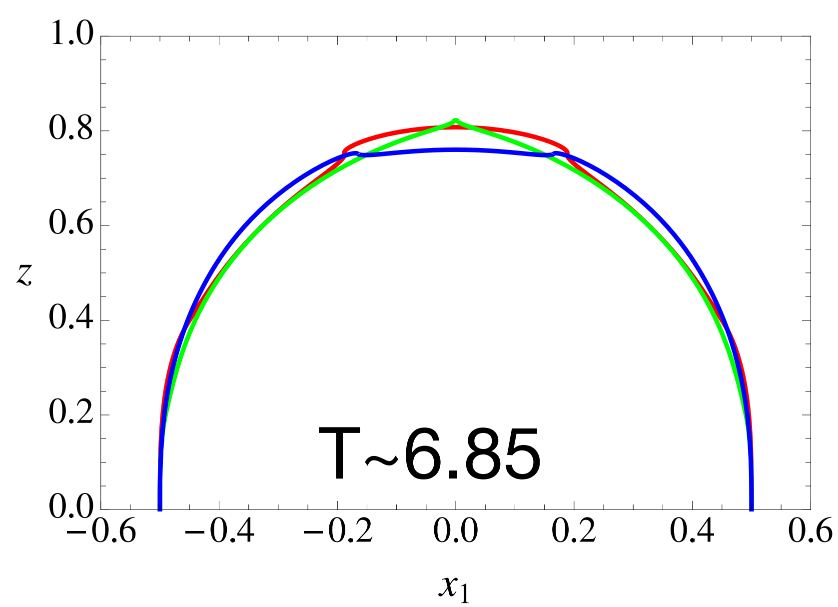
Z_2-symmetric quench

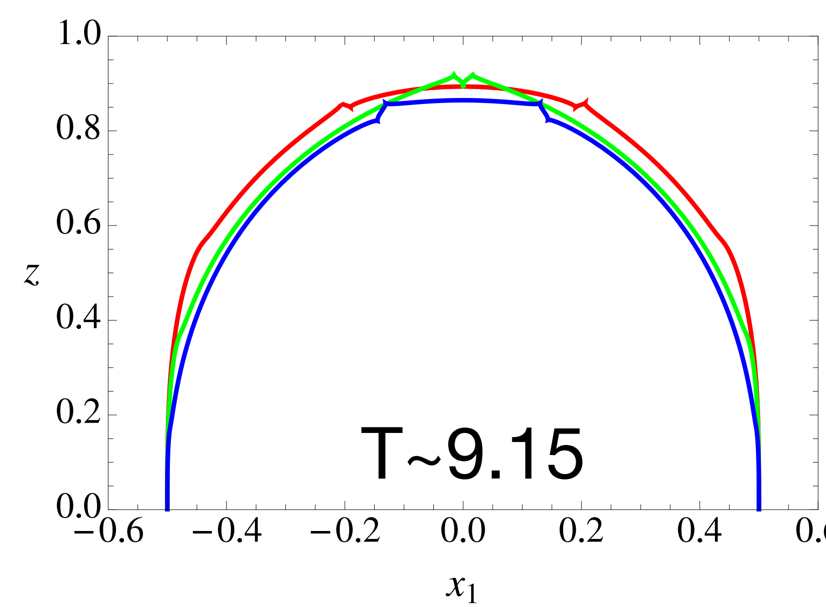




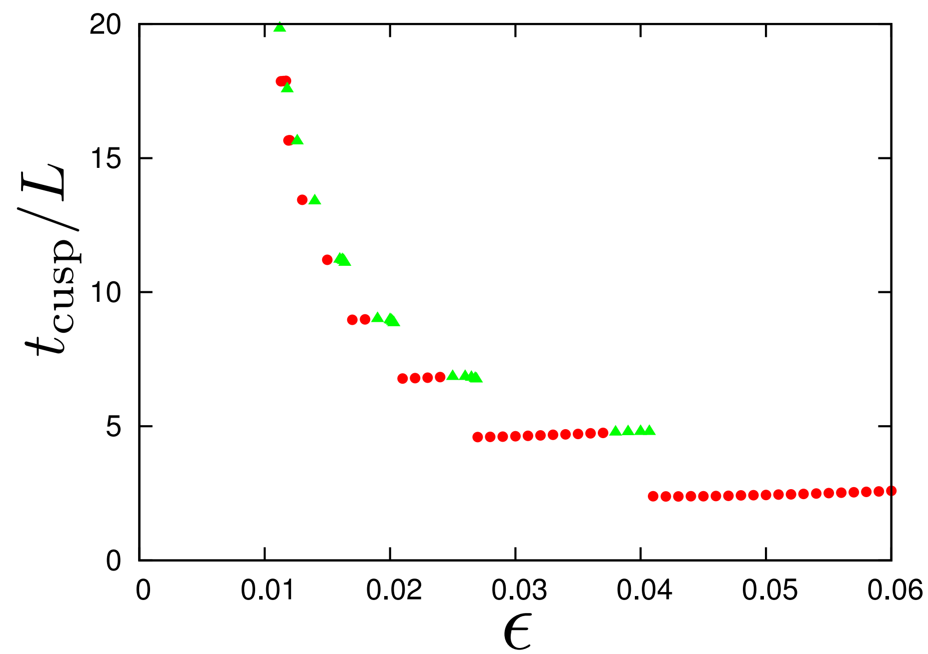
ε=0.025

