A Monotonicity Theorem for Two-dimensional Boundaries and Defects

Andy O'Bannon

UNIVERSITY OF OXFORD

THE ROYAL SOCIETY

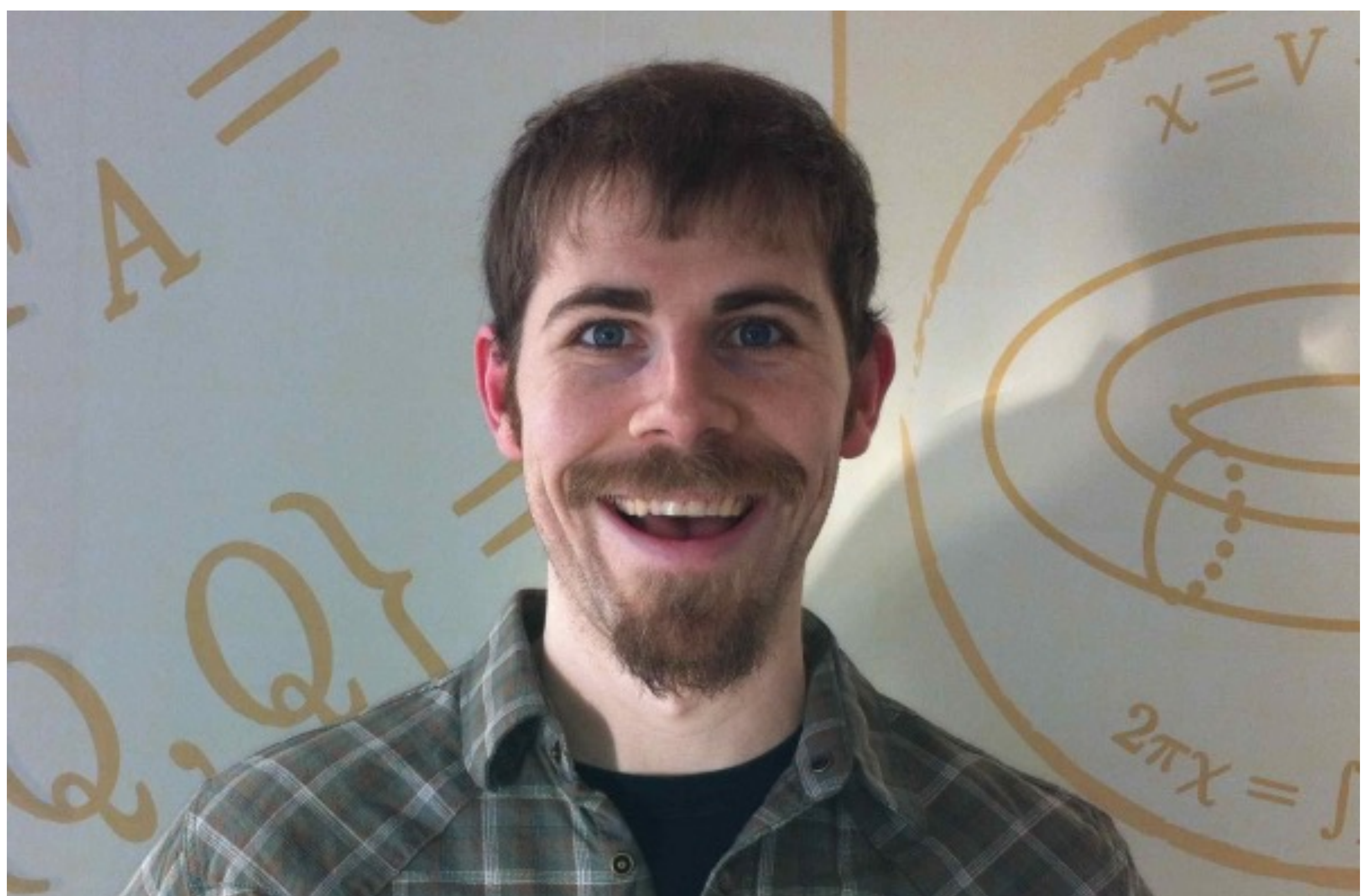
Gauge-Gravity Duality 2015
Galileo Galilei Institute, Florence
April 16, 2015

WORK IN PROGRESS

with

Kristan Jensen

Stony Brook University



DISCLAIMERS

No Gauge-Gravity Duality

Apologies for missed references

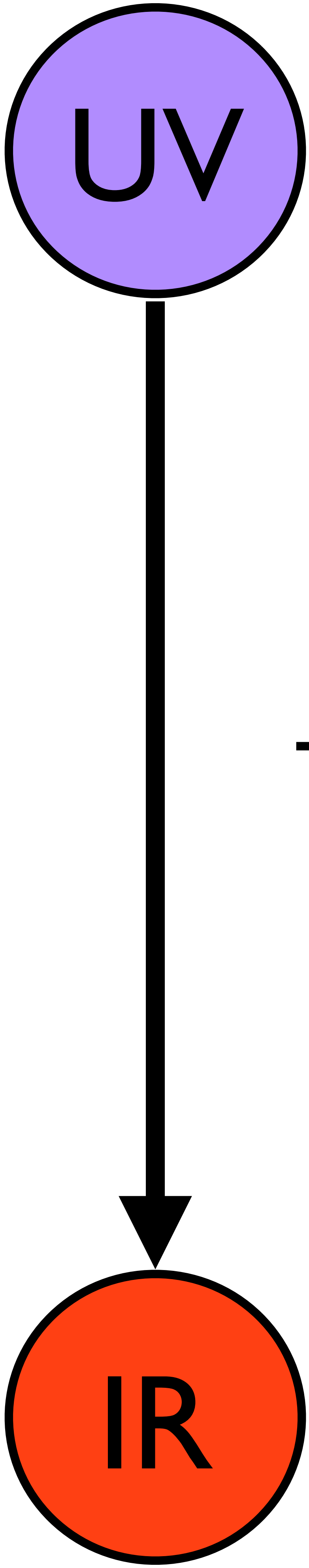
WORK IN PROGRESS

caveat emptor (buyer beware)

Suggestions welcome!

Outline:

- Review: Monotonicity Theorems
- The Systems
- The Proof
- Summary and Outlook

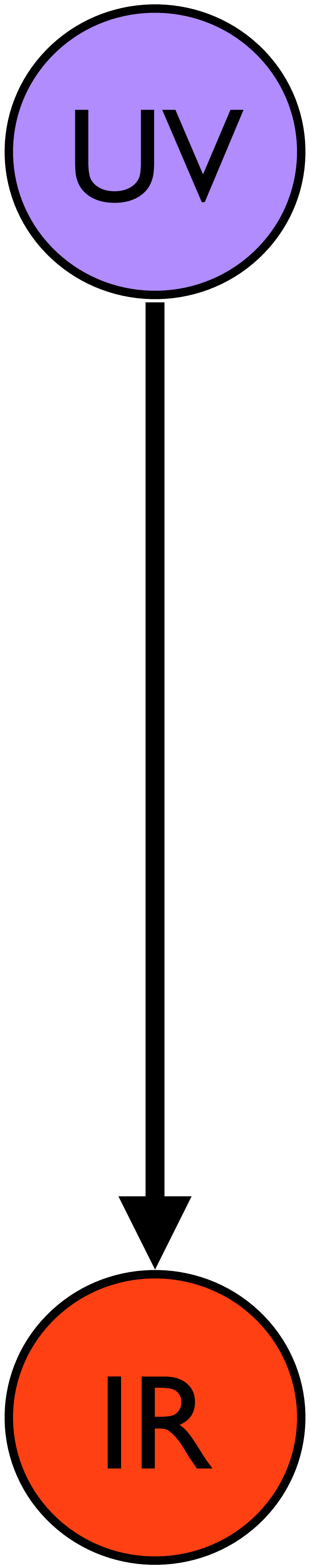


Microscopic/High-energy scales

Intuition:

The “number of degrees of freedom (DOF)” will DECREASE

Macroscopic/Low-energy scales



Example

Quantum Field Theory (QFT)

Wilsonian Renormalization Group (RG)

“integrate out” DOF

Below a mass threshold,
integrate out massive DOF

massive DOF
“decouples”

Monotonicity Theorems

Make our intuition precise, for RG flows in QFT

Provide a precise way to count number of DOF

Provide rigorous proof that the number of DOF DECREASES along RG flow

Place stringent theoretical constraints on what is possible in RG flows

for any coupling strength!

c-theorem
2-dimensional QFT
Zamolodchikov JETP 43, 12, 565, 1986

F-theorem
3-dimensional QFT
Jafferis, Klebanov, Pufu, and Safdi 1103.1181
Casini and Huerta 1202.5650

a-theorem
4-dimensional QFT
Cardy PLB 215 (1988) 749
Komargodski and Schwimmer 1107.3987

g-theorem
2-dimensional CFT
with a boundary
Affleck and Ludwig PRL 67 (1991) 161
Friedan and Konechny hep-th/0312197

c-, F-, a-theorems

Euclidean signature!

