Stringy Hadrons

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Introduction

- The holographic duality is an equivalence between certain bulk string theories and boundary field theories.
- Practically most of the applications of holography is based on relating bulk fields (not strings) and operators on the dual boundary field theory.
- This is based on the usual limit of $\alpha' \longrightarrow 0$ with which we go for instance from a closed string theory to a gravity theory .
- However, to describe hadrons in reality it seems that we need strings since after all in reality the string tension is not very large (λ of order one)

Introduction

- The main theme of this talk is that there is a wide sector of hadronic physical observables which cannot be faithfully described by bulk fields but rather require dual stringy phenomena
- It is well known that this is the case for Wilson, ‘t Hooft and Polyakov lines and also Entanglement entropy
- We argue here that in fact also the spectra, decays and other properties of hadrons:

mesons, baryons and glueballs

can be recast only by holographic stringy hadrons

Introduction

- The major argument against describing the hadron spectra in terms of fluctuations of fields like bulk fields or modes on probe branes is that they generically do not admit properly the Regge behavior of the spectra.
- For M^2 as a function of J we get from flavor branes only J=0,J=1 mesons and there will be a big gap of order λ in comparison to high J mesons if we describe the latter in terms of strings.
- The attempts to get the linearity between M^2 and n basically face problems whereas for strings it is an obvious property.

Outline:

- Confining backgrounds
- The condition for a Regge-like behavior.
- Holography and stringy mesons
- The classical spectra of stringy mesons
- On the quantization of the strings with massive endpoints
- Fits to mesonic data
- Stringy Holographic Baryons
- Fits to baryonic data
- Closed strings as gluballs.
- Identifying gluballs in flavorless data
- Summary

Confining backgrounds

Sufficient conditions for confinement

- We proved a theorem that sufficient conditions for a background to admit a confining Wilson line are if either (Y.kinar, E. Schreiber J.S)

(i) $(f(u))^2 = G_{00} G_{xx}(u)$ has a minimum at u_{min} and $f(u_{min})>0$

(i) $(g(u))^2 = G_{00} G_{uu}(u)$ diverges at u_{div} and $f(u_{div})>0$

Confining backgorunds

- There are handful of backgrounds that admit confining Wilson lines.
- There are bottom-up scenarios like the hard and soft wall
- Here we mention top-down models
- Most of the analysis here is model independent.
- A prototype model of the pure gauge sector is:
- The compactified D4 brane background:
- (i) The critical (10d) model (Witten)
- (ii) The non-critical (6d) model. (S. Kuperstein J.S)

Compactified D4 model (Witten's model)

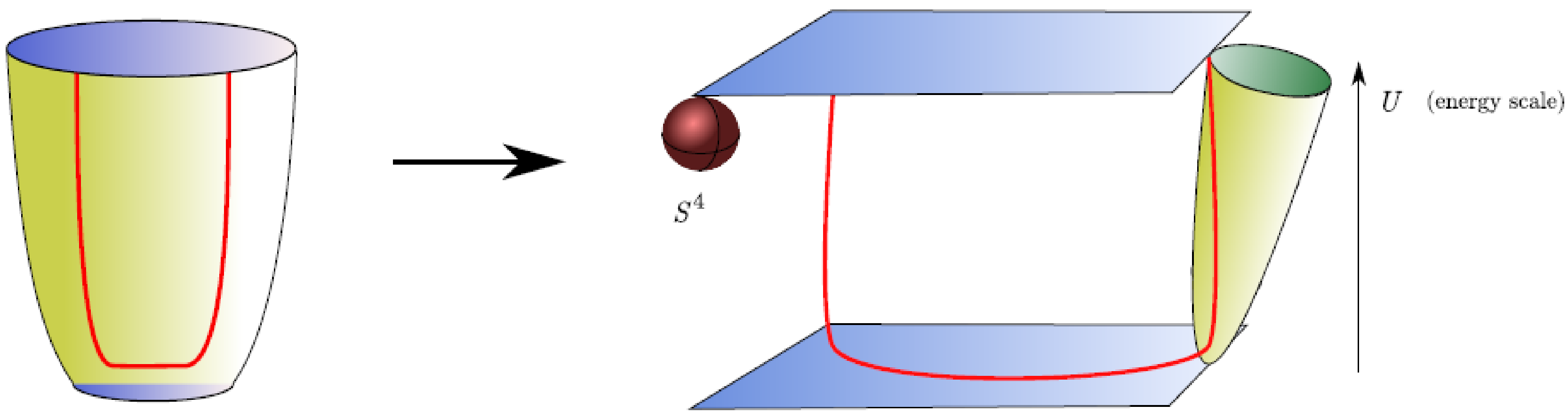
$$ds^2 = \left(\frac{U}{R_{D4}}\right)^{3/2}\left[\eta_{\mu\nu}dX^\mu dX^\nu + f(U)d\theta^2\right] + \left(\frac{R_{D4}}{U}\right)^{3/2}\left[\frac{dU^2}{f(U)} + U^2 d\Omega_4\right]$$

world-volume
our 3+1 world

$$f(U) = 1 - \left(\frac{U_\Lambda}{U}\right)^3$$

θ is a compact Kaluza-Klein circle

U: radial direction bounded from below $U \geq U_\Lambda$



•The gauge theory and sugra parametrs are related via

$$g_5^2 = (2\pi)^2 g_s l_s, \qquad g_4^2 = \frac{g_5^2}{2\pi R} = 3\sqrt{\pi}\left(\frac{g_s u_\Lambda}{N_c l_s}\right)^{1/2}, \qquad M_{gb} = \frac{1}{R},$$

$$T_{st} = \frac{1}{2\pi l_s^2}\sqrt{g_{tt} g_{xx}}\Big|_{u=u_\Lambda} = \frac{1}{2\pi l_s^2}\left(\frac{u_\Lambda}{R_{D4}}\right)^{3/2} = \frac{2}{27\pi}\frac{g_4^2 N_c}{R^2} = \frac{\lambda_5}{27\pi^2 R^3},$$

5d coupling — 4d coupling — glueball mass

String tension

•The gravity picture is **valid** only provided that $\lambda_5 >> R$

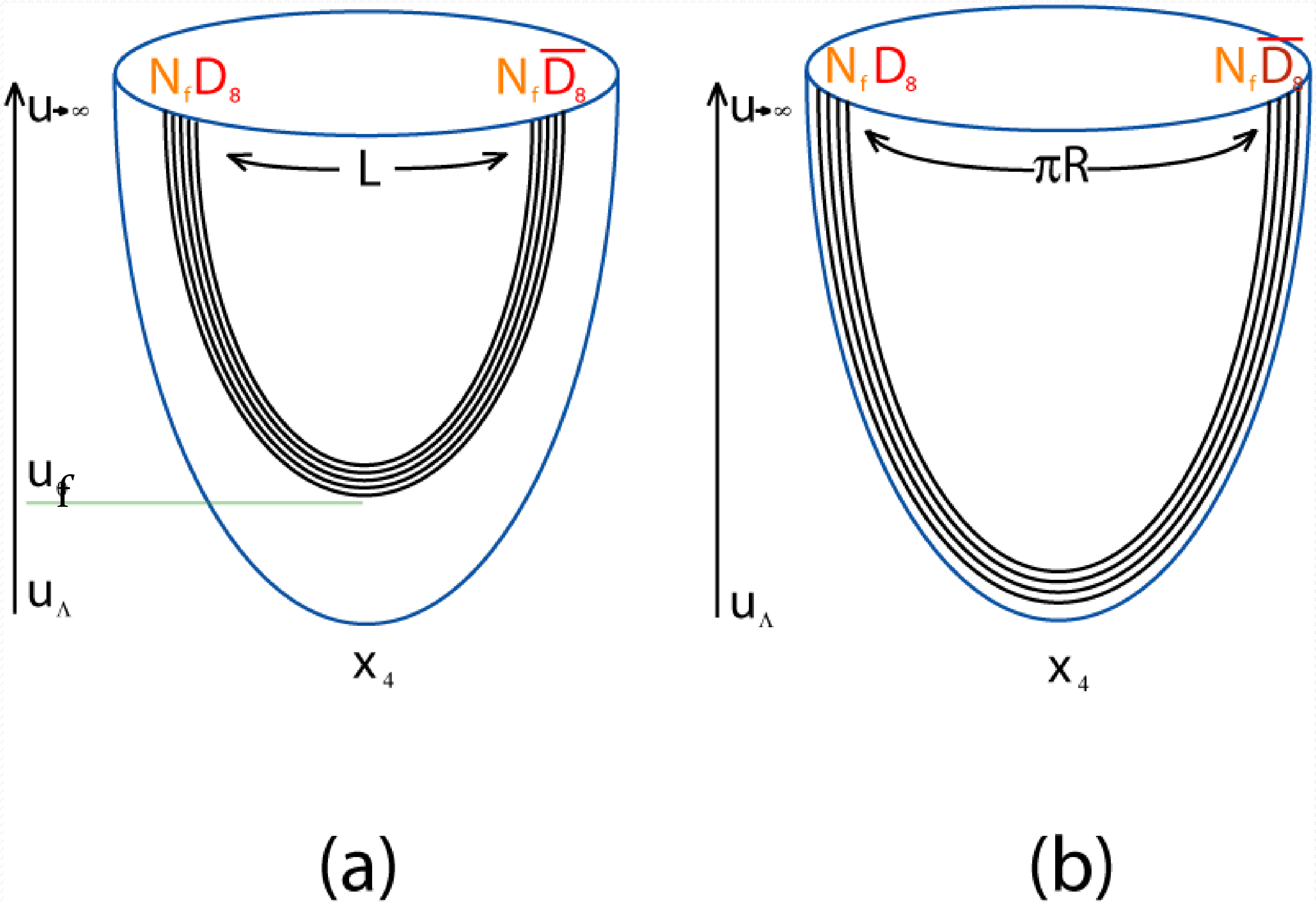
•At energies $E << 1/R$ the theory is **effectively 4d**.

•However it is not really QCD since $M_{gb} \sim M_{KK}$

•In the opposite limit of $\lambda_5 << R$ **we approach QCD**

Adding flavor: The Sakai Sugimoto model

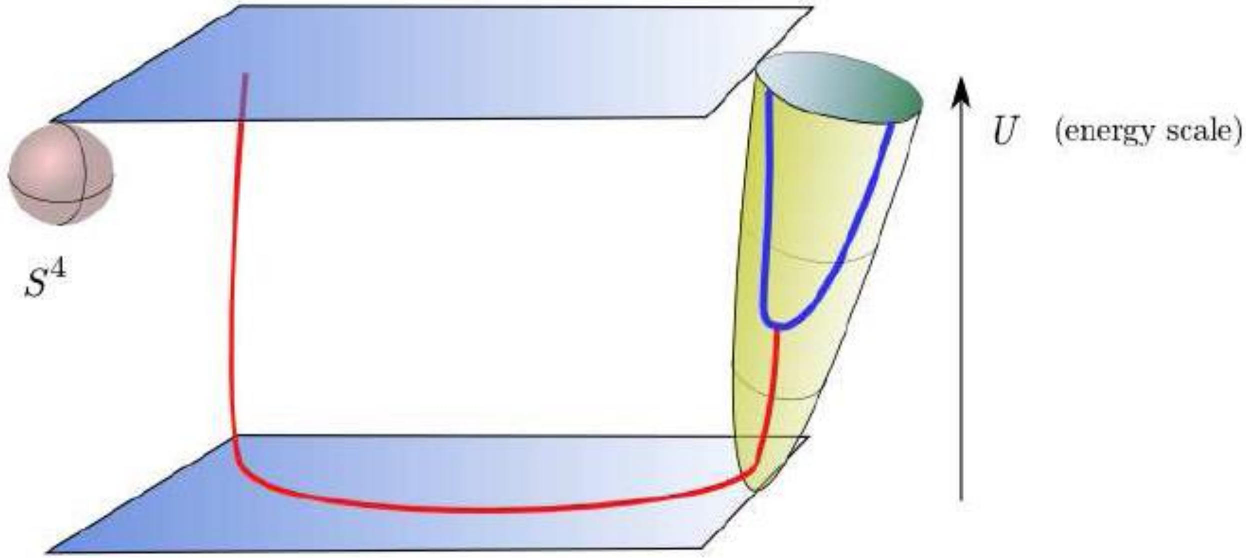
- Adding Nf D8 anti-D8 branes into Witten's model



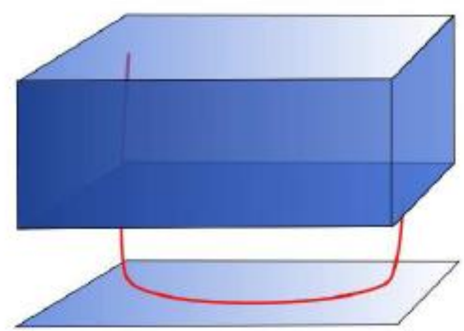
Generalized SS model

Sakai Sugimoto model

Adding flavor: The Sakai Sugimoto model



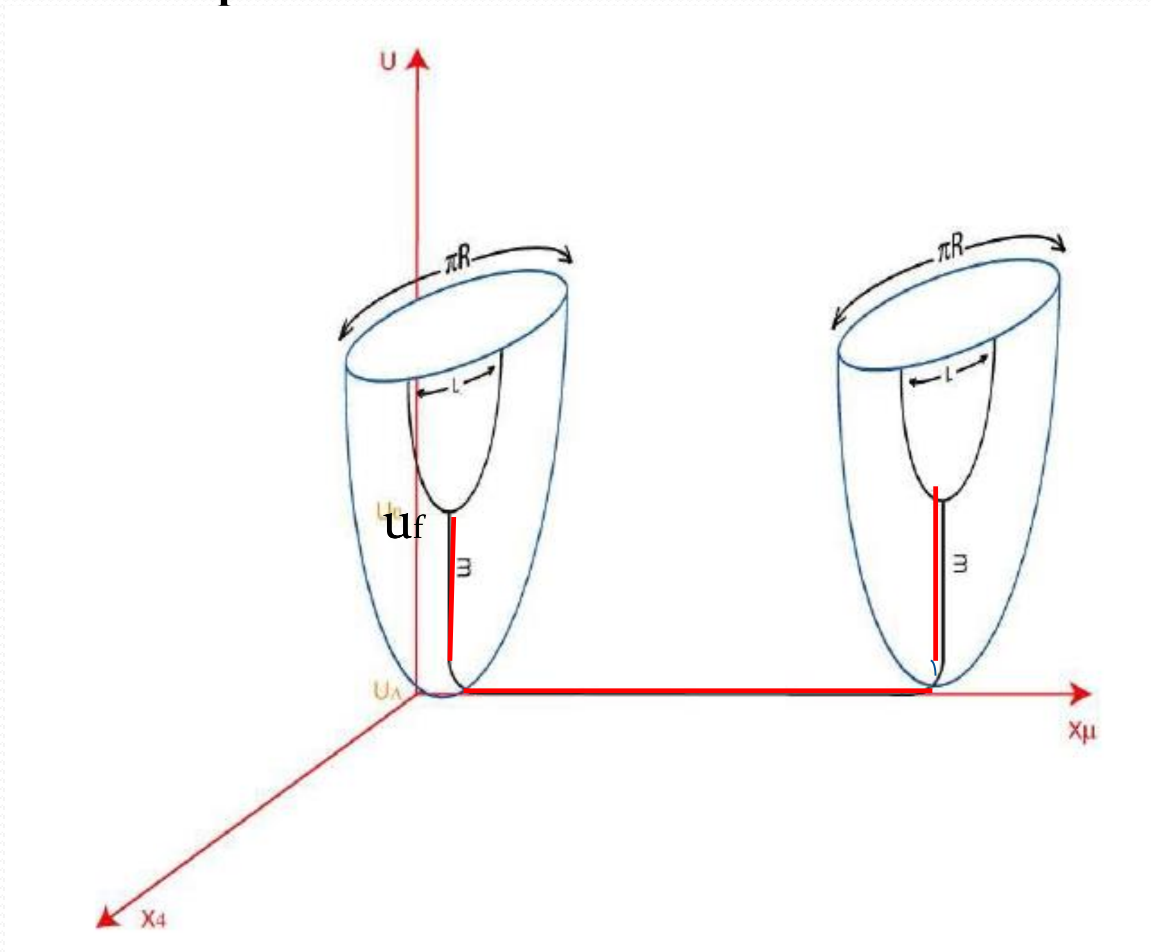
suppressing everything but U
and our 3+1d world:



Stringy holographic Mesons

Stringy meson in U shape flavor brane setup

- In the generalized Sakai Sugimoto model or its non-critical partner the meson looks like



Rotating Strings ending on flavor branes

- Consider a general background of the form

$$ds^2 = G_{mm}dx^m dx^m = -G_{00}dt^2 + G_{xx}dx^2 + G_{uu}du^2 + G_{yy}dy$$

- G_{mm} (u) is a function of the radial direction u
- We look for rotating solutions of the eom

$$x^0 = e\tau, \theta = e\omega\tau, R = R(\sigma), u = u(\sigma), Y^i = Y^i(u_f)$$

- We assume that $u_f > u > u_\Lambda$

Strings ending on flavor branes

- Denote $f \equiv G_{00}$ with $G_{00} = G_{xx}$ and $g^2 = G_{00}G_{uu}$.
- The action in the σ=R gauge than reads