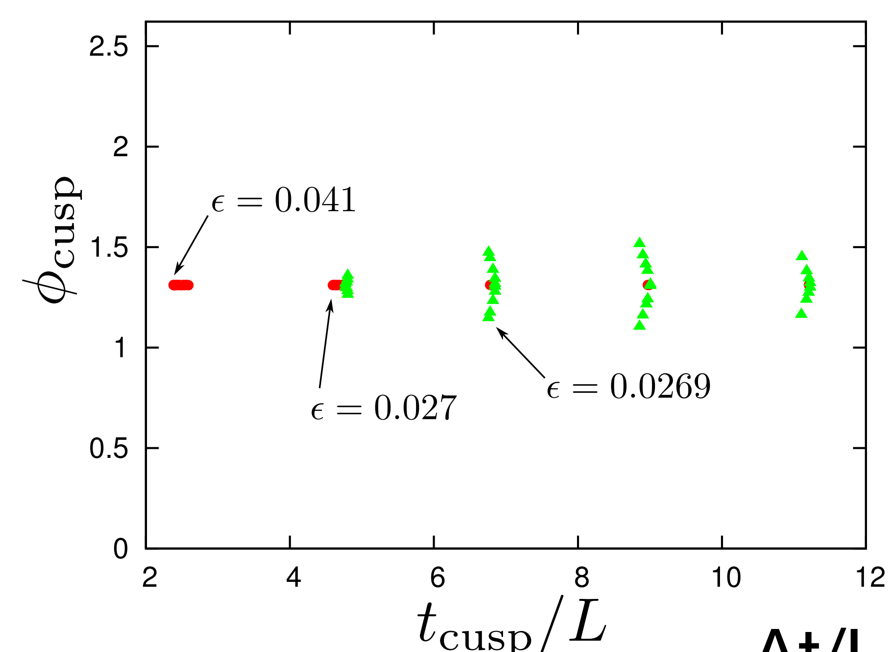
Δt/L=2





Z_2-symmetric quench

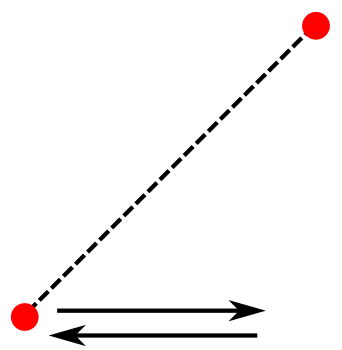


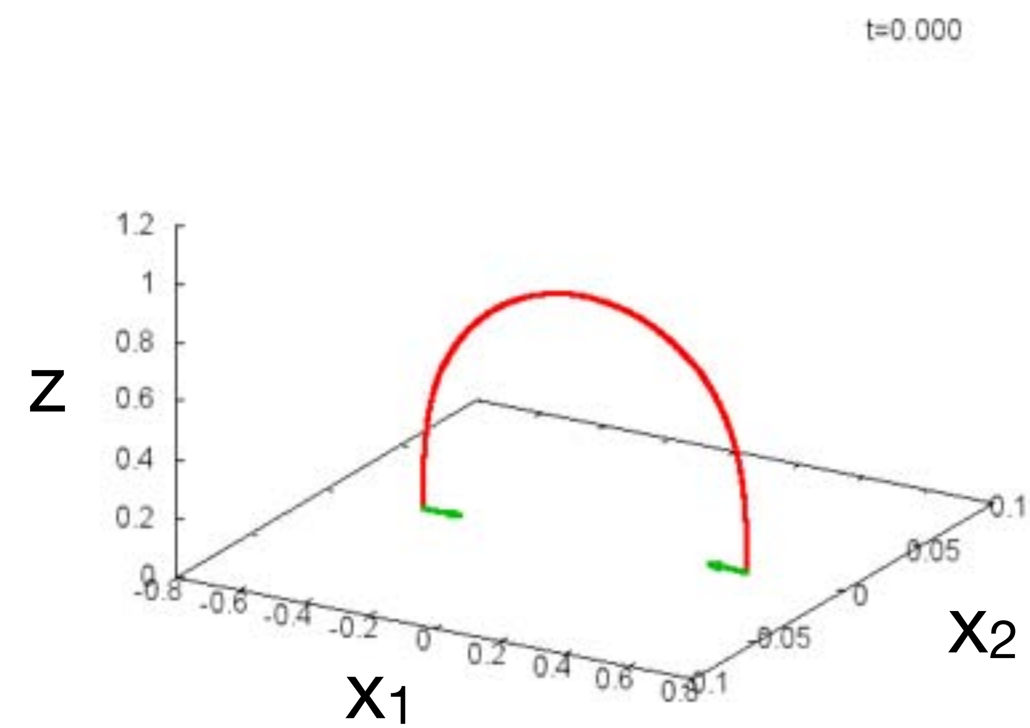


Δt/L=2

- More discretized formation times because of wave collisions
- First cusps by such collisions (red ●). The cusps are pair-created and annihilated.
- Traveling cusps can be formed first (green ▲)