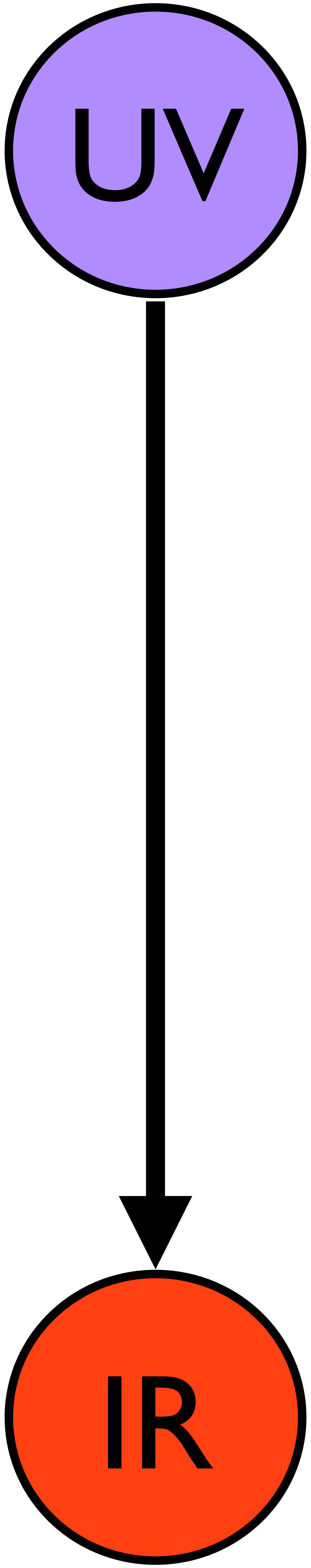
For QFTs that are

R ENORMALIZABLE

U NITARY

L OCAL

E UCLIDEAN SYMMETRIC



UV CFT

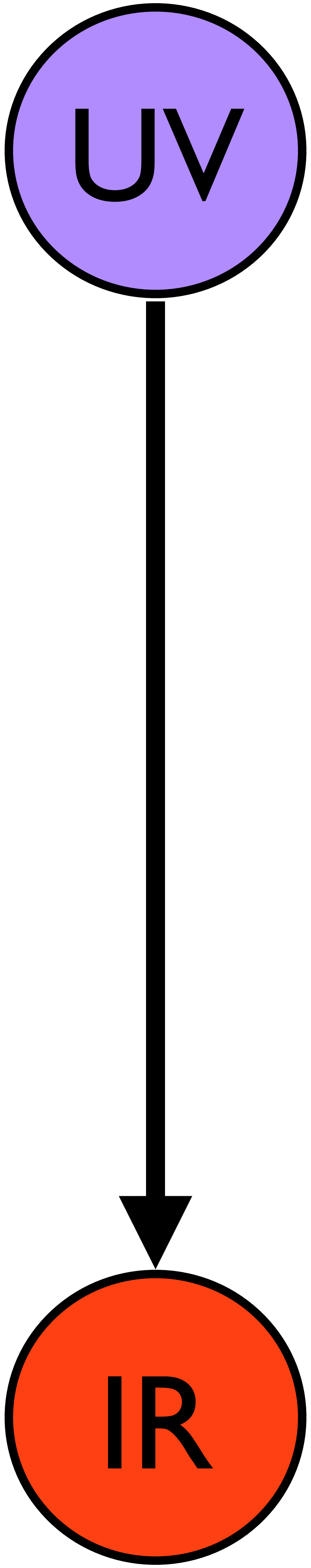
RG flow between fixed points

Conformal Field Theories
(CFTs)

$$g_{\mu\nu}(x) \to e^{2\Omega(x)} g_{\mu\nu}(x)$$

$$T_\mu{}^\mu = \frac{\delta}{\delta\Omega} \ln Z = 0$$

IR CFT



UV CFT

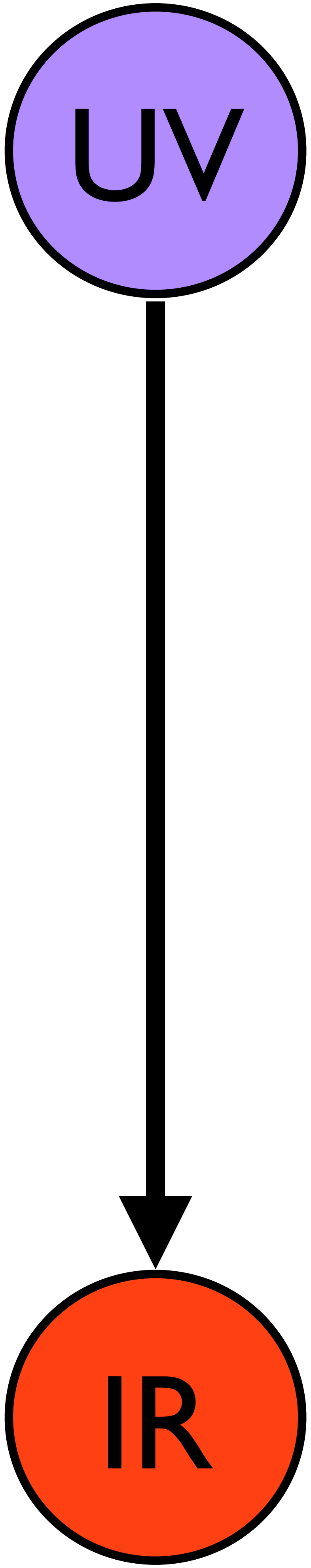
RG flow between fixed points

Conformal Field Theories (CFTs)

$$S_{\mathrm{CFT}}^{\mathrm{UV}} \to S_{\mathrm{CFT}}^{\mathrm{UV}} + \int d^d x \, \lambda_{\mathcal{O}} \, \mathcal{O}(\vec{x})$$