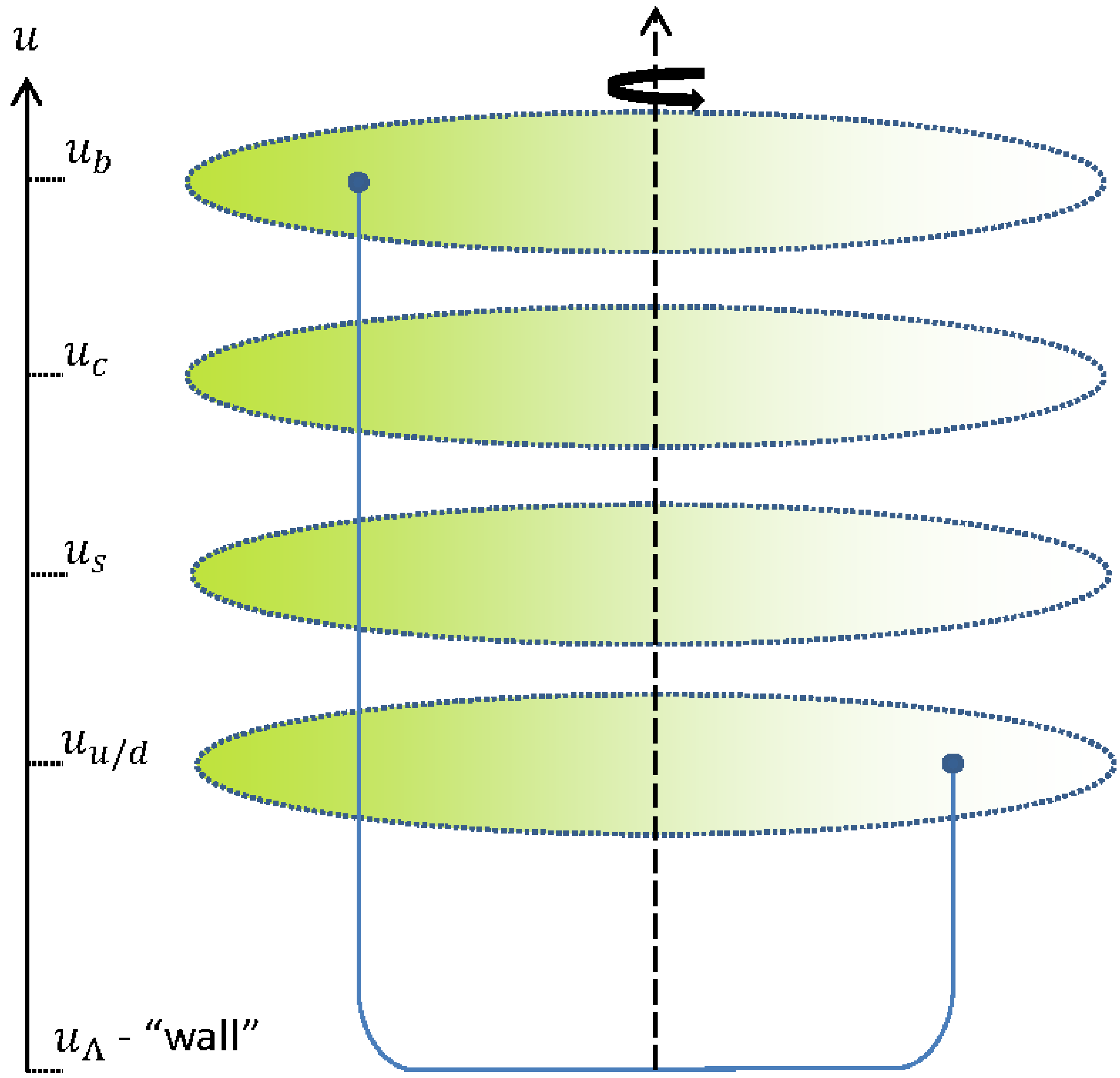
$$S_{NG} = T \int d\tau dR[(fe^2 - f(e\omega R)^2)(f + G_{uu}\dot{u}^2)]^{\frac{1}{2}}$$

- The equation of motion for u(R)

$$\partial_R[g^2 \frac{\epsilon \dot{u}}{G}] = \epsilon(\partial_u G))$$

where
$$\epsilon \equiv \sqrt{1 - v^2}$$
$$G \equiv \sqrt{f^2 + g^2\dot{u}^2} \qquad v = \omega R$$

Example: The B meson



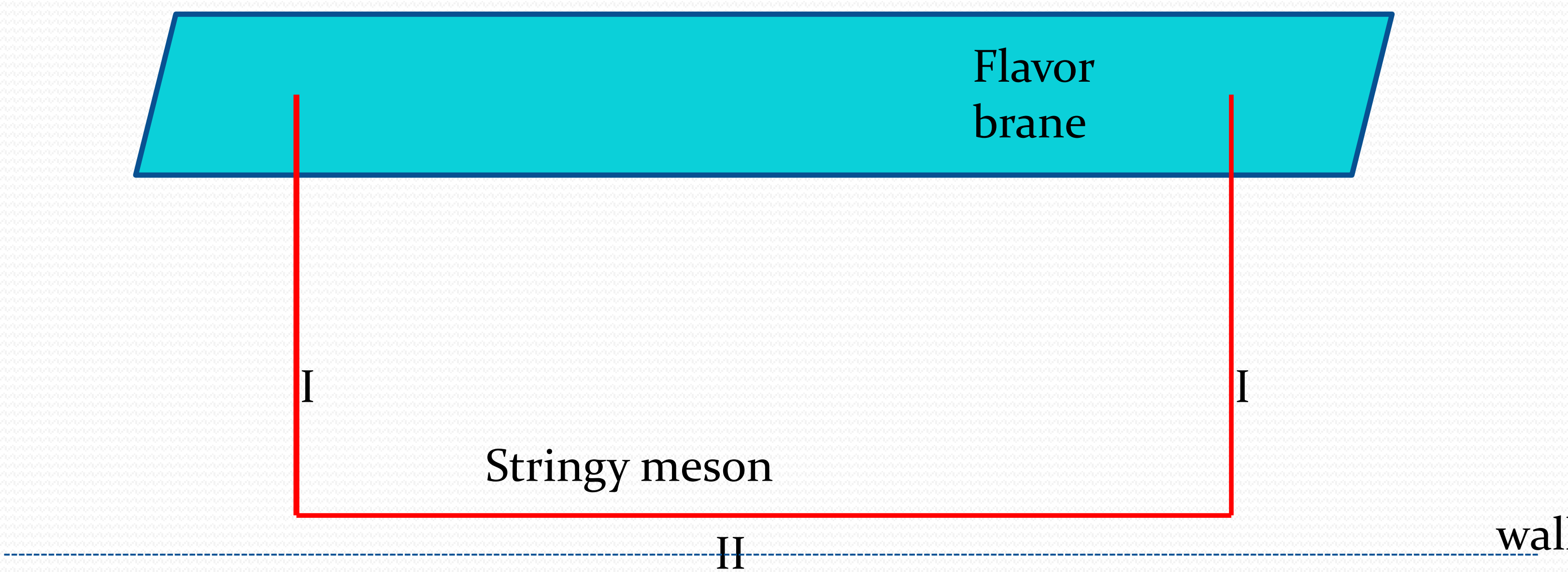
Strings ending on flavor branes

- We now separate the profile into two regions:
- Region (I) vertical $\dot{u} \longrightarrow \infty$

$$\sigma \in (-\pi/2, -\alpha), \sigma \in (\alpha, \pi/2)$$

- Region (II) horizontal $\dot{u} \longrightarrow 0,\ u = u_0$

$$\sigma \in (-\alpha, \alpha)$$



String end-point mass

- We define the string end-point quark mass

$$m_{sep} = T\int_{u_0}^{u_f} g(u)du = T\int_{u_0}^{u_f} \sqrt{G_{00}G_{uu}}du$$

- For δ S=o the system has to obey the condition

$$T_{eff}(1-v^2) = m_q\omega^2 R_0$$

$$T_{eff} = Tf = TG_{00}$$

$$\frac{T_{eff}}{\gamma} = m_{sep}\gamma\omega^2 R_0$$

- This requires that $G_{00}(u_0) > 0$

Condition for a stringy meson

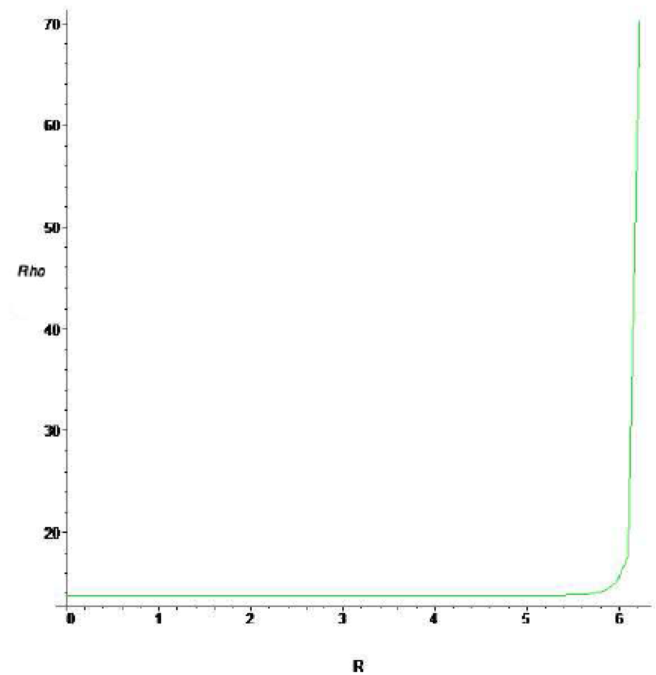
- The conditions to have a solution together read

$$\text{a) } \partial_u G_{00}(u_0) = 0 \text{ and } G_{00}(u_0) > 0 \text{ or}$$
$$\text{b) } G_{uu} \to \infty \text{ and } G_{00}(u_0) > 0$$

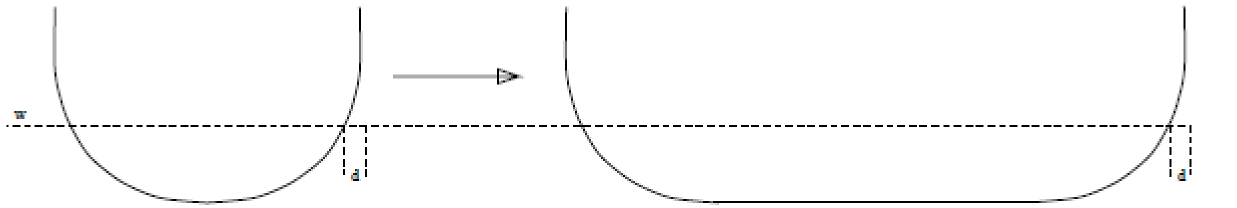
- The conditions to have mesons with Regge behavior in the limit of small m_{sep} are precisely the conditions to have a confining Wilson line

How close is the |_| string to the real holographic one

- This is a numerical calculation of the profile for a string with J=3 rotating in Witten's model background



- For the static case $d/L \leq O(L^{-\frac{1}{k/2-j}})$



Energy and Angular momentum

- The Noether charges associated with the shift of t and θ

$$E = T\int d\sigma \frac{\sqrt{f^2+g^2(u')^2}}{\mathcal{E}} = T\int d\sigma\gamma\sqrt{f^2+g^2(u')^2}$$

$$J = T\int d\sigma\omega R^2 \frac{\sqrt{f^2+g^2(u')^2}}{\mathcal{E}} = T\int d\sigma\omega R^2\gamma\sqrt{f^2+g^2(u')^2}$$

- The contribution of the vertical segments

$$E_I = = T\int_{u_\Lambda}^{u_f} du\gamma\sqrt{\frac{f^2}{(\dot{u})^2}+g^2} = 2\gamma_0 T\int_{u_\Lambda}^{u_f} dug(u) \equiv 2\gamma_0 m_{SEP}$$

$$J_I = = T\int d\sigma\omega R^2\gamma\sqrt{f^2+g^2(u')^2} = 2\gamma_0\omega R_0^2 T\int_{u_\Lambda}^{u_f} dug(u) \equiv 2\gamma_0 m_{SEP}\omega R_0^2$$

Energy and Angular momentum

- Recall the string end-point mass defined as

$$m_{sep} = T\int_{u_\Lambda}^{u_f} du g(u)$$

- The horizontal segment contributes

$$E_{II} = = T\int_{-R_0}^{R_0} dR\gamma\sqrt{f^2+g^2(\dot{u})^2} = f(u_\Lambda)T\int_{-R_0}^{R_0}\frac{dR}{\sqrt{1-\omega^2R^2}} = 2\frac{T_{eff}}{\omega}\arcsin(\omega R_0)$$

$$J_{II} = = T\int_{-R_0}^{R_0} dR\gamma\sqrt{f^2+g^2(\dot{u})^2} = f(u_\Lambda)Tw\int_{-R_0}^{R_0}\frac{dRR^2}{\sqrt{1-\omega^2R^2}} = 2T_{eff}[\frac{1}{\omega^2}\arcsin(\omega R_0) - \omega R_0\sqrt{1-\omega^2R_0^2}$$

- Combining together all the segments we get

$$E = 2m_{sep}\gamma_0 + 2\frac{T_{effe}}{\omega}\arcsin(\omega R_0)$$

$$J = m_{sep}\gamma_0\omega R_0^2 + \frac{T_{effe}}{\omega^2}\arcsin(\omega R_0)$$

Small and large mass approximations

- We can get such relations in the limits of
- Small mass

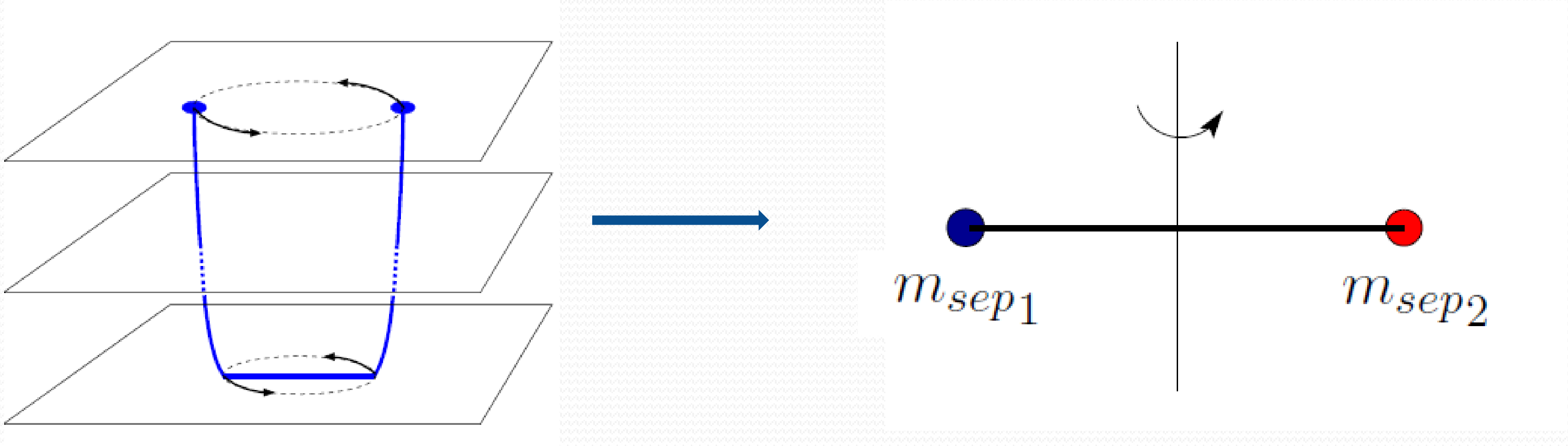
$$J = \alpha' E^2 - \alpha' \frac{4\pi^{1/2}}{3}(m_1^{3/2} + m_2^{3/2})\sqrt{E}$$

- Large mass

$$J_4 = \frac{2m^{1/2}}{T3\sqrt{3}}(E-2m)^{3/2} + \frac{7}{\sqrt{1083}m^{1/2}T}(E-2m)^{5/2} - \frac{1003}{\sqrt{2332803}Tm^{3/2}}(E-2m)^{7/2}$$

From holographic string to string with massive endpoints

- It is now clear that we can map the energy and angular momentum of the holographic spinning string to those of a string in flat space time with massive endpoints. The masses are m_{sep_1} and m_{sep_2}



(M. Kruczenski, L. Pando Zayas D. Vaman)

On the quantization of bosonic string with massive particles on its ends

The classical action

- There are two ways to write the bulk string action

$$S_{NG} = -T \int d\tau \int_{-\delta}^{\delta} d\sigma \sqrt{-h} \qquad h_{\alpha\beta} \equiv \eta_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$$

$$S_{Pol} = -\frac{T}{2} \int d\sigma d\tau \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu$$

- There are two ways to write the endpoints action

$$S_{psq} = -m \int d\tau \sqrt{-\dot{X}^2} \qquad \dot{X}^\mu \equiv \partial_\tau X^\mu$$

$$S_{pa} = \frac{1}{2} \int d\tau \left[\frac{(\dot{X})^2}{\eta} - \eta m^2 \right]$$

Possible classical actions

- Thus there are 4 possible ways for the combined action

$$(i)\ S_{(NG,psq)} \qquad (ii)\ S_{(NG,pa)} \qquad (iii)\ S_{(Pol,psq)} \qquad (iv)\ S_{(Pol,pa)}$$

- In fact there is also another Weyl invariant action

$$S_{Wi} = -\frac{T}{2}\int d\sigma d\tau \sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu + m\int d\tau \sqrt{\gamma_{\tau\tau}}\sqrt{\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu}|_{\sigma=-\delta,\sigma}$$

- For (iv) we associate η with γ_{00} or to take it independent

$$\eta(\tau) = \frac{\sqrt{-\gamma_{\tau\tau}(\sigma,\tau)}|_{\sigma=-\delta,\sigma=\delta}}{m^2}$$

The equations of motion

- The variation of the bulk of the NG action yields

$$\partial_\alpha \left(\sqrt{-h} h^{\alpha\beta} \partial_\beta X^\mu \right) = 0$$

- At the two boundaries we get

$$T\sqrt{-h}\partial^\sigma X^\mu \pm m\partial_\tau \left(\frac{\dot{X}^\mu}{\sqrt{-\dot{X}^2}} \right) = 0$$

- In (ii) the boundary equations and η equations are

$$T\sqrt{-h}\partial^\sigma X^\mu \pm \partial_\tau \left(\frac{\dot{X}^\mu}{\eta(\tau)} \right) = 0 \qquad \frac{(\dot{X})^2}{\eta(\tau))^2} + m^2 = 0$$

The equations of motion

- In (iii) the bulk equation is

$$\partial_a(\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_\beta X^\mu) = 0$$

- The boundary equation is

$$T\sqrt{-\gamma}\partial^\sigma X^\mu \pm m\partial_\tau\left(\frac{\dot{X}^\mu}{\sqrt{-\dot{X}^2}}\right) = 0$$

- The variation of the metric

$$\partial^\alpha X^\mu \partial^\beta X_\mu - \frac{1}{2}\gamma^{\alpha\beta}\partial_\delta X^\mu \partial^\delta X_\mu = 0$$