Transverse linear quench



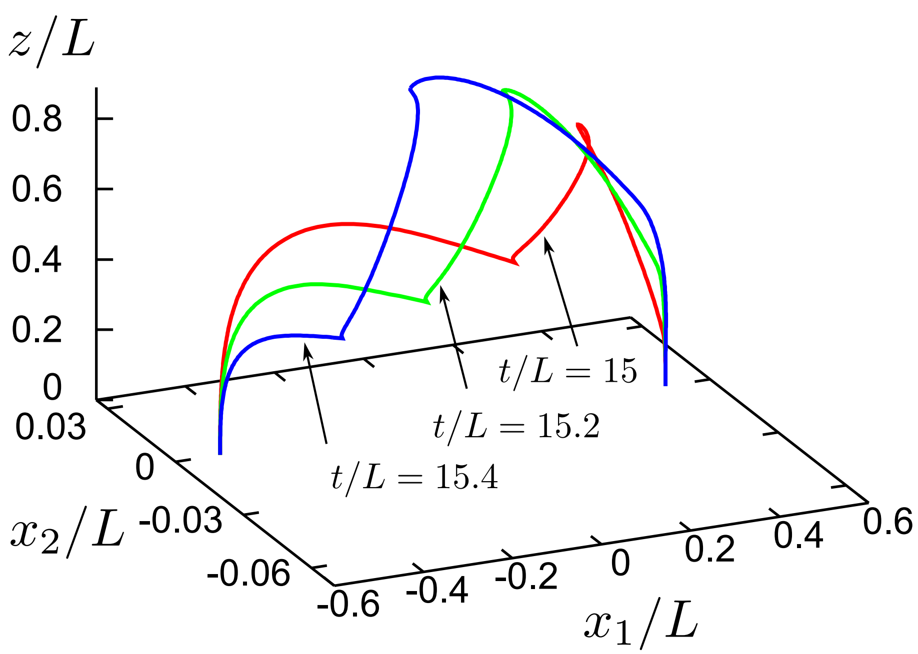


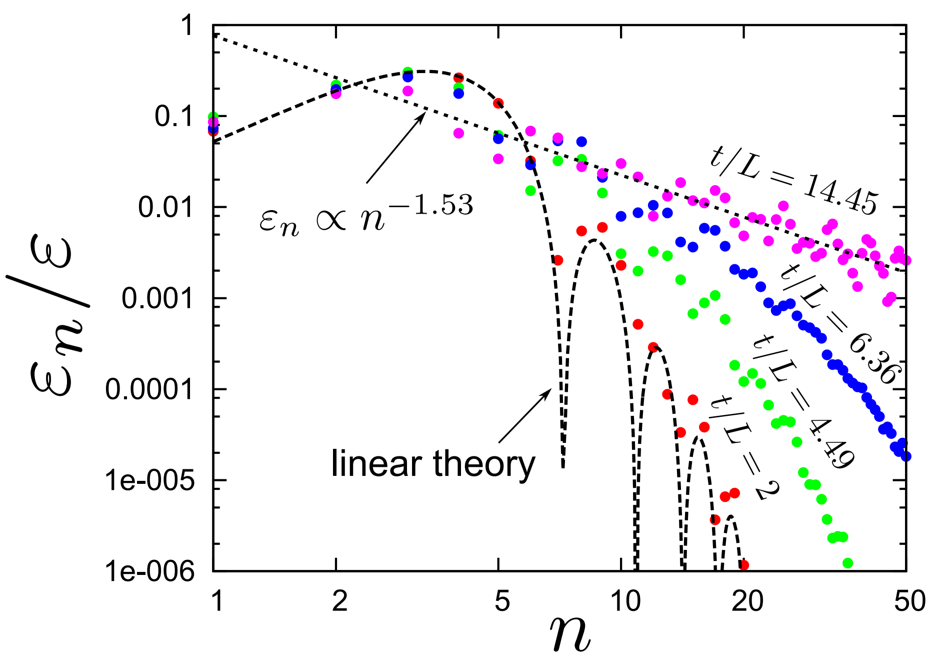
ε=0.03, Δt/L=2

***Green arrows: forces

String oscillates in 1+3 dim (t,z,x_1,x_2)