$$\Delta_{\mathcal{O}} < d$$

IR CFT



UV CFT

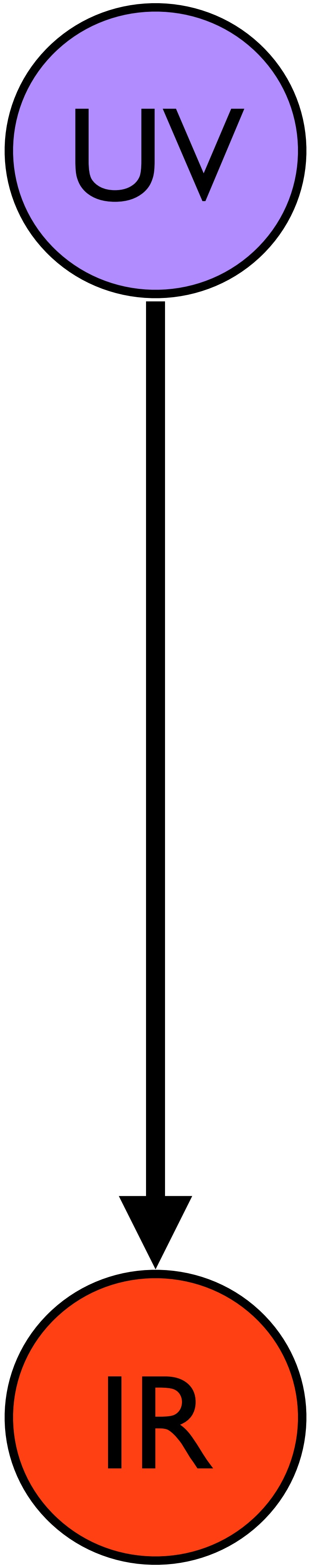
c-theorem

2-dimensional RULE QFTs

Zamolodchikov JETP 43, 12, 565, 1986

central charge c

IR CFT



UV CFT

c-theorem

2-dimensional RULE QFTs

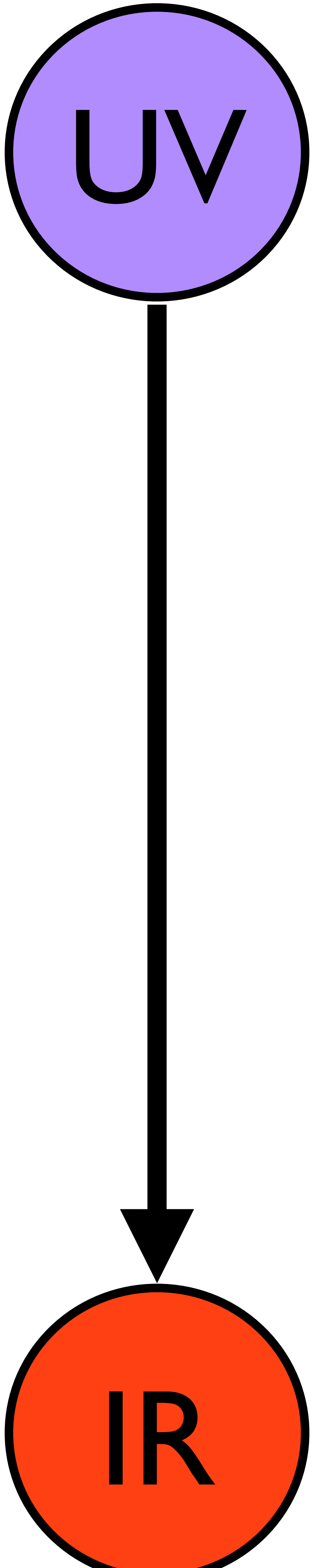
Zamolodchikov JETP 43, 12, 565, 1986

Weyl Anomaly

non-dynamical background metric $g_{\mu\nu}$

$$T_\mu{}^\mu \neq 0$$

IR CFT



UV CFT

c-theorem

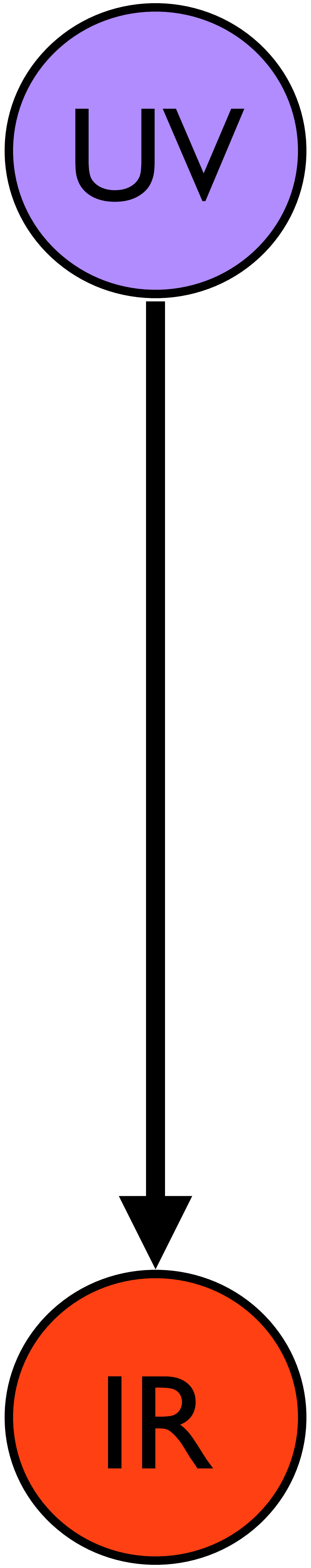
2-dimensional RULE QFTs

Zamolodchikov JETP 43, 12, 565, 1986

Weyl Anomaly

$$T_{\mu}^{\ \mu} = \frac{c}{24\pi} R$$

IR CFT



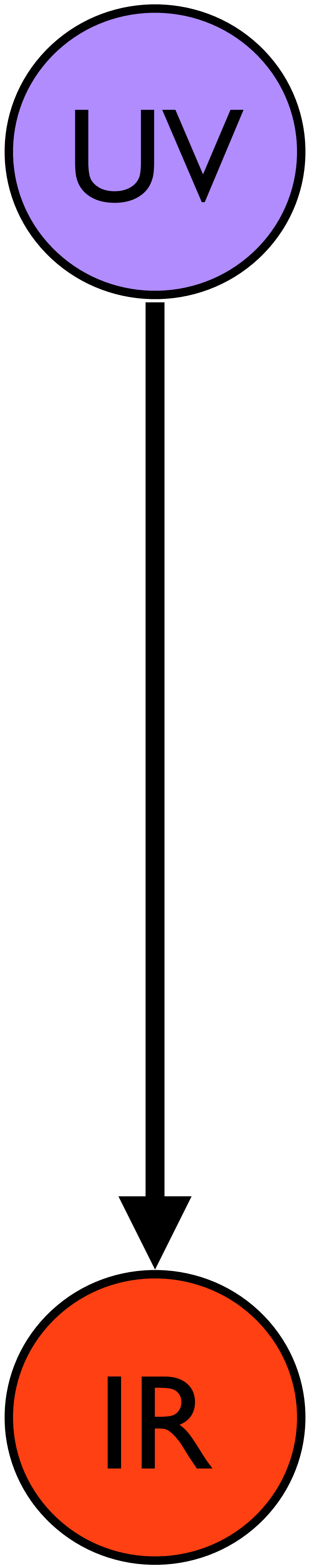
UV CFT

c-theorem
2-dimensional RULE QFTs
Zamolodchikov JETP 43, 12, 565, 1986

$$c_{\mathrm{UV}} \geq c_{\mathrm{IR}}$$

Proven directly,
using correlation functions of $T_{\mu\nu}$

IR CFT



UV CFT

c-theorem

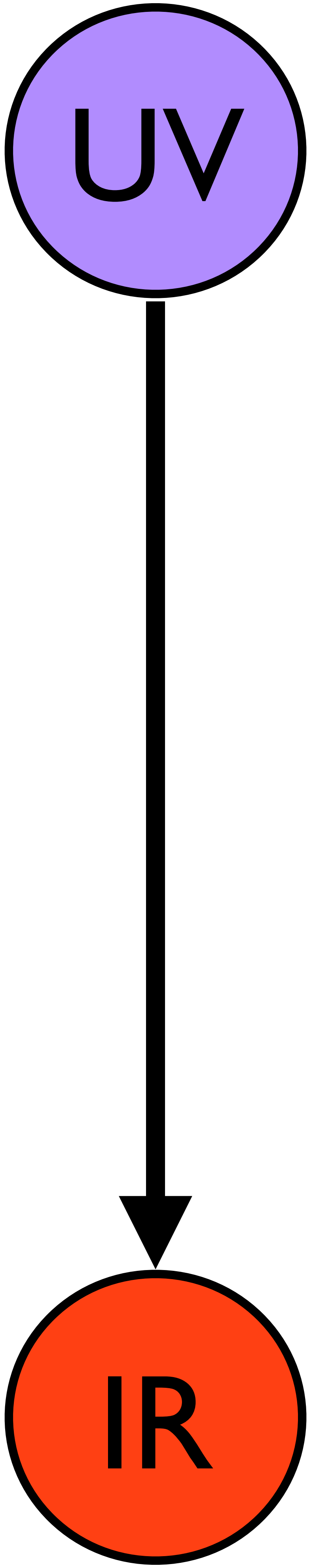
2-dimensional RULE QFTs

Zamolodchikov JETP 43, 12, 565, 1986

$$c_{\mathrm{UV}} \geq c_{\mathrm{IR}}$$

“Weak form”

IR CFT



UV CFT

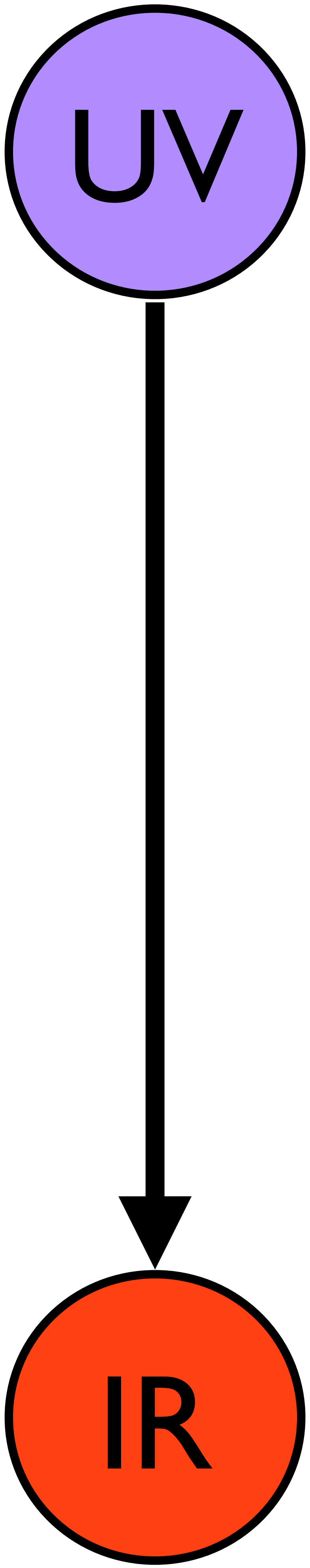
F-theorem
3-dimensional RULE QFTs

Jafferis, Klebanov, Pufu, and Safdi 1103.1181
Casini and Huerta 1202.5650

At fixed point: conformally map to S^3

$$F \equiv -\ln Z^{\text{ren}}_{S^3} = -\langle \mathbf{1} \rangle^{\text{ren}}_{S^3}$$

IR CFT



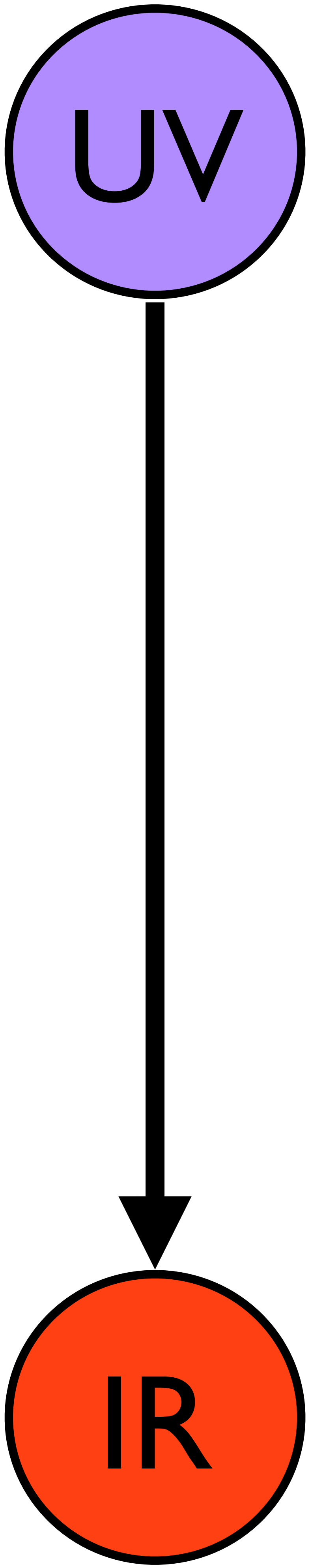
UV CFT

F-theorem
3-dimensional RULE QFTs
Jafferis, Klebanov, Pufu, and Safdi 1103.1181
Casini and Huerta 1202.5650

$$F_{\mathrm{UV}} \geq F_{\mathrm{IR}}$$

Proven using Strong Sub-Additivity (SSA) of Entanglement Entropy (EE)