The solutions of the equations of motion

- A rotating classical solution (τ, σ) in $\mathcal{R} \times [-\delta, \delta]$

$$X = L[\tau, \cos(w\tau)R(\sigma), \sin(w\tau)R(\sigma), 0]$$

- Correspondingly the boundary condition

$$\frac{m}{TL} = \frac{1 - R^2(\delta)}{R(\delta)}$$

- In particular

$$X = L[\tau, \cos(\tau)\sin(\sigma), \sin(\tau)\sin(\sigma), 0] \qquad \frac{m}{TL} = \frac{\cos^2\delta}{\sin\delta}$$

The quantum action

- Even without the massive endpoints the stringy actions in D space-time dimensions are not conformal invariant Q.M.
- Polyakov suggested to add the Liuville term

$$S_{\substack{\text{composite} \\ \text{Liouville}}} \equiv S_{\varphi} = \frac{\beta}{2\pi} \int d^2\sigma \sqrt{|g|} \left(g^{ab} \partial_a \varphi \partial_b \varphi - \varphi \mathcal{R}_{(2)} \right)$$

- For the NG case Polchinski and Strominger took

$$e^{\varphi} \equiv h_{+-} = \partial_+ X \cdot \partial_- X$$

- The PS action reads

$$S_{ps} = \frac{26-D}{24\pi} \int d\tau \int_{-\delta}^{\delta} d\sigma \frac{(\partial_+^2 X \cdot \partial_- X)(\partial_-^2 X \cdot \partial_+ X)}{(\partial_+ X \cdot \partial_- X)^2}$$

On the quantization (preliminary)

- The quantization of the holographic string is a difficult problem
- Instead we consider the quantization of an open string with massive endpoints.
- The exact solution for that question in D dimensions is not known
- There are two obvious limits of :

(i) The static case (v=0, $m \to \infty$)

(ii) The massless case (v=1, m=0)

On the quantization

- The energy of the quantized static open string with no massive endpoints in the D dimension is (Arvis)

$$E_n = \sqrt{(TL)^2 + 2\pi T\left(n - \frac{D-2}{24}\right)}$$

- A naïve generalization of the static to a rotating string with no massive endpoints

$$E_n = \sqrt{(2\pi T J) + 2\pi T\left(n - \frac{D-2}{24}\right)}$$

- Which translates to the Regge relation

$$n + J = \alpha' E_n^2 + a \qquad a = \frac{D-2}{24}$$

On the quantization (preliminary)

- For strings with massive endpoints there are two major differences:
- (i) The relation between J T and E is more complicated as we have seen above
- (ii) The eigenfrequencies are not anymore

$$\boxed{w_n = n}$$

- In addition one has to incorporate the PS non-critical term

On the quantization (preliminary)

- In the Polyakov formulation the solutions of the EQN are

$$X^\mu = x^\mu + l^2 p^\mu \tau + il\sqrt{2} \sum_{n \neq 0} \frac{1}{w_n} \alpha_n^\mu \cos(w_n \sigma + \phi_n) e^{-i w_n \tau}$$

- The eigenfrequencies and phases are given by

$$\tan(\phi_n) = \frac{m^2 w_n}{T} \qquad \tan(w_n \pi) + \frac{2T m^2 w_n}{T^2 - (w_n m^2)^2} = 0$$

- In the limit of massless and infinite mass we get

$$w_n = n.$$

The Casimir energy

- The Casimir energy (or the intercept) is given by

$$E_C(m) = \frac{1}{2}\sum_{n=1}^{\infty} w_n$$

- For the special cases

$$E_C(m=\infty) = E_C(m=0) = \frac{\pi}{2L}\sum_{n=1}^{\infty} n = -\frac{\pi}{24L}$$

- For finite mass $w_n = n + f(R)\frac{1}{n}$

and we cannot use the zeta function regularization

The Casimir energy

- How can we sum over the eigenfrequencies for the massive case?
- We use a contour integral to compute the sum using

(Lambiase Nesterenko)

$$\frac{1}{2\pi i}\oint_C dw w \frac{f'(w)}{f(w)} = \frac{1}{2\pi i}\oint_C dw w [Lan f(w)]' = \sum_k n_k w_k - \sum_l p_l \tilde{w}_l$$

we take zeros poles

$$f(w) = 2mTw cos(wL) - (m^2 w^2 - T^2) sin(wL) = 0$$

So the Casimir energy is

$$E_C(m) = \frac{1}{4\pi i}\oint_C dw w [Lan f(w)]'$$

The Casimir energy

- Where C is a contour that includes the real semi-axis where all the roots of f(w) occur.
- Since f(w) does not have poles we deform the contour to a semi-circle of radius Λ and a segment along the imaginary axis $(-i\Lambda, i\Lambda)$.
- The Casimir energy thus reads

$$
\begin{aligned}
E_C^{(reg)}(m,L) &= \frac{1}{2\pi}\int_0^{\Lambda} dy Lan\left[2mTy\cosh(yL) + (m^2y^2+T^2)\sinh(yL)\right. \\
&+ \frac{1}{4\pi}\left[wLan[f(w)]\right]_{-\Lambda}^{\Lambda} + I_{sc}(\Lambda)
\end{aligned}
$$

- To regularize and renormalize the result we subtract

$$E_C^{(ren)}(m,L) = \lim_{\Lambda\to\infty}\left[E_C^{(erg)}(m,L) - E_C^{(reg)}(m,L\to\infty)\right]$$

The Casimir energy

- The subtracted energy is

$$E_C^{(erg)}(m, L \to \infty) = \frac{1}{2\pi}\int_0^{\Lambda} dy Lan\left[e^{(yL)}\frac{(my+T)^2}{2}\right]$$

- The renormalized Casimir energy is thus

$$E_C^{(ren)}(m, L) = \int_0^{\infty} dx Lan\left[1 - e^{-2x}\left(\frac{(x-a)}{x+a)}\right)^2 \cosh(xL) + (m^2x^2 + T^2)\sinh(xL)\right]5.$$

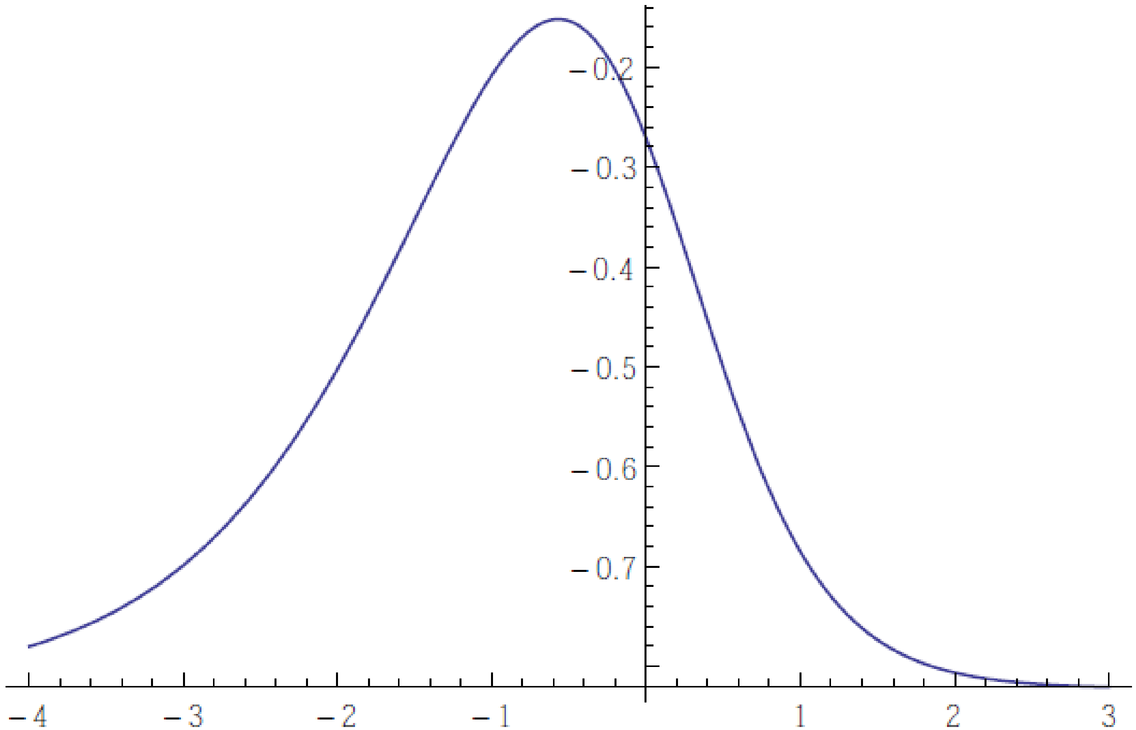
- For the massless and infinite mass cases

$$E_C^{(ren)}(m = 0, L) = E_C^{(ren)}(m = \infty, L) = \int_0^{\infty} dx Lan\left[1 - e^{-2xL}\right] = -\frac{\pi}{24L}$$

The Casimir energy

- Denoting a=m/Tl we definine the ratio

$$\eta(a) = \frac{E_c^{(ren)}(m, L)}{E_c^{(ren)}(m=\infty, L)} = -\frac{12}{\pi^2}\int_0^\infty dz Lan\left[1 - e^{-2z}\left(\frac{1-az}{1+az}\right)^2\right]$$



η as a function of $Lan_{10}(a)$

The Casimir energy

- For the rotating string we simply replace

$$TL = \sqrt{\frac{2}{\pi}}\sqrt{TJ}f(\frac{m_{sep}}{M})$$

- For the massless and small mass cases we have

$$f(\frac{m_{sep}}{M}) = 1 \quad for \quad a = 0 \qquad f(\frac{m_{sep}}{M}) = \sqrt{1 - \frac{8\sqrt{\pi}}{3}\left(\frac{m_{sep}}{M}\right)^{3/2}} \quad a << 1$$

The non criticality term: Liuville term

- The quantum string action is inconsistent for a non-critical D dimensions.
- In the Polyakov formulation for quantum conformal invariance one has to add a Liouville term.
- It can be built from a ``composite Liouville field"

$$\varphi \equiv -\frac{1}{2}\ln(g^{ab}\partial_a X^\mu \partial_b X_\mu)$$

- The action then reads

$$S = S_{\text{Polyakov}} + S_{\substack{\text{composite}\\ \text{Liouville}}}$$

- The Liouville term is

$$S_{\substack{\text{composite}\\ \text{Liouville}}} \equiv S_\varphi = \frac{\beta}{2\pi}\int d^2\sigma \sqrt{|g|}\,\left(g^{ab}\,\partial_a\varphi\,\partial_b\varphi - \varphi\,\mathcal{R}_{(2)}\right)$$

where

$$\beta \equiv \frac{26-D}{12}$$

The non criticality term: The Polchinsky Strominger term

- In the Nambu-Goto formulation the anomaly is cancelled by adding a Polchinky Strominger term
- For a classical rotating string parameterized as

$$X = l(\tau, \cos(\tau)\sin(\sigma), \sin(\tau)\sin(\sigma), 0)$$

- The induced metric is $h_{\alpha\beta} = l^2 \cos^2(\sigma)\eta_{\alpha\beta}$
- For the range of (τ,σ) $\mathcal{R} \times [-\delta, \delta]$
- The boundary condition is

$$\frac{m}{Tl} = \frac{\cos\delta}{\tan\delta}$$

- The PS term is

$$\begin{aligned} S_{ps} &= \int_{-\delta}^{\delta} \frac{26-D}{24\pi} \frac{(\partial_+^2 X \cdot \partial_- X)(\partial_-^2 X \cdot \partial_+ X)}{(\partial_+ X \cdot \partial_- X)} = -\frac{26-D}{24\pi} \int_{-\delta}^{\delta} d\sigma \tan^2(\sigma) \\ &= -\frac{26-D}{12\pi}(\tan(\delta) - \delta) \end{aligned}$$

The non-criticality term for the massless case

- Inserting the rotating classical string to the Liuville field one finds that
- The Liouville term= The Polchinsky Strominger term

- For the massless case δ=π/2 and hence the non-critical term diverges.
- Hellerman et al suggested a procedure to regularize and renormalize this divergence for the massless case.
- They found an amazing result that the intercept dose not depend on D

$$a = \frac{D-2}{24} + \frac{26-D}{24} = 1$$

- The generalization of this result to the massive case is under current investigation

Leading 1/m order quantum correction

- In the limit of large m/TL (v<<1) the boundary eom

$$\frac{TL}{\gamma} = m\gamma(wL)^2 \rightarrow \qquad (wL)^2 = \frac{TL}{m} << 1$$

- The classical trajectory

$$J \sim \frac{4\pi}{3\sqrt{3}}\alpha' \, m^{1/2}(E-2m)^{3/2} + O(E-2m)^{5/2}$$

- The quantum corrected trajectory involves

$$\alpha' E_{cl}^2 \rightarrow \alpha' E_{qm}^2 = \alpha' E_{cl}^2 + a = \alpha' E_{cl}^2 + (a_{Cas} + a_{PS})$$

Leading 1/m order quantum correction

- Thus the corrected trajectory reads

$$J \sim \frac{4\pi}{3\sqrt{3}}\alpha' \, m^{1/2} \left(\sqrt{E^2 + \frac{(a_{Cas} + a_{PS})}{\alpha'}} - 2m \right)^{3/2} + O(E - 2m)^{5/2}$$

- The contribution of Sps to the intercept for D=4

$$a_{ps} = -\frac{26 - D}{12\pi}(\tan(\delta) - \delta) = -\frac{11}{36\pi}\left(\frac{TL}{m}\right)^3$$

- We can replace the dependence on TL with

$$\frac{TL}{m} \simeq \left(\frac{3\sqrt{3}}{2}\frac{TJ}{m^2}\right)^{2/3} \qquad a_{ps} \simeq -\frac{2}{\pi}\left(\frac{TJ}{m^2}\right)^2$$

- We can approximate the a_{Cas}

$$a_{Cas} \simeq \frac{3}{2\pi}\left[0.18\left(\frac{TL}{m}\right) - 0.07\right] = 0.16\left(\frac{TJ}{m^2}\right)^{2/3} - 0.06$$

Fits of Stringy mesons with massive endpoints to experimental data

Fitting analysis

- Now we leave the holographic world and go down to earth to fit the data using the analogous string with massive endpoints.
- We confront the theoretical massive modified Regge relations (M^2, J, n) with experimental data.
- It is easier to analyze separately (M^2, J) and (M^2, n)
- For (M^2, J) we use the following models:
- (i) The linear original Regge relation

$$J_l = \alpha' E^2 + \alpha_0$$

Fitting analysis: (M^2, J)

- (2) The modified massive Regge relation

$$E_m = 2m\left(\frac{\omega R \arcsin(\omega R) + \sqrt{1-(\omega R)^2}}{1-(\omega R)^2}\right)$$

$$J_m = \frac{m^2}{T}\frac{(\omega R)^2}{(1-(\omega R)^2)^2}\left(\arcsin(\omega Rq) + \omega R\sqrt{1-(\omega R)(\omega R)^2} + \alpha_0\right.$$

The small and large mass limits

- In the small mass limit m/TL<<1 the trajectory reads

$$J_{sm} = \alpha' E^2 - \alpha' \frac{4\pi^{1/2}}{3}(m_1^{3/2} + m_2^{3/2})\sqrt{E} + \alpha_0$$

- The large mass limit

$$J_{lm} = \frac{2m^{1/2}}{T3\sqrt{3}}(E-2m)^{3/2} + \frac{7}{\sqrt{1083}m^{1/2}T}(E-2m)^{5/2} - \frac{1003}{\sqrt{2332803}Tm^{3/2}}(E-2m)^{7/2}$$

Fitting analysis (M^2, n)

- The original linear Regge relation

$$n_l = \alpha' E^2 + \alpha_0$$

- The WKB approximation

$$n_{WKB} = a + \frac{1}{\pi} \int_{x_-}^{x^+} dx \sqrt{(E - V(x; J_s))^2 - m^2 - (J_q/x)^2}$$

Extracted parameters

- The parameters we extract from the fits with the lowest χ^2 are
- (i) α' measured in Gev^{-2} or the string tension $T = \frac{1}{2\pi\alpha'}$
- (ii) α_0 the intercept (dimensionless)
- (iii) (m_1, m_2) the string endpoint masses