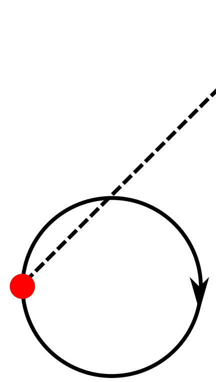
Transverse linear quench

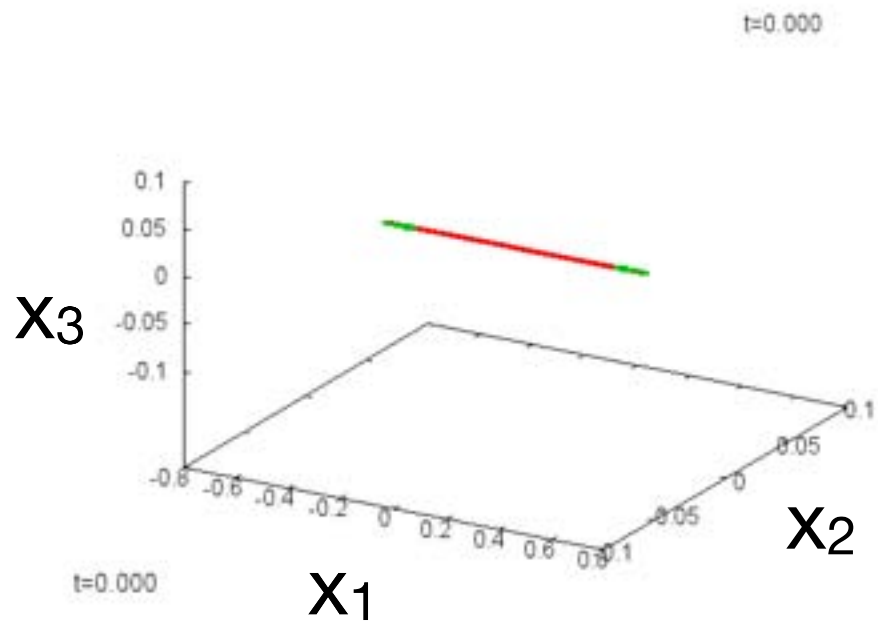




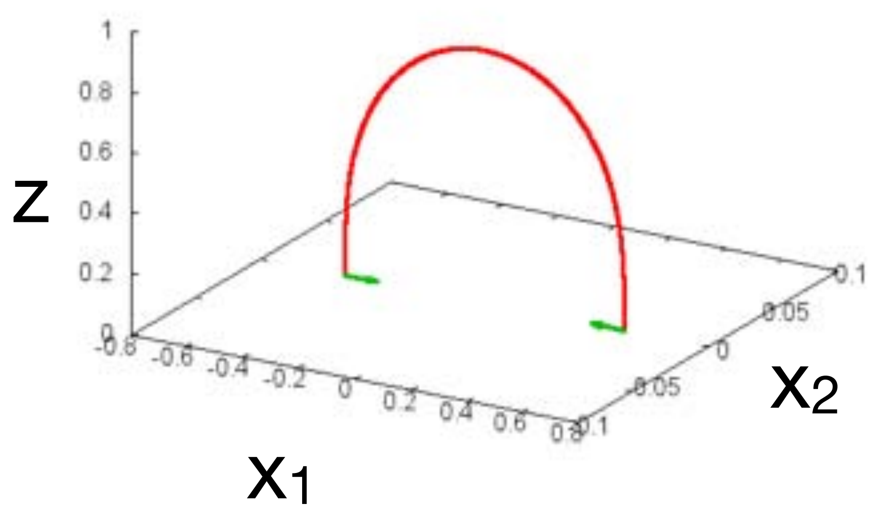
- Cusps are formed at T~14.45
- The energy spectrum shows a direct cascade