IR CFT



UV CFT

a-theorem
4-dimensional RULE QFTs

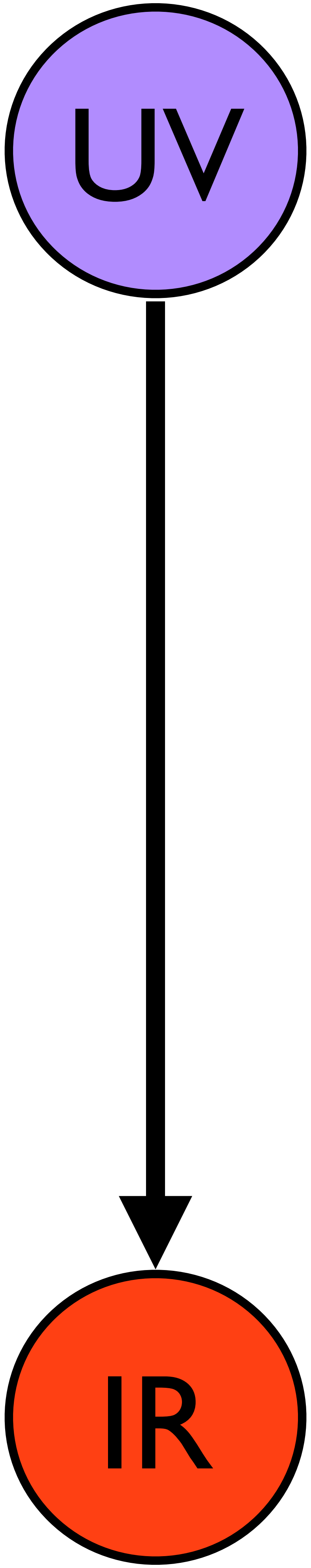
Cardy PLB 215 (1988) 749

Komargodski and Schwimmer 1107.3987

Weyl Anomaly

$$T_\mu{}^\mu = a\,E - c\,W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$$

IR CFT



UV CFT

a-theorem

4-dimensional RULE QFTs

Cardy PLB 215 (1988) 749

Komargodski and Schwimmer 1107.3987

$$a_{\rm UV} \geq a_{\rm IR}$$

Proven using "Weyl Anomaly Matching"

IR CFT

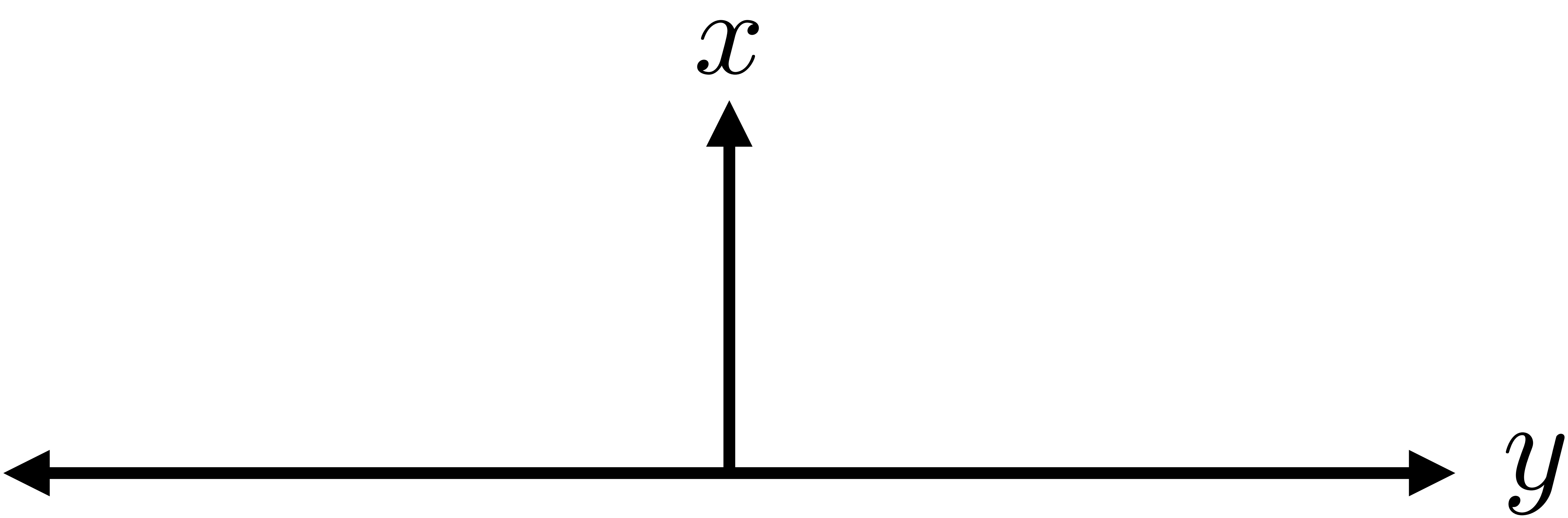
The g-theorem

Affleck and Ludwig PRL 67 (1991) 161

Friedan and Konechny hep-th/0312197

Renormalizable, Unitary, Local CFT in $d = 2$

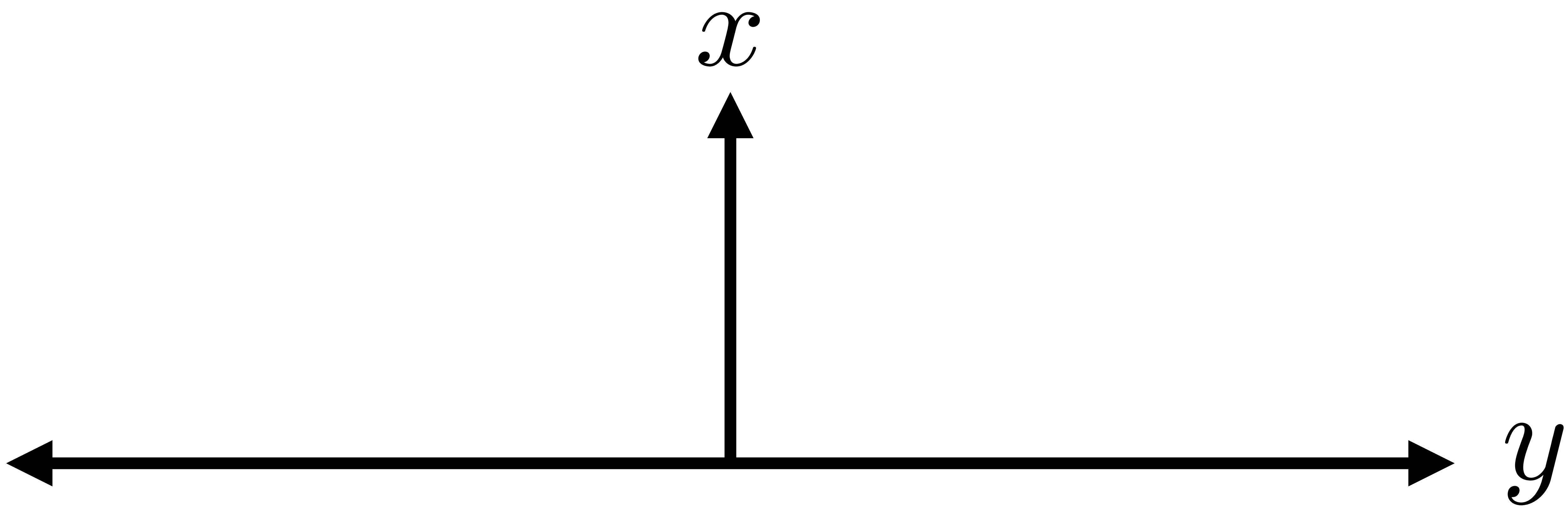
on a space with a boundary



The g-theorem

Conformal boundary conditions

Boundary CFT (BCFT)



The g-theorem

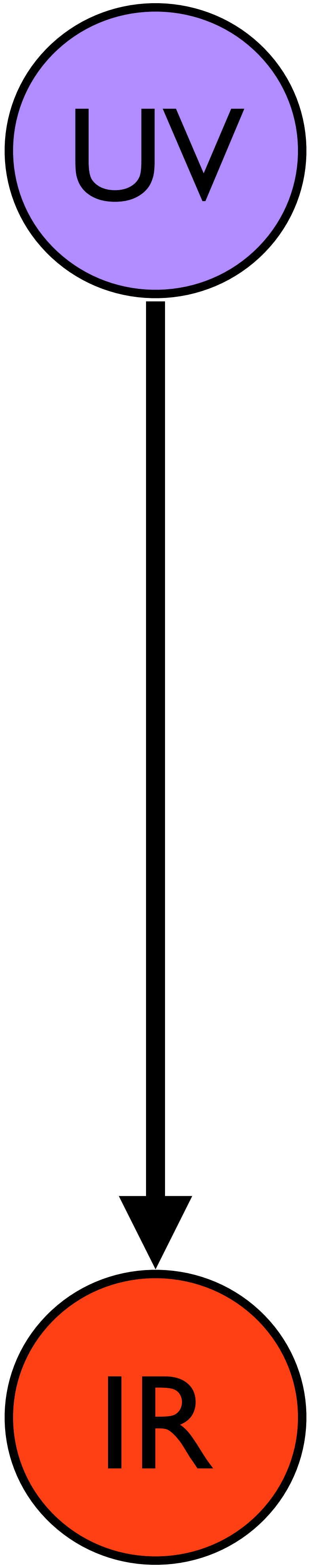
Boundary RG flows

$$S_{\mathrm{BCFT}} \to S_{\mathrm{BCFT}} + \int dx\, dy\, \delta(x) \lambda_{_\mathcal{O}} \mathcal{O}(y) \qquad \Delta_{\mathcal{O}} < 1$$

Bulk theory remains conformal

$$T_{\mu\nu} = [T_{\mu\nu}]_{\mathrm{bulk}} + \delta(x)\, [T_{\mu\nu}]_{\partial}$$

$$\left[T_\mu{}^\mu\right]_{\mathrm{bulk}} = 0 \qquad T_\mu{}^\mu = \delta(x)\, \left[T_\mu{}^\mu\right]_\partial \neq 0$$



UV BCFT

CFT with boundary condition "α"

Boundary RG flow

$$S^{\rm UV}_{\rm BCFT} \to S^{\rm UV}_{\rm BCFT} + \int dx\, dy\, \delta(x) \lambda_{\mathcal{O}} \mathcal{O}(y)$$

$$\Delta_{\mathcal{O}} < 1$$

IR BCFT

CFT with boundary condition "β"

The g-theorem

Affleck and Ludwig PRL 67 (1991) 161

Friedan and Konechny hep-th/0312197

At fixed point: conformally map to disk

conformal boundary condition "α"

$$g_\alpha \equiv \langle \mathbf{1} \rangle_\alpha^{\text{ren.}}$$

"Boundary entropy"

$$\ln g_\alpha$$

Counts DOF localized at the boundary

The g-theorem

Affleck and Ludwig PRL 67 (1991) 161

Friedan and Konechny hep-th/0312197

$$g_{\mathrm{UV}} \geq g_{\mathrm{IR}}$$

Proven directly,
using correlation functions of $T_{\mu\nu}$

Generalizations?