- We define χ^2 in a non-standard but convenient way

$$\chi^2 = \frac{1}{N-1}\sum_i \left(\frac{M_i^2 - E_i^2}{M_i^2}\right)^2$$

The meson trajectories fitted

- For (M^2, J) we compared with the following Regge trajectories

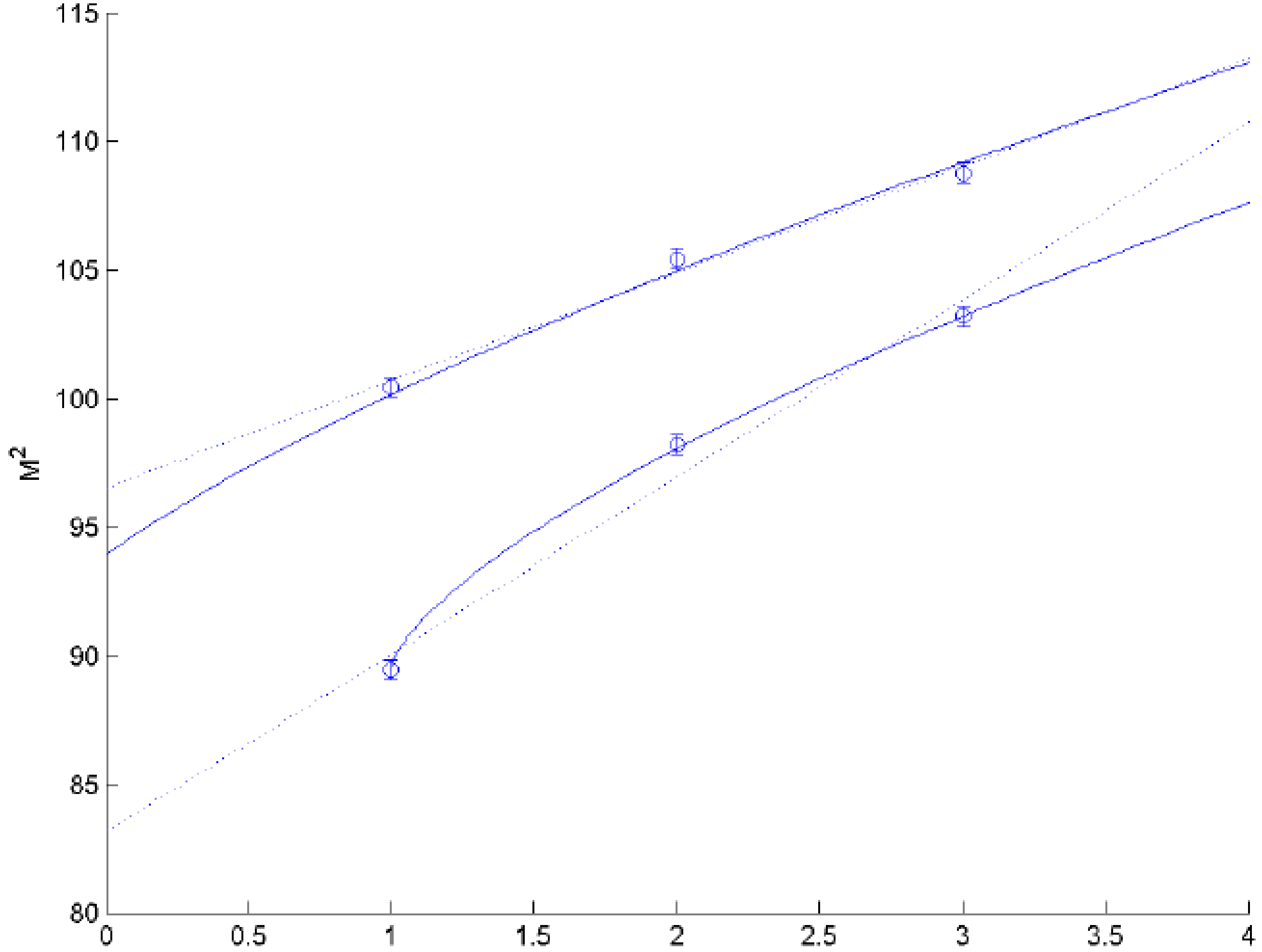
ρ **meson parent** $-$ $\rho(775.5 \pm 0.4)(1^{--}), a_2(1318.3 \pm 0.6)(2^{++}), \rho_3(1688.8 \pm 2.1)(3^{--}),$
$a_4(2001 \pm 10)(4^{++}), \rho_5(2350)(5^{--}), a_6(2450 \pm 130)(6^{++})$
ρ **meson daughter** $-$ $a_0(984.7 \pm 1.2)1^-(0^{++})\rho(1459 \pm 11)1^+(1^{--}), a_2(1732 \pm 16)1^-(2^{++}), \rho_3(1990)1^+(3^-$
ω **meson** $-$ $\omega(782.65 \pm 0.12)(1^{--}), f_2(1275.4 \pm 1.1)(2^{++}),$
$\omega_2(1667 \pm 4)(3^{--}), f_4(2025 \pm 10)(4^{++}), f_6(2465 \pm 50)$
$\mathbf{K^*}$ **meson** $-$ $K^*(891.66 \pm 0.26)(1^-), K_2^*(14256 \pm 1.5)(2^+),$
$K_3^*(1776 \pm 7)(3^-), K_4^*(2045 \pm 9)(4^+), K_5^*(2382 \pm 24)(5^-)$
$\mathbf{c\bar{c}}$ **meson** $-$ $\Psi(1S)(3.0969), \chi_{c2}(1P)(3.5563), Psi(1D)(3.836)$
$\mathbf{b\bar{b}}$ **meson** $-$ $\Upsilon(1s)(1^{--})(9.46), \chi(1P)(2^{++})(9.912), \psi(1D)(3^{--})(10.161)$

- For (M^2, n) the trajectories used

$\mathbf{c\bar{c}}$ **meson** $-$ $\Psi(1s)(3.0969), Psi(2s)(3.686), Psi(3s)(3.7699), Psi(4s)(4.04)$
$\mathbf{b\bar{b}}$ **meson** $-$ $\Upsilon(1s)(9.4604), \Upsilon(2s)(10.023), \Upsilon(3s)(10.3533), \Upsilon(4s)(10.580), \Upsilon(5s)(10.865)$

The botomonium trajectories

- To emphasize the deviation from the linearity we start with the botomonium trajectories

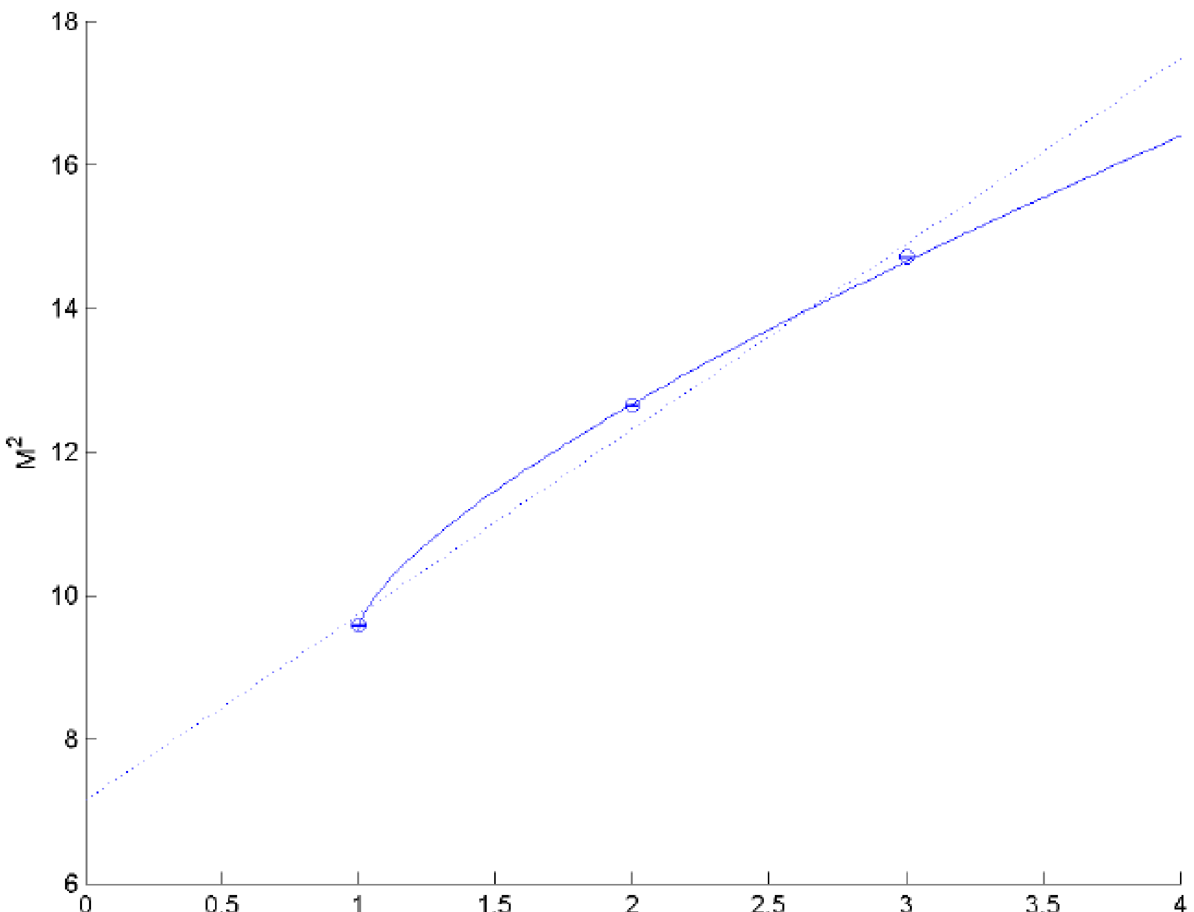


The charmonium trajectories

- For the charmonium trajectory we get

$$a = 1, \alpha' = 0.999, 2m = 3086$$

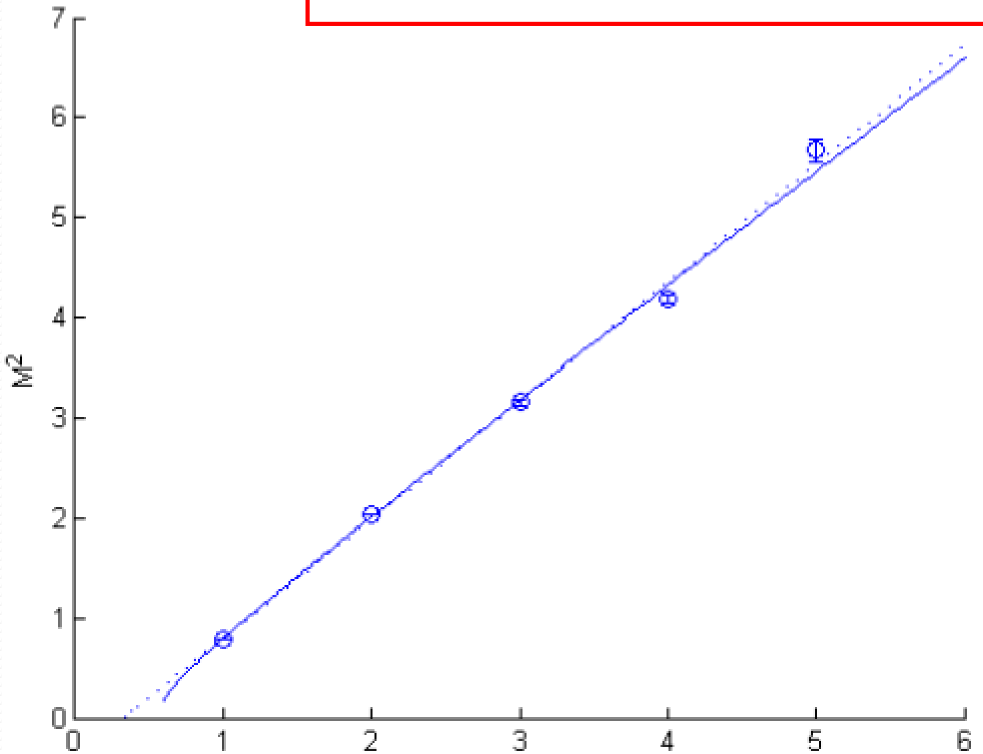
- Now the improvement over the linear is $\chi_c^2/\chi_l^2 = 0.041$



The K* trajectories

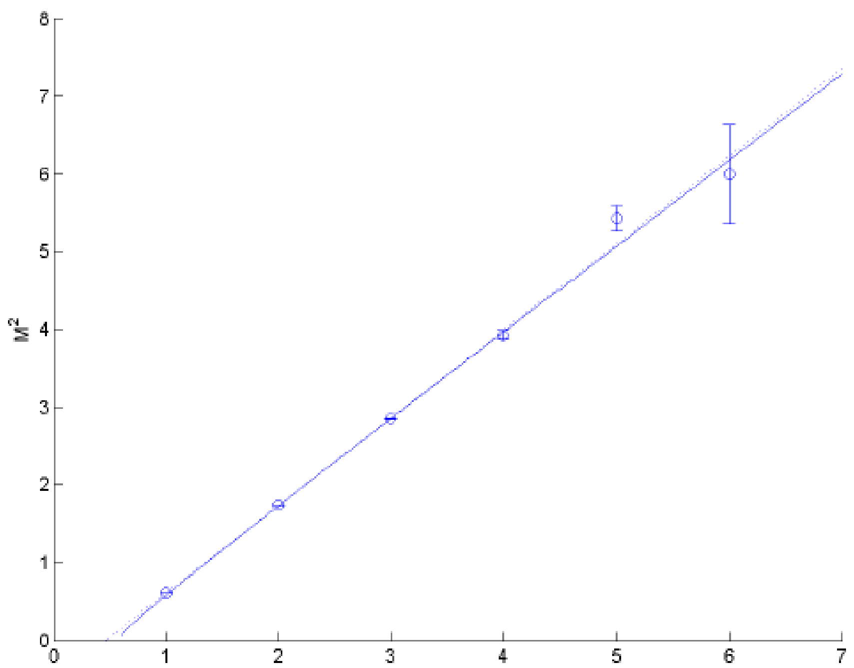
- The K* mesons $K^*(892), K_2^*(1430), K_3^*(1780), K_4^*(2045), K_5^*(2380)$ are constructed from $d\bar{s},\ u\bar{s},\ \bar{u}s,\ \text{or}\ \bar{d}s$ and have S=1
- The best fitted tension and intercept are $a = 0.6, \alpha' = 0.91$
- The best fitted masses are

$$m_{ud} = 115 \quad m_s = 410$$



The ρ trajectories

- For the ρ mesons trajectories the difference between the original linear and modified trajectories is the smallest
- For the linear $a = 0.45.\ \alpha' = 0.888$
- For the massive $a = 0.6, \alpha' = 0.916, 2m = 229$
- The ratio is $\chi_c^2/\chi_l^2 = 0.810.$



Optimization in the m α' plane for ρ and ω

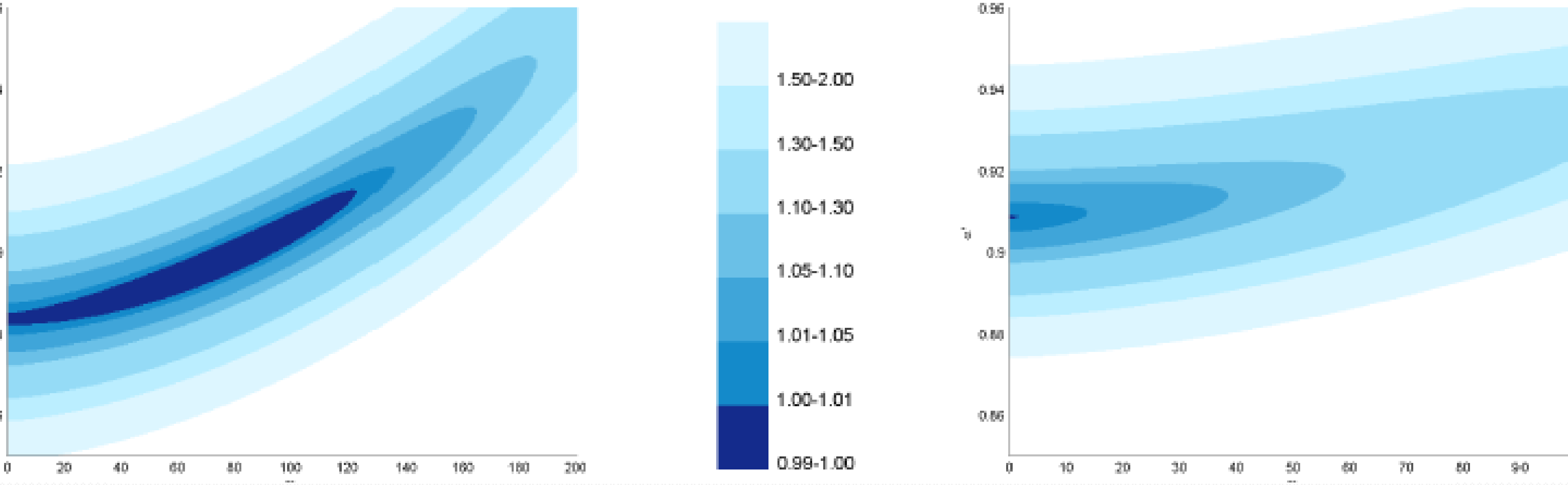


Figure 2. χ^2 as a function of α' and m for the (J, M^2) trajectory of the ρ (left) and ω (right) mesons. The intercept a is optimized to get a best fit for each point in the (α', m) plane. χ^2 in these plots is normalized so that the value of the optimal linear fit $(m = 0)$ is $\chi^2 = 1$.

Optimization in the m α’ plane : s quark

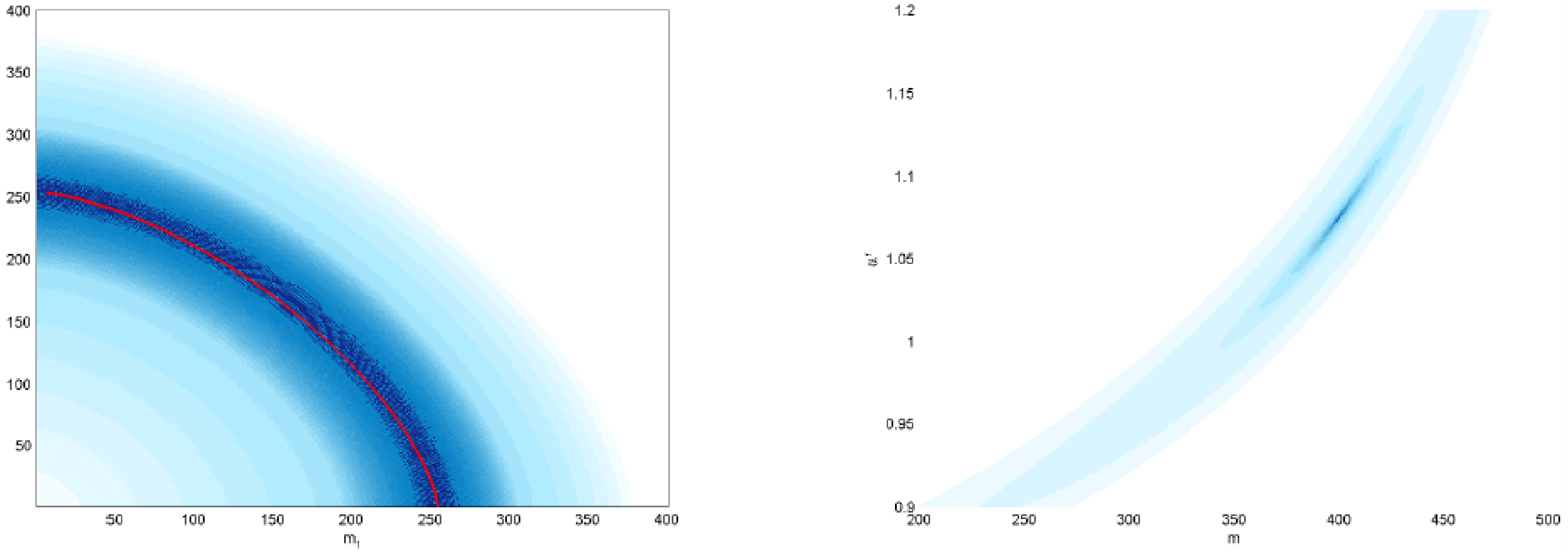


Figure 3. Left: χ^2 as a function of two masses for the K^* trajectory. a and α' are optimized for each point. The red line is the curve $m_1^{3/2} + m_2^{3/2} = 2 \times (160)^{3/2}$ along which the minimum (approximately) resides. The minimum is $\chi_m^2/\chi_l^2 = 0.925$ and the entire colored area has $\chi_m^2/\chi_l^2 < 1$. On the right is χ^2 as a function of α' and m for the (J, M^2) trajectory of the ϕ. The intercept a is optimized. The minimum is at $\alpha' = 1.07, m = 400$ with $\chi_m^2/\chi_l^2 < 10^{-4}$ at the darkest spot. The lightest colored zone still has $\chi_m^2/\chi_l^2 < 1$, and the coloring is based on a logarithmic scale.