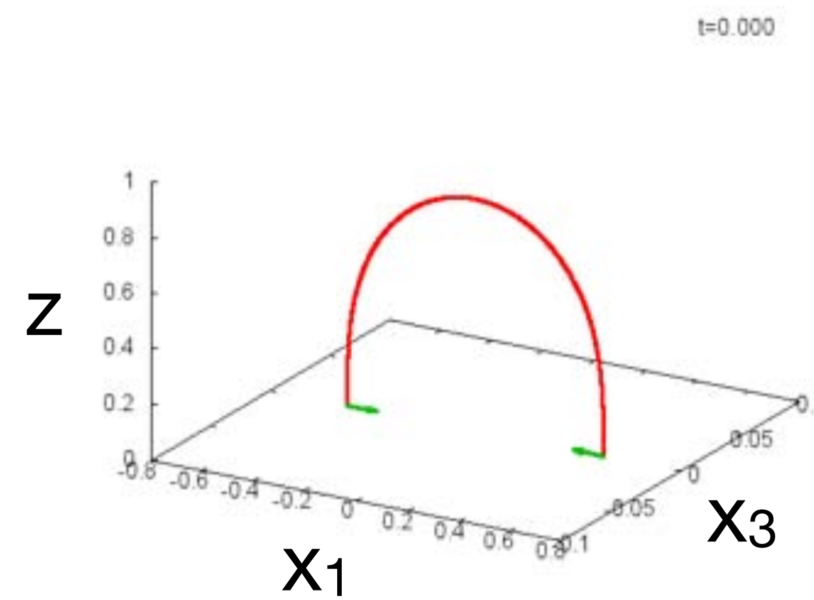
Transverse circular quench





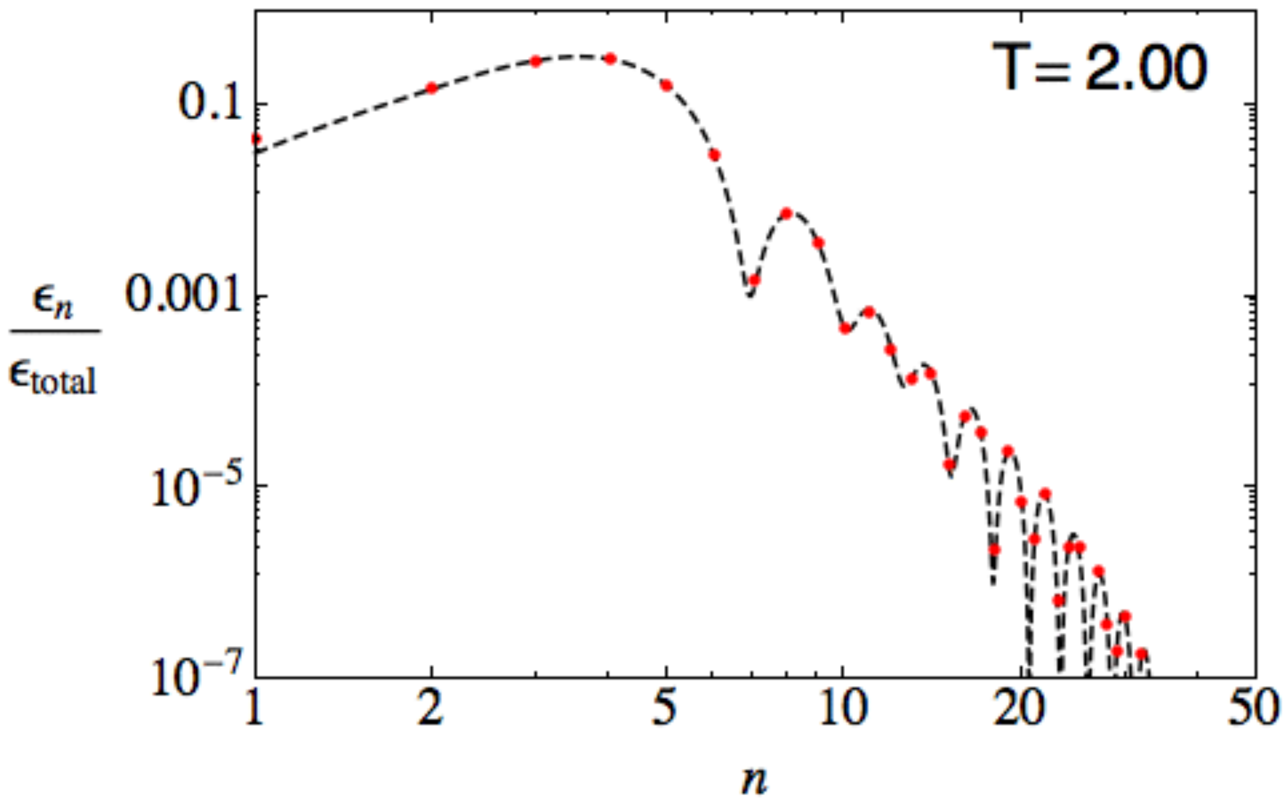
ε=0.02, Δt/L=2





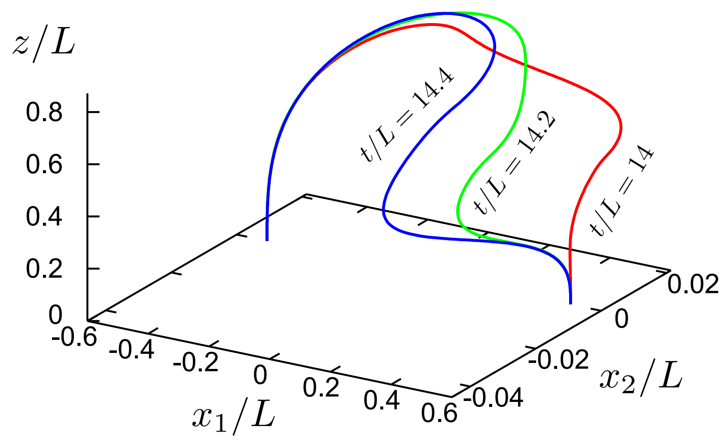
String oscillates in all 1+4 dim (t, z, x_1, x_2, x_3)