Higher-dimensional g-theorems?

Proposals

Yamaguchi	hep-th/0207171
Takayanagi et al.	1105.5165, 1108.5152, 1205.1573
Estes, Jensen, O'B., Tsatis, Wrase	1403.6475
Gaiotto	1403.8052

Many tests in particular examples

No proofs yet!

GOAL

Prove a g-theorem for BCFTs in $d = 3$

Proposals

Nozaki, Takayanagi, Ugajin 1205.1573

Estes, Jensen, O'B., Tsatis, Wrase 1403.6475

Method

"Weyl Anomaly Matching"

Komargodski and Schwimmer 1107.3987

Komargodski 1112.4538

Examples

Graphene with a boundary

Critical Ising model in $d = 3$ with a boundary

M-theory: M2-branes with a boundary

String theory: various brane intersections

Holographic BCFTs

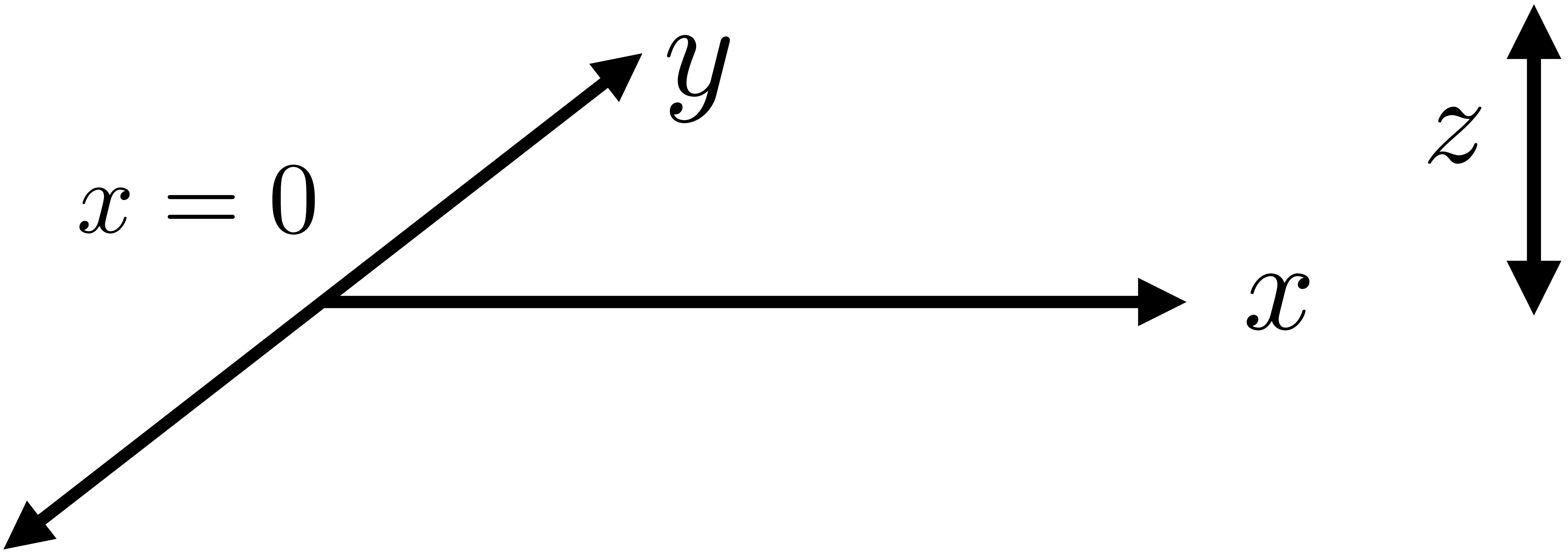
Outline:

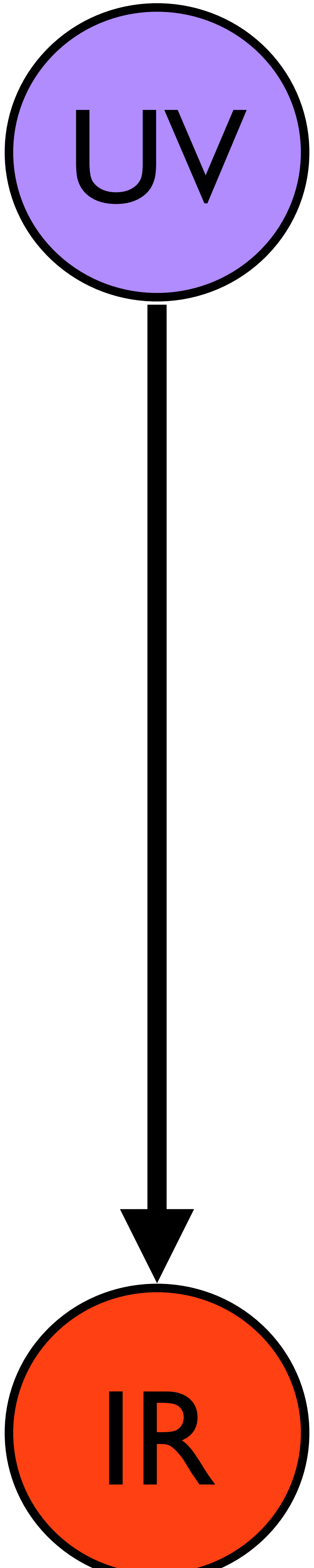
- Review: Monotonicity Theorems
- The Systems
- The Proof
- Summary and Outlook

The Systems

BCFT in $d = 3$

With a planar boundary





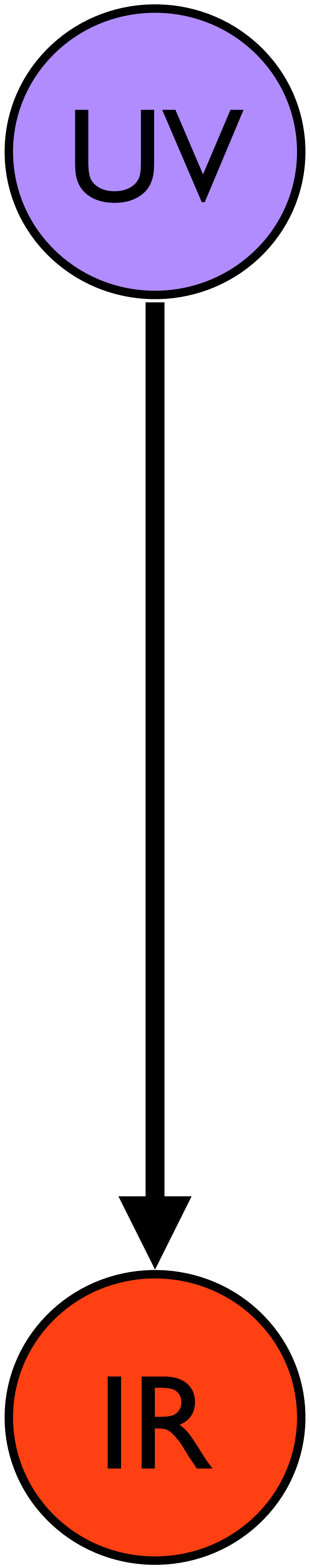
UV BCFT

Boundary RG Flows

$$S^{\mathrm{UV}}_{\mathrm{BCFT}} \to S^{\mathrm{UV}}_{\mathrm{BCFT}} + \int dx\, dy\, dz\, \delta(x)\, \lambda_{\mathcal{O}}\, \mathcal{O}(y,z)$$

scalar $\mathcal{O}$ with $\Delta_{\mathcal{O}} < 2$

IR BCFT



UV BCFT

Boundary RG Flows

Bulk theory remains conformal

$$T_{\mu\nu} = [T_{\mu\nu}]_{\text{bulk}} + \delta(x)\,[T_{\mu\nu}]_{\partial}$$

$$[T_{\mu}{}^{\mu}]_{\text{bulk}} = 0$$

$$T_{\mu}{}^{\mu} = \delta(x)\,[T_{\mu}{}^{\mu}]_{\partial} \neq 0$$

IR BCFT

Weyl Anomaly

BCFT in $d = 3$

non-dynamical background metric $g_{\mu\nu}(x)$

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\text{bulk}} + \delta(x) \left[T_\mu{}^\mu\right]_\partial$$

$$\left[T_\mu{}^\mu\right]_{\text{bulk}} = 0$$

What is the general form of $\left[T_\mu{}^\mu\right]_\partial$?

Geometry of Submanifolds

"worldsheet" σ^1, σ^2

"target space" x^μ

Embedding $x^\mu(\sigma^a)$

Induced metric

$$\hat{g}_{ab} = g_{\mu\nu} \frac{\partial x^\mu}{\partial \sigma^a} \frac{\partial x^\nu}{\partial \sigma^b}$$

$\hat{R}_{abcd}$ $\hat{R}_{ab}$ $\hat{R}$

Geometry of Submanifolds

Extrinsic Curvature
"Second Fundamental Form"

Gaussian Normal Coordinates

$$K_{ab} = \frac{1}{2}\partial_{\perp}\hat{g}_{ab}(x_{\perp}, \sigma)$$

Mean curvature

$$K \equiv \hat{g}^{ab} K_{ab}$$

Weyl Anomaly

What is the general form of $\left[T_{\mu}{}^{\mu}\right]_{\partial}$?