Toward a universal model

- The fit results for several trajectories simultaneously. The (J, M^2) trajectories of ρ, ω, K^*, ϕ D, and Ψ mesons
- We take the string endpoint masses

$$m_{u/d} = 60, m_s = 220, m_c = 1500$$

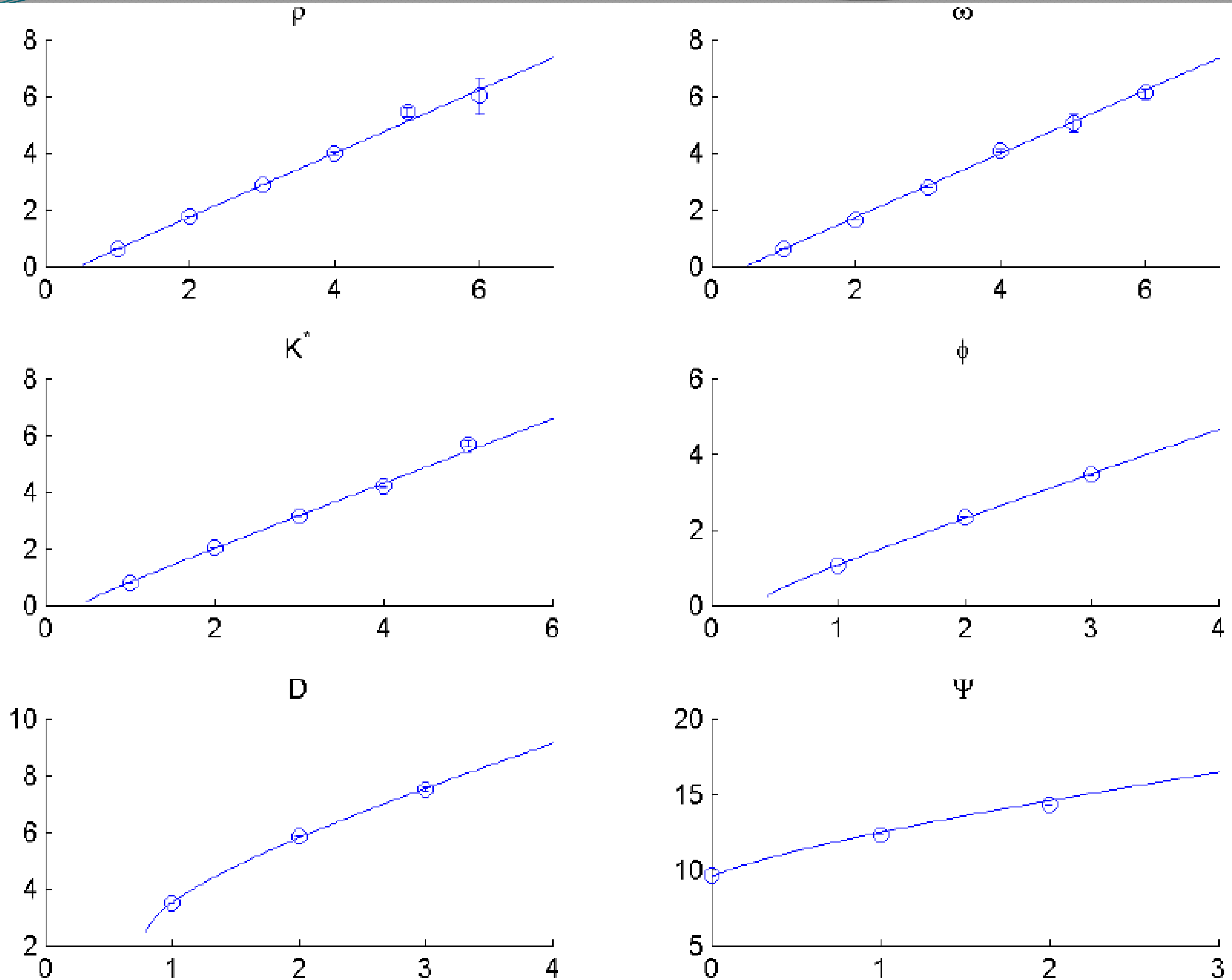
- Only the intercept was allowed to change. We got

$$\alpha' = 0.899$$

$$a_\rho = 0.51, a_\omega = 0.52, a_{K^*} = 0.49$$

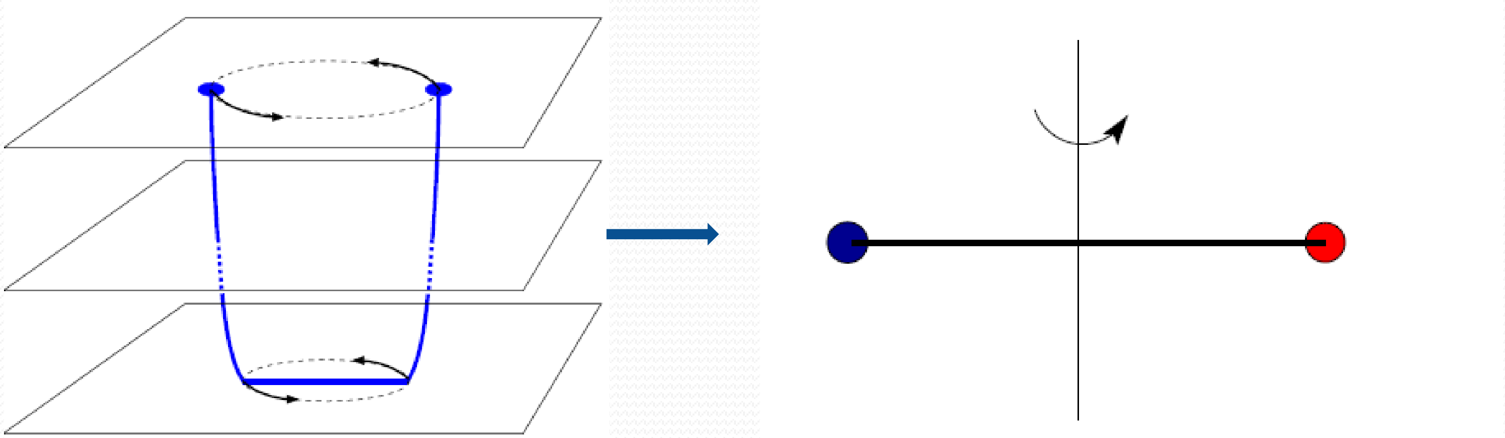
$$a_\phi = 0.44, a_D = 0.80, a_\Psi = 0.94$$

Toward a universal model



Holography versus massive endpoints toy model

- In the toy model of string with massive endpoints for vanishing orbital angular momentum J=0 the length of the string vanishes and hence only the quarks at the endpoints constitute the meson mass
- In holography we get non trivial contribution of the string even with no angular momentum
- Thus the comparison with data favors holography over the massive endpoints toy model .



Stringy holographic Baryons

Stringy Baryons in hologrphy

- How do we identify a baryon in holography ?

- Since a quark corresponds to a string, the baryon has to be a structure with N_c strings connected to it.

- Witten proposed a baryonic vertex in AdS_5xS^5 in the form of a wrapped D5 brane over the S^5.

- On the world volume of the wrapped D5 brane there is a CS term of the form

$$S_{cs} = \int_{S^5 \times R} a \wedge \frac{G_5}{2\pi}.$$

Baryonic vertex

- The flux of the five form is

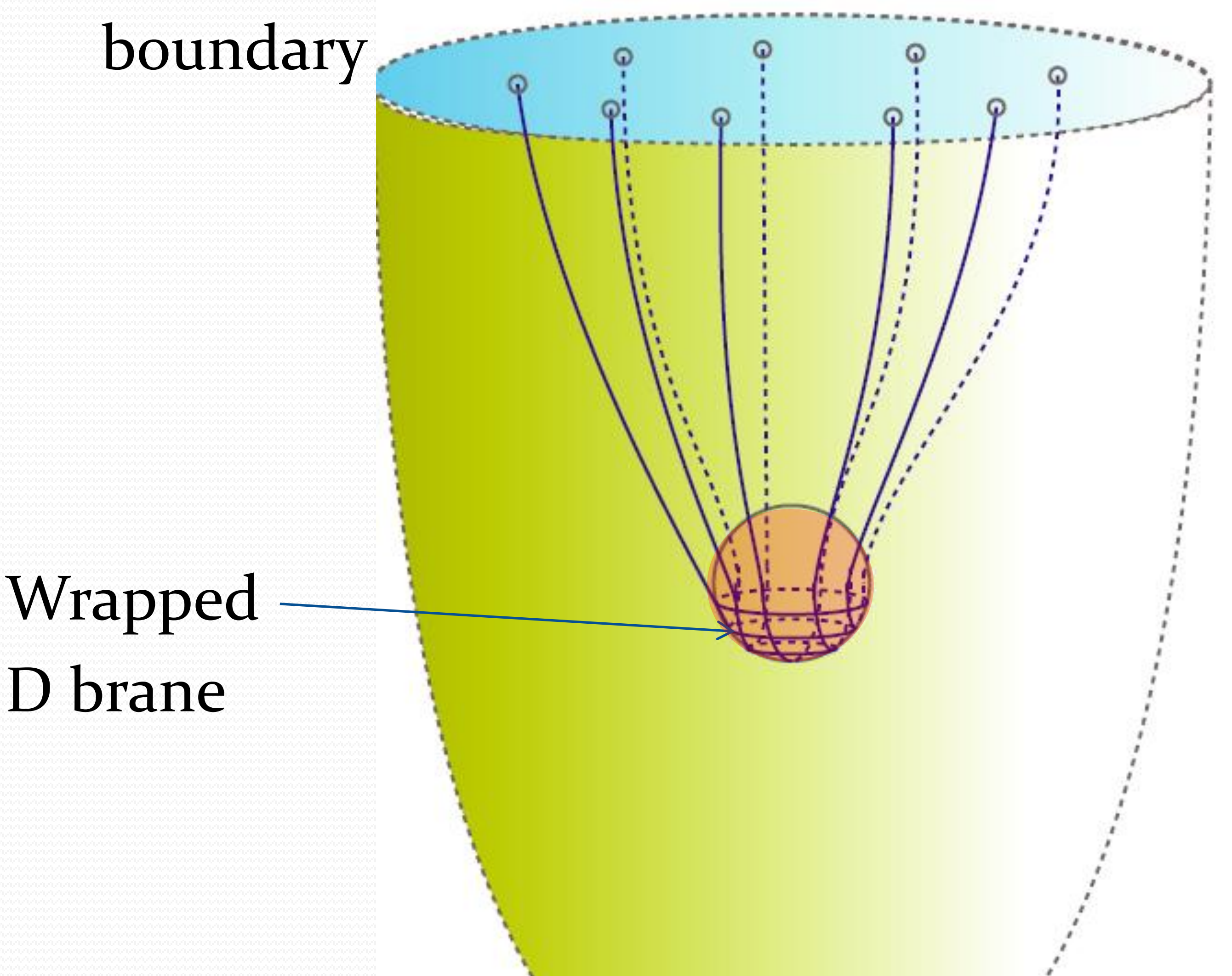
$$\int_{S^5} \frac{G_5}{2\pi} = N_c$$

- This implies that there is a charge N_c for the abelian gauge field. Since in a compact space one cannot have non-balanced charges there must be

N_c strings attached to it.

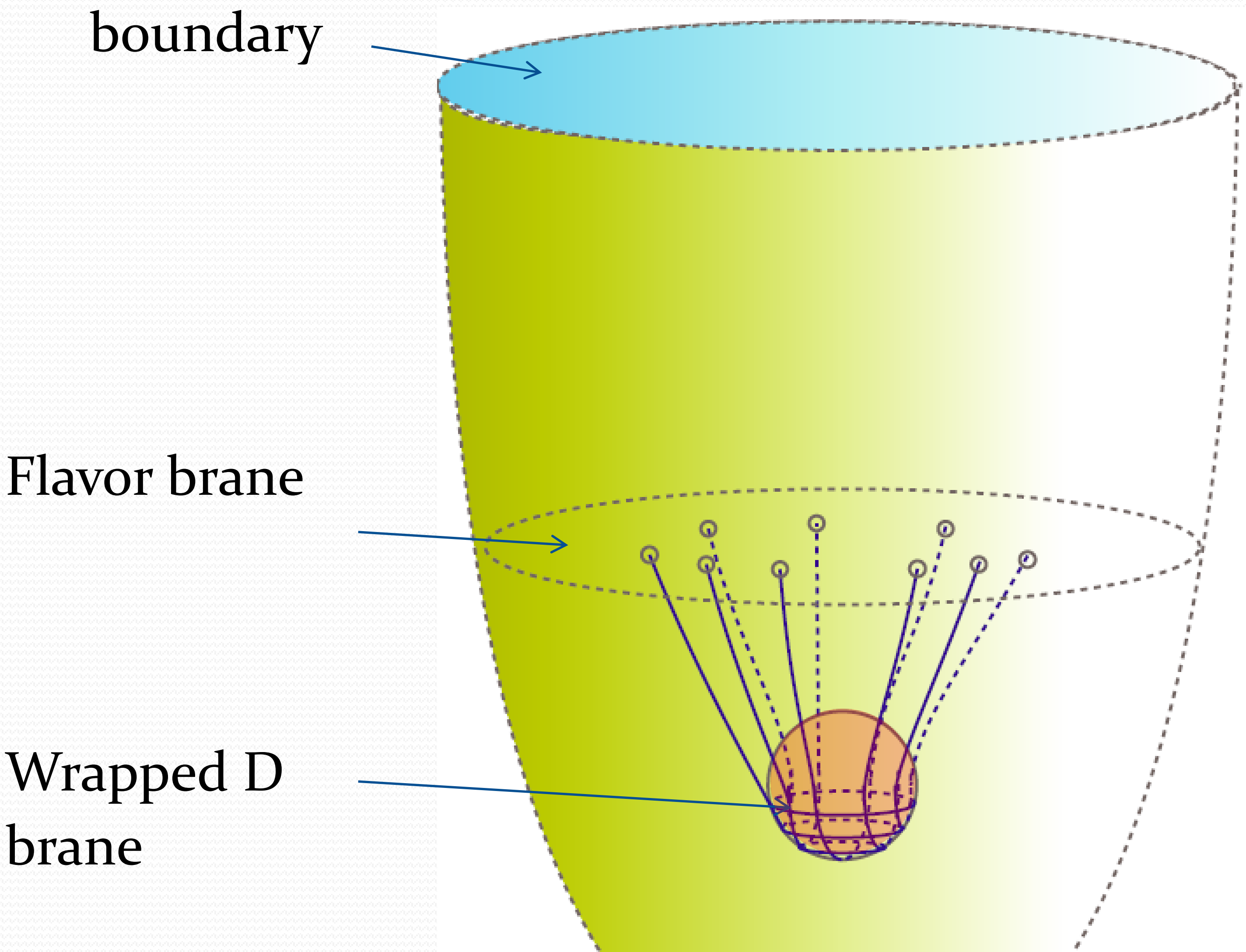
External baryon

- External baryon – Nc strings connecting the baryonic vertex and the boundary



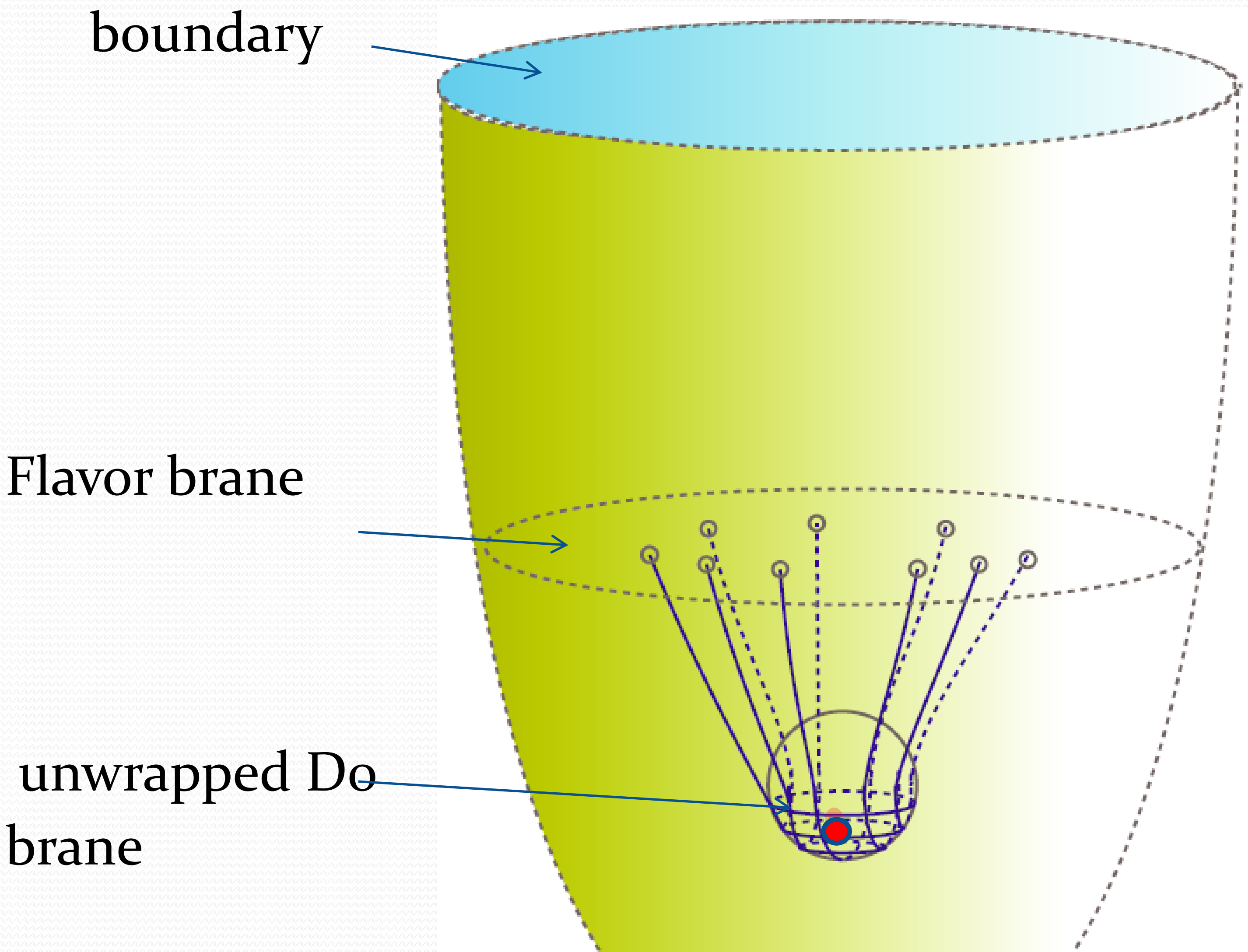
Dynamical baryon

- Dynamical baryon – Nc strings connecting the baryonic vertex and flavor branes



Dynamical baryon in the n-c Ads6 model

- In this model the baryonic vertex is a Do brane of the non-critical compact D4 brane background.