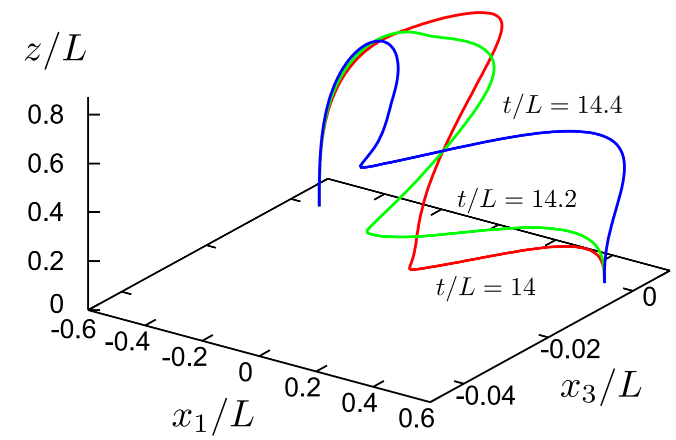
Energy spectrum (Log-log plot)



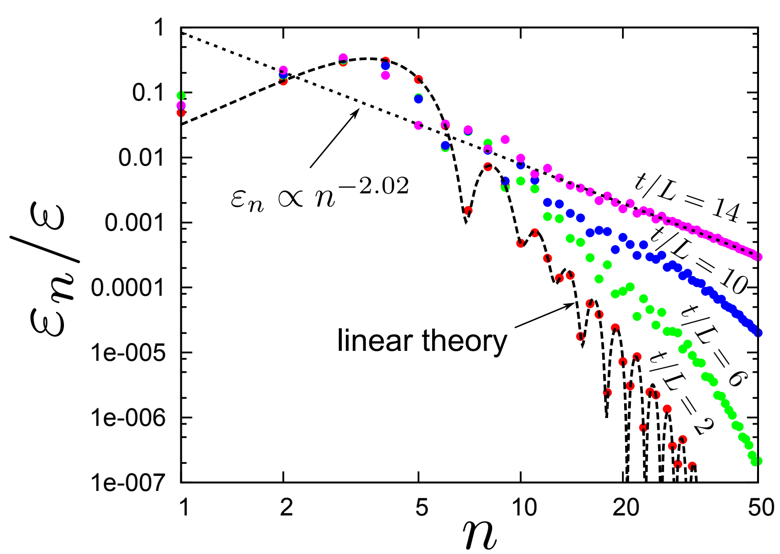
No cusp: no sustainable power law

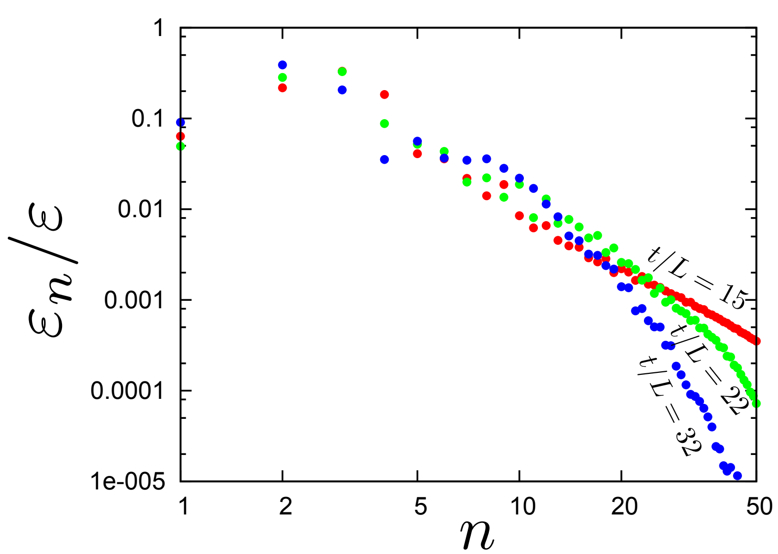
Transverse circular quench





Cuspy, but not real cusps





Direct cascade → inverse cascade

Summary

We computed nonlinear dynamics of the quark-antiquark fundamental string in AdS

- Cusps and turbulent behavior in ≦ 1+3 dim
- No cusp and an inverse cascade in 1+4 dim

Future works

- Large amplitude/finite temperature
- Non-conformal backgrounds
- and more

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