Schwimmer + Theisen 0802.1017

See also:

Berenstein, Corrado, Fischler, Maldacena hep-th/9809188

Graham + Witten hep-th/9901021

Henningson + Skenderis hep-th/9905163

Gustavsson hep-th/0310037, 0404150

Asnin 0801.1469

Weyl Anomaly

$$\left[T_\mu{}^\mu\right]_\partial = c_1\,\hat{R} + c_2(K_{ab}K^{ab} - \frac{1}{2}K^2)$$

Boundary "central charges"

c_1 and c_2

Weyl Anomaly

$$\left[T_\mu{}^\mu\right]_\partial = c_1\,\hat{R} + c_2\left(K_{ab}K^{ab} - \frac{1}{2}K^2\right)$$

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu}$$

$\int d^dx\,\sqrt{g}\,T_\mu{}^\mu$ **is invariant**

Weyl Anomaly

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu}$$

$$\left[T_\mu{}^\mu \right]_\partial = c_1 \hat{R} + c_2 (K_{ab}K^{ab} - \frac{1}{2} K^2)$$

Type A

$$\sqrt{\hat{g}} \hat{R} \to \sqrt{\hat{g}} \left[\hat{R} - 2\nabla^2 \Omega \right]$$

Changes by a total derivative

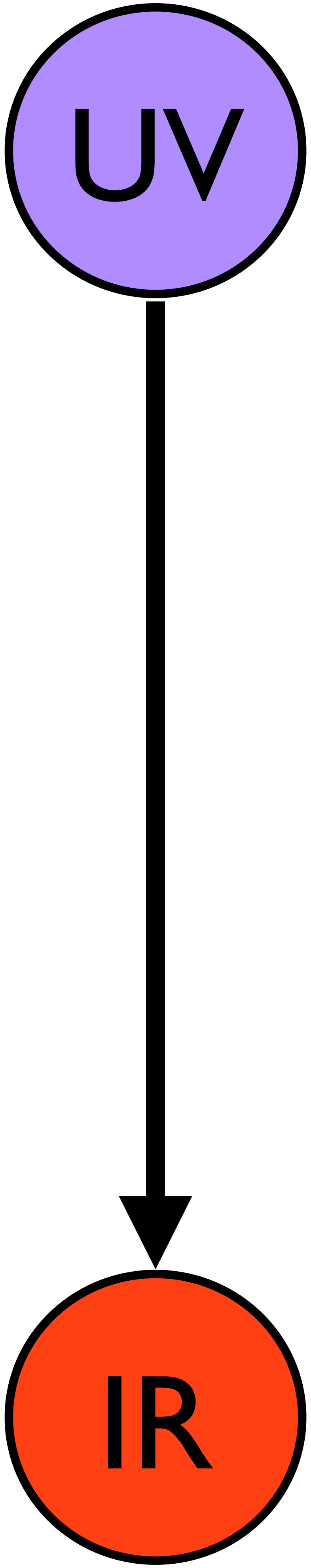
Type B

$$\sqrt{\hat{g}} (K_{ab}K^{ab} - \frac{1}{2} K^2)$$

Invariant

Outline:

- Review: Monotonicity Theorems
- The Systems
- The Proof
- Summary and Outlook



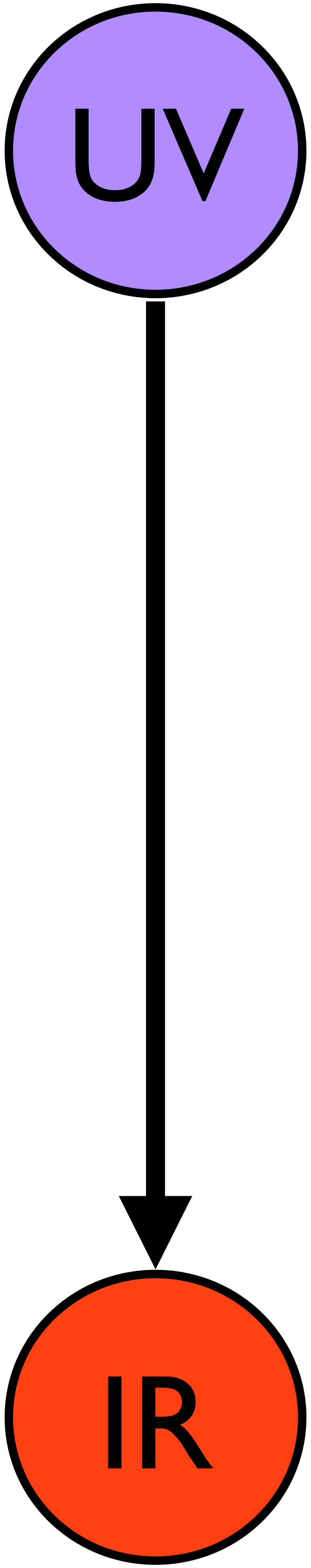
The Proof

Komargodski and Schwimmer 1107.3987

Komargodski 1112.4538

RULE QFT in any d

RG flow between fixed point CFTs



$$\left[T_\mu{}^\mu\right]^{\mathrm{UV}} = 0$$

$$T_\mu{}^\mu \neq 0$$

RG flow between fixed point CFTs

$$\left[T_\mu{}^\mu\right]^{\mathrm{IR}} = 0$$

Dilaton

non-dynamical background metric $g_{\mu\nu}(x)$

non-dynamical background scalar $\tau(x)$

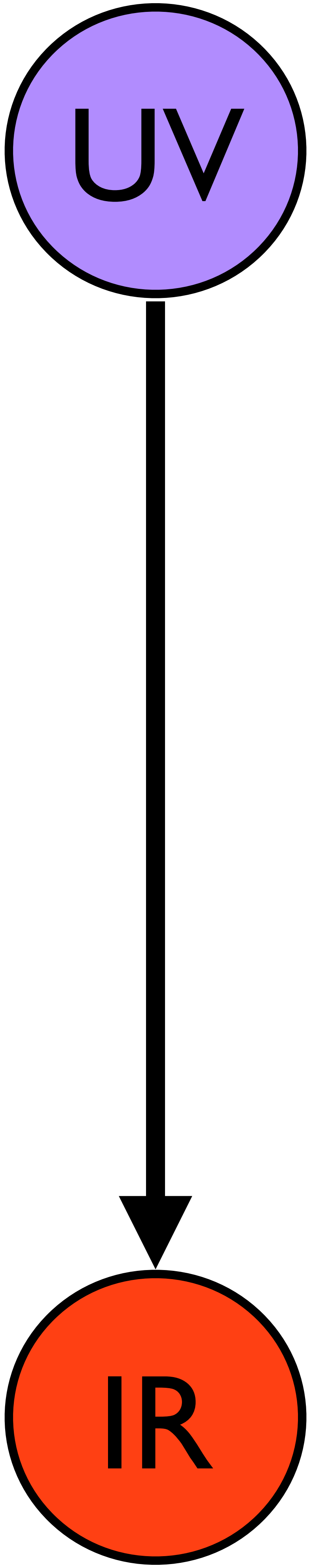
$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu} \qquad \tau \to \tau + \Omega$$

Dilaton

non-dynamical background metric $g_{\mu\nu}(x)$

non-dynamical background scalar $\tau(x)$

$$\lambda_{\mathcal{O}}\mathcal{O}(x) \to e^{(\Delta_{\mathcal{O}}-d)\tau(x)}\lambda_{\mathcal{O}}\mathcal{O}(x)$$



$$\left[T_\mu{}^\mu\right]^{\mathrm{UV}} = 0$$

$$g_{\mu\nu} = \delta_{\mu\nu}$$

$$\tau \neq 0$$

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\tau=0} + \left[T_\mu{}^\mu\right]_\tau = 0$$

$$\left[T_\mu{}^\mu\right]^{\mathrm{IR}} = 0$$

Dilaton

non-dynamical background metric $g_{\mu\nu}(x)$

non-dynamical background scalar $\tau(x)$

$$S_\tau = \int d^d x \, \sqrt{g} \, L(\lambda, \vec{x})_\tau$$

$$= \int d^d x \sqrt{g} \left[L(\lambda, \vec{x})_{\tau=0} + \tau \left[T_\mu{}^\mu \right]_{\tau=0} + \mathcal{O}(\tau^2) \right]$$