The location of the baryonic vertex

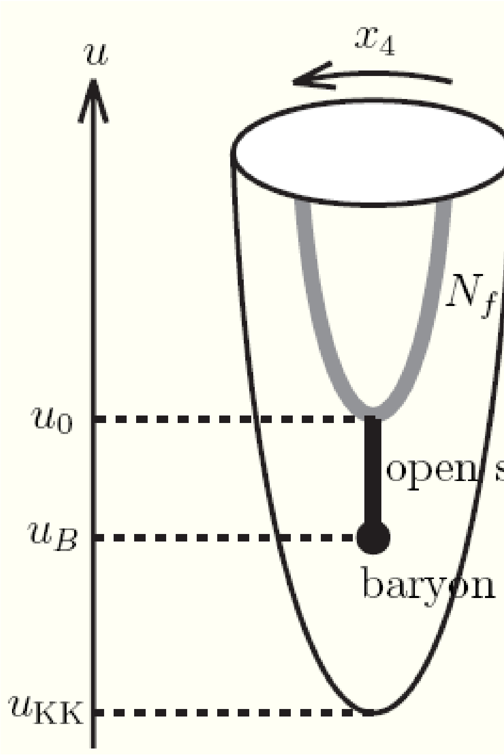
- We need to determine the location of the baryonic vertex in the radial direction.
- In the leading order approximation it should depend on the wrapped brane tension and the tensions of the Nc strings.
- We can do such a calculation in a background that corresponds to confining (like gSS) and to de-confining gauge theories. Obviously we expect different results for the two cases.

- The location of the baryonic vertex in the radial direction is determined by ``static equillibrium".

$$S = -T_4 \int dt d\Omega_4 e^{-\phi} \sqrt{-\det g_{\mathrm{D4}}} - N_c T_f \int dt du \sqrt{-\det g_{\mathrm{string}}}$$

- The energy is a decreasing function of x=uB/uKK and hence it will be located at the tip of the flavor brane

$$\mathcal{E}_{\mathrm{conf}}(x; x_0) = \frac{1}{3}x + \int_x^{x_0} \frac{dy}{\sqrt{1-y^{-3}}}$$

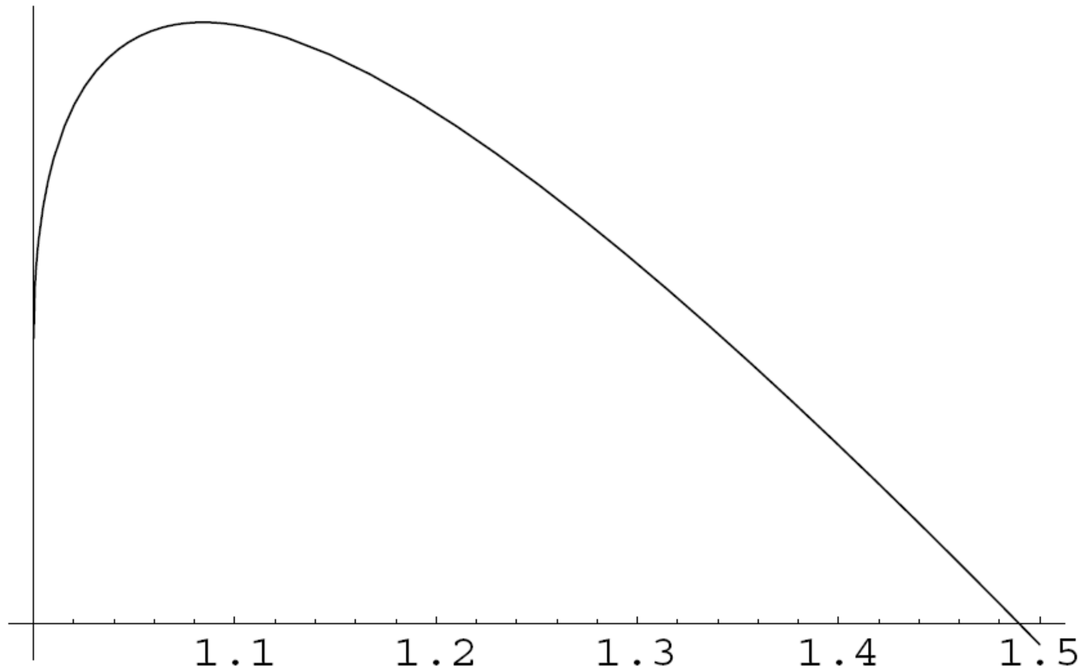


- It is interesting to check what happens in the deconfining phase.
- For this case the result for the energy is

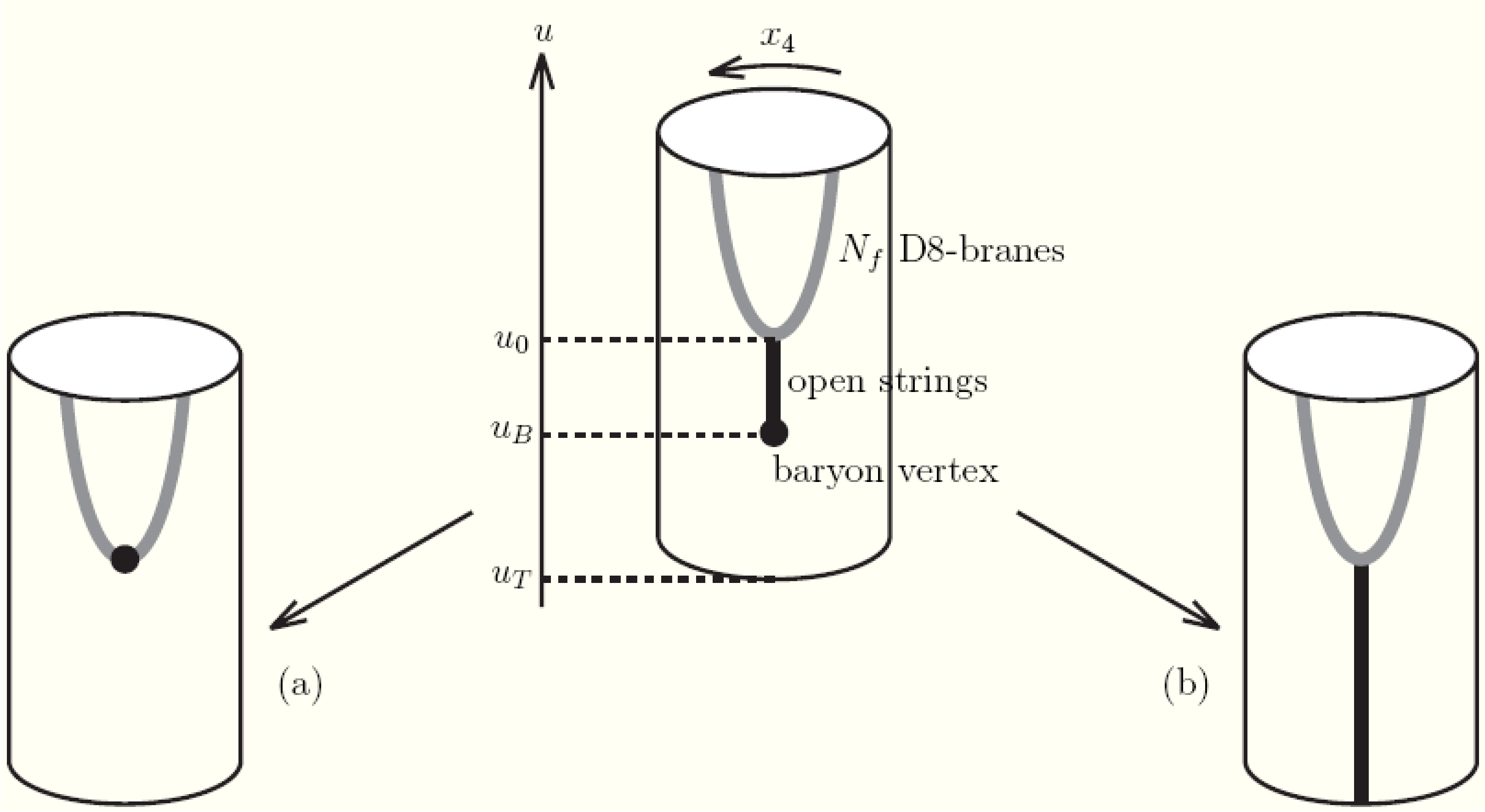
$$\mathcal{E}_{\rm deconf}(x;x_0) = \frac{1}{3}x\sqrt{1-\frac{1}{x^3}} + (x_0 - x)$$

- For $x>x_{cr}$ low temperature stable baryon
- For $x<x_{cr}$ high temperature dissolved baryon

The baryonic vertex falls into the black hole

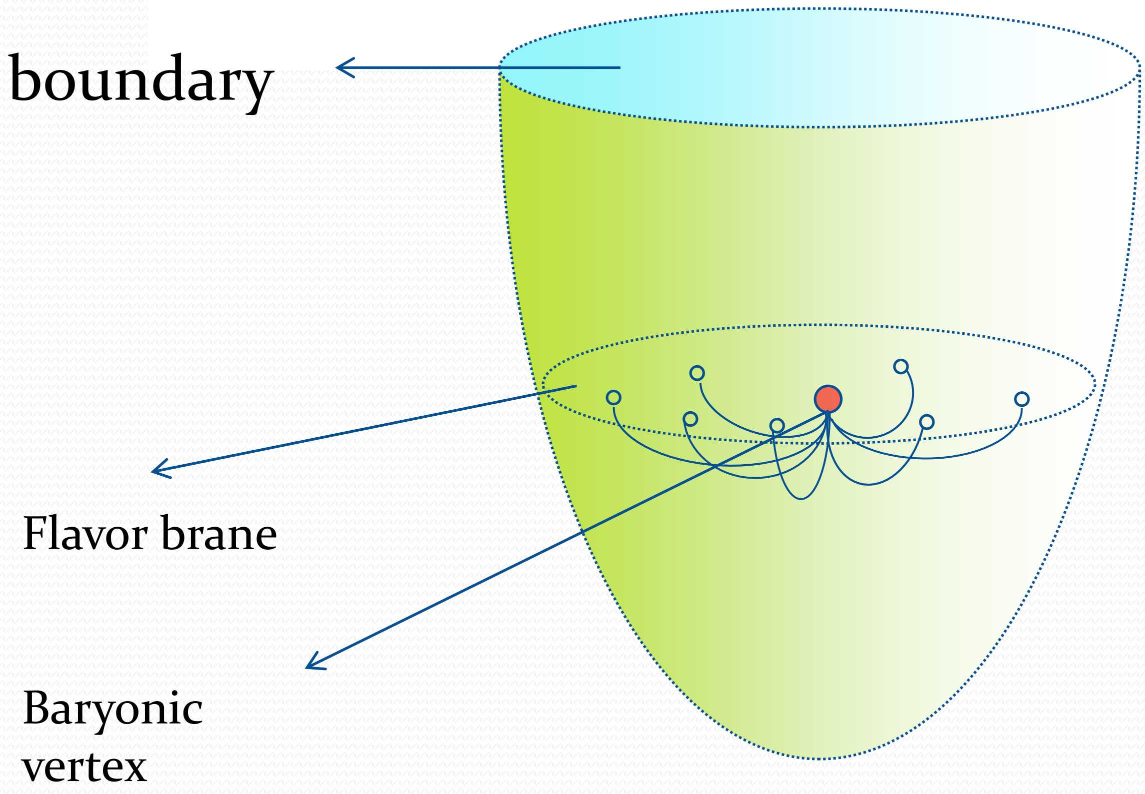


The location of the baryonic vertex at finite temperature



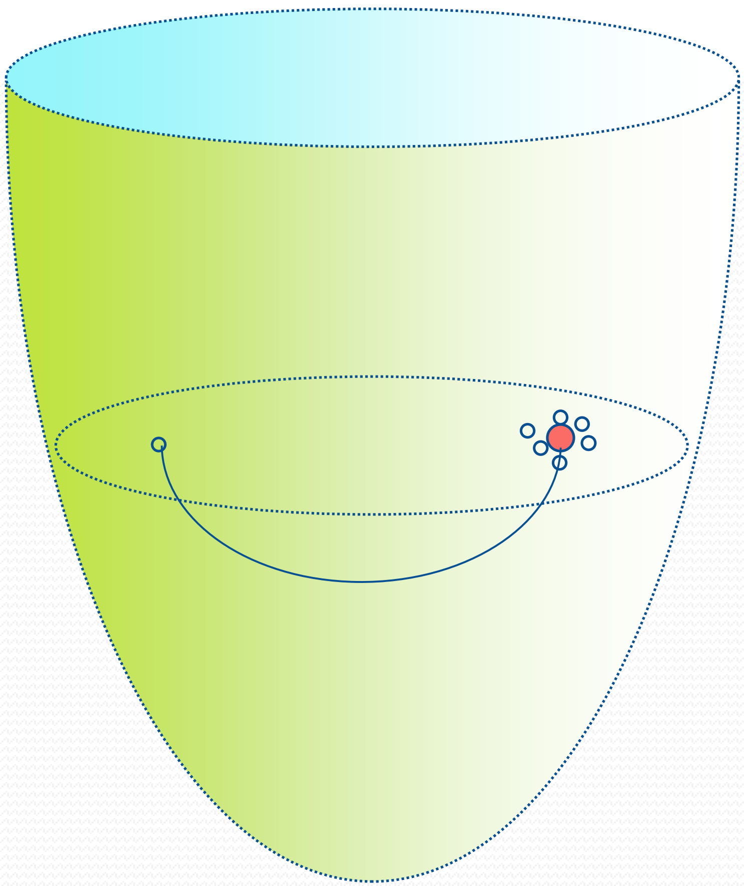
A possible baryon layout

- A possible dynamical baryon is with Nc strings connected to the flavor brane and to the BV which is also on the flavor brane.



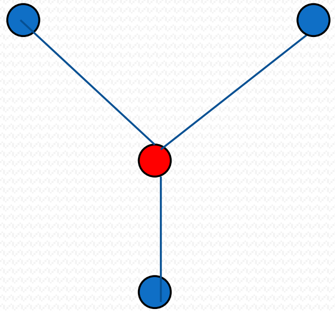
Nc-1 quarks around the Baryonic vertex

- Another possible layout is that of one quark connected with a string to the BV to which the rest of the Nc-1 quarks are attached.

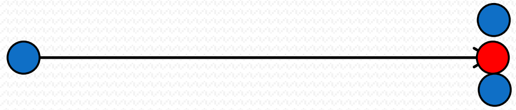


From large Nc to three colors

- Naturally the analog at Nc=3 of the symmetric configuration with a central baryonic vertex is the old Y shape baryon



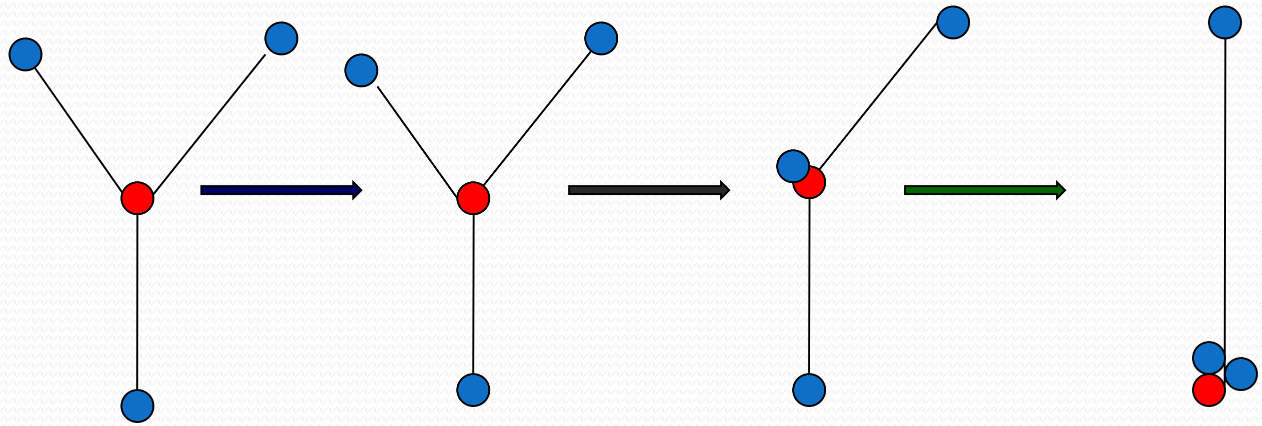
- The analog of the asymmetric setup with one quarks on one end and Nc-1 on the other is a straight string with quark and a di-quark on its ends.



Stability of an excited baryon

- Sharov and 't Hooft showed that the classical Y shape three string configuration is unstable. An arm that is slightly shortened will eventually shrink to zero size.
- We have examined Y shape strings with massive endpoints and with a massive baryonic vertex in the middle. G. Harpaz J.S
- The analysis included numerical simulations of the motions of mesons and Y shape baryons under the influence of symmetric and asymmetric disturbance.
- We indeed detected the instability
- We also performed a perturbative analysis where the instability does not show up.