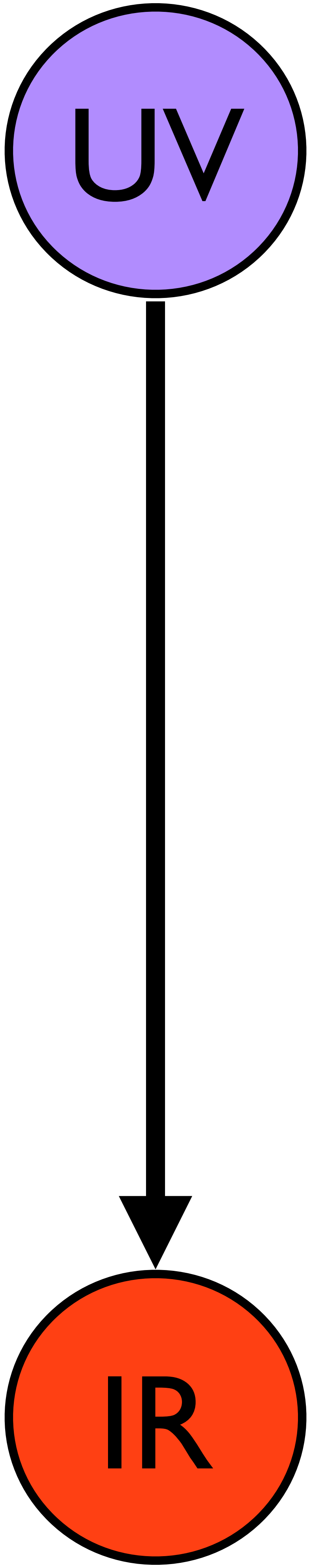
Dilaton

non-dynamical background metric $g_{\mu\nu}(x)$

non-dynamical background scalar $\tau(x)$

Weyl Anomaly Matching

d even



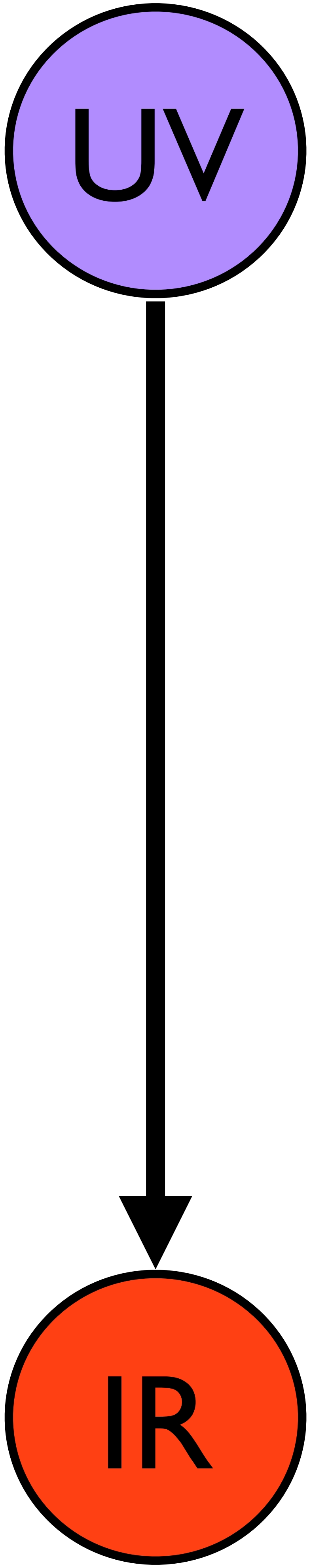
$$\left[T_\mu{}^\mu\right]^{\mathrm{UV}} \neq 0$$

$$g_{\mu\nu} \neq \delta_{\mu\nu}$$

$$\tau = 0$$

$$\left[T_\mu{}^\mu\right]^{\mathrm{UV}} \neq \left[T_\mu{}^\mu\right]^{\mathrm{IR}}$$

$$\left[T_\mu{}^\mu\right]^{\mathrm{IR}} \neq 0$$



$$\left[T_\mu{}^\mu\right]^{\mathrm{UV}} \neq 0$$

$$g_{\mu\nu} \neq \delta_{\mu\nu}$$

$$\tau \neq 0$$

Weyl Anomaly Matching

$$\left[T_\mu{}^\mu\right]^{\mathrm{UV}} = \left[T_\mu{}^\mu\right]^{\mathrm{IR}}_{\tau=0} + \left[T_\mu{}^\mu\right]_\tau$$

Dilaton

Integrate out massive DOF

Obtain low-energy effective action

$$S_{\text{eff}} \equiv -\ln Z_\tau$$

Regular and local in τ

Dilaton

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu} \qquad \tau \to \tau + \Omega$$

$$\frac{\delta S_{\rm eff}}{\delta \Omega} = -\frac{\delta}{\delta \Omega} \ln Z_\tau = -\sqrt{g}\, T_\mu{}^\mu$$

At the IR fixed point,

τ's contribution to $S_{\rm eff}$ must produce

$$\left[T_\mu{}^\mu\right]^\tau = \left[T_\mu{}^\mu\right]^{\rm UV}_{\tau=0} - \left[T_\mu{}^\mu\right]^{\rm IR}_{\tau=0}$$

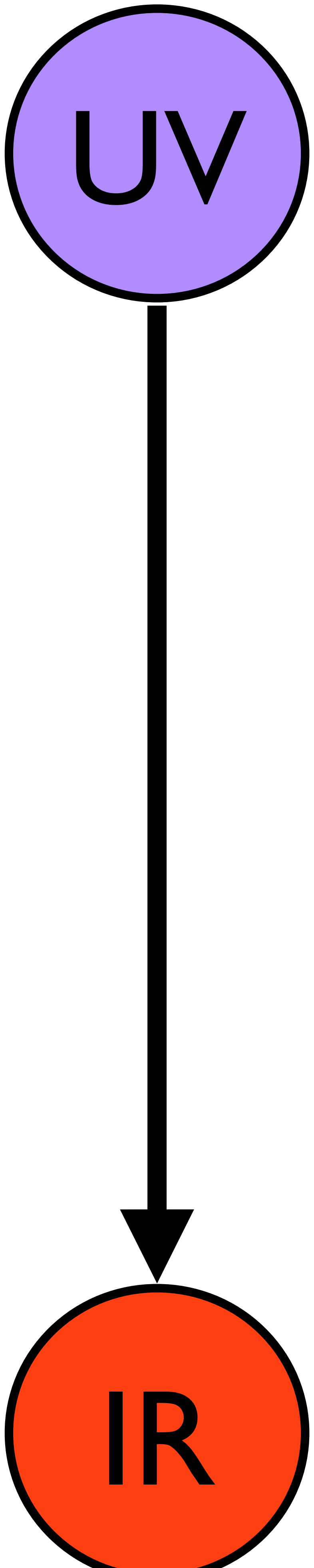
Dilaton

Boundary RG Flow

$$S^{\text{UV}}_{\text{BCFT}} \to S^{\text{UV}}_{\text{BCFT}} + \int dx\, dy\, dz\, \delta(x)\, \lambda_{\mathcal{O}} \mathcal{O}(y,z)$$

$$\lambda_{\mathcal{O}}\, \mathcal{O}(y,z) \to e^{(\Delta_{\mathcal{O}}-2)\tau(y,z)} \lambda_{\mathcal{O}}\, \mathcal{O}(y,z)$$

$\mathcal{T}$ is localized at the boundary,
and depends only on boundary coordinates!



$$\left[T_\mu{}^\mu\right]_\partial^{\rm UV} = c_1^{\rm UV}\,\hat{R} + c_2^{\rm UV}\left(K_{ab}K^{ab} - \frac{1}{2}K^2\right)$$

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\rm bulk} + \delta(x)\,\left[T_\mu{}^\mu\right]_\partial$$

Boundary RG Flow

$$S_{\rm BCFT}^{\rm UV} \to S_{\rm BCFT}^{\rm UV} + \int dx\,dy\,dz\,\delta(x)\,\lambda_{\mathcal{O}}\,\mathcal{O}(y,z)$$

$$\left[T_\mu{}^\mu\right]_{\rm bulk} = 0$$

$$\left[T_\mu{}^\mu\right]_\partial^{\rm IR} = c_1^{\rm IR}\,\hat{R} + c_2^{\rm IR}\left(K_{ab}K^{ab} - \frac{1}{2}K^2\right)$$

The Proof

Strategy

Komargodski and Schwimmer 1107.3987
Komargodski 1112.4538

In $S^{\tau}_{\rm eff}$ write the $\tau \nabla^2 \tau$ term in TWO WAYS

① Weyl anomaly matching

coefficient $\propto c_1^{\rm UV} - c_1^{\rm IR}$

② Unitarity $\Rightarrow$ coefficient ≥ 0

$$c_1^{\rm UV} \geq c_1^{\rm IR}$$

The Proof

$$S_{\text{eff}}^{\tau} = -\int d^3x\, \delta(x)\, \sqrt{g}\ \tau\ \left[(c_1^{\text{UV}} - c_1^{\text{IR}})\, \hat{R} + (c_2^{\text{UV}} - c_2^{\text{IR}})(K_{ab}K^{ab} - \frac{1}{2}K^2)\right]$$

$$+\mathcal{O}(\tau^2)$$

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu} \qquad \tau \to \tau + \Omega$$

$$\sqrt{\hat{g}}\hat{R} \to \sqrt{\hat{g}}\left[\hat{R} - 2\nabla^2\Omega\right]$$

$$\frac{\delta S_{\text{eff}}^{\tau}}{\delta\Omega} = -\left(\sqrt{g}\left[T_\mu{}^\mu\right]^{\text{UV}} - \sqrt{g}\left[T_\mu{}^\mu\right]^{\text{IR}}\right) + \sqrt{g}\,(c_1^{\text{UV}} - c_1^{\text{IR}})\, 2\nabla^2\tau$$

The Proof

$$S^{\tau}_{\text{eff}} = -\int d^3x\, \delta(x)\, \sqrt{g}\ \tau \left[(c_1^{\text{UV}} - c_1^{\text{IR}})\, \hat{R} + (c_2^{\text{UV}} - c_2^{\text{IR}})(K_{ab}K^{ab} - \frac{1}{2}K^2) \right]$$

$$- \int d^3x\, \delta(x)\, \sqrt{g}\ (c_1^{\text{UV}} - c_1^{\text{IR}})\, \tau \nabla^2 \tau \ + \ldots$$

$$\frac{\delta S^{\tau}_{\text{eff}}}{\delta \Omega} = -\left(\sqrt{g}\, \left[T_{\mu}{}^{\mu}\right]^{\text{UV}} - \sqrt{g}\, \left[T_{\mu}{}^{\mu}\right]^{\text{IR}} \right)$$

The Proof

$$S^{\tau}_{\rm eff} = -\int d^3x\, \delta(x)\, \sqrt{g}\ \tau \left[(c_1^{\rm UV} - c_1^{\rm IR})\, \hat{R} + (c_2^{\rm UV} - c_2^{\rm IR})(K_{ab}K^{ab} - \frac{1}{2}K^2) \right]$$

$$- \int d^3x\, \delta(x)\, \sqrt{g}\ (c_1^{\rm UV} - c_1^{\rm IR})\, \tau \nabla^2 \tau \ + \dots$$

Survives the flat-space limit
$g_{\mu\nu} \to \delta_{\mu\nu}$

$$\frac{\delta S^{\tau}_{\rm eff}}{\delta \Omega} = -\left(\sqrt{g}\, \left[T_{\mu}{}^{\mu} \right]^{\rm UV} - \sqrt{g}\, \left[T_{\mu}{}^{\mu} \right]^{\rm IR} \right)$$

The Proof

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Another form for the two-derivative term

$$Z = \int \mathcal{D}\,[\text{fields}]\, e^{-S(\lambda)_\tau}$$

$$S_\tau \;=\; \int d^3x \sqrt{g}\left[L(\lambda, \vec{x})_{\tau=0} + \tau\,\left[T_\mu{}^\mu\right]_{\tau=0} + \mathcal{O}(\tau^2)\right]$$

$$S^\tau_{\text{eff}} = -\ln Z = -\langle e^{-\int d^3x\,\tau(x) T_\mu{}^\mu(x) + \ldots}\rangle_{\tau=0}$$

The Proof

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Another form for the two-derivative term

$$\langle e^{-\int d^3x\, \tau(x) T_\mu{}^\mu(x) + \ldots} \rangle = 1 - \int d^3x\, \tau(x) \, \langle T_\mu{}^\mu(x) \rangle$$
$$+ \frac{1}{2} \int d^3x\, d^3y\, \tau(x)\tau(y) \, \langle T_\mu{}^\mu(x) T_\mu{}^\mu(y) \rangle + \ldots$$

↑

Taylor expand about x

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T_\mu^\mu(x) T_\mu^\mu(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\Big[\int d^3y\, (y-x)^\rho (y-x)^\sigma \langle T_\mu{}^\mu(x) T_\mu{}^\mu(y)\rangle\Big]$$

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\mathrm{bulk}} + \delta(x)\left[T_\mu{}^\mu\right]_\partial$$

$$\left[T_\mu{}^\mu\right]_{\mathrm{bulk}} = 0$$

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T_\mu^\mu(x) T_\mu^\mu(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\left[\int d^3y\,(y-x)^\rho(y-x)^\sigma \langle T_\mu{}^\mu(x) T_\mu{}^\mu(y)\rangle\right]$$

$$\langle T_\mu{}^\mu(x) T_\mu{}^\mu(y)\rangle = \delta(x)\delta(y)\langle \left[T_\mu{}^\mu(x)\right]_\partial \left[T_\mu{}^\mu(y)\right]_\partial\rangle$$

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T_\mu^\mu(x) T_\mu^\mu(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\left[\int d^3y\,(y-x)^\rho(y-x)^\sigma\langle T_\mu{}^\mu(x)T_\mu{}^\mu(y)\rangle\right]$$

Reflection Positivity

$$\langle T_\mu{}^\mu(x)T_\mu{}^\mu(y)\rangle = \delta(x)\delta(y)\langle\left[T_\mu{}^\mu(x)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle \geq 0$$

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T^{\mu}_{\mu}(x) T^{\mu}_{\mu}(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\Big[\int d^3y\, (y-x)^\rho (y-x)^\sigma \langle T_\mu{}^\mu(x) T_\mu{}^\mu(y)\rangle\Big]$$

Translation invariance along the boundary

$$\delta(x)\int d^3y\, \delta(y)\, (y-x)^\rho (y-x)^\sigma \langle \left[T_\mu{}^\mu(x)\right]_\partial \left[T_\mu{}^\mu(y)\right]_\partial\rangle$$

$$= \delta(x)\frac{1}{2}\delta^{\rho\sigma}\int d^3y\, \delta(y)\, y^2 \langle \left[T_\mu{}^\mu(0)\right]_\partial \left[T_\mu{}^\mu(y)\right]_\partial\rangle$$

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T^\mu_\mu(x) T^\mu_\mu(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\left[\int d^3y\, (y-x)^\rho (y-x)^\sigma \langle T_\mu{}^\mu(x) T_\mu{}^\mu(y)\rangle\right]$$

Reflection Positivity

$$\delta(x)\int d^3y\, \delta(y)\, (y-x)^\rho (y-x)^\sigma \langle \left[T_\mu{}^\mu(x)\right]_\partial \left[T_\mu{}^\mu(y)\right]_\partial\rangle$$

$$= \delta(x)\frac{1}{2}\delta^{\rho\sigma}\int d^3y\, \delta(y)\, y^2 \langle \left[T_\mu{}^\mu(0)\right]_\partial \left[T_\mu{}^\mu(y)\right]_\partial\rangle \geq 0$$

The Proof

$$\int d^3x\, \delta(x)\ \ \tau\nabla^2\tau\ \ (c_1^{\rm UV} - c_1^{\rm IR})$$

$$= \int d^3x\, \delta(x)\ \ \tau\nabla^2\tau\ \left[\frac{1}{8}\int d^3y\, \delta(y)\, y^2 \langle \left[T_\mu{}^\mu(0)\right]_\partial \left[T_\mu{}^\mu(y)\right]_\partial \rangle\right]$$

$$c_1^{\rm UV} - c_1^{\rm IR} = \left[\frac{1}{8}\int d^3y\, \delta(y)\, y^2 \langle \left[T_\mu{}^\mu(0)\right]_\partial \left[T_\mu{}^\mu(y)\right]_\partial \rangle\right] \geq 0$$

$$c_1^{\rm UV} \geq c_1^{\rm IR}$$

Defects

Renormalizable, Unitary, Local CFT in any $d \geq 3$

With a two-dimensional planar defect

Conformal defect

"Defect CFT" (DCFT)

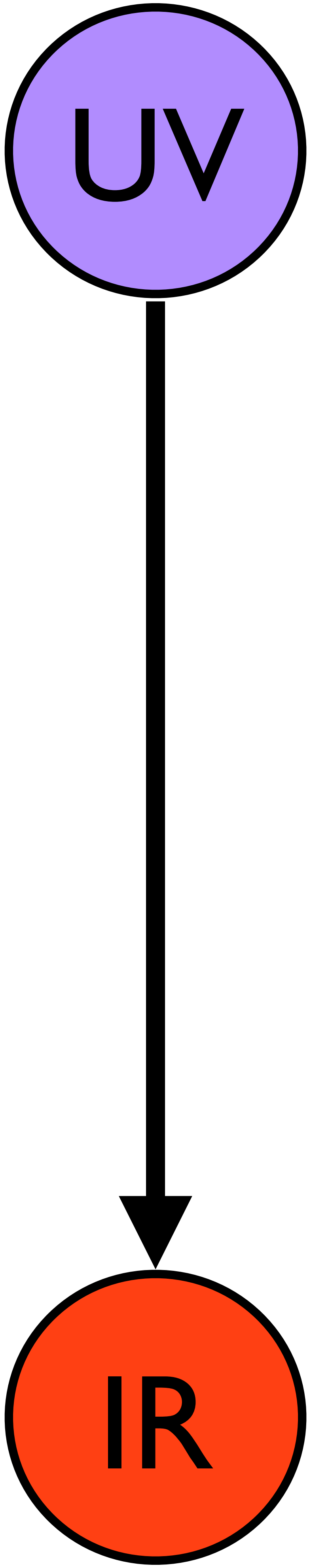
Defects

$$\left[T_\mu{}^\mu\right]_{\rm defect} = c_1\,\hat{R} + c_2 (K^\mu_{ab} K^{ab}_\mu - \frac{1}{2} K^\mu K_\mu) + c_3 \hat{g}^{ac} \hat{g}^{bd} W_{abcd}$$

Boundary “central charges”

c_1 c_2 c_3

$\hat{g}^{ac}\hat{g}^{bd}W_{abcd}$ is B-type



UV DCFT

Defect RG Flow

$$c_1^{\mathrm{UV}} \geq c_1^{\mathrm{IR}}$$

IR DCFT

Outline:

- Review: Monotonicity Theorems
- The Systems
- The Proof
- Summary and Outlook

Summary

Renormalizable, Unitary, Local, BCFT in $d = 3$

Renormalizable, Unitary, Local DCFT in $d \geq 3$ with two-dimensional defect

$$[T_\mu{}^\mu]_{\text{defect}} = c_1 \hat{R} + c_2 (K^\mu_{ab} K^{ab}_\mu - \frac{1}{2} K^\mu K_\mu) + c_3 \hat{g}^{ac} \hat{g}^{bd} W_{abcd}$$

Boundary or Defect RG Flows

$$c_1^{\text{UV}} \geq c_1^{\text{IR}}$$

Proof used only existing ingredients!

Outlook

Immediate questions

Can we prove a strong form?

Is c_1 bounded below?

Other methods of proof?

What about EE? Or holography?

Examples

Graphene with a boundary

Critical Ising model in $d = 3$ with a boundary

M-theory: M2-branes with a boundary

String theory: various brane intersections

Holographic BCFTs

Outlook

Prove more boundary/defect monotonicity theorems

Yamaguchi	hep-th/0207171
Estes, Jensen, O'B., Tsatis, Wrase	1403.6475
Gaiotto	1403.8052

Find a "universal" proof of monotonicity theorems?

Myers and Sinha	Giombi and Klebanov
1006.1263, 1011.5819	1409.1937

Thank You.