Baryonic instability



The conclusion from both the simulations and the qualitative analysis is that indeed the Y shape string configuration is unstable to asymmetric deformations.

Thus an excited baryon is an unbalanced single string with a quark on one side and a di-quark and the baryonic vertex on the other side.

Stringy holographic Baryons versus experimental data

Baryons are straigh strings!

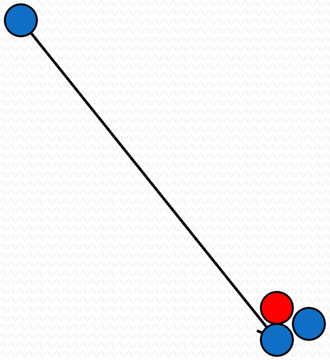
- It is straightforward to realize that the Y shape structure has

$$\alpha_Y' = 2/3\ \alpha_l'$$

A quick glance on the baryon trajectories shows that they admit roughly (5%) the same α' as that of the mesons. Thus we conclude that baryons are straight strings and not Y shape strings

Excited baryon as a single string

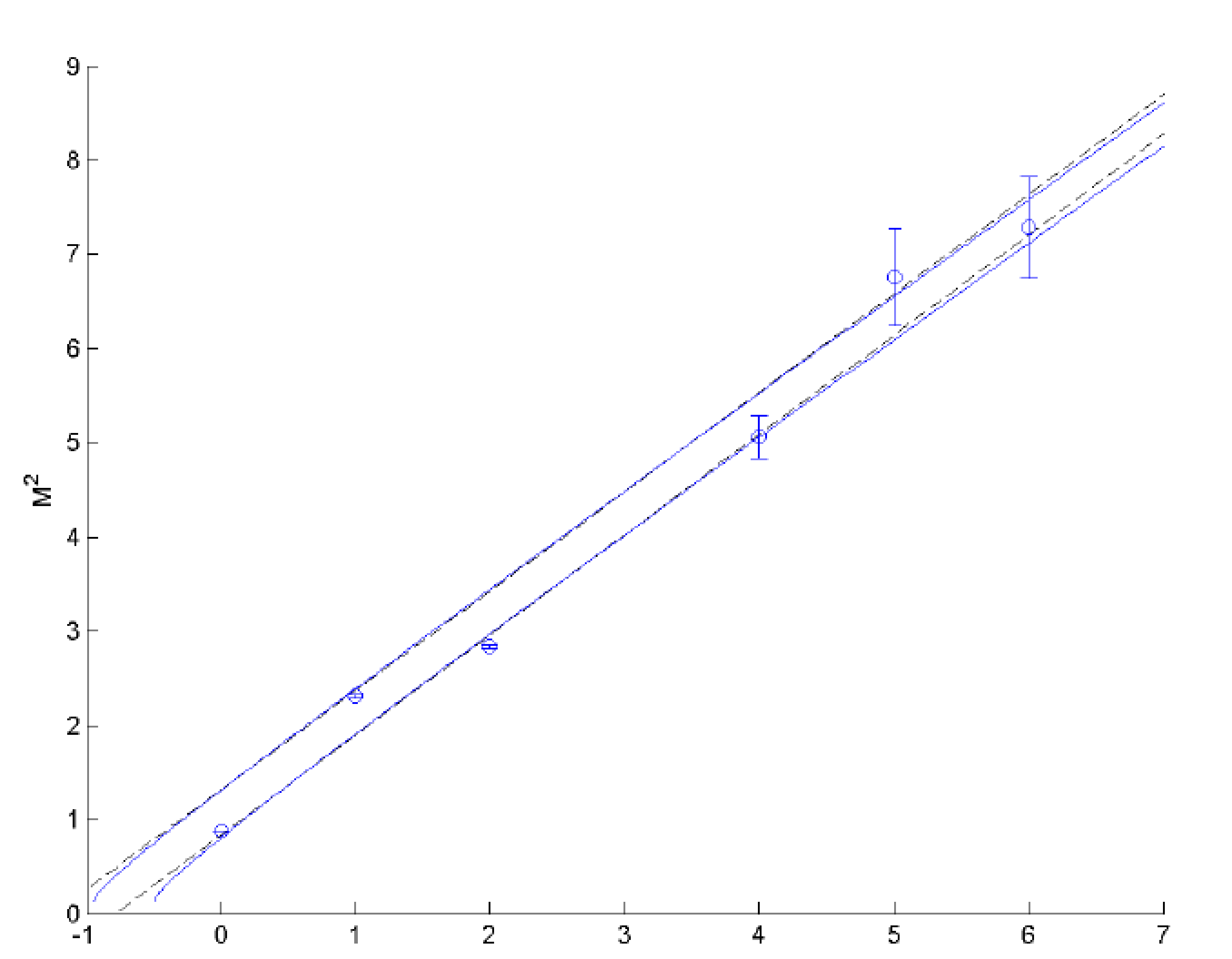
- Thus we are led to a picture where the baryon is a single string with a quark on one end and a di-quark (+ a baryonic vertex) at the other end.



- This is in accordance with stability analysis which shows that a small instability in one arm will cause it to shrink so that the final state is a single string

Fit to Regge trajectories of Nucleons

- Fit of the Regge trajectories of the Nucleons



Fitting the Nucleon trajectories

- Notice that there are separate trajectories for even L and for odd L.

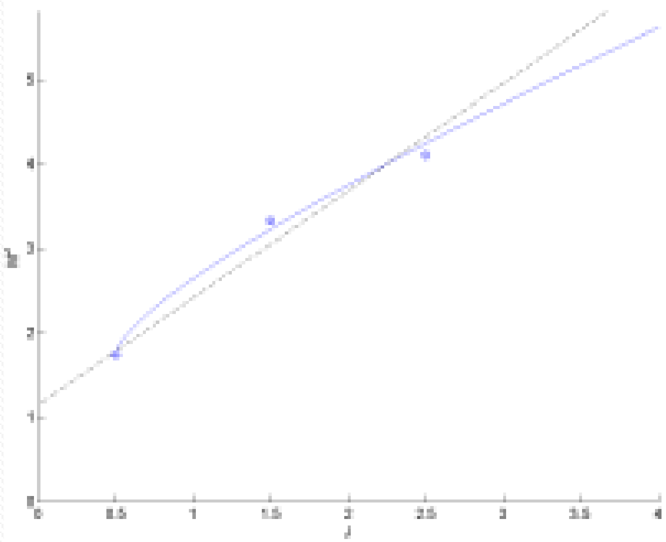
$$a_o = -0.95 \qquad a_e = -0.7, \alpha' = 0.966,$$

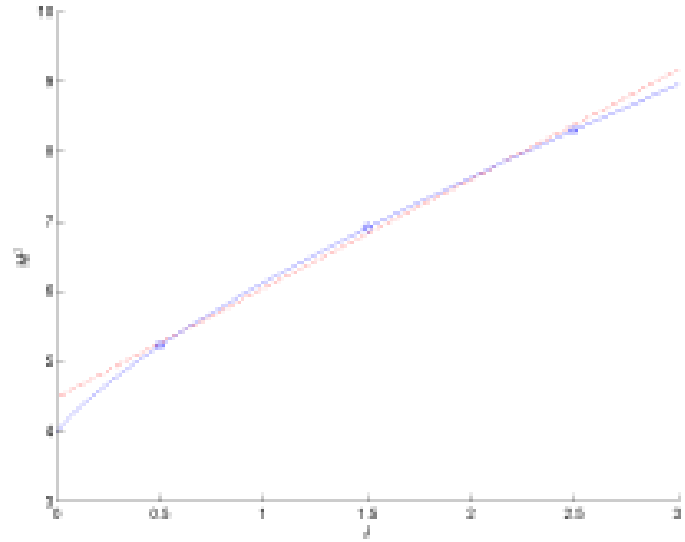
- Assuming that m1=115 Mev the best fit for m2= 57Mev with $\chi_c^2/\chi_l^2 = 0.564$

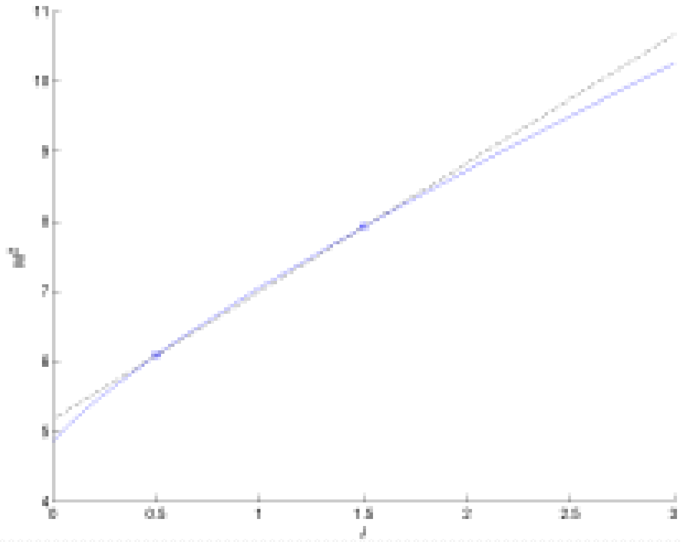
- The fit with m2=240 Mev is much worth

$$\chi_c^2/\chi_l^2 = 1.850$$

The trajectories of Ξ , Λc , Ξc

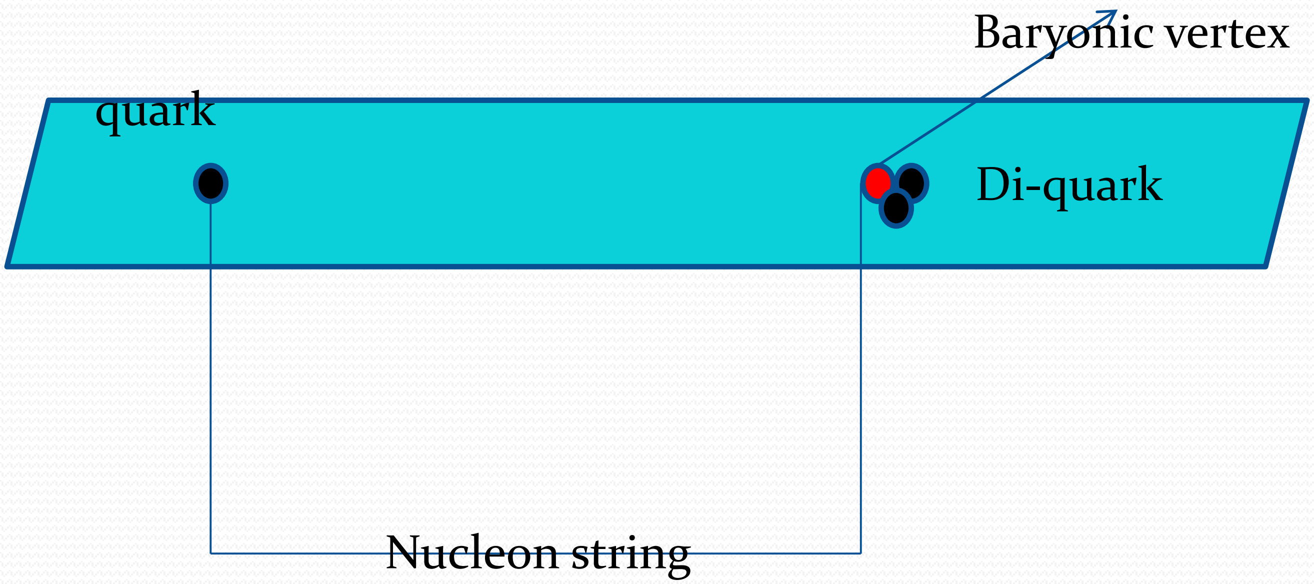






The structure of the stringy nucleon

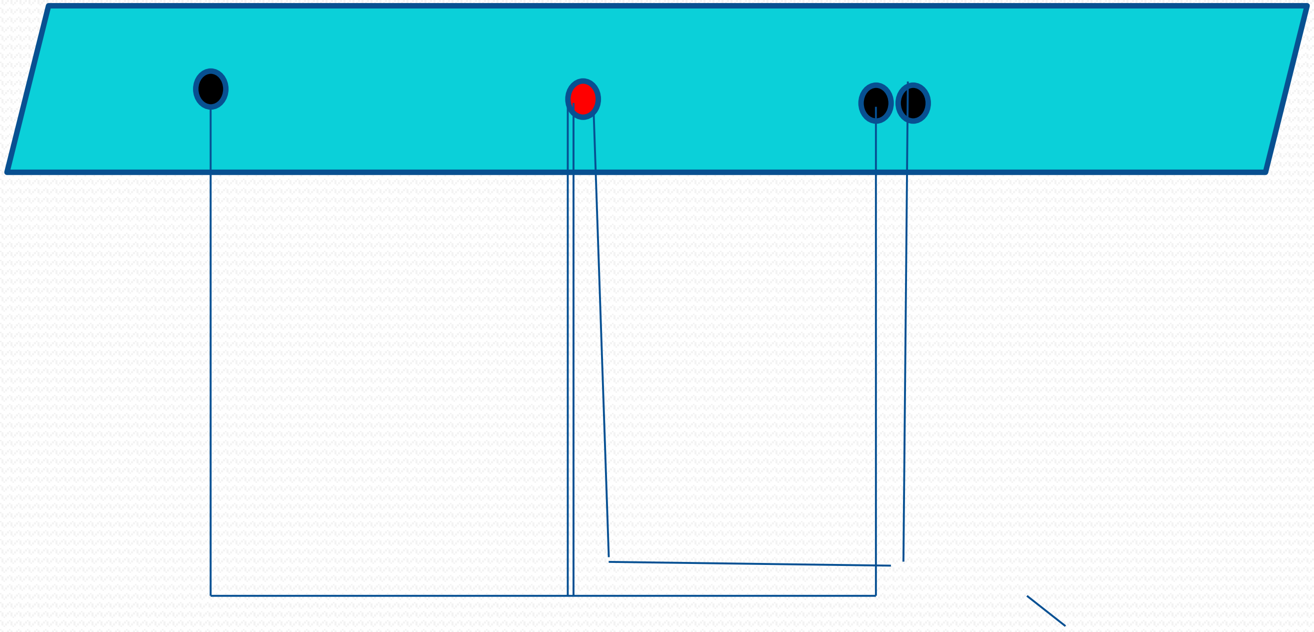
- We conclude that the setup is



- So in the right hand side we have m_q and not $2m_q$
- There does not seem to be a contribution to the mass from the Baryonic vertex

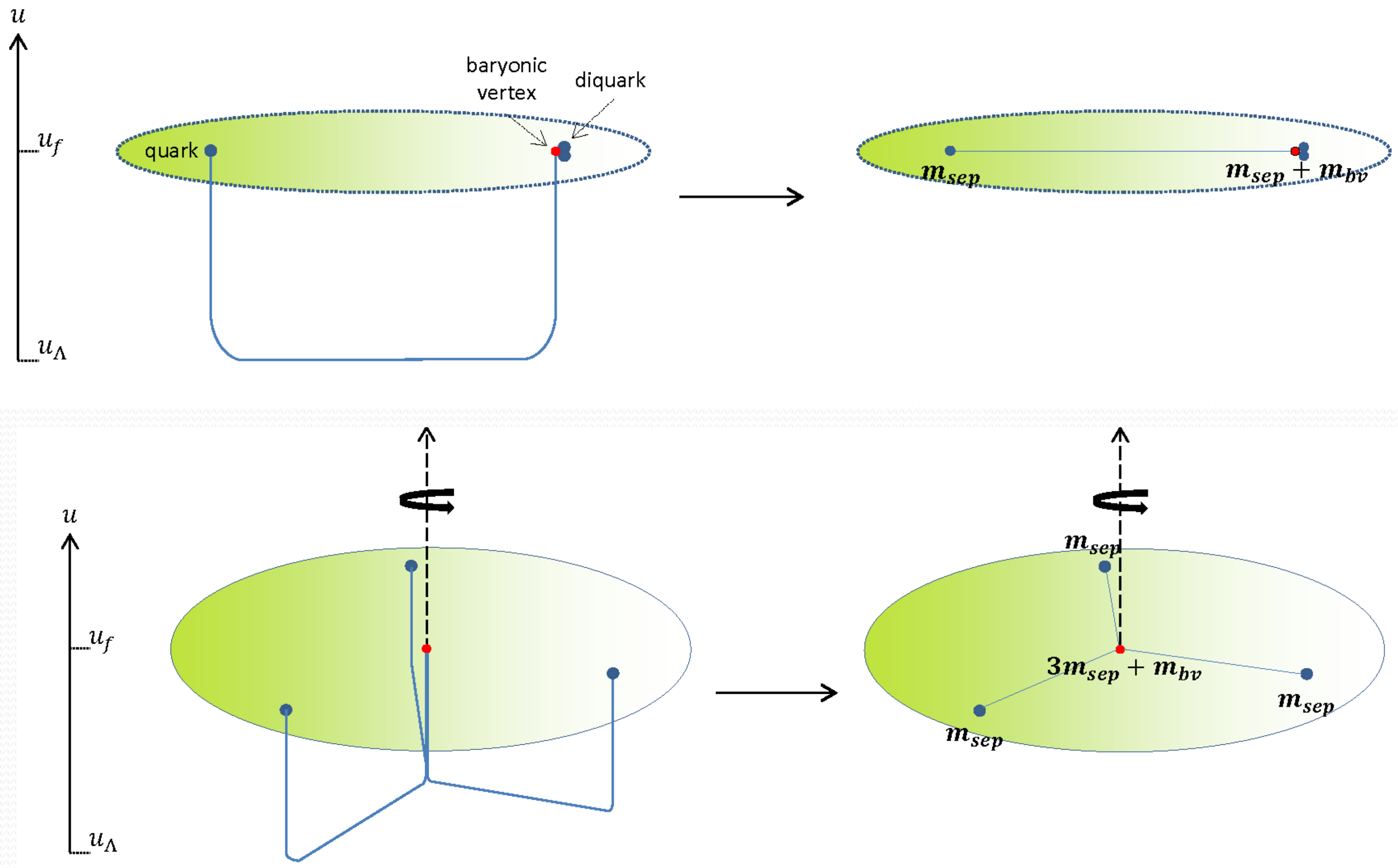
Central baryonic vertex is excluded

- The fit analysis definitely prefers the previous setup over a one with a central baryonic vertex

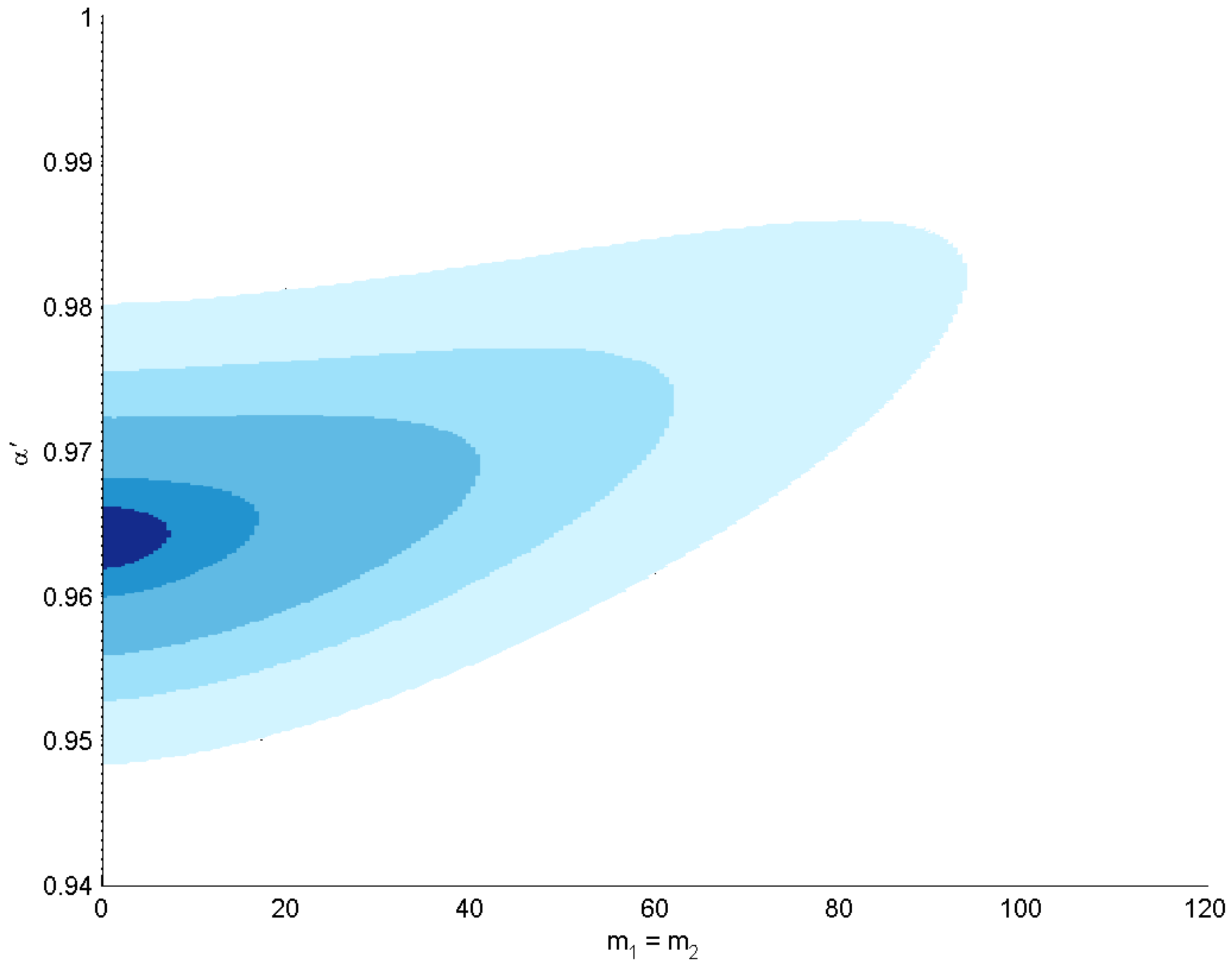


- A fit to such a scenario yields zero mass to the bayonic vertex and fails to see a $2m_{sep}$ on the rhs

From holographic to flat space-time configurations



α’ and 2m for the nucleon trajectory



Summary of the baryonic fits

(J, M^2) (n, M^2) $\Delta\ (3/2^+)$ $\Sigma(1385)\ 3/2^+$

Traj.	N	m	α'	a
N	7	$2m = 0 - 170$	$0.944 - 0.959$	$a_e = (-0.32) - (-0.23)$ $a_o = (-0.75) - (-0.65)$
$N^{[a]}$	4	$2m = 0 - 640$	$0.949 - 1.018$	$a_e = (-0.34) - 0.50$ $a_o = (-0.98) - (-0.13)$
$N^{[b]}$	15	$2m = 0 - 425$	$0.815 - 0.878$	$a_{1/2^+} = (-0.22) - 0.07$ $a_{3/2^-} = (-0.36) - (-0.06)$
Δ	7	$2m = 0 - 450$	$0.898 - 0.969$	$a_e = 0.14 - 0.54$ $a_o = (-0.84) - (-0.42)$
$\Delta^{[c]}$	3	$2m = 0 - 175$	$0.920 - 0.936$	$a = 0.11 - 0.21$
Λ	5	$2m = 0 - 125$	$0.946 - 0.955$	$a = (-0.68) - (-0.61)$
Σ	3	$2m = 1190$	1.502	$a = (-0.15)$
$\Sigma^{[d]}$	3	$2m = 1255$	1.459	$a = 1.37$
Ξ	3	$2m = 1320$	1.455	$a = 0.50$
Λ_c	3	$2m = 2010$	1.130	$a = 0.09$

- The range associates with χ^2 of 10%

Summary of the baryonic fits

- Fits for the optimal fixed $\alpha' = 0.950 \text{ GeV}^{-2}$

Traj.	N	m	a
N	7	$2m = 0 - 180$	$a_e = (-0.33) - (-0.22)$ $a_o = (-0.77) - (-0.65)$
Δ	7	$2m = 300 - 530$	$a_e = 0.31 - 0.66$ $a_o = (-0.71) - (-0.26)$
Λ	5	$2m = 0 - 10$	$a = (-0.68) - (-0.61)$
Σ	3	$2m = 530 - 690$	$a = (-0.29) - (-0.04)$
Σ^*	3	$2m = 435 - 570$	$a = 0.15 - 0.38$
Ξ	3	$2m = 750 - 930$	$a = (-0.22) - 0.10$
Λ_c	3	$2m = 1760$	$a = (-0.36)$
Ξ_c	2	$2m = 2060$	$a = (-1.13)$

- It is harder to construct a unified stringy model for the baryons than for the mesons.
- The model of a quark and a di-quark is best supported
- The mesonic and baryonic results are similar the Λ is an exception it prefers massless s quark

Glueballs as closed strings

Glueballs as closed strings

- Mesons are open strings with a massive quark and an anti-quark on its ends.
- Baryons are open strings with a massive quark on one end and a baryonic vertex and a di-quark on the other end.
- What are glue balls?
- Since they do not incorporate quarks it is natural to assume that they are rotating closed strings
- Angular momentum associates with rotation of folded closed strings

Closed strings versus open strings

- The spectrum of states of a closed string admits

$$M^2 = \frac{2}{\alpha'}\left(N + \tilde{N} + A + \tilde{A}\right)$$

- The spectrum of an open string

$$M^2_{open} = \frac{1}{\alpha'}(N + A)$$

- The slope of the closed string is ½ of the open one
- The closed string ground states has

$$M^2 = \frac{2}{\alpha'}(A + \tilde{A}) = \frac{2 - D}{6\alpha'}$$

- The intercept is 2

Closed strings versus open ones

- In the terminology of QCD the tension of the string associate with the Quadratic Casimir and hence the ratio

$$\frac{\alpha'_{gg}}{\alpha'_{q\bar{q}}} = \frac{C_2(\text{Fundamental})}{C_2(\text{Adjoint})} = \frac{N^2-1}{2N^2} = \frac{4}{9},$$

- This is in accordance with the ratio

Slope $_{\text{closed}}$ = ½ Slope$_{\text{open}}$

Phenomenology

- A rotating and exciting folded closed string admits in flat space-time a linear Regge trajectory

$$J + n = \alpha'_{gb} M^2 + a \qquad \alpha'_{gb} = \frac{1}{2}\alpha'$$

- The basic candidates of glueballs are flavorless hadrons f_0 of 0++ and f_2 of 2++. There are 9 (+3) fo and 12 (+5) f2.
- The question is whether one can fit all of them into meson and separately some glueball trajectories.
- We found various different possibilities of fits.

Glueball 0++ fits of experimental data

- Assignment with $f_0(1380)$ as the glueball groundstate

$$\text{Light}: \quad 1500, *1800, 2200$$

$$s\bar{s}: \quad 1710, 2100$$

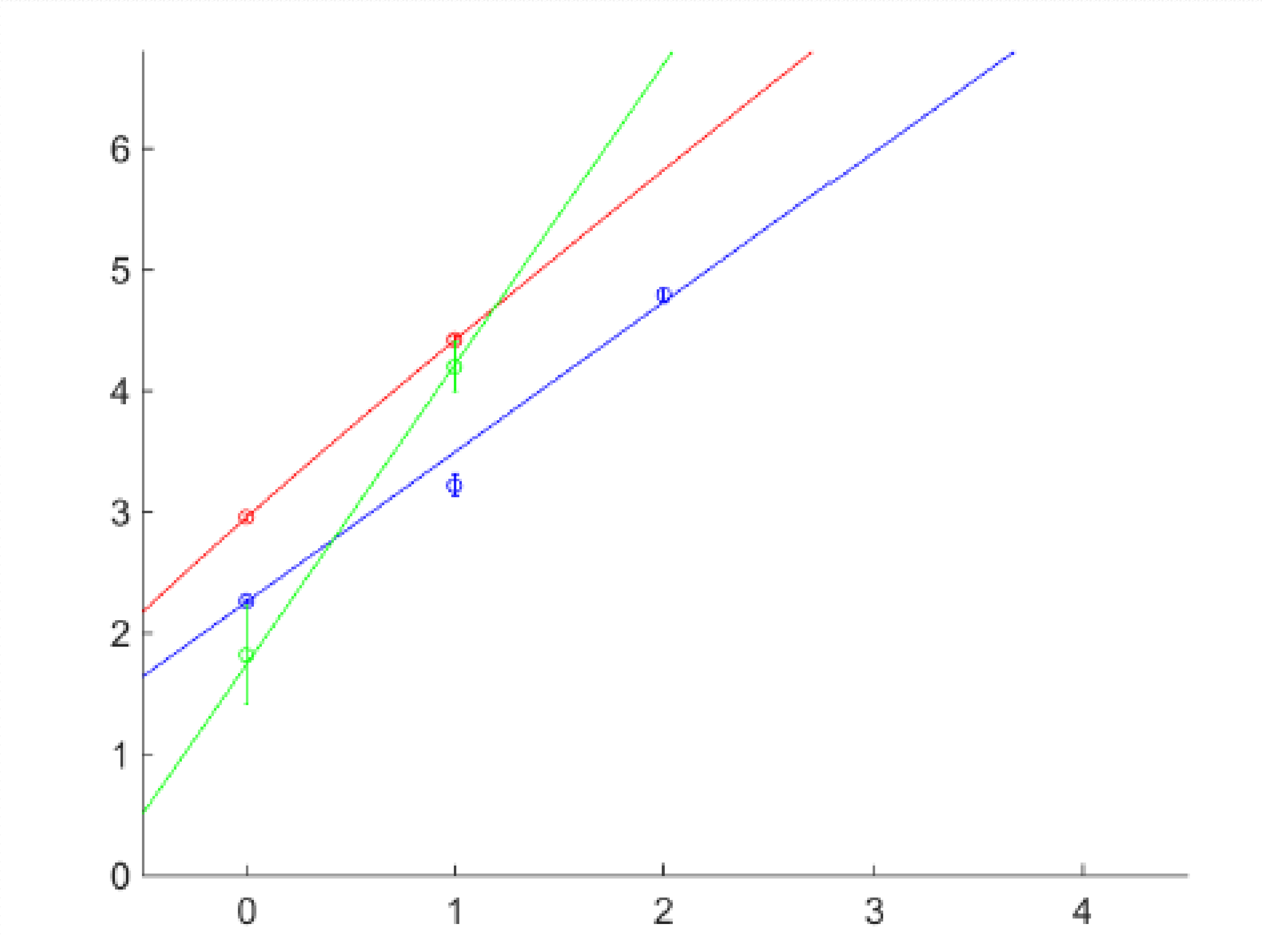
$$\text{Glue}: \quad 1370, *2060$$

Light		$s\bar{s}$		Glueball	
Exp.	Thry.	Exp.	Thry.	Exp.	Thry.
1505±6	1503	1720±6	1720	1350±150	1321
1795±25	1870	2103±8	2103	2050±50	2055
2189±13	2176				

Table 4. The results of the fit to the assignment with $f_0(1370)$ as the glueball ground state. The slope is $\alpha' = 0.808$ GeV^{-2} and the mass of the s quark $m_s = 439$ MeV. This fit has $\chi^2 = 1.76$. The intercepts obtained are (-1.81) for light mesons, (-1.17) for $s\bar{s}$, and (-0.71) for glueballs.

Glueball 0++ fits of experimental data

- The meson and glueball trajectories based on $f_0(1380)$ as a glueball lowest state.

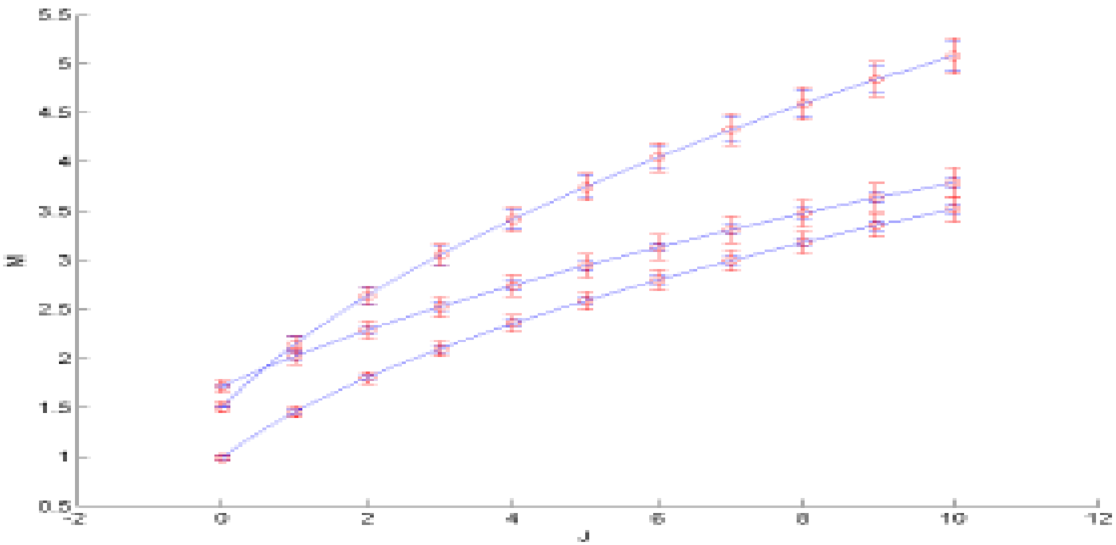


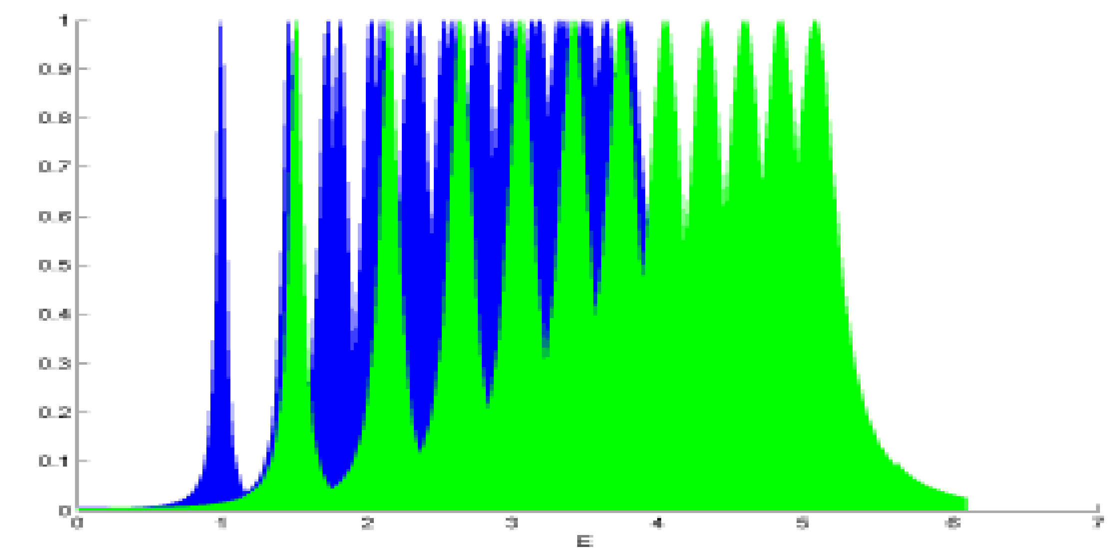
Predictions.

- Unfortunately there are only few confirmed flavorless hadrons with higher J and higher n.
- When we use the glueball slope we can fit at most 2 points. Higher points are already in a mass range where not much states have been confirmed.
- We can predict the locations of the higher glueballs and their width based on

$$\Gamma = M \frac{\Gamma_0}{M_0}$$

Predictions for glueball with f0(1500) ground state





Summary and Outlook

Summary and outlook

- Hadron spectra fit much better strings in holographic backgrounds rather than the spectra of bulk fields like fluctuations of flavor branes.
- Holographic Regge trajectories can be mapped into trajectories of strings with massive endpoints.
- Heavy quark mesons are described in a much better way by the holographic trajectories (or massive) than the original linear trajectories.
- Even for the u and d quark there is a non vanishing string endpoint mass of ~60 Mev.

Summary and outlook

- Baryons are also straight strings with tension which is the same as the one of mesons.
- The baryonic vertex is still mysterious since data prefers it to be massless.
- Glueballs can be described as rotating folded closed strings
- Open questions:
- Quantizing a string with massive endpoints
- Accounting for the spin and for the intercept
- Scattering amplitudes of mesons and baryons like (proton-proton scattering)
- Nuclear interaction and nuclear matter
- Incorporating leptons....